BAYESIAN ANALYSIS OF ^{187}Re DATA FROM THE MIBETA EXPERIMENT

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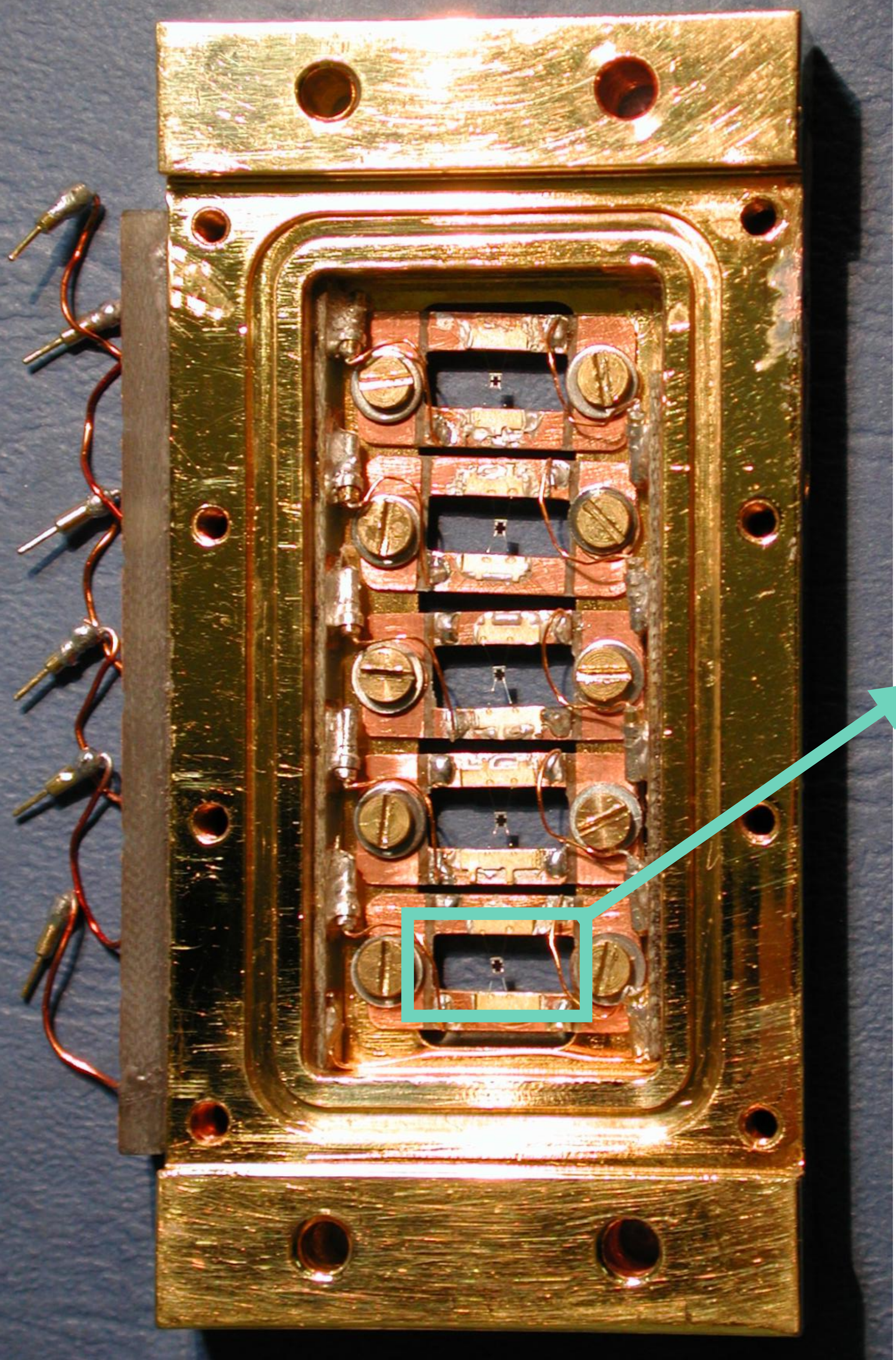
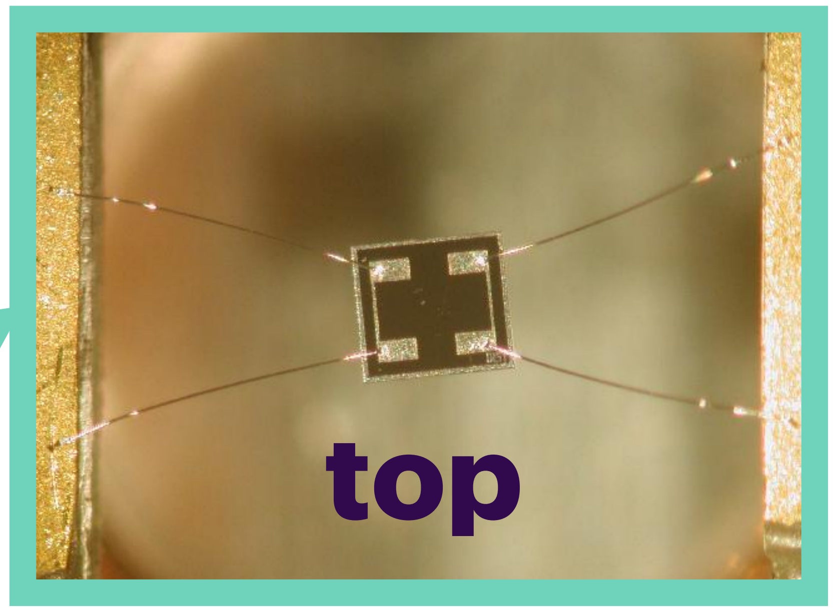
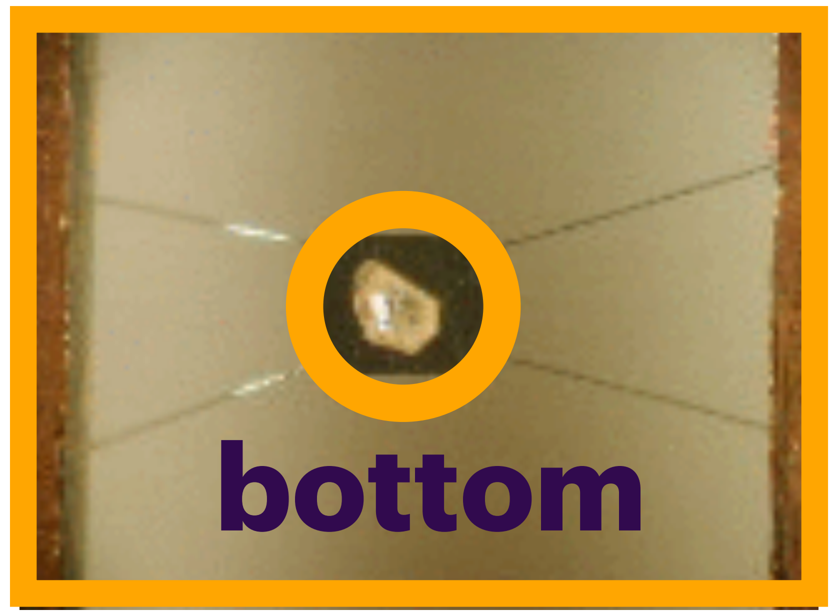
MIBETA (1996 - 2003)



- **Calorimetric** measurement of the electronic **anti-neutrino mass** from ^{187}Re beta decay (Q≈2470 eV).
- Detector: 2 arrays of 5 **microcalorimeters** consisting of an **AgReO4 crystal** and **Si thermistor** [1][2].



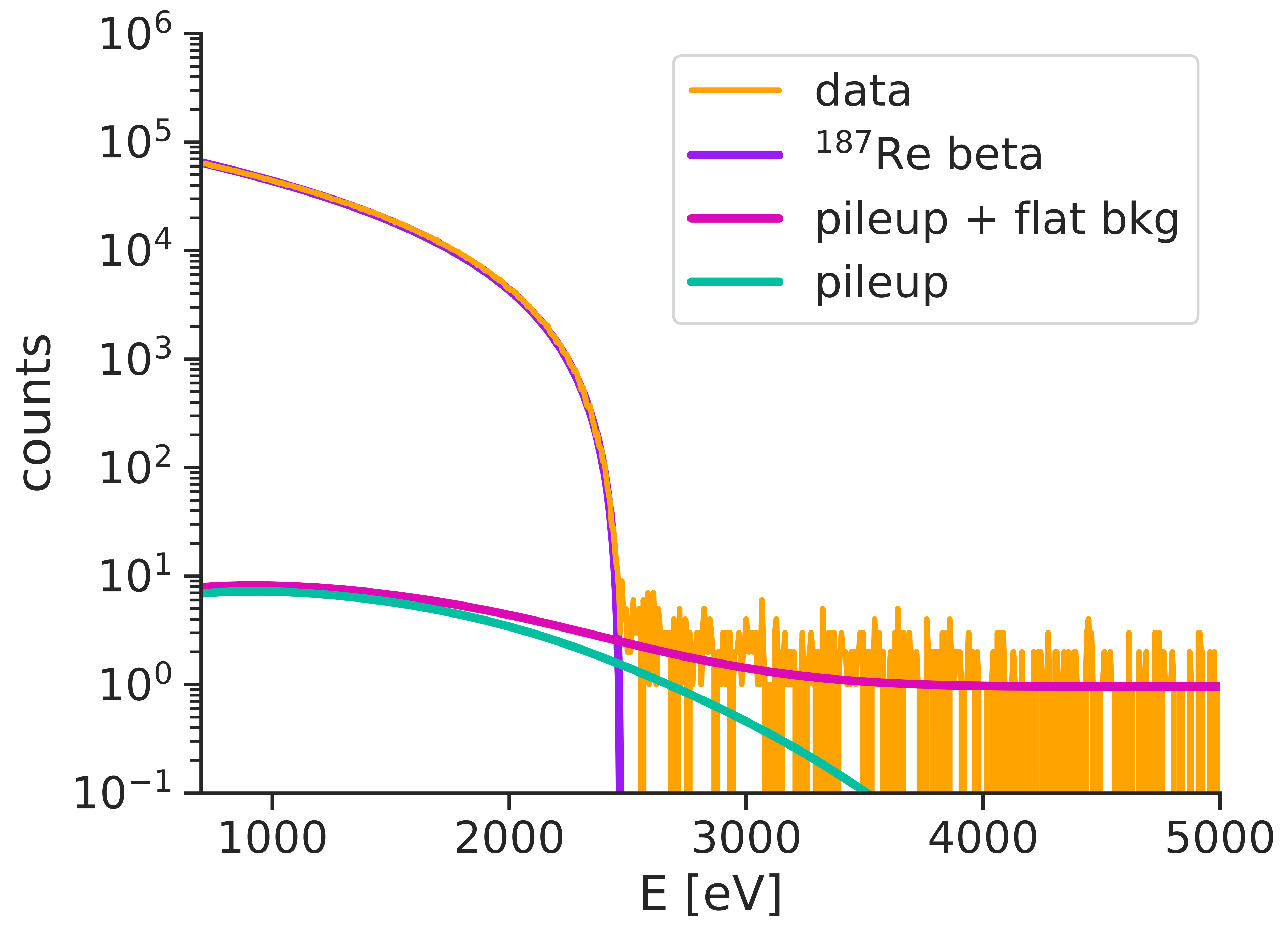


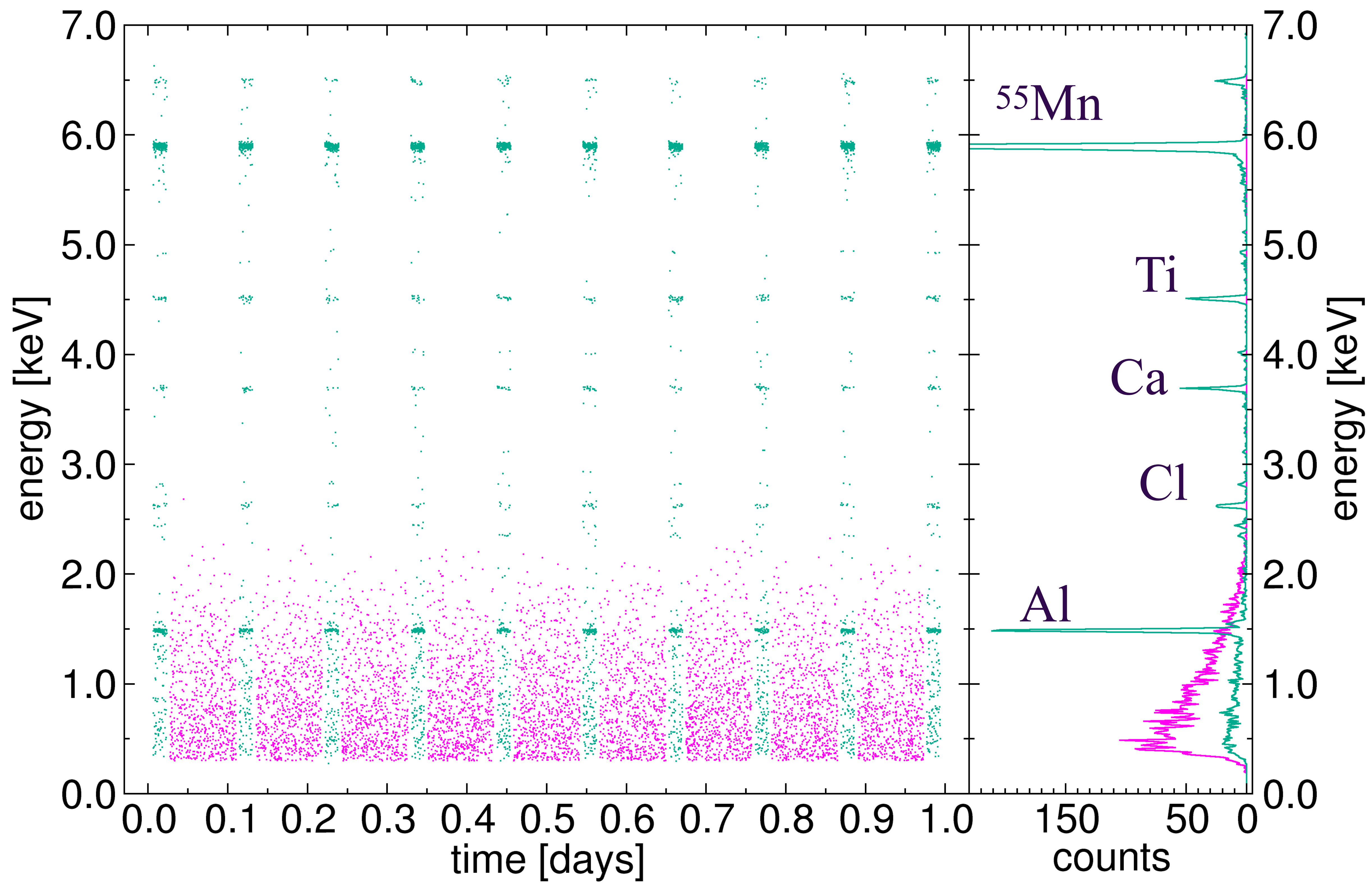
- **Detectors performance:** FWHM = 28 eV, ^{187}Re activity 0.15 Bq, rise time 0.5 ms
- **Frequentist result** (unpublished) of measurement campaign: $m_\nu \leq 15.3 \pm 2$ (syst) eV at 90% CL, $Q = 2465.3 \pm 0.5 \pm 1.5$ eV
- **This work:** treatment of additional systematics, build robust bayesian fitting procedure for the endpoint to use in future **HOLMES** analysis.

[1] M. Sisti and others, “New limits from the Milano neutrino mass experiment with thermal microcalorimeters,” Nucl. Instrum. Meth. A, vol. 520, pp. 125–131, 2004,
[2] C. Arnaboldi and others, “Bolometric bounds on the anti-neutrino mass,” Phys. Rev. Lett., vol. 91, p. 161802, 2003

AVAILABLE DATA

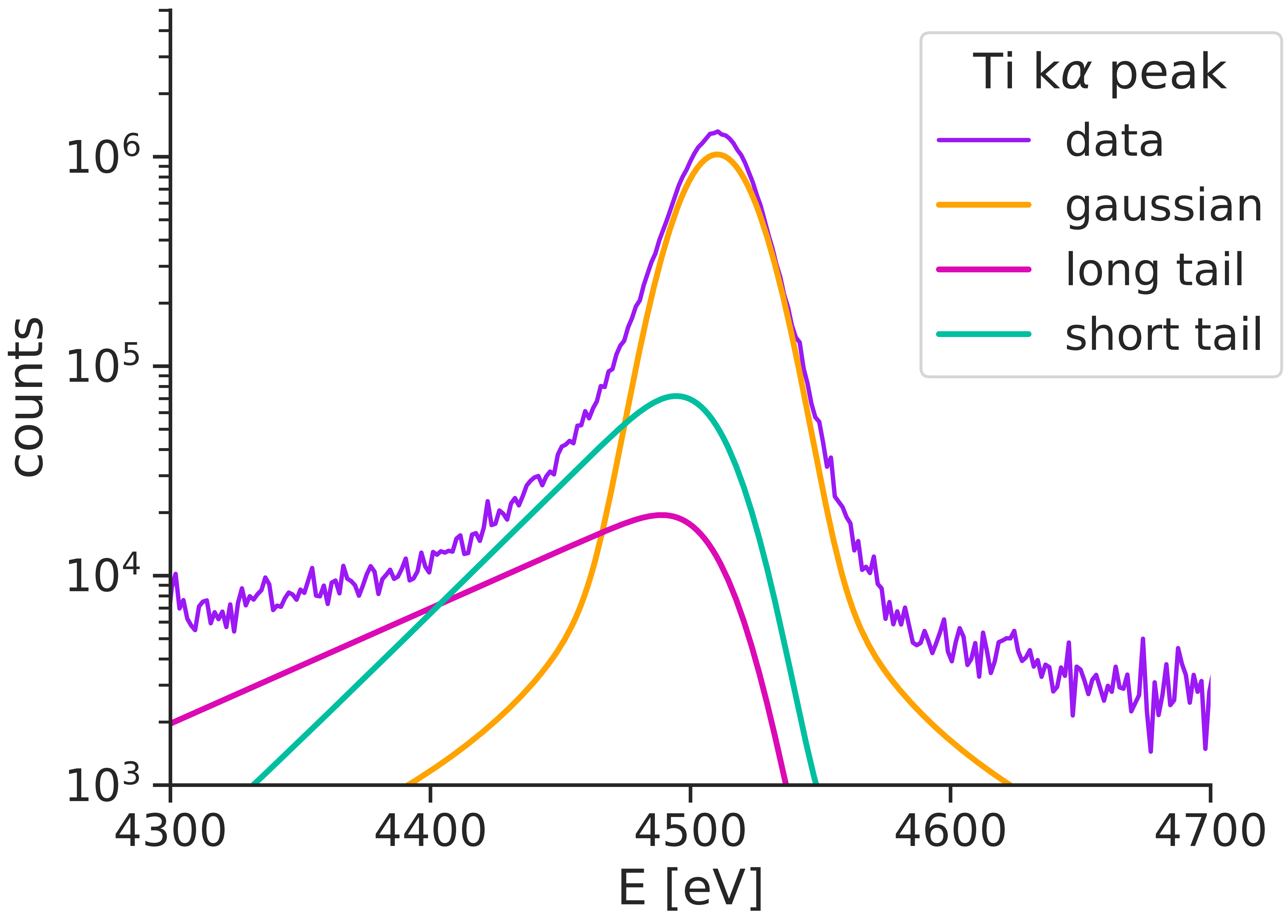
- $N_{tot} \sim 10^7$ beta events from 8 detectors in RUN 14 (5 months between 2002-2003).
- Data is **already calibrated and binned** with 0.6 eV bin width.





- Additional events from ^{55}**Fe calibration source** collected intermittently during the measurement.
- Considered **compound spectrum** of all detectors due to similar activity, FWHM and very small energy offsets from calibration.

EXPERIMENTAL RESPONSE



- Observed calibration peaks are **asymmetric** towards lower energies due to detector's response.
- Response can be described by mixture of a symmetric gaussian and its convolution with two different exponentials.

$$R(E, FWHM, A_1, A_2, \lambda_1, \lambda_2) =$$
$$(1 - A_{long} - A_{short}) Gauss(E,0,FWHM) +$$
$$A_{long} Exp(E, \lambda_{long}) \otimes Gauss(E,0,FWHM) +$$
$$A_{short} Exp(E, \lambda_{short}) \otimes Gauss(E,0,FWHM) +$$

- Long exponential tail attributed to surface effects and **limited to external source only [3]**.
- **Short tail must be included** in the internal detector response.

[3] E. Ferri and others, "Investigation of peak shapes in the MIBETA experiment calibrations." *The European Physical Journal A* 48, 2012

FIT LIKELIHOOD AND METHOD

- **Updated theoretical description [4]:** interpolation within 10^{-5} relative accuracy of spectrum considering atomic effects.

- **Analytic approximation of pileup** $y_{pu}(E, Q)$ obtained by auto-convolution of $(Q - E)^2$ dependence which was used in previous analyses.

$$y_{th}(E, Q, m_\nu) \propto F(E)pE(Q - E)\sqrt{(Q - E)^2 - m_\nu^2}D_{exc}(E)$$

$$y_{pu}(E, Q) \propto (Q - E)^2 \otimes (Q - E)^2$$

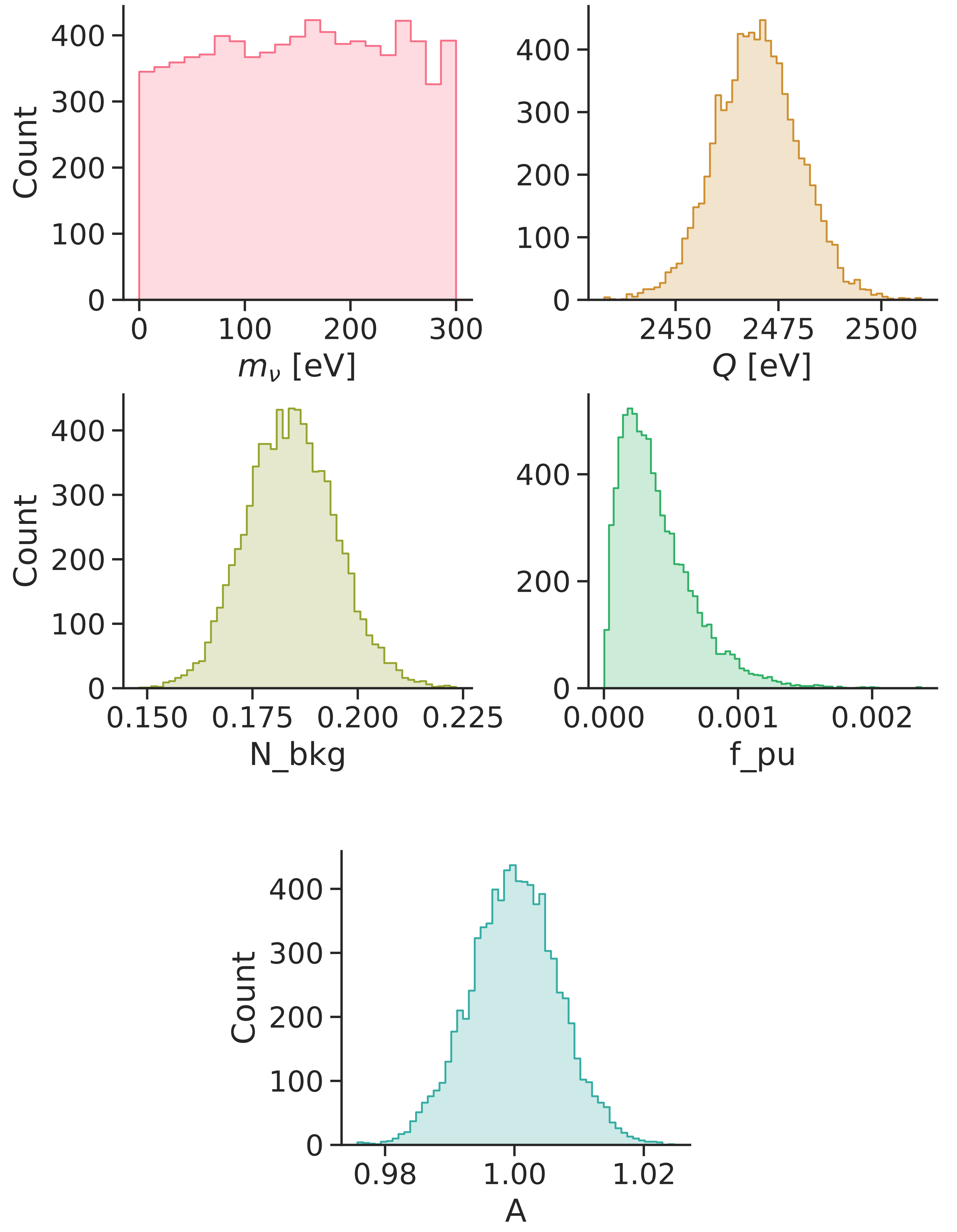
$$y(E, m_\nu, Q, N_{bkg}, f_{pu}, A, FWHM, \lambda, A_{exp}) \propto A[(1 - f_{pu})y_{th}(E, Q, m_\nu) + f_{pu}y_{pu}(E, Q) + N_{bkg}] \otimes R(E, FWHM, A_{exp}, \lambda)$$

- 8 parameters total: 5 parameters for the spectrum and 3 for the experimental response.
- Fit performed over binned data considering **poisson fluctuations** for each bin, 1.2 eV bins.
- Model implemented in **Stan,** an **Hamiltonian Monte Carlo** engine.
- Given a **model = priors + likelihood,** fit result consists of **samples from the posterior distribution.**

[4] O. Niţescu, R. Dvornický, and F. Šimkovic "Atomic corrections for the unique first-forbidden β transition of ^{187}Re"
Phys. Rev. C 109, 2023

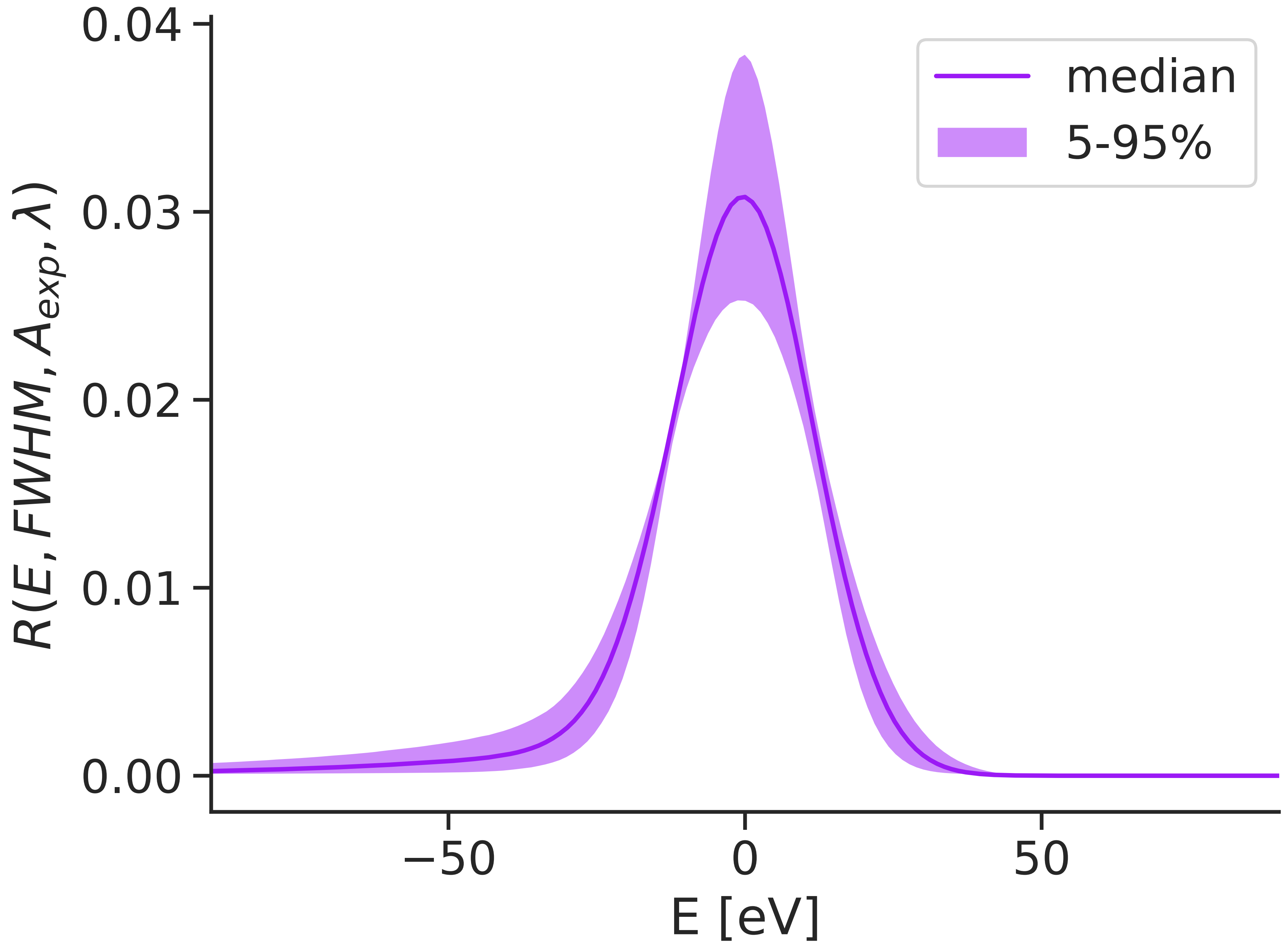
PRIORS - SPECTRUM

- **How to choose priors?** measurement, physical limits, model stability and regularization.
- Priors can be used to **model the systematics** of the experiment
- Flat prior for $m_\nu \sim \text{uniform}(0\text{ eV}, m_{max} \geq 300\text{ eV})$
- Prior for Q centered on PENTATRAP measurement [5]
  $Q = 2470.9 \pm 1.3\text{ eV} \rightarrow Q \sim \text{normal}(2740.9\text{ eV}, 10\text{ eV})$
- Background estimated from mean and number of events **between 4 keV and 6 keV**
  $N_{bkg} \sim \text{gamma}\left(\sum y(E_i), \text{len}(y(E_i))\right)$
- Pileup fraction prior centered on $A_{Re}\tau_R = 0.15 \times 0.5 \times 10^{-3}\text{s}$
  $f_{pu}(\times 10^4) \sim \text{gamma}(3.5, 0.5)$
- Normalization prior from Poisson statistics
  $A \sim \text{normal}(1, 1/\sqrt{N_{ev}})$



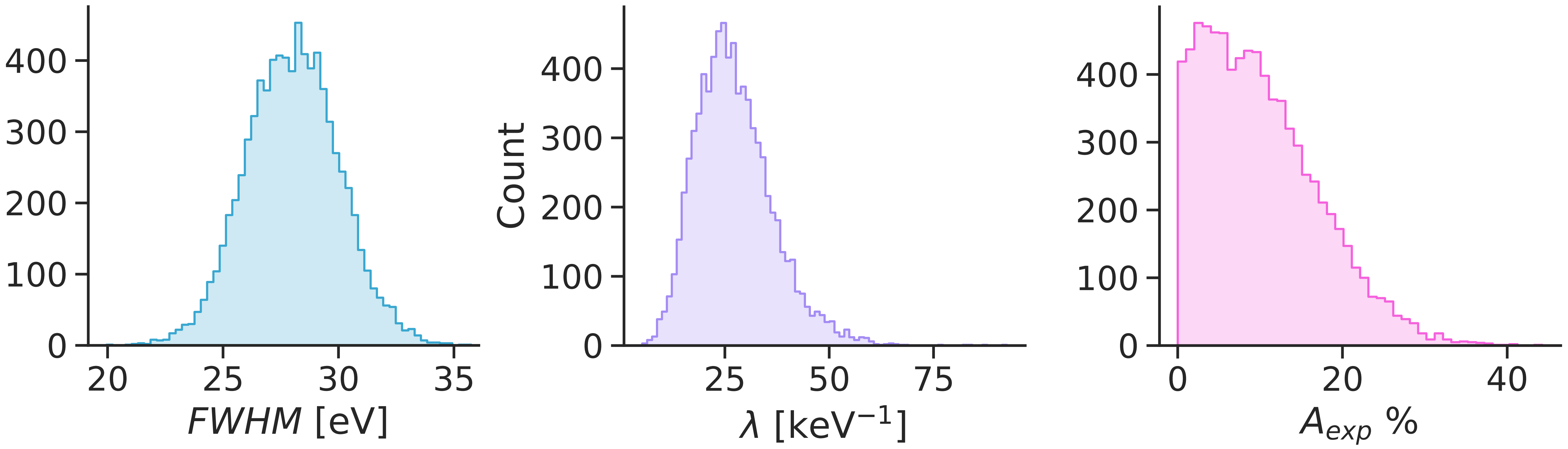
[5] P. Filianin and others, "Direct Q-Value Determination of the β− Decay of 187Re", Phys. Rev. Lett. 127, 2021

PRIORS - RESPONSE



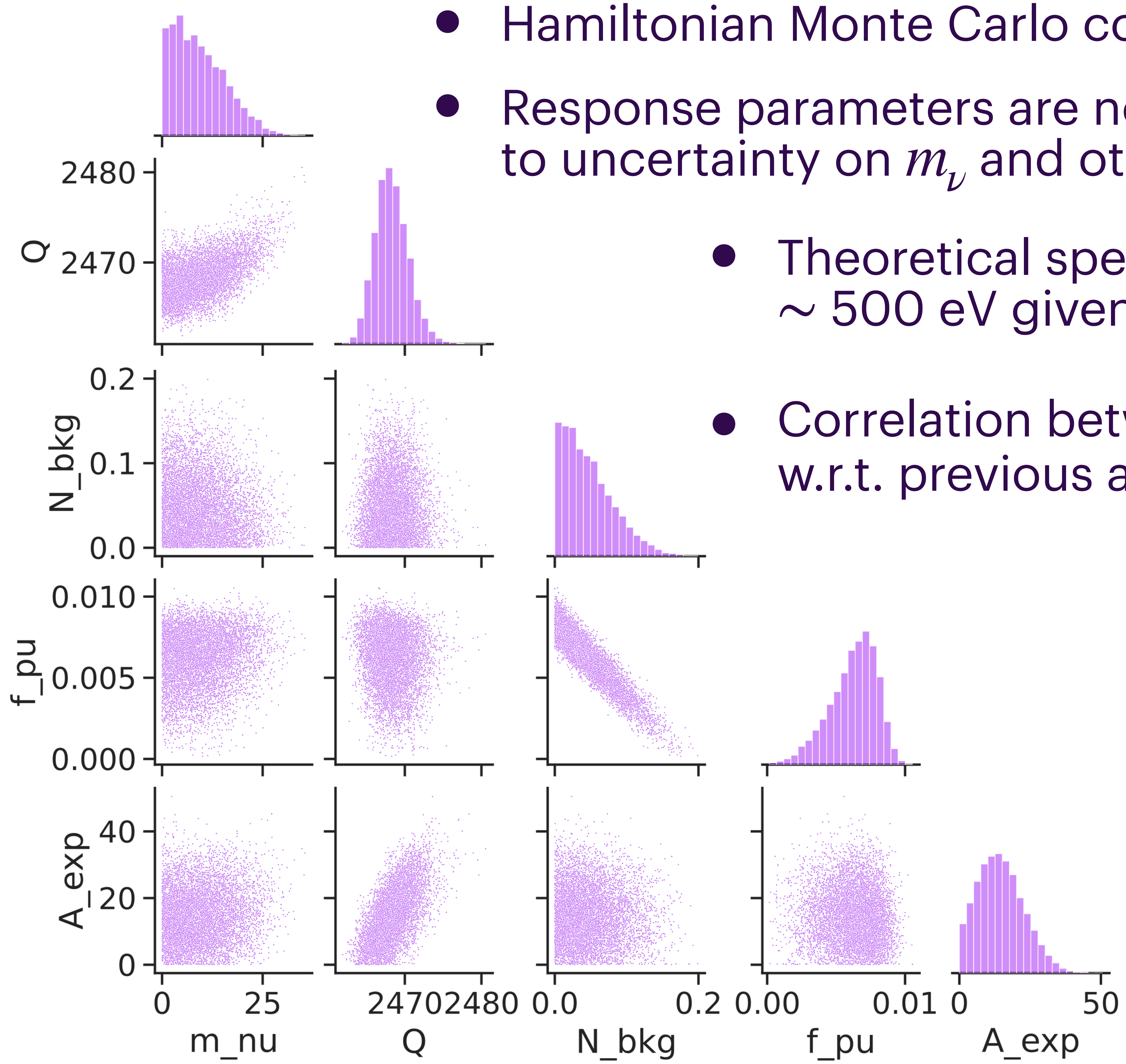
- Response priors obtained from **fit of calibration peaks.**
- $FWHM \sim$ normal(28.1 eV, 2.5 eV) considers extrapolation of resolution at endpoint (2470 .9 eV)
- $\lambda \sim$ normal(27 keV^{-1}, 10 keV^{-1}), $\lambda \geq 0$ keV
- $A_{exp} \sim$ normal(0.05, 0.1), $A_{exp} \geq 0$

- Range of predicted experimental response functions can be visualized with **prior predictive check.**

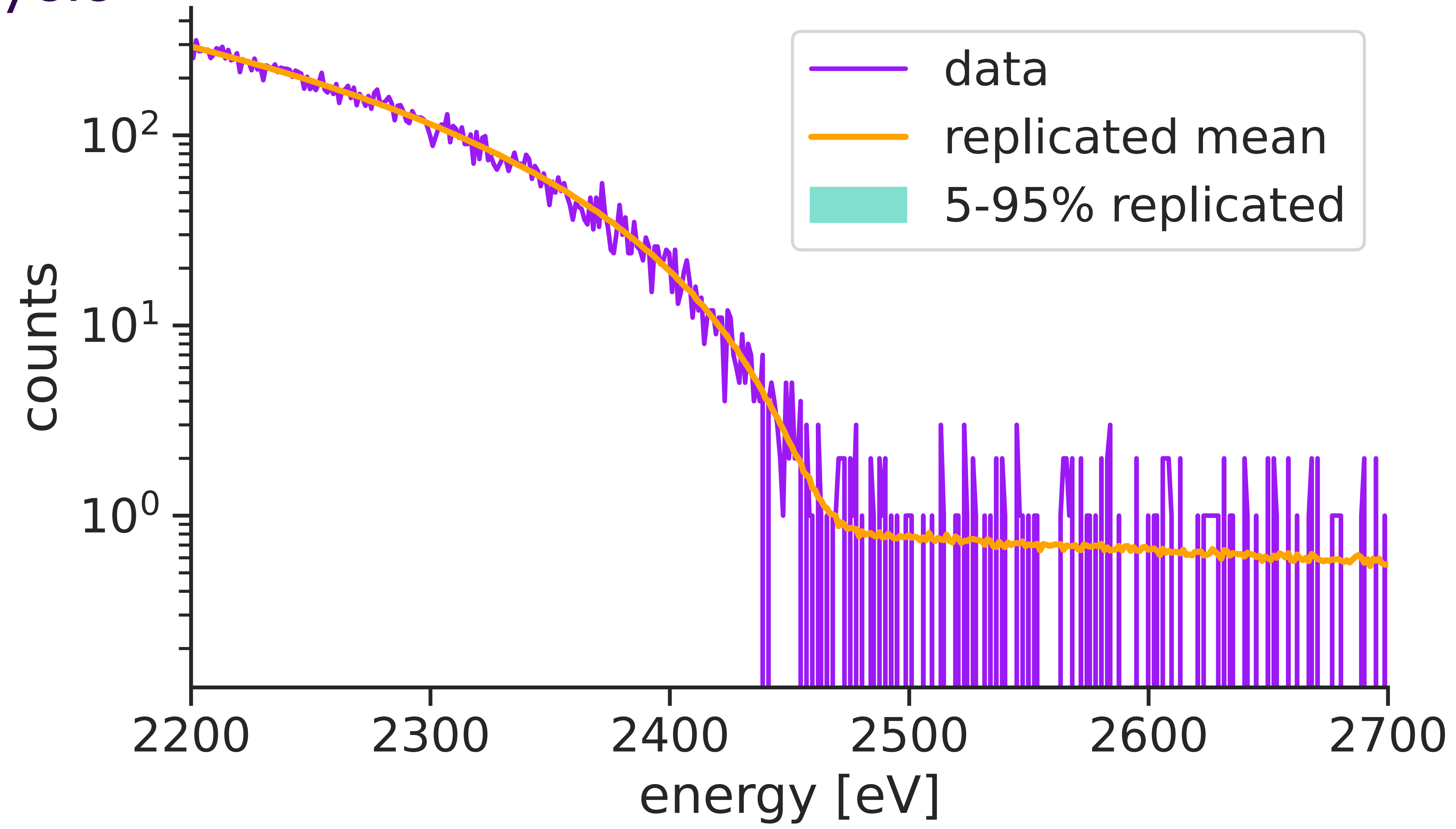


POSTERIOR AND RELEVANT CORRELATIONS

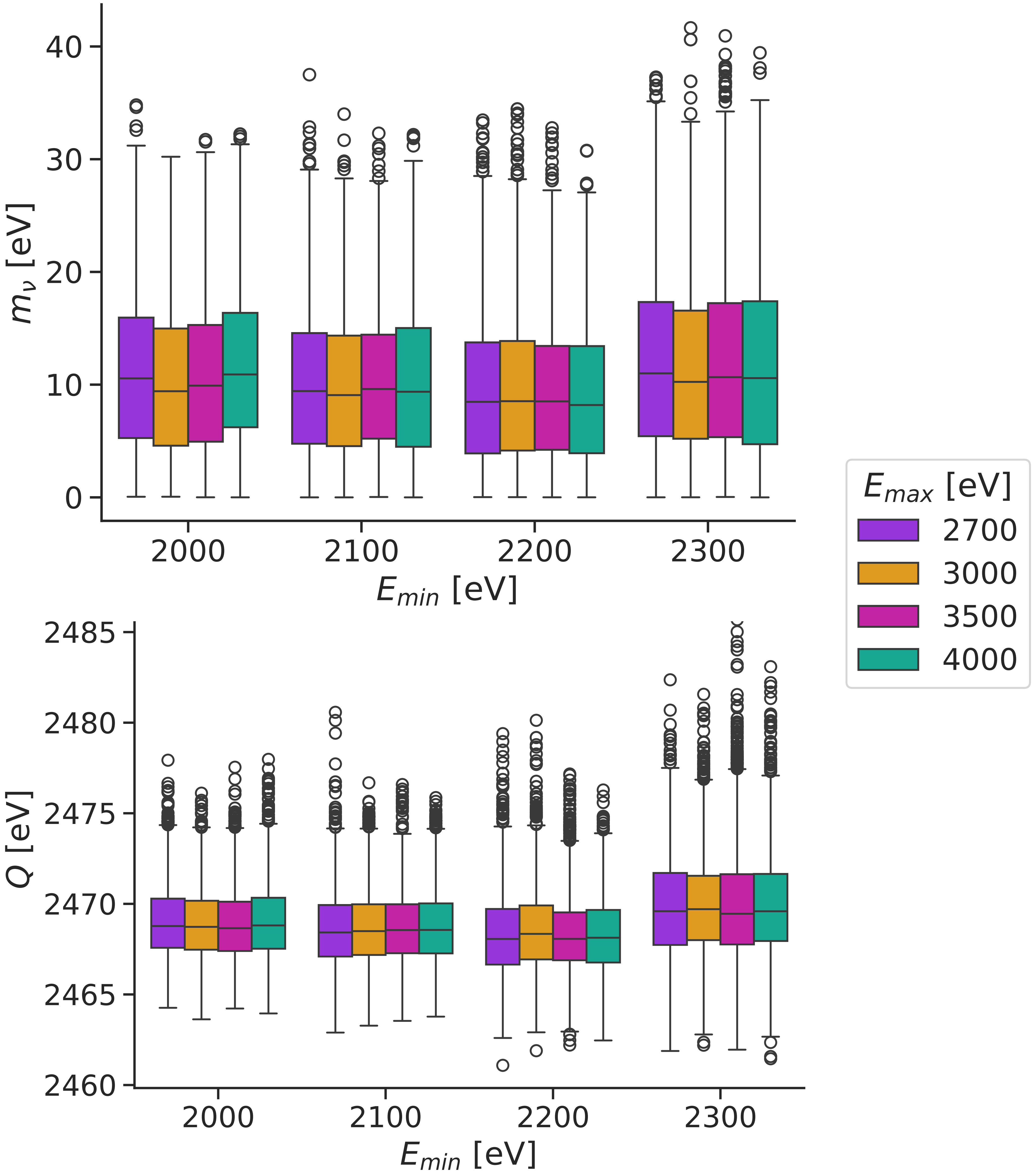
- Hamiltonian Monte Carlo converges without warnings.
- Response parameters are not influenced by the fit , however **they still contribute** to uncertainty on m_ν and other parameters.



- Theoretical spectrum **correctly describes** endpoint shape in the last $\sim$ 500 eV given available statistics.
- Correlation between Q and A_{exp} **shifts Q towards higher energies** w.r.t. previous analysis



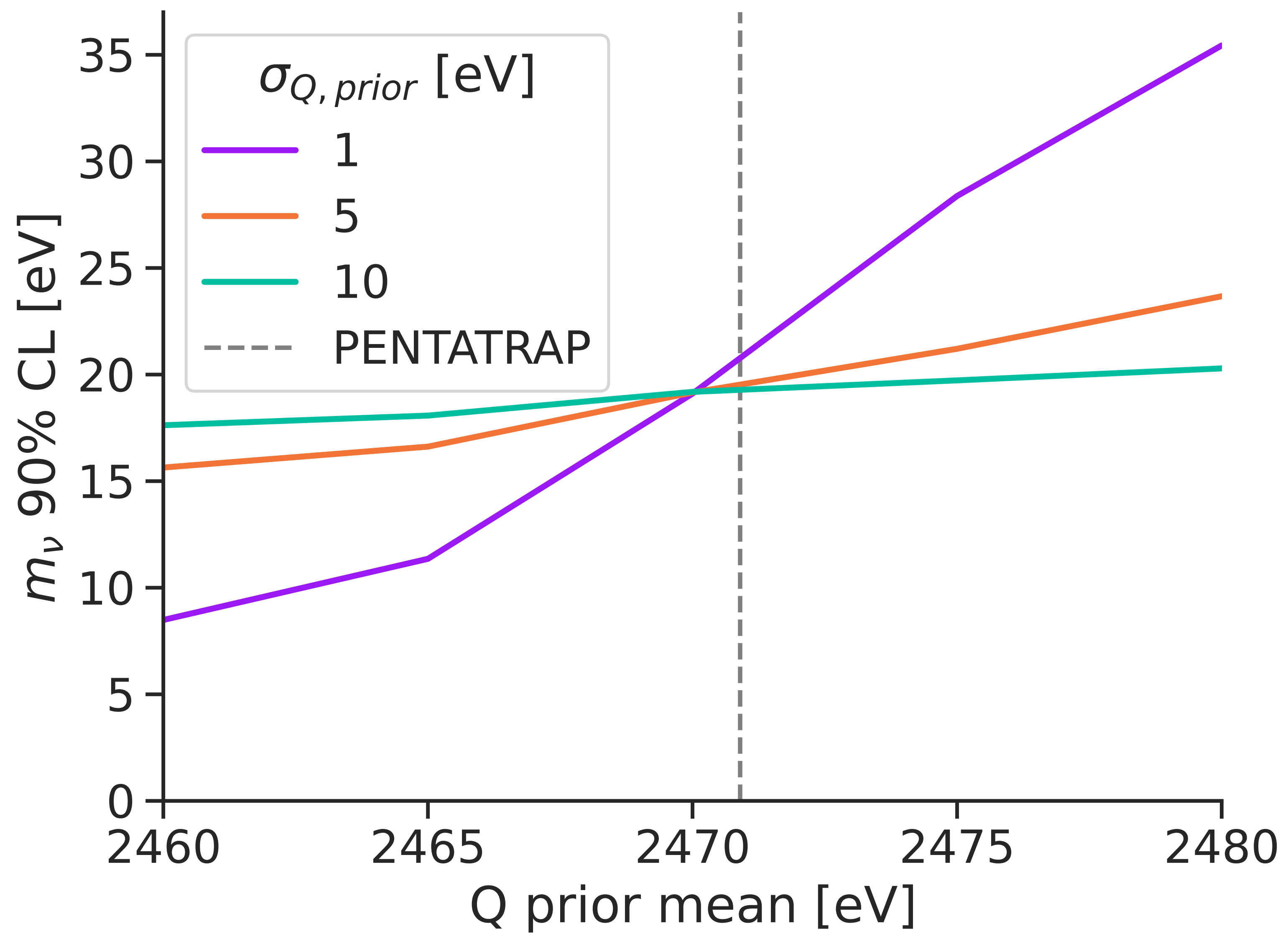
SENSITIVITY TO FIT INTERVAL

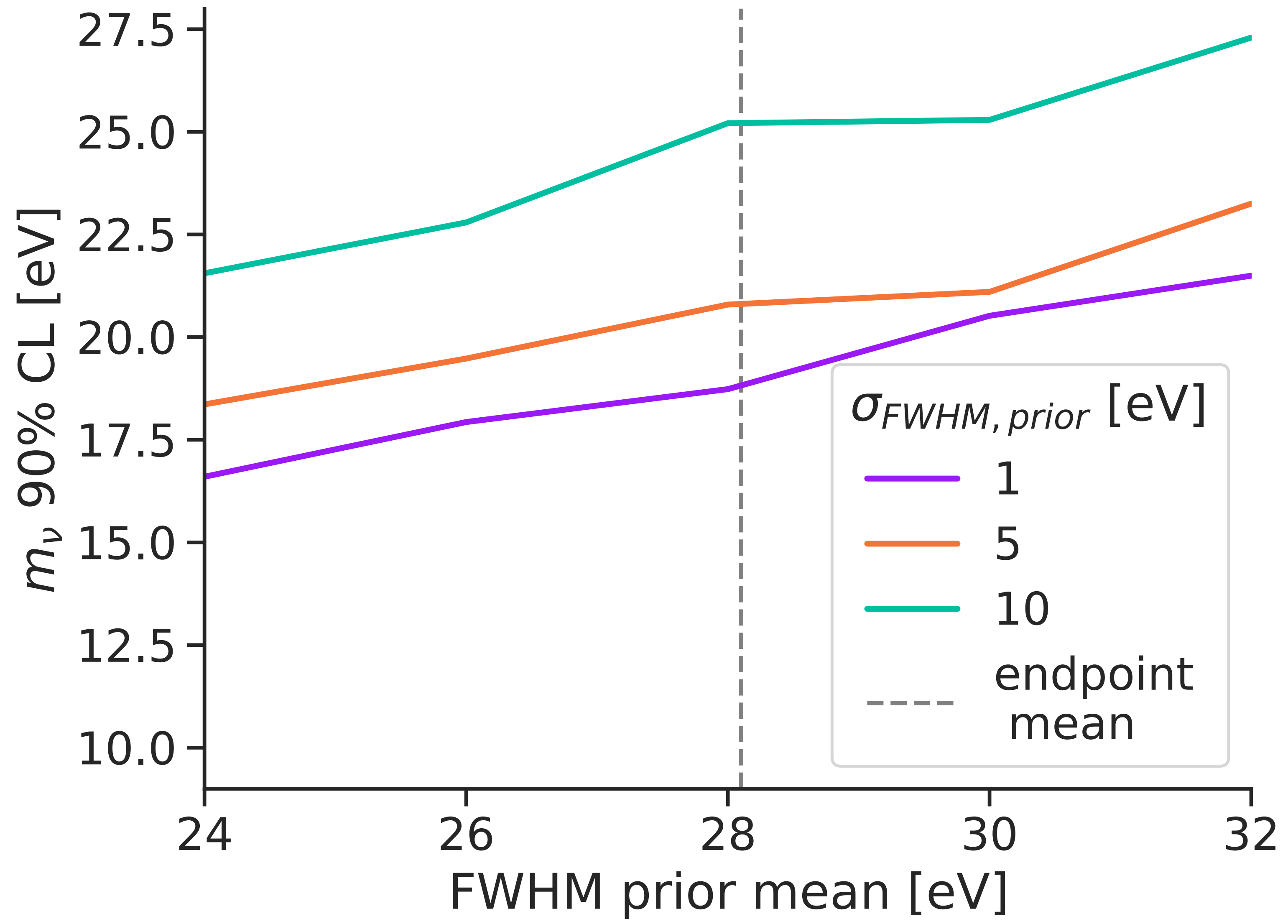


- Fit repeated for different lower and upper values of the ROI = $[E_{min}, E_{max}]$
- E_{min} < 2300 eV is required to correctly fit all the parameters of the model.
- Starting from $E_{min} = 2200$ eV, the 90% CL on m_ν worsens by 0.7 eV each time E_{min} is lowered by 100 eV.
- "Best fit" result $m_\nu \leq 17.8$ eV at 90% CL for ROI = [2200 eV, 4000 eV].

SENSITIVITY TO PRIORS

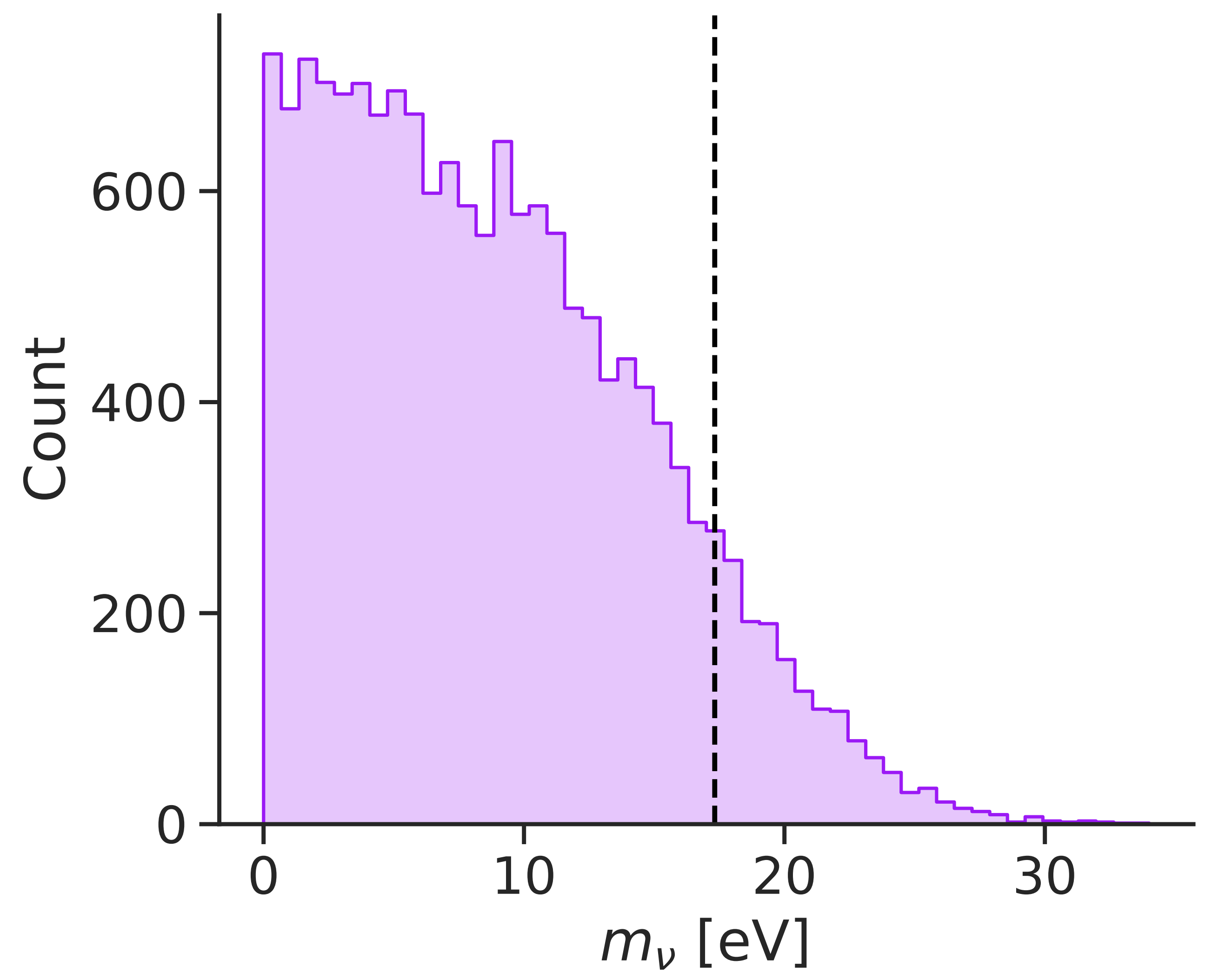
- The posterior of m_ν is not completely independent from the priors.
- Selecting a very strict prior, for example $Q \sim$ normal$(2470.9$ eV, 1.3 eV$)$ might **introduce a strong correlation.**
- In some cases, this dependence can be mitigated by **relaxing the prior,** possibly at the cost of an higher m_ν 90% CL upper limit.
- When not possible, analysis must consider reasonable range of variation in the priors and constrain systematics as much as possible.
- Overall, varying prior values in the expected range induces a $\sim$ 0.5 eV variation in the posterior of m_ν.

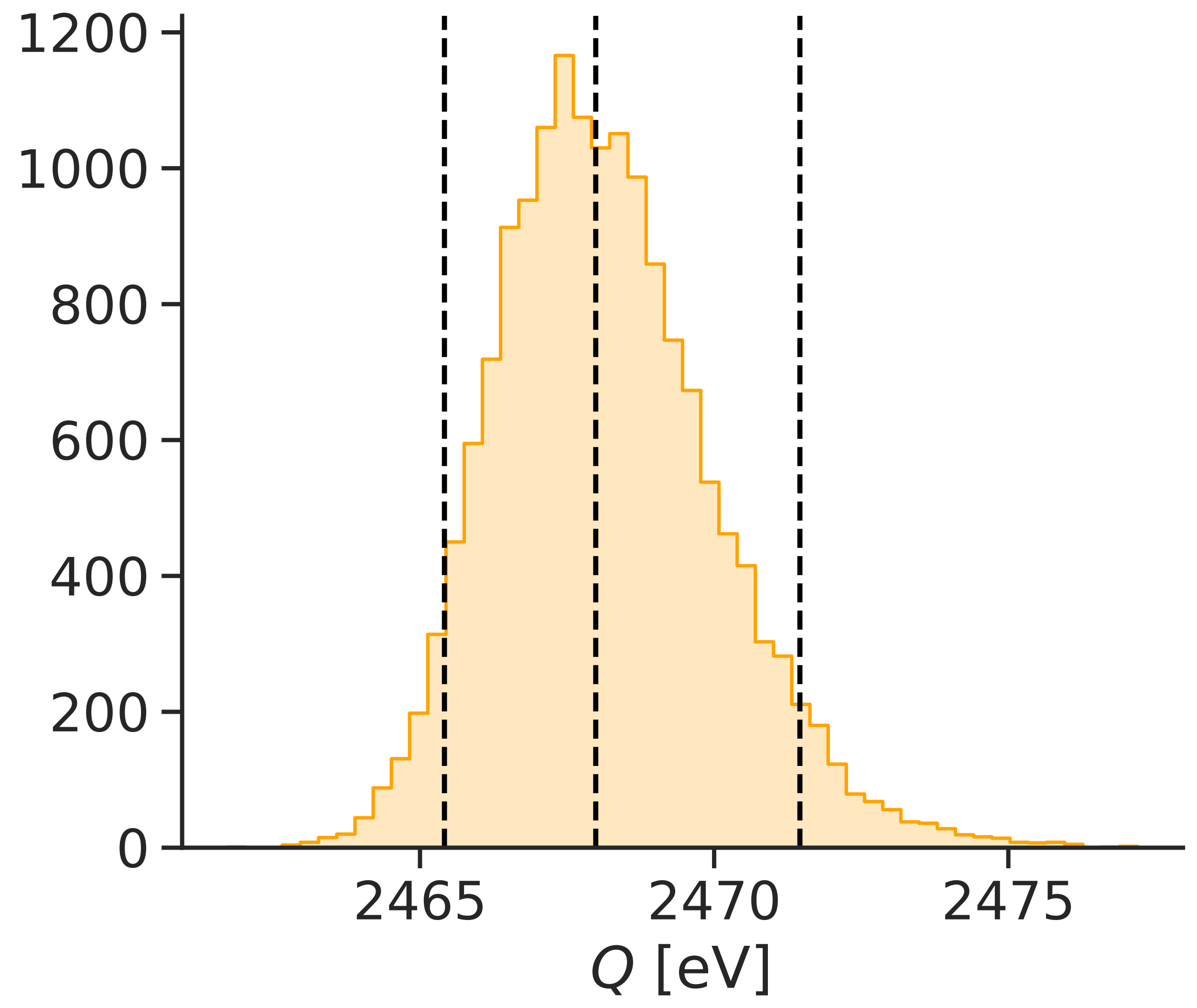


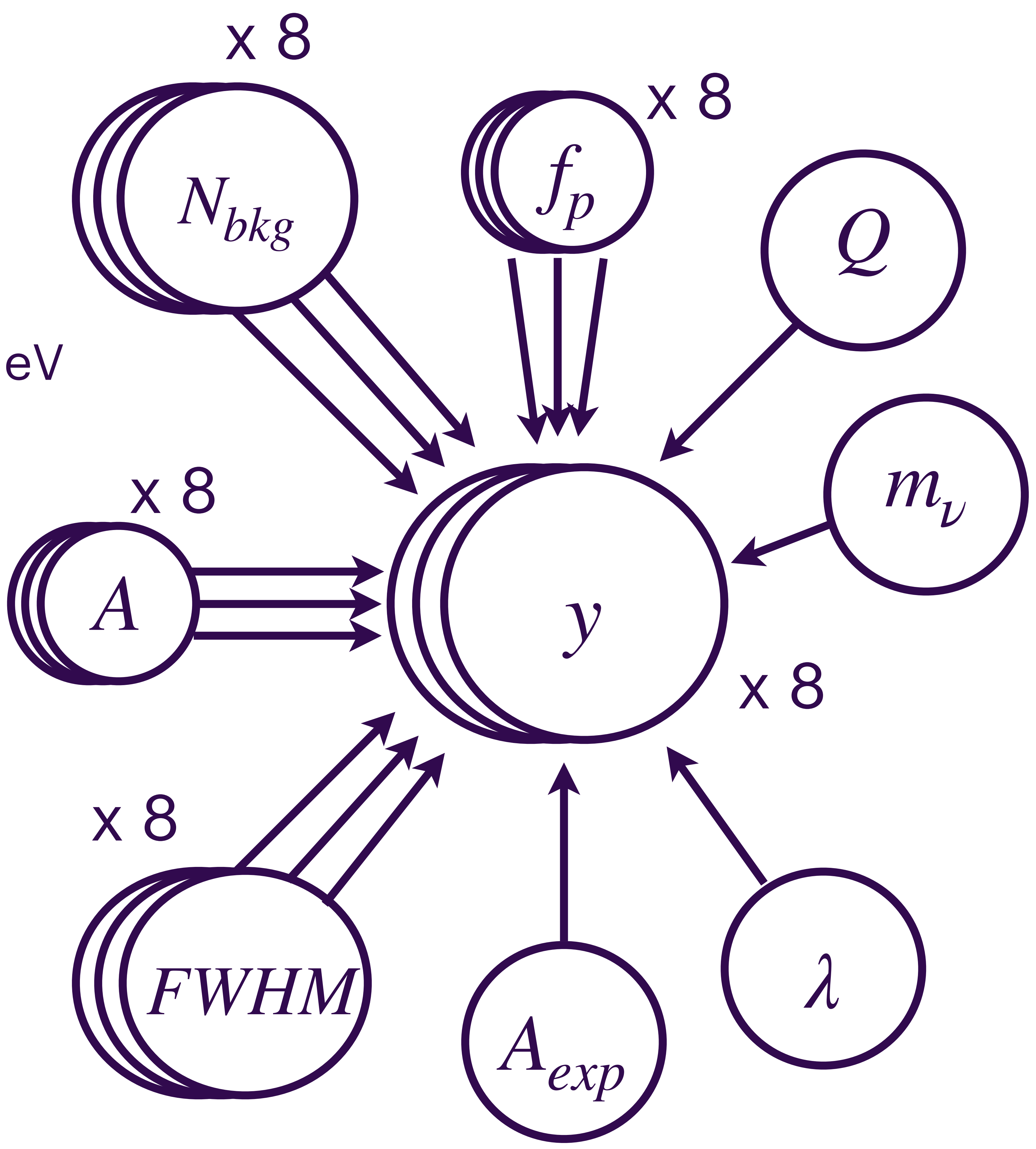


MULTI-DETECTOR MODEL

- Fit **separate** spectra of 8 detectors at the same time.
- **Four common parameters**, four detector-specific ones (36 total).
- **More stringent priors on FWHMs**, similar construction for all others.
- Multi detector model finds $m_\nu < 17.3$ eV at 90% CL, $Q = 2468.2 \pm 1.8$ eV
- **Better result w.r.t compound data is not guaranteed**, trade-off beween improved description and additional uncertainty introduced by more parameters.



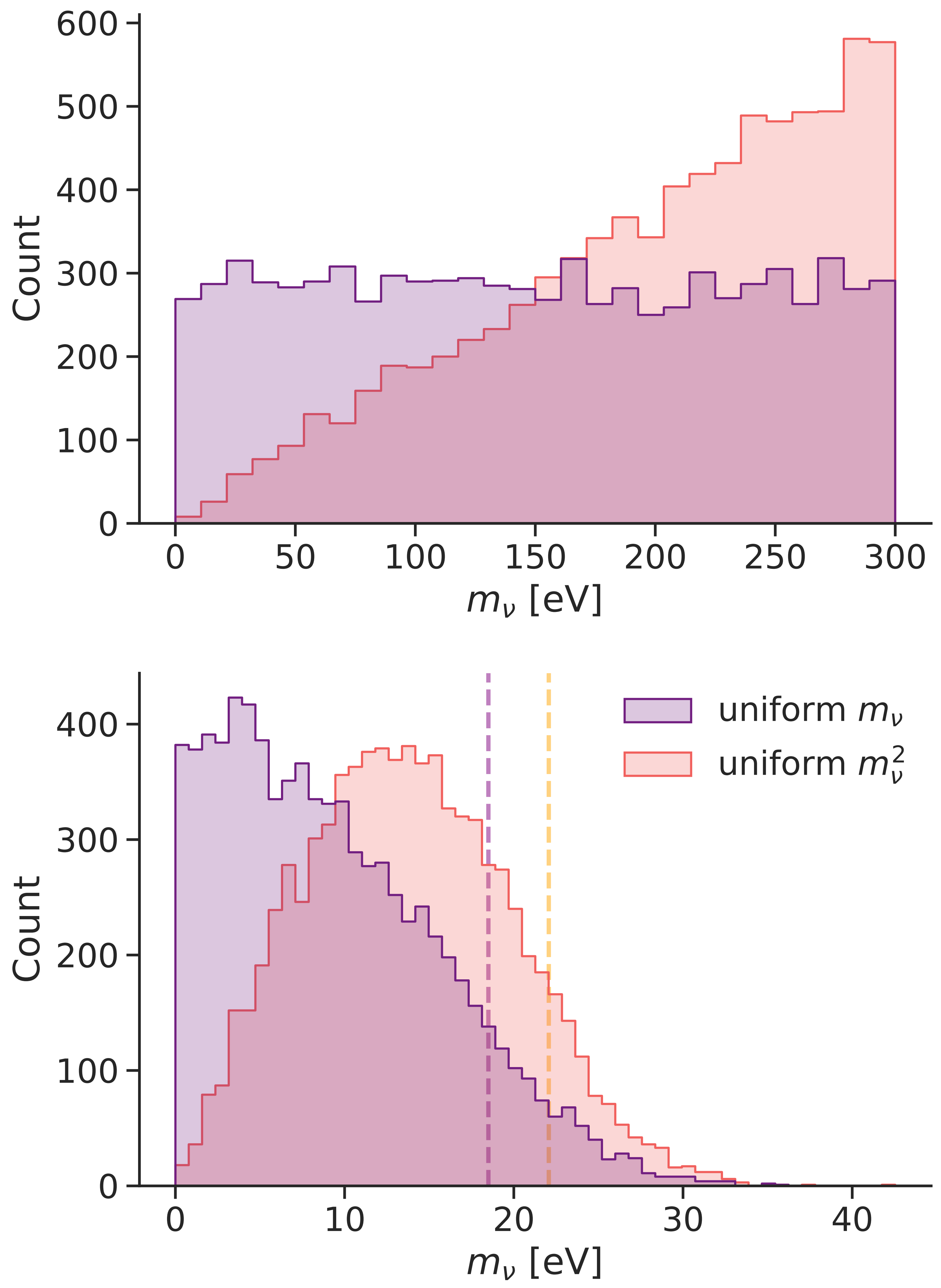




CONCLUSIONS

- Analyzed data from the MIBETA experiment with a Bayesian approach, considering **additional systematics and new theoretical spectrum**.
- In some cases, **relaxing priors improves the robustness** of the model.
- Fitting the detector's data separately provides the more stringent limit $m_\nu < 17.3$ eV at 90% CL. Result is **slighty more conservative** than frequentist approach.
- Updated value of $Q = 2468.2 \pm 1.8$ eV is closer to other independent measurements than previous result.
- Establishing and understaning interaction between parameters of endpoint model is crucial for **future HOLMES analysis**.

NEUTRINO MASS PRIOR



- The theoretical spectrum dependes explicitly from m_ν^2 only → use it as a parameter.
- Assigning a flat prior to m_ν^2 corresponds to favoring higher values of m_ν.
- 90% posterior CL with flat m_ν^2 prior is consistently 2 eV higher than with flat m_ν.
- In both cases results are independent from cutoff value of prior and interaction with other parmeters is similar.

BAYES' THEOREM

- Goal: infer value of parameter θ from data y
- Bayesian statistics: **probability** describes **degree of knowledge**
- **Prior**: initial knowlegde about θ
- **Likelihood**: relation between parameters and data (physical theory)
- **Posterior**: knowledge about θ having observed y
- A **model** is the combination of prior and likelihood

$$p(\theta \mid y) \propto p(y \mid \theta)\, p(\theta)$$