Power corrections and renormalons in pseudo- and quasi-GPDs

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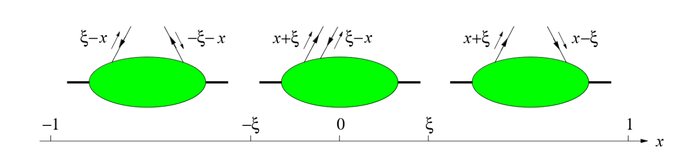
based on: VB, Maria Koller, Jakob Schönleber 2401.08012

Generalized Parton Distribution (GPD)



$$\langle p'|\bar{q}(\tfrac{z}{2}n)\not{n}q(-\tfrac{z}{2}n)|p\rangle = 2(Pn)I(\tau,\xi,\Delta^2;\mu)\,, \qquad n^2=0, \quad z\in\mathbb{R}$$

$$I(\tau,\xi,\Delta^2;\mu) = \int_{-1}^{1} dx\, e^{i\tau x} H(x,\xi,\Delta^2;\mu)\,, \qquad \tau = z(Pn)$$

⇐ The Wilson line between the quarks is implied

- Kinematic variables

$$P = \frac{1}{2}(p+p')\,, \qquad \Delta = (p'-p)\,, \qquad \xi = \frac{(np)-(np')}{(np)+(np')}$$

quasi-GPDs and pseudo-GPDs

Euclidean quantities, connected to GPDs via collinear factorization

- Position space

$$\langle p'|\bar{q}(\tfrac{z}{2}v)\gamma^\mu q(-\tfrac{z}{2}v)|p\rangle = 2(vP)\frac{v^\mu}{v^2}\,\mathcal{I}_\parallel(\tau,\xi,z^2) + 2\left(P^\mu - (vP)\frac{v^\mu}{v^2}\right)\mathcal{I}_\perp(\tau,\xi,z^2) + \Delta_\perp^\mu \mathcal{J}(\tau,\xi,z^2)$$

Factorization theorem

$$\mathcal{I}(\tau,\xi;z^2) = \int_0^1 du\, T_{\mathcal{I}}(u,\tau,\xi,z^2,\mu_F^2)\, I(u\tau,\xi,\mu_F^2) \;+\; \text{power corrections}$$

- Pseudo-GPDs

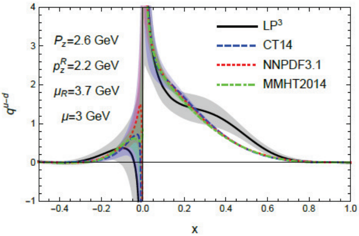
$$\mathcal{P}(x,\xi;z^2) = \int_{-\infty}^{\infty}\frac{d\tau}{2\pi}e^{-i\tau x}\mathcal{I}(\tau,\xi;z^2)$$

- Quasi-GPDs

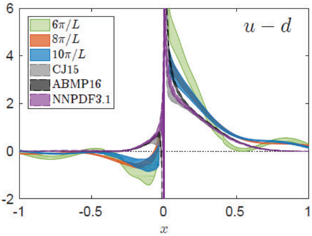
$$\mathcal{Q}(x,\xi;v\cdot P) = \int_{-\infty}^{\infty}\frac{d\tau}{2\pi}e^{-i\tau x}\mathcal{I}(\tau,\xi;\tfrac{\tau^2}{(vP)^2})$$

PDFs from qPDFs

Isovector quark unpolarized PDF



LP3, 1803.04393 & 1807.06566, $m_\pi \approx 135\ MeV, a = 0.09\ fm, L \approx 5.8 fm$

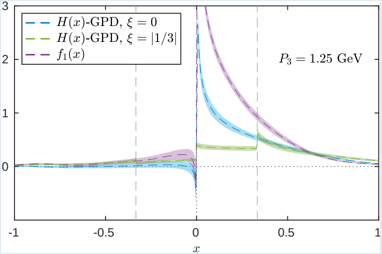


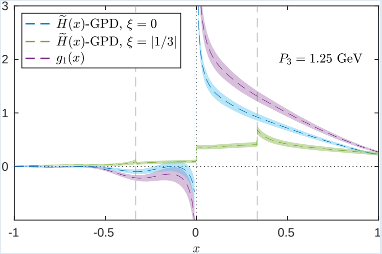
Alexandrou et al, 1803.02685, $m_\pi \approx 130\ MeV,\ a = 0.094\ fm, L \approx 4.5 fm$

thanks to J.-H. Zhang

- Power corrections ?

GPDs from qGPDs





C. Alexandrou *et al.*, PRL125 (2020) 262001

- Power corrections ?

Concept

M. Beneke, Phys.Rept. 317 (1999)
M. Beneke, VB, hep-ph/0010208

- Leading twist calculation "knows" about the necessity to add a power correction through a factorial divergence of the series in MS-type schemes

Example: Deep Inelastic Scattering

$$F_2(x,Q^2) = 2x\int_x^1 \frac{dy}{y} C(y,Q^2/\mu^2) q(\tfrac{x}{y},\mu^2) + \frac{1}{Q^2} D_2(x)$$

$$D_2(x) = \varkappa \Lambda^2_{\rm QCD} 2x \int_x^1 \frac{dy}{y} d_2(x) q(\tfrac{x}{y}), \qquad \varkappa = \mathcal{O}(1)$$

One-loop result:

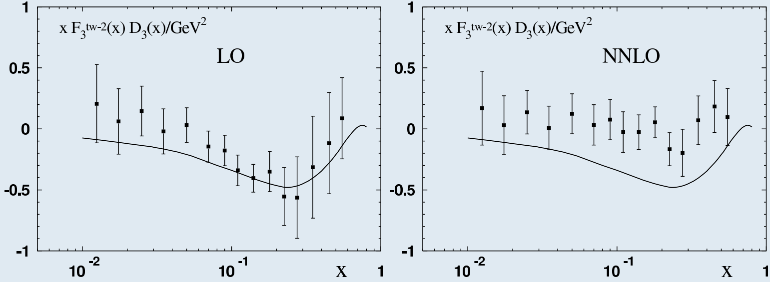
$$d_2^{(q)} = -\frac{4}{[1-x]_+} + 4 + 2x + 12x^2 - 9\delta(1-x) - \delta'(1-x)$$

Main conclusions:

- ratio twist-4/twist-2 is target-independent (if renormalon dominance correct)
- enhancement at $x \to 1$:

$$\left[\frac{\Lambda^2}{Q^2(1-x)}\right]^n$$

Example: CCFR data on $F_3(x, Q^2)$:



- Shape predicted; normalization arbitrary

Renormalons in Quasi- and Pseudo-distributions

- Pion pseudo-LCDA: VB, E. Gardi, S. Gottwald: NPB685 (2004) 171

- Pion pseudo- and quasi-PDFs: VB, A. Vladimirov, J.-H. Zhang, PRD99 (2019) 014013

— Power corrections for qPDFs have a generic behavior

$$\mathcal{Q}(x,p) = q(x)\left\{1 + \mathcal{O}\left(\frac{\Lambda^2}{p^2}\cdot\frac{1}{x^2(1-x)}\right)\right\}$$

— Power corrections for pPDFs have a generic behavior

$$\mathcal{P}(x,z) = q(x)\left\{1 + \mathcal{O}\left(z^2\Lambda^2(1-x)\right)\right\}$$

— Normalization to zero momentum in both cases strongly reduces power corrections apart from the $x \to 1$ region where they are enhanced by extra power of $1/(1-x)$.

- pseudo- and quasi-GPDs: this work

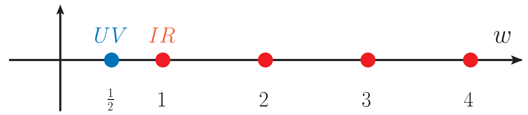
Methods

- Borel transform: a convenient way to handle a divergent series

$$H = \delta(1-\alpha) + \sum_{k=0}^{\infty} h_k a_s^{k+1}\,, \qquad a_s = \frac{\alpha_s(\mu)}{4\pi}\,, \qquad h_k \propto k!$$

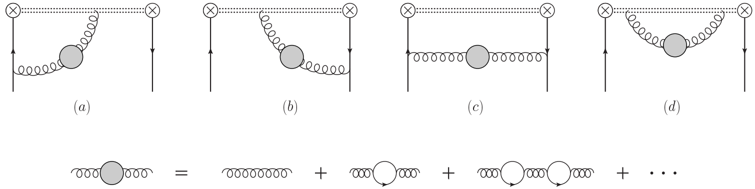
$$B[H](w) = \sum_{k=0}^{\infty} \frac{h_k}{k!}\left(\frac{w}{\beta_0}\right)^k \qquad\qquad H = \delta(1-\alpha) + \frac{1}{\beta_0}\int_0^\infty dw\, e^{-w/(\beta_0 a_s)} B[H](w)$$

- Integration may be obscured by singularites on the integration path — renormalons



Methods (2)

- Bubble-chain approximation 't Hooft, '77



- Borel-transform of the effective gluon propagator M. Beneke, NPB 405 (1993) 424

$$B[a_s D^{AB}(k)](w) = i\delta^{AB} \left(e^{5/3}\mu^2\right)^w \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{(-k^2 - i\epsilon)^{1+w}}\,.$$

- "Gluon mass" technique P. Ball, VB, M. Beneke, NPB 452 (1995) 563

$$\frac{1}{k^2-\lambda^2} = \frac{1}{2\pi i}\frac{1}{k^2}\int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} dw\, \Gamma(-w)\Gamma(1+w)\left(-\frac{\lambda^2}{k^2}\right)^w.$$

- Borel transform of the position-space quasi-distribution

$$B[\mathcal{I}(\tau,\xi,z^2)](w) = 2C_F e^{5/3w}\left(\frac{-z^2v^2\mu^2}{4}+i0\right)^w \frac{\Gamma(-w)}{\Gamma(w+1)}$$
$$\times\left\{\frac{2}{1+w}\int_0^1 d\alpha\,\alpha^{1+w}\,{}_2F_1(1,2-w,2+w,\alpha)\Big[\cos(\bar\alpha\xi\tau)I(\alpha\tau,\xi;\mu)-I(\tau,\xi;\mu)\Big]\right.$$
$$\left.-\frac{1}{1+w}I(\tau,\xi;\mu)-\underbrace{\frac{1}{1-2w}I(\tau,\xi;\mu)}_{\text{UV renormalon in Wilson line}}+\big[1\mp w\big]\int_0^1 d\alpha\,\alpha^w\frac{\sin(\bar\alpha\xi\tau)}{\tau\xi}I(\alpha\tau,\xi;\mu)\right\},$$

- removed e.g. via normalization at zero momentum:

$$\widehat{\mathcal{I}}(\tau,\xi,z^2)\sim\frac{\langle p'|\bar q(\frac{z}{2}v)\gamma^\mu q(-\frac{z}{2}v)|p\rangle}{\langle 0|\bar q(\frac{z}{2}v)\gamma^\mu q(-\frac{z}{2}v)|0\rangle}$$

Results(1): The leading $w = 1$ renormalon ambiguity in pseudo-GPDs

$$\mathcal{P}(x, \xi, z^2) = H(x, \xi) \pm \mathcal{N}\left(\Lambda^2 z^2 |v^2|\right) \delta_R \mathcal{P}(x, \xi)\,,$$

$$\begin{aligned}
2\delta_R \mathcal{P}^{\parallel}(x, \xi) = {} & \theta(x > \xi)\left[\int_x^1 dy\, \frac{H(y, \xi)}{y - \xi}\, \Phi\left(\frac{x - \xi}{y - \xi}\right) + \int_x^1 dy\, \frac{H(y, \xi)}{y + \xi}\, \Phi\left(\frac{x + \xi}{y + \xi}\right)\right] \\
& + \theta(-\xi < x < \xi)\left[-\int_{-1}^x dy\, \frac{H(y, \xi)}{y - \xi}\, \Phi\left(\frac{x - \xi}{y - \xi}\right) + \int_x^1 dy\, \frac{H(y, \xi)}{y + \xi}\, \Phi\left(\frac{x + \xi}{y + \xi}\right)\right] \\
& + \theta(x < -\xi)\left[-\int_{-1}^x dy\, \frac{H(y, \xi)}{y - \xi}\, \Phi\left(\frac{x - \xi}{y - \xi}\right) - \int_{-1}^x dy\, \frac{H(y, \xi)}{y + \xi}\, \Phi\left(\frac{x + \xi}{y + \xi}\right)\right]
\end{aligned}$$

$$2\delta_R \mathcal{P}^{\perp}(x, \xi) = 2\delta_R \mathcal{P}^{\parallel}(x, \xi) + \ldots$$

$$\Phi(\alpha) = \alpha + (1 - \alpha)\ln(1 - \alpha)$$

Results(2): The leading $w = 1$ renormalon ambiguity in quasi-GPDs

$$\mathcal{Q}(x,\xi,(vP)) = H(x,\xi) \pm \mathcal{N}\left(\frac{\Lambda^2|v^2|}{(vP)^2}\right)\delta_R\mathcal{Q}(x,\xi)$$

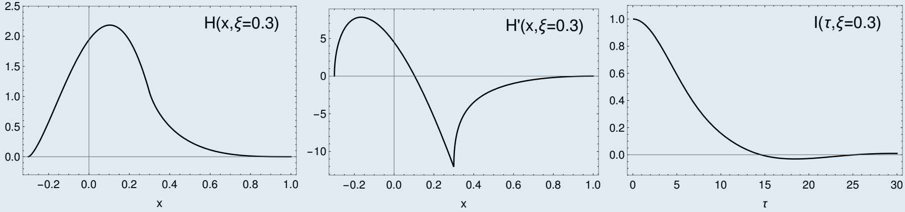
$$\begin{aligned}
2\delta_R\mathcal{Q}^{\|}(x,\xi) &= \left(1-\frac{\pi^2}{6}\right)\delta(x-\xi)[H'(\xi+\epsilon,\xi)-H'(\xi-\epsilon,\xi)]\\
&+\theta(x>\xi)\left[\frac{H'(x,\xi)}{x-\xi}+\frac{1}{(x-\xi)^2}\int_x^1 dy\left[\frac{x-\xi}{y-\xi}+\ln\left(1-\frac{x-\xi}{y-\xi}\right)\right]H'(y,\xi)\right]\\
&+\theta(x<\xi)\left[\frac{H'(x,\xi)}{x-\xi}-\frac{1}{(x-\xi)^2}\int_{-1}^x dy\left[\frac{x-\xi}{y-\xi}+\ln\left(1-\frac{x-\xi}{y-\xi}\right)\right]H'(y,\xi)\right]\\
&+\left(1-\frac{\pi^2}{6}\right)\delta(x+\xi)[H'(-\xi+\epsilon,\xi)-H'(-\xi-\epsilon,\xi)]\\
&+\theta(x>-\xi)\left[\frac{H'(x,\xi)}{x+\xi}+\frac{1}{(x+\xi)^2}\int_x^1 dy\left[\frac{x+\xi}{y+\xi}+\ln\left(1-\frac{x+\xi}{y+\xi}\right)\right]H'(y,\xi)\right]\\
&+\theta(x<-\xi)\left[\frac{H'(x,\xi)}{x+\xi}-\frac{1}{(x+\xi)^2}\int_{-1}^x dy\left[\frac{x+\xi}{y+\xi}+\ln\left(1-\frac{x+\xi}{y+\xi}\right)\right]H'(y,\xi)\right]
\end{aligned}$$

GPD model

- Toy model:

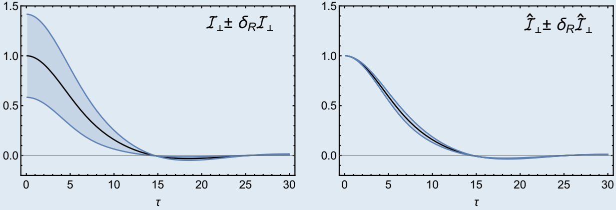
$$H(x,\xi) = \theta(x>-\xi)\frac{2+\lambda}{4\xi^3}\left(\frac{x+\xi}{1+\xi}\right)^{\lambda}[\xi^2 - x + \lambda\xi(1-x)]$$
$$-\theta(x>\xi)\frac{2+\lambda}{4\xi^3}\left(\frac{x-\xi}{1-\xi}\right)^{\lambda}[\xi^2 - x - \lambda\xi(1-x)]\,,$$

with $\lambda = 3/2$ corresponding to the valence quark PDF $q(x) \sim (1-x)^3/\sqrt{x}$

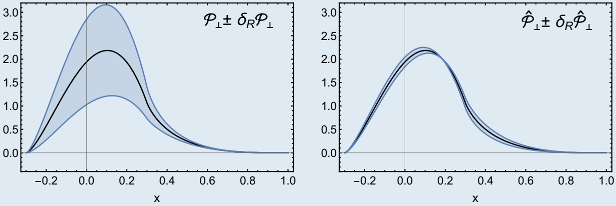


Renormalon ambiguity

- position space ($\xi = 0.3,\ z \sim 1$ fm)

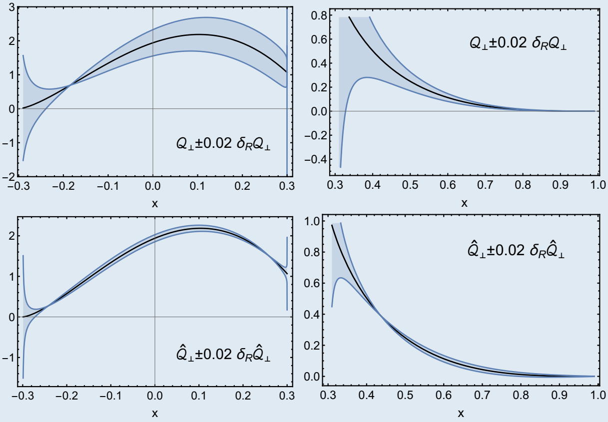


- pseudo-GPD ($\xi = 0.3,\ z \sim 1$ fm)



Renormalon ambiguity

- quasi-GPD ($\xi = 0.3$)



- $\delta_R \mathcal{Q}(x,\xi)/\mathcal{Q}(x,\xi)$ in the end-point regions:

ERBL: $\ln(\xi - x)$ for $x \to \xi$
$1/(x+\xi)^2$ for $x \to -\xi$

DGLAP: $1/\sqrt{x-\xi}$ for $x \to \xi$
$1/(1-x)$ for $x \to 1$

The $x = \xi$ kinematic point in the quasi-GPD approach

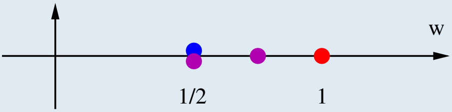
- Observe a series of singular terms

$$\ldots + \frac{1}{(vP)^2}\delta(x-\xi)\Delta H'(\xi,\xi) + \frac{1}{(vP)^4}\delta'(x-\xi)\Delta H''(\xi,\xi) + \ldots$$

- Change the order of limits:

  first expand in $1/(vP)^k$, then send $x \to \xi$ $\Rightarrow$ first set $x = \xi$, then expand in $1/(vP)^k$

- Result: New singularities (violet) at $w = 1/2$ and $w = 1 - p/2$ (for $q(x) \sim x^{-p}$)



$\Rightarrow$ Power corrections $1/(vP)^1$, $1/(vP)^{2-p}$ (at $x = \xi$)

- Coefficient of $1/(vP)^1$ is proportional to $H'(\xi+\epsilon,\xi) - H'(\xi-\epsilon,\xi)$

Intepretation

- The $1/(Pv)^2$ power correction (= renormalon ambiguity) to a qGPD is a distribution:

— contains a term $\delta(x-\xi)$
— contains a contribution $\sim 1/\sqrt{x-\xi}$ from the DGLAP region

- Consider a qPDF smeared over a narrow interval $x-\xi \sim \Lambda/P$

$$\widetilde{\mathcal{Q}}(x,\xi) = \int dx'\, \Theta(x'-x)\mathcal{Q}(x',\xi) \qquad\qquad \int dx\, \Theta(x) = 1$$

E.g. Gaussian smearing

$$\Theta(x) = \frac{1}{\sqrt{\pi}}\frac{(Pv)}{\Lambda}\exp\Big(-(Pv)^2x^2/\Lambda^2\Big)$$

- If this smearing is applied:

— the term with a δ-function gets promoted to a $1/(vP)$ correction
— the term $\sim 1/\sqrt{x-\xi}$ (for our GPD model) produces a $1/(Pv)^{3/2}$ contribution
$\leftarrow$ as found by the direct calculation at $x=\xi$

Cusp at $x = \xi$ from position space viewpoint

Assume that $H(x,\xi)$ is continuous and vanishes sufficiently fast at the end points, but has a cusp at $x = \xi$

$$I(\tau,\xi) = -\frac{1}{\tau^2}\int_{-\xi}^{1} dx\, H(x,\xi)\,\frac{d^2}{dx^2} e^{i\tau x}$$

$$= \frac{1}{\tau^2}\Big[H'(\xi-\epsilon,\xi) - H'(\xi+\epsilon,\xi)\Big] e^{i\tau\xi}$$

$$- \frac{1}{\tau^2}\left(\int_{-\xi}^{\xi-\epsilon} + \int_{\xi+\epsilon}^{1}\right) dx\, e^{i\tau x} H''(x,\xi)$$

The remaining integral is finite and decreases at $\tau \to \infty$ provided $H''(x,\tau)$ does not have a strong singularity $1/(x-\xi)^p$ with $p \geq 1$

- Cusp at $x = \xi$ gives rize to an oscillating $1/\tau^2 e^{i\tau\xi}$ tail at large Ioffe times

— Not a problem for pGPDs

— Nonperturbative corrections for qGPDs at $x = \xi$ come from large τ

Summary

Renormalons allow one to study functional dependence of power corrections in specific kinematic limits

- Normalization at zero momentum leads to a strong reduction of power corrections apart from the $x \to 1$ limit, both for pGPDs and qGPDs
- Nonperturbative corrections to (normalized) pseudo-GPDs for $x \to \xi$ are expected to be small
- Nonperturbative corrections to quasi-GPDs remain finite for $x \to \xi$, with the power changing from $1/(Pv)^2$ at $x - \xi = \mathcal{O}(1)$ to $1/(Pv)$ at $x - \xi \lesssim \mathcal{O}(\Lambda/(Pv))$

- Finite-$t/(vP)^2$ corrections can be significant and have to be taken into account