Helicity Parton Distribution Functions at NNLO: determination from DIS and SIDIS data and impact of heavy quarks

QCD Evolution 2024

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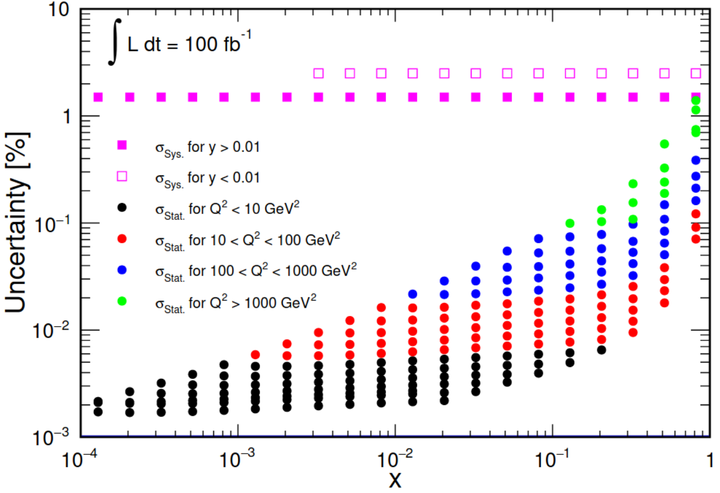
Precision



[Figure from the EIC Yellow Report arXiv:2103.05419]

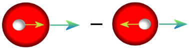
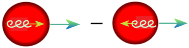
At the EIC, cross sections are expected to be measured with a precision of 1%

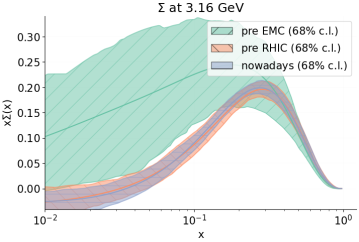
Accuracy

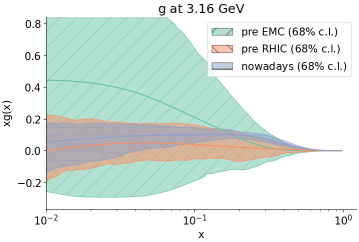
The accuracy of theoretical predictions must match the precision of the measurements

$$\mathcal{O}_I = \sum_{f=q,\bar{q},g} \Delta\mathcal{C}_{If}(y,\alpha_s(\mu^2)) \otimes \Delta f(y,\mu^2) + \text{p.s. corrections} \qquad f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

$$\Delta f(x) \equiv f^{\uparrow}(x) - f^{\downarrow}(x)\,, \qquad f = u, \bar{u}, d, \bar{d}, s, \bar{s}, g$$

$\Delta q(x) =$  $\qquad \Delta g(x) =$ 

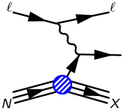
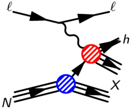
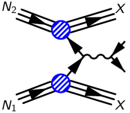




Accuracy of matrix elements

Accuracy of PDFs

Data: spin asymmetries

PROCESS	MEASURED ASYMMETRIES	SUBPROCESSES	PROBED PDFS
 DIS $\ell^\pm + N \to \ell^\pm + X$	$A_1 \approx \frac{\sum_q \Delta q(x)+\Delta\bar{q}(x)}{\sum_{q'} q'(x)+\bar{q}'(x)}$	$\gamma^* q \to q$	$\Delta q + \Delta\bar{q}$ Δg (NLO)
 SIDIS $\ell^\pm + N \to \ell^\pm h + X$	$A_1^h \approx \frac{\sum_q \Delta q(x)\otimes D_q^h(z)}{\sum_{q'} q'(x)\otimes D_{q'}^h(z)}$	$\gamma^* q \to q$	$\Delta u\ \Delta\bar{u}$ $\Delta d\ \Delta\bar{d}$ Δg (NLO)
	$A_{LL}^{\gamma N\to D_0 X} \approx \frac{\Delta g\otimes D_c^{D^0}(z)}{g(x)\otimes D_c^{D^0}(z)}$	$\gamma^* g \to c\bar{c}$	Δg
 pp $N_1 + N_2 \to \{j, W^\pm, h\} + X$	$A_{LL}^{jet} \approx \frac{\sum_{a,b=q,\bar{q},g} \Delta f_a(x_1)\otimes\Delta f_b(x_2)}{\sum_{a,b,c=q,\bar{q},g} f_a(x_1)\otimes f_b(x_2)}$	$gg \to qg$ $qg \to qg$	Δg
	$A_L^{W^+} \approx \frac{\Delta u(x_1)\bar{d}(x_2)-\Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2)+\bar{d}(x_1)u(x_2)}$	$u_L\bar{d}_R \to W^+$ $d_L\bar{u}_R \to W^+$	$\Delta u\ \Delta\bar{u}$ $\Delta d\ \Delta\bar{d}$
	$A_{LL}^h \approx \frac{\sum_{a,b,c=q,\bar{q},g} \Delta f_a(x_1)\otimes\Delta f_b(x_2)\otimes D_c^h(z)}{\sum_{a,b,c=q,\bar{q},g} f_a(x_1)\otimes f_b(x_2)\otimes D_c^h(z)}$	$gg \to qg$ $qg \to qg$	Δg

Theory: factorisation and evolution

1. Collinear factorisation of physical observables $\mathcal{O}_I$
   - a convolution between coefficient functions $\Delta\mathcal{C}_{If}(x, \alpha_s(\mu^2))$ and PDFs $\Delta f(x, \mu^2)$

$$\mathcal{O}_I = \sum_{f=q,\bar{q},g} \Delta\mathcal{C}_{If}(y, \alpha_s(\mu^2)) \otimes \Delta f(y, \mu^2) + \text{p.s. corrections}$$

   - coefficient functions allow for a perturbative expansion in terms of $a_s = \alpha_s/(4\pi)$

$$\Delta\mathcal{C}_{If}(y, \alpha_s) = \sum_{k=0} a_s^k \Delta C_{If}^{(k)}(y)$$

- DIS (up to NNLO) [NPB 417 (1994) 61]
- SIDIS (up to NNLO) [arXiv:2404.08597; arXiv:2404.09959]
- *pp* (up to (N)NLO) [PRD 70 (2004) 034010] [PLB 817 (2021) 136333] [PRD 67 (2003) 054004, ibidem 054005]

2. Evolution of parton distributions
   - a set of $(2n_f + 1)$ integro-differential equations, n_f is the number of active flavors

$$\frac{\partial}{\partial \ln \mu^2} \Delta f_i(x, \mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} \Delta\mathcal{P}_{ji}\left(z, \alpha_s(\mu^2)\right) \Delta f_j\left(\frac{x}{z}, \mu^2\right)$$

   - with perturbative computable splitting functions

$$\Delta\mathcal{P}_{ji}(z, \alpha_s) = \sum_{k=0} a_s^{k+1} \Delta P_{ji}^{(k)}(z)$$

- LO [NP B126 (1977) 298]
- NLO [ZP C70 (1996) 637, PR D54 (1996) 2023]
- NNLO [NP B889 (2014) 351]

Theory: constraints

1. Polarized PDFs must lead to positive cross sections [see S. Forte's talk]
   - at LO, polarized PDFs are bounded by their unpolarized counterparts

$$|\Delta f(x,\mu^2)| \leq f(x,\mu^2)$$

   - beyond LO, other relations hold, but are of limited effect [NP B534 (1998) 277]

2. Polarized PDFs must be integrable
   - *i.e.* require that the axial matrix elements of the nucleon are finite

$$\langle P,S|\bar{\psi}_q\gamma^\mu\gamma_5\psi_q|P,S\rangle \longrightarrow \text{finite for each flavor } q$$

3. Assume SU(2) and SU(3) symmetry
   - relate the octet of axial-vector currents to β-decay of spin-$1/2$ hyperons

$$a_3 = \int_0^1 dx\,\Delta T_3 = F + D \qquad a_8 = \int_0^1 dx\,\Delta T_8 = 3F - D$$

$$\Delta T_3 = (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d}) \qquad \Delta T_8 = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) - 2(\Delta s + \Delta\bar{s})$$

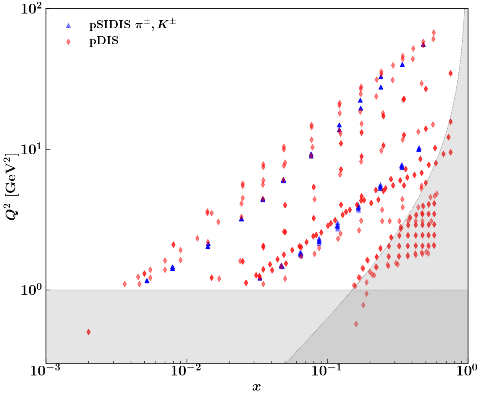
   - note: violations of SU(3) symmetry are advocated in the literature [ARNPS 53 (2003) 39]

1. Accuracy of PDFs: MAPPDFpol1.0

[arXiv:2404.04712]

https://github.com/MapCollaboration/Denali

The data set



kinematic cuts:

$Q^2 > 1$ GeV^2 $\quad W^2 > 6.25$ GeV^2

almost all JLab data are excluded by these cuts
(data not indicated in the table)

Data set	Obs.	$N_{\rm dat}$	Reference
EMC	g_1^p	10	[NPB 328 (1989) 1]
SMC	g_1^p	12	[PRD D58 (1998) 112001]
	g_1^d	12	[PRD 58 (1998) 112001]
E142	g_1^n	7	[PRD 54, (1996) 6620]
E143	g_1^p	25	[PRD 58 (1998) 112003]
	g_1^d	25	[PRD 58 (1998) 112003]
E154	g_1^n	11	[PRL 79 (1997) 26]
E155	g_1^p/F_1^p	22	[PLB 493 (2000) 19]
	g_1^n/F_1^n	22	[PLB 493 (2000) 19]
COMPASS	g_1^p	15	[PLB 753 (2016) 18]
	g_1^d	17	[PLB 769 (2017) 34]
HERMES	g_1^n	8	[PLB 404 (1997) 383]
	g_1^p	14	[PRD 75 (2007) 012007]
	g_1^d	14	[PRD 75 (2007) 012007]
COMPASS	$A_{1,p,d}^{\pi^+}$	22	[PLB 680 (2009) 217]
	$A_{1,p,d}^{\pi^-}$	22	[PLB 680 (2009) 217]
	$A_{1,p,d}^{K^+}$	22	[PLB 680 (2009) 217]
	$A_{1,p,d}^{K^-}$	22	[PLB 680 (2009) 217]
HERMES	$A_{1,p,d}^{\pi^+}$	18	[PRD 99 (2019) 112001]
	$A_{1,p,d}^{\pi^-}$	18	[PRD 99 (2019) 112001]
	$A_{1,d}^{K^+}$	9	[PRD 99 (2019) 112001]
	$A_{1,d}^{K^-}$	9	[PRD 99 (2019) 112001]
Total		362	

The methodology

Monte Carlo representation of data uncertainties into PDF uncertainties ($N_{\rm rep} = 150$)

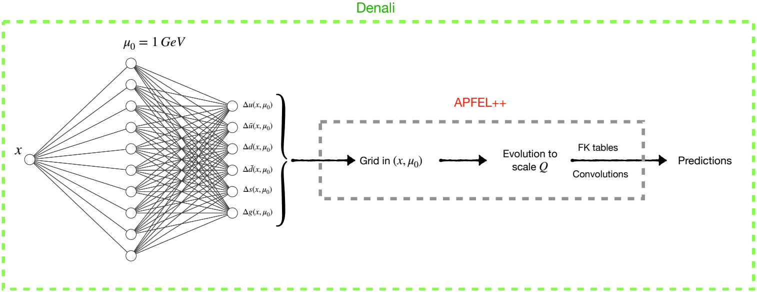
Neural Network parametrisation at $\mu_0 = 1$ GeV: $\{\Delta u, \Delta\bar{u}, \Delta d, \Delta\bar{d}, \Delta s, \Delta\bar{s}, \Delta g\}$

For each helicity PDF replica, input unpolarised PDFs and FFs are random replicas from NNPDF3.1 [EPJ C77 (2017) 663] and MAPFF1.0 [PL8 34 (2022) 137456]

Constraints on a_3 and a_8 implemented through pseudodata
$a_3 = 1.2756 \pm 0.0013 \qquad a_8 = 0.585 \pm 0.025$ [PTEP 2022 (2022) 083C01]

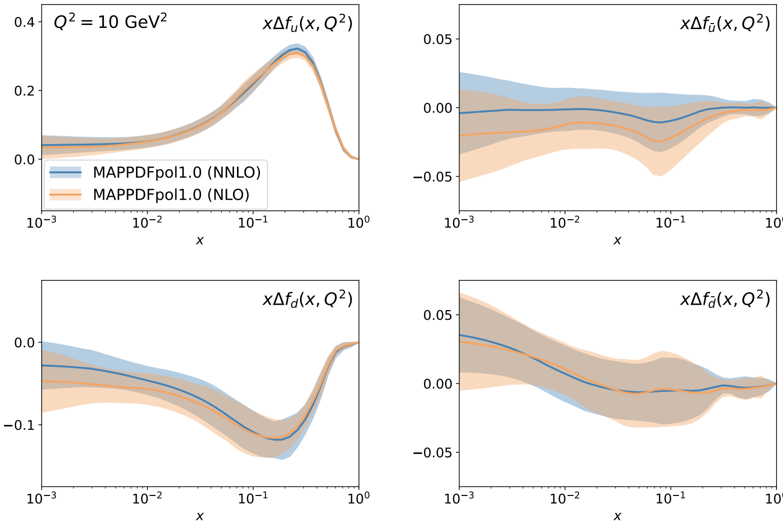
The last layer is bound to the positivity inequality by construction

NNLO theoretical predictions for SIDIS are approximate [PRD 104 (2021) 094046]



[Image credit: A. Chiefa]

Impact of perturbative corrections



$N_{\rm dat} = 362 \qquad \chi^2/N_{\rm dat} = 0.64$ (NLO) $\qquad \chi^2/N_{\rm dat} = 0.78$ (NNLO)

Impact of perturbative corrections generally moderate, except for Δg and Δs, $\Delta \bar{s}$

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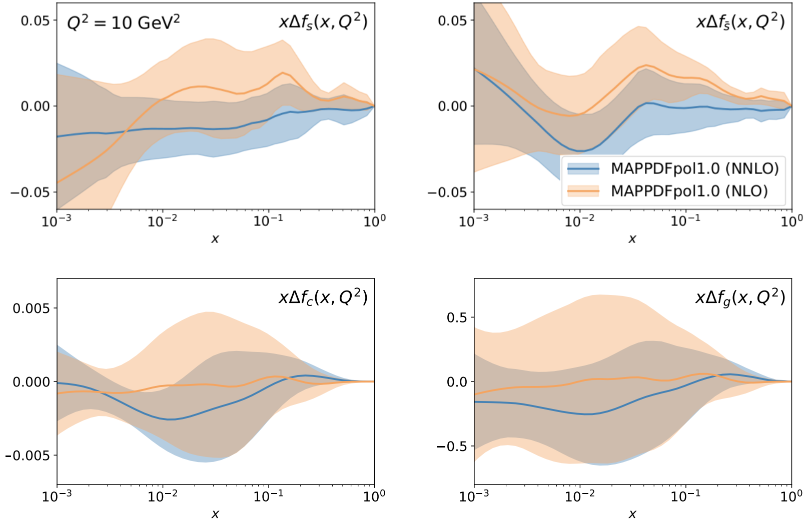

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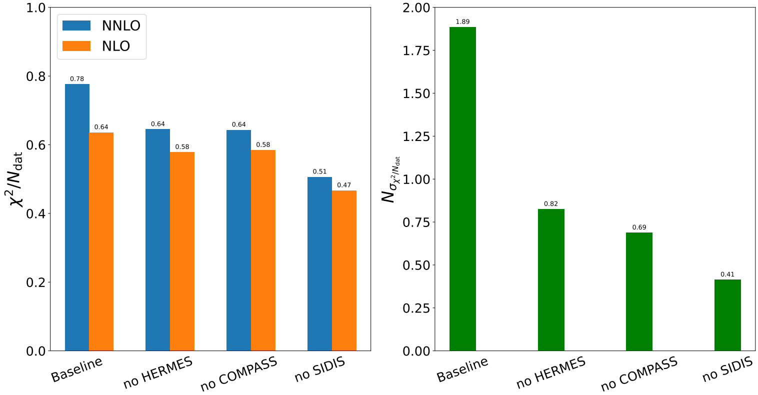
Impact of perturbative corrections generally moderate, except for Δg and Δs, $\Delta\bar{s}$

Impact of data

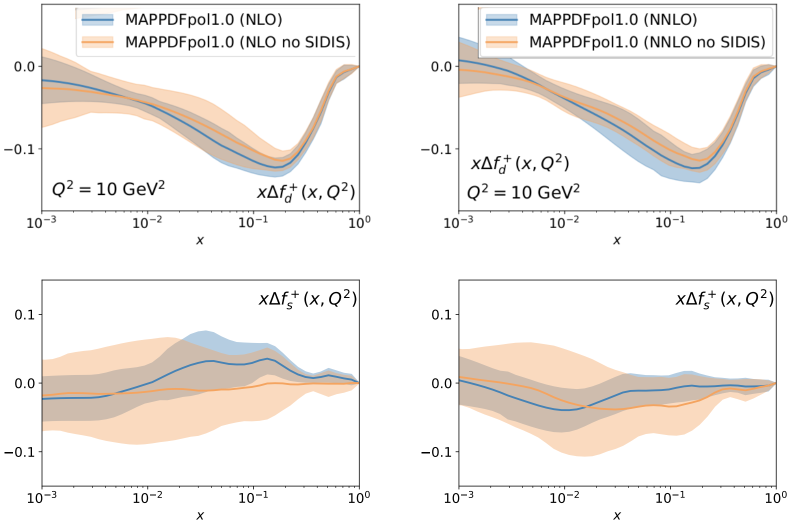


$$N_{\sigma_{\chi^2/N_{\rm dat}}} = (\chi^2_{\rm NNLO} - \chi^2_{\rm NLO})/\sqrt{2N_{\rm dat}}$$

SIDIS data sets are not described as well as the DIS ones

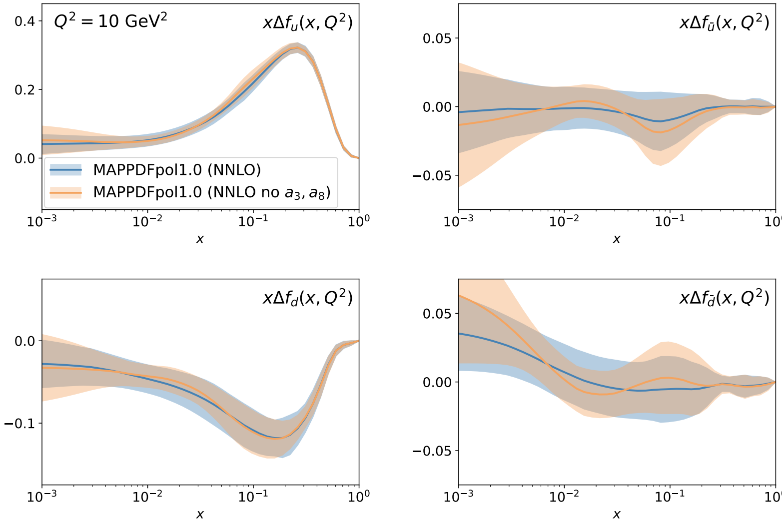
HERMES and COMPASS SIDIS data sets, while being equally well described separately, are no longer so when included together in the fit

Impact of data



Upon inclusion of SIDIS data, PDF uncertainties generally decrease at both NLO and NNLO (or remain unchanged, for Δf_u^+ and Δg), despite the χ^2 increase at NNLO

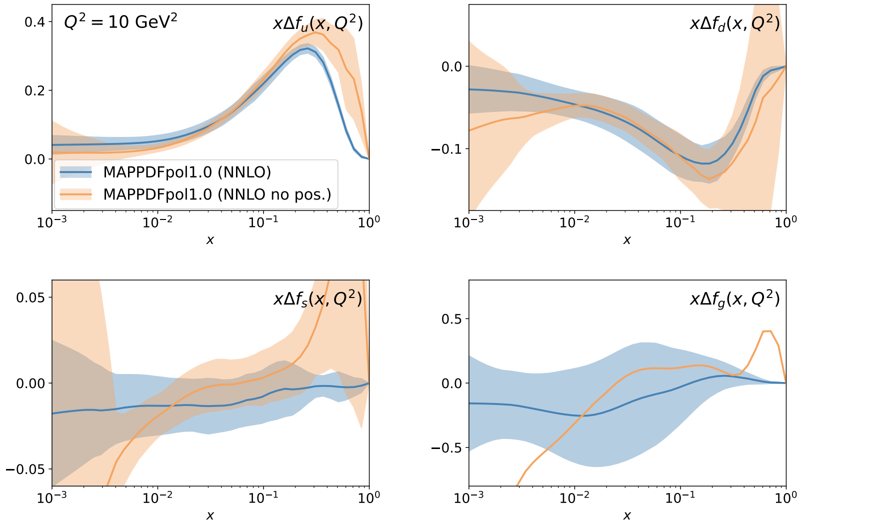
Impact of theoretical constraints: sum rules



NLO	$\chi^2/N_{\rm dat} = 0.64$	$\chi^2/N_{\rm dat} = 0.63$ (no a_3, no a_8)
NNLO	$\chi^2/N_{\rm dat} = 0.78$	$\chi^2/N_{\rm dat} = 0.74$ (no a_3, no a_8)

No evidence of violation of SU(2) and SU(3) symmetries

Impact of theoretical constraints: positivity



If one removes the positivity constraint, the fit falls apart

NNLO $\chi^2/N_{\rm dat} = 0.78$ $\chi^2/N_{\rm dat} = 0.66$ (no pos.) NLO $\chi^2/N_{\rm dat} = 0.64$

2. Accuracy of matrix elements: polarised FONLL

[Eur.Phys.J. C84 (2024) 189]

https://github.com/NNPDF/eko

https://github.com/NNPDF/yadism

How to treat heavy quarks in polarised DIS?

Let us restrict ourselves to the case of the charm quark

The current treatment of charm in polarised DIS: ZM-VFN scheme (charm is massless)

Extend the FONLL GM-VFN scheme [NPB 834 (2010) 116] to polarised DIS

$$g_1^{\rm FONLL} = g_1^{\rm FFNS,3} + g_1^{\rm FFNS,4} - g_1^{\rm double-counting}$$

$g_1^{\rm FFNS,3}$ retains all mass effects at a finite order

$g_1^{\rm FFNS,4}$ resums all collinear logs, but has no power-like terms

$g_1^{\rm double-counting}$ is the overlap between FFNS,3 and FFNS,4

NNLO splitting functions [NPB 889 (2014) 351; PLB 748 (2015) 432; JHEP 01 (2022) 193]

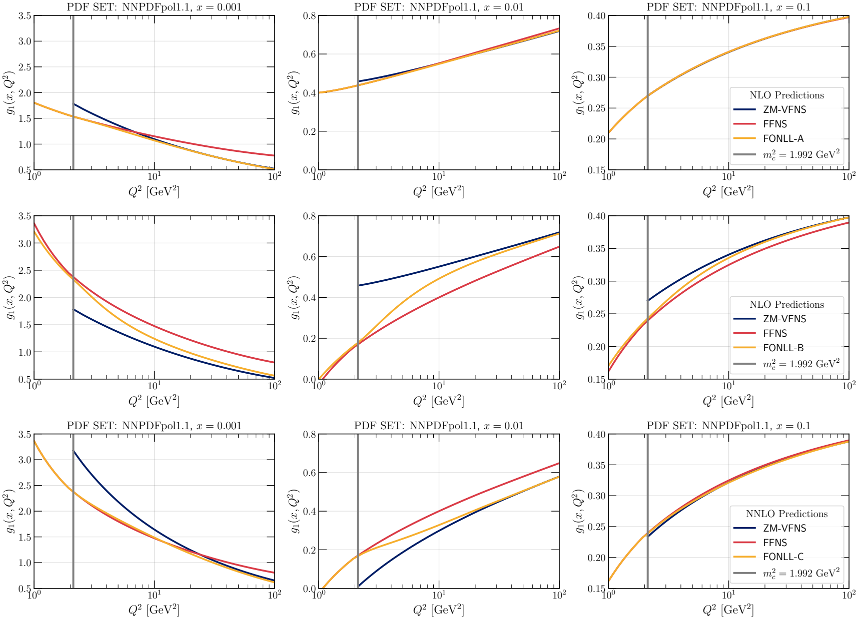
NNLO matching conditions [NPB 988 (2023) 116114]

NNLO massless coefficient functions [NPB 417 (1994) 61]

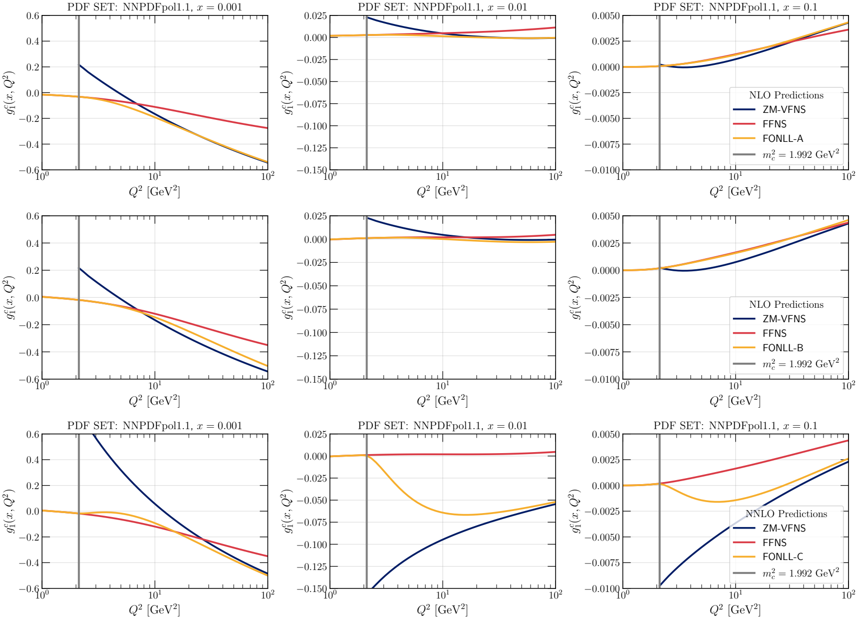
NNLO massive coefficient functions [PRD 98 (2018) 014018; NPB 897 (2015) 612; ibid. 953 (2020) 114945; ibid. 964 (2021) 115331; PRD 104 (2021) 034030; NPB 988 (2023) 116114; ibid. 999 (2024) 116427]

implemented in EKO [EPJ C82 (2022) 976] and YADISM [arXiv:2401.15187]

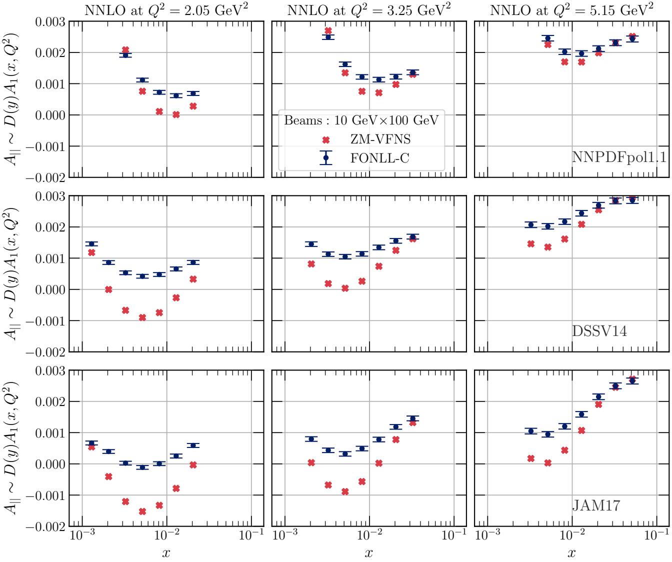
FONLL structure functions: g_1



FONLL structure functions: g_1^c

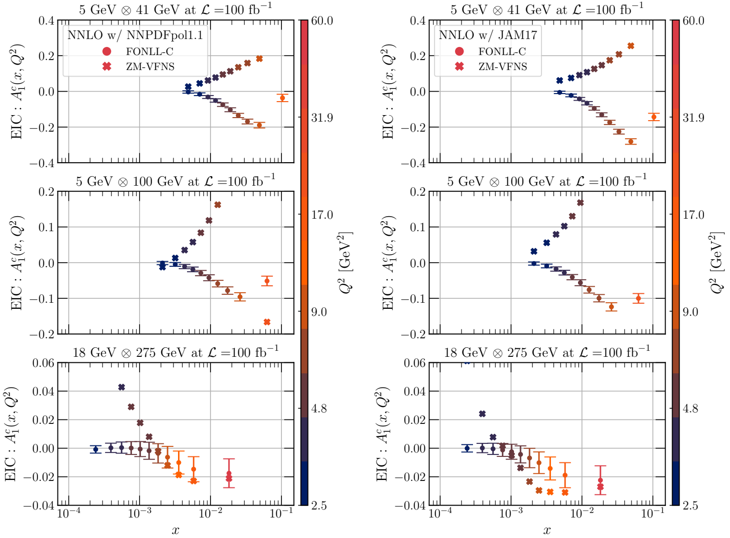


Impact of FONLL on asymmetries at the EIC: $A_\parallel$



The difference between predictions obtained with either the ZM-VFN or the FONLL schemes is larger than the projected experimental uncertainties (irrespective of the input PDF set)

Impact of FONLL on asymmetries at the EIC: A_1^c



The difference between predictions obtained with either the ZM-VFN or the FONLL schemes is larger than projected experimental uncertainties (irrespective of the input PDF set)

3. To conclude

Summary

MAPPDFpol1.0 is the first attempt of a global NNLO determination of polarised PDFs

The impact of NNLO corrections on PDFs is moderate

The impact of NNLO corrections on fit quality is more significant

There is a subtle interplay between NNLO corrections and experimental data

The sea-quark and gluon polarised PDFs remain unconstrained from DIS and SIDIS and essentially compatible with zero within their large uncertainties

The total polarised strange PDF turns out to be similar in the global determination and in the determination without SIDIS data

No evidence of any significant violation of the SU(2) and SU(3) flavour symmetries

Positivity constraints play a decisive role in constraining PDFs

The FONLL GM-VFN scheme has been extended to polarised DIS

Charm mass effects are sizeable on the scale of the precision of EIC experimental data

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Thank you