Leading-Twist Flavor Singlet Quark TMDs at small-x

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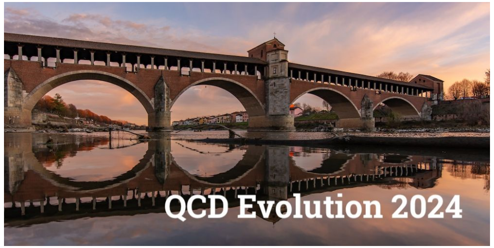
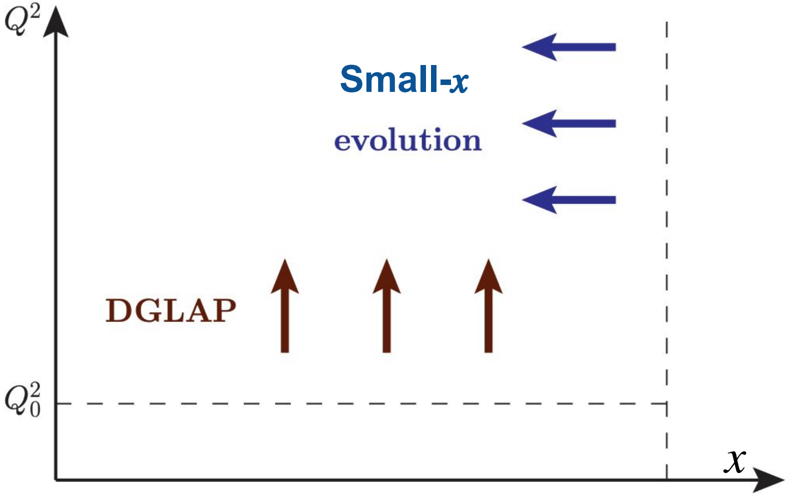
In collaboration with:
D. Adamiak and M. G. Santiago

Based on: 2108.03667, 2204.11898, 2209.03538, 2310.02231, and an **upcoming paper**

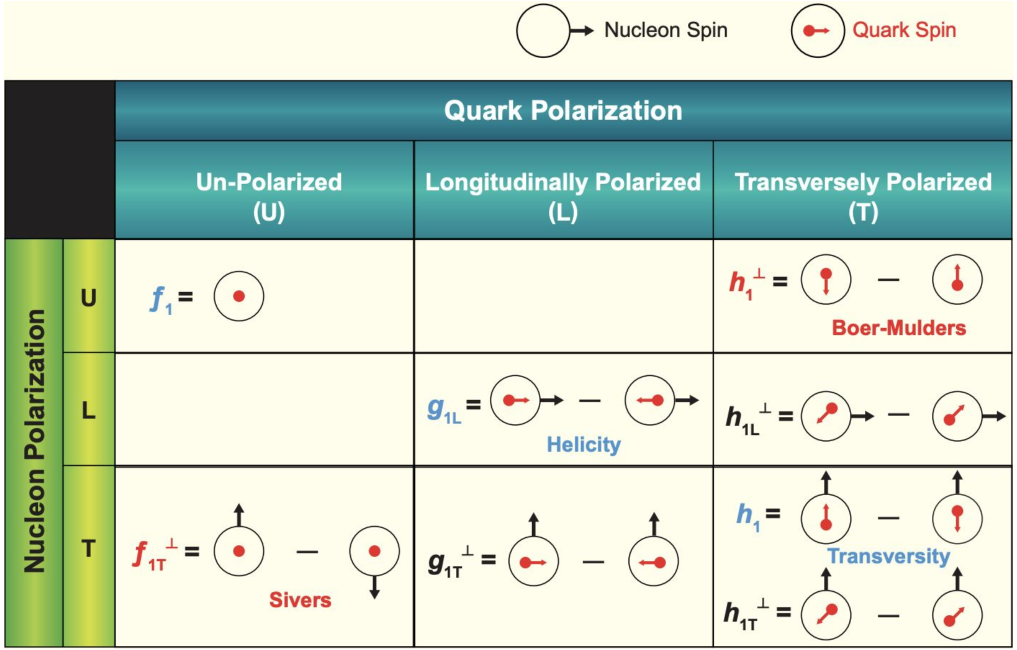


Hadron Structure at small x

- Difficult to study experimentally, as it requires large center-of-mass energy.
- **Small-x evolution** could bridge the gap: parton densities at small x can be written in terms of their values at moderate x, which can be inferred from available data.
- Upcoming high-luminosity, polarized scattering machines like the EIC can cross check our formalism.



Main Objectives



[Accardi et al, 1212.1701]

We study leading-twist quark TMDs at small x, with 2-fold objectives

- Small-x evolution
- Asymptotic behaviors as $x \to 0$

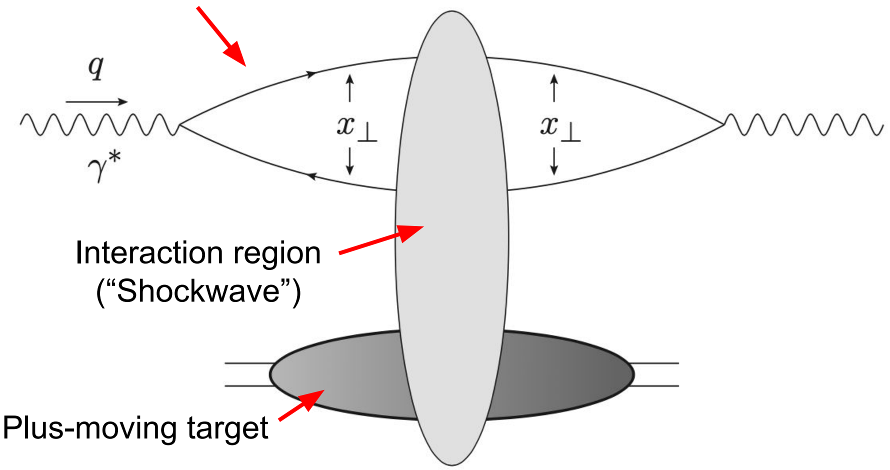
Evolution allows for global fit with small-x data ($x \lesssim 0.1$).

DIS at Small x: The Dipole Picture

- Unpolarized PDF and structure functions, $F_1(x, Q^2)$ and $F_2(x, Q^2)$, relate to the **s-matrix** of dipole-target scattering:

$$S(\underline{x}_1, \underline{x}_0, s) \equiv S_{10}(s) = \frac{1}{N_c} \left\langle \mathrm{tr}\left[V_{\underline{1}} V_{\underline{0}}^\dagger\right] \right\rangle (s)$$

Brackets: Averaging over target's state, including spin



where

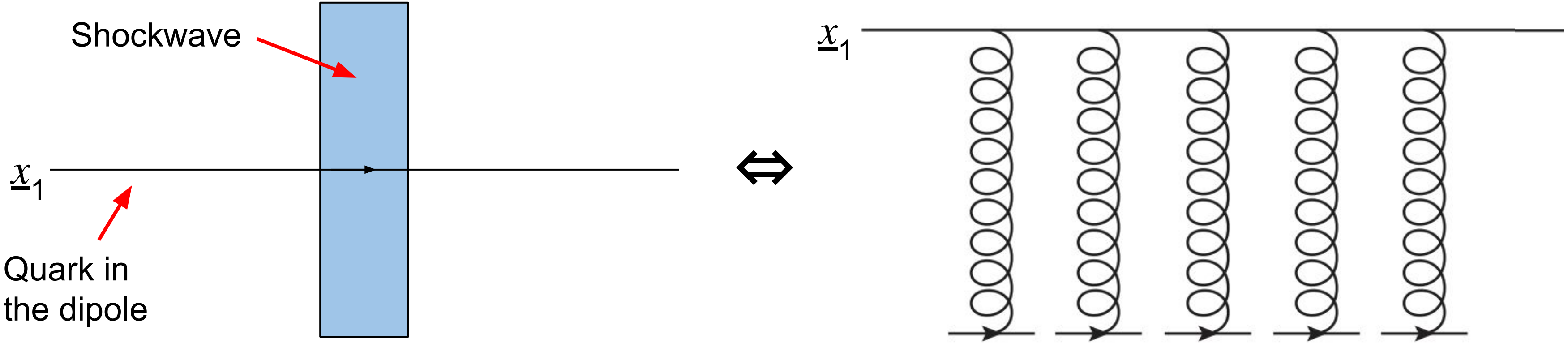
$$V_{\underline{1}}[x_f^-, x_i^-] \equiv V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp\left[ig \int_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1)\right]$$

$$V_{\underline{1}} \equiv V_{\underline{1}}[\infty, -\infty]$$

Light-cone (unpolarized) Wilson line

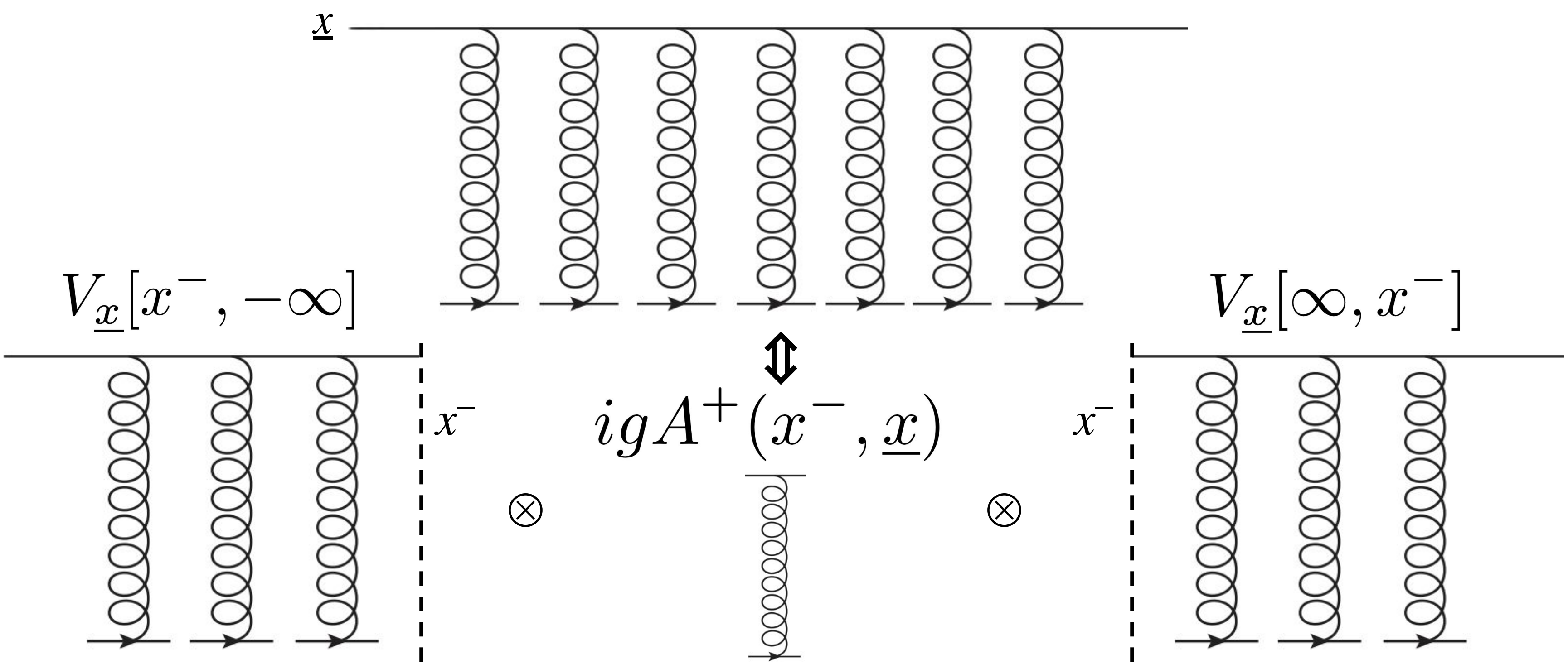
Unpolarized Dipole Amplitude

- Parton **unpolarized PDF,** $\Sigma(x, Q^2)$ and $G(x, Q^2)$, relate to **unpolarized dipole amplitude,** $S_{10}(s) = \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_{\underline{1}} V_{\underline{0}}^{\dagger}\right]\right\rangle(s)$, which obeys BFKL/BK/JIMWLK evolution.
- Quark going through the shockwave at $\underline{x}_1$: unpolarized Wilson line, .
- Multiple parton exchanges at **eikonal** level (leading order in x or CM energy).



Unpolarized Wilson Line

$$V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp \left[ig \int_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1) \right]$$



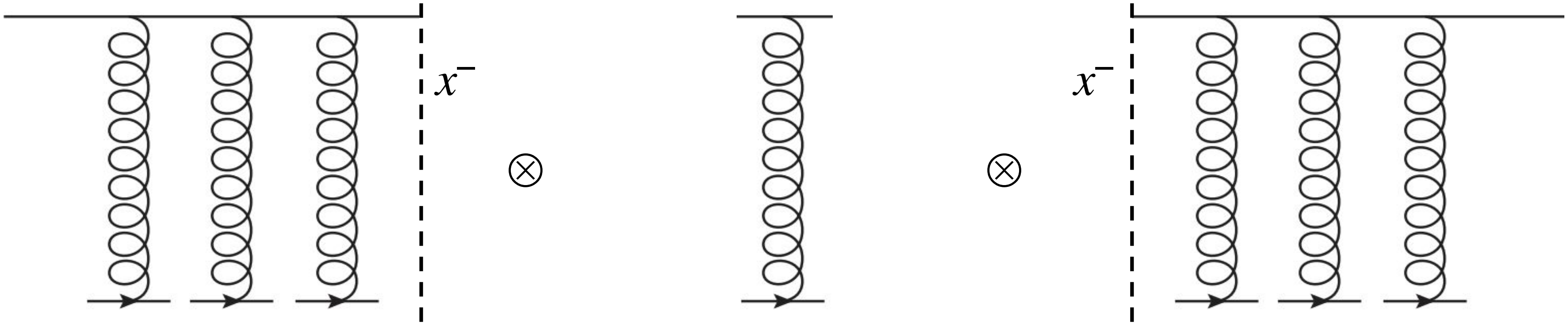
Unpolarized Wilson Line

$$V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp \left[ig \int\limits_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1) \right]$$

- Eikonal vertex insertion:

$$V_{\underline{x}} = ig \int_{-\infty}^{\infty} dx^- V_{\underline{x}}[\infty, x^-] \, A^+(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$

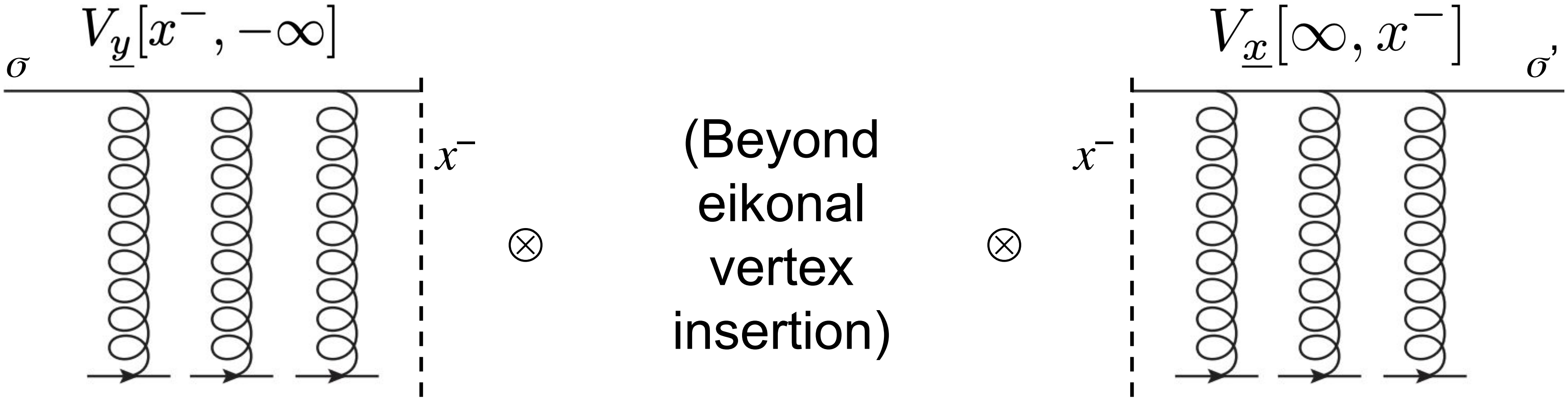
$V_{\underline{x}}[x^-, -\infty]$ $\quad$ $igA^+(x^-, \underline{x})$ $\quad$ $V_{\underline{x}}[\infty, x^-]$



Spin-Dependent Wilson Line

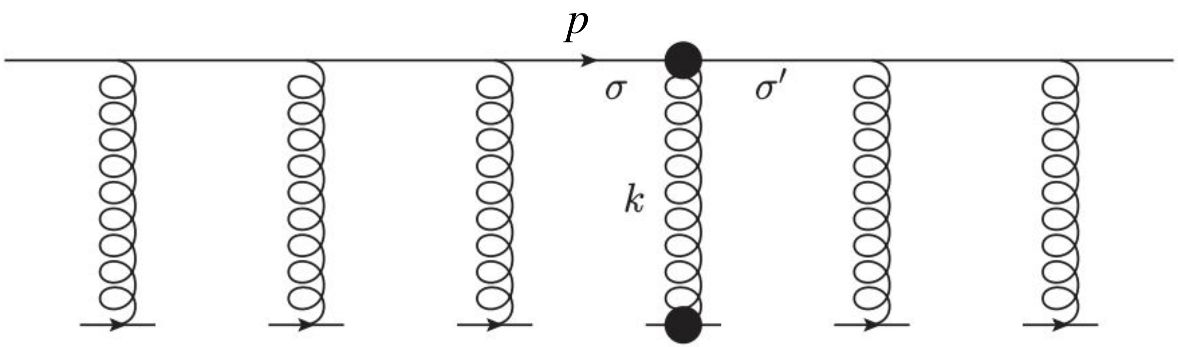
$$V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp \left[ig \int_{x_i^-}^{x_f^-} dx^- A^+(0^+, x^-, \underline{x}_1) \right]$$

- Insertion of beyond-eikonal (s-suppressed) vertex w/ non-trivial spin structures
- Denoted by $V_{\underline{x},\underline{y};\,\sigma',\sigma}$, no longer diagonal in transverse positions
- $V_{\underline{x},\underline{y};\,\sigma',\sigma} = V_{\underline{x}}\, \delta_{\sigma\sigma'}\, \delta^2(\underline{x}-\underline{y}) + V^{[1]}_{\underline{x},\underline{y}}\, \delta_{\sigma\sigma'} + V^{[2]}_{\underline{x},\underline{y}}\, \delta_{\sigma,-\sigma'} + V^{[3]}_{\underline{x},\underline{y}}\, \sigma\delta_{\sigma\sigma'} + V^{[4]}_{\underline{x},\underline{y}}\, \sigma\delta_{\sigma,-\sigma'}$



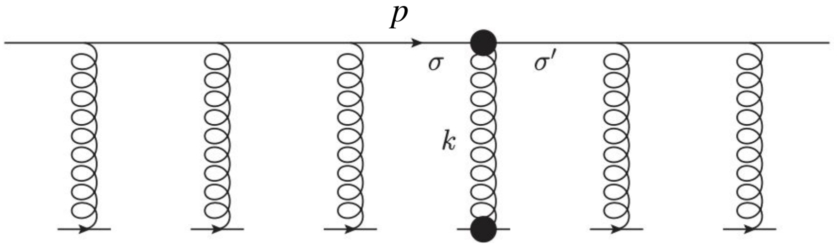
Gluon Vertex

- Starting from general structure: $\bar{u}_{\sigma'}(p+k)\, ig\not{A}(k)\, u_\sigma(p)$
- Take transverse Fourier transform and expand in powers of s
- Similar method for longitudinal and transverse spins, but with different basis spinors



[Kovchegov, Santiago, 2108.03667]

Gluon Vertex



$$V^{\mathrm{G}}_{\underline{x},\underline{y};\sigma',\sigma} = V_{\underline{x}}\,\delta^2(\underline{x}-\underline{y})\,\delta_{\sigma,\sigma'} + \int\limits_{-\infty}^{\infty} \mathrm{d}z^- \, d^2 z \; V_{\underline{x}}[\infty, z^-]\,\delta^2(\underline{x}-\underline{z})\,\mathcal{O}^{\mathrm{pol\ G}}_{\sigma',\sigma}(z^-,\underline{z})\,V_{\underline{y}}[z^-,-\infty]\,\delta^2(\underline{y}-\underline{z})$$

← 1 gluon vertex

$$+ \int\limits_{-\infty}^{\infty} \mathrm{d}z_1^- \, d^2 z_1 \int\limits_{z_1^-}^{\infty} \mathrm{d}z_2^- \, d^2 z_2 \sum_{\sigma''=\pm 1} V_{\underline{x}}[\infty, z_2^-]\,\delta^2(\underline{x}-\underline{z}_2)\,\mathcal{O}^{\mathrm{pol\ G}}_{\sigma',\sigma''}(z_2^-,\underline{z}_2)\,V_{\underline{z}_1}[z_2^-,z_1^-]\,\delta^2(\underline{z}_2-\underline{z}_1)$$

$$\times\, \mathcal{O}^{\mathrm{pol\ G}}_{\sigma'',\sigma}(z_1^-,\underline{z}_1)\,V_{\underline{y}}[z_1^-,-\infty]\,\delta^2(\underline{y}-\underline{z}_1).$$

← 2 gluon vertices

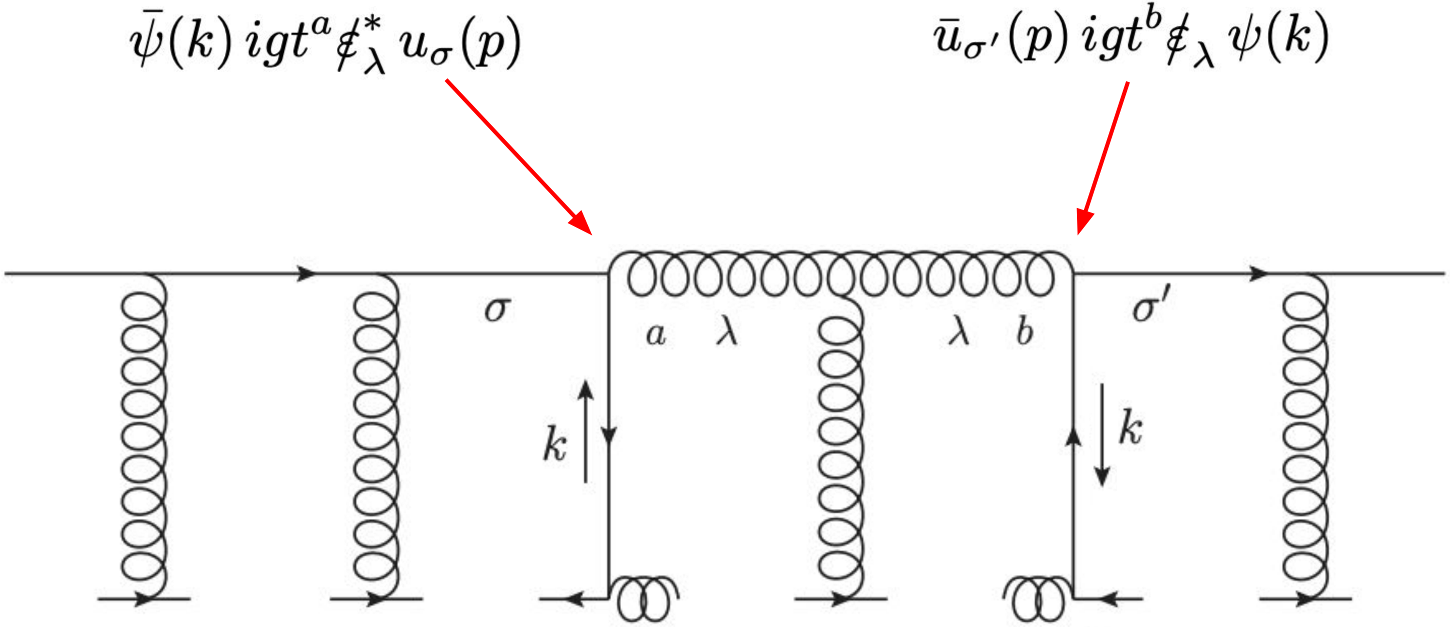
$$\mathcal{O}^{\mathrm{pol\ G}}_{\sigma',\sigma}(x^-,\underline{x}) = -i\,\delta_{\sigma,\sigma'}\left[\overleftarrow{D}^i \frac{1}{2(P_2^- + iD^-)} \vec{D}^i + \frac{m^2}{2(P_2^- + iD^-)}\right]$$

$$+ \frac{ig}{2}\left\{\sigma\,\delta_{\sigma,\sigma'}\left[F^{12}\frac{1}{P_2^- + iD^-} - \frac{i}{(P_2^-)^2}\epsilon^{ij}\,\overleftarrow{D}^i\,F^{-j}\right] + \delta_{\sigma,-\sigma'}\frac{m}{(P_2^-)^2}\,\epsilon^{ij} S^i F^{-j} - \sigma\delta_{\sigma,-\sigma'}\frac{im}{(P_2^-)^2}\,S^i\,F^{-i}\right\}$$

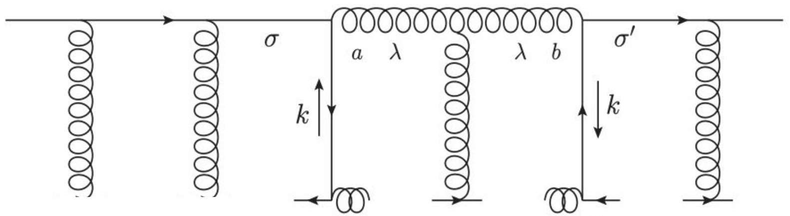
- Sub-eikonal
- Sub-sub-eikonal

[Kovchegov, Santiago, 2108.03667]

$q\bar{q}$ Vertices



$q\bar{q}$ Vertices



$$V^{\mathrm{pol\,q\bar{q}}}_{\underline{x},\underline{y};\sigma',\sigma} = \int\limits_{-\infty}^{\infty} dz_1^- \int\limits_{z_1^-}^{\infty} dz_2^- \, V_{\underline{x}}[\infty, z_2^-] \, \mathcal{O}^{\mathrm{pol\,q\bar{q}}}_{\sigma',\sigma}(z_2^-, z_1^-; \underline{x}, \underline{y}) \, V_{\underline{y}}[z_1^-, -\infty]$$

$$+ \int\limits_{-\infty}^{\infty} dz_1^- \int\limits_{z_1^-}^{\infty} dz_2^- \int\limits_{z_2^-}^{\infty} dz_3^- \int\limits_{z_3^-}^{\infty} dz_4^- \int d^2 z \sum_{\sigma''} V_{\underline{x}}[\infty, z_4^-] \, \delta^2(\underline{x} - \underline{z}_4) \, \mathcal{O}^{\mathrm{pol\,q\bar{q}}}_{\sigma',\sigma''}(z_4^-, z_3^-; \underline{x}, \underline{z}) \, V_{\underline{z}}[z_3^-, z_2^-]$$

$$\times \mathcal{O}^{\mathrm{pol\,q\bar{q}}}_{\sigma'',\sigma}(z_2^-, z_1^-; \underline{z}, \underline{y}) \, V_{\underline{y}}[z_1^-, -\infty]$$

1 $q\bar{q}$ vertex pair

2 pairs of $q\bar{q}$ vertices

$$\mathcal{O}^{\mathrm{pol\,q\bar{q}}}_{\sigma',\sigma}(z_2^-, z_1^-; \underline{z}_2, \underline{z}_1) = -\frac{g^2 p_1^+}{2s} \, t^b \psi_\beta(z_2^-, \underline{z}_2) \left[\delta^{b'b''} - \frac{i p_1^+ \vec{\mathcal{D}}^{b'b''-}_{z_2}}{s} \right] U^{b''a''}_{\underline{z}_2}[z_2^-, z_1^-] \, \delta^2(\underline{z}_2 - \underline{z}_1)$$

$$\times \left[\delta^{a''a'} - \frac{i p_1^+ \vec{\mathcal{D}}^{a''a'-}_{z_1}}{s} \right] \Bigg\{ \delta_{\sigma,\sigma'} \left[\underline{\gamma^+} - \frac{2 m p_1^+}{s} \right] \delta^{a'a} \delta^{bb'} - \sigma \delta_{\sigma,\sigma'} \left[\underline{\gamma^+ \gamma_5} - \frac{2 m p_1^+}{s} \, i \gamma^1 \gamma^2 \right] \delta^{a'a} \delta^{bb'}$$

$$+ \frac{p_1^+}{s} \delta_{\sigma,\sigma'} \left[(\gamma^1 + i\gamma_5\gamma^2) \left[i \, (\underline{S} \cdot \underline{\vec{D}}^{a'a}_{z_1}) - \gamma_5 \, (\underline{S} \times \underline{\vec{D}}^{a'a}_{z_1}) \right] \delta^{bb'} - (\gamma^1 - i\gamma_5\gamma^2) \left[i \, (\underline{S} \cdot \underline{\overleftarrow{D}}^{bb'}_{z_2}) + \gamma_5 \, (\underline{S} \times \underline{\overleftarrow{D}}^{bb'}_{z_2}) \right] \delta^{a'a} \right]$$

$$+ \frac{p_1^+}{s} \sigma \delta_{\sigma,\sigma'} \left[(i\gamma^2 + \gamma_5\gamma^1) \left[i \, (\underline{S} \cdot \underline{\vec{D}}^{a'a}_{z_1}) - \gamma_5 \, (\underline{S} \times \underline{\vec{D}}^{a'a}_{z_1}) \right] \delta^{bb'} + (i\gamma^2 - \gamma_5\gamma^1) \left[i \, (\underline{S} \cdot \underline{\overleftarrow{D}}^{bb'}_{z_2}) + \gamma_5 \, (\underline{S} \times \underline{\overleftarrow{D}}^{bb'}_{z_2}) \right] \delta^{a'a} \right]$$

$$- \frac{p_1^+}{s} \delta_{\sigma,-\sigma'} \left[\gamma^-\gamma^+ \left[i\gamma_5 \, (\underline{S} \cdot \underline{\vec{D}}_{z_1}) - (\underline{S} \times \underline{\vec{D}}_{z_1}) \right] + \gamma^+\gamma^- \left[i\gamma_5 \, (\underline{S} \cdot \underline{\overleftarrow{D}}_{z_2}) - (\underline{S} \times \underline{\overleftarrow{D}}_{z_2}) \right] \right] \delta^{a'a} \delta^{bb'}$$

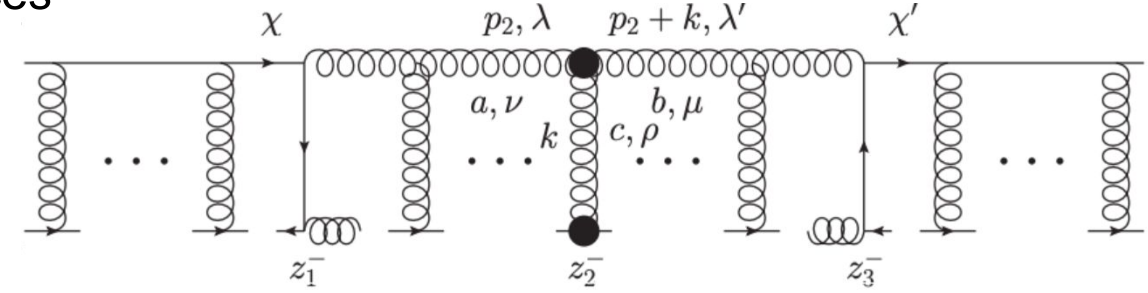
$$- \frac{p_1^+}{s} \sigma \delta_{\sigma,-\sigma'} \left[\gamma^-\gamma^+ \left[i \, (\underline{S} \cdot \underline{\vec{D}}_{z_1}) - \gamma_5 \, (\underline{S} \times \underline{\vec{D}}_{z_1}) \right] + \gamma^+\gamma^- \left[i \, (\underline{S} \cdot \underline{\overleftarrow{D}}_{z_2}) - \gamma_5 \, (\underline{S} \times \underline{\overleftarrow{D}}_{z_2}) \right] \right] \delta^{a'a} \delta^{bb'} \Bigg\}_{\alpha\beta}$$

$$\times \bar{\psi}_\alpha(z_1^-, \underline{z}_1) \, t^a + O\left(\frac{1}{s^3}\right)$$

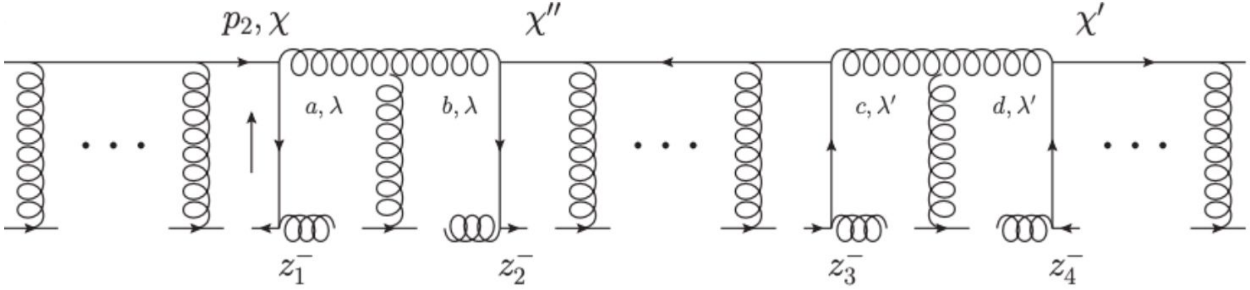
[Kovchegov, Santiago, 2108.03667]

Other contributions

- $qg\overline{q}$ vertices



- $qq\overline{q}\overline{q}$ vertices



Dominant contributions

- For each spin structure, the eikonality level of its dominant contribution is:

Helicity	
Spin structure	Dominant contribution
$\delta_{\sigma,\sigma'}$	Eikonal
$\sigma\delta_{\sigma,\sigma'}$	Sub-eikonal
$\delta_{\sigma,-\sigma'}$	Sub-sub-eikonal
$\sigma\delta_{\sigma,-\sigma'}$	Sub-sub-eikonal

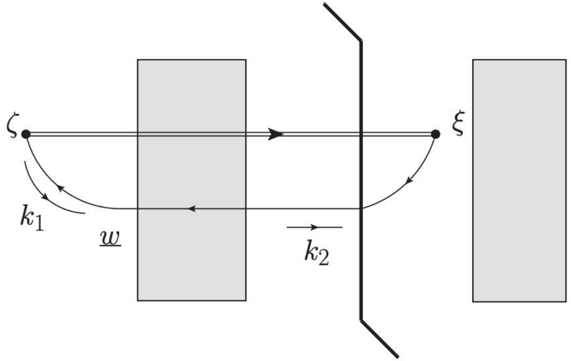
Transverse spin	
Spin structure	Dominant contribution
$\delta_{\chi,\chi'}$	Eikonal
$\chi\delta_{\chi,\chi'}$	Sub-sub-eikonal
$\delta_{\chi,-\chi'}$	Sub-eikonal
$\chi\delta_{\chi,-\chi'}$	Sub-sub-eikonal

Quark TMDs at Small x

[Kovchegov, Sievert, 1808.09010]
[Cougoulic, Kovchegov, Tarasov, YT, 2204.11898]
[Kovchegov, Santiago, 2209.03538]
[Santiago, 2310.02231]

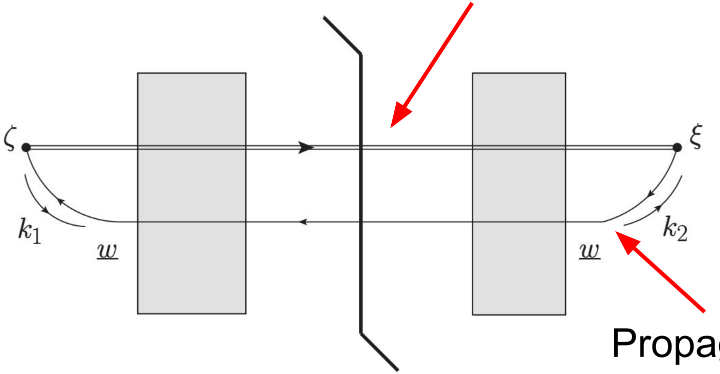
$$\Phi^{[\Gamma]}(x, k_\perp^2) = \int \frac{d^2r\, dr^-}{(2\pi)^3}\, e^{ik\cdot r}\, \langle P, S|\, \bar\psi(0)\, \mathcal{U}[0, r]\, \Gamma\, \psi(r)\, |P, S\rangle$$
$$= \frac{P^+}{4\pi^3} \sum_X \int d\xi^- d^2\xi\, d\zeta^- d^2\zeta\, e^{ik\cdot(\zeta-\xi)} (\Gamma)_{\alpha\beta} \left\langle \bar\psi_\alpha(\xi)\, V_{\underline{\xi}}[\xi^-, \infty]\, |X\rangle \langle X|\, V_{\underline{\zeta}}[\infty, \zeta^-]\, \psi_\beta(\zeta) \right\rangle$$

where Γ is a product of Dirac matrices



$\zeta^- < 0$ and $\xi^- > 0$

+



$\zeta^- < 0$ and $\xi^- < 0$

+ etc

[Kovchegov, Sievert, 1808.09010]
[Cougoulic, Kovchegov, Tarasov, YT, 2204.11898]
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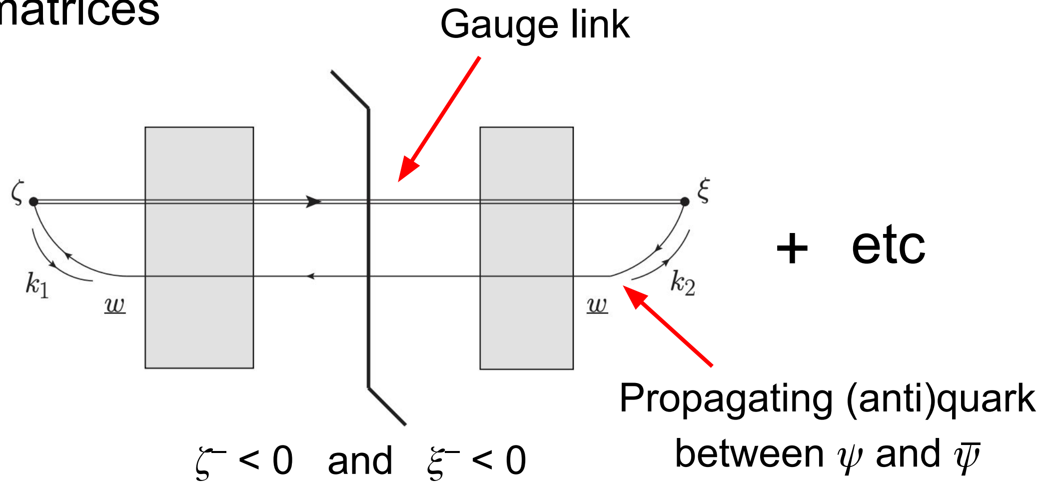
Quark TMDs at Small x

$$\Phi^{[\Gamma]}(x, k_\perp^2) = \int \frac{d^2r\, dr^-}{(2\pi)^3}\, e^{ik\cdot r} \langle P, S|\, \bar{\psi}(0)\, \mathcal{U}[0, r]\, \Gamma\, \psi(r)\, |P, S\rangle$$
$$= \frac{P^+}{4\pi^3} \sum_X \int d\xi^- d^2\xi\, d\zeta^- d^2\zeta\, e^{ik\cdot(\zeta-\xi)} (\Gamma)_{\alpha\beta} \left\langle \bar{\psi}_\alpha(\xi)\, V_{\underline{\xi}}[\xi^-, \infty]\, |X\rangle \langle X|\, V_{\underline{\zeta}}[\infty, \zeta^-]\, \psi_\beta(\zeta) \right\rangle$$

where Γ is a product of Dirac matrices

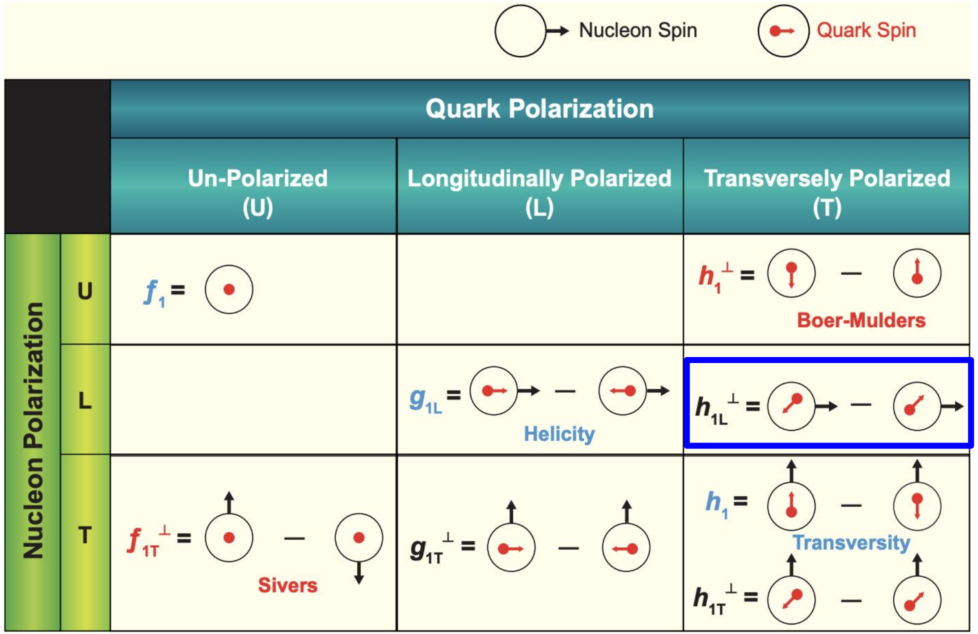
- Gauge link and quark prop through shockwave: Wilson line
- Expand these Wilson lines in eikonality to pick out the term of desired spin structure



+ etc

$\zeta^- < 0$ and $\xi^- < 0$

Quark TMDs at Small x: $h_{1L}^{\perp}$ example



$$\frac{k^j}{M} h_{1L}^{q\perp}(x, k_\perp^2) = \frac{1}{2} \sum_{S_L} S_L \int \frac{d^2r\, dr^-}{(2\pi)^3} e^{ik\cdot r} \langle P, S_L | \bar{\psi}(0) \mathcal{U}[0,r] \frac{\gamma^+ \gamma^j \gamma_5}{2} \psi(r) | P, S_L \rangle$$

$$h_{1L}^{\perp,S}(x,k_\perp) = \sum_f \left[h_{1L}^{\perp,q}(x,k_\perp) + h_{1L}^{\perp,\bar{q}}(x,k_\perp)\right]$$

Quark TMDs at Small x: $\mathrm{h}_{1L}^{\perp}$ example

- Following the process described previously, flavor-singlet worm-gear h is

$$\frac{\underline{k}^1}{M} h_{1L}^{\perp,\mathrm{S}}(x,k_\perp^2) = \frac{ixN_c}{2\pi^4}\int d^2x_1\, d^2x_0 \int \frac{d^2k_1}{(2\pi)^2}\, e^{i(\underline{k}+\underline{k}_1)\cdot\underline{x}_{10}}\, \frac{1}{k_{1\perp}^2 k_\perp^2}\left(\frac{1}{k_{1\perp}^2}+\frac{1}{k_\perp^2}\right)\int\frac{dz}{z}$$
$$\times\left\{\left[2(\underline{S}\cdot\underline{k})(\underline{S}\cdot\underline{k}_1)-(\underline{k}\cdot\underline{k}_1)\right]H_{10}^{1L}(z)+\left[(\underline{S}\cdot\underline{k})(\underline{S}\times\underline{k}_1)+(\underline{S}\times\underline{k})(\underline{S}\cdot\underline{k}_1)\right]H_{10}^{2L}(z)\right\}$$

where $$H_{10}^{1L}(z) = \frac{(zs)^2}{2N_c}\,\mathrm{Im}\left\langle \mathrm{T\,tr}\left[V_{\underline{0}}V_{\underline{1}}^{\mathrm{T}\dagger}\right]+\mathrm{T\,tr}\left[V_{\underline{0}}^\dagger V_{\underline{1}}^{\mathrm{T}}\right]\right\rangle$$

$$H_{10}^{2L}(z) = \frac{(zs)^2}{2N_c}\,\mathrm{Re}\left\langle \mathrm{T\,tr}\left[V_{\underline{0}}V_{\underline{1}}^{\mathrm{T}\perp\dagger}\right]+\mathrm{T\,tr}\left[V_{\underline{0}}^\dagger V_{\underline{1}}^{\mathrm{T}\perp}\right]\right\rangle$$

(Longitudinal) spin-dependent averaging: $\frac{1}{2}\sum_{\mathcal{S}_L}\mathcal{S}_L\,\langle\cdots\rangle$

Generally, depends on target's spin

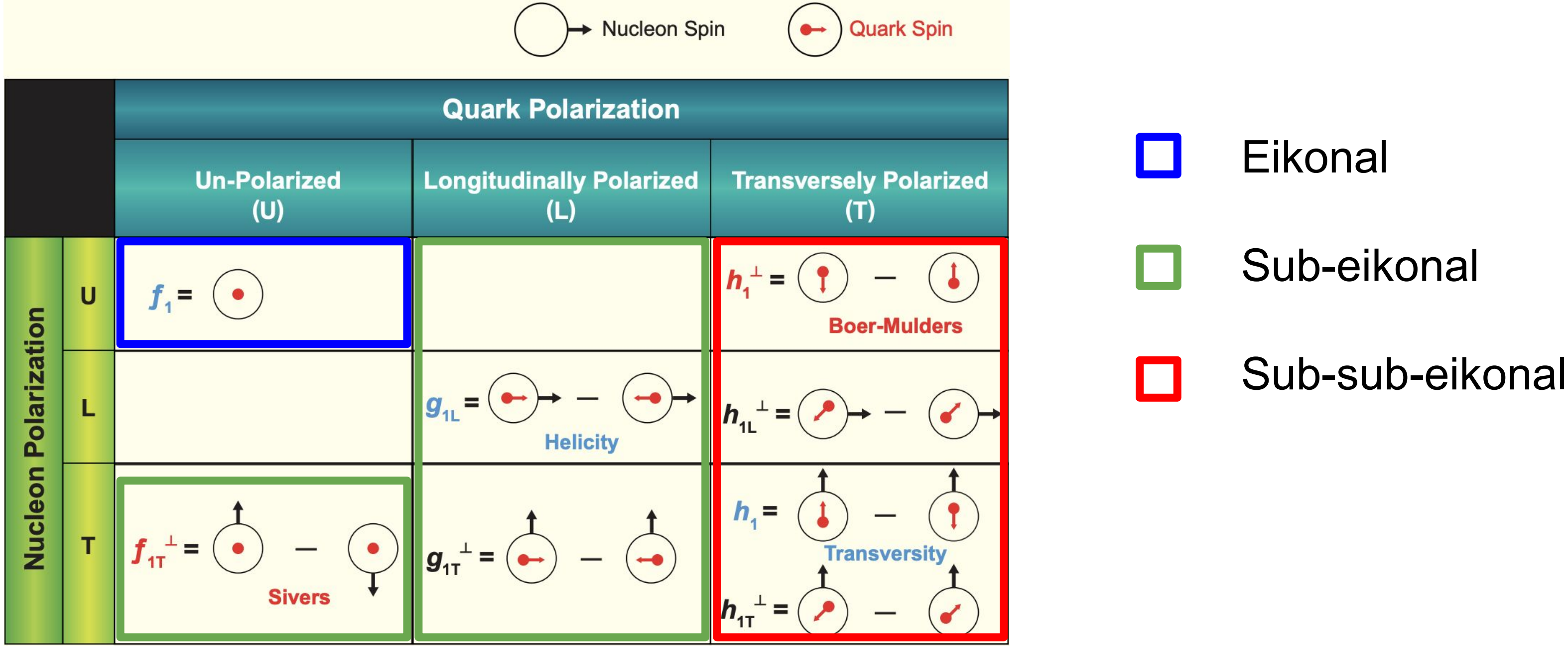
$$V_{\underline{z}}^{\mathrm{T}} = \frac{g^2(p_1^+)^2}{2s^2}\int_{-\infty}^{\infty}dz_1^-\int_{z_1^-}^{\infty}dz_2^-\, V_{\underline{z}}[\infty,z_2^-]\,t^b\psi_\beta(z_2^-,\underline{z})\,U_{\underline{z}}^{ba}[z_2^-,z_1^-]$$
$$\times\left\{\gamma^-\gamma^+\left[i\gamma_5\,(\underline{S}\cdot\vec{\underline{D}}_z)-(\underline{S}\times\vec{\underline{D}}_z)\right]+\gamma^+\gamma^-\left[i\gamma_5\,(\underline{S}\cdot\overleftarrow{\underline{D}}_z)-(\underline{S}\times\overleftarrow{\underline{D}}_z)\right]\right\}_{\alpha\beta}\bar\psi_\alpha(z_1^-,\underline{z})\,t^a\,V_{\underline{z}}[z_1^-,-\infty],$$

$$V_{\underline{z}}^{\mathrm{T}\perp} = -\frac{g^2(p_1^+)^2}{2s^2}\int_{-\infty}^{\infty}dz_1^-\int_{z_1^-}^{\infty}dz_2^-\, V_{\underline{z}}[\infty,z_2^-]\,t^b\psi_\beta(z_2^-,\underline{z})\,U_{\underline{z}}^{ba}[z_2^-,z_1^-]$$
$$\times\left\{\gamma^-\gamma^+\left[i\,(\underline{S}\cdot\vec{\underline{D}}_z)-\gamma_5\,(\underline{S}\times\vec{\underline{D}}_z)\right]+\gamma^+\gamma^-\left[i\,(\underline{S}\cdot\overleftarrow{\underline{D}}_z)-\gamma_5\,(\underline{S}\times\overleftarrow{\underline{D}}_z)\right]\right\}_{\alpha\beta}\bar\psi_\alpha(z_1^-,\underline{z})\,t^a\,V_{\underline{z}}[z_1^-,-\infty],$$

[Santiago, 2310.02231]
[Adamiak, Santiago, YT, in preparation]

Summary of Eikonality (Flavor Singlet)

- For each TMD, the eikonality level of its dominant contribution is:



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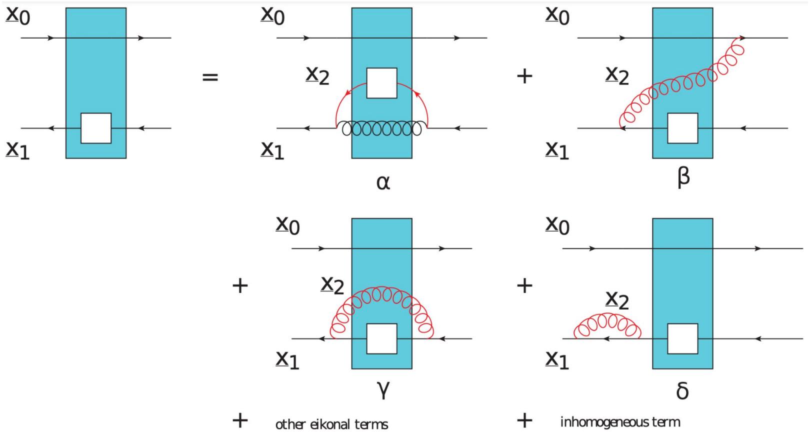
[Santiago, 2310.02231]

Small-x Evolution

- Quark TMD → polarized dipoles → small-x evolution
- Impose kinematic constraints with **neighbor dipoles** (lifetime ≠ dipole size)
- The evolution resums $\alpha_s \ln^2(1/x)$
- Solving the evolution equation asymptotically gives small-x behavior of TMDs.



Small-x Evolution

- Generally, the evolution equation is not closed, generating hierarchy of multiquark correlators
- In the large N_c limit, we have a closed and linear system of equations. Schematically:

$$\begin{pmatrix} H(x_{10}^2, zs) \\ \Gamma(x_{10}^2, x_{21}^2, zs) \end{pmatrix} = \begin{pmatrix} H(x_{10}^2, zs) \\ \Gamma(x_{10}^2, x_{21}^2, zs) \end{pmatrix}^{(0)} + \int \frac{dz'}{z'} \int \frac{d^2 x_{32}}{x_{32}^2} \mathcal{K}(x_{10}, x_{21}, x_{32}) \otimes \begin{pmatrix} H(x_{32}^2, z's) \\ \Gamma(x_{10}^2, x_{32}^2, z's) \end{pmatrix}$$

Initial condition:
deduced from moderate-x data

Evolution kernel:
a matrix-valued function of parent and daughter dipole sizes

Small-x Evolution

- 4 families of evolution equation

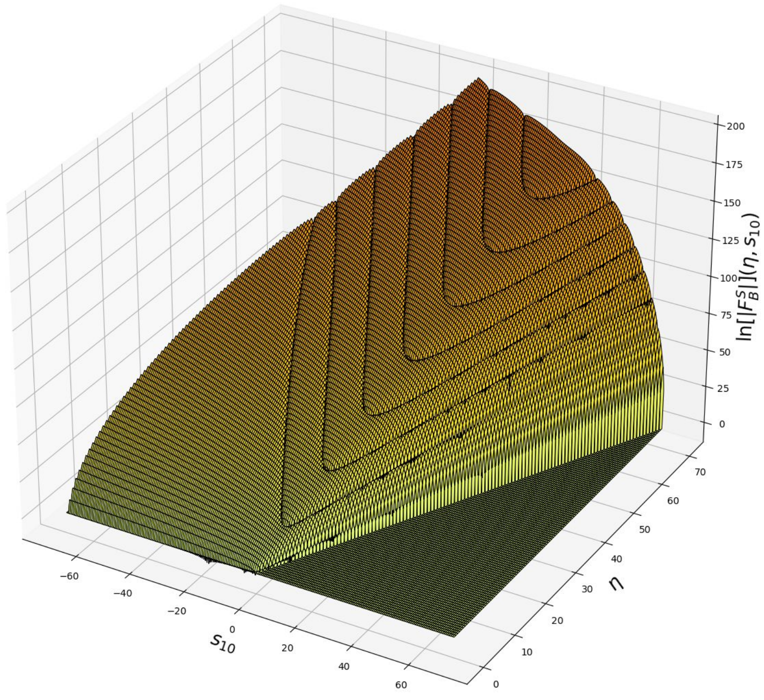
Corresponding flavor singlet quark TMDs	Eikonality
Helicity (g_{1L})	Sub-eikonal
Sivers ($f_{1T}^{\perp}$) and Worm-gear ($g_{1T}^{\perp}$)	Sub-eikonal
Boer-Mulders ($h_1^{\perp}$) and Worm-gear ($h_{1L}^{\perp}$)	Sub-sub-eikonal
Transversity (h_1) and pretzelosity ($h_{1T}^{\perp}$)	Sub-sub-eikonal

See Monday talk by Yuri Kovchegov

[Adamiak, Santiago, YT, in preparation]

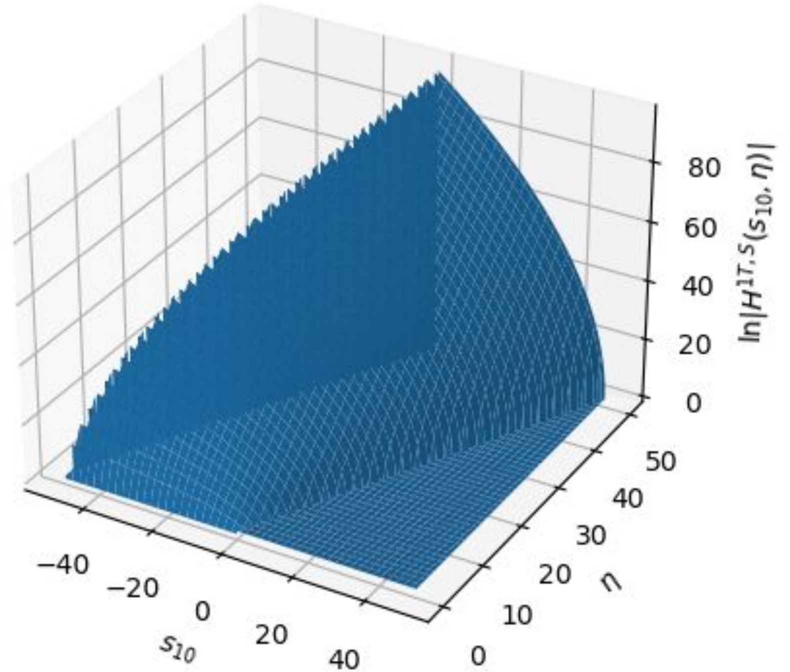
Sivers and Worm-Gear g

- Solved iteratively at large-N_c
- The TMDs grow as a power law ~ $(1/x)^{2.8\sqrt{\alpha_s N_c/4\pi}}$ with oscillatory behavior in $\ln(1/x)$. However, the period is *extremely* large.
- At sufficiently small x, we need to include single-logs, $\alpha_s \ln(1/x)$, together with unpolarized evolution



Transversity and Pretzelosity

- Solved **analytically** at large-N_c
- The difference between dipoles and neighbor dipoles is suppressed at small x.
- The TMDs grow as a power law ~ $x\ (1/x)^{2\sqrt{\alpha_s N_c/2\pi}}$
- The extra factor of x comes from the TMD's relation with the dipoles



Boer-Mulders and Worm-Gear h

- Solved **analytically** at large-N_c
- At small-x, the TMDs oscillate as ~ $x\ J_1[\ln(1/x)]$, which has decaying amplitude. Thus, there is no significant contribution from small x.
- Overall, we can write TMD ~ x.

Small-x Asymptotic Behaviors

- Flavor singlet [Cougoulic, Kovchegov, Tarasov, YT, 2204.11898; Adamiak, Santiago, YT, in preparation]

Leading Twist Quark TMDs				
		Quark Polarization		
		U	L	T
Nucleon Polarization	U	$f_1^{\mathrm{S}} \sim x^{-\frac{4\alpha_s N_c}{\pi}\ln(2)}$		$h_1^{\perp\mathrm{S}} \sim x$
	L		$g_1^{\mathrm{S}} \sim x^{-3.66\sqrt{\alpha_s N_c/2\pi}}$	$h_{1L}^{\perp\mathrm{S}} \sim x$
	T	$f_{1T}^{\perp\mathrm{S}} \sim x^{-2.8\sqrt{\alpha_s N_c/4\pi}}$	$g_{1T}^{\mathrm{S}} \sim x^{-2.8\sqrt{\alpha_s N_c/4\pi}}$	$h_1^{\mathrm{S}} \sim h_{1T}^{\perp\mathrm{S}} \sim x^{1-2\sqrt{\frac{\alpha_s N_c}{2\pi}}}$

$$\Phi^{\mathrm{S}}(x,k_\perp) = \sum_{f=u,d,s}\left[\Phi^{\mathrm{q}}(x,k_\perp) + \Phi^{\bar{\mathrm{q}}}(x,k_\perp)\right]$$

Small-x Asymptotic Behaviors

- Flavor non-singlet [Kovchegov, Sievert, Pitonyak, 1610.06197; Santiago, 2310.02231]

Leading Twist Quark TMDs				
		Quark Polarization		
		U	L	T
Nucleon Polarization	U	$f_1^{\mathrm{NS}} \sim x^{-\sqrt{2\alpha_s C_F/\pi}}$		$h_1^{\perp\mathrm{NS}} \sim x$
	L		$g_1^{\mathrm{NS}} \sim x^{-\sqrt{\alpha_s N_c/\pi}}$	$h_{1L}^{\perp\mathrm{NS}} \sim x$
	T	$f_{1T}^{\perp\mathrm{NS}} \sim C_{\mathcal{O}} x^{-1} + C_1 x^{-3.4\sqrt{\alpha_s N_c/4\pi}}$	$g_{1T}^{\mathrm{NS}} \sim x^0$	$h_1^{\mathrm{NS}} \sim h_{1T}^{\perp\mathrm{NS}} \sim x^{1-2\sqrt{\alpha_s N_c/2\pi}}$

Eikonal contribution from spin-dependent odderon

$$\Phi^{\mathrm{NS}}(x, k_\perp) = \Phi^{\mathrm{q}}(x, k_\perp) - \Phi^{\bar{\mathrm{q}}}(x, k_\perp)$$

- Next step: gluon TMDs

Conclusion

- We have developed a framework that relates spin-dependent quark TMDs to beyond-eikonal corrections of the light-cone Wilson line, in term of which the small-x evolution equations have been developed (1 system of eqns per TMD)
- The equations are at DLA, resumming $\alpha_s \ln^2(1/x)$.
- Asymptotic behaviors have been determined for all quark TMDs. (This work: flavor singlet; previous work: flavor non-singlet)
- The framework can be extended to gluon TMDs (to do next).