Transverse Momentum Moments: From TMDs to collinear distributions

arxiv 2402.01836 (February, 2024)

QCD Evolution 2024

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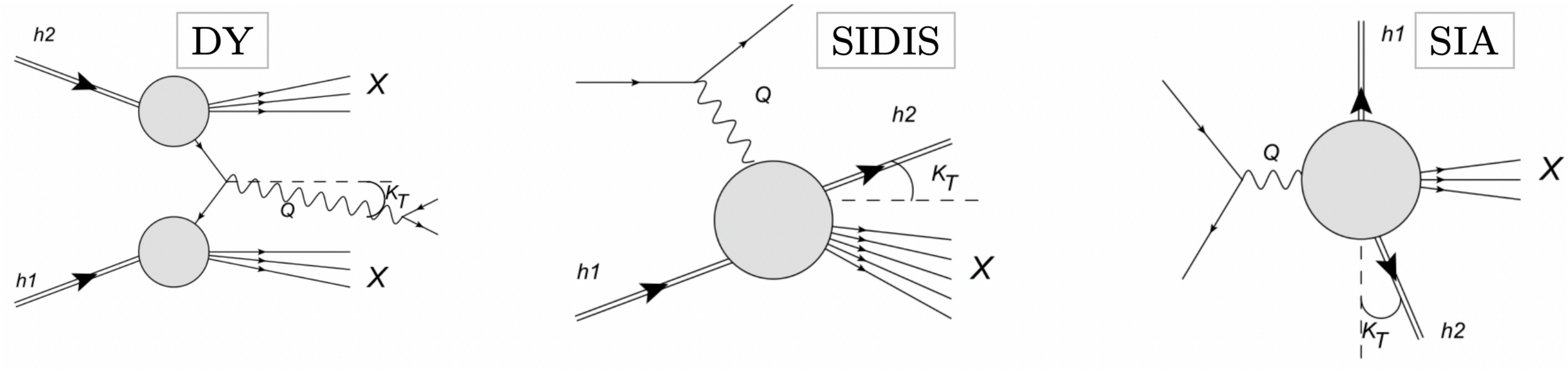
Outline

- Parton Distribution Functions (PDFs)
- Transverse Momentum Dependent Parton Distribution Functions (TMDs)
- Transverse Momentum Moments (TMMs)
    - Zeroth Moment
    - First Moment
    - Second Moment
- Conclusions

Transverse Momentum Dependent Factorization



- Inelastic processes ⇒ Structure of Hadrons (PDFs, TMDPDFs, TMDFFs,...)

- Cross sections **factorize** into different blocks (in the regime where $Q^2 \gg \Lambda^2_{QCD}$ and $Q^2 \gg k_T^2$)

$$\frac{d\sigma}{d[\ldots]dQdk_T} \simeq \sigma_0 \int \frac{d^2 b_T}{(2\pi)^2} e^{-ib_T k_T} \underbrace{|H_1(Q)|^2}_{\text{Hard}} \underbrace{F_1(x_1, b_T; \mu, \zeta)}_{\text{TMD}} \underbrace{F_2(x_2, b_T; \mu, \zeta)}_{\text{TMD}}$$

Distribution Functions for Partons

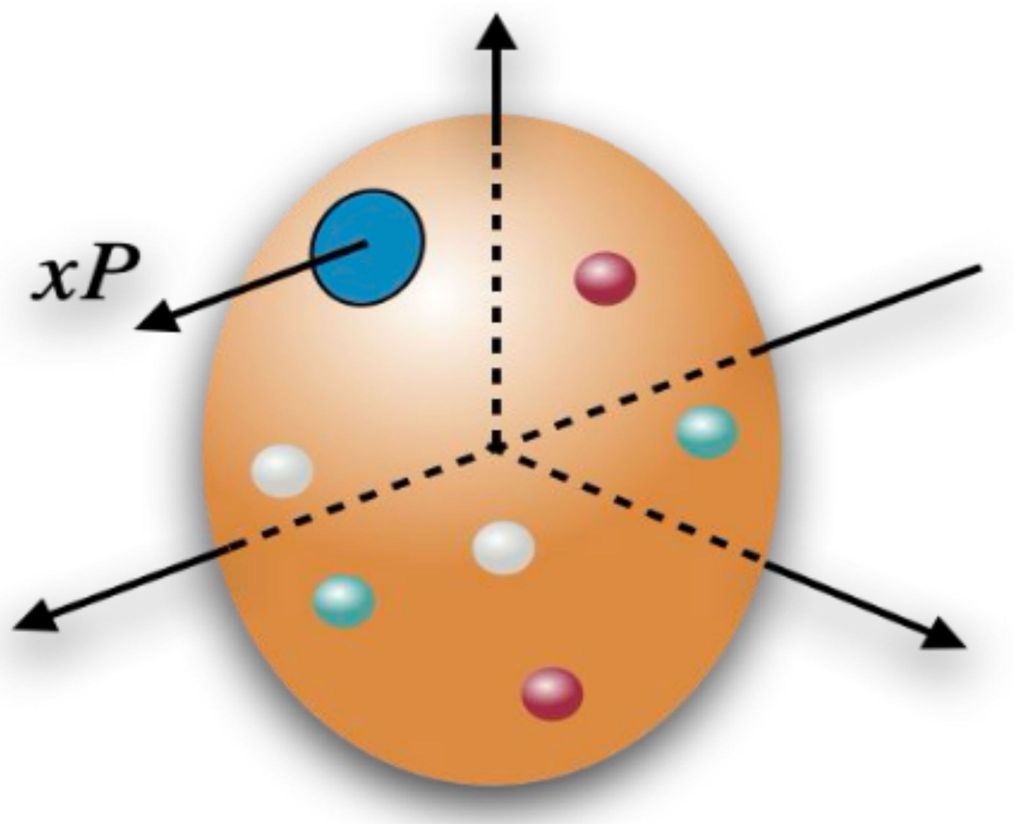
- **Unsubtracted PDF** operator definition

$$f^{[\Gamma]}(x) = \int \frac{db^+}{2\pi} e^{-ib^+(xP^-)} \langle P, S \,|\, \bar{q}^j(b^+) \Gamma_{ji} [b^+, 0] q^i(0) \,|\, P, S \rangle$$

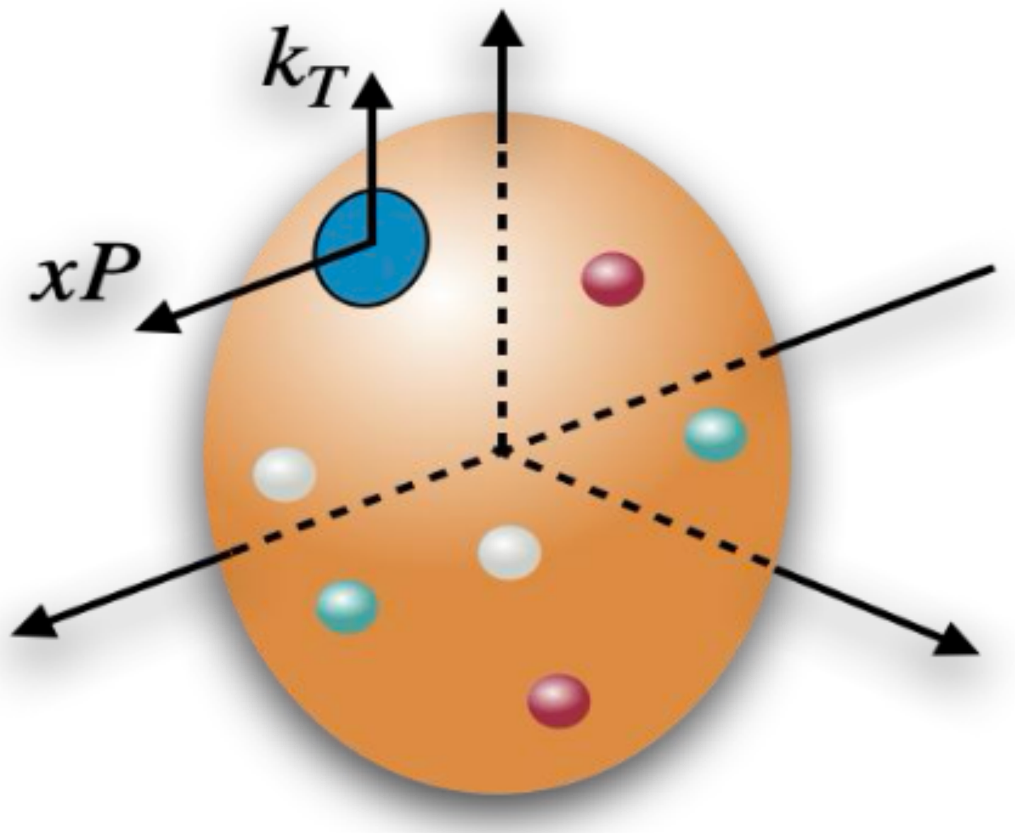
- **Unsubtracted TMD**

$$F^{[\Gamma]}(x, b_T) = \int \frac{db^+}{2\pi} e^{-ib^+(xP^-)} \langle P, S \,|\, \bar{q}^j(b) \Gamma_{ji} [b, b + s\infty][b + s\infty, s\infty][s\infty, 0] q^i(0) \,|\, P, S \rangle$$

- Probability that a parton q carries a fraction $\mathbf{x}$ of the total momentum of the hadron (and transverse momentum k_T for TMD)



PDF



TMD

Distribution Functions for Partons

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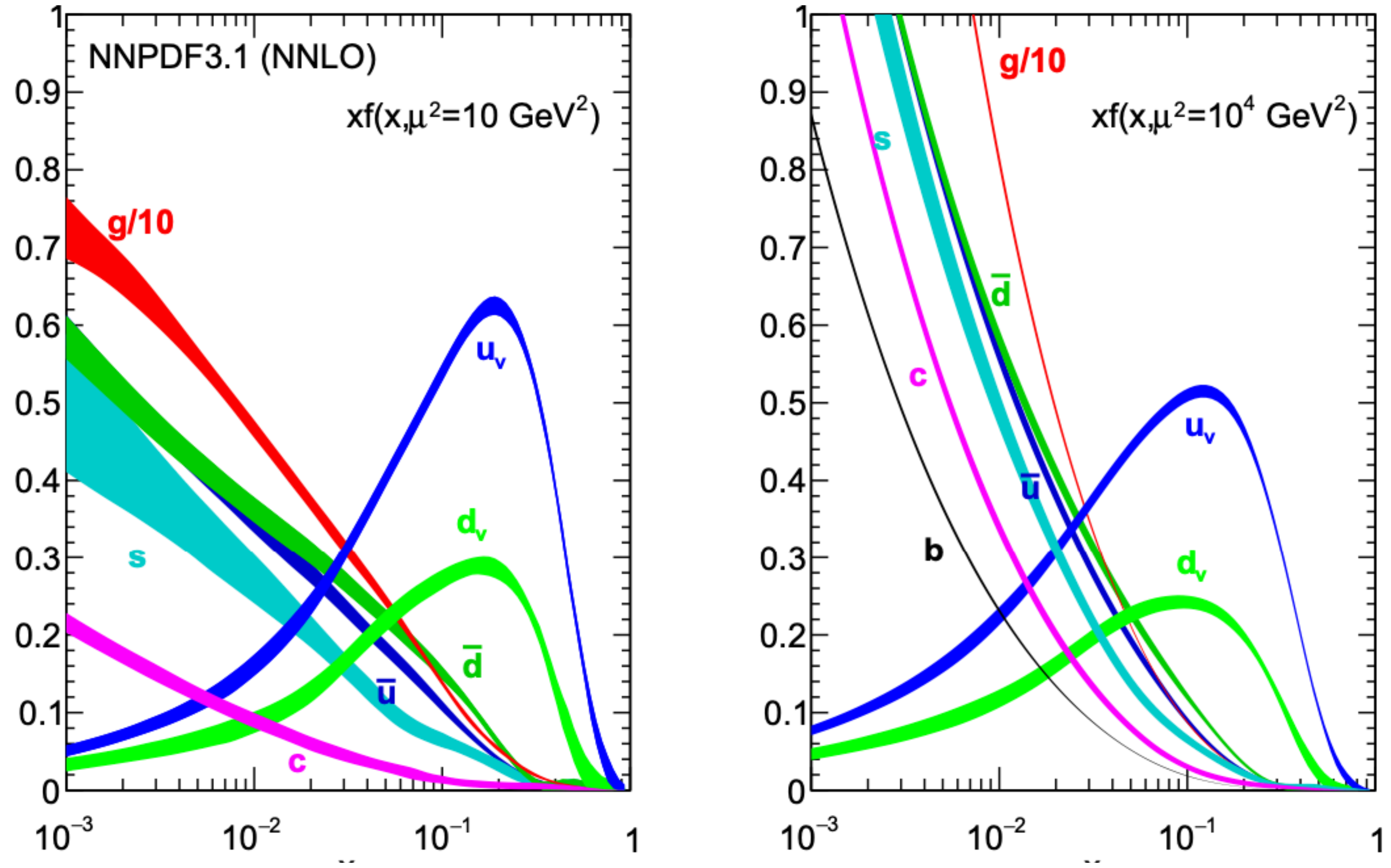
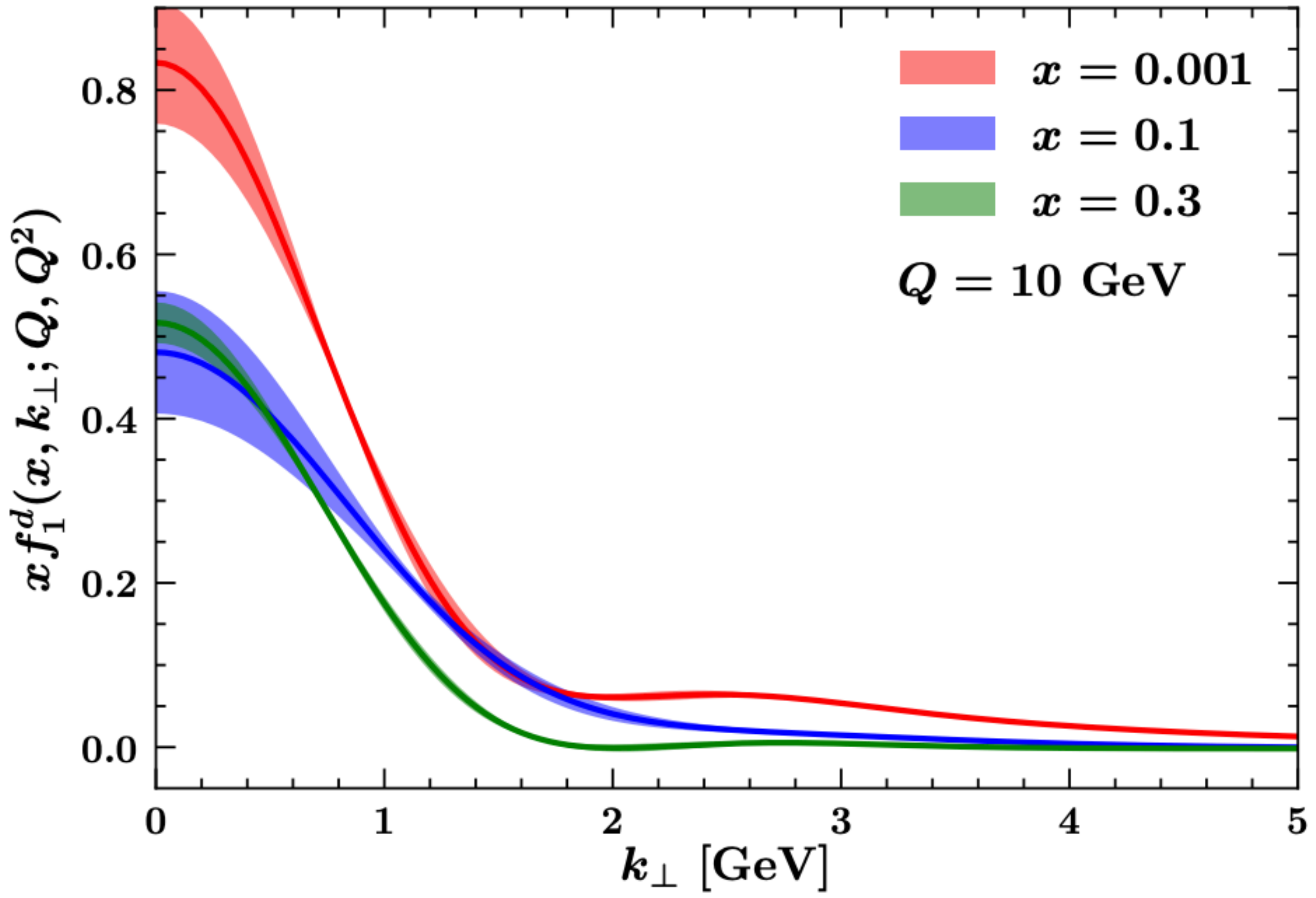
$$F^{[\Gamma]}(x, b_T) = \int \frac{db^+}{2\pi} e^{-ib^+(xP^-)} \langle P, S \,|\, \bar{q}^j(b) \Gamma_{ji} [b, b + s\infty][b + s\infty, s\infty][s\infty, 0] q^i(0) \,|\, P, S \rangle$$

- Probability that a parton q carries a fraction $\mathbf{x}$ of the total momentum of the hadron (and transverse momentum k_T for TMD)

- Combine data from many experiments to extract PDFs and TMDs ⇒ **Global fits** (e.g. NNPDF, Pavia 19, SV19,...)

NNPDF Collaboration

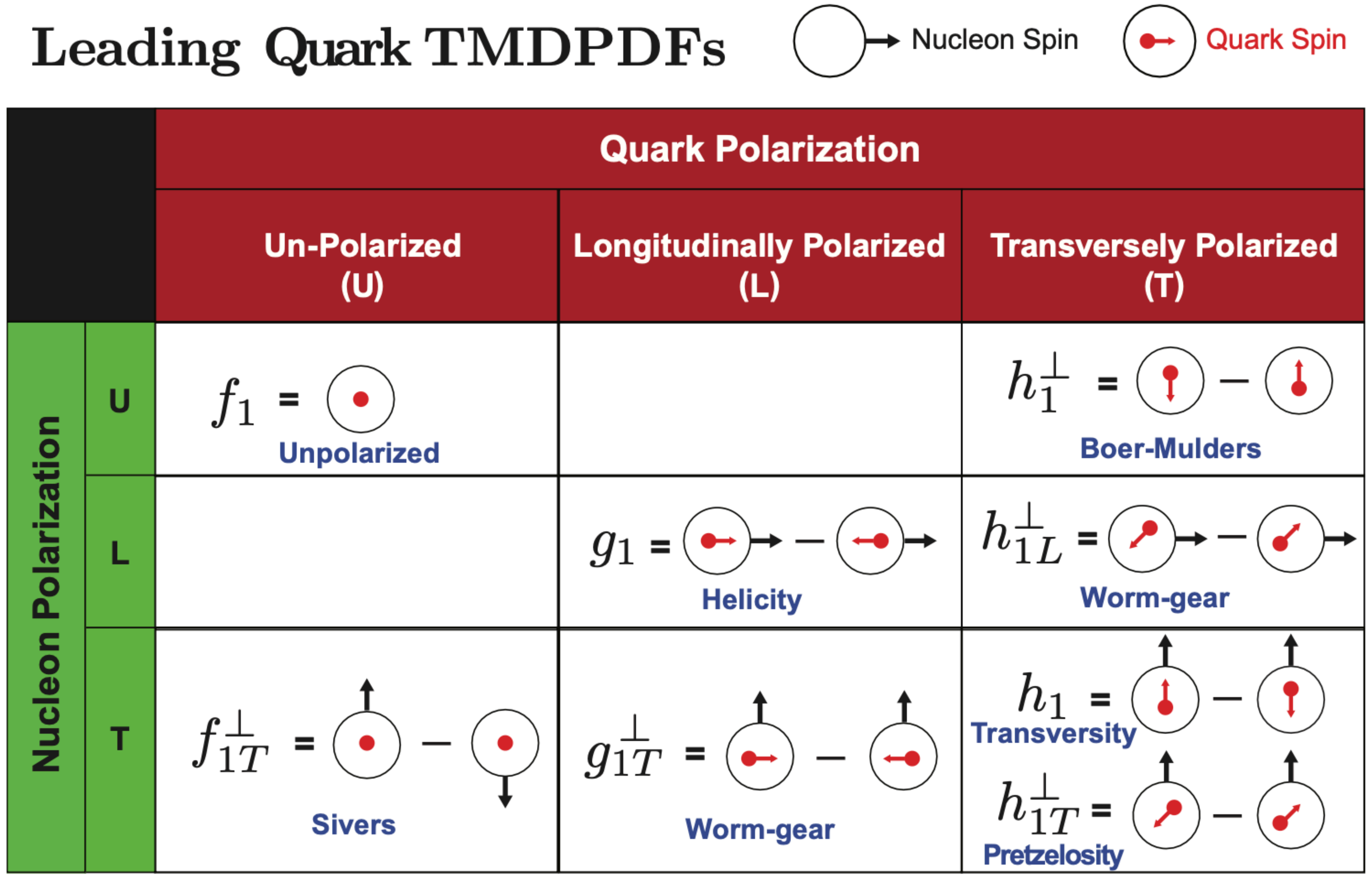




Credit: Pavia19

Parametrization of TMDPDFs

- In momentum space we can decompose TMDPDFs further



TMD Handbook

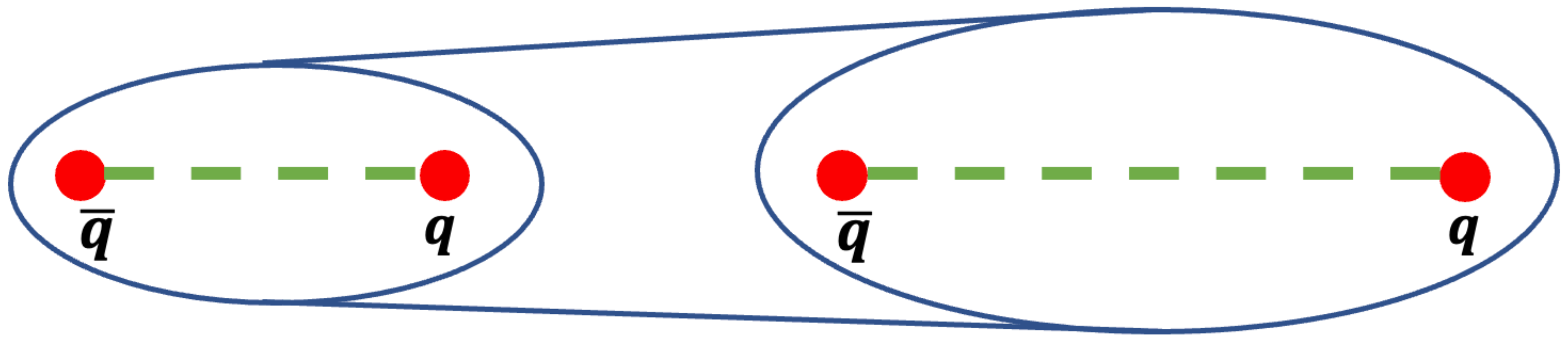
$$F^{[\gamma^+]}(x, k_T) = f_1^{(0)} - \epsilon_T^{\mu\nu} \frac{k_{T\mu} S_{T\nu}}{M} f_{1T}^{\perp(1)}$$

$$F^{[\gamma^+\gamma^5]}(x, k_T) = S_L\, g_1^{(0)} - \frac{(k_T \cdot S_T)}{M} g_{1T}^{\perp(1)}$$

$$F^{[i\sigma^{\alpha+}\gamma^5]}(x, k_T) = S_T^\alpha\, h_1^{(0)} + \frac{S_L k_T^\alpha}{M} h_{1L}^{\perp(0)} - \frac{\epsilon_T^{\alpha\mu} k_{T\mu}}{M} h_1^{\perp(1)} - \frac{\mathbf{k}_T^2}{M^2} \left(\frac{g_T^{\alpha\mu}}{2} + \frac{k_T^\alpha k_T^\mu}{\mathbf{k}_T^2} \right) S_{T\mu}\, h_{1T}^{\perp(2)}$$

- Explicit index for large k_T asymptotic behavior: $F^{(n)}(x, k_T) \propto (k_T^2)^{-n-1}$

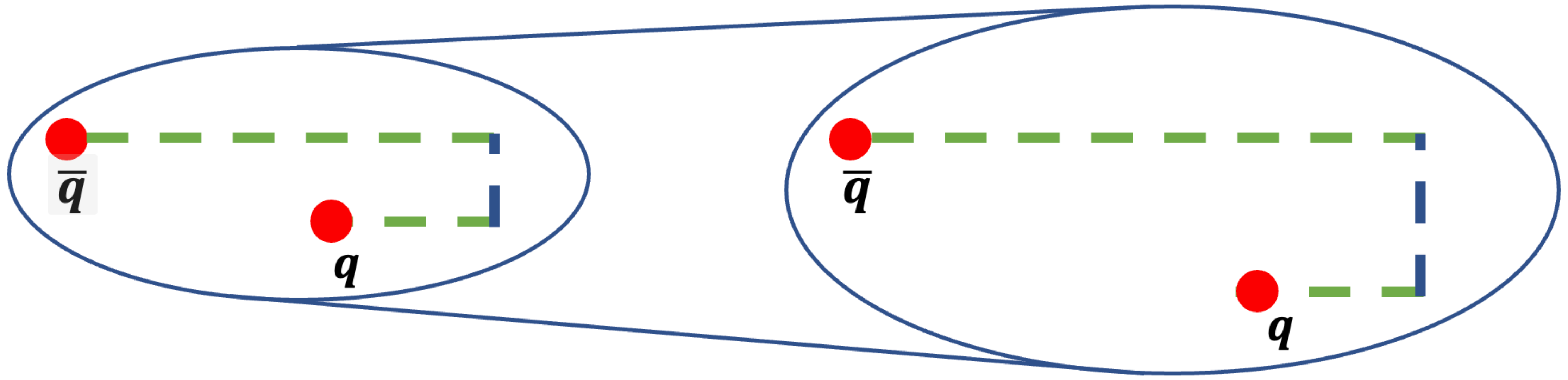
PDF and TMD Evolution



- UV Renormalization ⇒ Scale μ
- PDF evolution ⇒ **DGLAP equation**

$$\frac{df(x,\mu)}{d\ln\mu^2} = \int_x^1 dy\, P\left(\frac{x}{y}\right) f(y,\mu) \equiv P \otimes f(x,\mu)$$

- Known up to 4 loop accuracy (N^{4}LO)

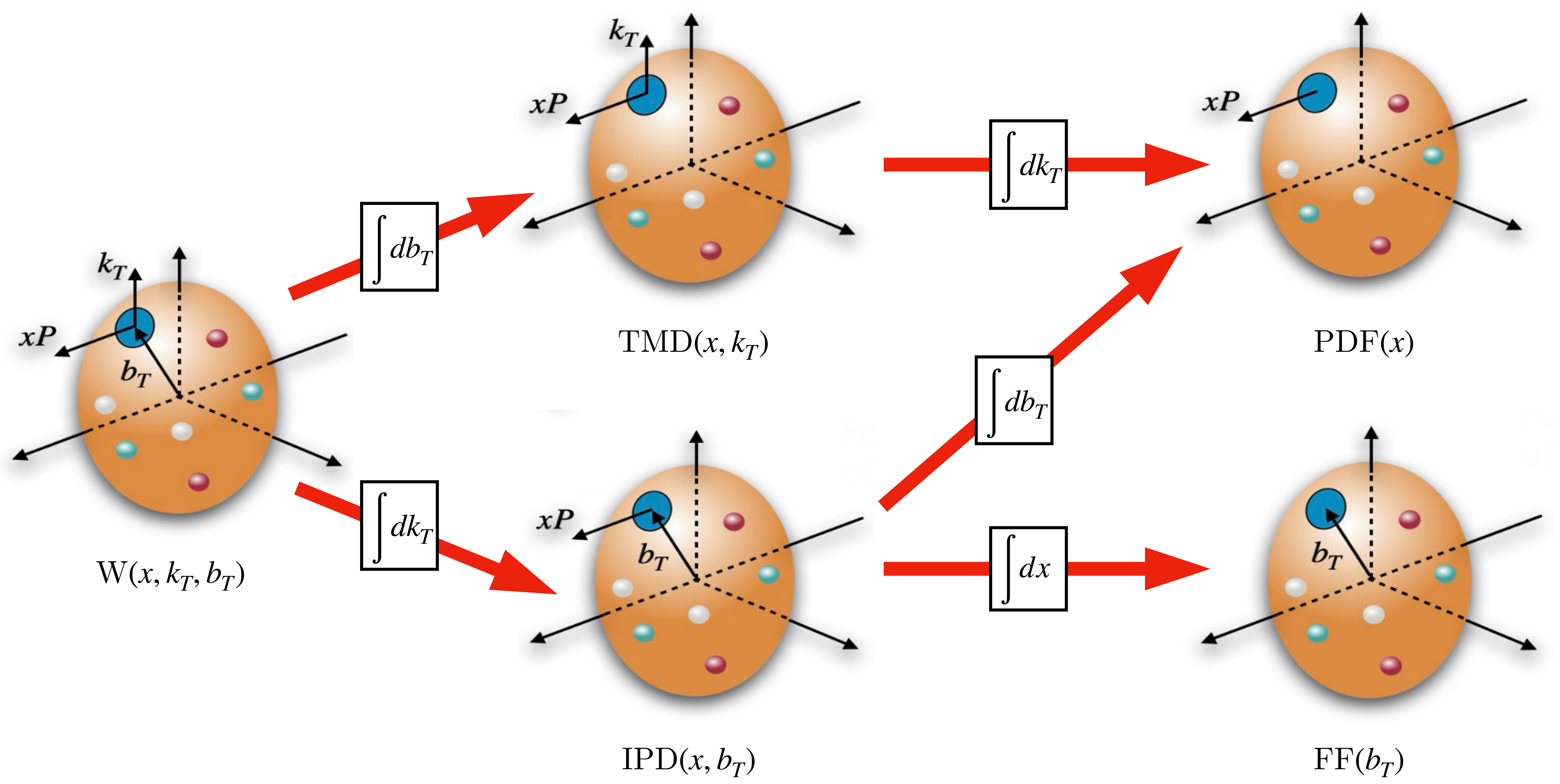


- UV+Rapidity regularization ⇒ Scales μ and ζ
- **Two scales** to evolve ⇒ **UV R.G. + CS**

$$\frac{d\ln F(x,b_T;\mu,\zeta)}{d\ln\mu^2} = \frac{\gamma_F(\mu,\zeta)}{2}; \quad \frac{d\ln F(x,b_T;\mu,\zeta)}{d\ln\zeta} = -\mathscr{D}(b_T,\mu)$$

- **Collins-Soper kernel** $\mathscr{D}(b_T,\mu)$ (New ingredient)

The hadron structure landscape



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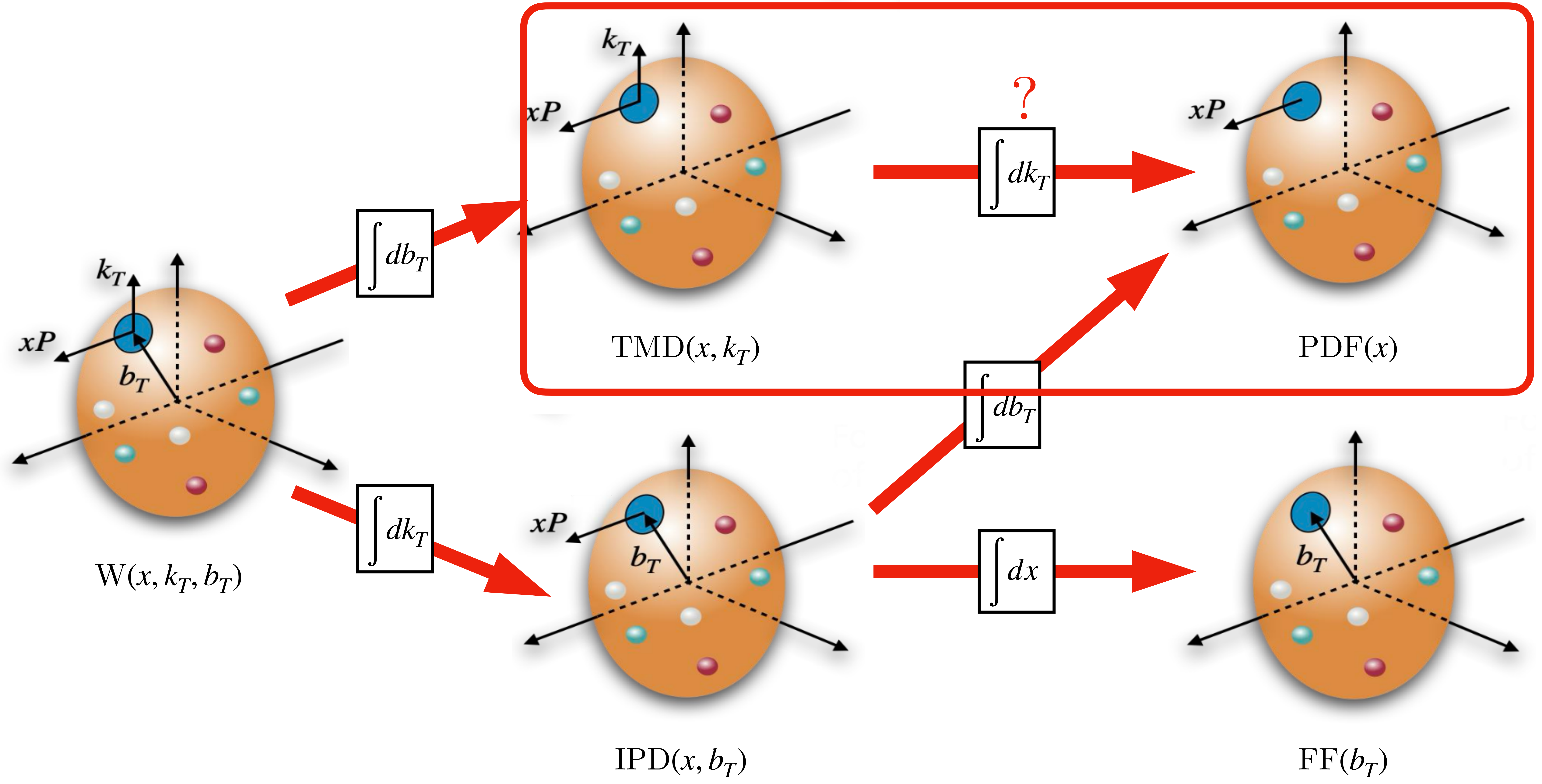

Integrated TMDs

- Collinear matrix elements of interest

$$\mathbb{M}^{[\Gamma]}_{\nu_1\ldots\nu_r}(x) \equiv \int \frac{db^+}{2\pi} e^{-ib^+(xP^-)} \langle P,S\,|\,\bar{q}^j_f(b^+)\overleftarrow{D}_{\nu_1}\ldots\overleftarrow{D}_{\nu_r}\Gamma_{ji}[b^+,0]q^i_f(0)\,|\,P,S\rangle \xrightarrow{\;r=0\;} f^{[\Gamma]}(x)\ \text{(PDF)}$$

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- Renormalization procedures $\Rightarrow$ Mismatch in scale dependence

$$\mathbb{M}^{[\Gamma]}_{\nu_1\ldots\nu_r}(x,\mu) \quad \xleftrightarrow{??} \quad \int d^2\mathbf{k}_T\,\mathbf{k}_{T\nu_1}\ldots\mathbf{k}_{T\nu_r}\,F^{[\Gamma]}(x,k_T,\mu,\zeta)$$

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- In position space (FT: $k_T \to b_T$) ⇒ Related by the **Operator Product Expansion** (OPE) in small b_T regime

$$F^{[\Gamma]}(x,b_T,\mu,\zeta) = \underbrace{C(x,\mu_{OPE},\mu,\zeta)}_{\text{Matching Coeff.}} \otimes f^{[\Gamma]}(x,\mu_{OPE}) + \mathcal{O}(b_T^2)$$

Transverse Momentum Moments (TMMs)

- **Weighted integrals** with a momentum cut-off $|k_T| < \mu$, two possibilities:
  - For TMDs evaluated in the **ζ-prescription**

$$\tilde{M}^{[\Gamma]}_{\nu_1 \ldots \nu_r}(x, \mu) \equiv \int^{\mu} d^2\mathbf{k}_T \, \mathbf{k}_{T\nu_1} \ldots \mathbf{k}_{T\nu_r} \, \tilde{F}^{[\Gamma]}(x, k_T)$$

  - For TMDs in general scales we can also achieve cancellation of C.S. kernel for $\zeta = \mu^2$

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Transverse Momentum Moments (TMMs)

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Optimal TMD (Independent on both scales!)

See: JHEP 06 (2020) 137

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- TMMs coincide with collinear quantities in a minimal subtraction scheme different from $\overline{\text{MS}}$-scheme:

$$\underbrace{\tilde{M}^{[\Gamma]}_{\nu_1 \ldots \nu_r}(x, \mu)}_{\text{TMM}} = \underbrace{\mathbb{M}^{[\Gamma]}_{\nu_1 \ldots \nu_r}(x, \mu)}_{\text{Collinear}} + \mathcal{O}(\mu^{-2}) \quad \Longrightarrow \quad \frac{d\tilde{M}^{[\Gamma]}_{\nu_1 \ldots \nu_r}(x, \mu)}{d \ln \mu^2} = P' \otimes \tilde{M}^{[\Gamma]}_{\nu_1 \ldots \nu_r}(x, \mu)$$

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M. A. Ebert, et al. JHEP 07, 129 (2022)

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DGLAP kernel in "TMD-scheme"

Zeroth Transverse Momentum Moment

- Integrals to calculate are of the form

$$\mathcal{G}_{m,n}[\tilde{F}](x,\mu) \equiv \int^{\mu} d^2\mathbf{k}_T \left(\frac{\mathbf{k}_T^2}{2M^2}\right)^m \tilde{F}^{(n)}(x,k_T)$$

- In the large-μ regime this integral is logarithmic divergent if $m = n$ ($\mathcal{G}_{n,n}[F](x,\mu) \propto \ln(\mu)$), power-like divergent for $m > n$ ($\mathcal{G}_{n+l,n}[F](x,\mu) \propto \mu^{2l}$ for $l > 0$) and convergent for $m < n$

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- For the zeroth transverse moment we recover twist-two PDFs

$$\tilde{M}^{[\gamma^+]}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T \tilde{F}^{[\gamma^+]}(x,k_T) = \mathscr{G}_{0,0}[f_1^{(0)}](x,\mu)$$

$$\tilde{M}^{[\gamma^+\gamma^5]}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T \tilde{F}^{[\gamma^+\gamma^5]}(x,k_T) = S_L\, \mathscr{G}_{0,0}[g_1^{(0)}](x,\mu)$$

$$\tilde{M}^{[i\sigma^{\alpha+}\gamma^5]}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T \tilde{F}^{[i\sigma^{\alpha+}\gamma^5]}(x,k_T) = S_T^{\alpha}\, \mathscr{G}_{0,0}[h_1^{(0)}](x,\mu)$$

$\Rightarrow$

$$\mathscr{G}_{0,0}[f_1^{(0)}](x,\mu) = q_{(TMD)}(x,\mu) + \mathcal{O}(\mu^{-2})$$

$$\mathscr{G}_{0,0}[g_1^{(0)}](x,\mu) = \Delta q_{(TMD)}(x,\mu) + \mathcal{O}(\mu^{-2})$$

$$\mathscr{G}_{0,0}[h_1^{(0)}](x,\mu) = \delta q_{(TMD)}(x,\mu) + \mathcal{O}(\mu^{-2})$$

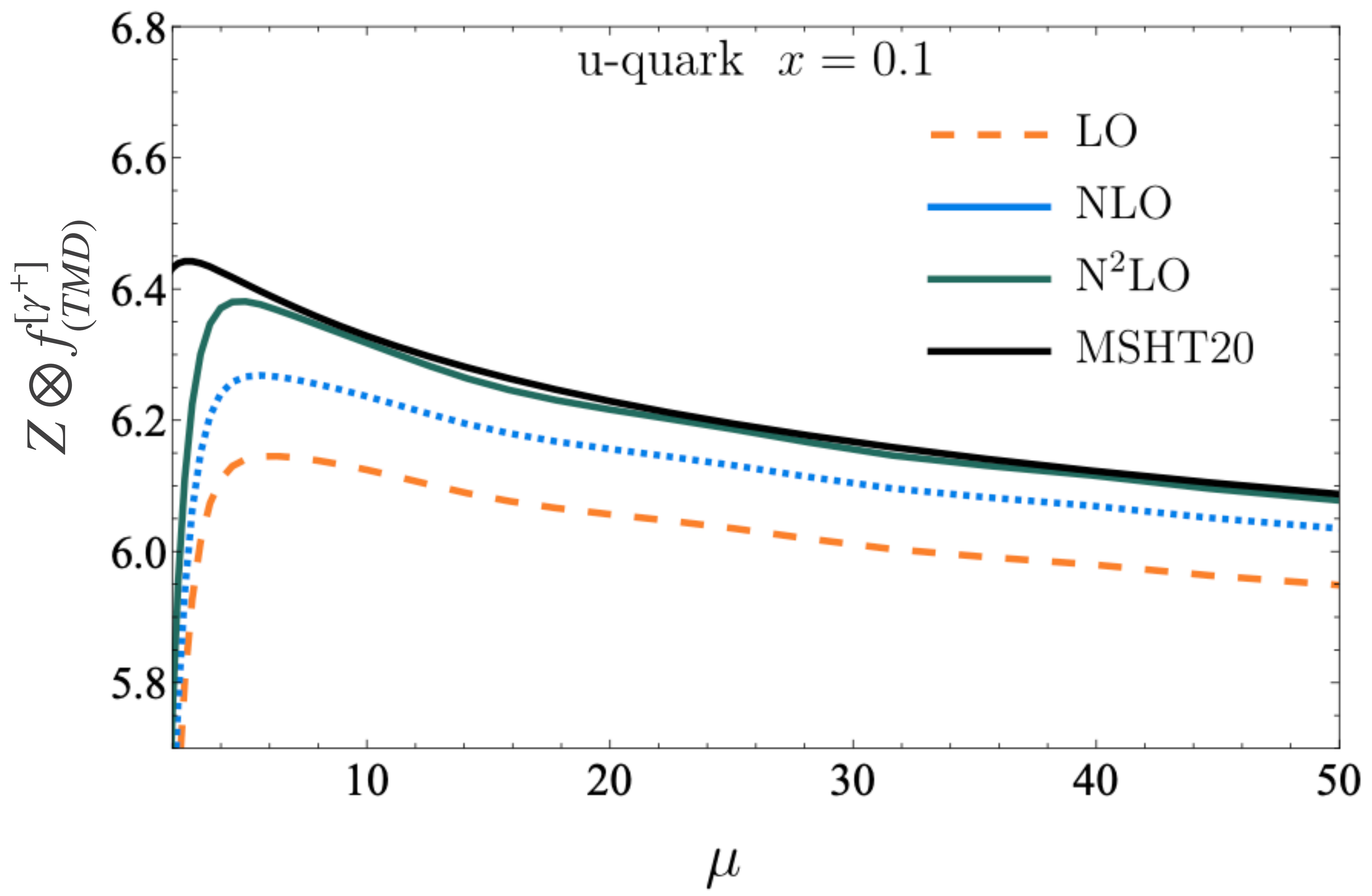
Zeroth Transverse Momentum Moment

- We apply transformation to the matching

$$\mathcal{G}_{0,0}[C \otimes f^{[\Gamma]}_{(\overline{MS})}](x,\mu) = f^{[\Gamma]}_{(TMD)}(x,\mu) + \mathcal{O}(\mu^{-2})$$

- Evolution deviates from $\overline{MS}$ DGLAP at NLO

$$\frac{d f^{[\Gamma]}_{(TMD)}}{d \ln \mu^2}(x,\mu) = P' \otimes f^{[\Gamma]}_{(TMD)}(x,\mu) \quad \Rightarrow \quad P' - P = \mathcal{O}(\alpha_s^2)$$

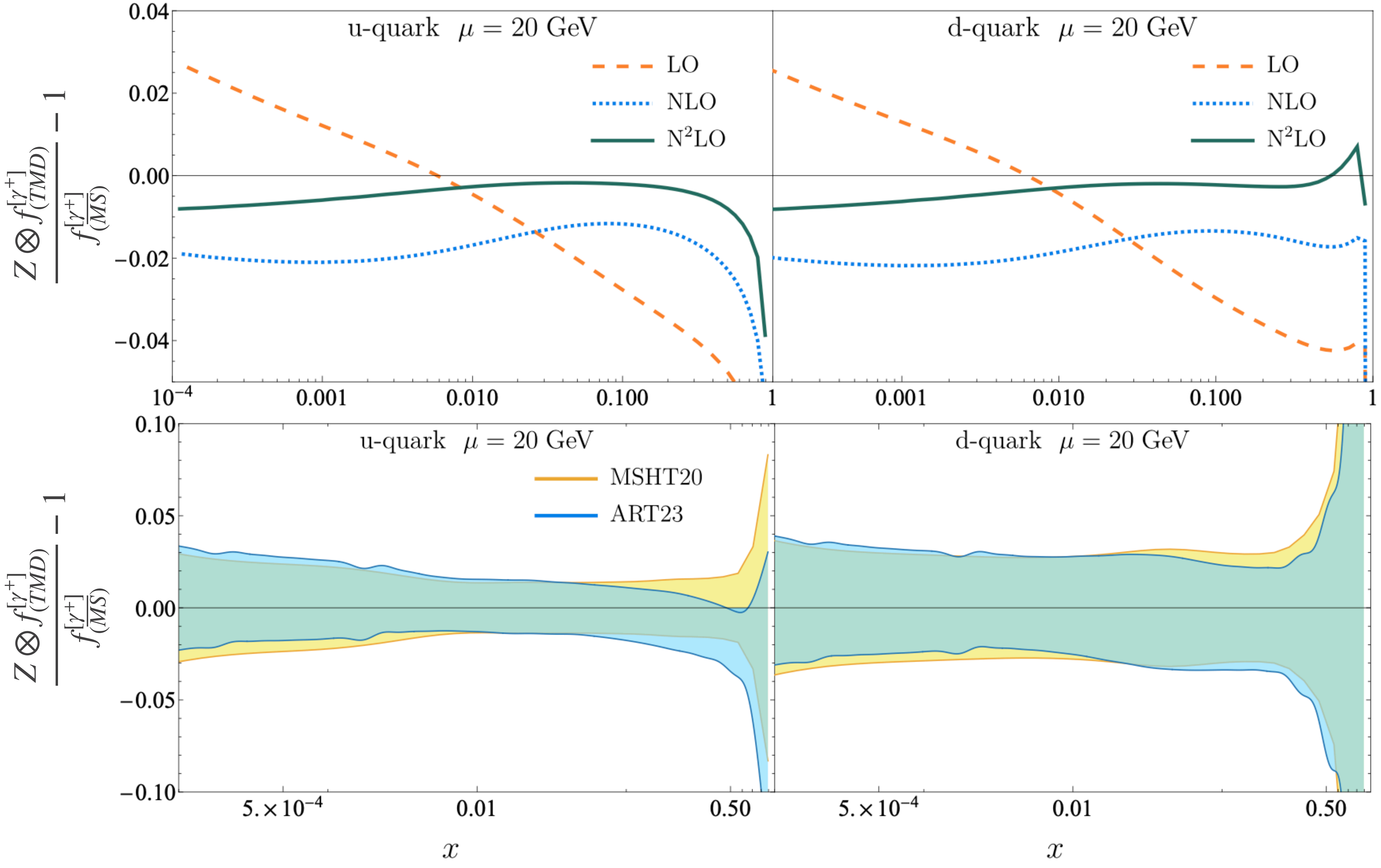


- Transition from TMD-scheme achieved by factor Z (matrix in flavor space)

$$f^{[\Gamma]}_{(\overline{MS})}(x,\mu) = Z \otimes f^{[\Gamma]}_{(TMD)}(x,\mu) = (\mathbf{1} + \alpha_s Z_1(x) + \alpha_s^2 Z_2(x) + \mathcal{O}(\alpha_S^3)) \otimes f^{[\Gamma]}_{(TMD)}(x,\mu)$$

Zeroth Transverse Momentum Moment

- If we compare the collinear distributions at a fixed (large-enough) μ we see great agreement



First Transverse Momentum Moment

- The first TMMs now involve $f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$, $h_{1L}^{\perp(1)}$, and $h_1^{\perp(1)}$

$$\tilde{M}_\mu^{[\gamma^+]}(x,\mu) = \int^\mu d^2\mathbf{k}_T\, \mathbf{k}_{T\mu} \tilde{F}^{[\gamma^+]}(x,k_T) = -\,\epsilon_{T,\mu\nu}\, S_T^\nu\, M \mathcal{G}_{1,1}[f_{1T}^{\perp(1)}](x,\mu)$$

$$\tilde{M}_\mu^{[\gamma^+\gamma^5]}(x,\mu) = \int^\mu d^2\mathbf{k}_T\, \mathbf{k}_{T\mu} \tilde{F}^{[\gamma^+\gamma^5]}(x,k_T) = -\,S_{T\mu}\, M \mathcal{G}_{1,1}[g_{1T}^{\perp(1)}](x,\mu)$$

$$\tilde{M}_\mu^{[i\sigma^{\alpha+}\gamma^5]}(x,\mu) = \int^\mu d^2\mathbf{k}_T\, \mathbf{k}_{T\mu} \tilde{F}^{[i\sigma^{\alpha+}\gamma^5]}(x,k_T) = -\,S_L\, g_{T,\mu\alpha}\, M \mathcal{G}_{1,1}[h_{1L}^{\perp(1)}](x,\mu) - \epsilon_{T,\mu\alpha}\, M \mathcal{G}_{1,1}[h_1^{\perp(1)}](x,\mu)$$

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- The OPE for these TMDs relates them to several collinear distributions $f_t(x,\mu)$ (with different twist t)

$$\tilde{F}^{(1)}(x,b_T) = \sum_t R_t \otimes C_t(x_1,\ldots,x_i,\mu_{OPE}) \otimes f_t(x_1,\ldots,x_i,\mu_{OPE}) + \mathcal{O}(b_T^2)$$

↖ Projection operator of momentum fractions x_i to x

First Transverse Momentum Moment

- Evolution also deviates from $\overline{MS}$ at NLO

$$\frac{d\,\mathcal{G}_{1,1}[\tilde{F}^{(1)}](x,\mu)}{d\ln\mu^2}(x,\mu) = R_t \otimes P_t' \otimes \mathcal{G}_{1,1}[\tilde{F}^{(1)}](x,\mu) \quad \Longrightarrow \quad P_t' - P_t = \mathcal{O}(\alpha_s^2)$$

- Projection operator R_t prevents transformation to $\overline{MS}$-scheme

First Transverse Momentum Moment

- Evolution also deviates from $\overline{MS}$ at NLO

$$\frac{d\,\mathcal{G}_{1,1}[\tilde{F}^{(1)}](x,\mu)}{d\ln\mu^2}(x,\mu) = R_t \otimes P'_t \otimes \mathcal{G}_{1,1}[\tilde{F}^{(1)}](x,\mu) \longrightarrow P'_t - P_t = \mathcal{O}(\alpha_s^2)$$

- Projection operator R_t prevents transformation to $\overline{MS}$-scheme

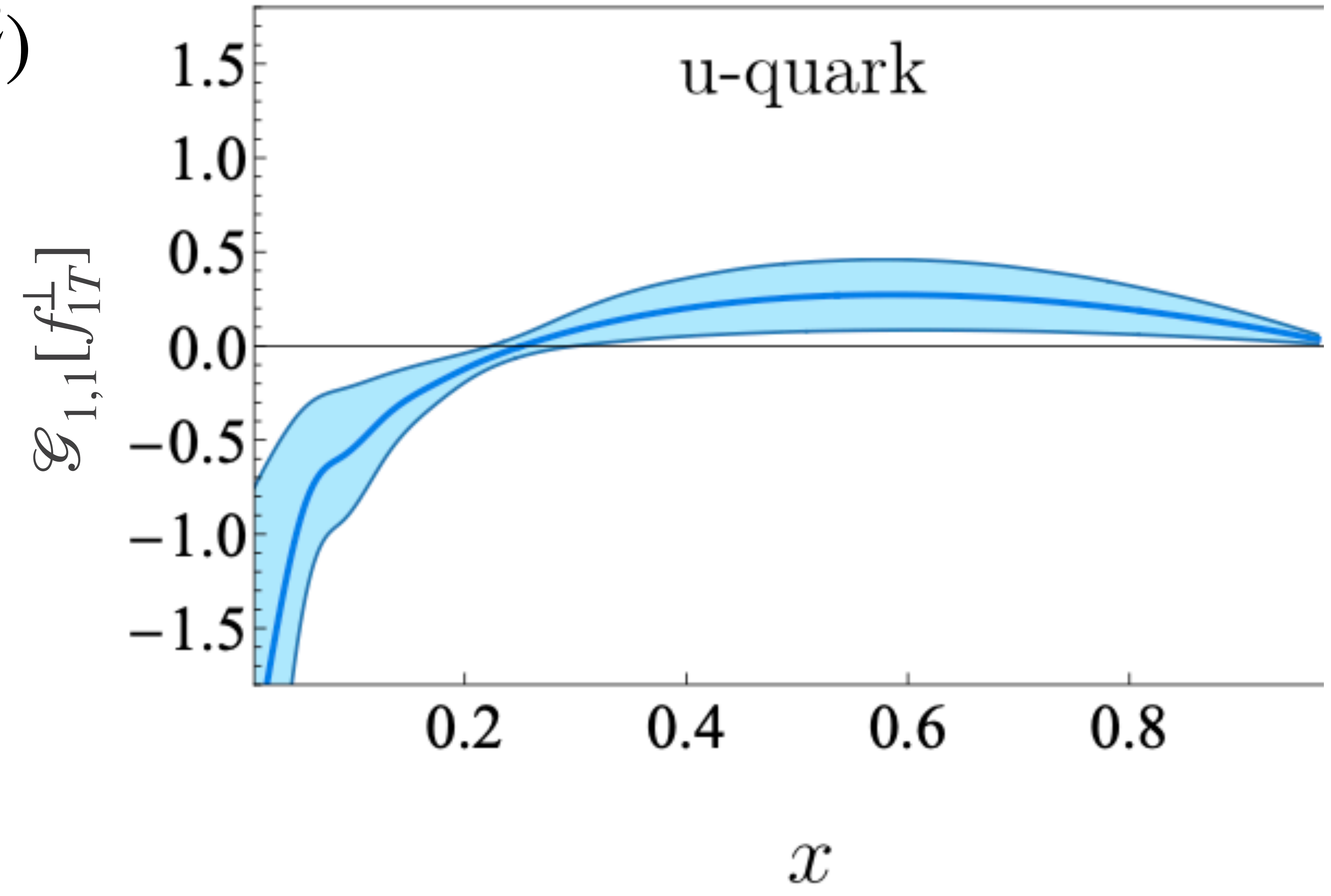
- First TMM of **Sivers**: $\mathcal{G}_{1,1}[f_{1T}^{\perp(1)}] = \pm\pi T_{(TMD)} + \mathcal{O}(\mu^{-2})$ (Qiu-Sterman)

- **Mean transverse momentum shift** of a parton inside a transversely polarized nucleon

$$\langle \mathbf{k}^f_{T,\nu}\rangle(\mu) = -\epsilon_{T,\nu\rho}S_T^\rho M\int_0^1 dx\,\mathcal{G}_{1,1}[f_{1T}^{\perp(1)}](x,\mu)$$

$$\longrightarrow \langle \mathbf{k}^u_{T,1}\rangle(\mu) = -0.011\text{GeV}$$

$\mu = 10\,\text{GeV}\ (\nu = 1)$



Second Transverse Momentum Moment

- The second TMMs have the general form

$$\tilde{M}^{[\Gamma]}_{\mu\nu,div}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T\, \mathbf{k}_{T\mu}\mathbf{k}_{T\nu}\, \tilde{F}^{[\Gamma]}_{P_S}(x,k_T) = C^{[\Gamma]}_{\mu\nu}(S)M^2\mathcal{G}_{1,0}[f^{(0),[\Gamma]}](x,\mu) + D^{[\Gamma]}_{\mu\nu}(S)M^2\mathcal{G}_{2,2}[f^{(2),[\Gamma]}](x,\mu)$$

- Operation $\mathcal{G}_{1,0}$ has **power-like divergences** $\propto \mu^2 \longrightarrow$ Subtraction

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- Operation $\mathcal{G}_{1,0}$ has **power-like divergences** $\propto \mu^2 \longrightarrow$ Subtraction

- The OPE for these TMDs is

$$\tilde{F}^{(0)}(x,b_T) = [C \otimes f](x) + b_T^2 \sum_t [C_t \otimes f_t](x) + \mathcal{O}(b_T^4)$$

$$\mathcal{G}_{1,0}[\tilde{F}^{(0)}](x,\mu) \equiv \frac{\mu^2}{2M^2} AS[\mathcal{G}_{1,0}[\tilde{F}^{(0)}]](x,\mu) + \overline{\mathcal{G}}_{1,0}[\tilde{F}^{(0)}](x,\mu)$$

Second Transverse Momentum Moment

- Subtracting asymptotic term we get

$$\tilde{M}^{[\gamma^+]}_{\mu\nu}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T\, \mathbf{k}_{T\mu}\mathbf{k}_{T\nu}\, \tilde{F}^{[\gamma^+]}_{P_S}(x,k_T) = -\, g_{T,\mu\nu}\, M^2 \overline{\mathcal{G}}_{1,0}[f_1^{(0)}](x,\mu)$$

$$\tilde{M}^{[\gamma^+\gamma^5]}_{\mu\nu}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T\, \mathbf{k}_{T\mu}\mathbf{k}_{T\nu}\, \tilde{F}^{[\gamma^+\gamma^5]}_{P_S}(x,k_T) = -\, S_L\, g_{T,\mu\nu}\, M^2 \overline{\mathcal{G}}_{1,0}[g_1^{(0)}](x,\mu)$$

$$\tilde{M}^{[i\sigma^{\alpha+}\gamma^5]}_{\mu\nu}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T\, \mathbf{k}_{T\mu}\mathbf{k}_{T\nu}\, \tilde{F}^{[i\sigma^{\alpha+}\gamma^5]}_{P_S}(x,k_T) = -\, S_{T,\alpha}\, g_{T,\mu\nu}\, M^2 \overline{\mathcal{G}}_{1,0}[h_1^{(0)}](x,\mu) + (g_{T,\mu\alpha}S_{T,\nu} + g_{T,\nu\alpha}S_{T,\mu} - g_{T,\mu\nu}S_{T,\alpha})\frac{M^2}{2}\mathcal{G}_{2,2}[h_{1T}^{\perp(2)}](x,\mu)$$

Second Transverse Momentum Moment

- Subtracting asymptotic term we get

$$\tilde{M}^{[\gamma^+]}_{\mu\nu}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T\, \mathbf{k}_{T\mu}\mathbf{k}_{T\nu}\, \tilde{F}^{[\gamma^+]}_{P_S}(x,k_T) = -\, g_{T,\mu\nu}\, M^2 \overline{\mathcal{G}}_{1,0}[f_1^{(0)}](x,\mu)$$

$$\tilde{M}^{[\gamma^+\gamma^5]}_{\mu\nu}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T\, \mathbf{k}_{T\mu}\mathbf{k}_{T\nu}\, \tilde{F}^{[\gamma^+\gamma^5]}_{P_S}(x,k_T) = -\, S_L\, g_{T,\mu\nu}\, M^2 \overline{\mathcal{G}}_{1,0}[g_1^{(0)}](x,\mu)$$

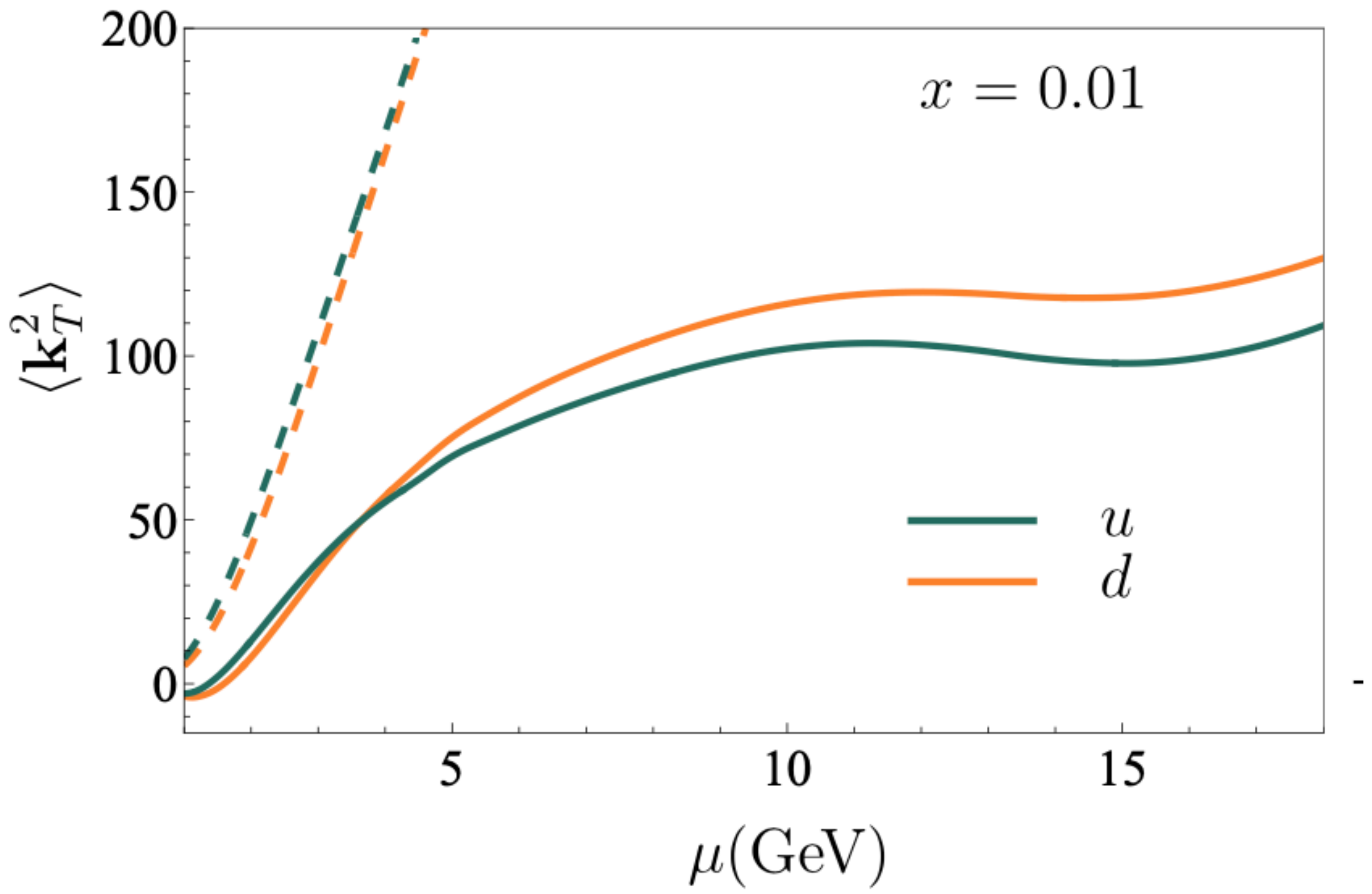
$$\tilde{M}^{[i\sigma^{\alpha+}\gamma^5]}_{\mu\nu}(x,\mu) = \int^{\mu} d^2\mathbf{k}_T\, \mathbf{k}_{T\mu}\mathbf{k}_{T\nu}\, \tilde{F}^{[i\sigma^{\alpha+}\gamma^5]}_{P_S}(x,k_T) = -\, S_{T,\alpha}\, g_{T,\mu\nu}\, M^2 \overline{\mathcal{G}}_{1,0}[h_1^{(0)}](x,\mu)$$
$$+(g_{T,\mu\alpha}S_{T,\nu} + g_{T,\nu\alpha}S_{T,\mu} - g_{T,\mu\nu}S_{T,\alpha})\frac{M^2}{2}\mathcal{G}_{2,2}[h_{1T}^{\perp(2)}](x,\mu)$$

- We can estimate **average width** of the TMDs

$$\langle \mathbf{k}_T^2 \rangle(x,\mu) = -\, g_T^{\mu\nu}\, \tilde{M}^{[\gamma^+]}_{\mu\nu}(x,\mu) = 2M^2 \overline{\mathcal{G}}_{1,0}[f_1^{(0)}](x,\mu)$$

Second Transverse Momentum Moment

- Representation of average width for a fixed x confirms that power growth (dashed line) is canceled



Second Transverse Momentum Moment

- Representation of average width for a fixed x confirms that power growth (dashed line) is canceled

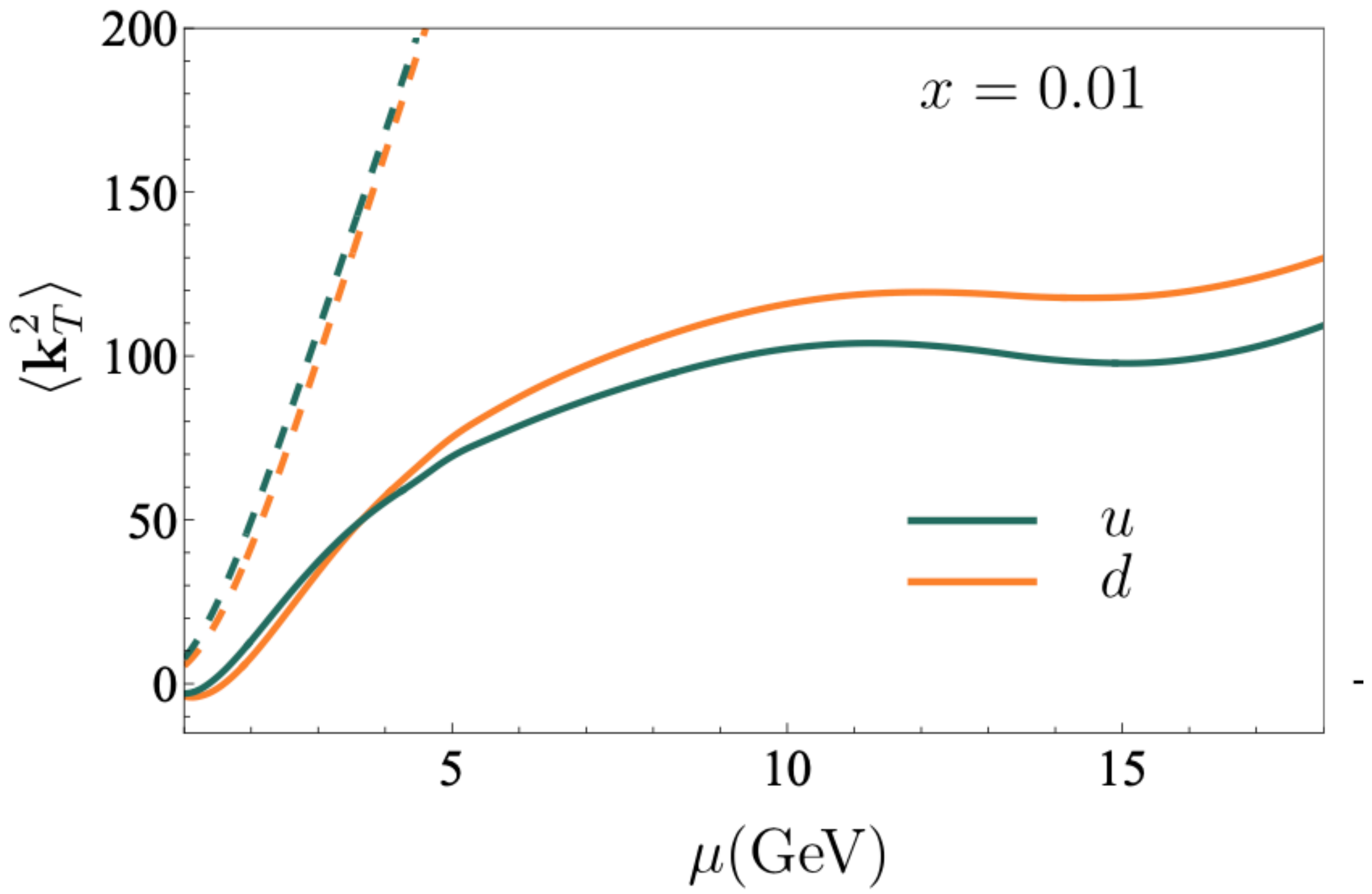
- **Width averaged with x**

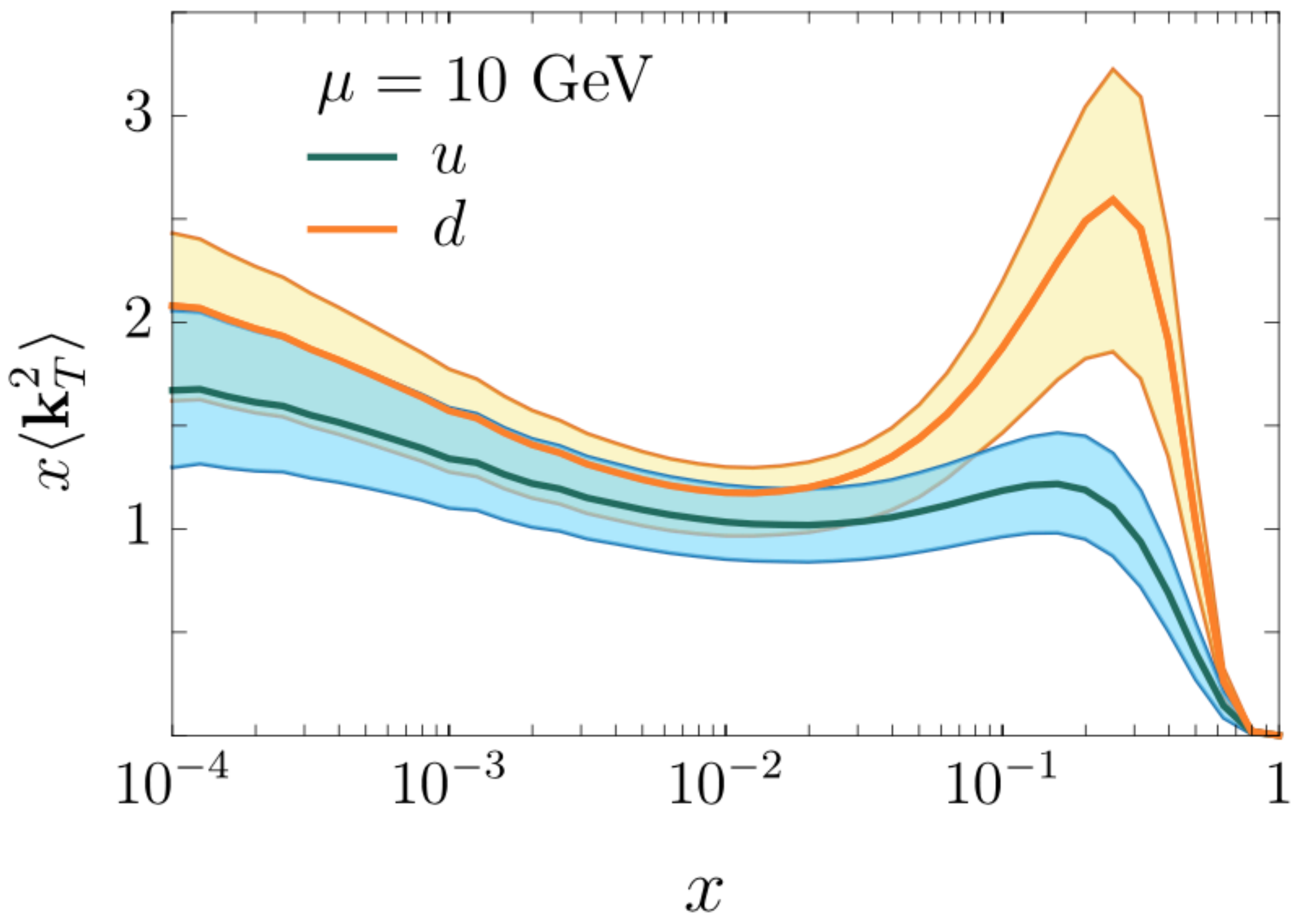
$$\langle x\mathbf{k}_T^2\rangle(\mu) = 2M^2 \int_0^1 dx\, x\, \overline{\mathscr{G}}_{1,0}[f_1](x,\mu)$$

- Inclusion of x-weight facilitates convergence for ART23 extraction

- At $\mu = 10$ GeV we obtain

$$\langle x\mathbf{k}_T^2\rangle_u = 0.52 \pm 0.12 \text{ GeV}^2 \qquad \langle x\mathbf{k}_T^2\rangle_d = 1.10 \pm 0.28 \text{ GeV}^2$$





Conclusions

- Weighted integrals (TMMs) establish a theoretically solid relation between TMDs and different collinear quantities
- TMMs of TMDs in ζ-prescription (or all scales set equal) obey DGLAP evolution in some minimal subtraction scheme (TMD-scheme)
- Transformation to $\overline{MS}$-scheme is done via a perturbatively calculable coefficient Z
- Zeroth TMM is related to leading power PDFs
- First TMMs are related to the average transverse momentum shift of partons
- Second TMMs provide information on the average width of partons inside the hadron

Thank you for your attention!

Factorization Power Corrections

$$\frac{d\sigma}{d[\ldots]dQdk_T} = H_1(Q^2) \otimes F_1(x_1, k_T) \otimes F_2(x_2, k_T) \quad \leftarrow \textbf{LP}$$

$$+\frac{k_T}{Q} H_2(Q^2) \otimes F_3(x_3, k_T) \otimes F_4(x_4, k_T) \quad \leftarrow \textbf{NLP}$$

$$+\frac{k_T^2}{Q^2} H_3(Q^2) \otimes F_5(x_5, k_T) \otimes F_6(x_6, k_T) + \ldots \quad \leftarrow \textbf{NNLP}$$

- Leading power of factorization is widely studied
- Different theoretical formalisms to study sub-leading contributions (SCET, small-x, background field method,…)
- New contributions present more fields (more than one momentum fraction x_i), we order them by **twist** ($t = D - S$)

The ζ-prescription construction

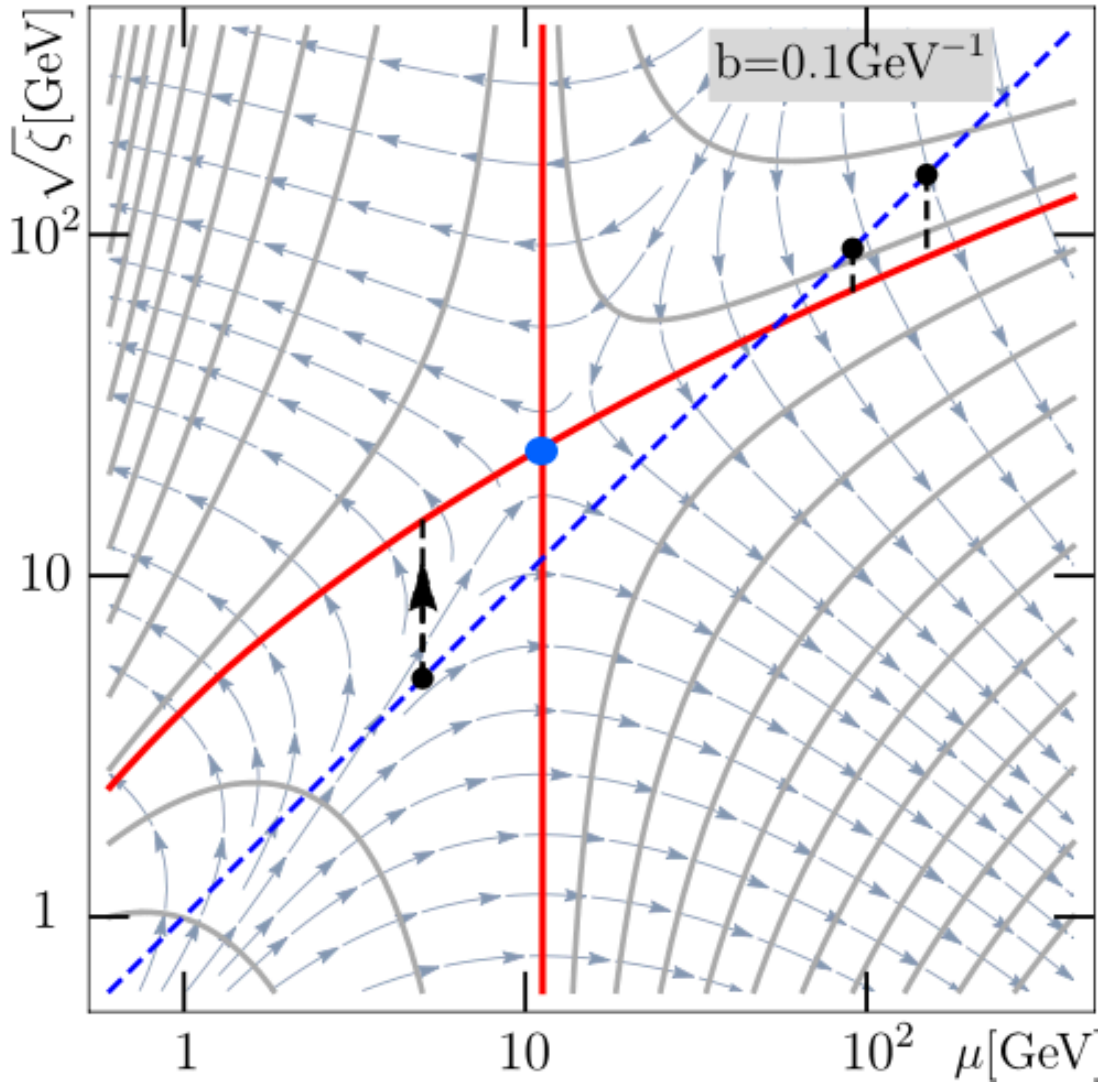
- Evolution equations can be expressed as

$$\nabla F = \left(\frac{d}{d\ln\mu^2}, \frac{d}{d\ln\zeta}\right) F = \left(\frac{\gamma_F(\mu,\zeta)}{2}, -\mathcal{D}(b_T,\mu)\right) F = \mathbf{E}\, F$$

- $\nabla \times \mathbf{E} = 0 \longrightarrow$ **Equipotential (null-evolution) lines**

$$\gamma_F(\mu, \zeta_\mu(b_T)) = 2\mathcal{D}(b_T,\mu)\frac{d\ln\zeta_\mu(b_T)}{d\ln\mu^2}$$

- **Saddle point** $(\mu_0, \zeta_0) \longrightarrow \mathbf{E}(\mu_0,\zeta_0) = (0{,}0) \longrightarrow$ **Special line**



Credit: SV19

- **Optimal TMD** is defined in this special equipotential line

$$F(x, b_T; \mu_0, \zeta_{\mu_0}(b_T)) \equiv \tilde{F}(x, b_T)$$

No scale dependence!

Evolution

$$F\left(x, b_T; Q, Q^2\right) = \left(\frac{Q^2}{\zeta_Q(b_T)}\right)^{-\mathcal{D}(b_T,Q)} \tilde{F}(x, b_T)$$