Progress in Understanding Heavy Quark Fragmentation in the Transverse Plane

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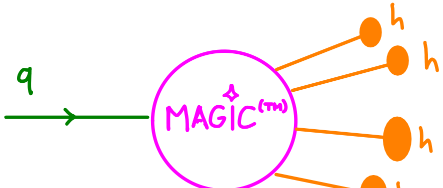
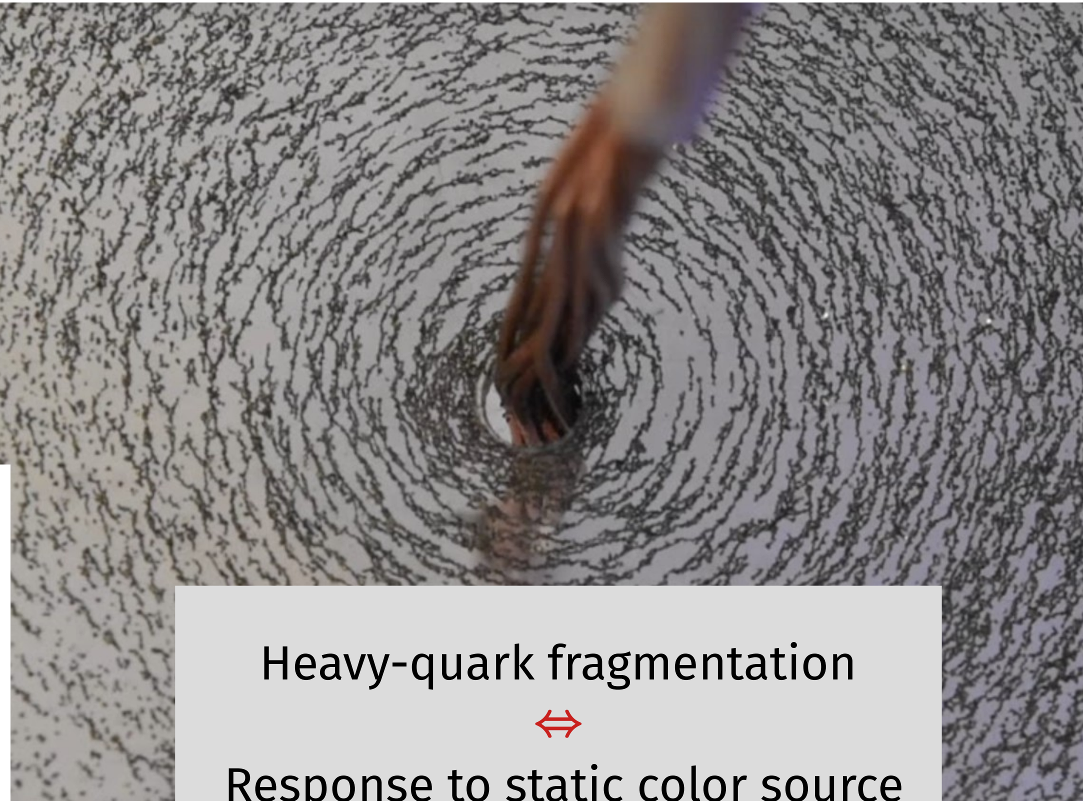
[2305.15461, 2404.08622, and work in progress]

QCD Evolution 2024

Pavia, May 27, 2024

Motivation: Probe Hadronization in Depth

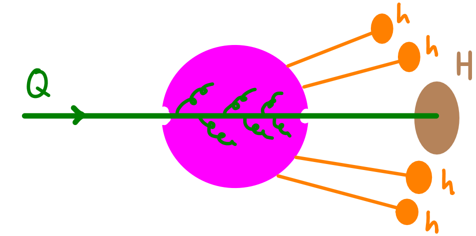




Heavy-quark fragmentation

⇔

Response to static color source

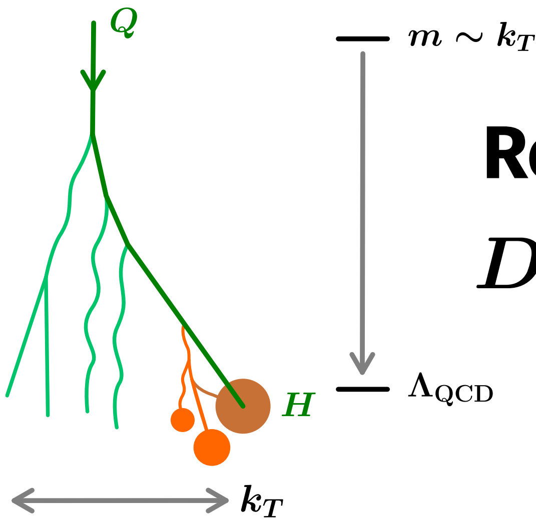


Heavy-quark TMD FFs $D_{1\,H/Q}(z_H, k_T)$ $\begin{cases} \ldots \text{provide a 3D picture of this} \\ \ldots \text{are a universal building block} \end{cases}$

Overview of 2305.15461: Heavy-quark TMD FFs

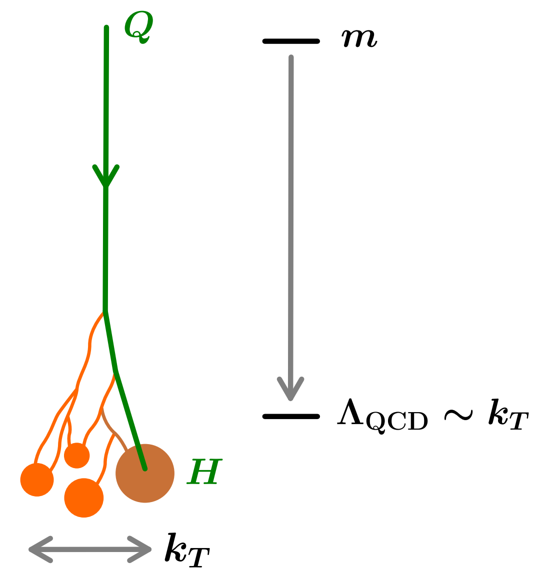
JHEP 09 (2023) 205 with Rebecca von Kuk (DESY) & Zhiquan Sun (MIT)

[→ See e.g. talk at CFNS TMD Workshop in June '23]



Regime (a)

$$D_{1\,H/Q}(z_H, k_T) = d_{1\,Q/Q}(z_H, k_T)\,\chi_H$$



Regime (b)

$$D_{1\,H/Q} = \delta(\bar z_H)\, C_m(m)\, N_{H/\ell}\, \chi_{1,\ell}(k_T)$$

$$H^{\perp}_{1\,H/Q} = \delta(\bar z_H)\, C_m(m)\, N_{H/\ell}\, \chi^{\perp}_{1,\ell}(k_T)$$

$\bar z_H \equiv 1 - z_H$

HQET "TMD Fragmentation Factors"

[See talks by S. Romero, L. Maxia on Wed for progress on TMD FFs from NRQCD]

Overview of today's talk

1. Transverse Momentum-Dependent
   Heavy-Quark Fragmentation at Next-to-Leading Order

2. How Many Structure Functions Characterize Polarized
   Heavy Quark Fragmentation in the Transverse Plane?

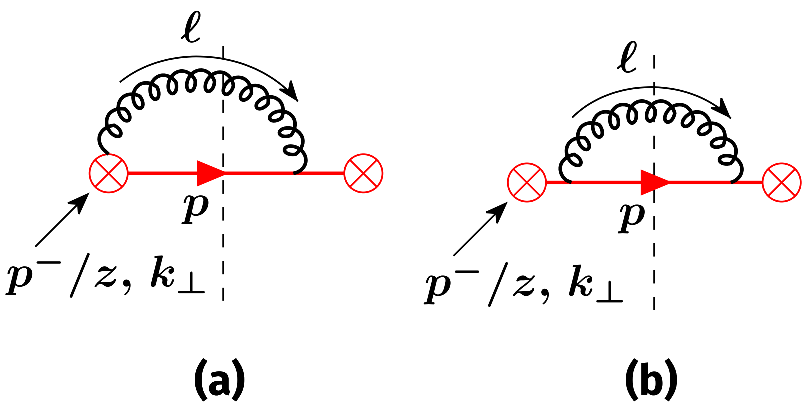
1. Transverse Momentum-Dependent Heavy-Quark Fragmentation at Next-to-Leading Order

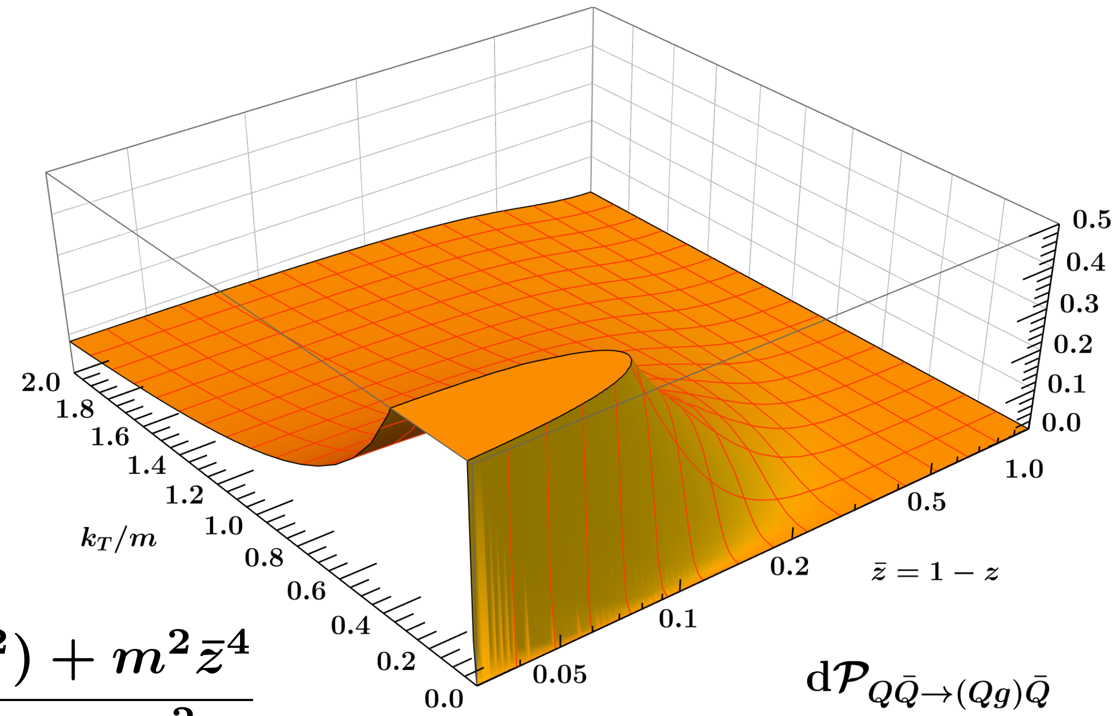
   2404.08622 with Rebecca von Kuk (DESY) & Zhiquan Sun (MIT)

2. How Many Structure Functions Characterize Polarized Heavy Quark Fragmentation in the Transverse Plane?

Regulating the quasi-collinear splitting probability

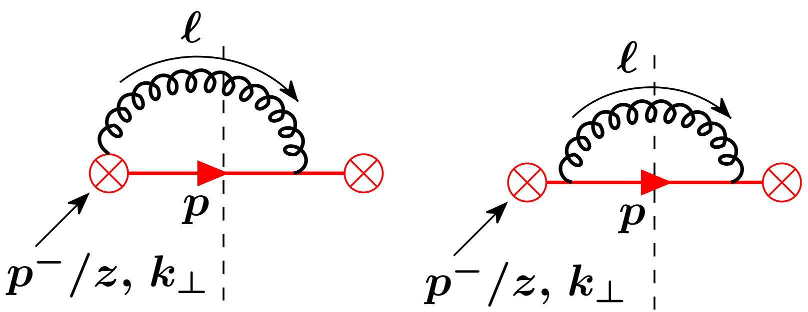




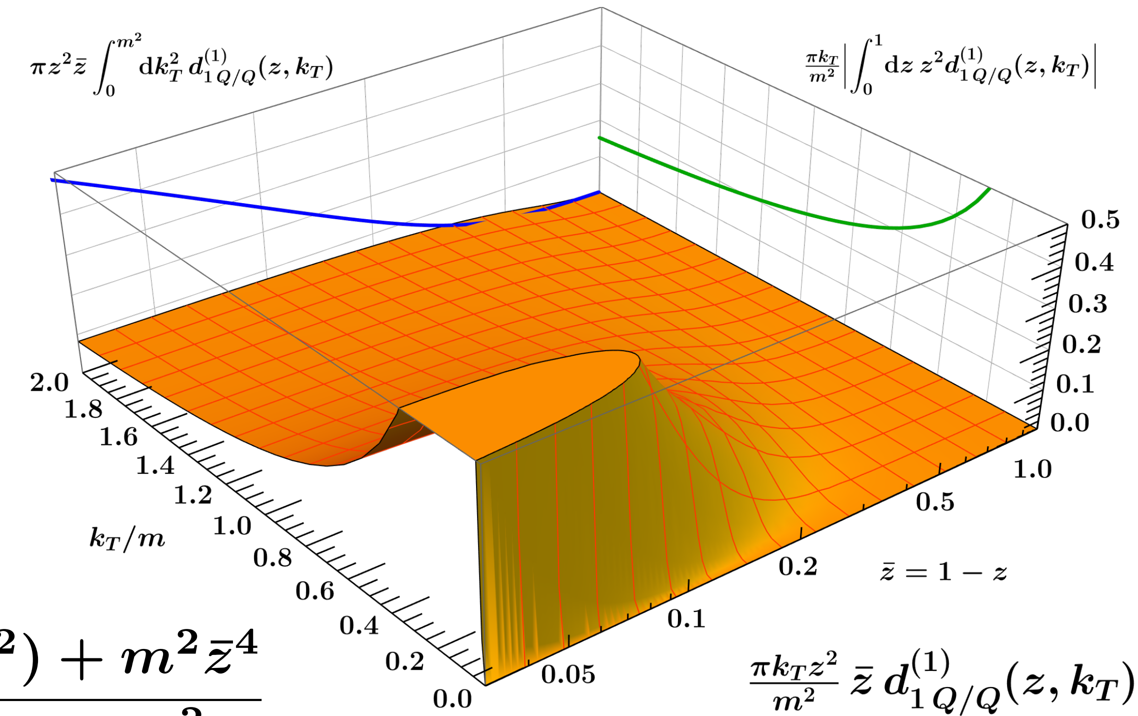
$\bar{z} \equiv z_g = 1 - z > 0$ and $k_T > 0$:

$$\frac{\mathrm{d}\mathcal{P}_{Q\bar{Q}\to(Qg)\bar{Q}}}{\mathrm{d}z\,\mathrm{d}(k_T^2)} = \frac{\alpha_s C_F}{2\pi}\,\frac{k_T^2 z^2(1+z^2) + m^2\bar{z}^4}{\bar{z}\left[k_T^2 z^2 + m^2\bar{z}^2\right]^2}$$

Regulating the quasi-collinear splitting probability



(a) **(b)**



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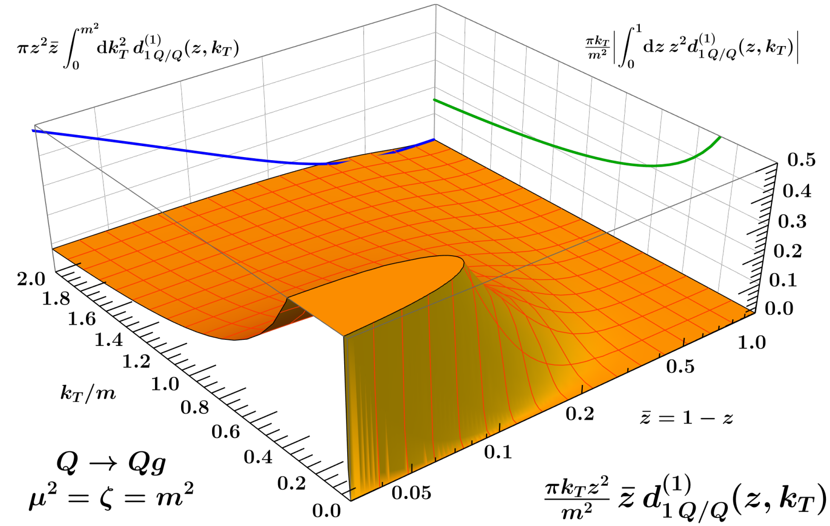
$$\iint = \frac{\alpha_s C_F}{4\pi}\left(5 - \frac{\pi}{2} - \frac{2\pi^2}{3}\right)$$

$$d^{(a)}_{1\,Q/Q} = \frac{\alpha_s C_F}{4\pi}\, \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \left(\frac{\mu}{k_T}\right)^{2\epsilon} \left(\frac{\sqrt{\zeta}}{\nu}\right)^{-\eta} \frac{1}{\pi z^{2-2\epsilon}}\, \frac{z^\eta}{\bar{z}^{1+\eta}}\, \frac{4z^3}{k_T^2 z^2 + m^2 \bar{z}^2}$$

$$d^{(b)}_{1\,Q/Q} = \frac{\alpha_s C_F}{4\pi}\, \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \left(\frac{\mu}{k_T}\right)^{2\epsilon} \frac{1}{\pi z^{2-2\epsilon}}\, 2z^2 \bar{z} \left[\frac{k_T^2 z^2 + m^2(1 - 4z + z^2) - \epsilon(k_T^2 z^2 + m^2 \bar{z}^2)}{(k_T^2 z^2 + m^2 \bar{z}^2)^2}\right]$$

Expanding in 2D plus distributions

- $\dfrac{z^\eta}{\bar{z}^{1+\eta}} = -\dfrac{\delta(\bar{z})}{\eta} + \mathcal{O}(\eta^0)$ cancels as for massless



$\Rightarrow$ Left to expand: $f(x, z, \epsilon)$ $\qquad x \equiv k_T^2/m^2$

Recall: $f(x, \epsilon) = \big[f(x, 0) + \mathcal{O}(\epsilon)\big]_+ + \delta(x)\, F(\epsilon)$

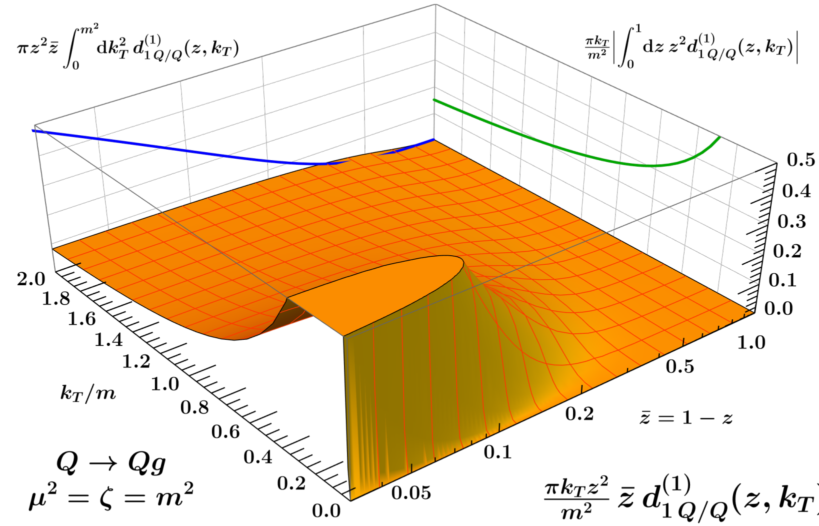
$$F(\epsilon) = \int_0^1 \mathrm{d}x'\, f(x', \epsilon) = \frac{1}{\epsilon} + \dots$$

Generalize to 2D

$$f(x, z, \epsilon) = \big[f(x, z, 0) + \mathcal{O}(\epsilon)\big]_{+,+} + \delta(x)\big[F_x(z, \epsilon)\big]_+ + \delta(\bar{z})\big[F_z(x, \epsilon)\big]_+ + \delta(x)\,\delta(\bar{z})\, F_{xz}(\epsilon)$$

$$F_x(z, \epsilon) \equiv \int_0^1 \mathrm{d}z'\, f(x, z', \epsilon) \qquad F_z(x, \epsilon) \equiv \int_0^1 \mathrm{d}x'\, f(x', z, \epsilon) \qquad F_{xz}(\epsilon) \equiv \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z'\, f(x, z', \epsilon)$$

$$\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z\, \big[f(x, z)\big]_{+,+}\, g(x, z) \equiv \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z\, f(x, z)\, \big[g(x, z) - g(0, z) - g(x, 0) + g(0, 0)\big]$$



$$
\begin{aligned}
& d_{1\,Q/Q}^{(1)}(z, k_T, \mu, \zeta) \\
&= \frac{\alpha_s C_F}{4\pi} \frac{1}{\pi z^2} \Bigg\{ \delta(\bar{z}) \bigg[2 \ln \frac{\zeta}{\mu^2} \mathcal{L}_0(k_T^2, \mu^2) \\
&\quad + 2 \ln \frac{\mu^2}{m^2} \mathcal{L}_0(k_T^2, m^2) \\
&\quad - \ln^2 \frac{\mu^2}{m^2} \delta(k_T^2) + 3 \ln \frac{\mu^2}{m^2} \delta(k_T^2) \bigg] \\
&\quad + \frac{1}{m^2} \left[\frac{2 x z^4 (1 + z^2) + 2 z^2 \bar{z}^4}{\bar{z} (x z^2 + \bar{z}^2)^2} \right]_{+,+} + \delta(k_T^2) \left[\frac{2(1 + z^2)}{\bar{z}} \ln\!\left(1 + \frac{z^2}{\bar{z}^2}\right) - \frac{4 z^3}{\bar{z}(1 - 2\bar{z} z)} \right]_+ \\
&\quad + \delta(\bar{z}) \frac{1}{m^2} \left[- \frac{2 + 3x + 4x^2 + 3x^3 + \pi \sqrt{x}(2 + 7x + x^2) + x(1 + 7x + 2x^2) \ln x}{x(1 + x)^3} \right]_+ \\
&\quad + \delta(\bar{z})\, \delta(k_T^2) \left(5 - \frac{\pi}{2} - \frac{2\pi^2}{3}\right) \Bigg\}
\end{aligned}
$$

$x \equiv k_T^2 / m^2$

→ Confirms expected renormalization by $\gamma_\mu(\zeta, \cancel{m}, \mu)$ and $\gamma_\zeta(k_T, m, \mu)$

[Secondary quark mass effects in the CS kernel show up at NNLO.]

Results in b_T space

Plus distributions domains align with primary variables $\Rightarrow$ easy to take integral transforms!

$$\int_0^\infty \mathrm{d}(k_T^2)\, J_0(b_T k_T)\, \big[f(k_T^2, z)\big]_{+,+} = \Big[\int_0^\infty \mathrm{d}(k_T^2)\big[J_0(b_T k_T) - 1\big] f(k_T^2, z)\Big]_+$$

only z

$$\Downarrow$$

$$d^{(1)}_{1\,Q/Q}(z, b_T, \mu, \zeta) = \frac{\alpha_s C_F}{4\pi}\frac{1}{z^2}\Big\{\delta(1-z)\Big[-2L_b \ln\frac{\zeta}{m^2} + 4\ln^2\frac{\mu}{m} + 6\ln\frac{\mu}{m} - L_y^2 + 4 - \frac{\pi^2}{6}\Big] - 4(1+L_y)\,\mathcal{L}_0(1-z) - 8\mathcal{L}_1(1-z) + \tilde{\mathcal{R}}(z, b_T m)\Big\}$$

$$\tilde{\mathcal{R}}(z, y) \equiv y\int_0^\infty \mathrm{d}t\, J_1(ty)\,\mathcal{R}(z, t^2) = \frac{4}{\bar z}\Big[1 + L_y + (1+z^2)K_0\Big(\frac{y\bar z}{z}\Big) - y\bar z K_1\Big(\frac{y\bar z}{z}\Big) + 2\ln\bar z\Big] = \mathcal{O}(\bar z^0)$$

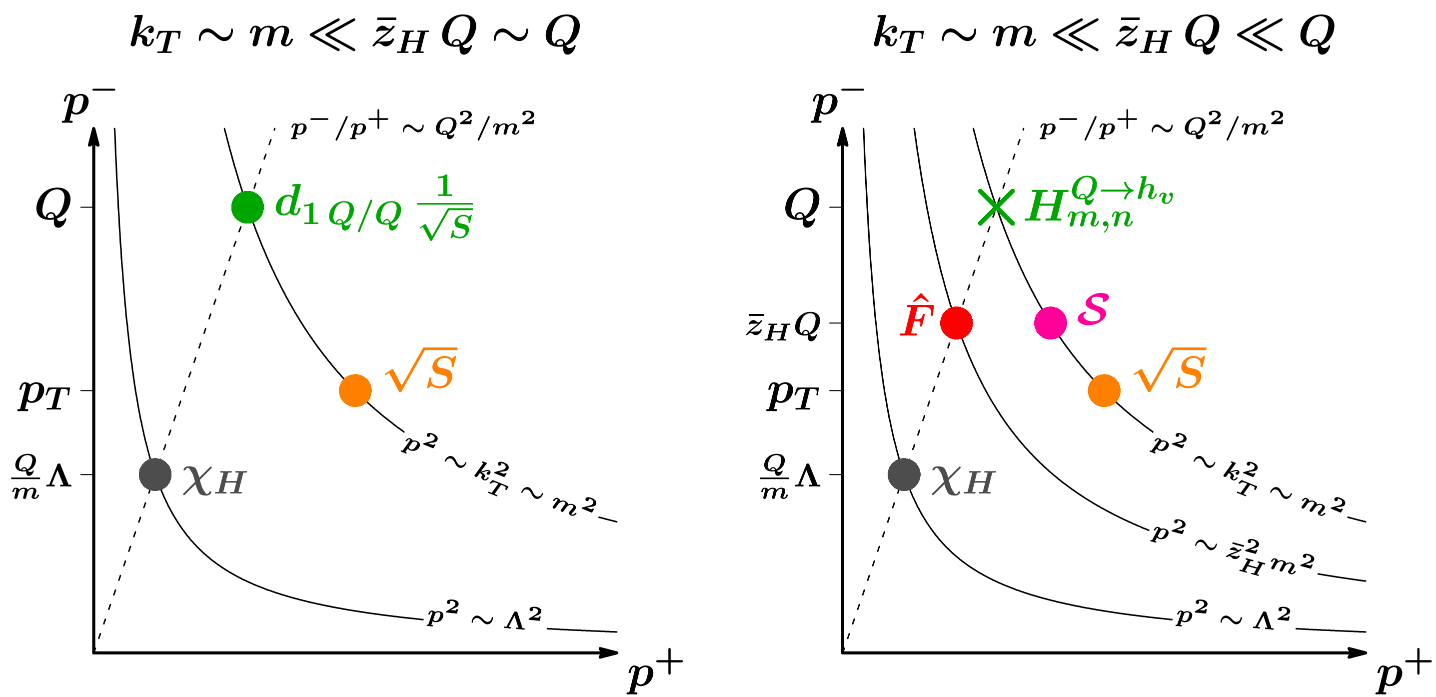
$$L_y \equiv \frac{m^2 b_T^2}{(2e^{-\gamma_E})^2} \qquad L_b \equiv \frac{\mu^2 b_T^2}{(2e^{-\gamma_E})^2}$$

Full agreement with Dai, Kim Leibovich, 2310.19207, who used a completely orthogonal organization of k_T singularities. ✓

Why so simple as $z \to 1$? Why only logarithms of $m\, b_T$?

Joint resummation for massive quark fragmentation

[Joint resummation: Laenen, Sterman, Vogelsang ‘01; Lustermans, Waalewijn, Zeune ‘16; Kang, Samanta, Shao, Zeng ‘22]



$$d_{1\,Q/Q}\Big(1-\frac{k^-}{\omega}, b_T, \mu, \omega^2\Big) = H_{m,n}^{Q\to h_v}\Big(m, \mu, \frac{\omega}{\nu}\Big)\,\mathcal{S}(k^-, b_T, m, \mu, \nu) \otimes_{k^-} \hat{F}\big(k^-, \bar{n}\cdot v\,\mu\big)$$
$$\times \sqrt{S}(b_T, m, \mu, \nu) + \mathcal{O}\big[(1-z_H)^0\big]$$

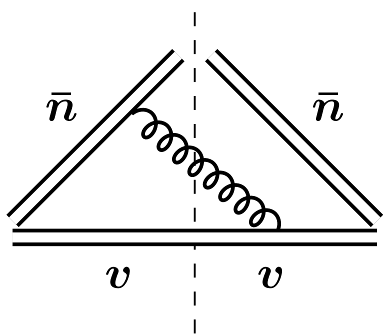
✓

[Full two-loop RHS known: Becher, Neubert ‘06; Pietrulewicz, Samitz, Spiering, Tackmann ‘17]

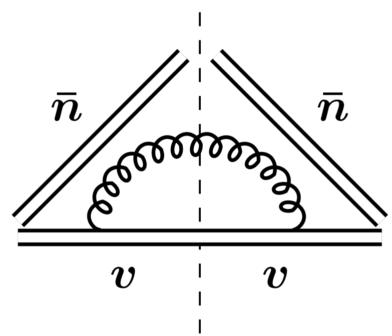
Large-mass limit and renormalization of HQET TMD FFs

$$\sum_H D_{1\,H/Q}(z, b_T, \mu, \zeta) = \delta(1-z)\, C_m(m, \mu, \zeta)\, \chi_1\Big(b_T, \mu, \frac{\sqrt{\zeta}}{m}\Big) + \mathcal{O}\Big(\frac{1}{m}\Big)$$

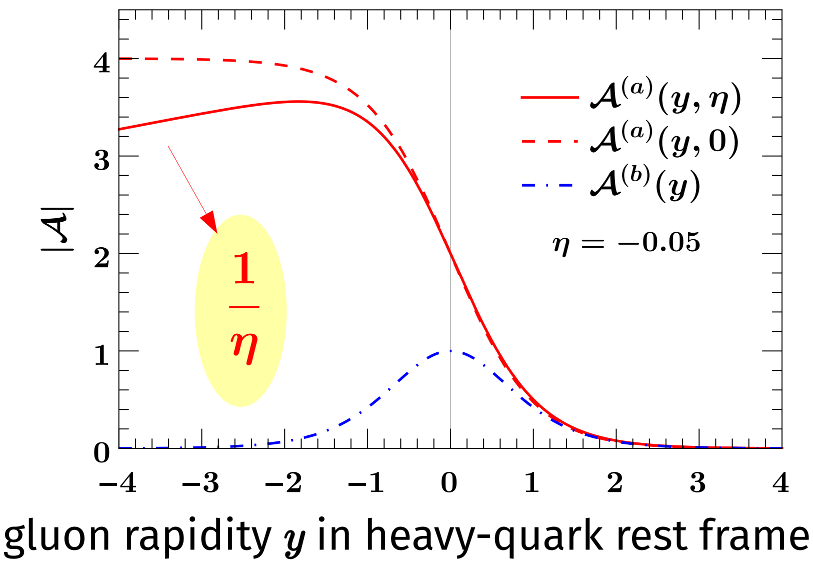
$$\chi_1^{\text{bare}}(b_T, \epsilon, \eta, \bar{n}\cdot v) = \frac{1}{N_c}\,\text{Tr}\big\langle 0\big| W_\eta^\dagger(b_\perp)\, Y_v(b_\perp)\, Y_v^\dagger(0)\, W_\eta(0)\big|0\big\rangle$$



(a)



(b)



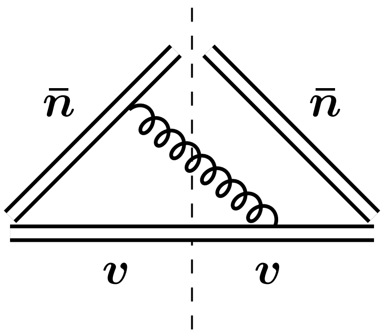
$$\chi_1(b_T, \mu, \rho) = \lim_{\epsilon\to 0} Z_{\chi_1}^{-1}(\mu, \rho, \epsilon) \lim_{\eta\to 0}\Big[\chi_1^{\text{bare}}(b_T, \epsilon, \eta, \rho)\sqrt{S^{(n_\ell)}(b_T, \epsilon, \eta, \nu)}\Big]$$

$\rho \equiv \bar{n}\cdot v$

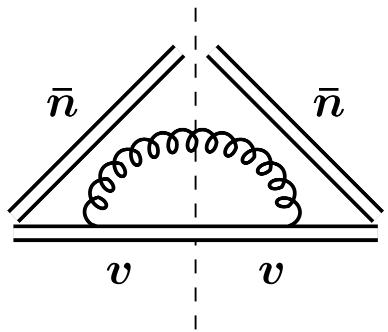
$$= 1 + \frac{\alpha_s C_F}{4\pi}(-L_b)\big(4\ln\rho - 2\big) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\Lambda_{\text{QCD}}^2 b_T^2)$$

$$\sum_H D_{1\,H/Q}(z, b_T, \mu, \zeta) = \delta(1-z)\, C_m(m, \mu, \zeta)\, \chi_1\Big(b_T, \mu, \frac{\sqrt{\zeta}}{m}\Big) + \mathcal{O}\Big(\frac{1}{m}\Big)$$

$$\chi_1^{\text{bare}}(b_T, \epsilon, \eta, \bar{n}\cdot v) = \frac{1}{N_c}\, \text{Tr}\big\langle 0 \big| W_\eta^\dagger(b_\perp)\, Y_v(b_\perp)\, Y_v^\dagger(0)\, W_\eta(0) \big| 0 \big\rangle$$



(a)



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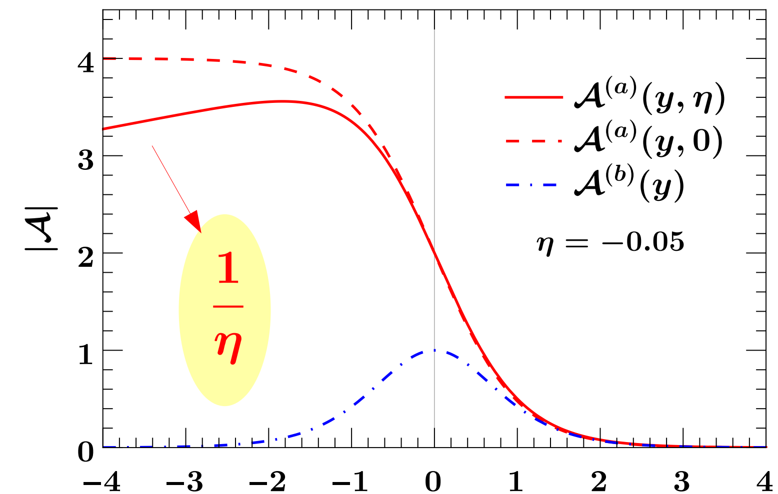


$$\chi_1(b_T, \mu, \rho) = \lim_{\epsilon\to 0} Z_{\chi_1}^{-1}(\mu, \rho, \epsilon) \lim_{\eta \to 0}\Big[\chi_1^{\text{bare}}(b_T, \epsilon, \eta, \rho)\sqrt{S^{(n_\ell)}}(b_T, \epsilon, \eta, \nu)\Big]$$

$\rho \equiv \bar{n}\cdot v$

$$= 1 + \frac{\alpha_s C_F}{4\pi}(-L_b)\big(4\ln\rho - 2\big) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\Lambda_{\text{QCD}}^2 b_T^2)$$

Large-mass limit and renormalization of HQET TMD FFs

$$\sum_H D_{1\,H/Q}(z, b_T, \mu, \zeta) = \delta(1-z)\, C_m(m, \mu, \zeta)\, \chi_1\Big(b_T, \mu, \frac{\sqrt{\zeta}}{m}\Big) + \mathcal{O}\Big(\frac{1}{m}\Big) \quad ✓$$

$$\chi_1^{\mathrm{bare}}(b_T, \epsilon, \eta, \bar{n}\cdot v) = \frac{1}{N_c} \mathrm{Tr}\langle 0 | W_\eta^\dagger(b_\perp)\, Y_v(b_\perp)\, Y_v^\dagger(0)\, W_\eta(0) | 0 \rangle$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln \chi_1(b_T, \mu, \rho) = \gamma_{\chi_1}\big[\alpha_s(\mu), \rho\big] = \gamma_\mu^q(\mu, \zeta) - \gamma_{C_m}(\mu, m, \zeta)$$

TMD μ anom. dim.

$$\rho \frac{\mathrm{d}}{\mathrm{d}\rho} \ln \chi_1(b_T, \mu, \rho) = \gamma_\zeta^{(n_\ell)}(b_T, \mu)$$

CS kernel (n_ℓ light quarks)

$$\rho \frac{\mathrm{d}}{\rho} \gamma_{\chi_1}\big[\alpha_s(\mu), \rho\big] = \mu \frac{\mathrm{d}}{\mu} \gamma_\zeta^{(n_\ell)}(\mu, b_T) = -2\Gamma_{\mathrm{cusp}}\big[\alpha_s(\mu)\big]$$

$$\sqrt{\zeta} = \bar{n}\cdot P_H / z \overset{!}{=} m\, \bar{n}\cdot v = m\,\rho$$

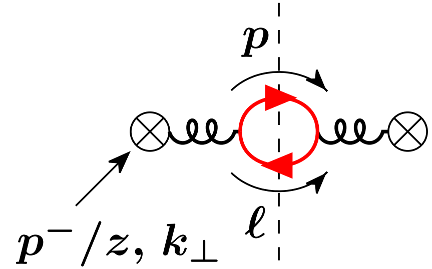
$$\chi_1(b_T, \mu, \rho) = \lim_{\epsilon\to 0} Z_{\chi_1}^{-1}(\mu, \rho, \epsilon) \lim_{\eta \to 0}\Big[\chi_1^{\mathrm{bare}}(b_T, \epsilon, \eta, \rho) \sqrt{S^{(n_\ell)}(b_T, \epsilon, \eta, \nu)}\Big] \quad ✓$$

$$= 1 + \frac{\alpha_s C_F}{4\pi}(-L_b)\big(4 \ln\rho - 2\big) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2 b_T^2)$$

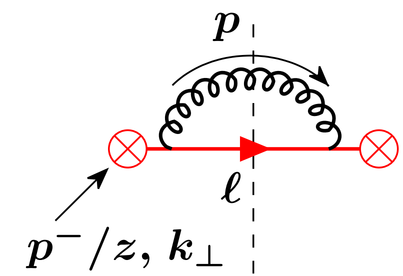
$\rho \equiv \bar{n}\cdot v$

- Nonvalence channels & small-mass limit ✓

$$d_{1\,Q/i}(z, b_T, \mu, \zeta) = \frac{1}{z^2} \sum_j \mathcal{J}_{j/i}(z, b_T, \mu, \zeta) \otimes_z d_{Q/j}(z, \mu) + \mathcal{O}(m^2 b_T^2)$$



$$\mathcal{J}_{k/i}(z, b_T, m, \mu, \zeta) = \frac{1}{z^2} \sum_j \mathcal{J}_{j/i}(z, b_T, \mu, \zeta) \otimes_z \mathcal{M}_{k/j,T}(z, m, \mu) + \mathcal{O}(m^2 b_T^2)$$



[NLO massless matching coefficients: Echevarria, Idilbi, Scimemi, 1402.0869]

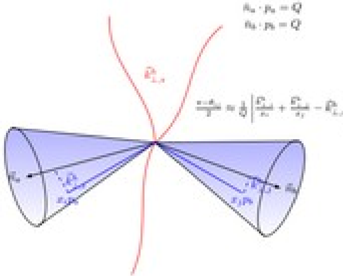
- Full mass dependence in NNLL Energy-Energy-Correlator in the back-to-back limit:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}z_\chi} = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(1)}_{ee\to q\bar{q}}}{\mathrm{d}z_\chi} + 2\mathcal{H}^{(0)}_{ee\to Q\bar{Q}}\frac{\alpha_s C_F}{4\pi}\left\{8(2\ln\rho - 1)\,\mathcal{L}_0(\bar{z}_\chi) + 8\left(2\ln^2\rho + \ln\rho + \frac{2\pi^2}{3} - 2\right)\delta(\bar{z}_\chi)\right.$$
$$\left. + 4\rho^2\frac{\pi(-3 - 18x + x^2) + \sqrt{x}(9 + x - 9x^2 - x^3) - \sqrt{x}(1 + 24x + 9x^2 + 2x^3)\ln x}{\sqrt{x}(1+x)^4}\right\}$$
$$+ \mathcal{O}(\alpha_s^2) + \bar{z}^0_\chi + \mathcal{O}\left(\frac{1}{\rho}\right)$$

Light-quark EEC [see talk by G. Ferrera on Tuesday!]

NEW

[Cf. Craft, Lee, Meçaj, Moult, 2210.09311 for NLO mass effects in the collinear limit of the EEC.]



1. Transverse Momentum-Dependent Heavy-Quark Fragmentation at Next-to-Leading Order

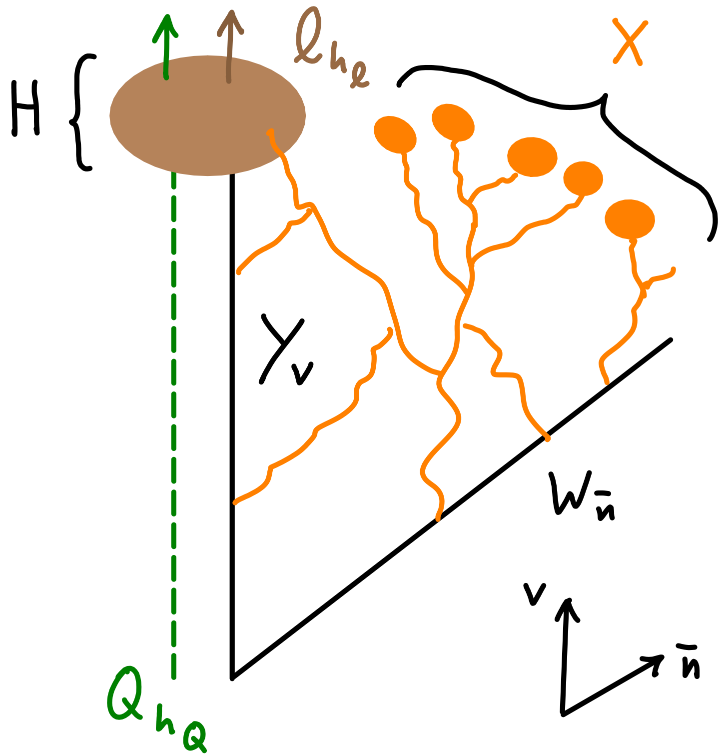
2. How Many Structure Functions Characterize Polarized Heavy Quark Fragmentation in the Transverse Plane?

Work in progress with Rebecca von Kuk (DESY), Kyle Lee (MIT), Zhiquan Sun (MIT)

TMD light-spin density matrix

$$\rho_{\ell,h_\ell h'_\ell}(b_\perp, z^\mu) \equiv \frac{1}{N_c}\,\mathrm{Tr}\sum_X\!\!\!\!\!\!\int \langle 0|W^\dagger(b_\perp)\,Y_v(b_\perp)|s_\ell, h_\ell, f_\ell; X\rangle\langle s_\ell, h'_\ell, f_\ell; X|Y_v^\dagger(0)\,W(0)|0\rangle$$



Useful unit vector choices:

$$x^\mu = \frac{b_\perp^\mu}{b_T}$$

$$z^\mu = v^\mu - \frac{\bar{n}^\mu}{\bar{n}\cdot v}$$

$$y^\mu = x_\nu \epsilon_\perp^{\mu\nu}$$

Building a basis

$$\rho_{\ell,h_\ell h'_\ell}(b_\perp,z) \equiv \frac{1}{N_c}\,\mathrm{Tr}\sum\!\!\!\!\!\!\!\int_X \langle 0|W^\dagger(b_\perp)\,Y_v(b_\perp)|s_\ell,h_\ell,f_\ell;X\rangle\langle s_\ell,h'_\ell,f_\ell;X|Y_v^\dagger(0)\,W(0)|0\rangle$$

- Hermiticity: $\rho_\ell^\dagger(b_\perp,z) = \rho_\ell(-b_\perp,z)$
- Parity: $\rho_\ell(b_\perp,z) = \rho_\ell(-b_\perp,-z)$
- Rotations: $\rho_\ell(R_{\vec\alpha}b_\perp,R_{\vec\alpha}z) = e^{+\mathrm{i}\alpha\cdot\Sigma_\ell}\rho_\ell(b_\perp,z)e^{-\mathrm{i}\alpha\cdot\Sigma_\ell}$

$$x^\mu = \frac{b_\perp^\mu}{b_T}$$

$$z^\mu = v^\mu - \frac{\bar n^\mu}{\bar n\cdot v}$$

$$y^\mu = x_\nu\epsilon_\perp^{\mu\nu}$$

$$\Sigma_\ell^{\mu_1\cdots\mu_N} \equiv \left\{\Sigma_\ell^{\mu_1},\ldots,\Sigma_\ell^{\mu_N}\right\} - \text{Gram-Schmidt} \qquad N \le 2s_\ell$$

NEW

$$\Rightarrow \quad \rho_\ell(b_\perp,z) = \sum_N\sum_n \chi_{1,\ell}^{(N,n)}(b_T)\,\Sigma^{\mu_1,\ldots,\mu_N} \times \begin{cases} x^{\mu_1}\ldots x^{\mu_n}z^{\mu_{n+1}\cdots z^{\mu_N}}, & N\in 2\mathbb{Z} \\ \mathrm{i}x^{\mu_1}\ldots x^{\mu_{n-1}}y^{\mu_n}z^{\mu_{n+1}\cdots z^{\mu_N}}, & N\notin 2\mathbb{Z}\end{cases}$$

So how many structure functions are there?

At light spin s_ℓ, rank $N = 2s_\ell$, add only $2\lfloor s_\ell \rfloor + 1$ structure functions!

Hadron multiplet	s_ℓ	# LP TMD FFs	# $\chi_{1,\ell}^{(N,n)}$'s	# $\chi_{1,\ell}^{(N,0)}$'s ($b_T \to 0$)
Λ_b	0	8	1	1
B, B^*	$1/2$	$2+12$	2	1
Σ_b, Σ_b^*	1	$8+16$	5	2
B_1, B_2^*	$3/2$	$12+20$	8	2

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⇒ Can get complete spin ½ density matrix from unpolarized hadrons!

⇒ Nice target e.g. for an LHCb polarized hadron-in-jet measurement!

Stay tuned for final results for TMD FFs!

Summary

- Showed complete NLO results for unpolarized heavy-quark TMD FFs, with nontrivial momentum-space singularities.
- Checked vs. large-mass, small-mass, and new joint threshold/transverse momentum resummation regime
- HQET fragmentation factors feature CS-like rapidity evolution with respect to a *dimensionless* boost parameter $\rho = \sqrt{\zeta}/m = \bar{n} \cdot v$.
- Showed how many structure functions characterize all possible polarized heavy-quark TMD FFs in the HQET limit (pleasingly few!).

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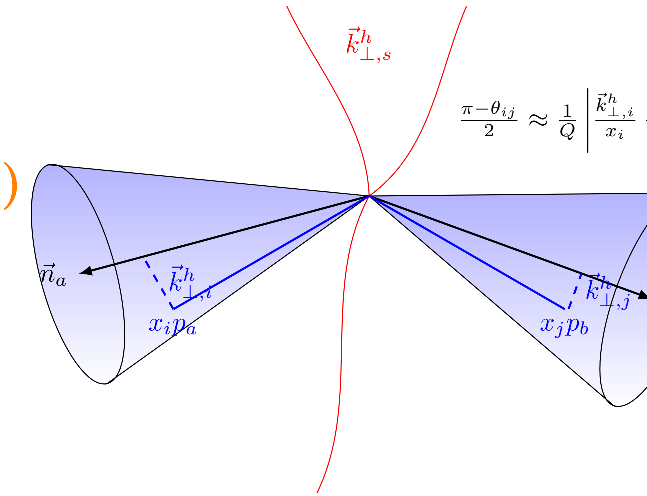
Thank you for your attention!

Backup

Application: Back-to-back EEC with heavy quarks

$$\frac{1}{\sigma_0}\frac{d\sigma}{dz_\chi} = \sum_{i,j} \mathcal{H}_{ee\to ij}(Q^2,\mu)\, Q^2 \int_0^\infty db_T\, b_T\, J_0\big(\sqrt{1-z_\chi}\, b_T Q\big)$$
$$\times\, J_i(b_T, m, \mu, Q/\nu)\, J_j(b_T, m, \mu, Q/\nu)\, S(b_T, m, \mu, \nu)$$

$$i, j = q, Q, g$$



$\Downarrow$

[See Craft, Lee, Meçaj, Moult, 2210.09311
for mass effects in the collinear limit.]

[Figure credit:
Moult, Zhu, 1801.02627]

$$\frac{1}{\sigma_0}\frac{d\sigma^{(1)}}{dz_\chi} = \frac{1}{\sigma_0}\frac{d\sigma^{(1)}_{ee\to q\bar{q}}}{dz_\chi} + 2\mathcal{H}^{(0)}_{ee\to Q\bar{Q}}\frac{\alpha_s C_F}{4\pi}\Big\{8(2\ln\rho - 1)\,\mathcal{L}_0(\bar{z}_\chi) + 8\Big(2\ln^2\rho + \ln\rho + \frac{2\pi^2}{3} - 2\Big)\,\delta(\bar{z}_\chi)$$
$$+ 4\rho^2\frac{\pi(-3-18x+x^2) + \sqrt{x}(9+x-9x^2-x^3) - \sqrt{x}(1+24x+9x^2+2x^3)\ln x}{\sqrt{x}(1+x)^4}\Big\}$$
$$+ \mathcal{O}(\alpha_s^2) + \bar{z}_\chi^0 + \mathcal{O}\Big(\frac{1}{\rho}\Big)$$

NEW

[Cf. Lepenik, Mateu, 1912.08211 for analytic SCET$_{\mathrm{I}}$ massive event shapes at one loop.]

Fun Fact: The heavy-quark EEC at nonperturbative values

Sum over heavy hadrons is built into EEC ...

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}z_\chi}\bigg|_{\text{heavy}} = \mathcal{H}_{ee\to Q\bar{Q}}(Q^2,\mu)\, C_m^2(m,\mu,Q^2) \times Q^2\int_0^\infty \mathrm{d}b_T\, b_T\, J_0\big(\sqrt{1-z_{\chi,H}}\, b_T Q\big)\left[\chi_1\left(b_T,\mu,\frac{Q}{m}\right)\right]^2$$

... which lets us complete the sum over states in the EFT! [2305.15461]

$$\chi_1(b_T) \equiv \frac{1}{N_c}\,\mathrm{Tr}\langle 0 \big| W^\dagger(b_\perp)\, Y_v(b_\perp)\, Y_v^\dagger(0)\, W(0) \big| 0\rangle$$

$\Rightarrow$ Plain Wilson line correlators, also nonperturbatively!