POSITIVITY CONSTRAINTS
ON
PARTON DISTRIBUTIONS

Stefano Forte
Università di Milano & INFN

UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

INFN
Istituto Nazionale di Fisica Nucleare

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A TRIVIALITY
CROSS-SECTION POSITIVITY

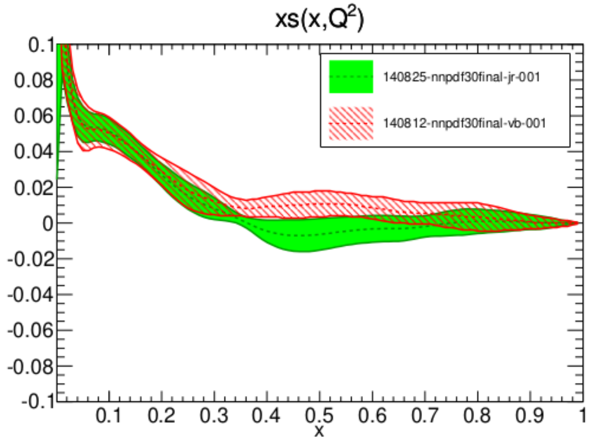
- CROSS-SECTION $\Rightarrow$ PROBABILITY "POSITIVE" (= NON-NEGATIVE) $\sigma \geq 0$
- (LONGITUDINALLY) POLARIZED ASYMMETRY $A = \frac{\Delta\sigma}{\sigma} = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}} \leq 1$

IMPLICATION FOR PDFs

- LO PDFs=CROSS-SECTION $\Rightarrow$ PDFs POSITIVE UP TO NLO
- PDFs GROW AT SMALL x IN THE PERTURBATIVE REGION
- UNPOLARIZED GROWTH FASTER
- CONSTRAINT RELEVANT AT LARGE x BOTH IN POLARIZED AND UNPOLARIZED CASE

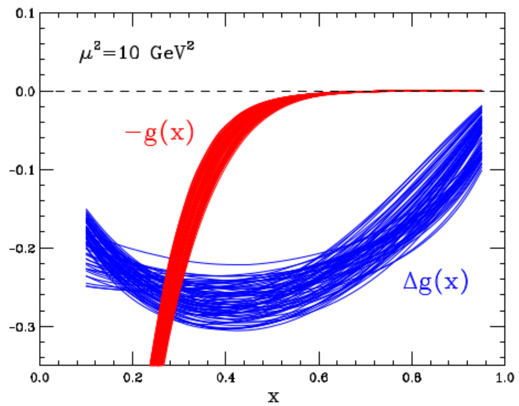
POSITIVE VS NON-POSITIVE XSECT:

NNPDF3.0

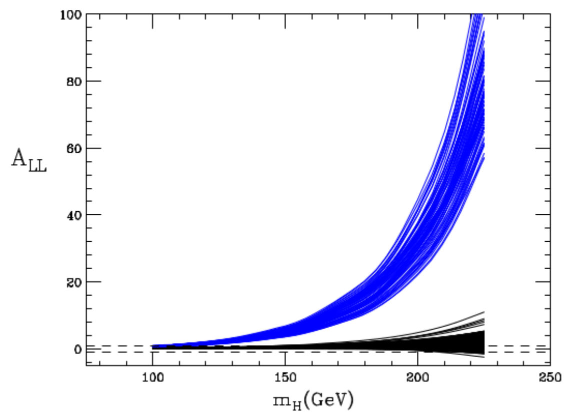


JAM GLUON PDFs: NO POSITIVITY

POLARIZED VS - UNPOLARIZED



ASYMMETRY



(De Florian, S.F., Vogelsang, 2024)

LESS TRIVIAL
PDF POSITIVITY

$$\sigma(x,Q^2) = \sigma_0 \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}, \alpha_s(Q^2)\right) q\left(y, Q^2\right);\ C(x,\alpha_S(Q^2)) = \delta(1-x) + \alpha_s(Q^2)C^{(1)}(x) + \dots$$

$$\sigma(x,Q^2) = \sigma_0 q(x,Q^2) + \alpha_s(Q^2)[C^{(1)} \otimes q](x) + \dots$$

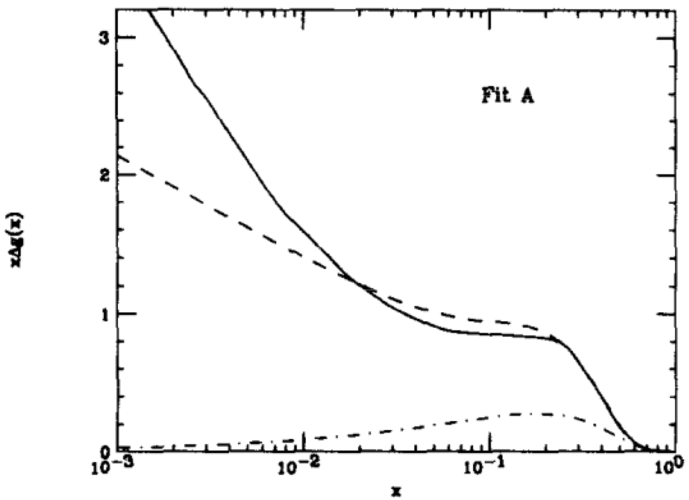
LO PDFS

- PROPORTIONAL TO CROSS-SECTIONS
- GENERALIZE TO COMPLETE PDF SET: GLUON $\Leftrightarrow$ DIS HIGGS PRODUCTION $\to$ POSITIVE PDFS
- (LONGITUDINALLY) POLARIZED: $C \Rightarrow \Delta C$

NLO AND BEYOND

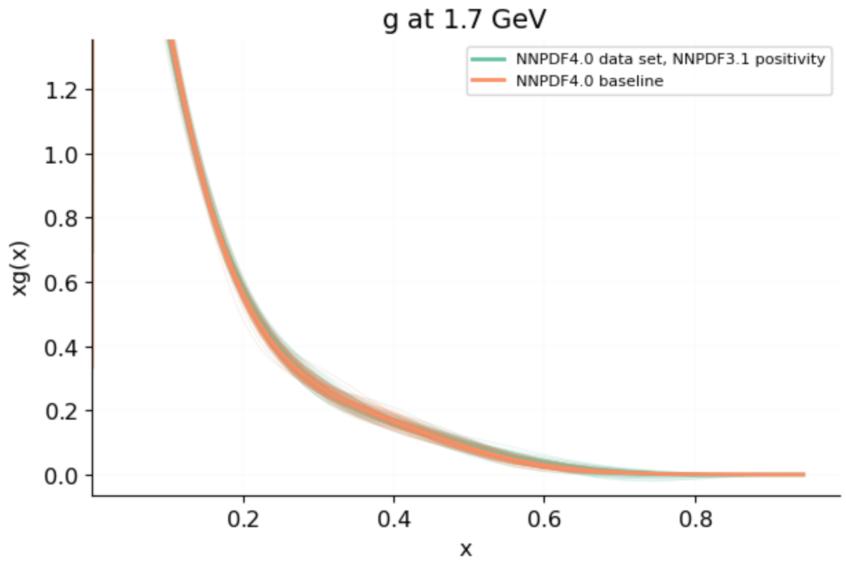
- POSITIVITY OF PDF $\Leftrightarrow$ FACTORIZATION SCHEME
  CAN BE MORE OR LESS RESTRICTIVE THAN XSECT
- PICK COMPLETE SET OF PROCESSES $\Rightarrow$ BOUNDS AT NLO (AND BEYOND)
- XSECT POSITIVITY PROCESS-DEP, MUST HOLD FOR ALL PROCESSES

NLO VS LO BOUND ON Δg VS BEST-FIT



(Altarelli, S.F., Ridolfi, 1998)

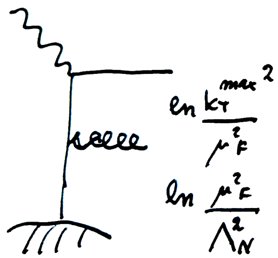
POSITIVE GLUON PDF VS XSECT

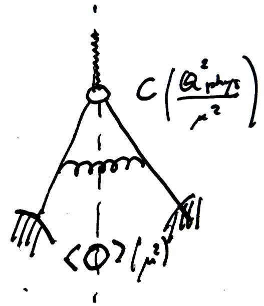


WHY SHOULD PDFs BE NEGATIVE?

AND CAN WE DO SOMETHING ABOUT IT?

- REAL EMISSION $\Rightarrow$ COLLINEAR SINGULARITIES, CUT OFF BY HADR. SCALE Λ_N: $\ln \frac{k_t^{2\,\max}}{\Lambda_N^2}$
- FACTORIZED IN PDF DOWN TO SCALE μ_F^2 $\Rightarrow \ln \frac{k_t^{2\,\max}}{\mu_F^2}$ IN HARD CROSS-SECTION





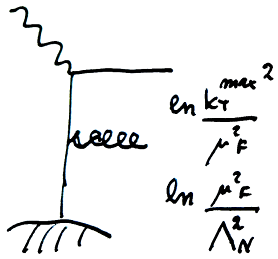
- IN $\overline{\rm MS}$ SCHEME $\mu_F^2 = \kappa Q^2$, AT LARGE x (THRESHOLD) $k_t^{2\,\max} < Q^2$
  EXAMPLE: DIS $k_t^{2\,\max} = Q^2\,\frac{1-x}{4x}$
- $\mu_F^2 > k_t^{2\,\max} \Rightarrow \ln \frac{k_t^{2\,\max}}{\mu_F^2} < 0 \Rightarrow$ NEGATIVE CONTRIB. TO COEFFICIENT FCTN
  EXAMPLE: DIS $\ln \frac{1-x}{4x}$
- NEGATIVE COEFFICIENT FCTN $\Rightarrow$ NEGATIVE PDF
- PROBLEM ONLY IN NONDIAGONAL (GLUON INITIATED), DIAGONAL $\Rightarrow$ COEFF OF LOG NEGATIVE
- SOLUTION (?) $\Rightarrow$ DEFINE "POSITIVE" FACTORIZATION SCHEME $\mu_F^2 = k_t^{2\,\max}$

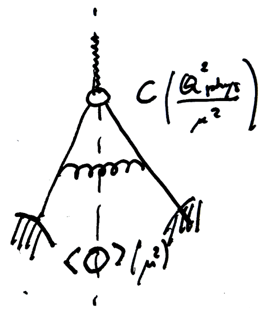
(Candido, S.F., Hekhorn, 2020)

WHAT DID COLLINS SAY?

(Collins, Rogers, Sato, 2022)

- IN OPERATOR APPROACH $\Rightarrow$ PROTON OME $\langle O \rangle$
  UV DIVERGENT
- UV RENORMALIZATION
  $\Rightarrow$ COLLINEAR SINGULARITY IN COEFF. $\ln \frac{\mu^2}{\Lambda_N^2}$
- COLLINEAR FACTORIZ $\Rightarrow$ FINITE PDF $\ln \frac{\mu_F^2}{\Lambda_N^2}$





- PDF BECOMES NEGATIVE WHEN $\mu_F^2 \lesssim \Lambda_N^2$
  UV SCALE LOWER THAN NUCLEON SCALE
- WHATEVER THE FACTORIZATION SCHEME, ALWAYS HAPPENS AT LOW ENOUGH SCALE Q^2
- Λ_N^2 NONPERTURBATIVE $\Rightarrow$ DIFFICULT TO DETERMINE

A PERTURBATIVE PDF POSITIVITY CONDITION

(Candido, S.F., Giani, Hekhorn, 2023)

ARE $\overline{\mathrm{MS}}$ NLO PDFS POSITIVE AND IF SO AT WHICH SCALES?

- "PHYSICAL" SCHEME: PDFS $\Leftrightarrow$ OBSERVABLES
  EXAMPLE: $u(x, Q^2) = \frac{9}{4} F_2^u(x, Q^2)$ (Diemoz, Ferroni, Longo, Martinelli, 1988; Catani, 1996; Altarelli, SF, Ridolfi 1998)
- VECTOR $f(x, Q^2)$ OF PDFS $\Rightarrow f^{\mathrm{phys}}(x, Q^2) = \frac{1}{\sigma_0} \sigma(x, Q^2)$
- $\sigma = \sigma_0 C^{\overline{\mathrm{MS}}} \otimes f^{\overline{\mathrm{MS}}} \Rightarrow f^{\overline{\mathrm{MS}}} = \left[C^{\overline{\mathrm{MS}}}\right]^{-1} f^{\mathrm{phys}}; \quad \left[C^{\overline{\mathrm{MS}}}\right]^{-1} \otimes \left[C^{\overline{\mathrm{MS}}}\right] = \delta(1-x)$

DOES CONVOLUTION WITH $\left[C^{\overline{\mathrm{MS}}}\right]^{-1}$ PRESERVE POSITIVITY?

PERTURBATIVITY

PERTURBATIVE INVERSION

- FROM phys TO $\overline{\rm MS}$: $f^{\rm PHYS}(x,Q^2) = \left[1 + \frac{\alpha_s}{2\pi} C^{(1),\,\overline{\rm MS}}(x)\otimes\right] f^{\overline{\rm MS}}(Q^2) + \mathcal{O}(\alpha_s^2)$
- FROM $\overline{\rm MS}$ TO phys: $f^{\overline{\rm MS}}(x,Q^2) = \left[1 - \frac{\alpha_s}{2\pi} C^{(1),\,\overline{\rm MS}}(x)\otimes\right] f^{\rm PHYS}(Q^2) + \mathcal{O}(\alpha_s^2)$
- POSITIVITY CONDITION $\left|\frac{\alpha_s}{2\pi} C^{(1),\overline{\rm MS}} \otimes f^{\rm PHYS}\right| \leq |f^{\rm PHYS}|$
- PERTURBATIVITY VIOLATED BY SUDAKOV LOGS $\left[\frac{\ln(1-x)}{1-x}\right]_+ +$

EXACT INVERSION IN SOFT LIMIT $\left[\delta(1-x) + \frac{\alpha_s}{2\pi}\left[\frac{\ln(1-x)}{1-x}\right]_+ + C_D^{(1),\,\overline{\rm MS}}(x)\right]^{-1} =$

$$= \delta\,(1-x) - 2C_F\,\frac{\alpha_s}{2\pi}\left(\frac{\ln(1-x)}{\left(1+C_F\,\frac{\alpha_s}{2\pi}\,\ln^2(1-x)\right)^2}\,\frac{1}{1-x}\right)_+ + \mathrm{NLL}(1-x)$$

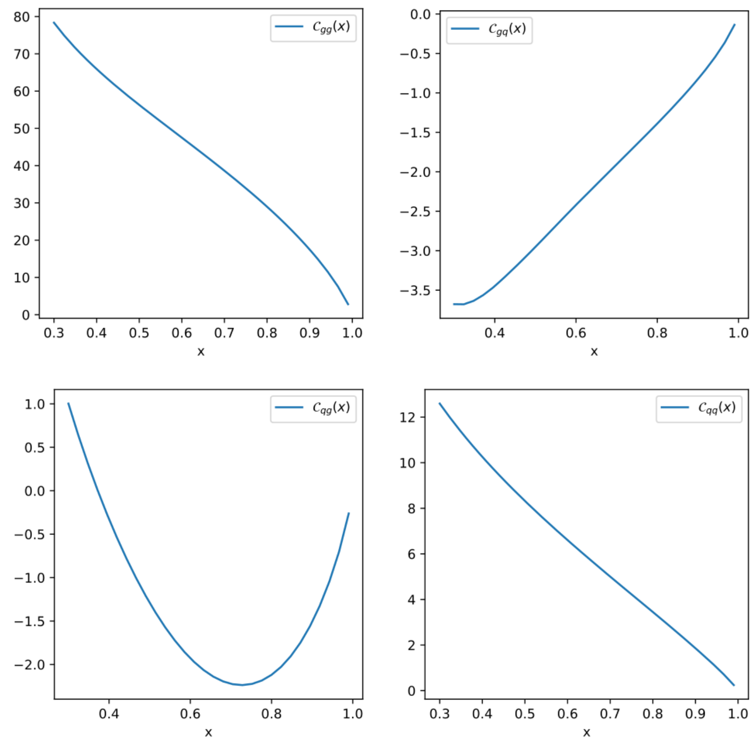
THE POSITIVITY CONDITION

- SUDAKOV CONTRIBUTION $\Rightarrow$ PRESERVES POSITIVITY

- PERTURBATIVE RESIDUE
  $\left|\int_x^1 \frac{dy}{y} C^{(1),\overline{\mathrm{MS}}}(y) f^{\mathrm{PHYS}}\left(\frac{x}{y}\right)\right| \leq \left|f^{\mathrm{PHYS}}(x) \int_x^1 \frac{dy}{y} C^{(1),\overline{\mathrm{MS}}}(y)\right| \leq |f^{\mathrm{PHYS}}(x)|$: $f^{\mathrm{PHYS}}(x)$
  DROPS WITH INCREASING x

- PDF-INDEP CONDITION ON CUMULANT $\frac{\alpha_s}{2\pi}\left|\sum_j \int_x^1 \frac{dy}{y} C_{ij}^{(1),\overline{\mathrm{MS}}}(y)\right| \leq 1$

THE CUMULANT MATRIX



- CONDITION DOMINATED BY C_{gg}

- SIGNIFICANT AT LARGE x,
  UNRELIABLE AS $x \to 1$

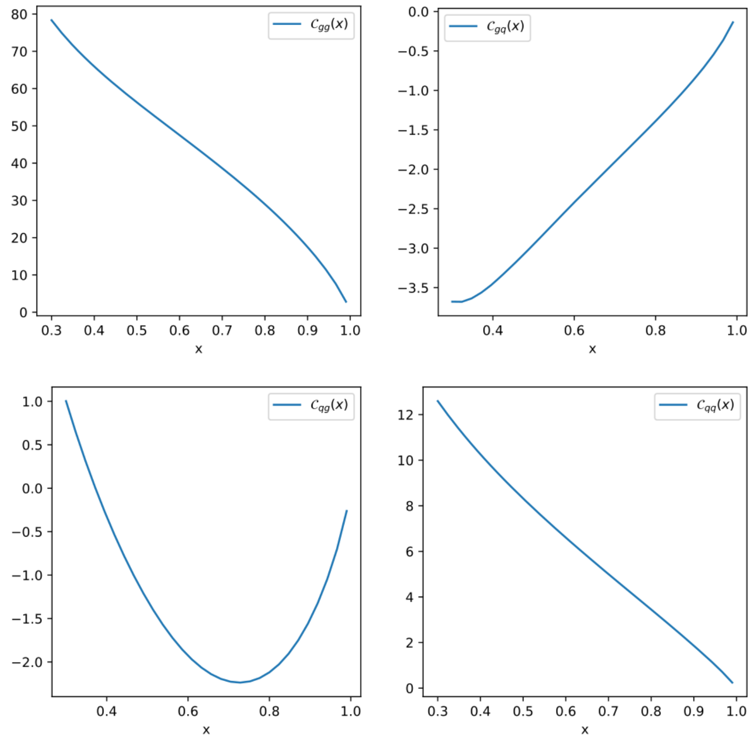
- IMPOSE AT $x = 0.8 \Rightarrow Q^2 \gtrsim 5\ \mathrm{GeV}^2$

WHAT ABOUT THE POLARIZED CASE?

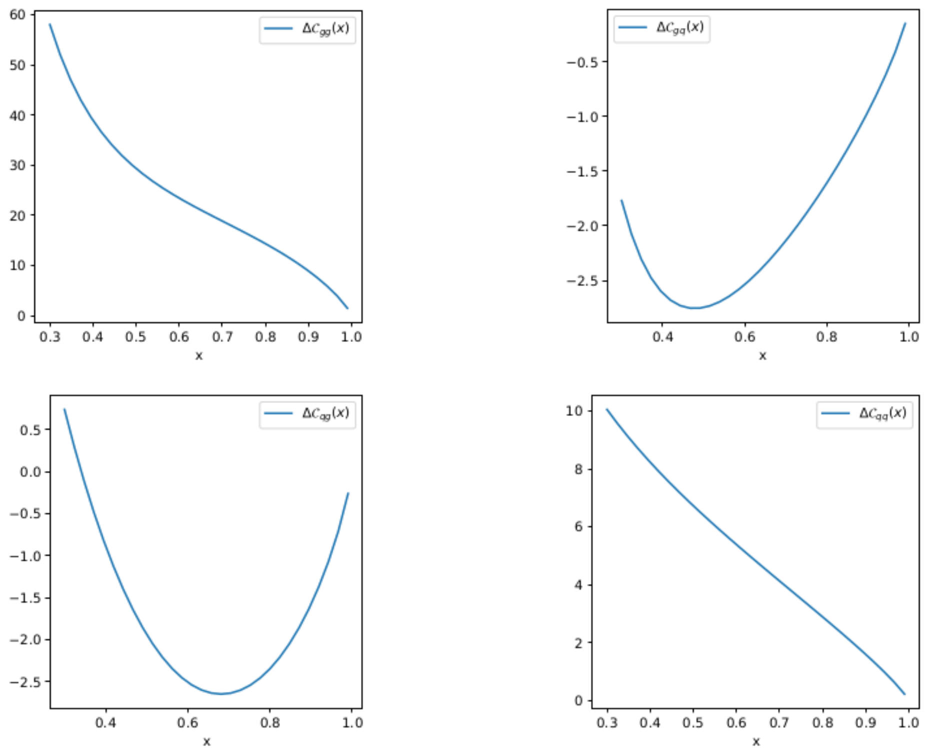
- $\Delta\sigma = \Delta\sigma_0 + \Delta C^{(1)} \otimes \Delta q$
- $\Delta C = C_{++} - C_{+-}$; $\Delta C \approx C$ AT LARGE $x \Rightarrow C_{++} \approx C$, $C_{+-} \ll C_{++}$
- UNPOLARIZED POSITIVITY $\Rightarrow$ POLARIZED POSITIVITY

THE CUMULANT MATRIX

UNPOLARIZED



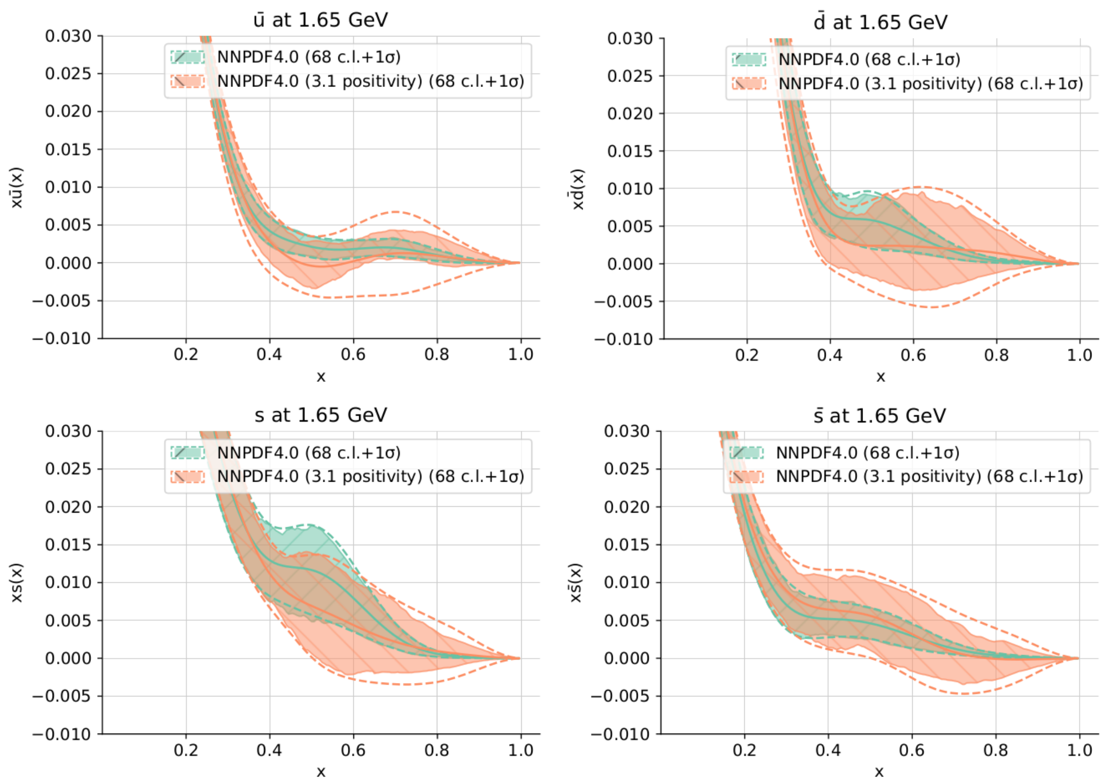
POLARIZED



PHENOMENOLOGICAL IMPLICATIONS

- HIGHER TWIST DIAGNOSTICS
  - POSITIVITY NOT IMPOSED
  - NEGATIVE PDFS FOUND $\Rightarrow$ SIZABLE HT CONTRIBUTIONS
- FIT STABILIZATION
  - HT NOT SIGNIFICANT
  - POSITIVITY IMPOSED $\Rightarrow$ MORE ACCURATE PDFS

NNPDF4.0: PDF VS XSECT POSITIVITY



CONCLUSION

ELSEVIER

Nuclear Physics B (Proc. Suppl.) 74 (1999) 138–141

Are Parton Distributions Positive? *

Stefano Forte[a]†, Guido Altarelli[b], Giovanni Ridolfi[c]

posi-
tivity bounds at large scales are trivial, but they should not be imposed at very low scale either, where they are unreliable. However, if imposed at the boundary of validity of perturbation theory, positivity bounds can be phenomenologically relevant in providing complementary information which is useful in the determination of the shape of polarized parton distribution.