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Analysis methods & tools at KATRIN

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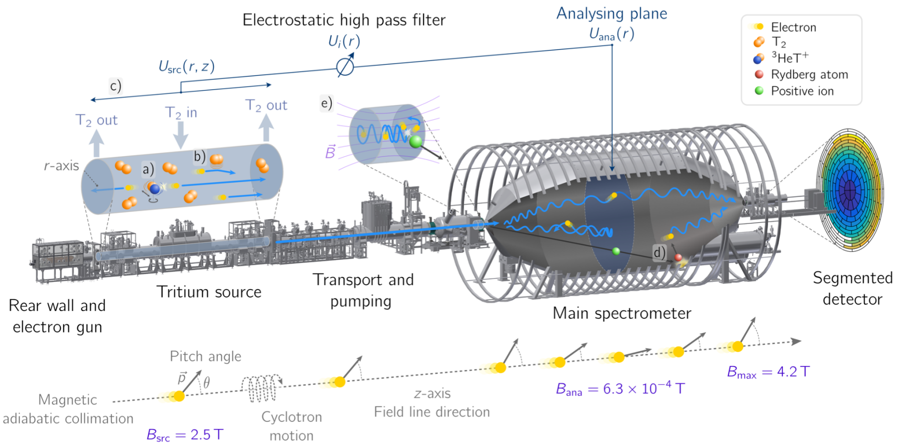
NuMass workshop, Feb. 29, 2024

Outline

- KATRIN analysis overview
- KATRIN analysis challenges and solutions
- Introduction to KATRIN analysis methods

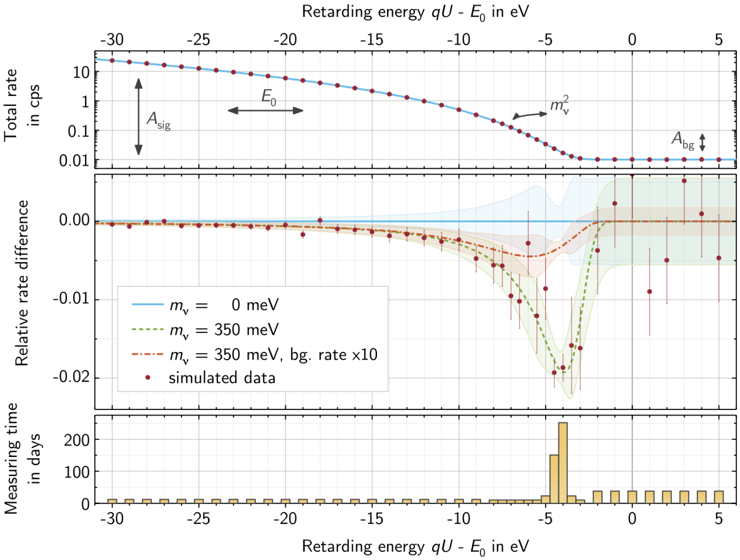
The KATRIN beamline



Pixel-wise field configuration and detector response

For details, see Volker's talk, Monday 14:20

The KATRIN spectrum



KATRIN dataset

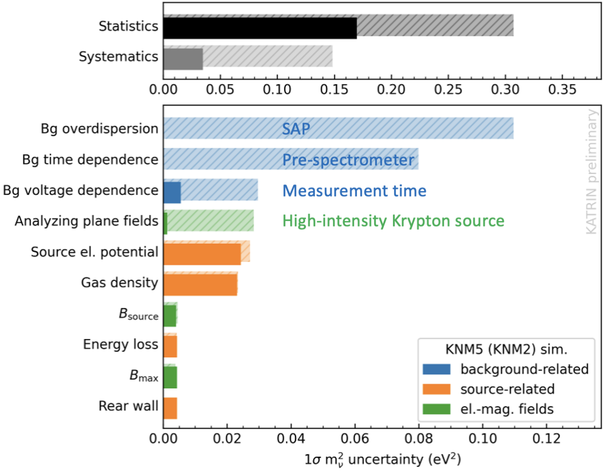
Beta scans:

- Subruns: 10∼1000 seconds of data collected at fixed high voltage
  – Pixel-wise event rates corrected by detector efficiency
- Runs: combination of scans over all high voltage set points (∼3 hours)
  – Scan direction: up, down, random
- Beta scans form measurement campaigns: collection of runs with the same measurement configuration, several months
  – Five campaigns for the upcoming data release, with different field strengths, tritium column density and measurement time distribution
  – Golden runs selected with monitored slow control data

Calibration and monitoring scans:

- Electron gun measurement of column density and energy loss
- Krypton calibration of source and field

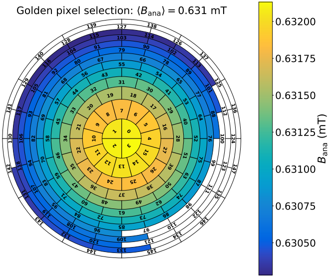
KATRIN systematics from calibration

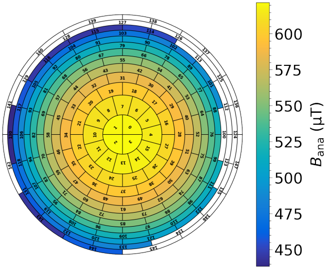


See Joscha's talk, Monday 15:00 and Benedikt's talk, Friday 10:45

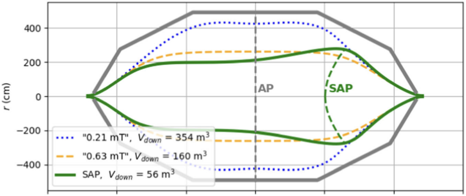
Stacking of KATRIN beta scan data

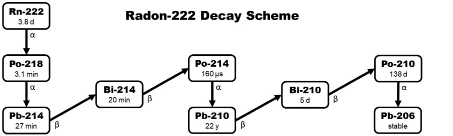
- $\sim$1000 runs $\times$ 147 pixels, need a reduction of data size and number of fitting parameters
- Stack pixel-wise data at the same high voltage set point
- Combine detector pixels with similar transmission conditions into patches
- Negligible systematic contribution

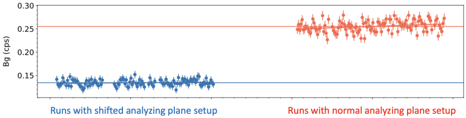




The shifted analyzing plane configuration







- A shifted analyzing plane configuration has been adopted since April 2020, leading to a higher radial inhomogeneity in the magnetic fields
- Uniform pixel combination no longer valid; instead use 14 "patches" for each campaign → model evaluation complicated

KATRIN analysis challenges

- Computation time increases more than linearly with respect to number of patches
  – 14 times more data points for each likelihood evaluation
  – 14 times more free fitting parameters A_{sig}, A_{bkg} and E_0
  – Slower convergency

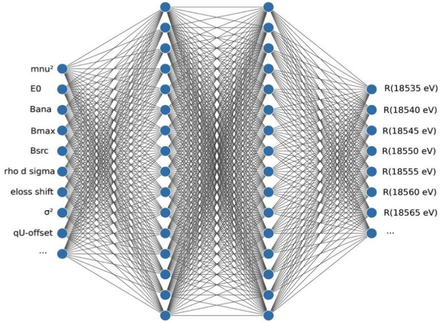
- Two independent solutions
  – The KaFit team: highly optimized, direct model evaluation with block caching
  – The Netrium team: fast model prediction with a neural network

KaFit optimizations

- Block caching of detector response function and tritium decay spectrum
- Convolution of multiple scattering energy loss in Fourier space
- Shared dependent models among detector pixels
- Numerical Gauss-Legendre integration
- Fitting time reduced by a factor of $\sim$10k for one campaign, O(100) CPU hours for combined analysis using O(10) cores

Model prediction with neural network

- Network training with $O(10^6)$ Monte Carlo spectra
  – $\sim$100k CPU hours in network preparation
- Fast analytical model prediction in given parameter range
  – $O(10)$ CPU hours in fitting
- Regenerate neural networks for new parameter ranges



See *Eur. Phys. J. C 82, 439 (2022)* for more details

Introduction to KATRIN analysis: blinding

- Blind neutrino mass from analysis
  - Artificial final state distributions generated to randomly shift neutrino mass in its uncertainty range
- Comparison between fitting teams with Monte Carlo data
  - Investigation on systematic effects
  - Upon agreement between teams, freeze all inputs to the model
- Run-wise fit on real data with $m_\nu = 0$ fixed
  - Check endpoint, signal and background stability
- Unblind neutrino mass with real data
  - Data analysis with artificial final state distributions
  - Upon agreement between teams, freeze all analysis methods and use true final state distributions for final results

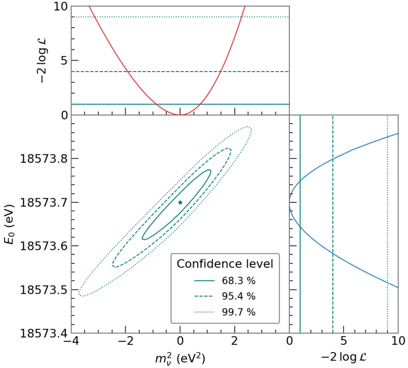
- Likelihood function:

$$-2\log\mathcal{L}_{\text{nap}} = \sum_i \frac{(R^{\text{calc}}(qU_i) - R^{\text{meas}}(qU_i))^2}{\sigma_R^2},$$

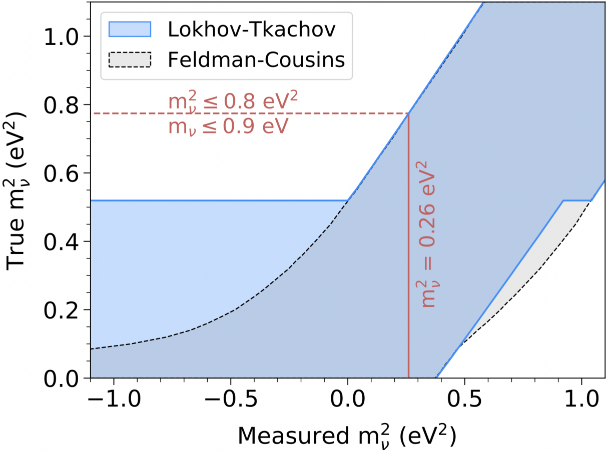
$$-2\log\mathcal{L}_{\text{sap}} = \sum_{i,k} 2\left(R_k^{\text{calc}}(qU_i)\cdot t_i - N_{i,k} + N_{i,k}\cdot\ln\frac{N_{i,k}}{R_k^{\text{calc}}(qU_i)\cdot t_i}\right),$$

$$-2\log\mathcal{L}_{\text{sys}} = (\vec{\eta} - \vec{\eta}_{\text{ext}})^T\cdot\Theta_{\text{cov}}^{-1}\cdot(\vec{\eta} - \vec{\eta}_{\text{ext}}).$$

  - Patch-wise model k for SAP
  - Normal distribution for NAP
- Model extension for negative m_ν^2
- For beyond standard neutrino interaction searches, see Shailaja's talk, Thursday 11:25



Confidence belt: Lokhov-Tkachov construction



Estimator: $\max(\hat{m}_\nu^2, 0)$. See *Phys. Part. Nucl. 46, 347-365 (2015)*

The Bayesian approach

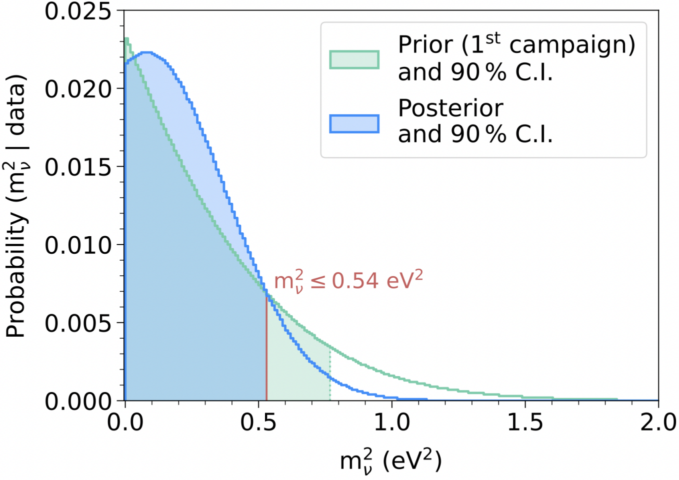
- The Bayes' theorem:

$$\pi(\theta_j|y) \propto \int_{\theta_1}\int_{\theta_2}\cdots\int_{\theta_{j-1}}\int_{\theta_{j+1}}\cdots\int_{\theta_n} \pi(y|\vec{\theta})\,\pi(\theta_j)\cdot\prod_{i\neq j}^{n}\pi(\theta_i)d\theta_i$$

- Physical, non-negative priors on neutrino mass

- Markov Chain Monte Carlo method for multi-dimensional integration
  - Efficiency critical in parameter space of Dim. ~ 400
  - KaFit: Affine-invariant stretch move and parallel sampling (**emcee**)
    Require likelihood function evaluation
  - Netrium: Hamiltonian move with Netrium using **BAT**
    Require first derivatives of the likelihood function

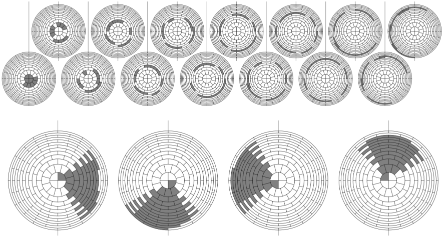
Data combination among campaigns

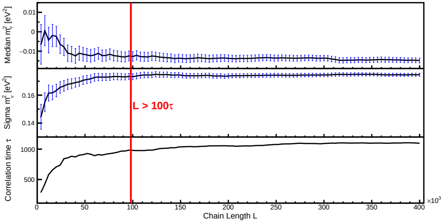


Posterior(n^{th} campaign) $\rightarrow$ Prior($n+1^{th}$ campaign)

- Frequentist:
  - Fitted neutrino mass trend with different energy ranges
  - Split data set by scan direction and detector patches
  - Best-fit evolution w.r.t systematic contributions

- Bayesian:
  - Convergency criteria
  - Sampling error





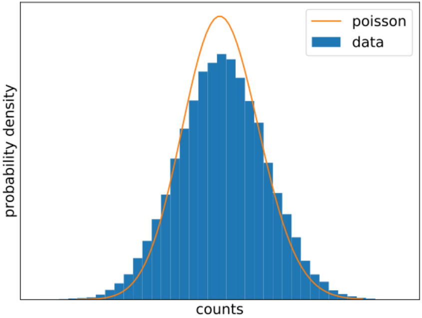
Summary

- Software optimizations and neural network approach solve the KATRIN analysis challenge
- Frequentist results with blind analysis coming out soon
- Bayesian analysis in good path

Thanks for your attention!

Backup: Non-poissonian background overdispersion

- KeV electrons from ^{219}Rn decay trapped in main spectrometer
- Trapped electrons scatter off residual gas and produce correlated secondaries
- Described by a combination of Poisson and normal distribution, characterized by their ratio
- No longer observed in the asymmetric SAP setup



Backup: prior dependency in Bayesian analysis

- Posterior weakly constrained by data with a strong prior
  - $\sum_i m_i < 0.12\,\text{eV}$ from cosmology
- Weakly informative priors
  - Gaussian prior from Mainz and Troitsk results
- Uninformative priors
  - Flat in m_ν^2
  - Flat positive in m_ν^2
  - Flat positive in m_ν
- Least informative prior
  - Constructed to maximize constraint from data

- Flat positive prior in m_ν^2 suggested
  - Mode converging to the frequentist best-fit
  - No unphysical model extension
  - Equivalent to truncated flat prior in m_ν^2 after chain convergence