Complete 1-loop study of exclusive J/ψ and Υ photoproduction with full GPD evolution

J.P. Lansberg with C. Flett, S. Nabeebaccus, M. Nefedov, P. Sznajder and J. Wagner

IJCLab Orsay – Paris-Saclay U. – CNRS

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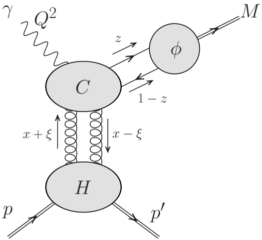
Exclusive quarkonium photoproduction and GPDs

Collinear factorisation at the *amplitude* level:

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz\, H(x)\phi(z) C(x,z)$$

$H(x)$: Generalised parton distribution (GPD)
$\phi(z)$: Distribution amplitude
$C(x,z)$: Coefficient function



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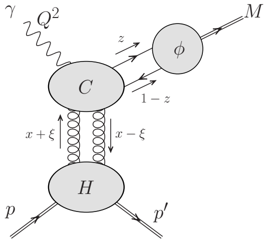

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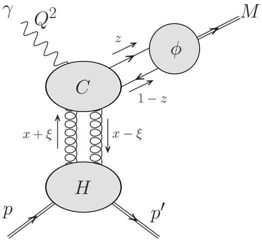

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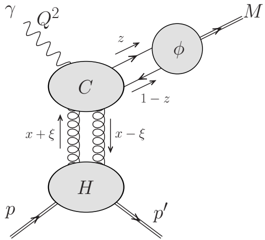

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- No all-order proof of factorisation but correct IR-pole structure at *NLO*

[D. Ivanov, A. Schafer, L. Szymanowski, G. Krasnikov EPJC 34 (2004) 297]

Leading order amplitude

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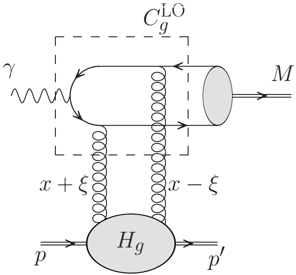

$$\mathcal{A} = \epsilon_\gamma^\mu \epsilon_M^{*\nu} \mathcal{T}^{\mu\nu}$$

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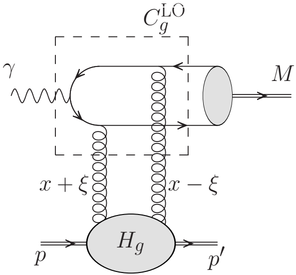

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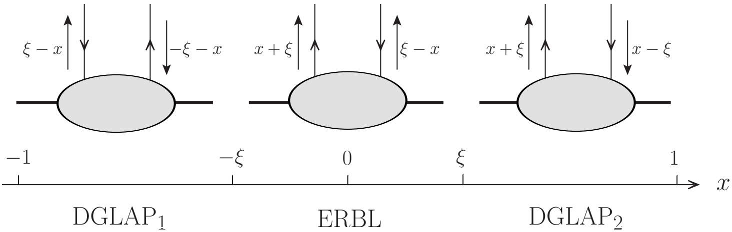
Large $W_{\gamma p}$ (small x in inclusive physics) $\leftrightarrow$ *small ξ*

Imaginary part of amplitude

DGLAP and ERBL regions



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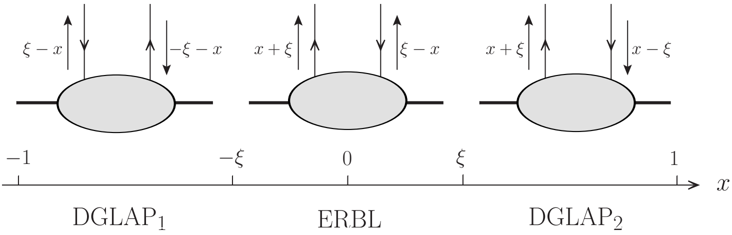

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- ERBL region shrinks as $W_{\gamma p}$ increases.

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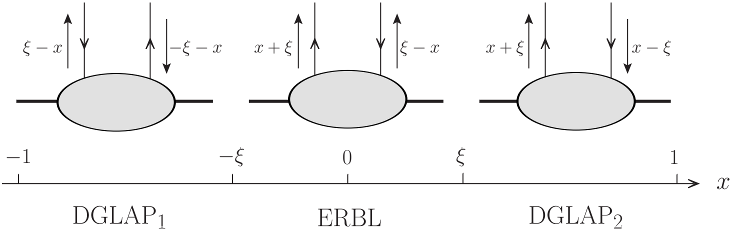


For LO amplitude:

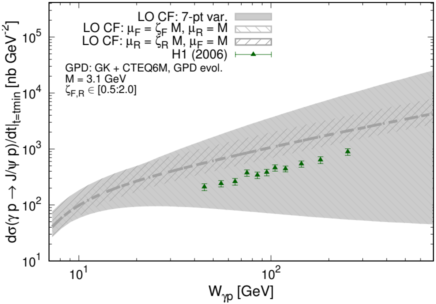
- Picks up *imaginary part* at $x = \pm\xi$.

$$\mathrm{Im}C_g^{\mathrm{LO}}\left(\frac{\xi}{x}\right) = -\pi\frac{F_{LO}}{2}\left[\delta\left(\frac{\xi}{x} - 1\right) + \delta\left(\frac{\xi}{x} + 1\right)\right]$$

$$\mathrm{Im}\mathcal{T}_{\mathrm{LO}}^{\mu\nu} = \pi\frac{g_\perp^{\mu\nu} F_{LO}}{\xi} H_g(\xi, \xi)$$

- Otherwise, amplitude fully real (principal value contribution).

LO cross section



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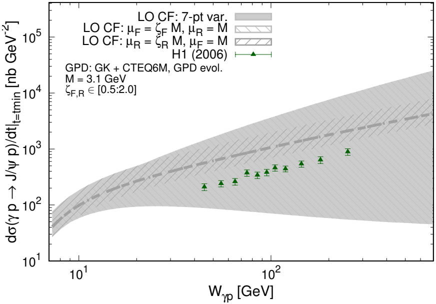

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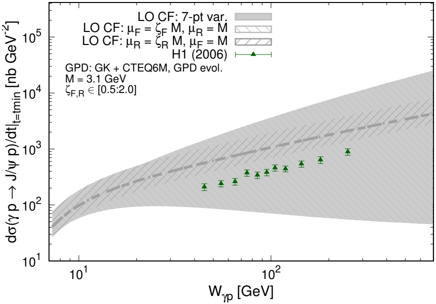

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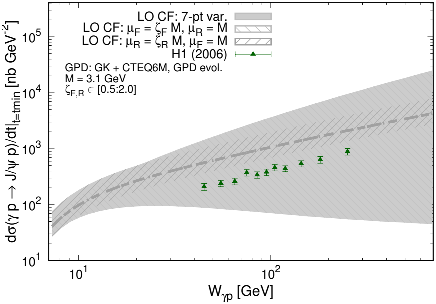

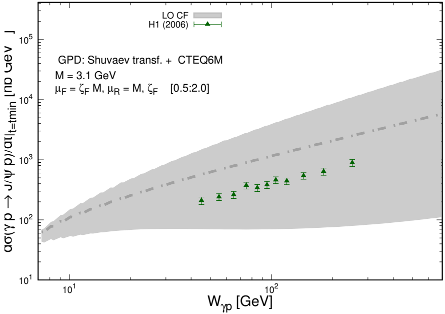

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Alternative (right) GPDs based on *Shuvaev transform* also using CTEQ6M

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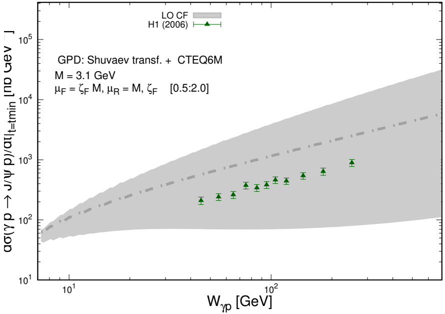
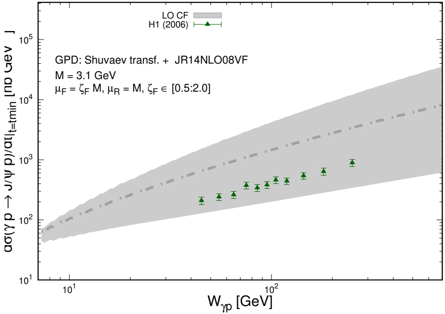
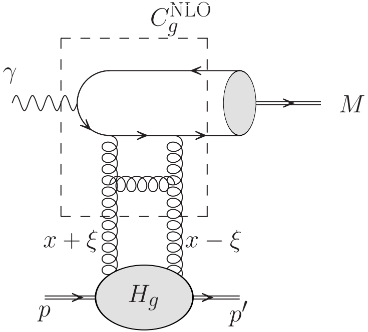


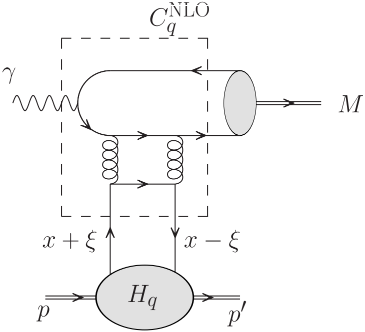


Weird behaviour absent with JR14 (right plot)

NLO amplitude

NLO amplitude has contributions from *both* quark and gluon GPDs:





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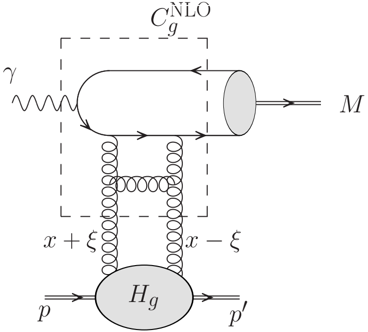

Imaginary part comes fully from the *DGLAP region* ($\xi \leq |x| \leq 1$)

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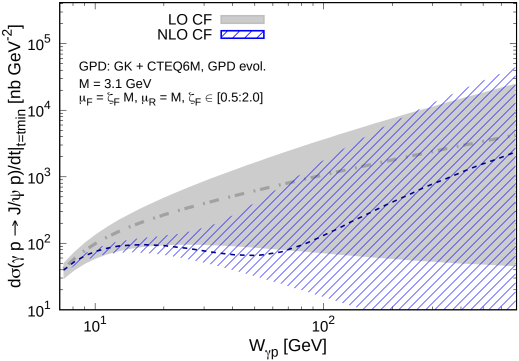

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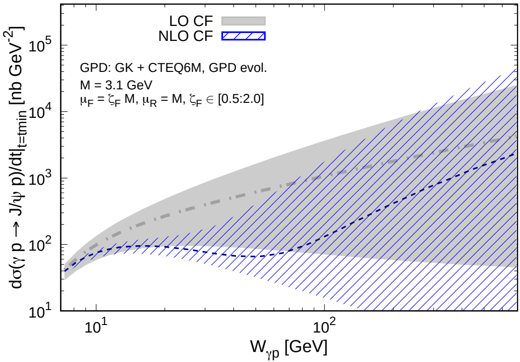

Oscillating energy dependence combined with a fast increasing uncertainty

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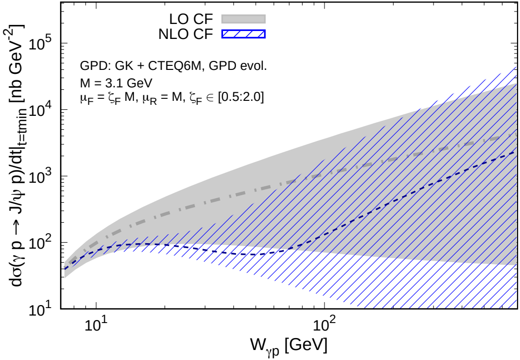

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Confirmation of an issue already uncovered twenty years ago

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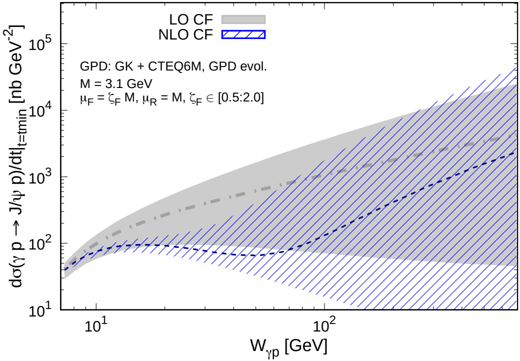

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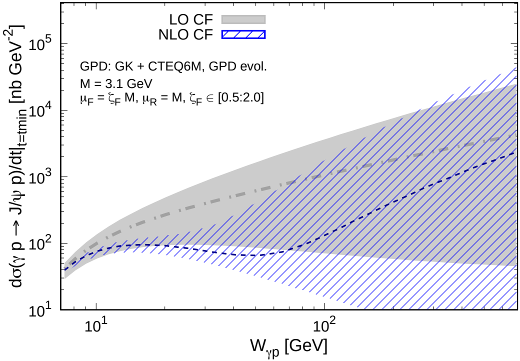

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how to solve this ?

Origin of the problem for NLO cross sections

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NLO correction of opposite sign to LO for $\mu_F = M (> M/2)$

cancellation then strong μ_F dependence at large W

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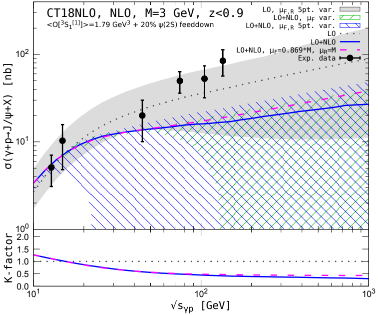
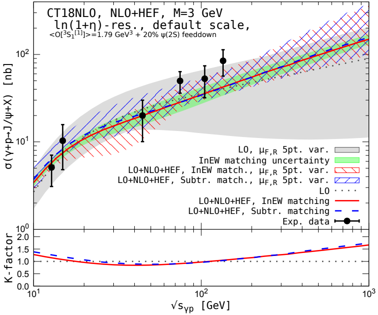
$\Longrightarrow$ Hints towards a solution through *resummation* of these logarithms...

Instabilities in inclusive cases, e.g. $\gamma p \to \mathcal{Q} + X$





Perturbative instabilities leading to negative cross sections in inclusive quarkonium production known since 90's:

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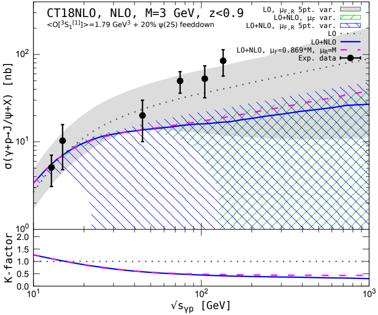

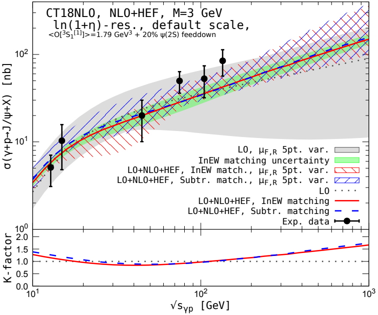

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JPL, M.A. Ozcelik, EPJC 81 (2021) 6, 497; A. Colpani Serri et al. PLB 835 (2022) 137556

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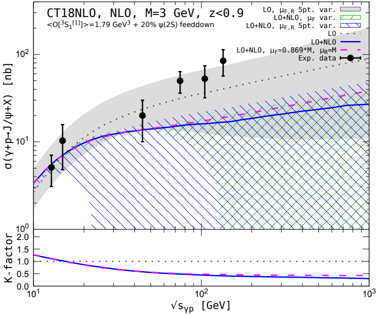

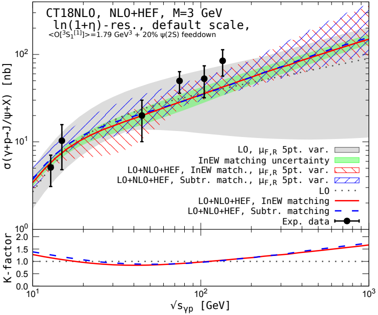

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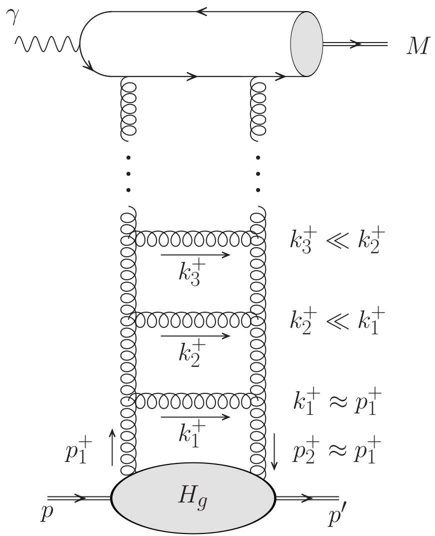
Solved by matching High Energy Factorisation (HEF) to Collinear Factorisation

J.P. Lansberg, M. Nefedov, M.A.Ozcelik, JHEP 05 (2022) 083 & EPJC 84 (2024) 351

Multiple gluon emissions:

BFKL ladder and resummation



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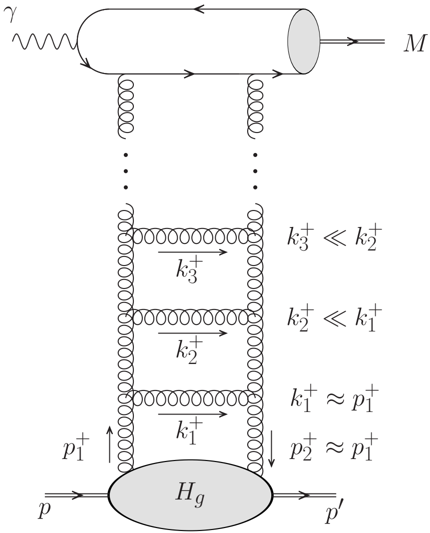

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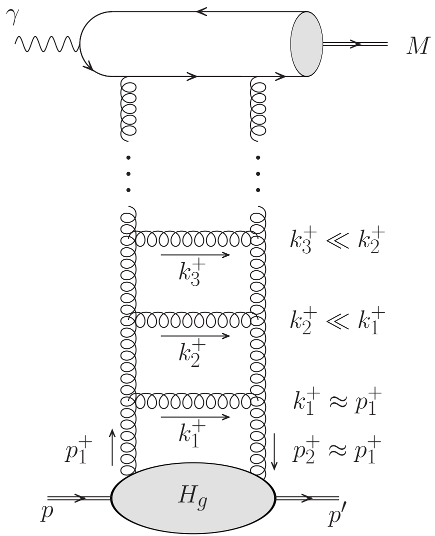

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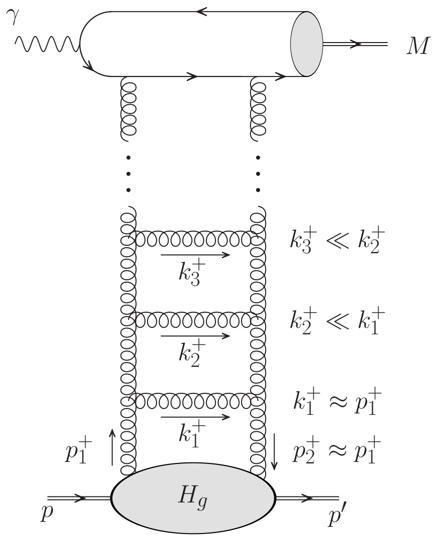

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- We implement a *resummation* of these BFKL-type logs using HEF, consistent with fixed-order evolution of GPD:
  $\Longrightarrow$ *Doubly-logarithmic approximation (DLA)*

Implementation of resummation: $C^{\text{CF}} \to C^{\text{HEF}}$

$$C_g^{\text{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi\hat{\alpha}_s F_{\text{LO}}}{2\left|\frac{\xi}{x}\right|}\sqrt{\frac{L_\mu}{L_x}}\left\{I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty}\text{Li}_{2k}(-1)\left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right)\right\},$$

where $L_\mu = \ln[M^2/(4\mu_F^2)]$, $L_x = \hat{\alpha}_s \ln\left|\frac{x}{\xi}\right|$ and $\hat{\alpha}_s = \alpha_s(\mu_R)C_A/\pi$.

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This yields, when expanded in α_s,

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- First two terms in α_s corresponds to the LO/NLO CF computation at small x.

Implementation of resummation: $C^{\rm CF} \to C^{\rm HEF}$

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Quark coefficient function:

$$C_q^{\rm HEF}\left(\frac{\xi}{x}\right) = \frac{2C_F}{C_A}C_g^{\rm HEF}\left(\frac{\xi}{x}\right),$$

Matching

We use *subtractive matching*:

$$\begin{aligned} C^{\text{match.}}_{g,q}\left(\frac{\xi}{x}\right) &= C^{\text{NLO CF}}_{g,q}\left(\frac{\xi}{x}\right) - C^{\text{asy.}}_{g,q}\left(\frac{\xi}{x}\right) + C^{\text{HEF}}_{g,q}\left(\frac{\xi}{x}\right), \\ C^{\text{asy.}}_{g}\left(\frac{\xi}{x}\right) &= \frac{C_A}{2C_F} C^{\text{asy.}}_{q}\left(\frac{\xi}{x}\right) \\ &= \frac{-i\pi F_{\text{LO}}}{2}\left[\delta\left(\left|\frac{\xi}{x}\right| - 1\right) + \frac{\hat{\alpha}_s}{\left|\frac{\xi}{x}\right|}\ln\left(\frac{M^2}{4\mu_F^2}\right)\right]. \end{aligned}$$

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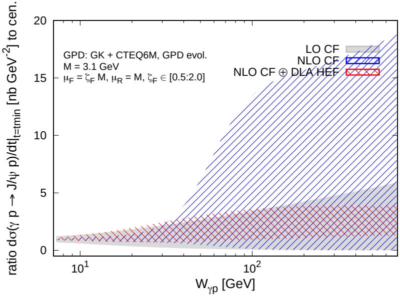
- Matching performed before x-integration.

Results: scale uncertainties for J/ψ



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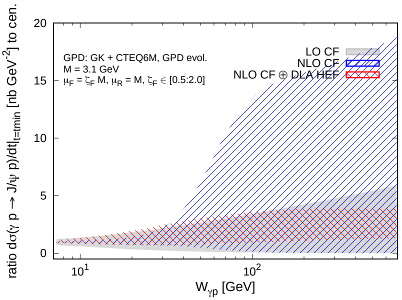

- μ_F uncertainties (left)
  - NLO CF $\oplus$ DLA HEF much better than the pathological NLO CF
  - NLO CF $\oplus$ DLA HEF significantly better than LO CF, even at large W

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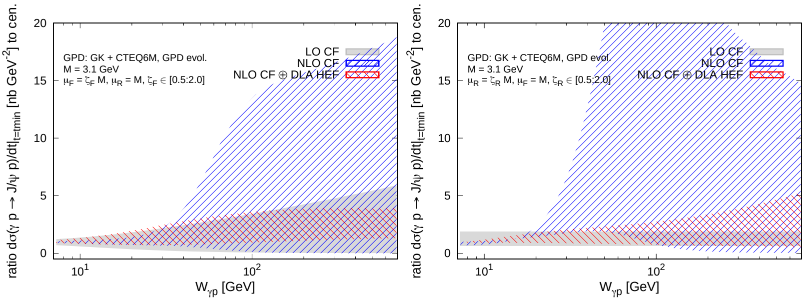

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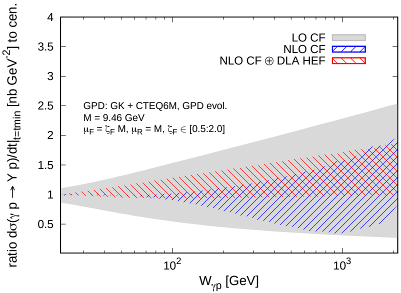
[at large W, $\propto W^{\alpha_s(\mu_R)}$]

Results: scale uncertainties for Υ



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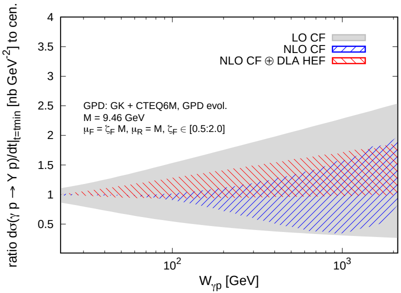

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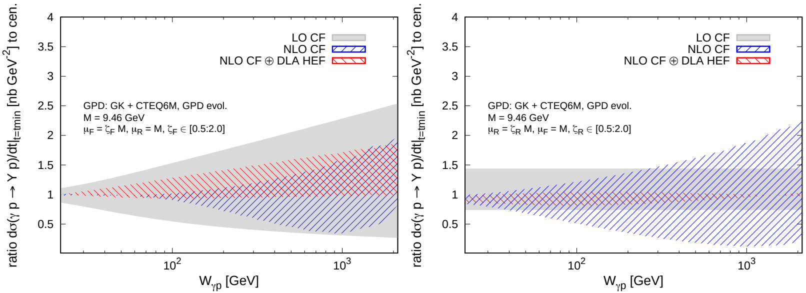

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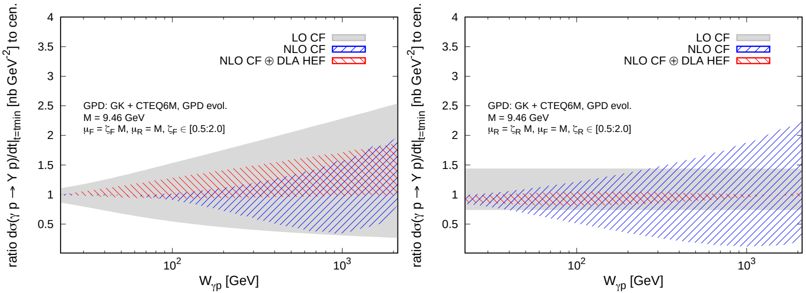

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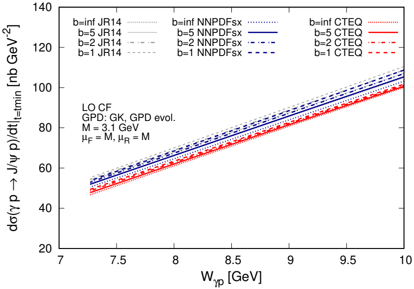
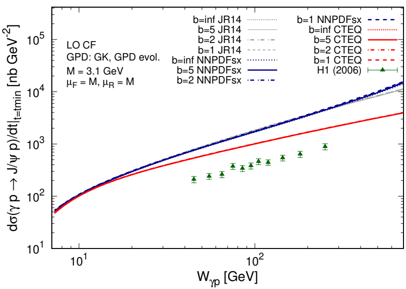
Perturbative instabilities are also present for Υ !

Results: comparing different GPD inputs for LO CF





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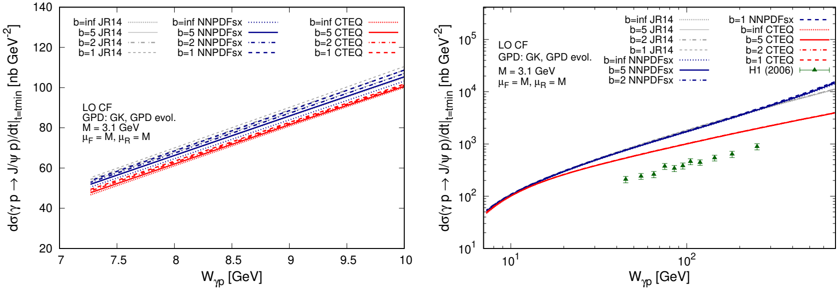

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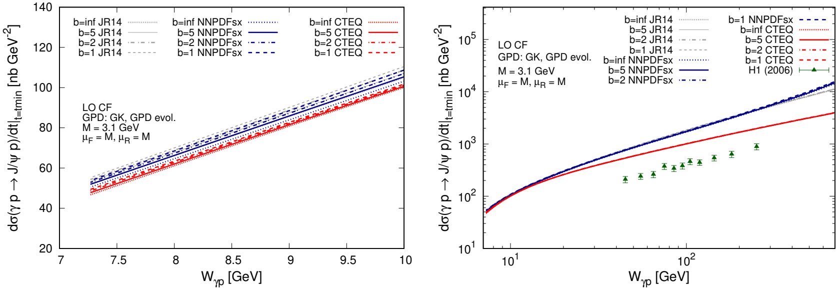

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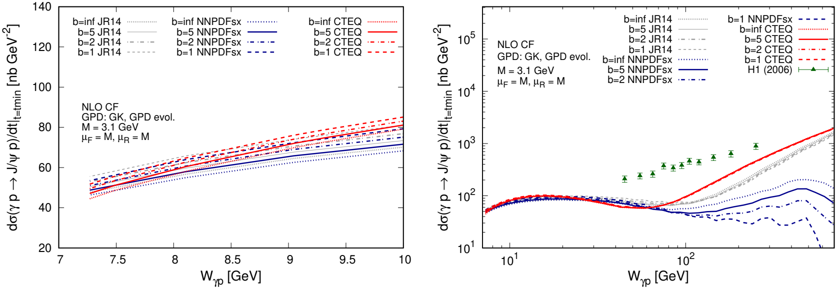
Different GPD inputs for NLO CF



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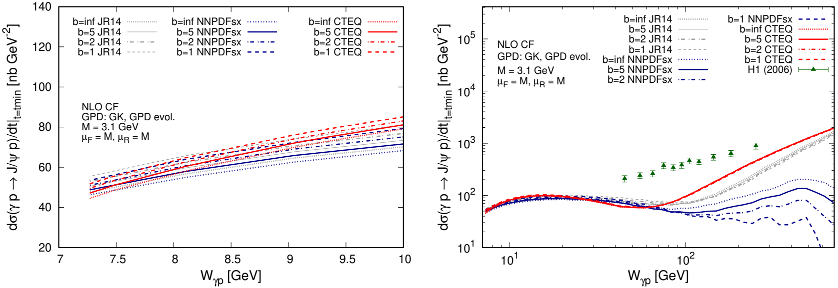

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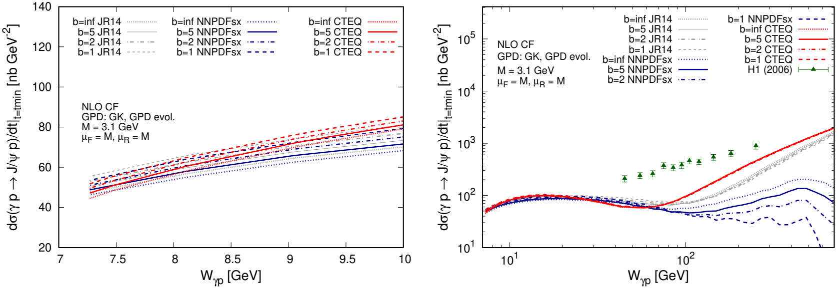

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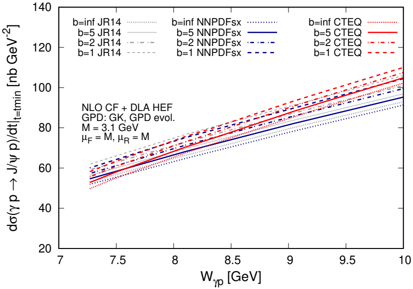
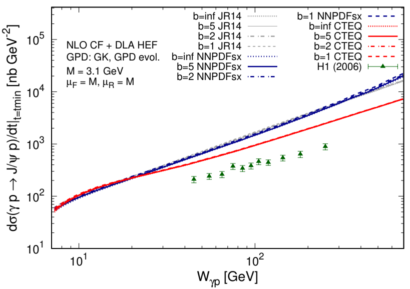
Different GPD inputs for NLO CF $\oplus$ HEF DLA





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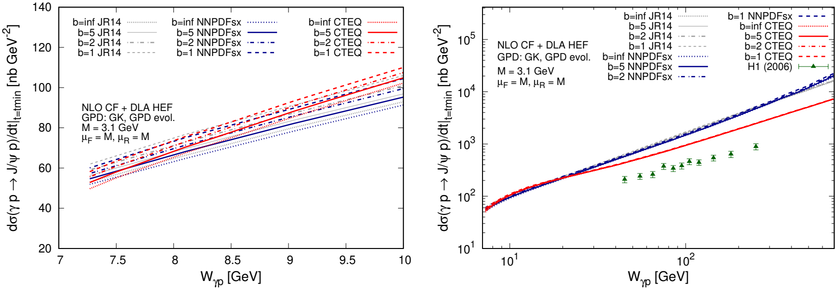

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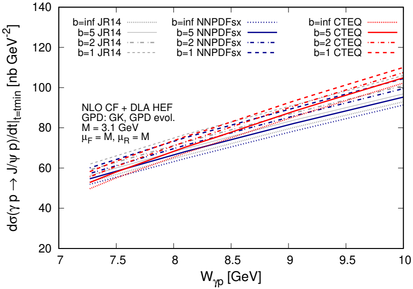

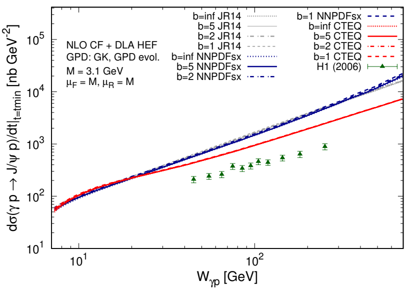

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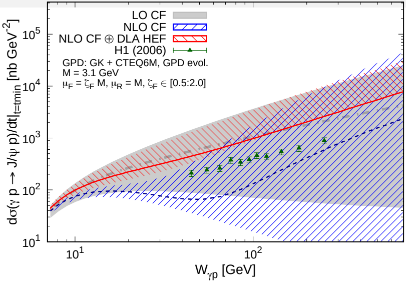

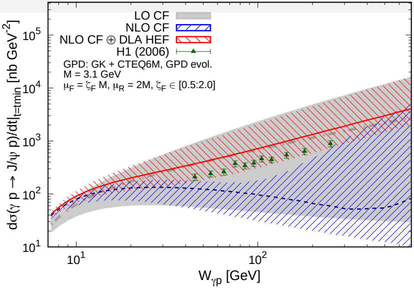

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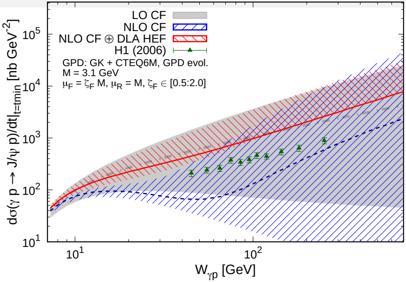

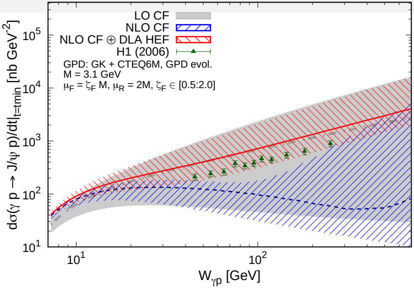

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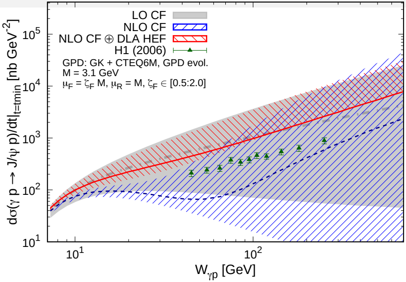

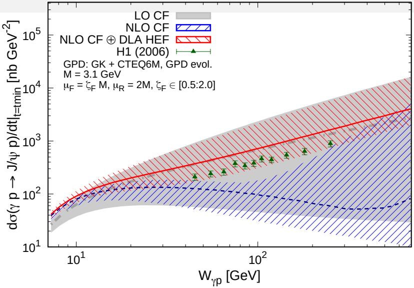

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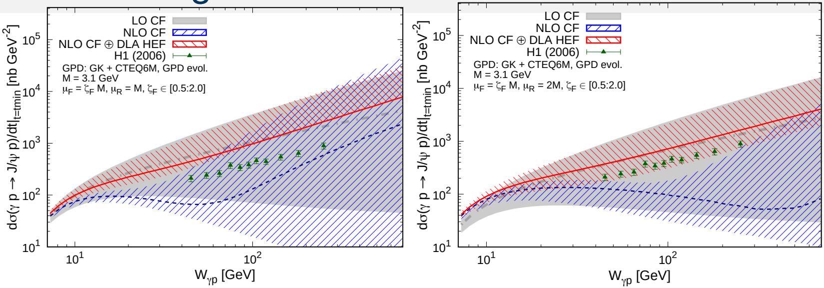

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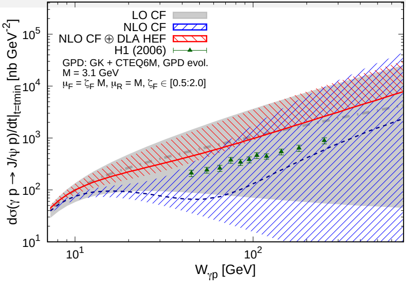

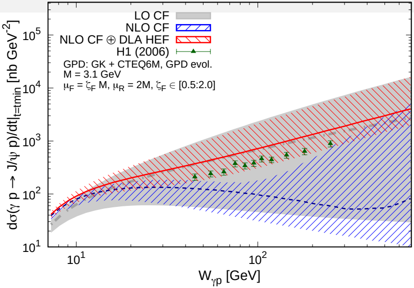

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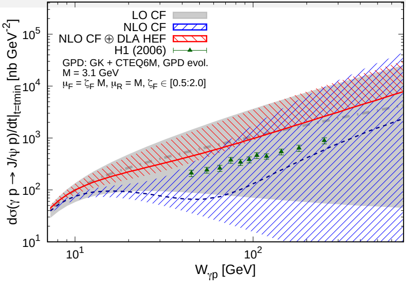
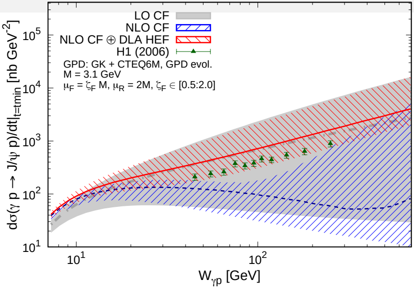
Going beyond DD ?

Summarising our results





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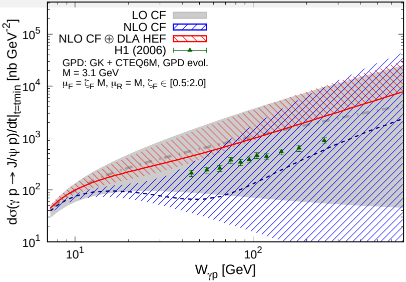
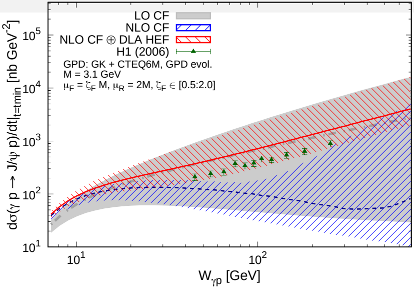
Going beyond DD ?

  - Impact of DA (relativistic corrections) vs strict static limit of NRQCD ?

Summarising our results





- *NLO CF $\oplus$ HEF DLA* cures the instabilities of NLO CF
- Theory in the ballpark of the H1 data, but clearly less precise

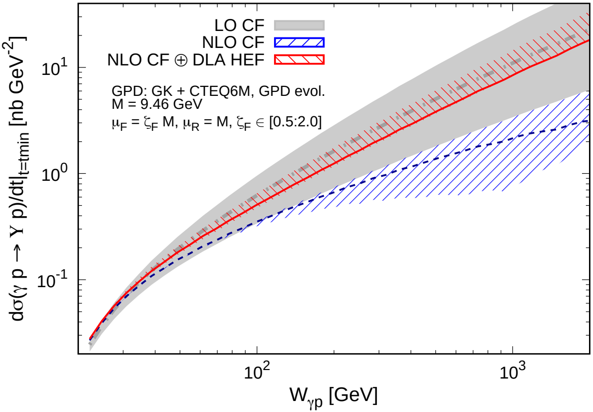
We only considered data at small t

- A priori *better agreement if $\mu_R \approx 2M$* (right plot) but
  - Impact of x/ξ GPD dependence unclear; varying b has negligible effect

    Going beyond DD ?
  - Impact of DA (relativistic corrections) vs strict static limit of NRQCD ?
  - Now ready for a full PDF uncertainty study

    Only comparisons of central PDFs

Results for exclusive Υ photoproduction



- GPD based on CTEQ6M PDF input, *full LO evolution of GPDs*
- Significant corrections wrt NLO and reduction of the uncertainties
- Extrapolating $d\sigma/dt|_{t=t_{\min}}$ from measurements at $W = 100$ GeV, one gets 0.7 nb GeV^{-2} in agreement with the red band

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- *No new non-perturbative ingredients are introduced !*
- Like in the inclusive case, the matched NLO+HEF results are *stable* and agree with data within the (large) theoretical uncertainties.
- The next step is to see how to fit (gluon) GPDs from such observables.
- Topic for future synergies between both VAs of STRONG2020: *NLOAccess and PARTONS*

Backup

BACKUP SLIDES

Implementation of high-energy resummation

HEF resummation of LLA contributions $\sim \alpha_s^n \ln^{n-1}(\frac{x}{\xi})$ *at integrand level* to the imaginary part of the $C_g(\frac{\xi}{x})$:

$$
\begin{aligned}
C_g^{\mathrm{HEF}}\left(\frac{\xi}{x}\right) &= \frac{-i\pi}{2}\frac{F_{\mathrm{LO}}}{\left(\frac{\xi}{x}\right)}\int_0^\infty d\mathbf{q}_T^2\,\mathcal{C}_{gi}\left(\frac{\xi}{x},\mathbf{q}_T^2,\mu_F,\mu_R\right)h(\mathbf{q}_T^2),\\
h(\mathbf{q}_T^2) &= \frac{M^2}{M^2+4\mathbf{q}_T^2}.
\end{aligned}
$$

Implementation of high-energy resummation

Resummation factor, $\mathcal{C}_{gi}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F, \mu_R\right)$ in the *Doubly-Logarithmic Approximation* (DLA) (in order to be consistent with fixed-order evolution of GPD) is given by the Blümlein-Collins-Ellis formula [hep-ph/9506403]

$$\mathcal{C}_{gg}^{(\mathrm{DL})}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) = \frac{\hat{\alpha}_s}{\mathbf{q}_T^2} \begin{cases} J_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{x}{\xi}\right) \ln\left(\frac{\mu_F^2}{\mathbf{q}_T^2}\right)}\right) & \text{if } \mathbf{q}_T^2 < \mu_F^2, \\ I_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{x}{\xi}\right) \ln\left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)}\right) & \text{if } \mathbf{q}_T^2 > \mu_F^2. \end{cases}$$

$\Longrightarrow$ resums terms scaling like $\left(\alpha_s \ln\left(x/\xi\right) \ln(\mu_F^2/\mathbf{q}_T^2)\right)^n$ to all orders in perturbation theory.

For the quark channel, the resummation factor is given in the DLA by:

$$\mathcal{C}_{gq}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) = \frac{C_F}{C_A}\left[\mathcal{C}_{gg}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) - \delta\left(1 - \frac{\xi}{x}\right)\delta(\mathbf{q}_T^2)\right].$$

Implementation of high-energy resummation

Useful representation in Mellin space:

$$\mathcal{C}_{gg}^{(\mathrm{DL})}(N, \mathbf{q}_T^2, \mu_F^2, \mu_R^2) \quad = \quad R(\gamma_{gg}) \, \frac{\gamma_{gg}}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}} .$$

γ_{gg} is the solution to the equation

$$\frac{\hat{\alpha}_s}{N}\chi(\gamma_{gg}) = 1, \quad \chi(\gamma) = 2\varphi(1) - \varphi(\gamma) - \varphi(1-\gamma), \quad \varphi(\gamma) = \frac{d \ln \Gamma(\gamma)}{d\gamma}$$

$$\gamma_{gg} = \frac{\hat{\alpha}_s}{N} + \mathcal{O}\left(\frac{\hat{\alpha}_s^4}{N^4}\right), \quad R(\gamma_{gg}) = 1 + \mathcal{O}\left(\hat{\alpha}_s^3\right)$$

DLA $\Longrightarrow$ *Drop terms in red*: $\gamma_{gg} \to \gamma_N \equiv \frac{\hat{\alpha}_s}{N}$.

Mellin transform maps logarithms $\ln\left(\frac{x}{\xi}\right)$ to the poles at $N = 0$:

$$\frac{x}{\xi} \ln^{k-1}\left(\frac{x}{\xi}\right) \leftrightarrow \frac{(k-1)!}{N^k}.$$

Shuvaev transform

[0812.3558]

$$H_q(x,\xi) \;=\; \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\,\mathrm{Im}\int_0^1 \frac{\mathrm{d}s}{y(s)\,\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{q(x')}{|x'|}\right),$$

$$H_g(x,\xi) \;=\; \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\,\mathrm{Im}\int_0^1 \frac{\mathrm{d}s(x+\xi(1-2s))}{y(s)\,\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{g(x')}{|x'|}\right),$$

$$y(s) = \frac{4s(1-s)}{x+\xi(1-2s)}\,.$$