Meson structures through collinear and transverse momentum dependent distributions

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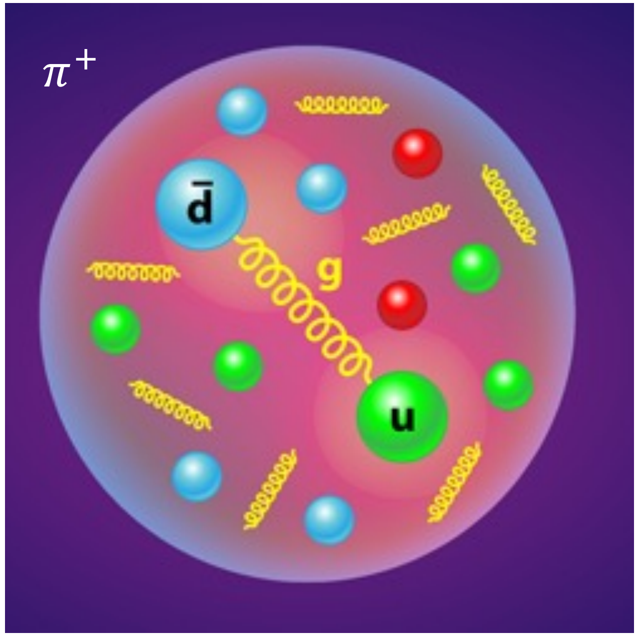
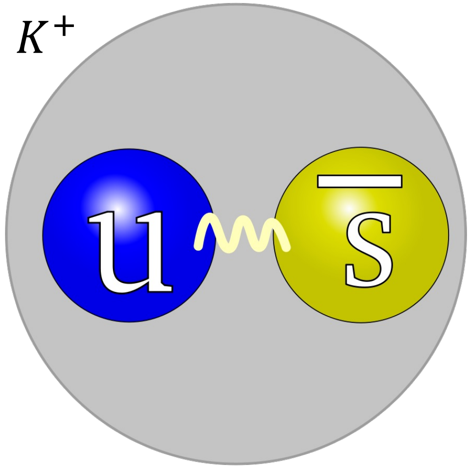
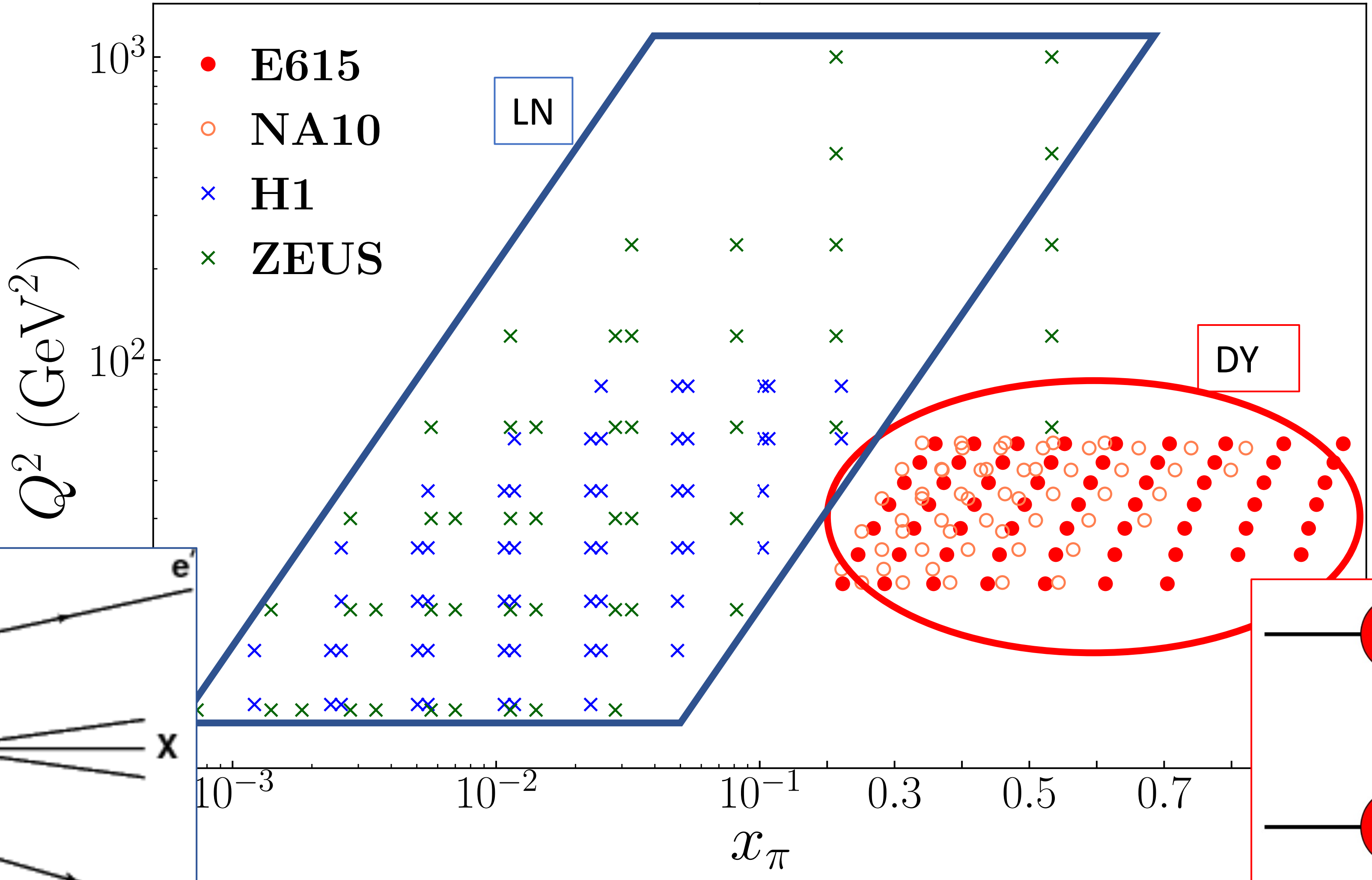
Non-proton structures - **Mesons**

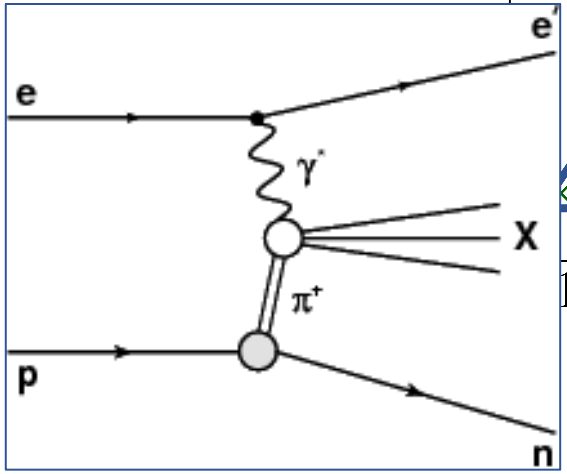


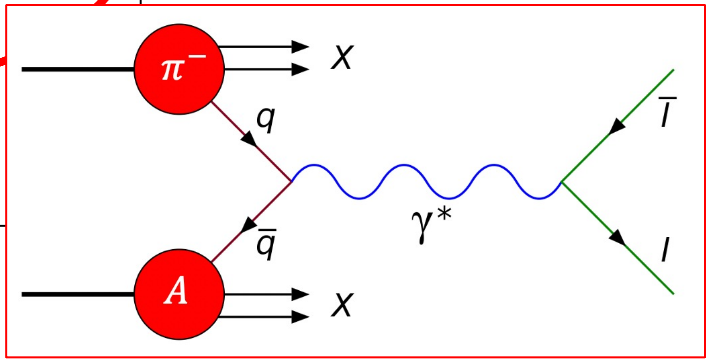
- Pion is the Goldstone boson associated with SU(2) chiral symmetry breaking
- Kaon – SU(3)
- Simultaneously a $q\bar{q}$ bound state
- Studying these structures provides another angle to probe QCD and effective confinement scales
- More available data is desperately needed



Available datasets for pion structures

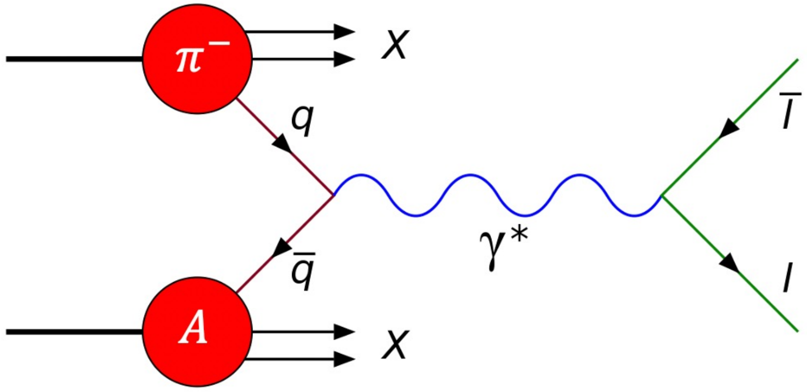




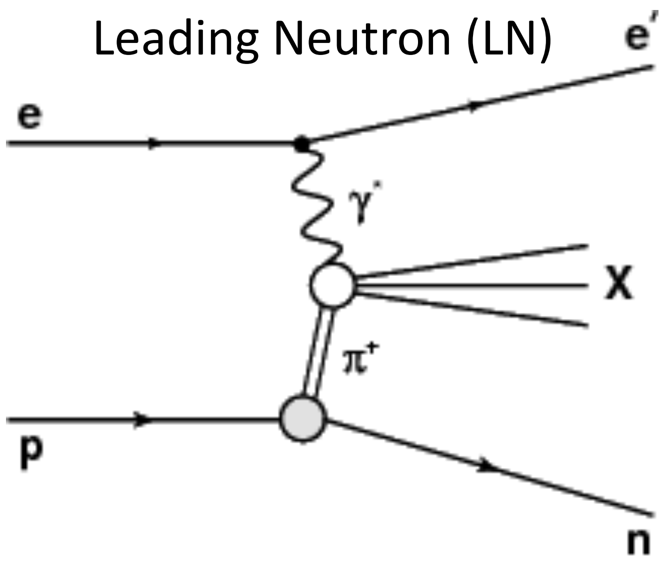


Pion PDFs in JAM

Drell-Yan (DY)



Leading Neutron (LN)

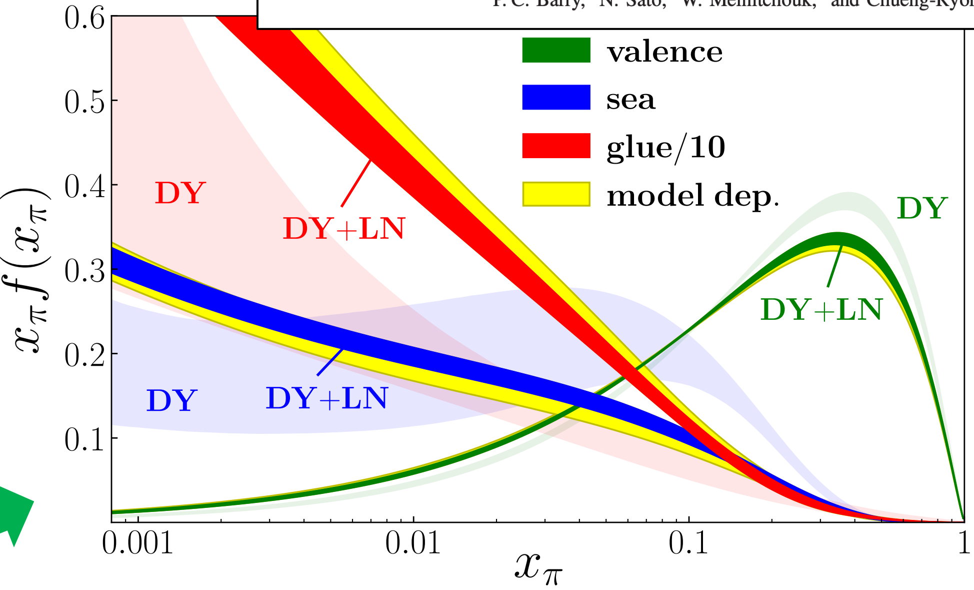


PHYSICAL REVIEW LETTERS **121,** 152001 (2018)

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

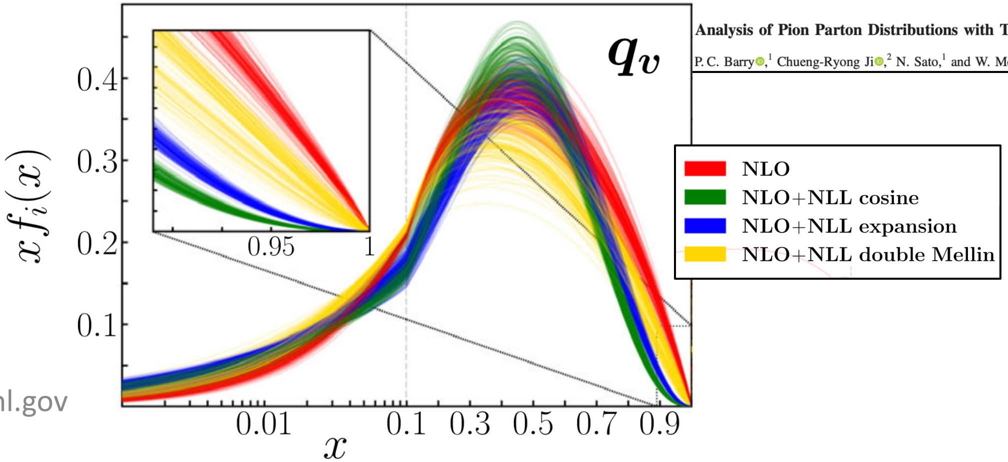


Threshold resummation in DY

PHYSICAL REVIEW LETTERS **127,** 232001 (2021)

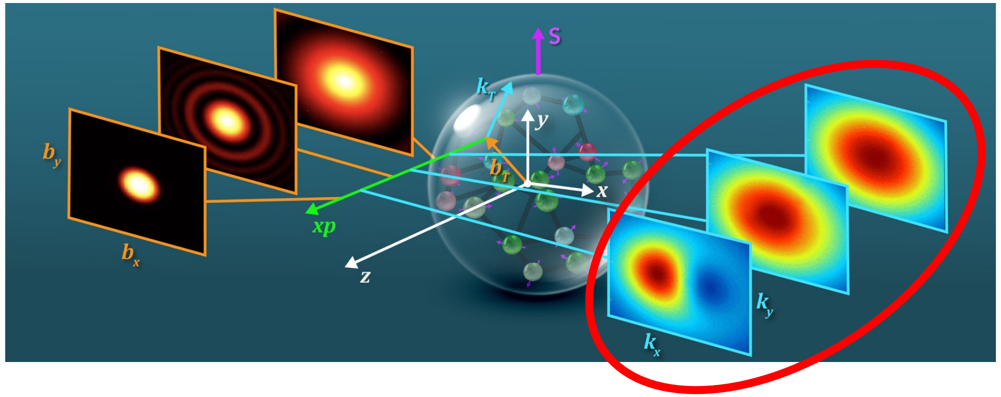
Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]



Part 1: TMDs in the Pion and proton

Unpolarized TMD PDF



$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}[\langle \mathcal{N} | \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) | \mathcal{N} \rangle]$$

$$b \equiv (b^-, 0^+, \boldsymbol{b}_T)$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- Coordinate space correlations of quark fields in hadrons can tell us about their transverse momentum dependence
- Modification needed for UV and rapidity divergences; acquire regulators: $\tilde{f}_{q/\mathcal{N}}(x, b_T) \to \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$

TMD PDF within the b_* prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Low-b_T: perturbative
high-b_T: non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)$$
$$\times \exp\left\{ -g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln\frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q) \right\}$$

Relates the TMD at small-b_T to the **collinear** PDF
$\Rightarrow$ TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative TMD structure of the hadron $\mathcal{N}(A)$
g_K: universal non-perturbative Collins-Soper kernel – same in all hadrons

- In this analysis, we use the MAP collaboration's parametrizations

Controls the perturbative evolution of the TMD

Factorization for low-q_T Drell-Yan

- Cross section has hard part and two functions that describe structure of beam and target
- So called "W"-term, optimized at low-q_T

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\tau \mathrm{d}Y \mathrm{d}q_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2,\mu) \int \mathrm{d}^2 b_T \, e^{i b_T \cdot q_T}$$

$$\times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2) \, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2) + \mathcal{O}\left(\frac{q_T}{Q}\right)$$

- Because of nuclear background, we **simultaneously** fit: π *and* p TMDs, π collinear PDFs, CS kernel, and nuclear TMD parameter

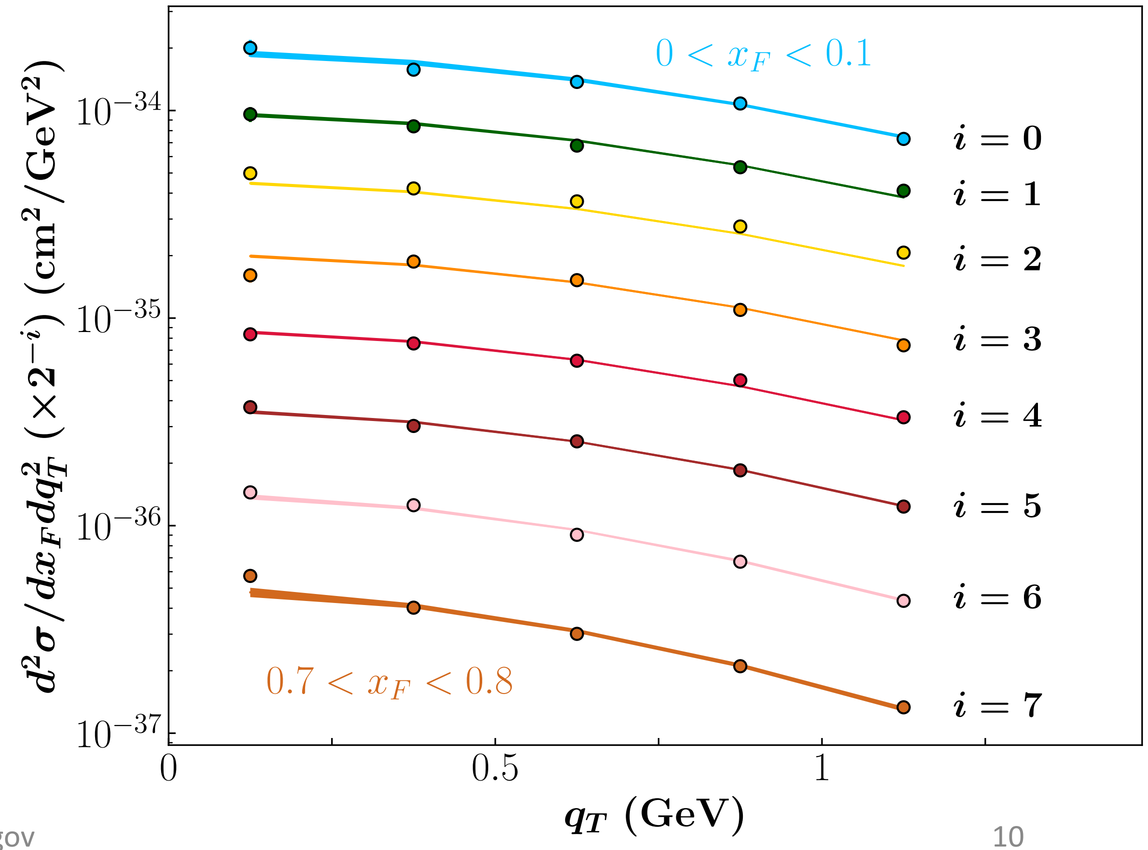
Two brief notes about E615 πA DY data

1. Provide $\frac{d\sigma}{dx_F d\sqrt{\tau}}$ and $\frac{d\sigma}{dx_F dq_T}$, but no way to treat correlations within datasets
   - **Equate** the fitted normalization parameter because the projections of each cross section come from the same events
2. We use NLO theory in the collinear observable
   - **Disagreement** in the analysis when we use NLO + NLL in collinear DY
   - **No** large-x **threshold corrections** in the q_T-dependent OPE to account for adjusted PDF

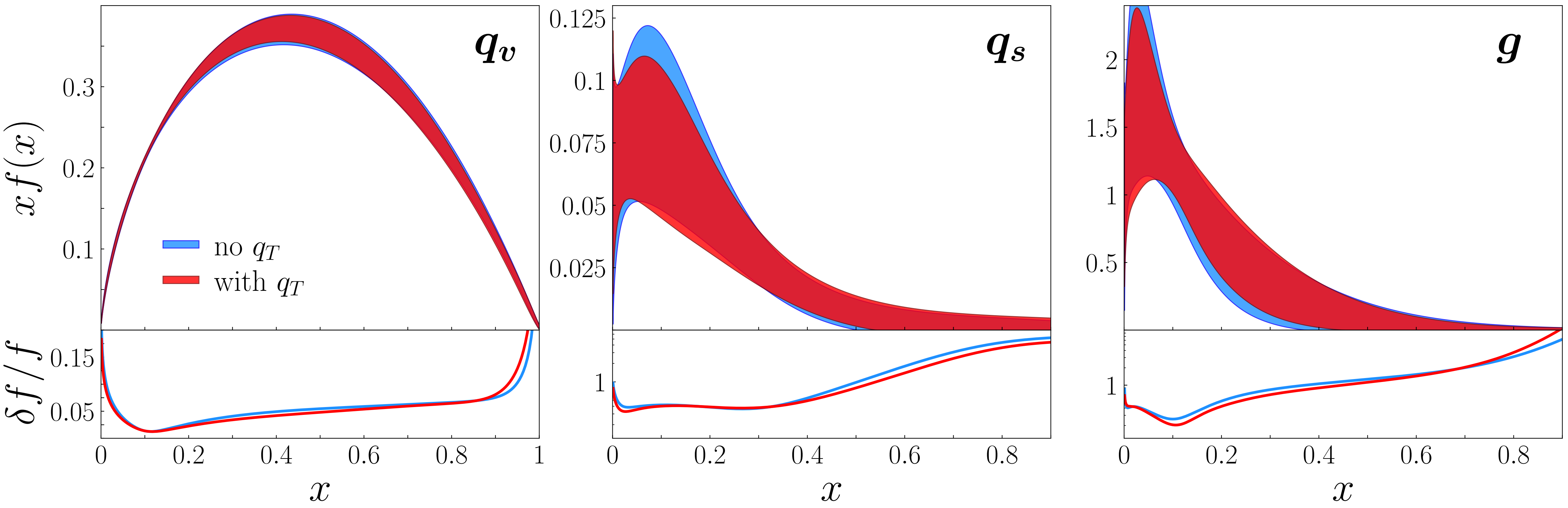
Data and theory agreement

- Fit both pA and πA DY data and achieve good agreement to both

Process	Experiment	$\sqrt{s}$ (GeV)	χ^2/N	Z-score
TMD				
q_T-dep. pA DY	E288 [90]	19.4	1.07	0.34
$pA \to \mu^+\mu^- X$	E288 [90]	23.8	0.99	0.05
	E288 [90]	24.7	0.82	0.99
	E605 [91]	38.8	1.22	1.03
	E772 [92]	38.8	2.54	5.64
(Fe/Be)	E866 [93]	38.8	1.10	0.36
(W/Be)	E866 [93]	38.8	0.96	0.15
q_T-dep. πA DY	E615 [94]	21.8	1.45	1.85
$\pi W \to \mu^+\mu^- X$	E537 [95]	15.3	0.97	0.03
collinear				
q_T-integr. DY	E615 [94]	21.8	0.90	0.48
$\pi W \to \mu^+\mu^- X$	NA10 [96]	19.1	0.59	1.98
	NA10 [96]	23.2	0.92	0.16
leading neutron	H1 [97]	318.7	0.36	4.59
$ep \to enX$	ZEUS [98]	300.3	1.48	2.15
Total			1.12	1.86



Extracted pion PDFs

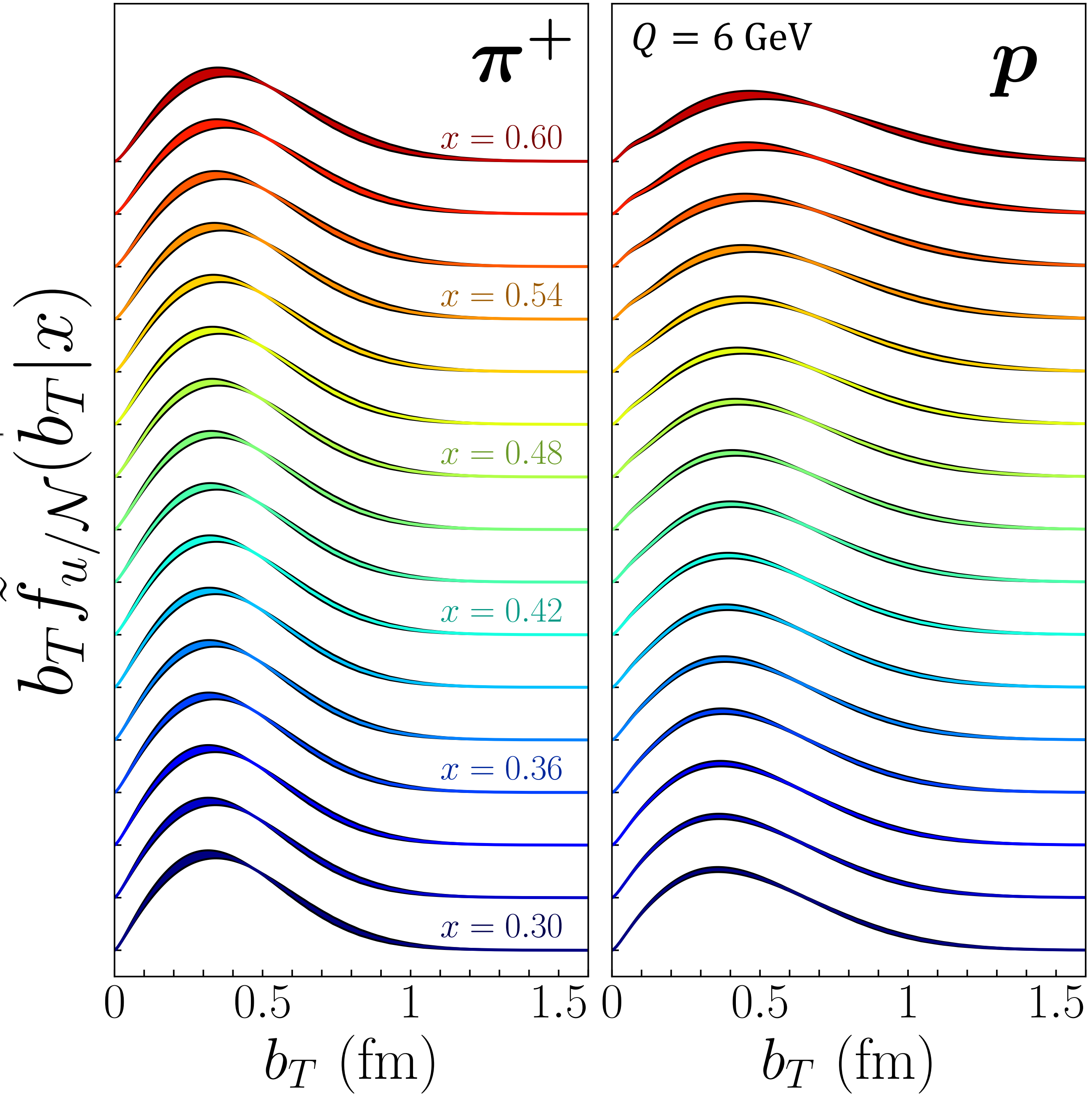


- The small-q_T data do not constrain much the PDFs

Resulting TMD PDFs of proton and pion

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$

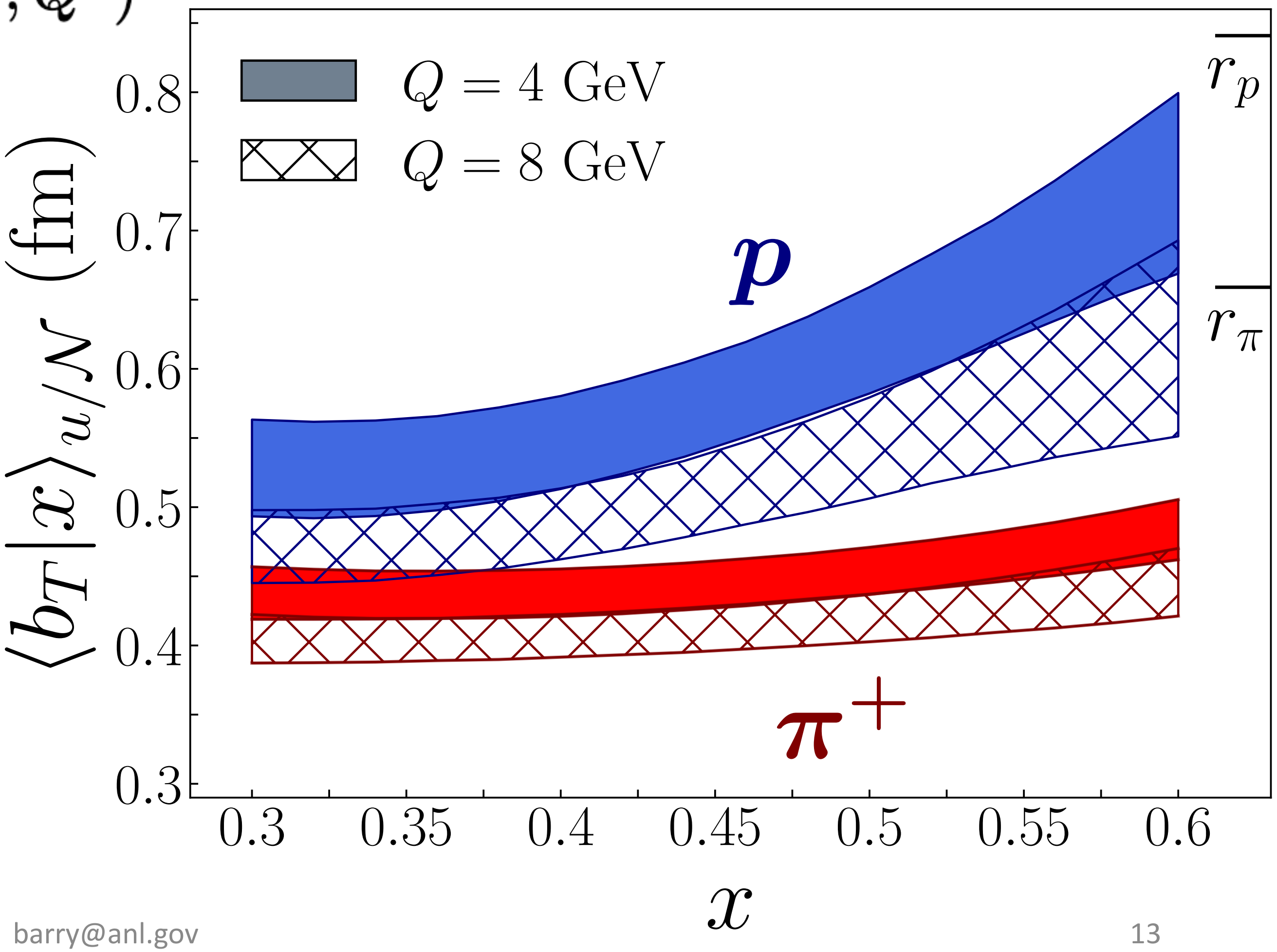
- Broadening appearing as x increases
- Up quark in pion is narrower than up quark in proton



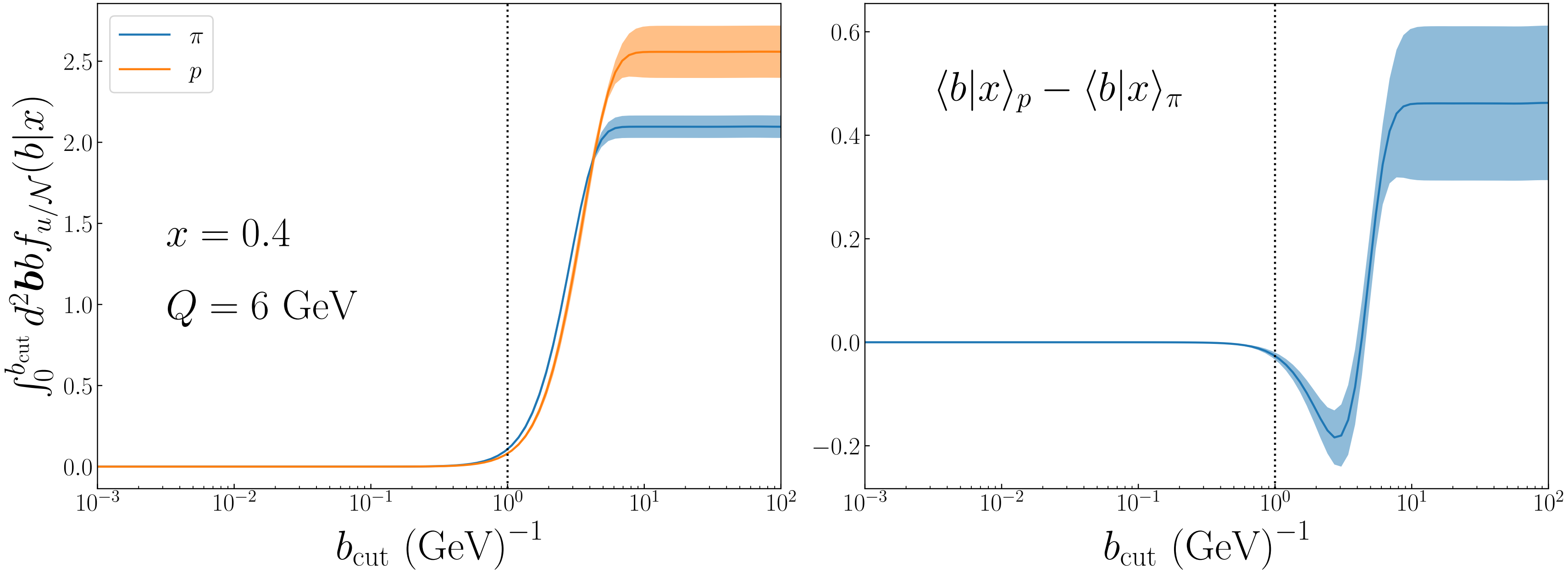
Resulting average b_T

$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int \mathrm{d}^2\boldsymbol{b}_T\, b_T\, \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2)$$

- Average transverse spatial correlation of the up quark in proton is $\sim$ 1.2 times bigger than that of pion
- Pion's $\langle b_T|x\rangle$ is $4 - 5.2\sigma$ smaller than proton in this range
- Decreases as x decreases



Emphasis on nonperturbative effects



- The $\langle b_T|x\rangle$ grows appreciably in the large-b_T region
- Saturation well beyond a perturbative scale
- Differences between proton and pion are in the nonperturbative region

Part 2: Collinear kaon PDFs

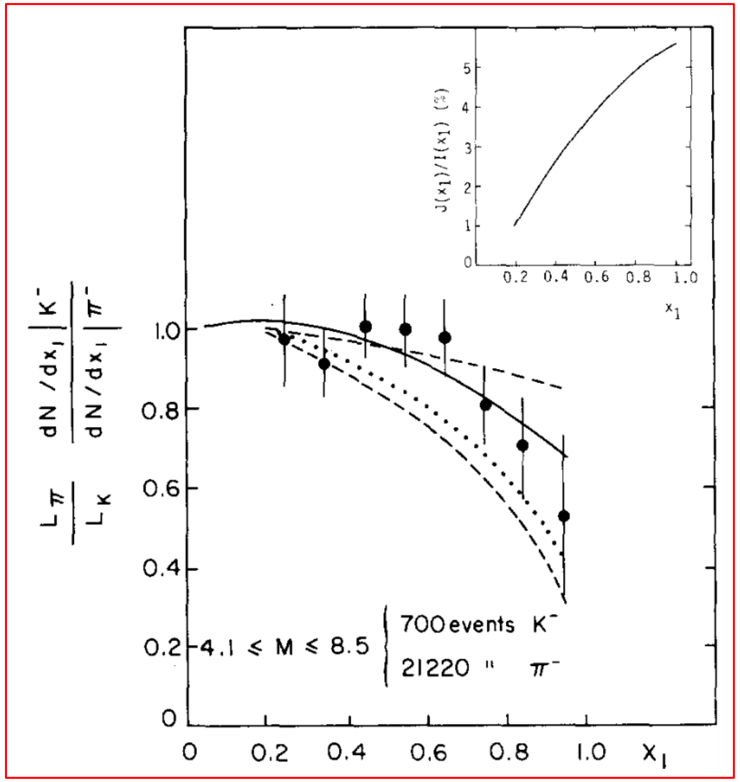
Kaon motivation (Collinear)

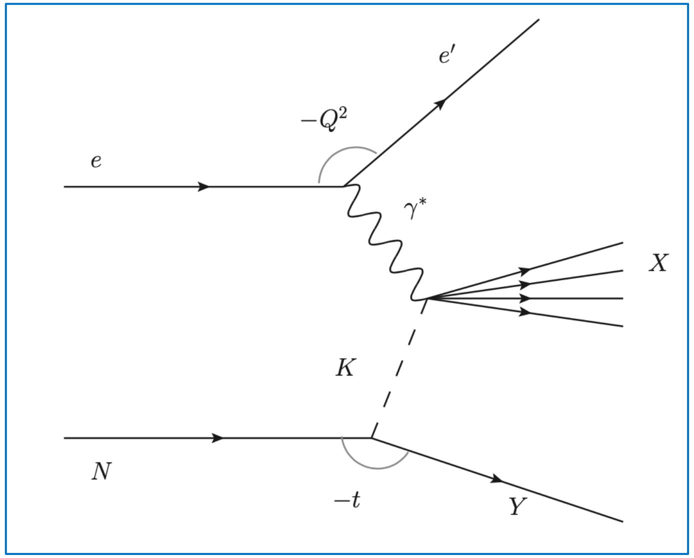
Existing measurements: Drell-Yan – NA3

- Ratios of KA to πA DY cross sections

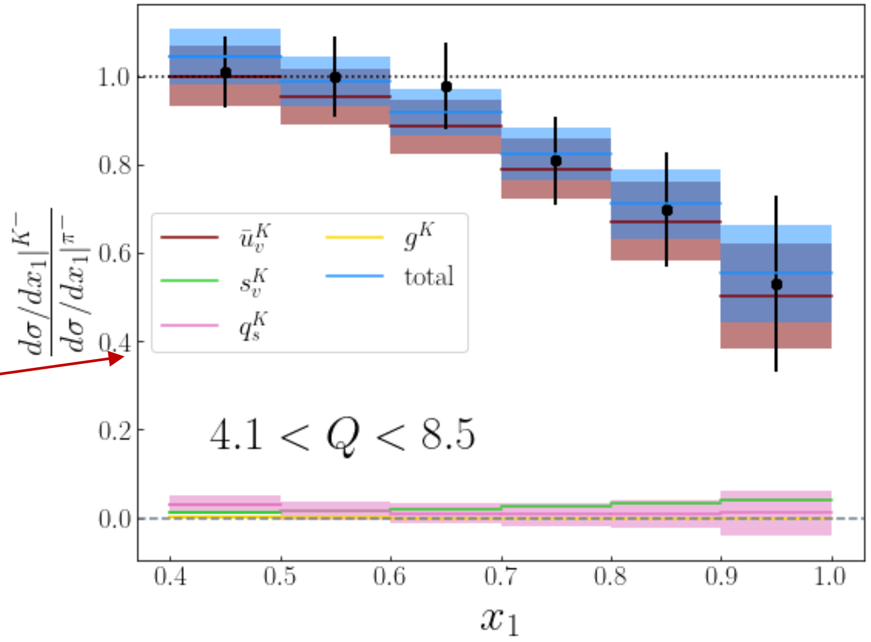
Future measurements:

- AMBER – kaon-induced DY
- JLab and EIC – tagged DIS
    - Existing $pp \to \Lambda X$ data provides constraints on the $pK\Lambda$ splitting function





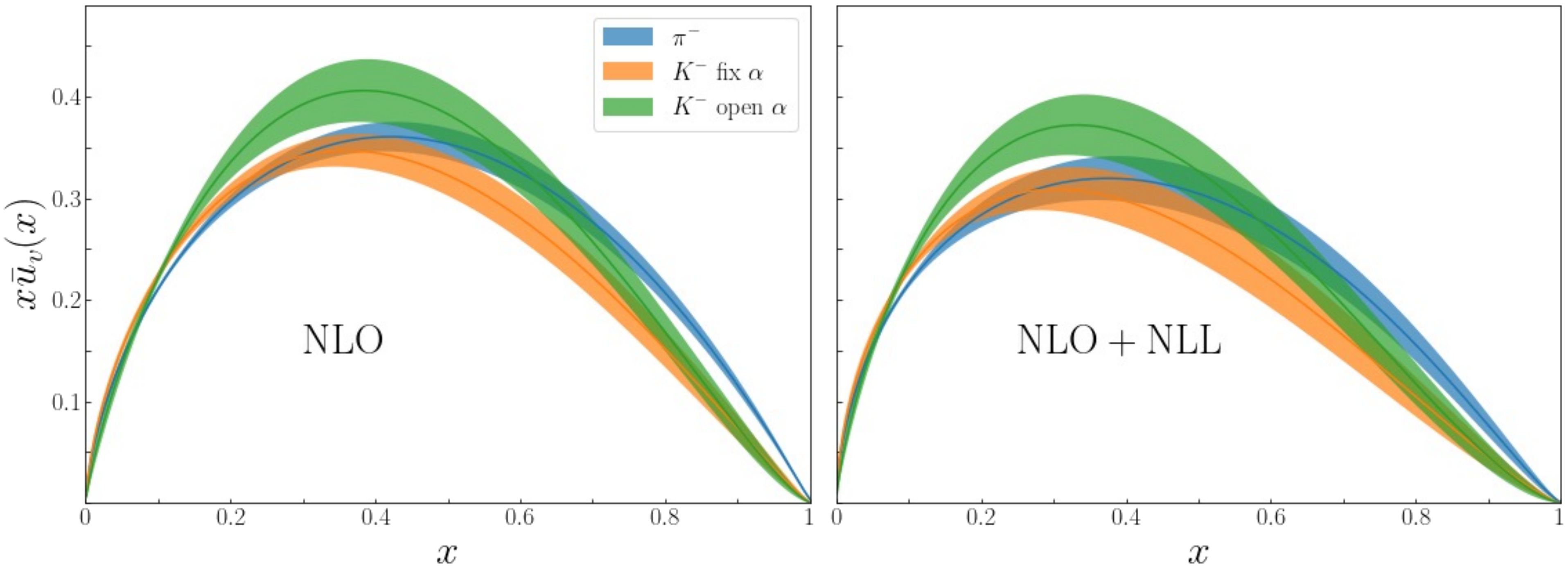
Parametrization of kaon PDFs



Almost entire signal comes from $\bar{u}_v^K$

- Parametrize $\bar{u}_v^K = N\, x^{\alpha}(1-x)^{\beta}$ with **open $\boldsymbol{\beta}$** in two ways:
  1. Fix α from the $\bar{u}_v^{\pi}$ α parameter
  2. Open α
- Equate $q_s^K = q_s^{\pi}$ and $g^K = g^{\pi}$ (no real constraints from data)
- Satisfy momentum sum rule through $s_v^K(x) = 2\bar{u}_v^{\pi}(x) - \bar{u}_v^K(x)$

Results from simultaneous K and π PDFs



- Different scenarios of fitting the kaon PDF gives large variance

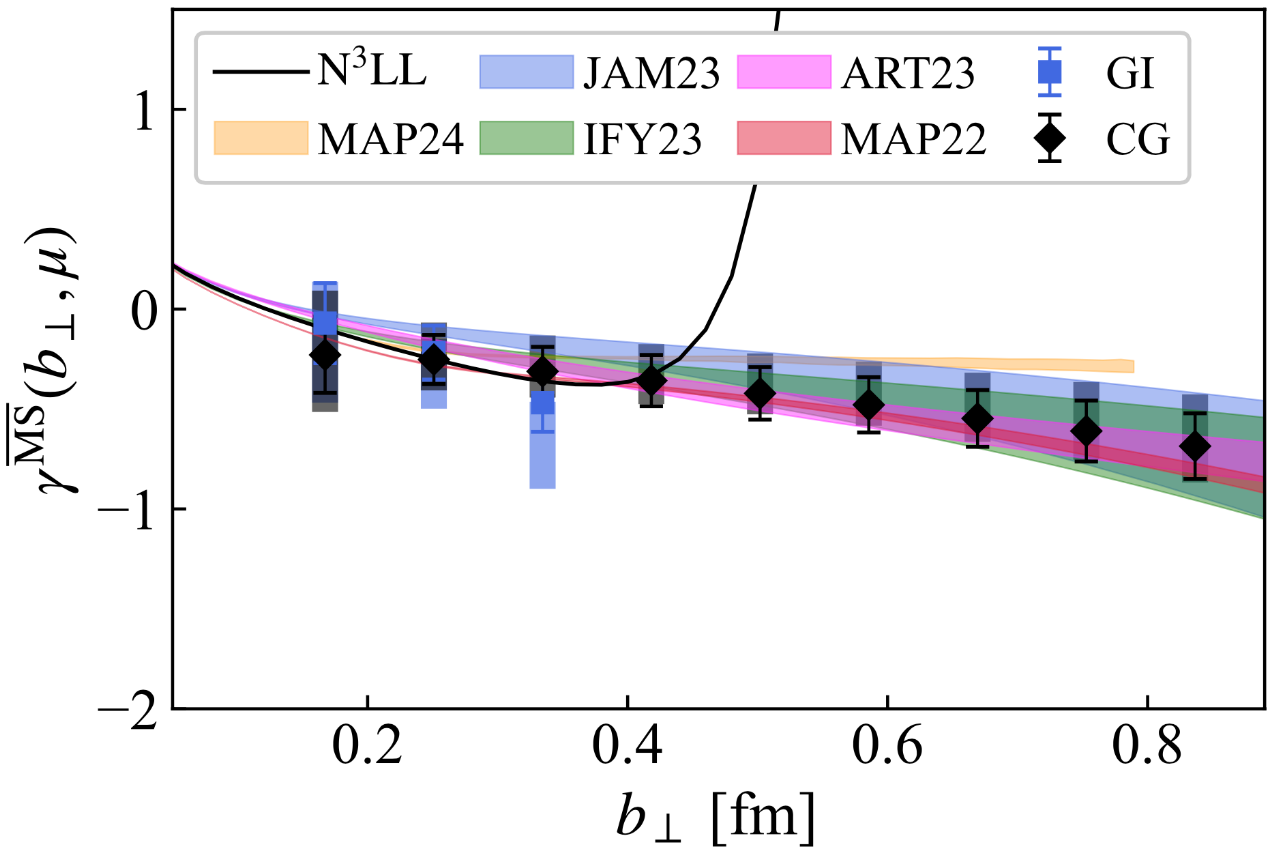
Takeaways and Outlook

- Pions and protons have significantly different **nonperturbative TMD structure** as evidenced from the low-energy data
- Fits of kaon PDFs are possible but largely unconstrained from available data
- High energy data from the TeVatron and LHC provide further constraints on the proton TMDs and potentially collinear PDFs

Backup

CS kernel

- Agreement with other phenomenological analyses, but with larger errors
- Good agreement with recent lattice data Phys. Lett. B **852**, 138617 (2024)



Courtesy of Xiang Gao

MAP parametrization

- The MAP collaboration (JHEP 10 (2022) 127) used the following form for the non-perturbative function

$$f_{1\,NP}(x, \boldsymbol{b}_T^2; \zeta, Q_0) = \frac{g_1(x)\, e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\, g_{1B}^2(x) \left[1 - g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right] e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\, g_{1C}(x)\, e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x) + \lambda^2\, g_{1B}^2(x) + \lambda_2^2\, g_{1C}(x)} \left[\frac{\zeta}{Q_0^2}\right]^{g_K(\boldsymbol{b}_T^2)/2}, \quad (38)$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

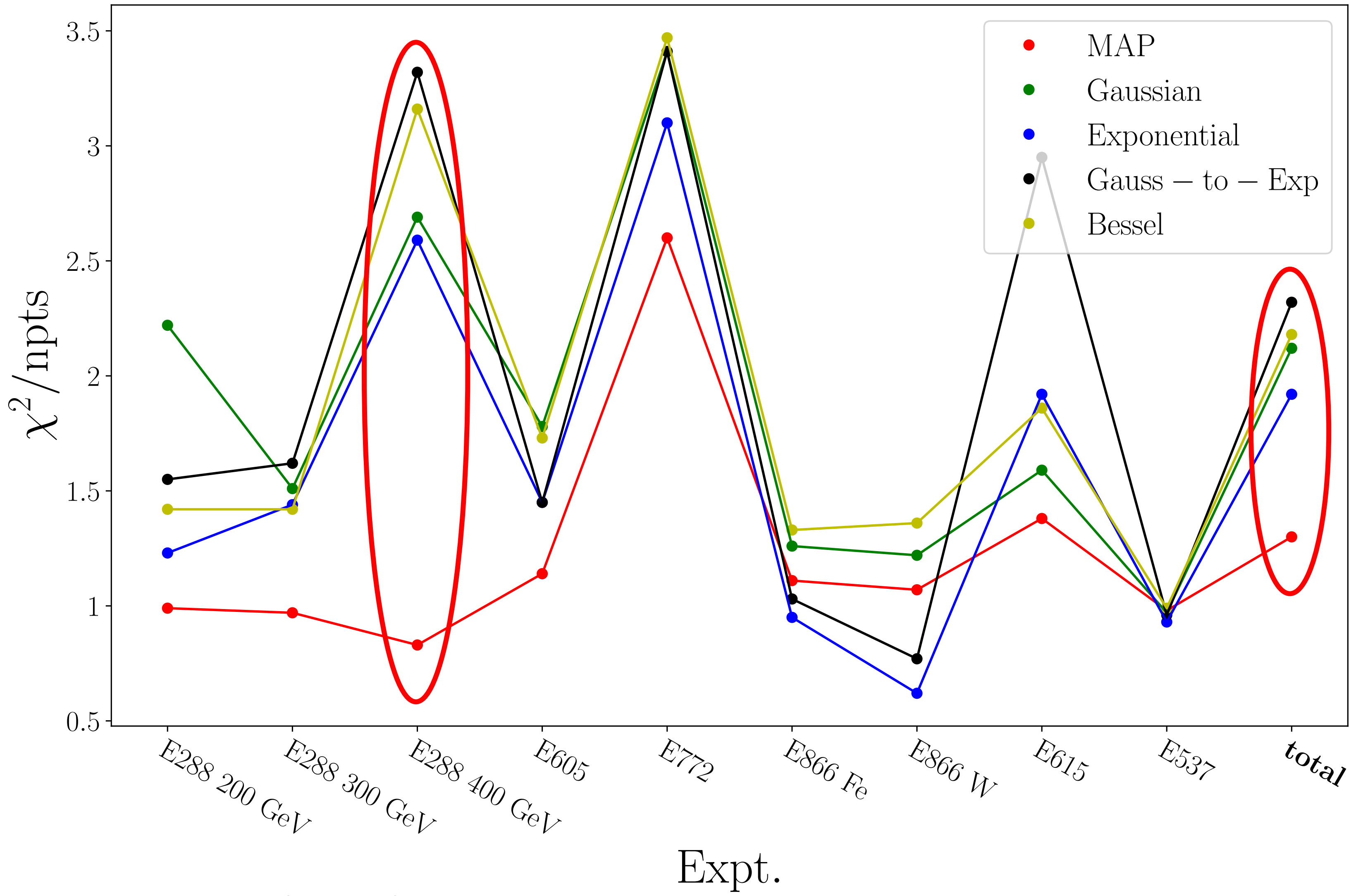
$$g_K(\boldsymbol{b}_T^2) = -g_2^2 \frac{\boldsymbol{b}_T^2}{2}$$

CS kernel

- 11 free parameters for each hadron (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)

Resulting χ^2 for each parametrization

- Tried multiple parametrizations for non-perturbative TMD structures
- MAP parametrization is able to describe better all the datasets



Nuclear TMD PDFs – working hypothesis

- We must model the nuclear TMD PDF from proton

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Each object on the right side independently obeys the CSS equation
  - **Assumption** that the bound proton and bound neutron follow TMD factorization
- Make use of isospin symmetry in that $u/p/A \leftrightarrow d/n/A$, etc.

Nuclear TMD parametrization

- Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett **129**, 242001 (2022).

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}} \left(1 - a_{\mathcal{N}} \left(A^{1/3} - 1\right)\right)$$

- Where $a_{\mathcal{N}}$ is an additional parameter to be fit

Bayesian Inference

- Minimize the χ^2 for each replica

Normalization parameter

$$\chi^2(\boldsymbol{a}, \text{data}) = \sum_e \left(\sum_i \left[\frac{d_i^e - \sum_k r_k^e \beta_{k,i}^e - t_i^e(\boldsymbol{a})/n_e}{\alpha_i^e} \right]^2 + \left(\frac{1 - n_e}{\delta n_e} \right)^2 + \sum_k \left(r_k^e \right)^2 \right)$$

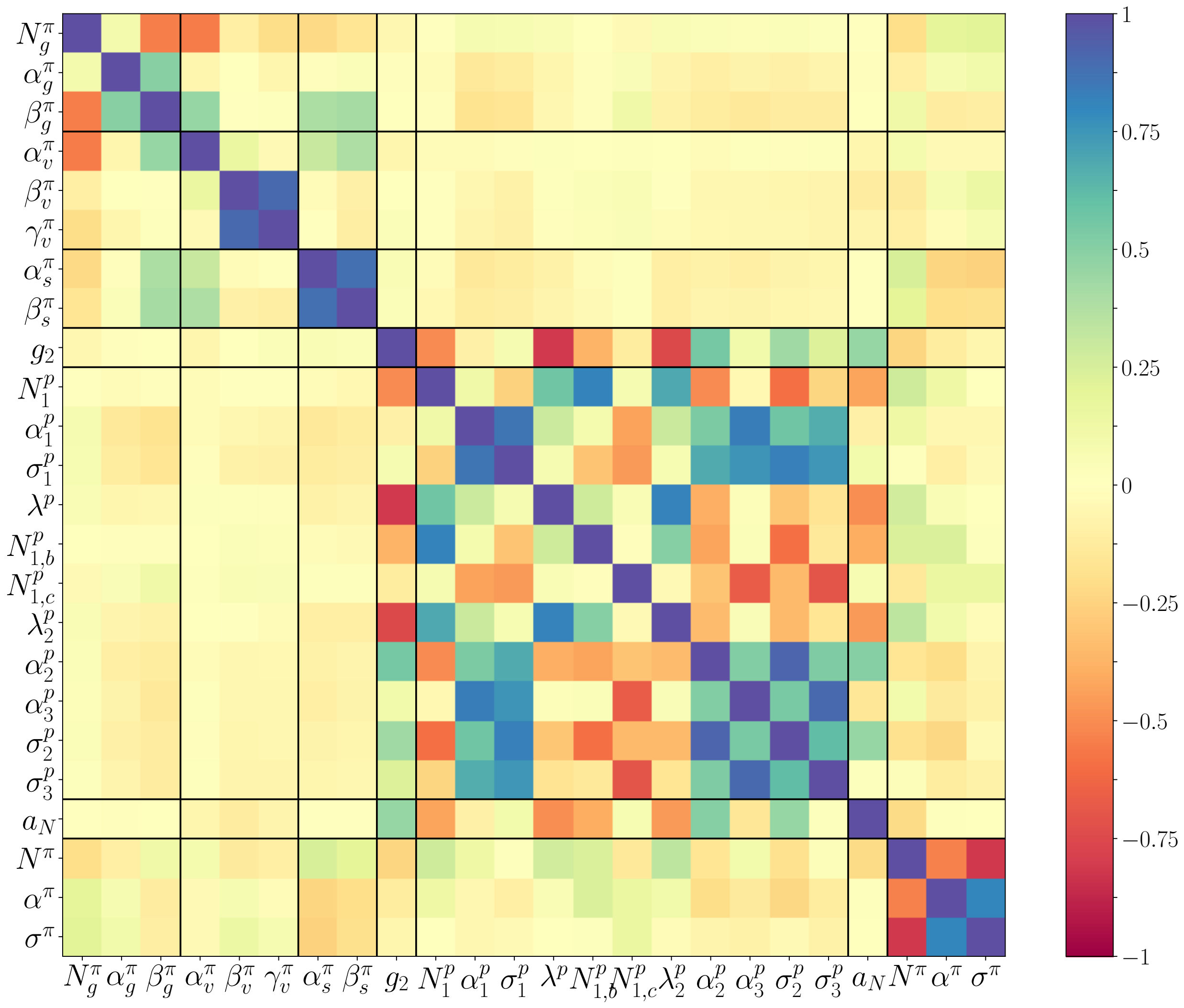
- Perform N total χ^2 minimizations and compute statistical quantities

Expectation value $$\mathrm{E}[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(\boldsymbol{a}_k),$$

Variance $$\mathrm{V}[\mathcal{O}] = \frac{1}{N} \sum_k \left[\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}] \right]^2,$$

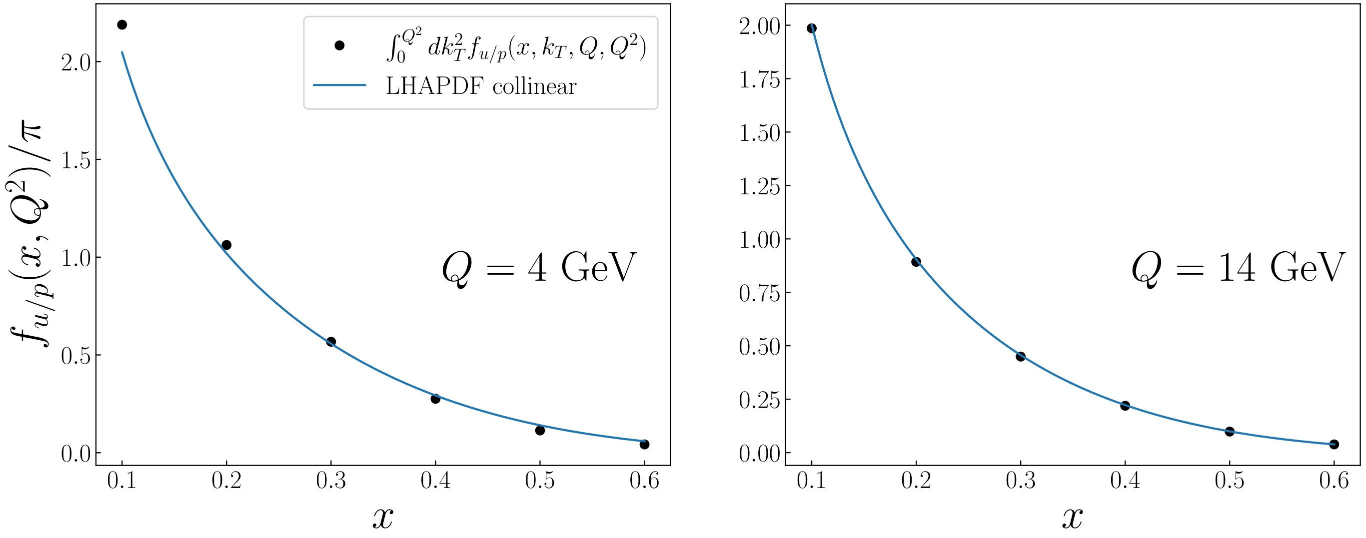
Correlations

- Level at which the distributions are correlated with each other
- Different distributions are largely correlated only within themselves



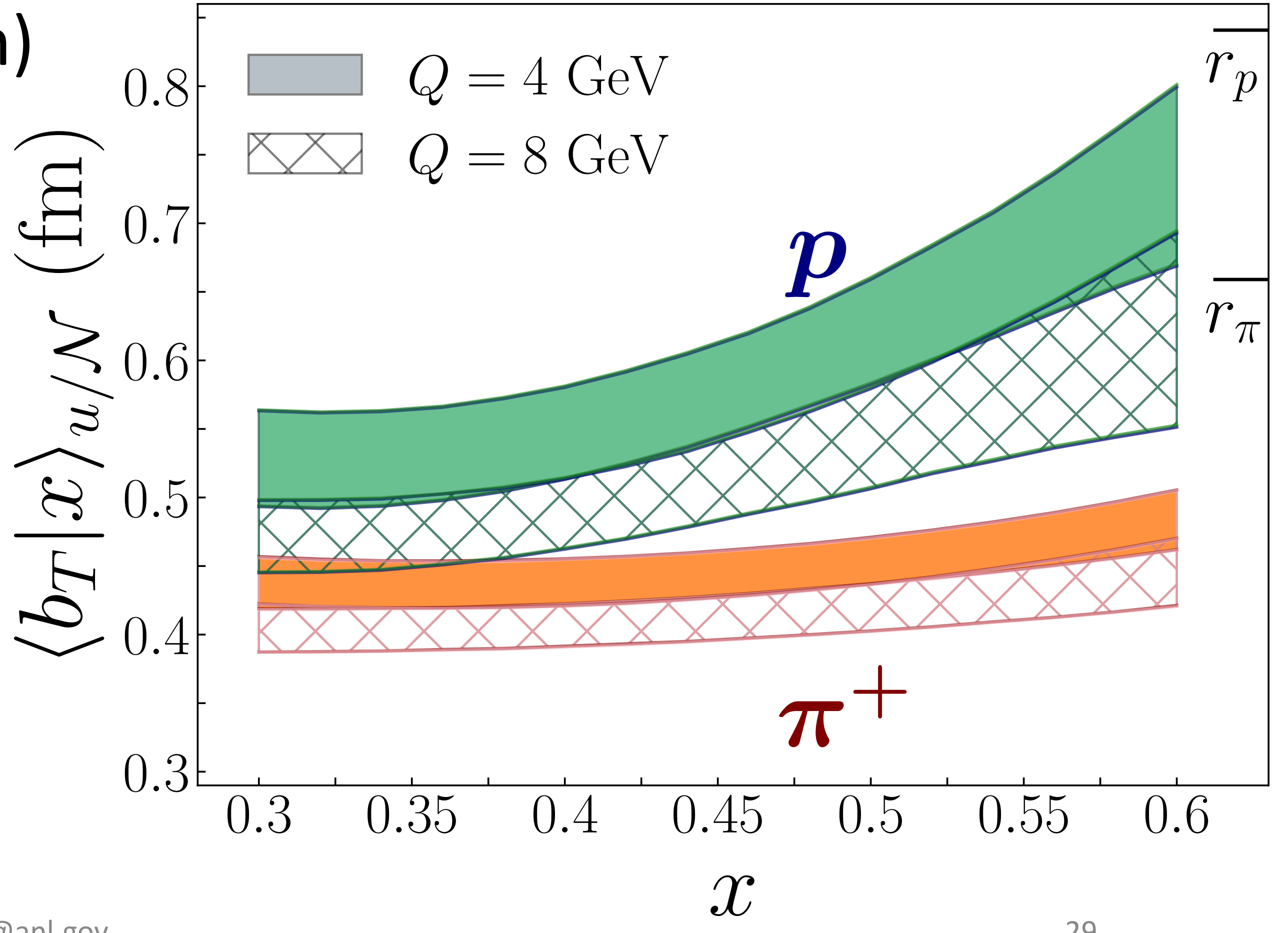
Collinear relation

- The TMD formalism requires that the integral over k_T^2 of the TMD gives the collinear PDF up to higher order corrections
- We demonstrate this for example in the proton case
- At larger Q, the power corrections are less important



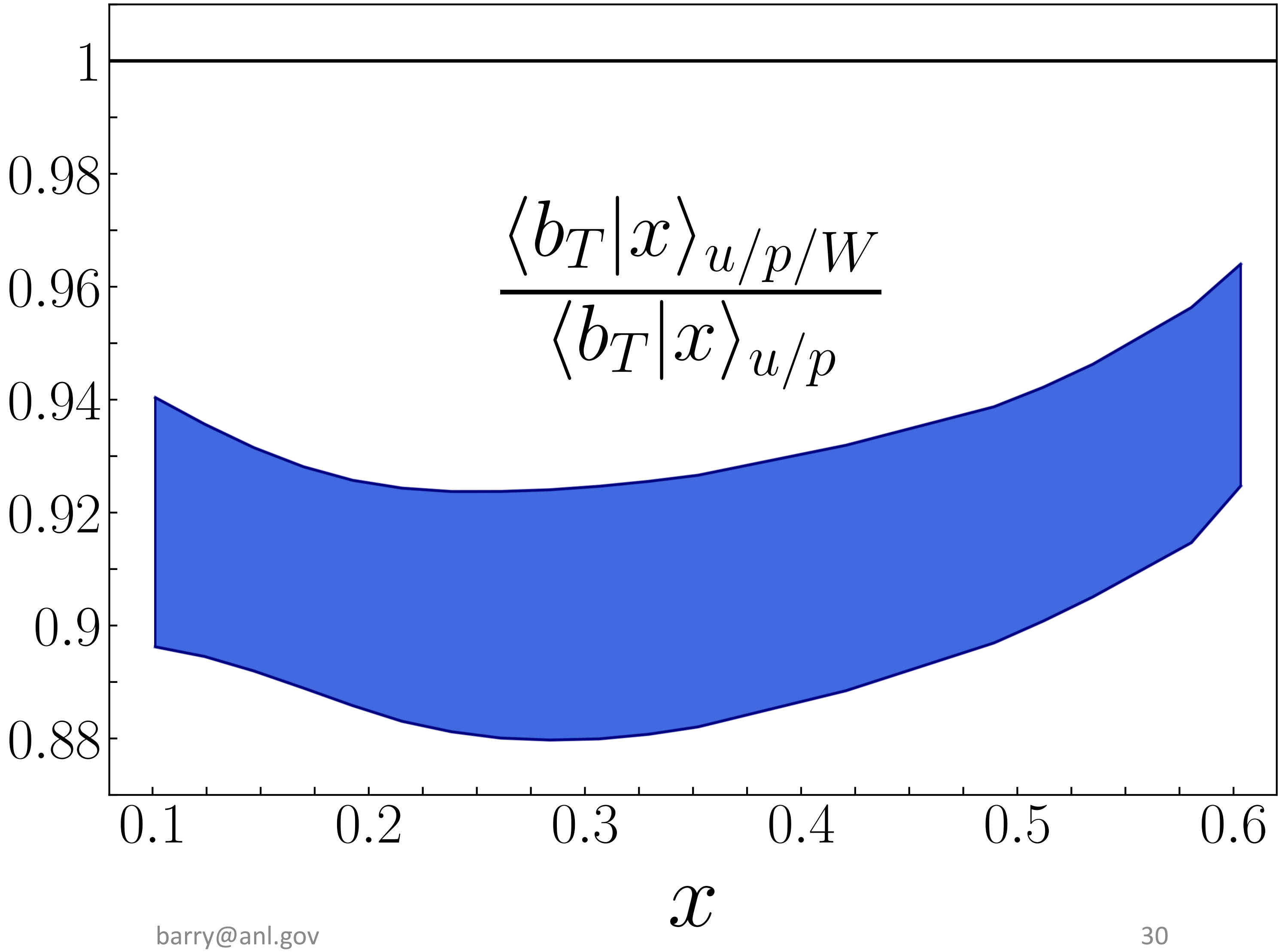
Emphasis on nonperturbative effects

- We vary the collinear PDFs
  p: CT14nlo (blue) → MMHT14 (green)
  π: JAM (red) → xFitter (orange)
- No change in the quantity!



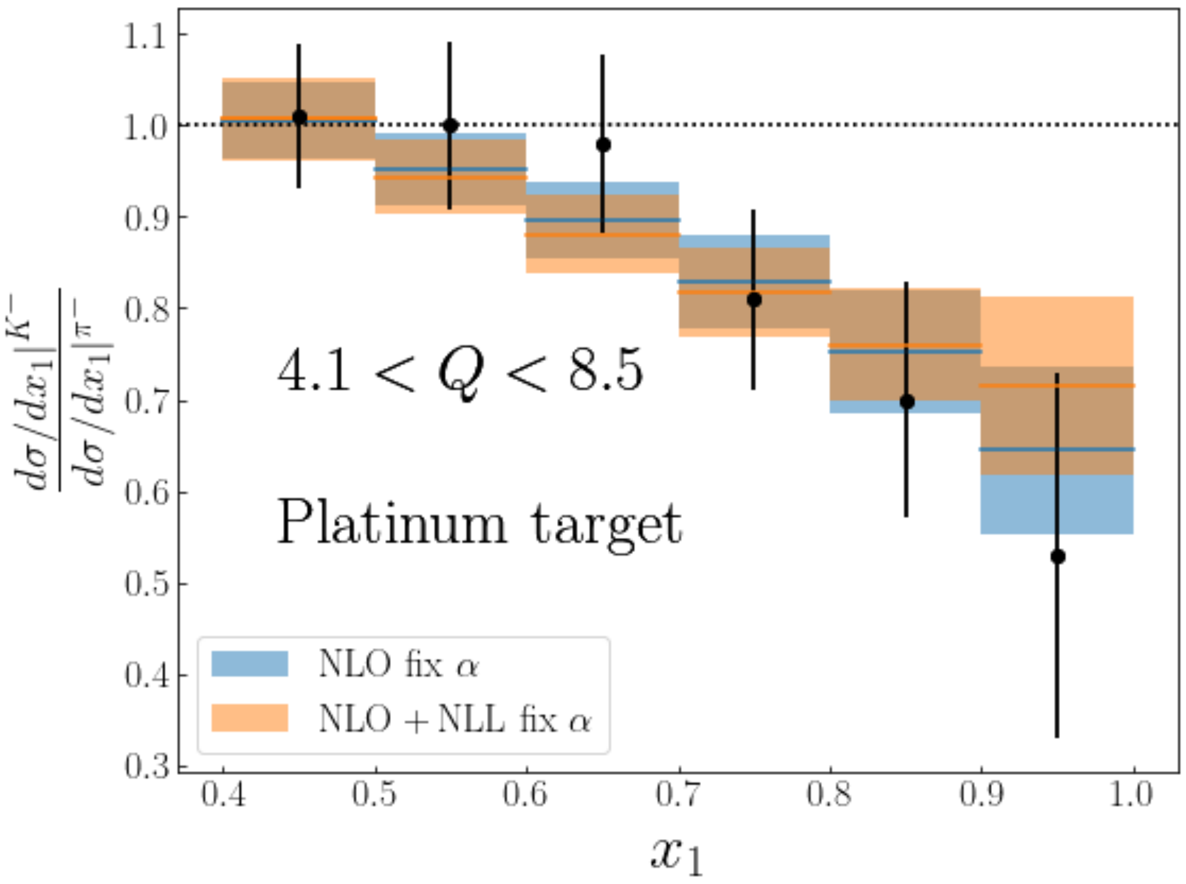
Transverse EMC effect

- Compare the average b_T given x for the up quark in the bound proton to that of the free proton
- Less than 1 by $\sim 5-12\%$ over the x range

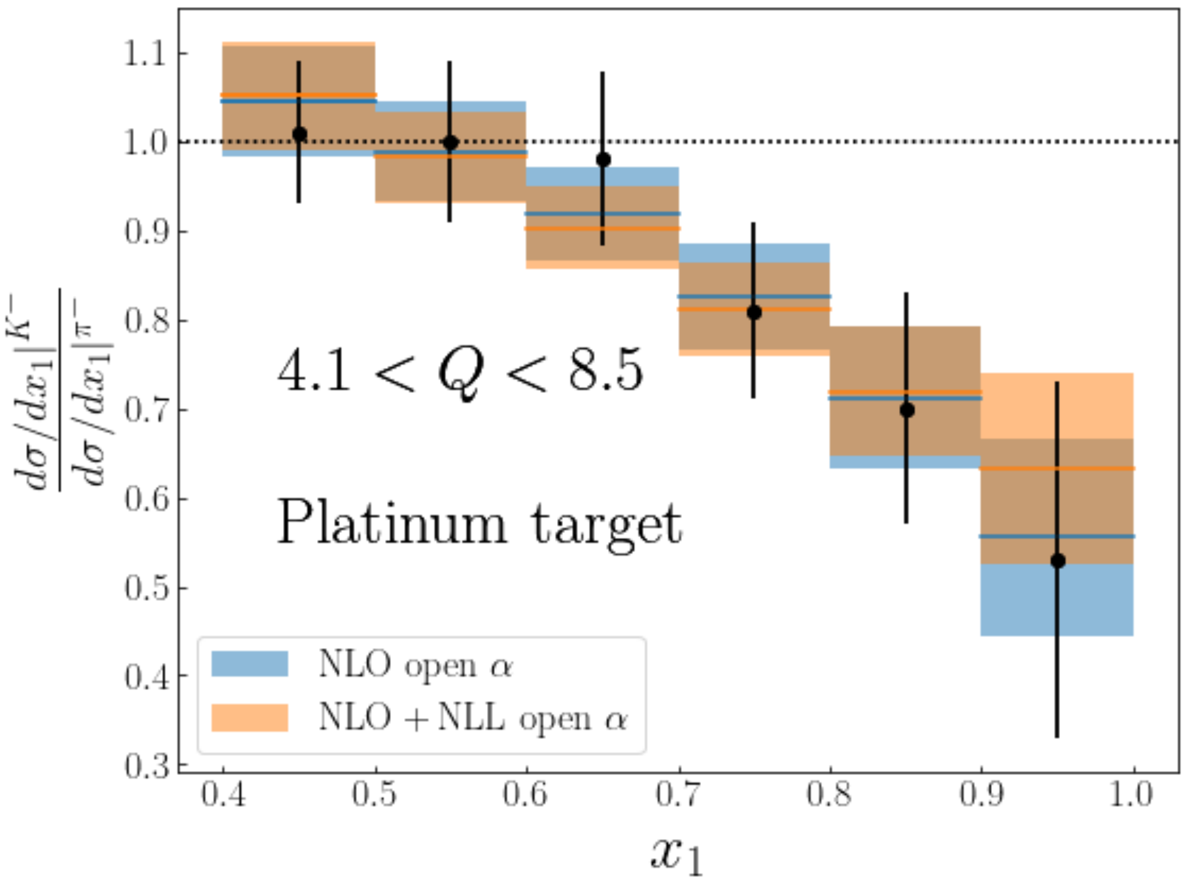


Results from simultaneous K and π PDFs

Fix α parameter



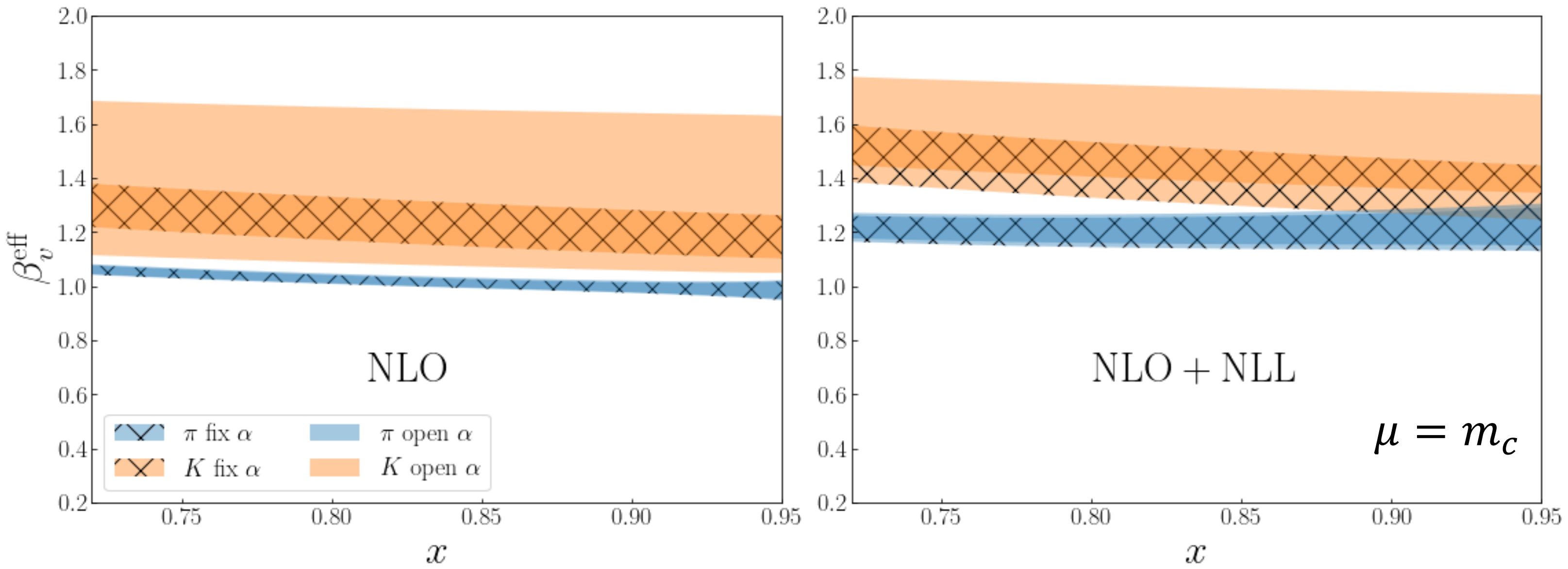
Open α parameter



Effective β_v parameter

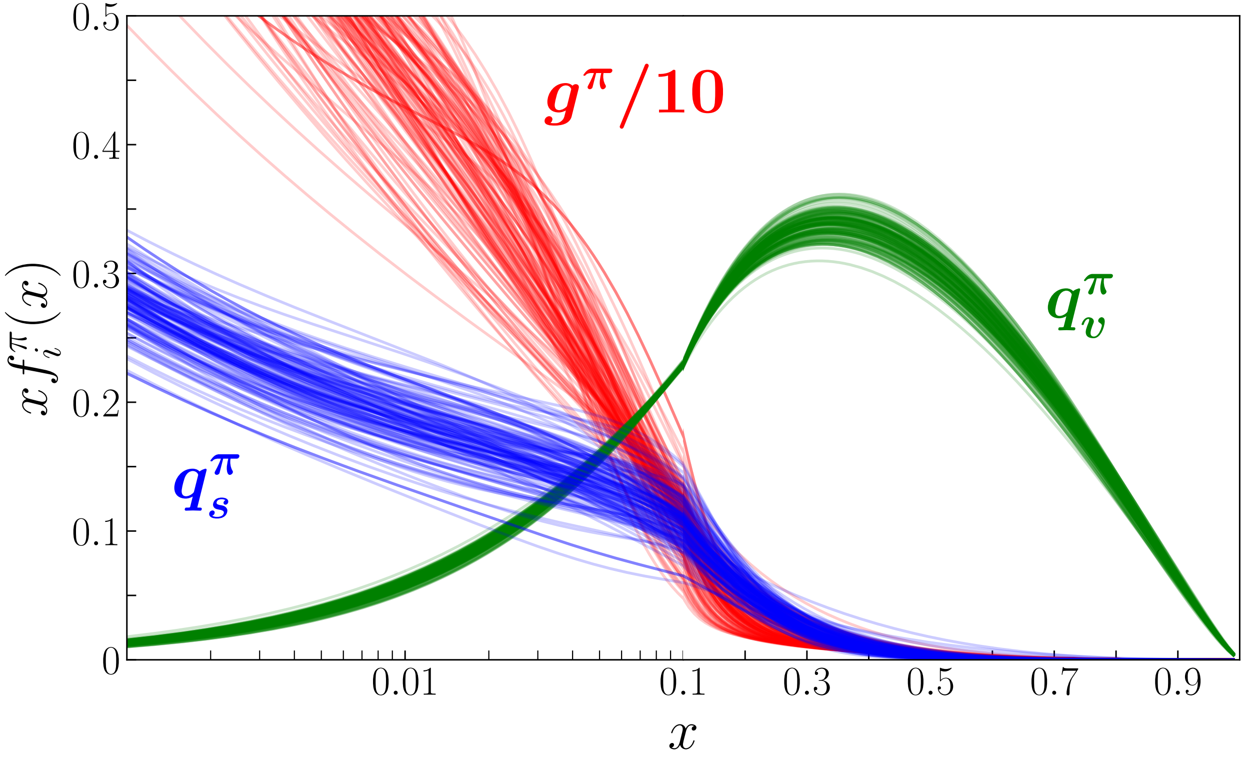
- Large-x behavior of the valence quark distribution

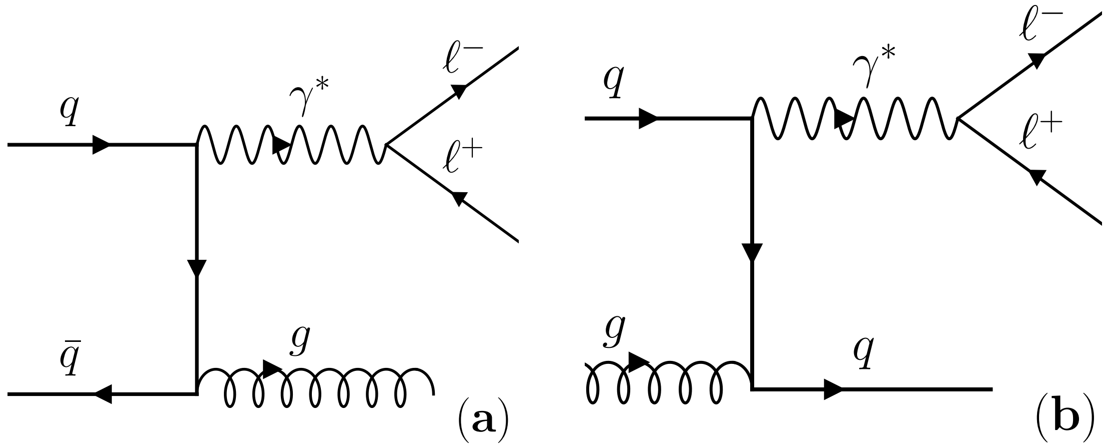
$$\beta_v^{\text{eff}}(x,\mu) = \frac{\partial \log |q_v(x,\mu)|}{\partial \log(1-x)}$$



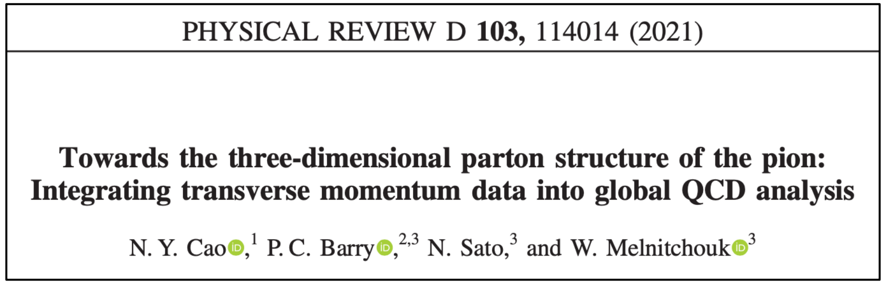
- Kaon is softer than the pion (with large uncertainties) – not yet $\beta = 2$

Large-p_T DY data





- Does not dramatically affect the PDF
- Successfully describe data with a scale $\mu = p_T/2$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Towards the three-dimensional parton structure of the pion: Integrating transverse momentum data into global QCD analysis

N. Y. Cao,[1] P. C. Barry,[2,3] N. Sato,[3] and W. Melnitchouk[3]