# Angular structure of Drell-Yan reaction in TMD factorization

Alexey Vladimirov

Complutense University of Madrid

MINISTERIO DE CIENCIA E INNOVACIÓN
Financiado por la Unión Europea NextGenerationEU
Plan de Recuperación, Transformación y Resiliencia
AGENCIA ESTATAL DE INVESTIGACIÓN

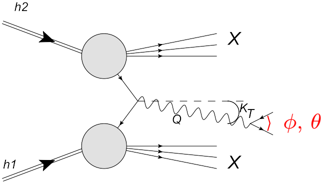
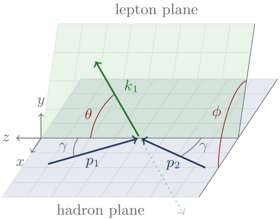



figure from [Gehrmann–De Ridder et al. 2301.11827]




| | |
|---|---|
| $Q^2$ | momentum of hard probe |
| $q_T$ | transverse momentum |
| $y$ | rapidity of hard probe |
| $\theta$, $\phi$ | angles of lepton pair (in CS frame) |

## Angular decomposition of EW-boson production

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

8 angular structure functions $A_{0,\dots,7}$ + 1 integrated (normalization)

**alternative notations:**

$$\frac{d\sigma}{d^4q d\Omega} = \frac{\alpha^2}{2\pi N_c Q^2 s^2}\left(W_T\left(1+\cos^2\theta\right) + W_L\left(1-\cos^2\theta\right) + W_\Delta \sin 2\theta\cos\phi + W_{\Delta\Delta}\sin^2\theta\cos 2\phi\right)$$

$$\frac{dN}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\Big(1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi$$
$$+\ \tau\sin\theta\cos\phi+\eta\cos\theta+\xi\sin^2\theta\sin 2\phi+\zeta\sin 2\theta\sin\phi+\chi\sin\theta\sin\phi\Big)$$

## Angular decomposition of EW-boson production

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

8 angular structure functions $A_{0,\dots,7}$ + 1 integrated (normalization)

**Measurements (proton Drell-Yan):**

| | | |
|---|---|---|
| CMS | $A_{0,..,7}$ | large $q_T$-bins, 2 bins in $y$ |
| LHCb | $A_{0,..,3}$ | large $q_T$-bins, $y > 2$ |
| ATLAS | $A_{0,..,7}$ | 2.5GeV bins, 3 bins in $y$ |
| E866 | $A_{0,..,3}$ | 1D measurement |

## Angular decomposition of EW-boson production

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

8 angular structure functions $A_{0,...,7}$ + 1 integrated (normalization)

Practically, they are not studied in TMD approach

double-BM effects

[Barone, Melis, Prokudin, 1009.3423]

[Lu, Schmidt, 1107.4693]

[Liu, Ma, 1201.2472]

[Wang, Mao, Lu, 1805.03017]

## Angular decomposition of EW-boson production

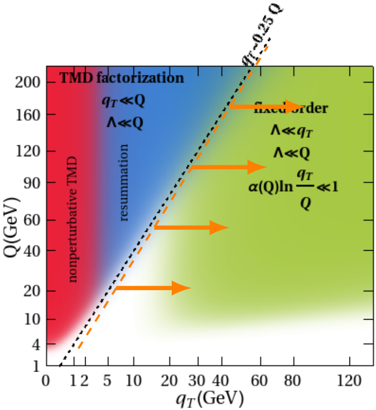
$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

8 angular structure functions $A_{0,\ldots,7}$ + 1 integrated (normalization)

**Leading power TMD factorization**
gives only 3 non-zero coefficients $A_{2,4,5}$.

**The majority of coefficients are power-suppressed**

## Different types of power corrections



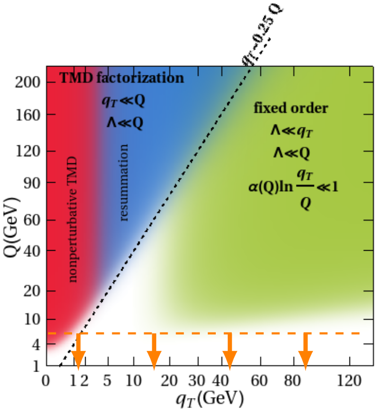

1. $q_T/Q$-corrections
   $Y$-term

2. $\Lambda/Q$ & $M/Q$-corrections
   higher-twist
   target-mass

3. $k_T/Q$-corrections
   kinematic

[AV,2307.13054]

## Different types of power corrections



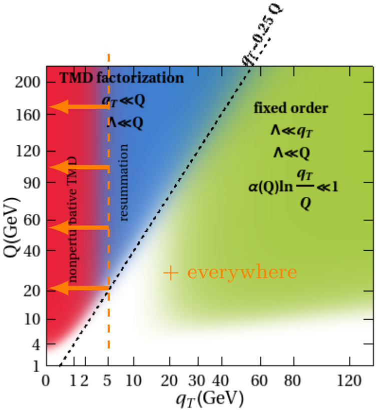

**1.** $q_T/Q$-corrections
$Y$-term

**2.** $\Lambda/Q$ & $M/Q$-corrections
higher-twist
target-mass

**3.** $k_T/Q$-corrections
kinematic

[AV,2307.13054]

## Different types of power corrections




**1.** $q_T/Q$-corrections
$Y$-term

**2.** $\Lambda/Q$ & $M/Q$-corrections
higher-twist
target-mass

**3.** $k_T/Q$-corrections
kinematic

[AV,2307.13054]

**Kinematic power corrections (KPC)** are largest corrections in the "natural" domain of TMD factorization $q_T \ll Q$

from QCD evolution 2023

$$D \sim \frac{\partial}{\partial b^\mu}$$
$\Phi_N$ is TMD of twist-N

$$W^{\mu\nu} = \frac{1}{N_c} \int \frac{d^2b}{(2\pi)^2} e^{-i(q_T b)} \Big\{$$

$$\Phi_2 \times \Phi_2$$

$$+\frac{1}{Q}\Big(D \;\; \Phi_2 \times \Phi_2 + \quad \Phi_2 \times \Phi_3\Big)$$

$$+\frac{1}{Q^2}\Big(D^2\Phi_2 \times \Phi_2 + D \;\; \Phi_2 \times \Phi_3 + \quad \Phi_3 \times \Phi_3 + \quad \Phi_2 \times \Phi_4 + \frac{\Phi_2 \times \Phi_2}{b^2}\Big)$$

$$+\frac{1}{Q^3}\Big(D^3\Phi_2 \times \Phi_2 + \quad \ldots \quad + \quad \Phi_3 \times \Phi_4 + \quad \ldots \quad + \frac{D\Phi_2 \times \Phi_2}{b^2}\Big)$$

$$+\frac{1}{Q^4}\Big(D^4\Phi_2 \times \Phi_2 + \quad \ldots \quad + \quad \Phi_2 \times \Phi_5 + \quad \ldots \quad + \frac{D\Phi_2 \times \Phi_2}{b^2} + \frac{\Phi_2 \times \Phi_2}{b^4}\Big)$$

$$+\cdots \Big\}^{\mu\nu},$$

from QCD evolution 2023

$D \sim \frac{\partial}{\partial b^\mu}$

$\Phi_N$ is TMD of twist-N

$$W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2}\, e^{-i(q_T b)} \Big\{$$

$$\Phi_2 \times \Phi_2 \qquad \text{LP} \checkmark$$

$$+\frac{1}{Q}\Big(D\ \Phi_2\times\Phi_2 + \Phi_2\times\Phi_3\Big) \qquad \text{NLP} \checkmark$$

$$+\frac{1}{Q^2}\Big(D^2\Phi_2\times\Phi_2 + D\ \Phi_2\times\Phi_3 + \Phi_3\times\Phi_3 + \Phi_2\times\Phi_4 + \frac{\Phi_2\times\Phi_2}{b^2}\Big) \qquad \text{NNLP}$$

Balitsky, tomorrow

$$+\frac{1}{Q^3}\Big(D^3\Phi_2\times\Phi_2 + \quad ... \quad + \Phi_3\times\Phi_4 + \quad ... \quad + \frac{D\Phi_2\times\Phi_2}{b^2}\Big)$$

$$+\frac{1}{Q^4}\Big(D^4\Phi_2\times\Phi_2 + \quad ... \quad + \Phi_2\times\Phi_5 + \quad ... \quad + \frac{D\Phi_2\times\Phi_2}{b^2} + \frac{\Phi_2\times\Phi_2}{b^4}\Big)$$

$$+\cdots\Big\}^{\mu\nu},$$

from QCD evolution 2023

$$W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2} e^{-i(q_T b)} \Big\{ \Phi_2 \times \Phi_2$$
$$+\frac{1}{Q}\Big(D\ \Phi_2 \times \Phi_2 + \Phi_2 \times \Phi_3\Big)$$
$$+\frac{1}{Q^2}\Big(D^2\Phi_2 \times \Phi_2 + D\ \Phi_2 \times \Phi_3 + \Phi_3 \times \Phi_3 + \Phi_2 \times \Phi_4 + \frac{\Phi_2 \times \Phi_2}{b^2}\Big)$$
$$+\frac{1}{Q^3}\Big(D^3\Phi_2 \times \Phi_2 + \ldots + \Phi_3 \times \Phi_4 + \ldots + \frac{D\Phi_2 \times \Phi_2}{b^2}\Big)$$
$$+\frac{1}{Q^4}\Big(D^4\Phi_2 \times \Phi_2 + \ldots + \Phi_2 \times \Phi_5 + \ldots + \frac{D\Phi_2 \times \Phi_2}{b^2} + \frac{\Phi_2 \times \Phi_2}{b^4}\Big)$$
$$+\cdots\Big\}^{\mu\nu},$$

resummed KPC
this talk

- Same TMDs as at LP
- Same perturbative part as LP
- Non-vanishing $q_T = 0$, and larger than $\frac{\Lambda}{Q}$
- Restore gauge and frame-invariance

## DY-hadron tensor with resummed KPC [AV,2307.13054]

$$W^{\mu\nu}_{\text{KPC}} = -\frac{4p_1^+p_2^-}{N_c}C_0\left(\frac{Q^2}{\mu^2}\right)\int d\xi_1 d\xi_2 \int d^4k_1 d^4k_2\, \delta^4(q-k_1-k_2)$$

$$\delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-)\delta(k_1^2)\delta(k_2^2)\Big\{$$

$$((k_1k_2)g^{\mu\nu} - k_1^\mu k_2^\nu - k_2^\mu k_1^\nu)\left(\Phi_{\bar{n}11}^{[\gamma^+]}\Phi_{n11}^{[\gamma^-]} + \Phi_{\bar{n}11}^{[\gamma^+\gamma^5]}\Phi_{n11}^{[\gamma^-\gamma^5]}\right)$$

$$+i\epsilon^{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta\left(\Phi_{\bar{n}11}^{[\gamma^+]}\Phi_{n11}^{[\gamma^-\gamma^5]} - \Phi_{\bar{n}11}^{[\gamma^+\gamma^5]}\Phi_{n11}^{[\gamma^-]}\right) + t^{\mu\nu}_{\alpha\beta}\Phi_{\bar{n}11}^{[i\sigma^{\alpha+}\gamma^5]}\Phi_{n11}^{[i\sigma^{\beta-}\gamma^5]}\Big\}$$

$\Phi^{[\Gamma]}$ is ordinary TMD distribution $\sim \langle p|\bar{q}...\Gamma...q|r\rangle$

## DY-hadron tensor with resummed KPC [AV,2307.13054]

$$W^{\mu\nu}_{\mathrm{KPC}} = -\frac{4p_1^+p_2^-}{N_c} C_0\left(\frac{Q^2}{\mu^2}\right) \int d\xi_1 d\xi_2 \int d^4k_1 d^4k_2\, \delta^4(q-k_1-k_2)$$

$$\delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-)\delta(k_1^2)\delta(k_2^2)\Big\{$$

$$\Big((k_1k_2)g^{\mu\nu} - k_1^\mu k_2^\nu - k_2^\mu k_1^\nu\Big)\left(\Phi_{\bar n 11}^{[\gamma^+]}\Phi_{n11}^{[\gamma^-]} + \Phi_{\bar n 11}^{[\gamma^+\gamma^5]}\Phi_{n11}^{[\gamma^-\gamma^5]}\right)$$

$$+i\epsilon^{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta\left(\Phi_{\bar n 11}^{[\gamma^+]}\Phi_{n11}^{[\gamma^-\gamma^5]} - \Phi_{\bar n 11}^{[\gamma^+\gamma^5]}\Phi_{n11}^{[\gamma^-]}\right) + t^{\mu\nu}_{\alpha\beta}\Phi_{\bar n 11}^{[i\sigma^{\alpha+}\gamma^5]}\Phi_{n11}^{[i\sigma^{\beta-}\gamma^5]}\Big\}$$

$$\int d^2k_{1T}d^2k_{2T}\delta^{(2)}(q_T - k_{1T} - k_{2T})$$

at LP

## DY-hadron tensor with resummed KPC [AV,2307.13054]

$$W^{\mu\nu}_{\text{KPC}} = -\frac{4p_1^+p_2^-}{N_c} C_0\left(\frac{Q^2}{\mu^2}\right)\int d\xi_1 d\xi_2 \int d^4k_1 d^4k_2\, \delta^4(q-k_1-k_2)$$

$$\delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-)\delta(k_1^2)\delta(k_2^2)\Big\{$$

$$\boxed{((k_1k_2)g^{\mu\nu} - k_1^\mu k_2^\nu - k_2^\mu k_1^\nu)}\left(\Phi_{\bar n 11}^{[\gamma^+]}\Phi_{n11}^{[\gamma^-]} + \Phi_{\bar n 11}^{[\gamma^+\gamma^5]}\Phi_{n11}^{[\gamma^-\gamma^5]}\right)$$

$$+i\epsilon^{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta\left(\Phi_{\bar n 11}^{[\gamma^+]}\Phi_{n11}^{[\gamma^-\gamma^5]} - \Phi_{\bar n 11}^{[\gamma^+\gamma^5]}\Phi_{n11}^{[\gamma^-]}\right) + t^{\mu\nu}_{\alpha\beta}\Phi_{\bar n 11}^{[i\sigma^{\alpha+}\gamma^5]}\Phi_{n11}^{[i\sigma^{\beta-}\gamma^5]}\Big\}$$

$g_T^{\mu\nu}$ at LP

## DY-hadron tensor with resummed KPC [AV,2307.13054]

$$W^{\mu\nu}_{\text{KPC}} = -\frac{4p_1^+p_2^-}{N_c} C_0\left(\frac{Q^2}{\mu^2}\right)\int d\xi_1 d\xi_2 \int d^4k_1 d^4k_2\, \delta^4(q-k_1-k_2)$$
$$\delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-)\delta(k_1^2)\delta(k_2^2)\Big\{$$
$$((k_1k_2)g^{\mu\nu} - k_1^\mu k_2^\nu - k_2^\mu k_1^\nu)\left(\Phi^{[\gamma^+]}_{\bar n 11}\Phi^{[\gamma^-]}_{n11} + \Phi^{[\gamma^+\gamma^5]}_{\bar n 11}\Phi^{[\gamma^-\gamma^5]}_{n11}\right)$$
$$+i\epsilon^{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\left(\Phi^{[\gamma^+]}_{\bar n 11}\Phi^{[\gamma^-\gamma^5]}_{n11} - \Phi^{[\gamma^+\gamma^5]}_{\bar n 11}\Phi^{[\gamma^-]}_{n11}\right) + t^{\mu\nu}_{\alpha\beta}\Phi^{[i\sigma^{\alpha+}\gamma^5]}_{\bar n 11}\Phi^{[i\sigma^{\beta-}\gamma^5]}_{n11}\Big\}$$

Same as at LP!

- Argument $Q^2$ [instead of $2q^+q^-$ at LP]
- $\mu$-dependence canceled by $\Phi(x, k_T; \mu, Q^2)$
- $\zeta\bar\zeta = Q^4$ [instead of $(2q^+q^-)^2$ at LP]
- The summation of KPC is done at $\zeta/\bar\zeta = 1$ (general form see [AV,2307.13054])

Main difference with LP is the convolution integral

KPC
$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\rho_{pp}(\text{polarizations}) F(\xi_1(x_1, \mathbf{k}_{1,2}), \mathbf{k}_1^2, Q, Q^2) F(\xi_2(x_2, \mathbf{k}_{1,2}), \mathbf{k}_2^2, Q, Q^2)$$

LP
$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\rho_{pp}^{\text{LP}}(\text{polarizations}) F(x_1, \mathbf{k}_1^2, Q, Q^2) F(x_2, \mathbf{k}_2^2, Q, Q^2)$$

Main difference with LP is the convolution integral

KPC
$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\underline{\rho_{pp}(\text{polarizations})} F(\xi_1(x_1, \mathbf{k}_{1,2}), \mathbf{k}_1^2, Q, Q^2) F(\xi_2(x_2, \mathbf{k}_{1,2}), \mathbf{k}_2^2, Q, Q^2)$$

LP
$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\underline{\rho_{pp}^{\text{LP}}(\text{polarizations})} F(x_1, \mathbf{k}_1^2, Q, Q^2) F(x_2, \mathbf{k}_2^2, Q, Q^2)$$

$$\rho_{pp} = \rho_{pp}^{\text{LP}} \underbrace{+\frac{\mathbf{k}_T}{Q} + \frac{\mathbf{k}_T^2}{Q^2} + ...}_{\text{restoration of g.inv.}}$$

Main difference with LP is the convolution integral

KPC

$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\rho_{pp}(\text{polarizations}) F(\boxed{\xi_1(x_1, \mathbf{k}_{1,2})}, \mathbf{k}_1^2, Q, Q^2) F(\boxed{\xi_2(x_2, \mathbf{k}_{1,2})}, \mathbf{k}_2^2, Q, Q^2)$$

LP

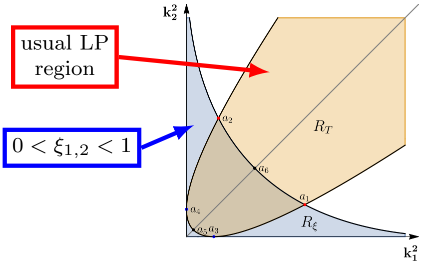
$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\rho_{pp}^{\text{LP}}(\text{polarizations}) F(\underline{x_1}, \mathbf{k}_1^2, Q, Q^2) F(\underline{x_2}, \mathbf{k}_2^2, Q, Q^2)$$

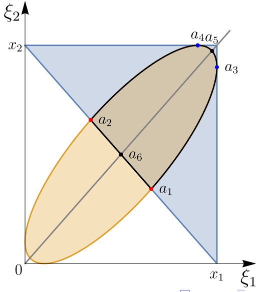
$$\xi_1 = \frac{x_1}{2}\left(1 + \frac{\mathbf{k}_1^2 - \mathbf{k}_2^2 + \sqrt{\lambda(\mathbf{k}_1^2, \mathbf{k}_2^2, \tau^2)}}{\tau^2}\right), \qquad \xi_2 = \frac{x_2}{2}\left(1 + \frac{\mathbf{k}_2^2 - \mathbf{k}_1^2 + \sqrt{\lambda(\mathbf{k}_1^2, \mathbf{k}_2^2, \tau^2)}}{\tau^2}\right)$$

Main difference with LP is the convolution integral

KPC
$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\rho_{pp}(\text{polarizations}) F(\xi_1(x_1, \mathbf{k}_{1,2}), \mathbf{k}_1^2, Q, Q^2) F(\xi_2(x_2, \mathbf{k}_{1,2}), \mathbf{k}_2^2, Q, Q^2)$$

LP
$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\rho_{pp}^{\text{LP}}(\text{polarizations}) F(x_1, \mathbf{k}_1^2, Q, Q^2) F(x_2, \mathbf{k}_2^2, Q, Q^2)$$

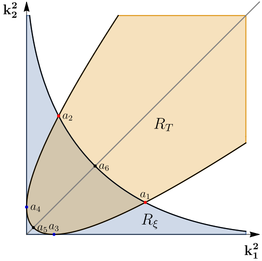


Main difference with LP is the convolution integral

KPC
$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\rho_{pp}(\text{polarizations}) F(\xi_1(x_1,\mathbf{k}_{1,2}),\mathbf{k}_1^2,Q,Q^2) F(\xi_2(x_2,\mathbf{k}_{1,2}),\mathbf{k}_2^2,Q,Q^2)$$

LP
$$\frac{d\sigma}{dq_T} = |C_V(Q)|^2 \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\rho_{pp}^{\text{LP}}(\text{polarizations}) F(x_1,\mathbf{k}_1^2,Q,Q^2) F(x_2,\mathbf{k}_2^2,Q,Q^2)$$



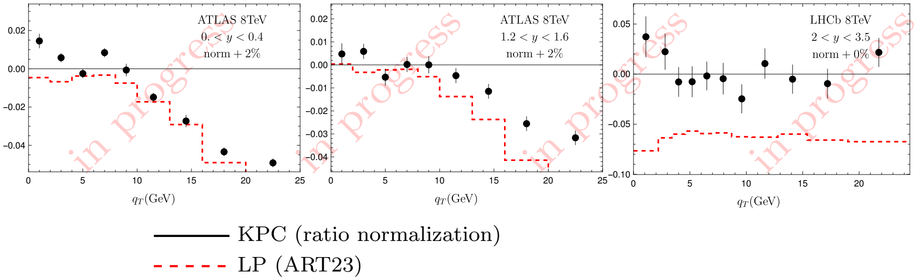

effectively **+1 integration**

artemide v3.0$\alpha$
main DY set ($\sim$ 600pt)
+bin integrations +accept.corr.

**LP**
accurate $\sim$ 1min
approx. $\sim$ 5sec.

**KPC**
accurate $\sim$ 2hours
approx. $\sim$ 4min

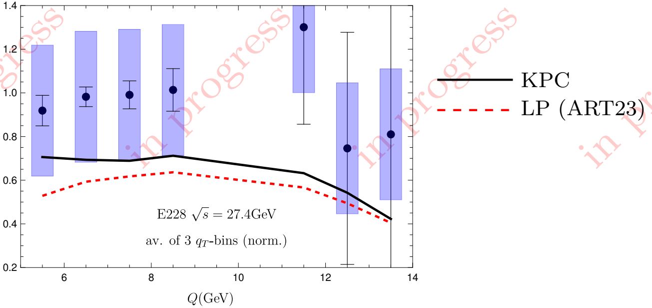
**Unpolarized DY** = normalization for angular coefficients
Computation with ART23 input [V.Moos,et al,2305.07473]




**Main effects are at**

- Large $q_T$-tale
- Normalization at forward rapidity & low energy

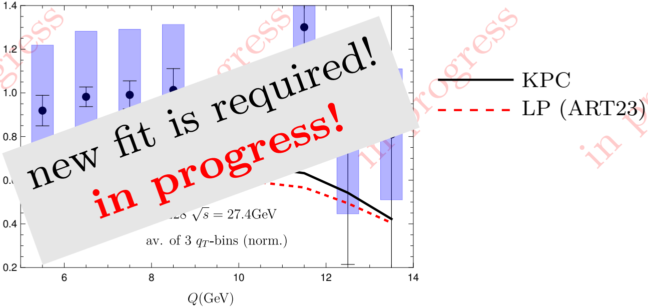
**Unpolarized DY** = normalization for angular coefficients
Computation with ART23 input [V.Moos,et al,2305.07473]




**Main effects are at**

- Large $q_T$-tale
- Normalization at forward rapidity & low energy

**Unpolarized DY** = normalization for angular coefficients
Computation with ART23 input [V.Moos,et al,2305.07473]




**Main effects are at**

- Large $q_T$-tale
- Normalization at forward rapidity & low energy

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \boxed{\frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

I use ART23 TMDPDF as unpolarized input.
Some tension with data is due to it.

$A_4$

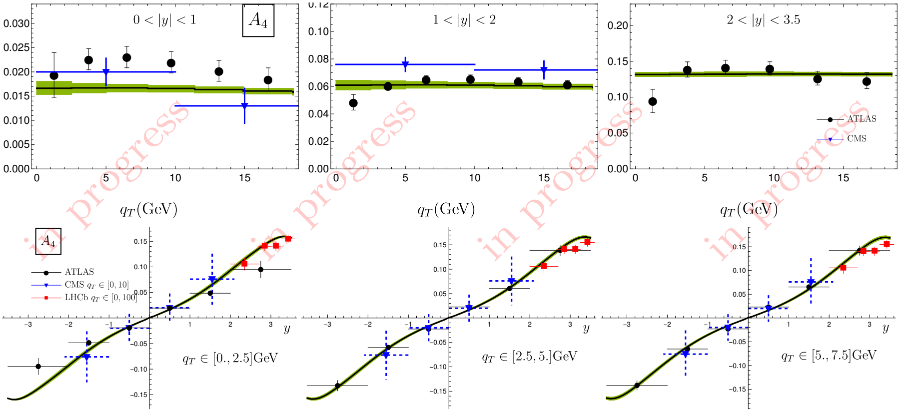
$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + \boxed{A_4\ \cos\theta}$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

$$A_4 \sim (av)_l(av)_q\left[2 - \frac{\mathbf{k}_1^2 + \mathbf{k}_2^2}{Q^2}\right][f_{1q}f_{1\bar{q}} - f_{1\bar{q}}f_{1q}]$$

- Leading power **(!!!)**
- Sensitive to a different EW-coupling
- Only unpolarized contribution
- Sensitive to a **difference** of quark/anti-quark distributions **(!!!)**

why we did not include it into standard TMD fits?

# $A_4$



0 < |y| < 1
A4
1 < |y| < 2
2 < |y| < 3.5
ATLAS
CMS
qT(GeV)
in progress
CMS qT ∈ [0, 10]
LHCb qT ∈ [0, 100]
qT ∈ [0., 2.5]GeV
qT ∈ [2.5, 5.]GeV
qT ∈ [5., 7.5]GeV
y

# $A_2$

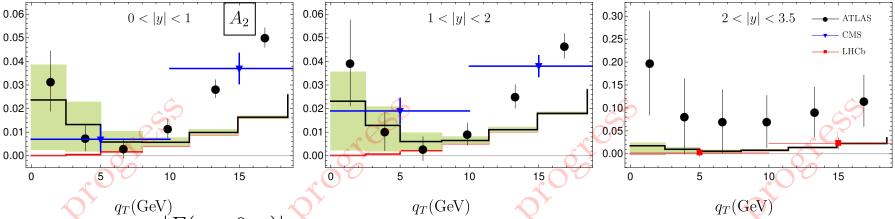
$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}\boxed{A_2\ \sin^2\theta\ \cos 2\phi} + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

$$A_2 \sim (v^2+a^2)_l\Big[2(\hat{q}\mathbf{k}_1)(\hat{q}\mathbf{k}_2) - (\mathbf{k}_1\mathbf{k}_2) + ...\Big]\Big[(v^2-a^2)_q\frac{h_1^\perp h_1^\perp}{M^2} + (v^2+a^2)_q\frac{f_1 f_1}{Q^2}\Big]$$

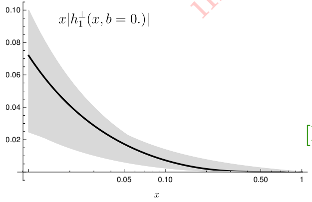
- Double BM at leading power
- Mixing with $f_1 f_1$

# Double Boer-Mulders part at LHC

$A_2$




$x|E(-x,0,x)|$

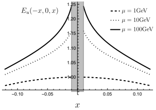



$$h_1^\perp = N e^{-0.1 b^2} x^a (1-x)^5$$
$$h_{1q}^\perp = -h_{1\bar{q}}^\perp$$

$$N \simeq (3 \pm 1) \cdot 10^{-3}, \qquad a = -1.5 \pm 0.2$$

[Kovchegov, Santiago,2209.03538] $\Rightarrow \quad a \sim +1$

but see Rodini's talk on Wednesday

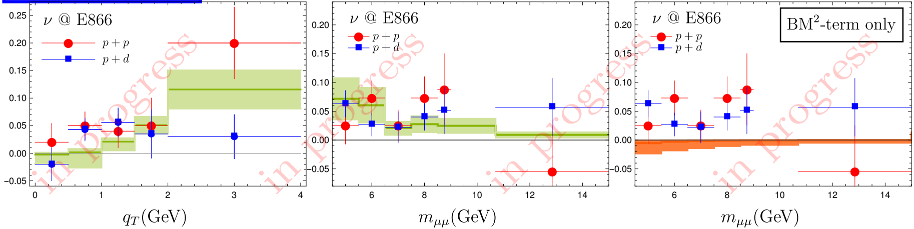


## Double Boer-Mulders part at low-energy

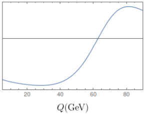
$$\nu = \frac{2A_2}{2 + A_0}$$

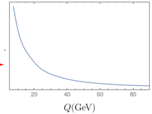
$\longleftarrow$ complicated composition of terms...

**Warning!**
E866 is 1D measurement
i.e. partially outside of fac.region




$$A_2 \sim \left[ \underbrace{(v^2 - a^2)_q}_{\substack{<0 \text{ for Z} \\ >0 \text{ for } \gamma}} \frac{h_1^\perp h_1^\perp}{M^2} + (v^2 + a^2)_q \underbrace{\frac{f_1 f_1}{Q^2}}_{\substack{\text{grows at} \\ Q \to 0}} \right]$$



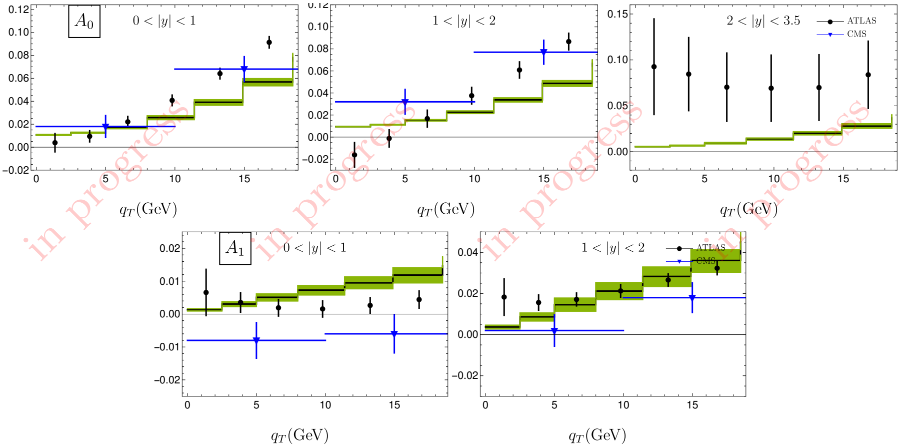
# $A_{0,1}$

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2} A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2} A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

$$A_0 \sim (v^2+a^2)_l \left[\frac{\mathbf{k}_1^2+\mathbf{k}_2^2}{Q^2}\right]\left[(v^2-a^2)_q \frac{h_1^\perp h_1^\perp}{M^2} + (v^2+a^2)_q \frac{f_1 f_1}{Q^2}\right]$$
$$A_1 \sim (v^2+a^2)_l \left[\frac{(\hat{q}\mathbf{k}_1)-(\hat{q}\mathbf{k}_2)}{Q}\right]\left[(v^2-a^2)_q \frac{h_1^\perp h_1^\perp}{M^2} + (v^2+a^2)_q \frac{f_1 f_1}{Q^2}\right]$$

- $A_0$ and $A_1$ are similar.
- $A_0 \sim Q^{-2}$, symmetric in $y$
- $A_1 \sim Q^{-1}$, anti-symmetric in $y$
- Non-zero at $q_T = 0$ (although small)

# $A_{0,1}$




**Double Boer-Mulders part is negligible,** $A_0^{hh} \sim 10^{-4}$

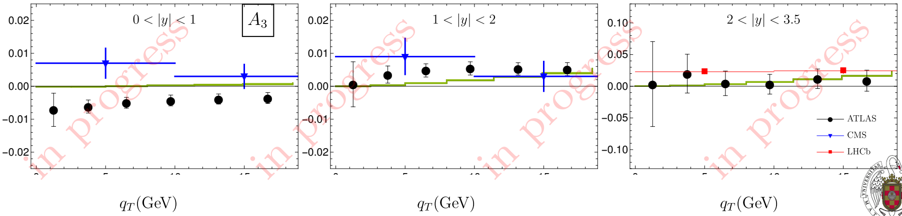
$A_1^{hh} = 0$ since $h_q = h_{\bar{q}}$

# $A_3$

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi + \frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + \boxed{A_3\ \sin\theta\ \cos\phi} + A_4\ \cos\theta + A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

$$A_3 \sim (av)_l (av)_q \left[\frac{(\hat{q}\mathbf{k}_1) - (\hat{q}\mathbf{k}_2)}{Q}\right][f_{1q}f_{1\bar{q}} - f_{1\bar{q}}f_{1q}]$$

- Similar to $A_4$ but with $Q^{-1}$ suppression



# $A_{5,6,7}$

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

$$A_5 \sim (v^2+a^2)_l(va-av)_q\left[\frac{2(\hat{q}\mathbf{k}_1)(\hat{q}\mathbf{k}_2)-(\mathbf{k}_1\mathbf{k}_2)}{M^2}\right]\left[h_{1q}^\perp h_{1\bar{q}}^\perp - h_{1\bar{q}}^\perp h_{1q}^\perp\right]$$
$$A_6 \sim (v^2+a^2)_l(va-av)_q\left[\frac{(\mathbf{k}_1^2+\mathbf{k}_2^2)((\hat{q}\mathbf{k}_1)+(\hat{q}\mathbf{k}_2))}{QM^2}\right]\left[h_{1q}^\perp h_{1\bar{q}}^\perp - h_{1\bar{q}}^\perp h_{1q}^\perp\right]$$
$$A_7 = 0$$

- $A_{5,6,7}$ are T-odd
- $A_5$ is leading power [$A_5$ for $A_2$ alike $A_4$ for $\sigma$],
- $A_6 \sim Q^{-1}$
- $A_{5,6}$ are non-zero only due to $Z/\gamma$ interference, thus extra $\Gamma_Z/Q$-factor
- ATLAS measured $A_5 \lesssim 0.2-1\%$, $A_6 \lesssim 0.-0.3\%$,$A_7 \lesssim 0.-0.1\%$ (+uncertainties)

## Conclusion

Angular coefficients provide extra information for TMD studies.
Each coefficient gives access to a unique portion of information.

### Main observations

- TMD factorization with included KPC work very well but an update of main fits is required.
- $A_4$ coefficient should be added to the main $f_1$-DY-data pool.
- $A_2$ measured at LHC indicates non-zero Boer-Mulders function (with $h_q h_{\bar{q}} < 0$)
- Power suppressed effects (such as terms $\sim f_1 f_1$ in $A_2$) became numerically significant/dominant at $Q \sim 10$GeV.

**Work still in progress!**

# Backup

## Remainder: EM gauge invariance

$$q^\mu W_{\mu\nu} = 0, \qquad \Leftrightarrow \qquad \left(q^+ \bar{n}^\mu + q^- n^\mu - i\frac{\partial}{\partial b_\mu}\right)\widetilde{W}_{\mu\nu} = 0$$

## Remainder: frame (reparametrization) invariance

$$\text{I:} \begin{cases} n^\mu \to n'^\mu = n^\mu + \dfrac{\Delta^\mu}{q^-} - \dfrac{\Delta^2}{2(q^-)^2}\bar{n}^\mu, \\ \bar{n}^\mu \to \bar{n}'^\mu = \bar{n}^\mu, \end{cases} \qquad \text{II:} \begin{cases} n^\mu \to n'^\mu = n^\mu, \\ \bar{n}^\mu \to \bar{n}'^\mu = \bar{n}^\mu + \dfrac{\overline{\Delta}^\mu}{q^+} - \dfrac{\overline{\Delta}^2}{2(q^+)^2}n^\mu, \end{cases}$$

For physical variables in implies (here for I)

$$k_{1T}^\mu \to k'^\mu_{1T} = k_{1T}^\mu, \qquad k_{2T}^\mu \to k'^\mu_{2T} = k_{2T}^\mu - \Delta^\mu - ((k_2\Delta) - \Delta^2)\frac{\bar{n}^\mu}{q^-}.$$

$$x_1 \to x_1' = x_1\left(1 + \frac{2(k_2\Delta) - \Delta^2}{2q^+q^-}\right), \qquad x_2 \to x_2' = x_2,$$

These invariances are restored by KPCs!