Simultaneous Bayesian reweighting of TMDs

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M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin,
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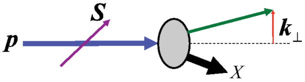
  - probing TMD effects in complementary kinematical regions with respect to semi-inclusive processes
  - studying universality and factorization breaking effects
- plenty of data available from RHIC

Introduction - the Sivers function



D.W. Sivers, PRD 41 (1990) 83 & PRD 43 (1991) 261

$$
\begin{aligned}
&f_{a/p^\uparrow}(x,\mathbf{k}_\perp) - f_{a/p^\uparrow}(x,-\mathbf{k}_\perp) \\
&= \Delta^N f_{a/p^\uparrow}(x,k_\perp^2)\,\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot\mathbf{S}}{|\mathbf{k}_\perp|} \quad (TO-CA) \\
&= -\frac{2|\mathbf{k}_\perp|}{M} f_{1T}^{\perp a}(x,k_\perp^2)\,\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot\mathbf{S}}{|\mathbf{k}_\perp|} \quad (Amsterdam)
\end{aligned}
$$

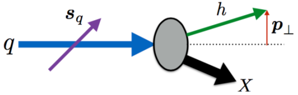
- genuine TMD distribution
- express correlation between proton transverse polarization and parton intrinsic $\mathbf{k}_\perp$
- extracted from SIDIS azimuthal asymmetries:

$$A_{UT}^{\sin(\phi_h-\phi_S)} \equiv \frac{F_{UT}^{\sin(\phi_h-\phi_S)}}{F_{UU,T}} = \frac{\mathcal{C}\left[f_{1T}^{\perp q} D_1^q\right]}{\mathcal{C}\left[f_1^q D_1^q\right]}$$

- modified universality/process dependence:

$$f_{1T}^{\perp q[DY]}(x,k_\perp^2) = -f_{1T}^{\perp q[SIDIS]}(x,k_\perp^2)$$

Introduction - the Collins function



J.C. Collins, NPB 396:161-182 (1993)

$$D_{h/q^\uparrow}(z,\mathbf{p}_\perp) - D_{h/q^\uparrow}(z,-\mathbf{p}_\perp)$$
$$= \Delta^N D_{h/q^\uparrow}(z,p_\perp^2)\,\frac{(\hat{\mathbf{p}}_q \times \mathbf{p}_\perp)\cdot \mathbf{s}_q}{|\mathbf{p}_\perp|} \quad (TO-CA)$$
$$= -\frac{2|\mathbf{p}_\perp|}{z m_h} H_1^{\perp q}(z,p_\perp^2)\,\frac{(\hat{\mathbf{p}}_q \times \mathbf{p}_\perp)\cdot \mathbf{s}_q}{|\mathbf{p}_\perp|} \quad (Amsterdam)$$

- genuine TMD fragmentation function
- express correlation between quark transverse polarization and produced hadron $\mathbf{p}_\perp$
- extracted, together with TMD transversity h_1^q, in global fits of SIDIS azimuthal asymmetries:

$$A_{UT}^{\sin(\phi_h+\phi_S)} = \frac{2(1-y)}{1+(1-y)^2}\,\frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU,T}} \sim \frac{\mathcal{C}\left[h_1^q H_1^{\perp q}\right]}{\mathcal{C}\left[f_1^q D_1^q\right]}$$

and $e^+e^- \to h_1 h_2 X$ azimuthal asymmetries (double ratio):

$$\frac{R_0^U}{R_0{}^{L(C)}} = \frac{1+P_0^U\cos(2\phi_1)}{1+P_0^{L(C)}\cos(2\phi_1)} \simeq 1+\cos(2\phi_1)A_0^{UL(UC)} \sim \mathcal{C}\left[H_1^{\perp\bar{q}} H_1^{\perp q}\right]$$

Introduction - formalism

U. D'Alesio, F. Murgia PRD 70 (2004) 074009; M. Anselmino *et al.*, PRD 73 (2006) 014020
L. Gamberg, Z.-B. Kang, PLB 696 (2011) 109; U. D'Alesio *et al.*, PRD 96 (2017) 036011 ...

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- A_N in $p^\uparrow p \to h X$:

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\Delta\sigma}{2d\sigma} \simeq \frac{d\Delta\sigma_{\rm Siv} + d\Delta\sigma_{\rm Col}}{2d\sigma}$$

with

$$d\Delta\sigma_{\rm Siv}^{\rm CGI\text{-}GPM} \propto \sum_{a,b,c,d} f_{1T}^{\perp a}(x_a, k_{\perp a}) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes H_{ab\to cd}^{\rm Inc} \otimes D_{h/c}(z, k_{\perp h})$$

$$d\Delta\sigma_{\rm Col} \propto \sum_{a,b,c,d} h_{1a}(x_a, k_{\perp a}) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes d\Delta\sigma^{a^\uparrow b \to c^\uparrow d} \otimes H_1^{\perp c}(z, k_{\perp h})$$

and

$$d\sigma \propto \sum_{a,b,c,d} f_{a/p}(x_a, k_{\perp a}) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes H_{ab\to cd}^{U} \otimes D_{h/c}(z, k_{\perp h})$$

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- GPM results (Sivers): $H^{\text{Inc}}_{ab\to cd} \to H^{U}_{ab\to cd}$

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- GPM results (Sivers): $H^{\text{Inc}}_{ab\to cd} \to H^{U}_{ab\to cd}$
- gluon Sivers effect negligible in the region of moderate and forward rapidity

The reweighting method

W.T. Giele, S. Keller PRD 58 (1998) 094023; R.D. Ball *et al.*, NPB 849 (2011) 112
N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020; H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

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how to extend the Bayesian reweighting to multiple, independent fits?

Simultaneous Bayesian reweighting

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- f and g extracted from fits to independent datasets $\boldsymbol{E}^a$ and $\boldsymbol{E}^b$ through χ^2-minimization:

$$\chi_a^2 \equiv \chi^2[\boldsymbol{a}; \boldsymbol{E}^a] = \sum_{i,j=1}^{N_{\rm dat}^a} (T_i[\boldsymbol{a}] - E_i^a)\,(C_{ij}^a)^{-1}(T_j[\boldsymbol{a}] - E_j^a)$$

$$\chi_b^2 \equiv \chi^2[\boldsymbol{b}; \boldsymbol{E}^b] = \sum_{i,j=1}^{N_{\rm dat}^b} (T_i[\boldsymbol{b}] - E_i^b)\,(C_{ij}^b)^{-1}(T_j[\boldsymbol{b}] - E_j^b)$$

and probability density functions $\pi(\boldsymbol{a})$, $\pi(\boldsymbol{b})$ reconstructed by generating $N_{\rm set}^a$ $\boldsymbol{a}_k$ and $N_{\rm set}^b$ $\boldsymbol{b}_l$ MC sets respectively

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- a new dataset $\boldsymbol{E}$ is measured; data described by *e.g.* $T_i[\boldsymbol{a},\boldsymbol{b}] \equiv \alpha T_i[\boldsymbol{a}] + \beta T_i[\boldsymbol{b}]$. Define

$$\chi_{\rm new}^2[\boldsymbol{a},\boldsymbol{b};\boldsymbol{E}] = \sum_{i,j=1}^{N_{\rm dat}} (T_i[\boldsymbol{a},\boldsymbol{b}] - E_i)\,C_{ij}^{-1}(T_j[\boldsymbol{a},\boldsymbol{b}] - E_j)$$

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$$\chi^2_{\rm new}[\boldsymbol{a},\boldsymbol{b};\boldsymbol{E}] = \sum_{i,j=1}^{N_{\rm dat}} (T_i[\boldsymbol{a},\boldsymbol{b}] - E_i)\,C^{-1}_{ij}(T_j[\boldsymbol{a},\boldsymbol{b}] - E_j)$$

- uncertainty on $T_i[\boldsymbol{a},\boldsymbol{b}]$ by taking all possible $(N^a_{\rm set} \times N^b_{\rm set})$ combinations
  $\Rightarrow (N^a_{\rm set} \times N^b_{\rm set})$ values $\chi^2_{\rm new} \equiv \chi^2_{kl,\rm new} = \chi^2_{\rm new}[\boldsymbol{a}_k,\boldsymbol{b}_l;\boldsymbol{E}]$

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- Posterior density through Bayes theorem:

$$\mathcal{P}(\boldsymbol{a},\boldsymbol{b}|\boldsymbol{E}) = \frac{\mathcal{L}(\boldsymbol{E}|\boldsymbol{a},\boldsymbol{b})\,\pi(\boldsymbol{a},\boldsymbol{b})}{Z}$$

with factorized prior $\pi(\boldsymbol{a},\boldsymbol{b}) = \pi(\boldsymbol{a})\pi(\boldsymbol{b})$ (extractions *a priori* independent)

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- we take $\mathcal{L}(\boldsymbol{E}|\boldsymbol{a},\boldsymbol{b})\, d\boldsymbol{E}$ as probability to find new data confined in a differential volume $d\boldsymbol{E}$ around $\boldsymbol{E}$; weights are then defined as [H. Paukkunen, P. Zurita, JHEP 12 (2014) 100]

$$w_{kl}(\chi^2_{\text{new}}) = \exp\left\{-\frac{1}{2}\frac{\chi^2_{kl,\text{new}}}{\Delta\chi^2}\right\} \Bigg/ \sum_{k',l'} \exp\left\{-\frac{1}{2}\frac{\chi^2_{k'l',\text{new}}}{\Delta\chi^2}\right\}$$

[$\Delta\chi^2$ for $n_a + n_b$ parameters at a given CL]

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- expectation value and variance for a quantity $\mathcal{O}$ (symmetric)

$$\mathrm{E}[\mathcal{O}] = \sum_{k=1}^{N^a_{\rm set}} \sum_{l=1}^{N^b_{\rm set}} w_{kl}\,\mathcal{O}(\boldsymbol{a}_k,\boldsymbol{b}_l) \qquad \mathrm{V}[\mathcal{O}] = \sum_{k=1}^{N^a_{\rm set}} \sum_{l=1}^{N^b_{\rm set}} w_{kl}\left(\mathcal{O}(\boldsymbol{a}_k,\boldsymbol{b}_l) - \mathrm{E}[\mathcal{O}]\right)^2$$

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- if $\mathcal{O}$ depends only on $\boldsymbol{a}$ or $\boldsymbol{b}$, then use weights

$$w_k = \sum_{l=1}^{N^b_{\rm set}} w_{kl} \qquad w_l = \sum_{k=1}^{N^a_{\rm set}} w_{kl}$$

A compression procedure

B. Bauer, D. Pitonyak, C.Shay, PRD 107 (2023) 014013
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A compression procedure

B. Bauer, D. Pitonyak, C.Shay, PRD 107 (2023) 014013
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712

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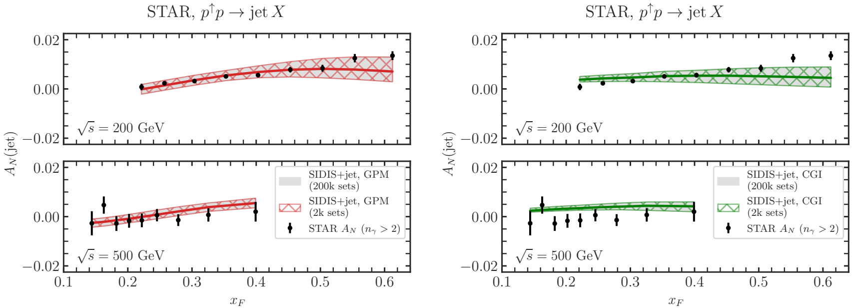
- $|t|$ with corresponding p-value $\gtrsim 0.1 \Rightarrow$ statistically equivalent distributions
- underlying assumption: Gaussian probability distributions
- our strategy:
  - employ t-test to find an optimal size for a representative sample
  - compare also median and asymmetric uncertainty of samples

A compression procedure - validation

J. Adam *et al.* (STAR Collaboration), PRD 103 (2021) 092009
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



- we apply the compression procedure and reproduce reweighting performed on A_N for jet production at STAR with a reduced sample of MC sets
- reweighted predictions from full sample (200k sets, gray) and reduced sample (2k sets, GPM and CGI-GPM)
- 2000 sets enough to reproduce same results!

same happens for unweighted predictions (not shown)

Priors from SIDIS and e^+e^- data

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712

- We perform updated extractions of $f_{1T}^{\perp q}$, h_1^q, $H_1^{\perp q}$ from SIDIS and e^+e^- data using most recent data from HERMES, COMPASS, JLab

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    M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, JHEP 07 (2018) 148
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- apply compression procedure by randomly sampling 2000 sets for $f_{1T}^{\perp q}$ and 2000 sets for h_1^q & $H_1^{\perp q}$
  h_1^q fulfil the Soffer Bound, applied a posteriori U. D'Alesio, CF, A. Prokudin, PLB 803 (2020) 135347

Results

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712

- The simultaneous reweighting is perfomed on A_N data:
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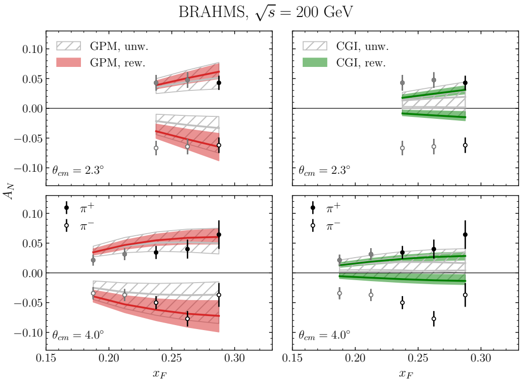
- $P_T > 1\,\text{GeV}$ as hard scale of the process
- median as cental value, 2σ CL asymmetric uncertainties; a total of 13 parameters $\Rightarrow \Delta\chi^2 = 22.69$ in the computation of w_{kl}

Results - BRAHMS

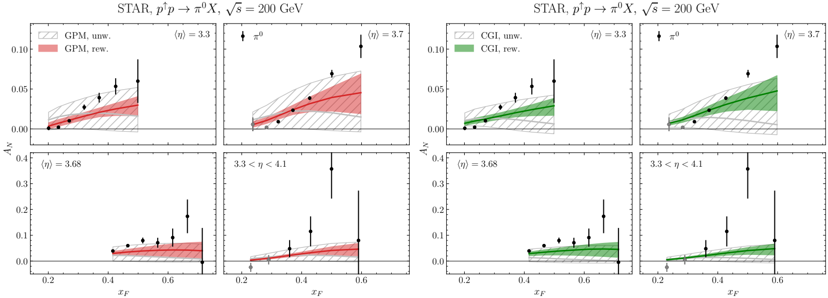
J. H. Lee, F. Videbæk, AIP Conf. Proc. 915, 533–538 (2007)
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712



- reweighted curves with reduced uncertainties
- GPM describes these data better than CGI-GPM
- quality of description increases if data with $P_T < 1.5$ GeV (gray points) is not considered

Results - STAR (I)

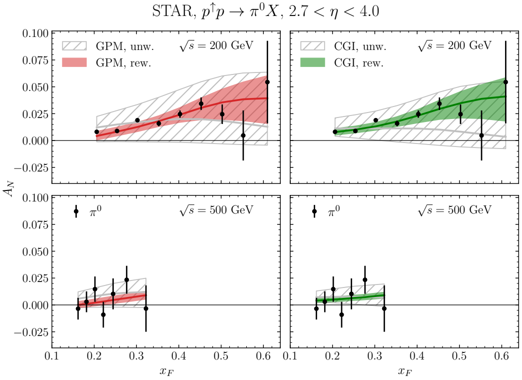
B. I. Abelev *et al.*, PRL 101, 222001 (2008); J. Adams *et al.*, PRL 92 171801 (2004); L. Adamczyk *et al.*, PRD 86 (2012) 051101
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712



- both GPM and CGI-GPM in qualitative agreement with the data
- reweighted bands able to describe data at moderate x_F
- shape better representing the steady increase of A_N at large x_F

Results - STAR (II)

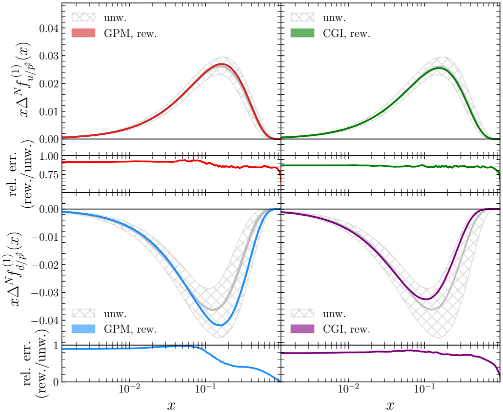
J. Adams *et al.*, PRD 103 (9) (2021) 092009
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712



- data not showing the usual steady increase at large x_F
- reweighted curves describe the data
- if reweighting was performed on these data solely, bands would be flatter

Results - Sivers function

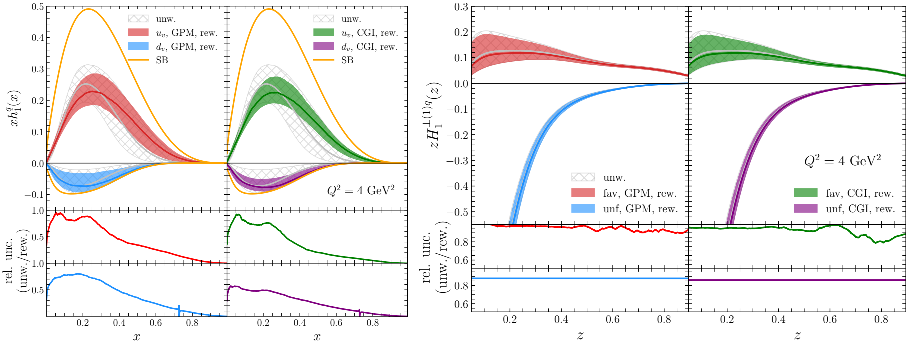
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712



- reduced uncertainties, especially at large x
- relative reduction up to $20-30\%$ for $f_{1T}^{\perp u}$ and $40-90\%$ for $f_{1T}^{\perp d}$

Results - transversity & Collins

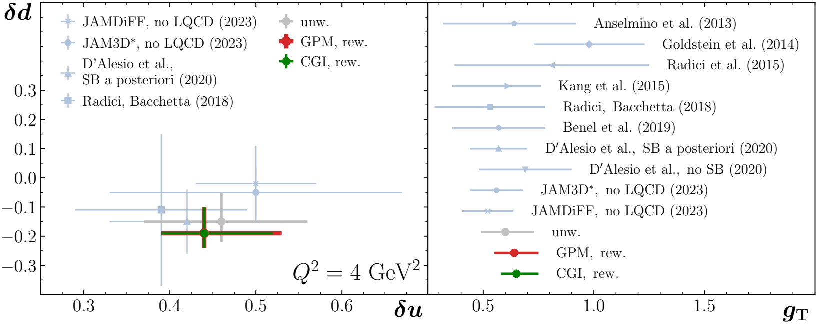
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712



- A_N data mainly affecting the transversity function
- reweighted transversity functions follow Soffer Bound rather closely at large x
- uncertainty reduction up to $80-90\%$ for h_1^q at large x, $\sim 10-15\%$ for $H_1^{\perp q(1)}$
- dominant contribution to A_N from the Collins mechanism

Results - tensor charges

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712



$$\delta q = \int_0^1 \left[h_1^q(x) - h_1^{\bar{q}}(x) \right] dx \,, \quad g_T = \delta u - \delta d$$

- consistency of different h_1^q extractions within different approaches exploiting a variety of experimental data

Conclusions and outlook

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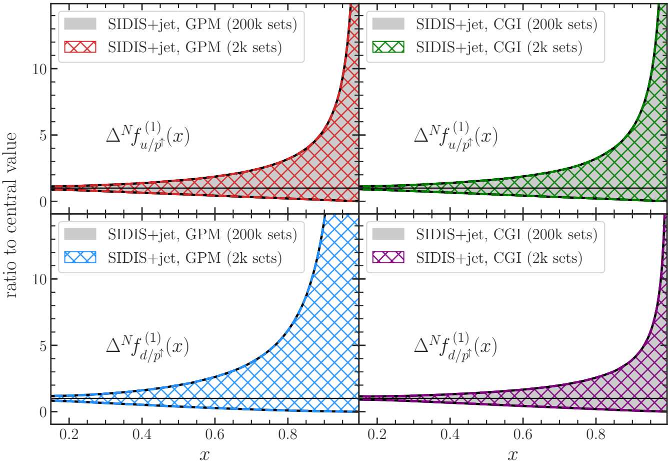
Thank you

Backup

Validating the compression algorithm

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712



correctly reproduce median and asymmetric uncertainties with only 2000 sampled sets!

Priors - parametrizations

- MSHT20nlo proton PDFs and DEHSS FFs
- Sivers (5 parameters):

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = \frac{4M_p k_\perp}{\langle k_\perp^2 \rangle_S} \Delta^N f_{q/p^\uparrow}^{(1)}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S} \quad (q = u, d)$$

$$\Delta^N f_{q/p^\uparrow}^{(1)}(x) = N_q (1-x)^{\beta_q}$$

- transversity and Collins (8 parameters)

$$h_1^q(x, k_\perp^2) = h_1^q(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle} \quad (q = u_v, d_v)$$

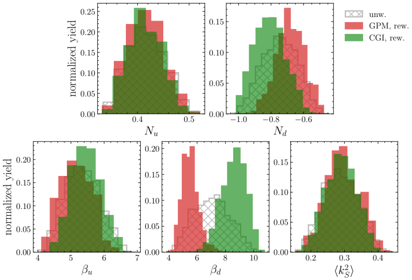
$$h_1^q(x, Q_0^2) \equiv \mathcal{N}_q^T(x)\, \mathrm{SB}(x, Q_0^2), \quad \mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta}$$

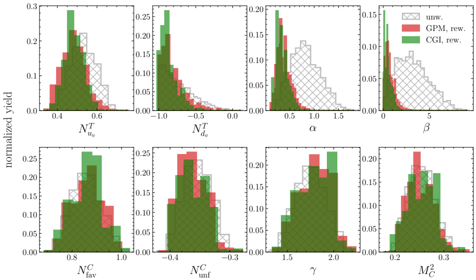
$$H_1^{\perp q}(z, p_\perp^2) = \mathcal{N}_q^C(z) \frac{z m_h}{M_C} \sqrt{2e}\, e^{-p_\perp^2/M_C^2} D_{h/q}(z, p_\perp^2) \quad (q = \mathrm{fav}, \mathrm{unf})$$

$$\mathcal{N}_{\mathrm{fav}}^C(z) = N_{\mathrm{fav}}^C z^\gamma, \qquad \mathcal{N}_{\mathrm{unf}}^C(z) = N_{\mathrm{unf}}^C$$

Results - parameter distributions

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712





Results - N_{eff}

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712

$$N_{\text{eff}} = \exp\left\{\sum_{k=1}^{N_{\text{set}}} w_k \ln\left(\frac{1}{w_k}\right)\right\}$$

- N_{eff} from the reweighting procedure on BRAHMS, older and latest STAR data:

	GPM	CGI-GPM
$f_{1T}^{\perp q}$	547	706
h_1^q & $H_1^{\perp q}$	285	110

- N_{eff} from the reweighting procedure on latest STAR data only:

	GPM	CGI-GPM
$f_{1T}^{\perp q}$	1807	1961
h_1^q & $H_1^{\perp q}$	1877	1514

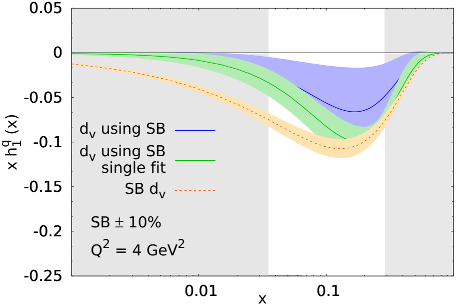
Results - tensor charges

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712

Tensor charges at $Q^2 = 4$ GeV^2:

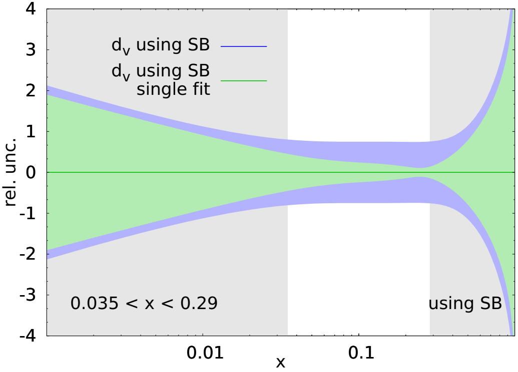
	unw.	rew. (GPM)	rew. (CGI-GPM)
δu	$0.46^{+0.10}_{-0.09}$	$0.47^{+0.09}_{-0.07}$	$0.47^{+0.08}_{-0.05}$
δd	$-0.15^{+0.10}_{-0.07}$	$-0.18^{+0.10}_{-0.06}$	$-0.19^{+0.07}_{-0.05}$
g_T	$0.60^{+0.13}_{-0.11}$	$0.64^{+0.11}_{-0.09}$	$0.65^{+0.10}_{-0.07}$

Transversity and Collins fit - role of the SB



- "using SB single fit": apply SB a priori – automatic fulfillment of the SB throughout the fit
  $\Rightarrow$ $N^T_{d_v}$ saturates at its lower value, MINUIT underestimates the uncertainty on $N^T_{d_v}$ $\Rightarrow$ uncertainty for $h_1^{d_v}$ underestimated
- "using SB": apply SB a posteriori $\Rightarrow$ minimizator explores other configurations in the parameter space, compatible with the SB, that were not seen due to the bias introduced in the parametrization

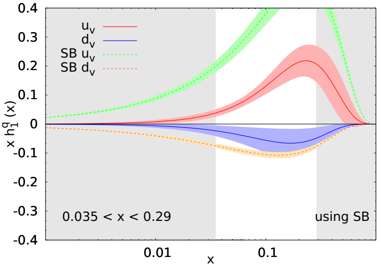
Fit results - using SB

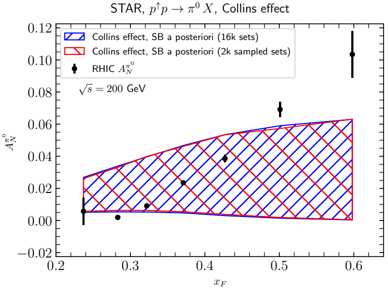


- auomatic fulfillment of the SB brings to underestimate the uncertainty
- underestimation is more severe in the region of fitted data

Results - Soffer Bound

U. D'Alesio, CF, A. Prokudin, PLB 803 (2020) 135347
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712





- SB applied a posteriori with constraints on fit parameters $\Rightarrow$ less biased estimate of uncertainty
- out of $\mathcal{O}(10^5)$ sets, $\sim$ 10% respect the SB $\Rightarrow$ sampled 2000 sets using the compression algorithm
- predictions for π^0 production at STAR (full vs sampled) are compatible within uncertainty
- large asymmetries for the Collins effect not seen in the past due to direct SB enforcement