THE THEORY OF PTOLEMY

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PTOLEMY

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OUTLINE

- Neutrino mass from β-decay
- Radial excitations of $^3He^+$
- Vibrational degrees of freedom
- Cosmic neutrinos
- Outlook

m_ν IN A NUTSHELL

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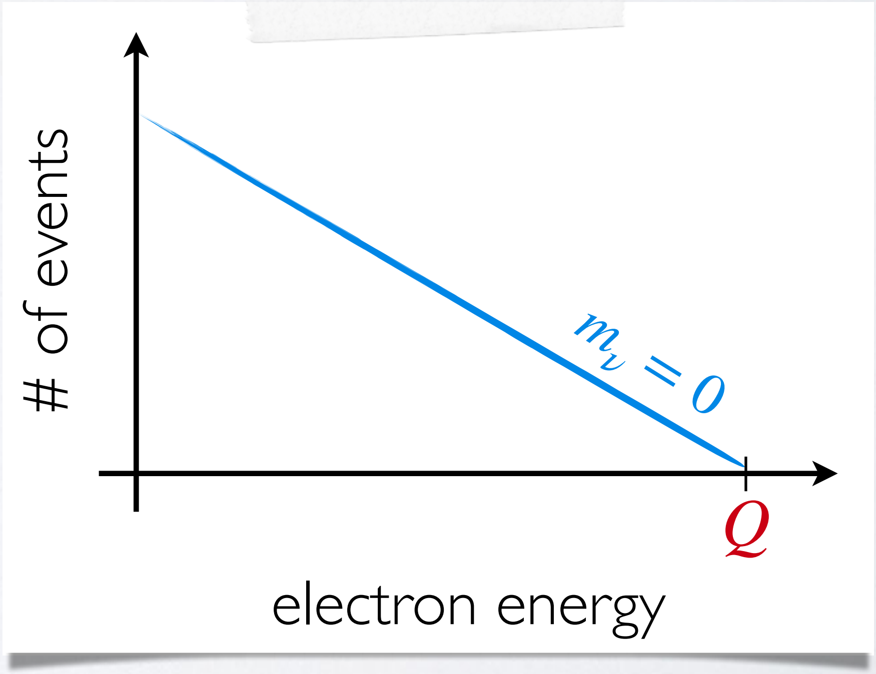

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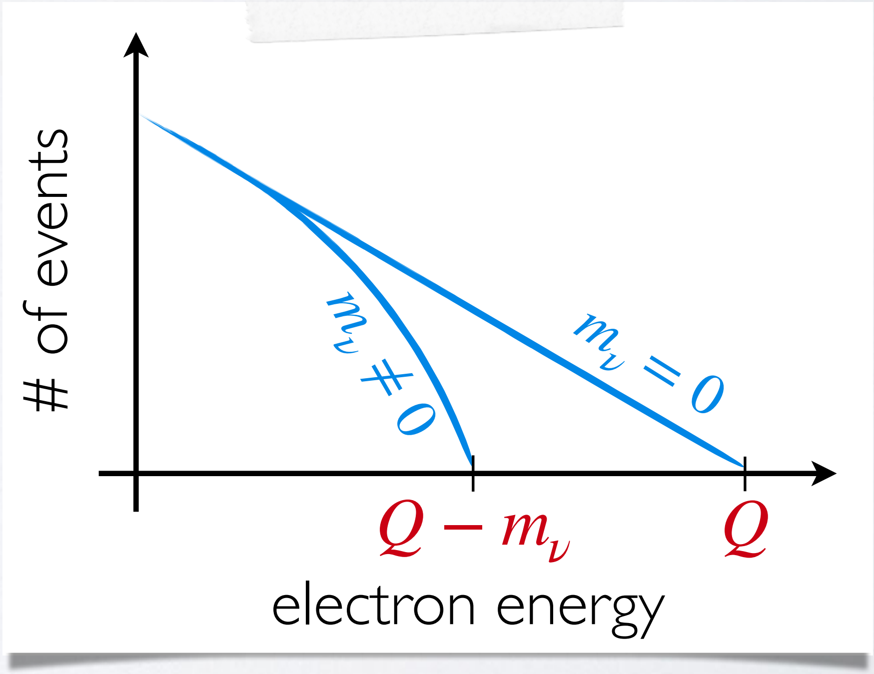

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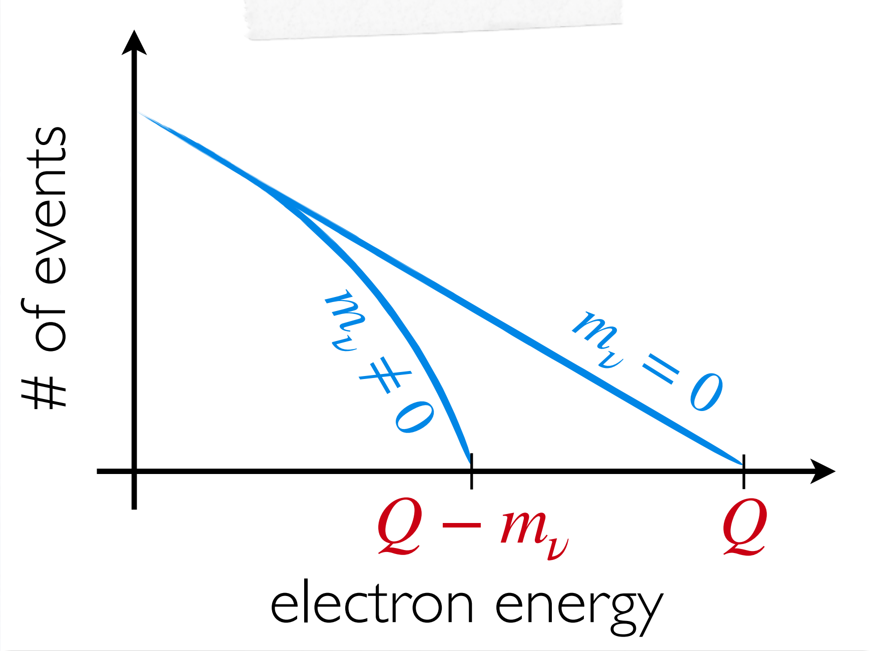
Measure the end-point region

↓

Extract m_ν



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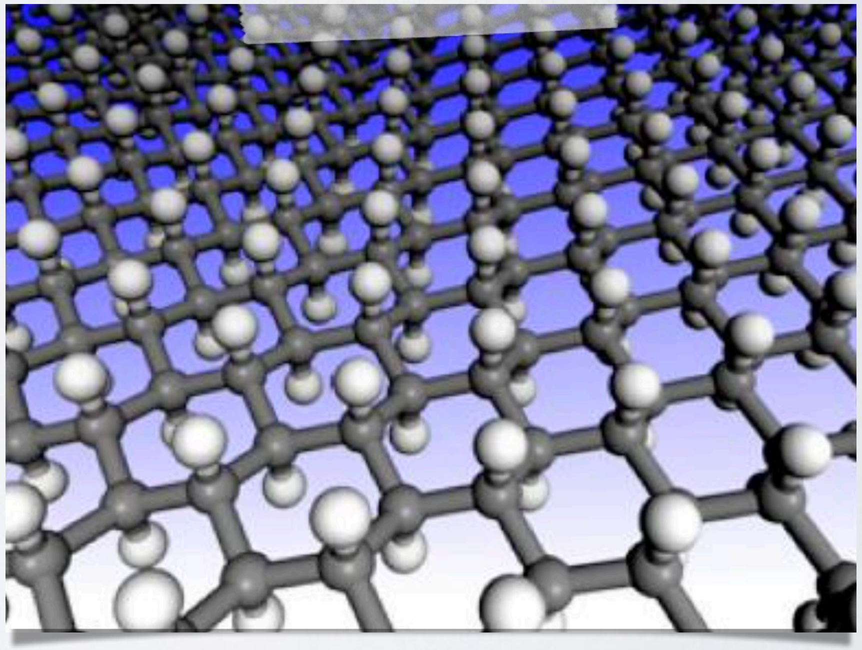

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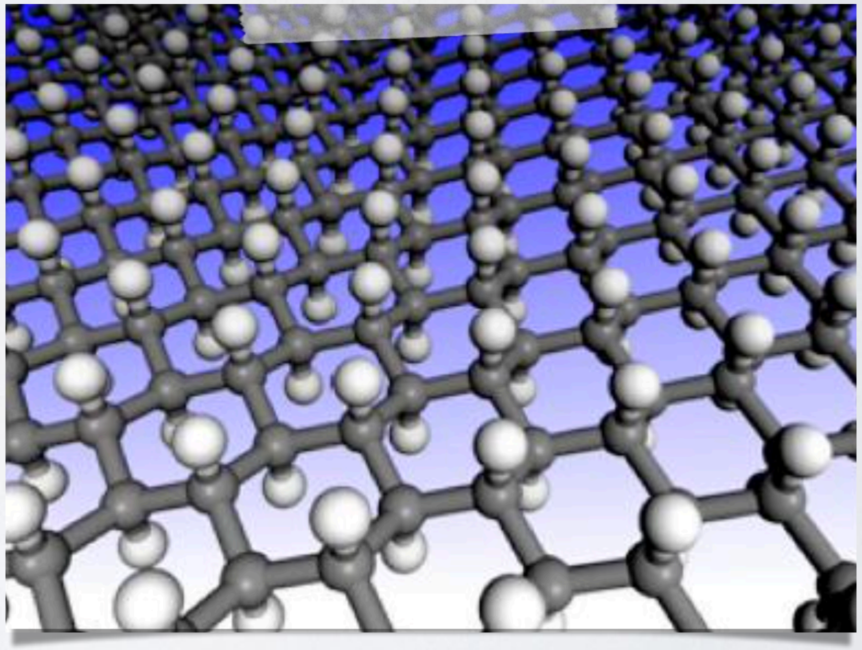

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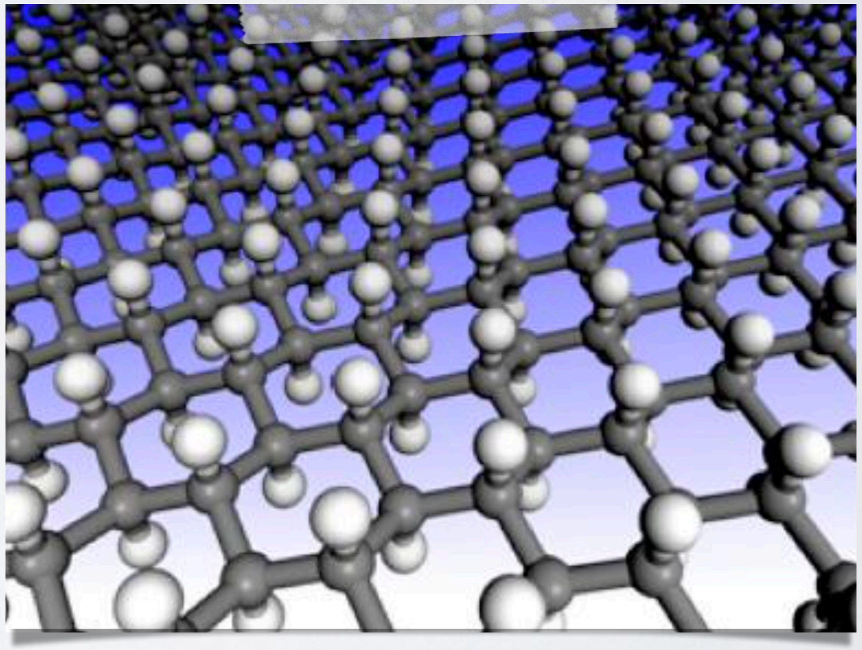

- Experimentally realized nanoporous graphene with 90% coverage

[see Betti et al. — Nano Lett. 2022]

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- 2D graphene satisfies both requirements

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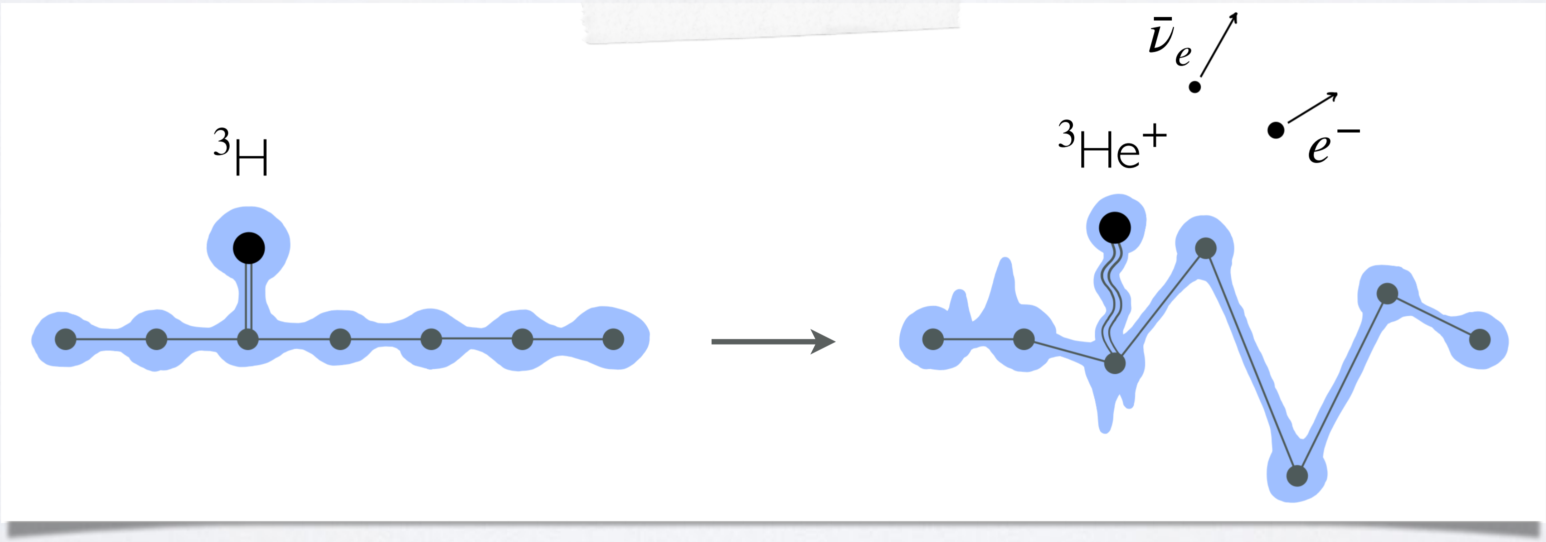

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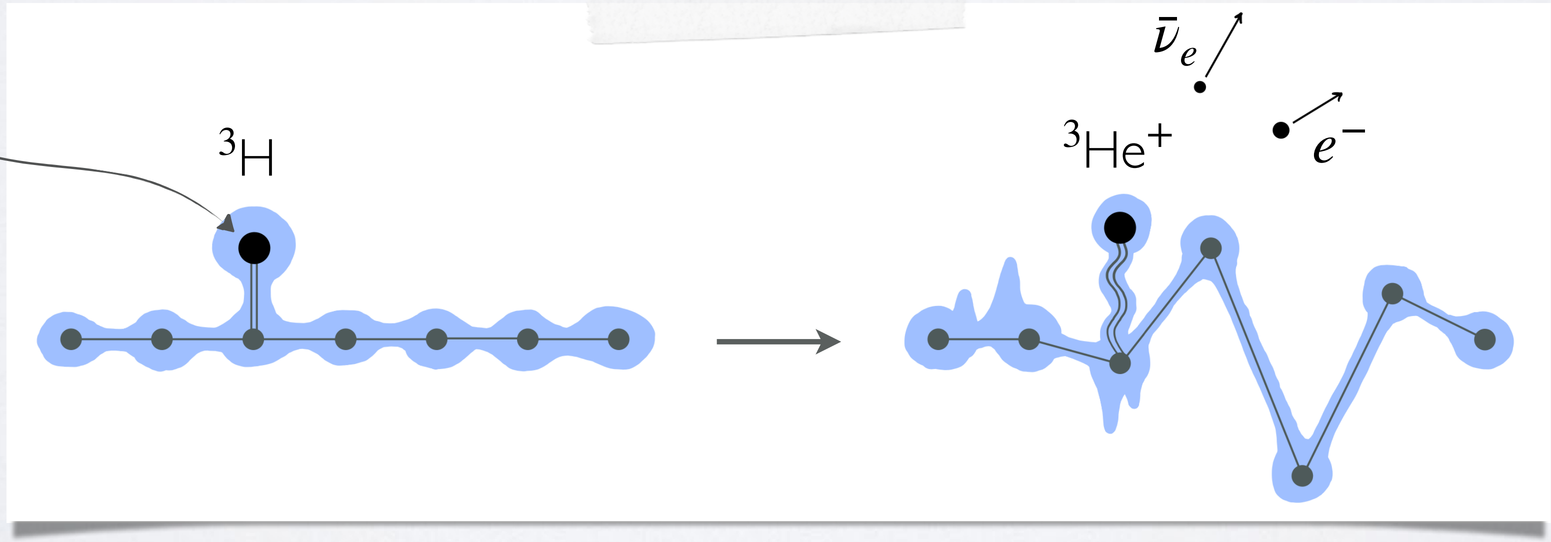

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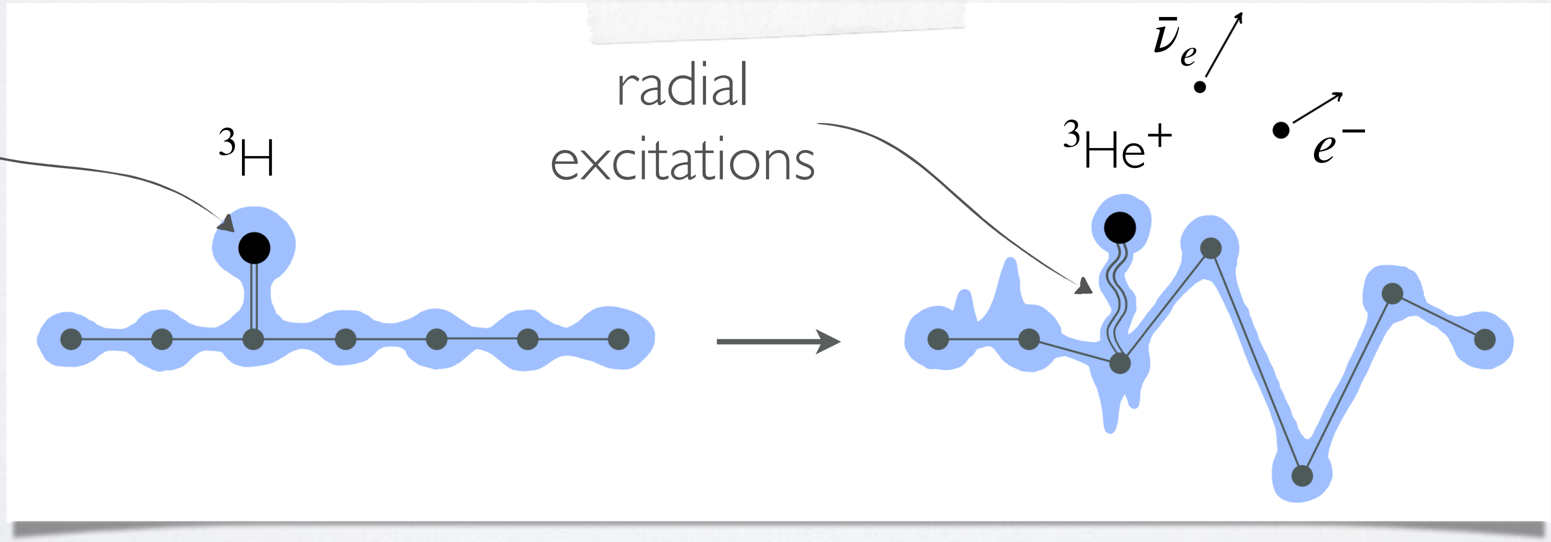

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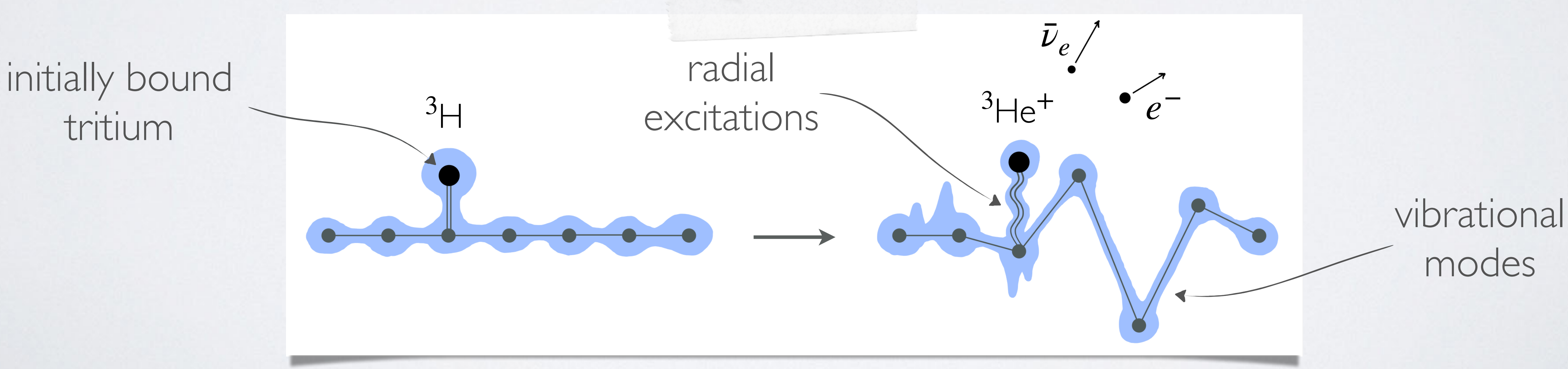

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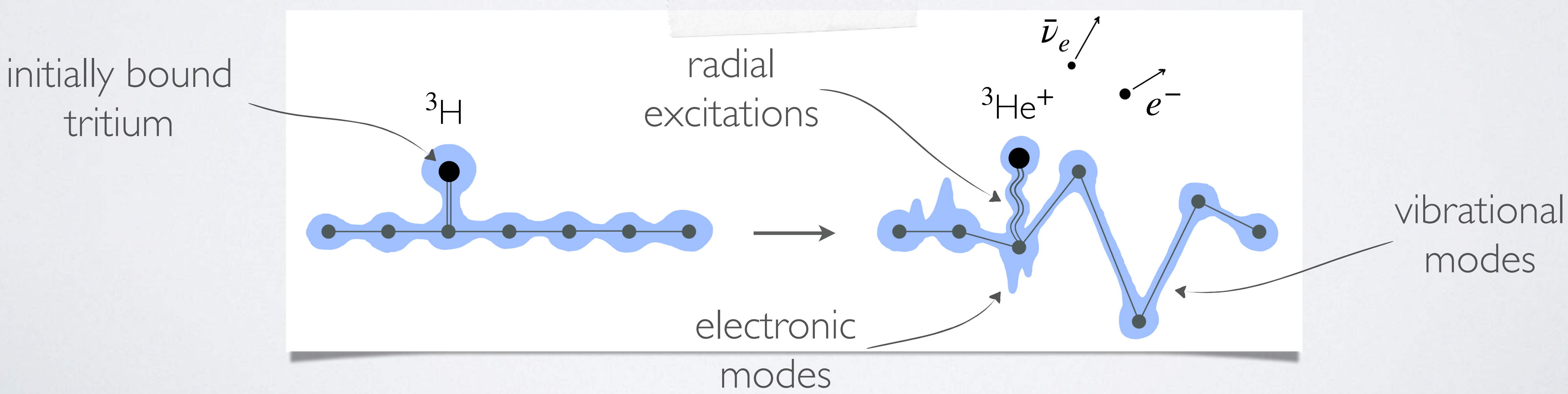

ROADMAP TO m_ν

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$\uparrow$

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Initial ^3H wave functions and binding energies

Final state effects

Inclusive on $^3He^+$ state, phonons and electrons

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Cond. mat. theory

- binding potential
- known uncert.
- …

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- structure factor
- energy loss funct.
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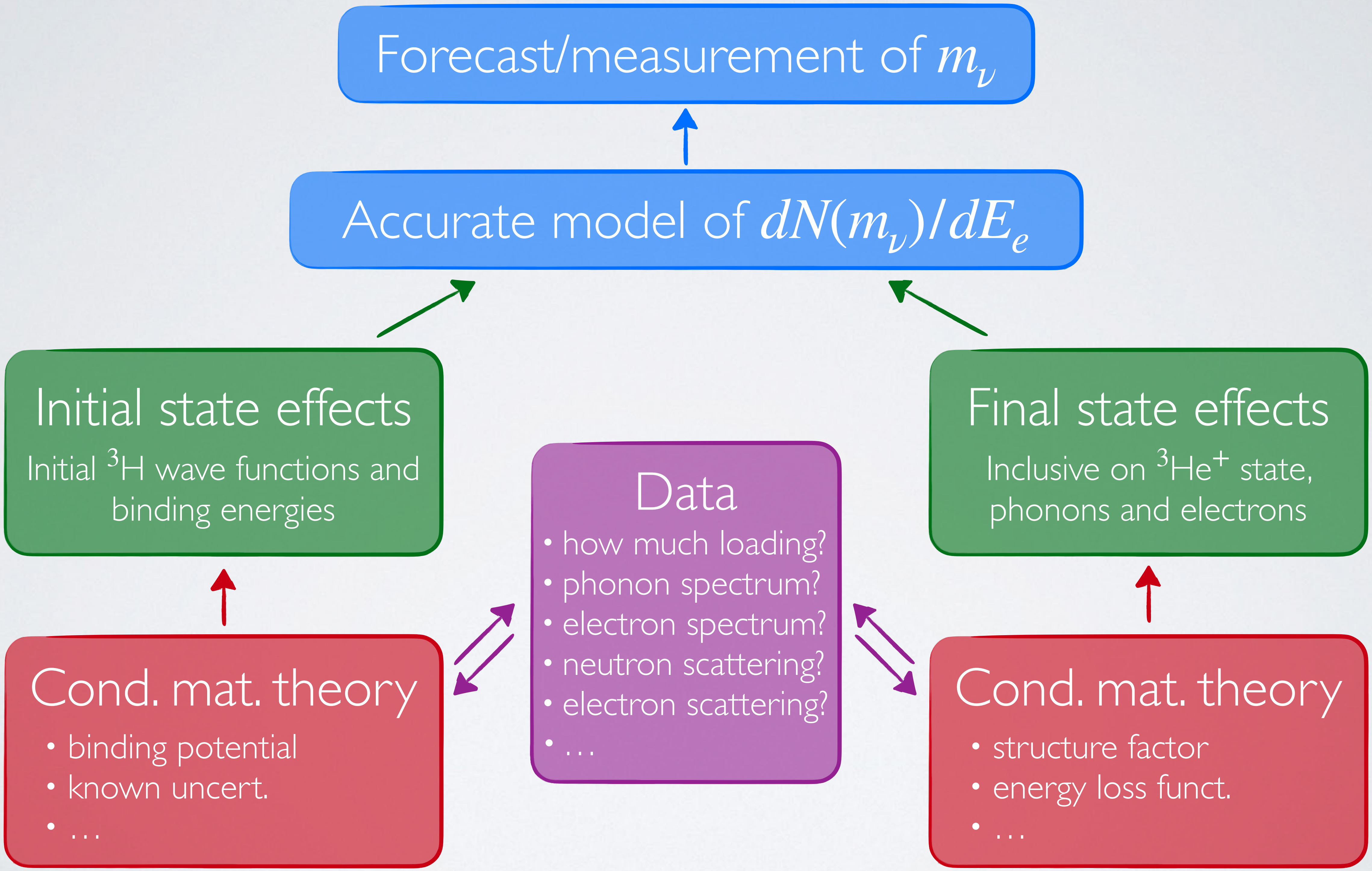

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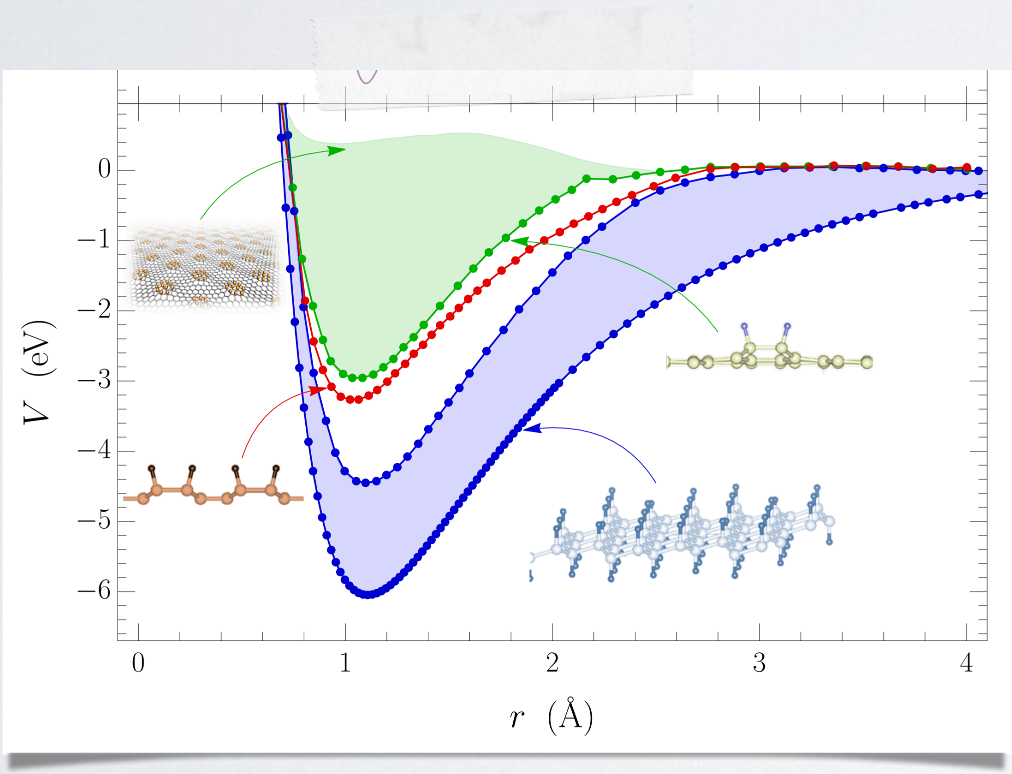

[PTOLEMY — PRD 2022, 2203.11228]

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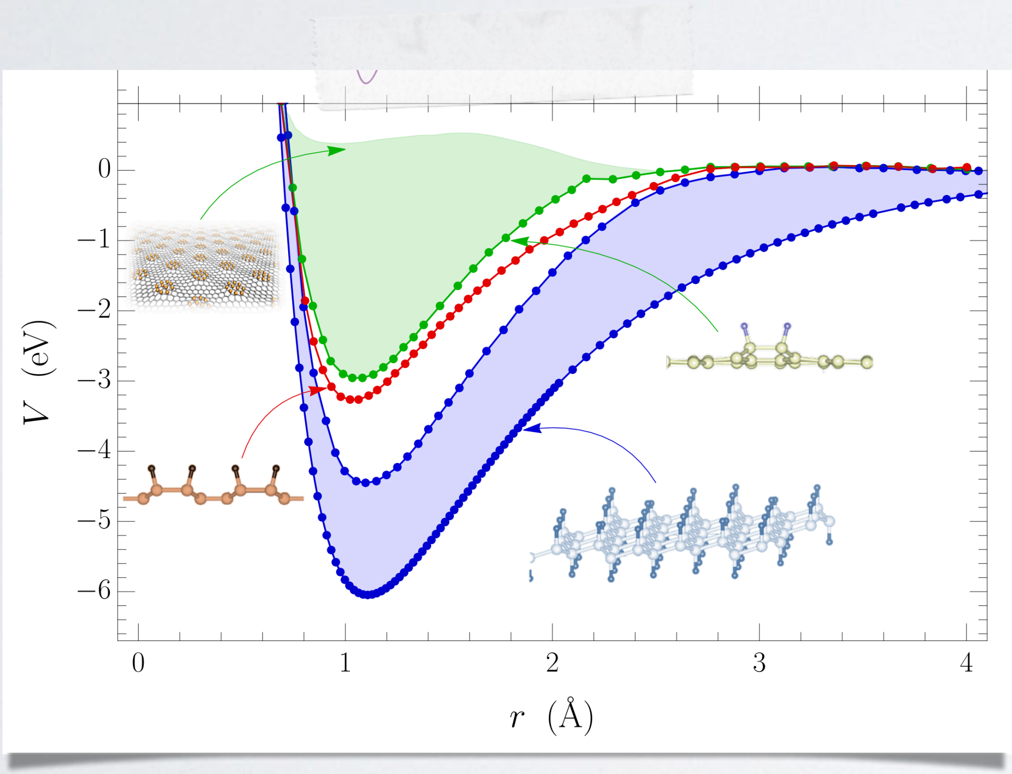

[PTOLEMY – PRD 2022, 2203.11228]

Roughly harmonic oscillator + Morse:

$$V\left(\mathbf{x}_{\parallel}, z\right) = \frac{1}{2} m_T\, \omega_{\parallel}^2\, x_{\parallel}^2 + D\left[1 - e^{-(z-z_0)/a}\right]$$

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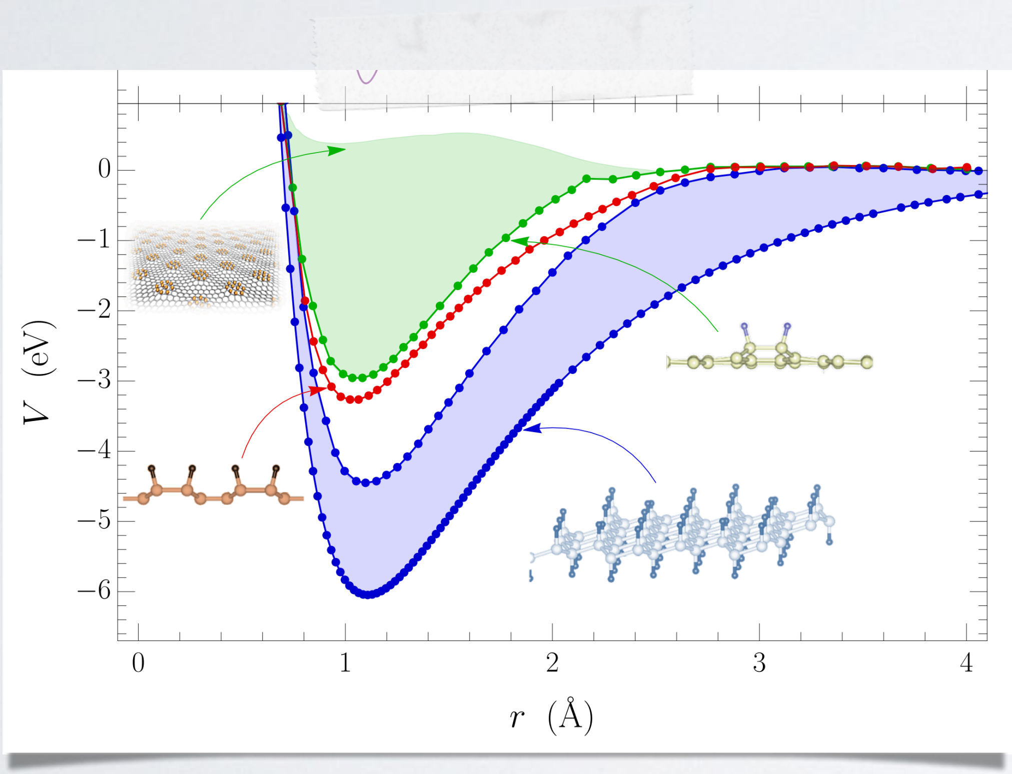

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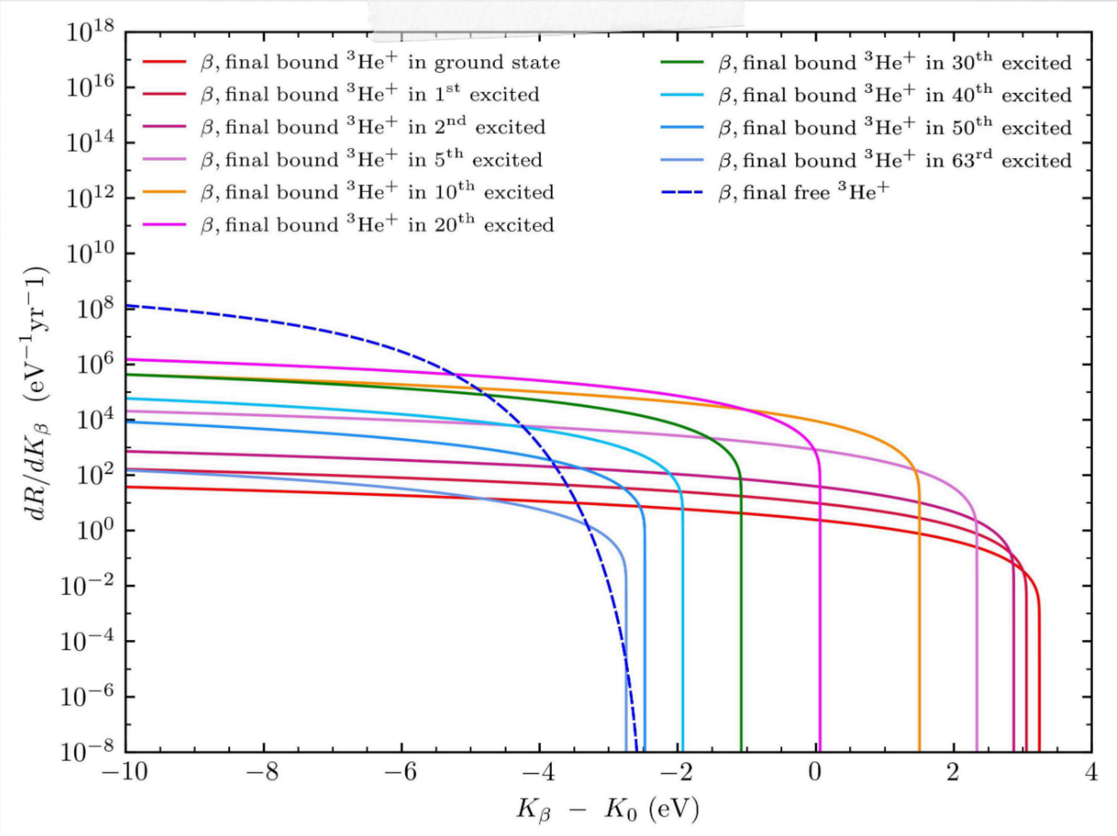

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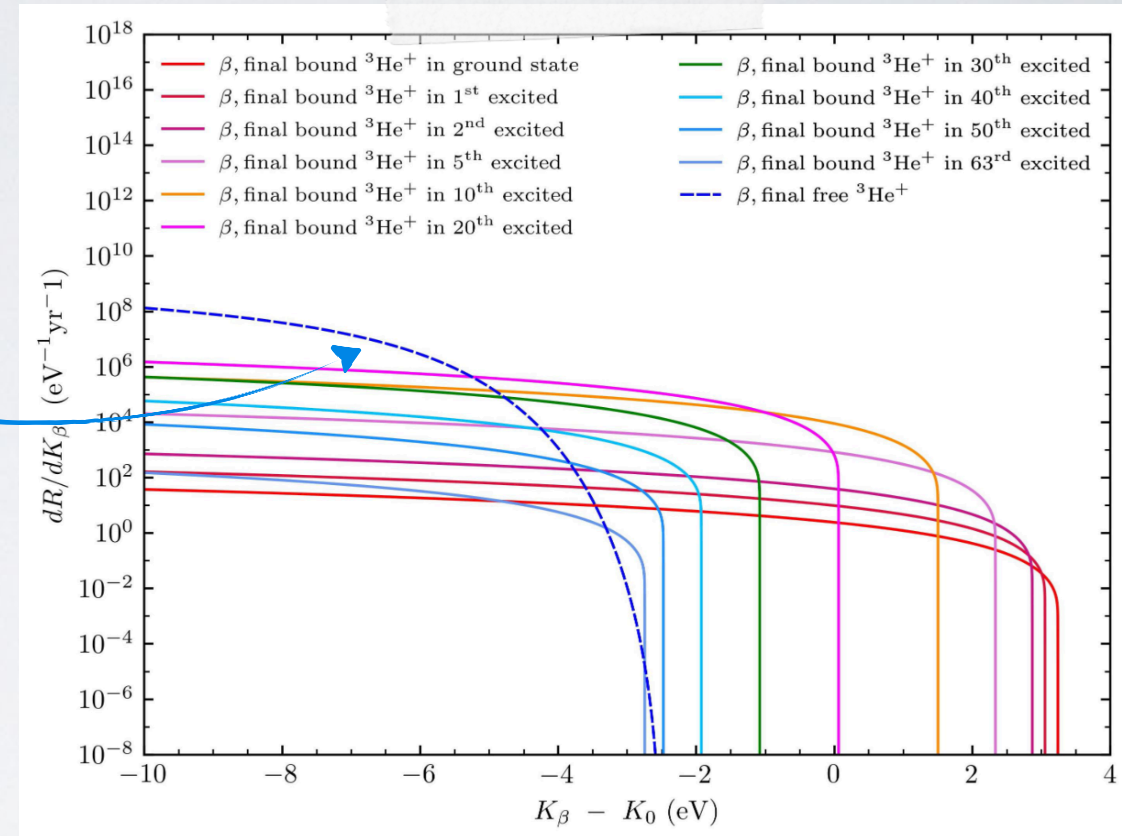

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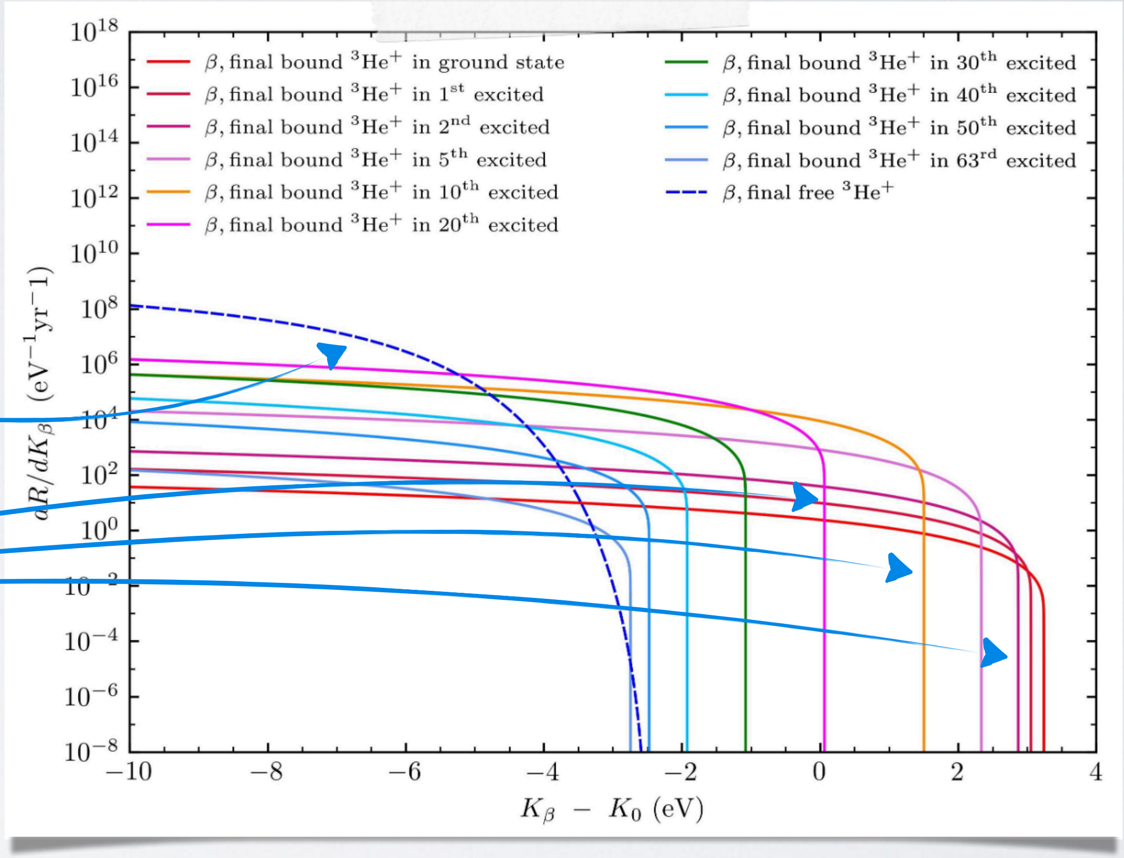

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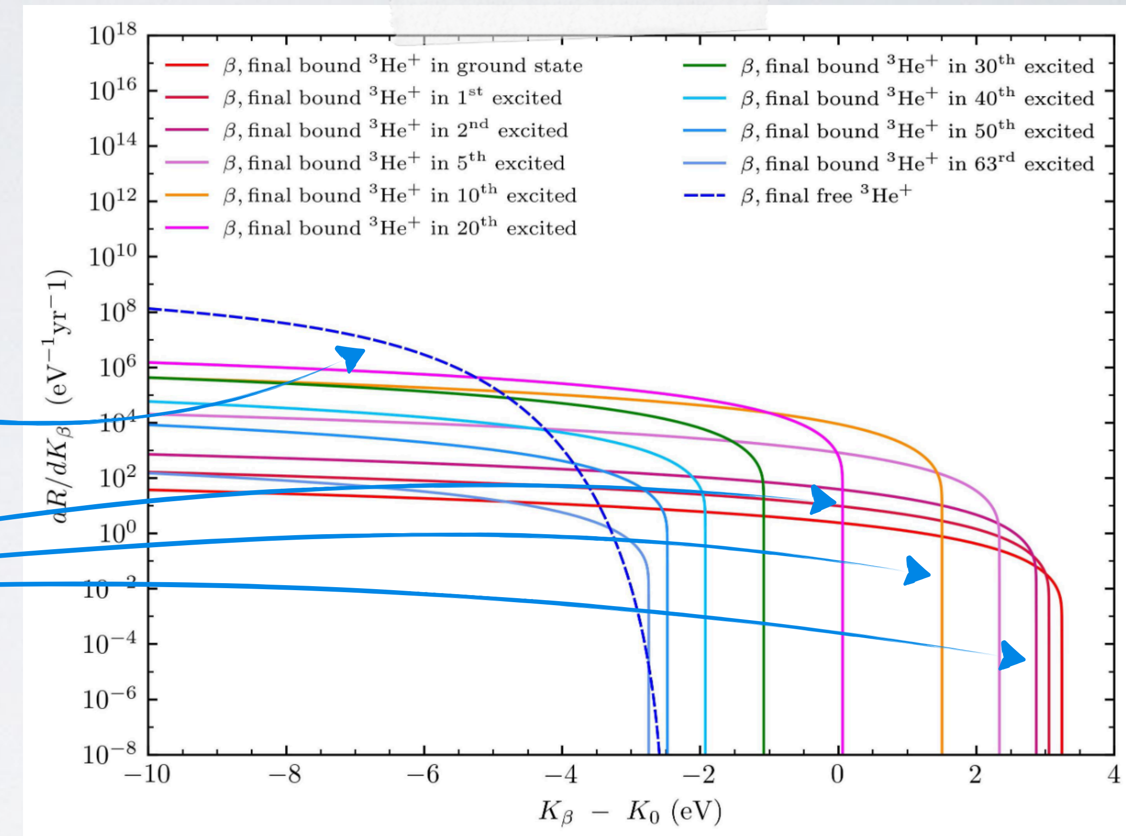

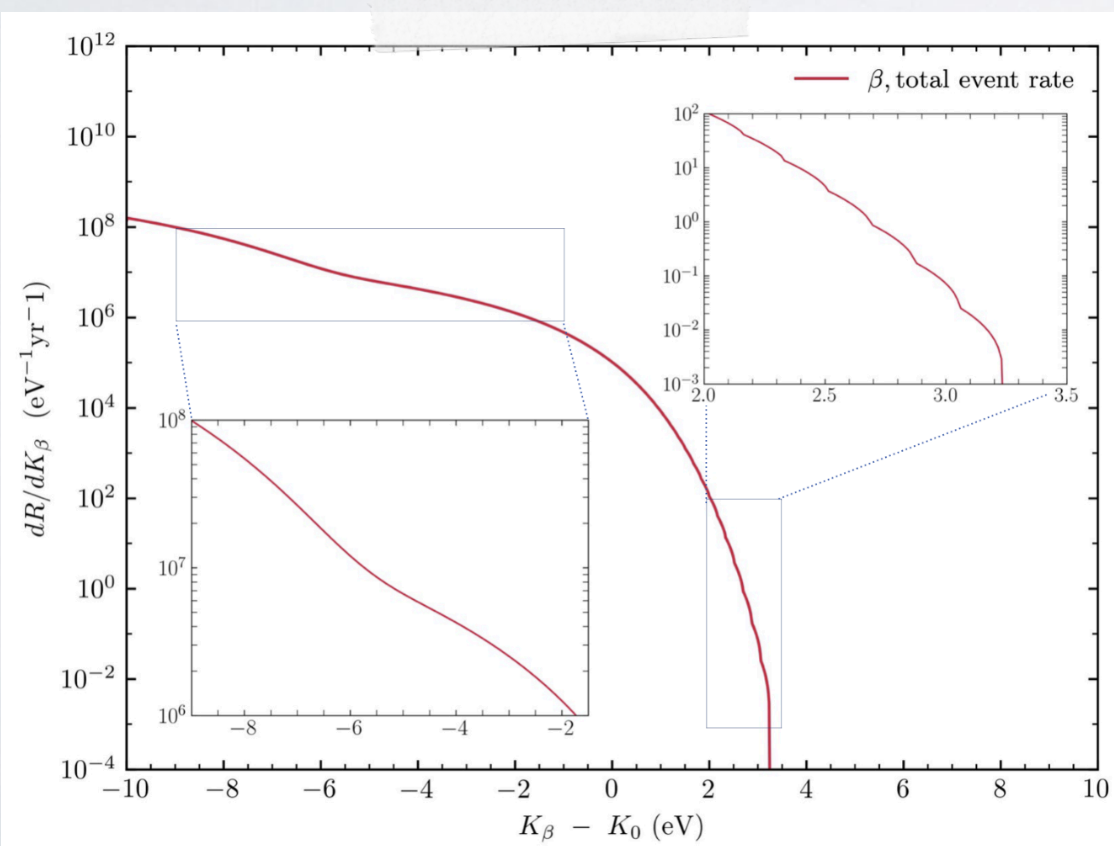

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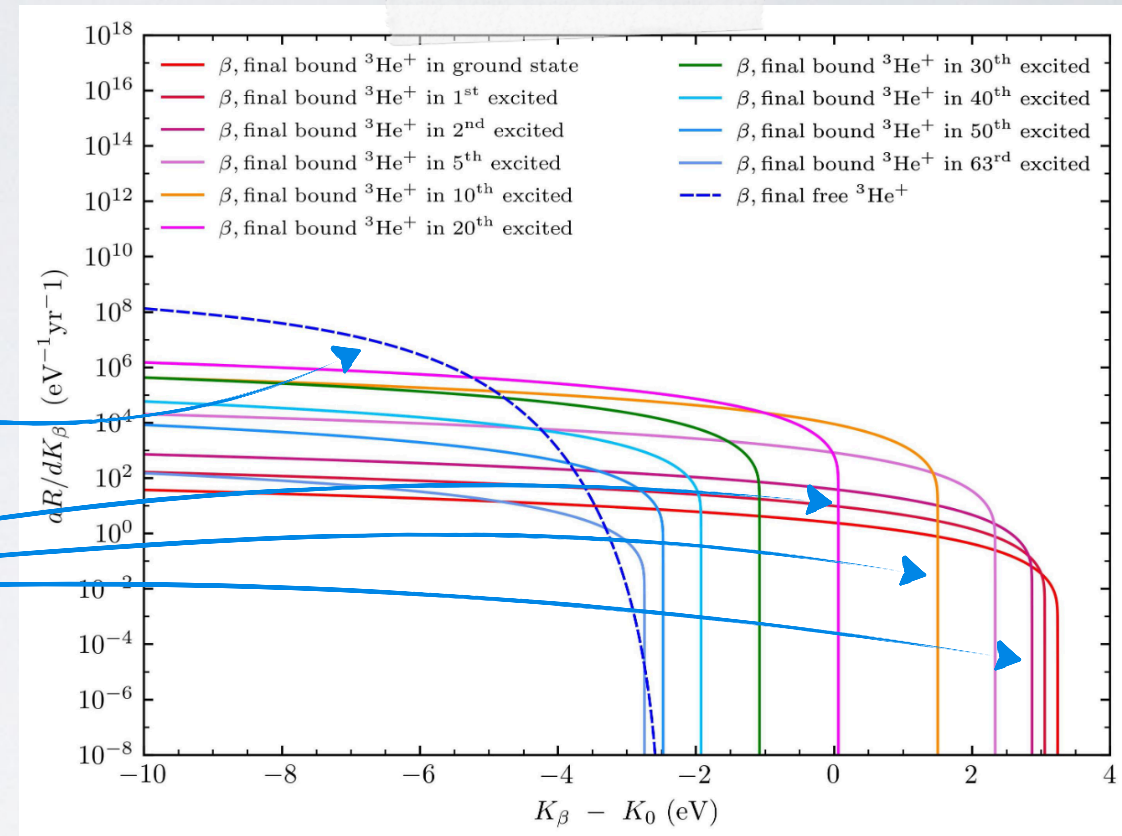

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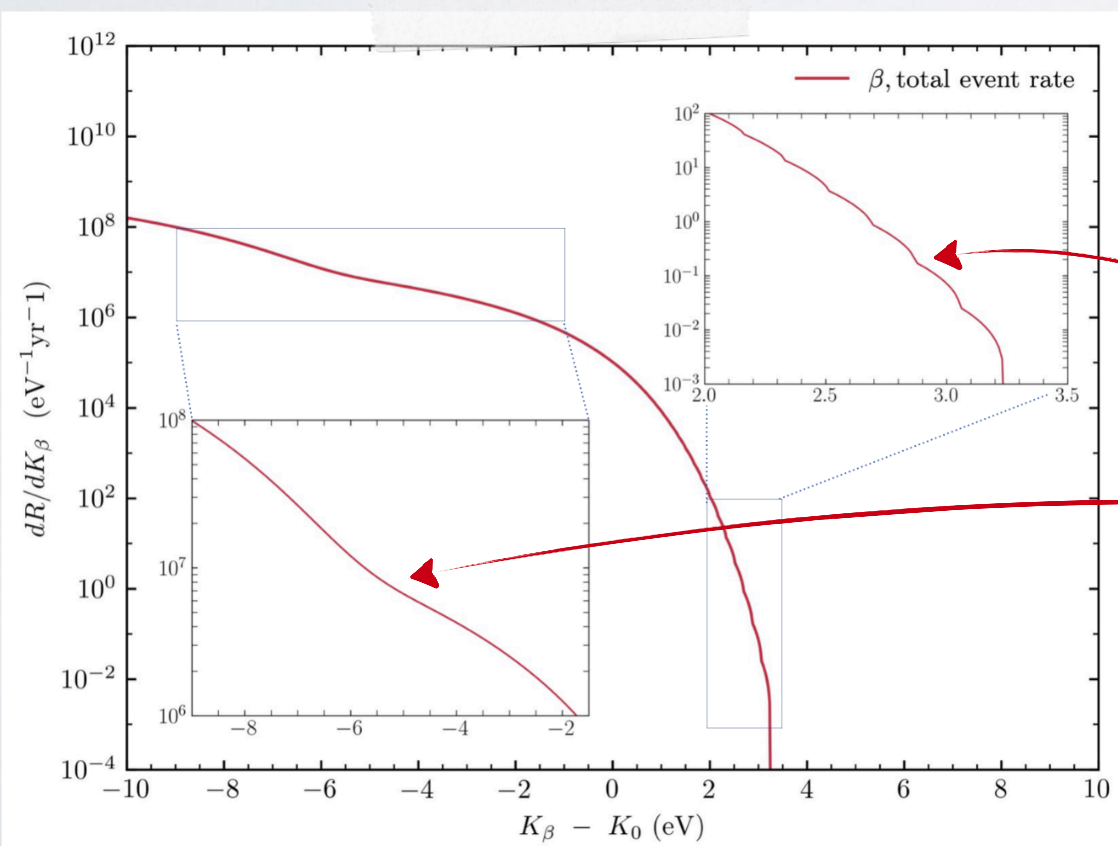

features are sensitive to m_ν

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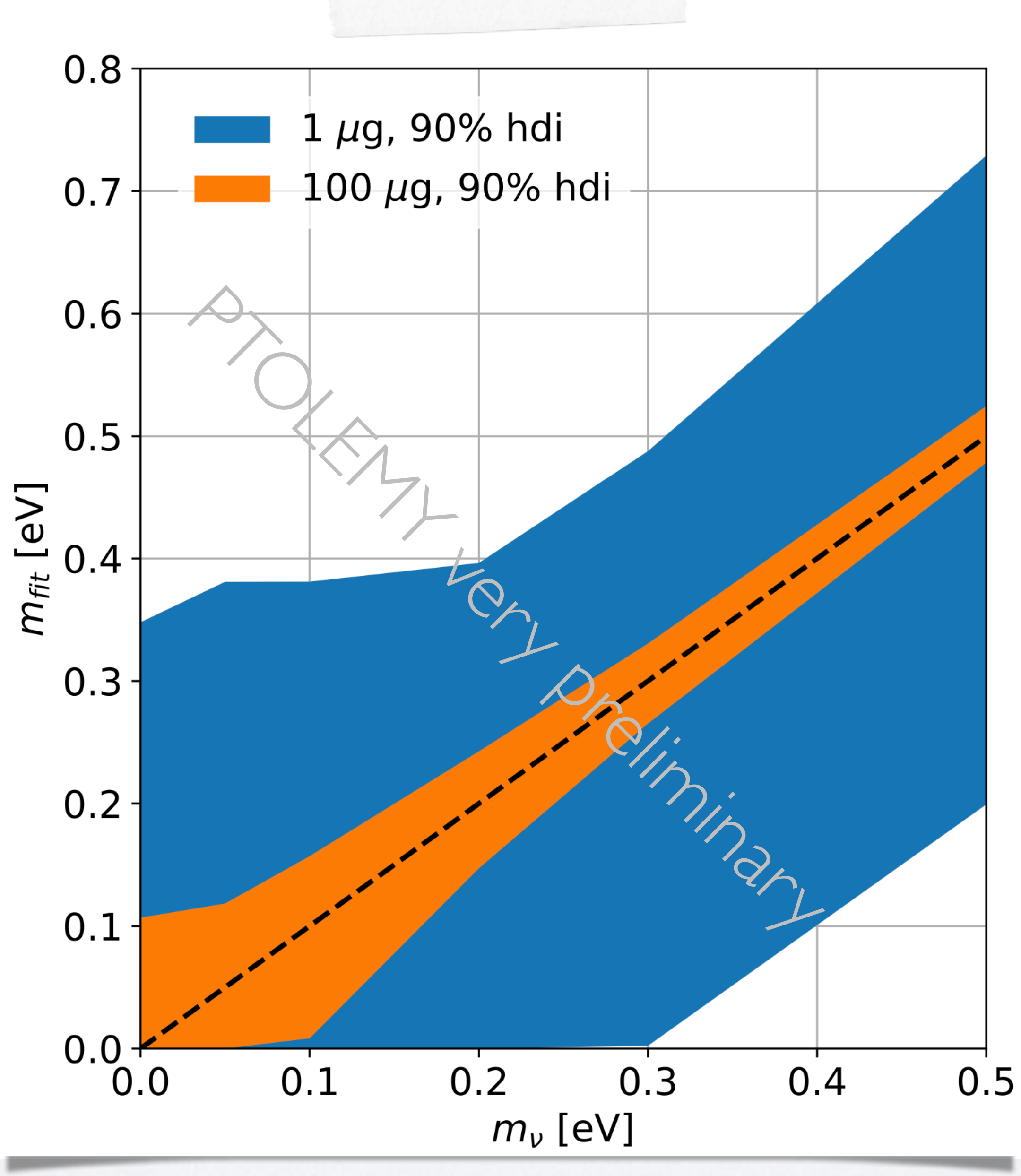

[P. Campana, A. Nucciotti — work in progress]

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3. Can we indeed assume that the DFT potential is the same before and after the decay?

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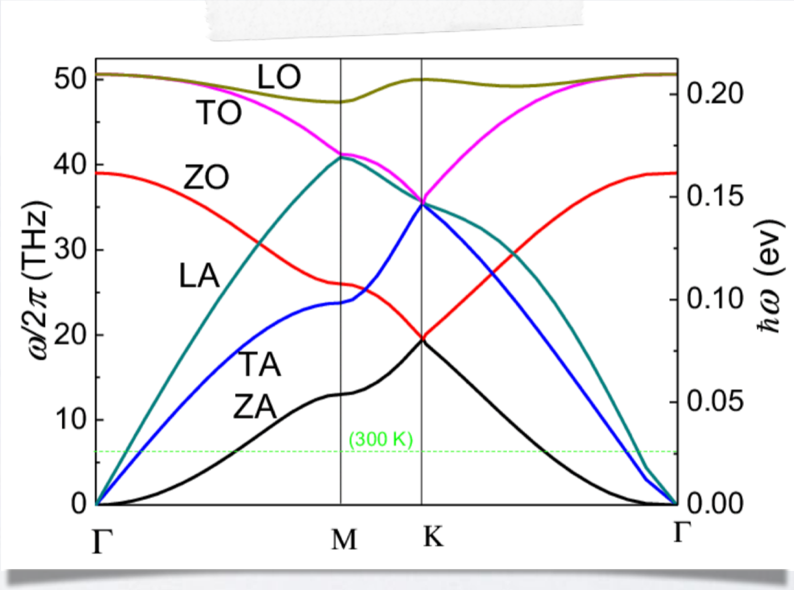

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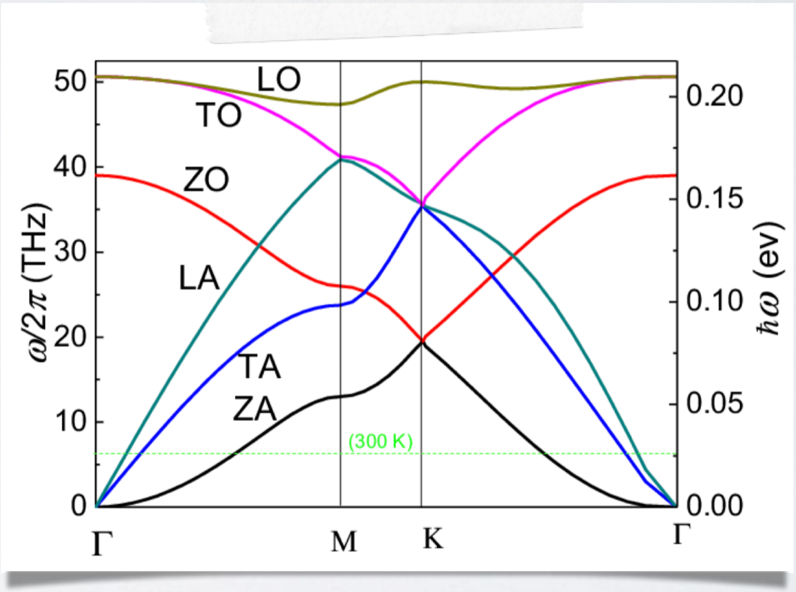

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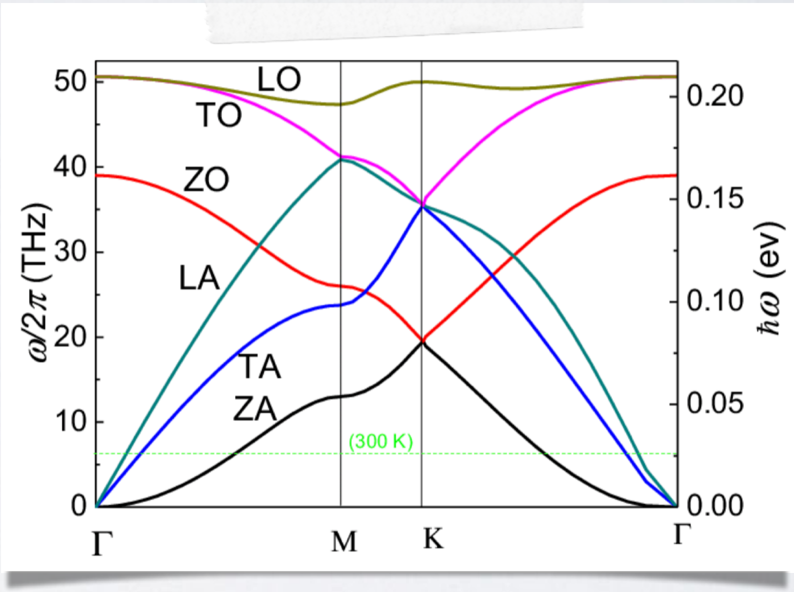

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$$\frac{dN}{dE_e} \sim \int d\omega d\mathbf{q} \, \frac{dN_{\text{no vibr}}}{dE_e d\mathbf{q}} \otimes S\left(\mathbf{q}, E_e - \omega\right)$$

[e.g., Berghaus, **AE**, Essig, Sholapurkar — JHEP 2020, 2210.06490; Squires — *Introduction to the theory of thermal neutron scattering*]

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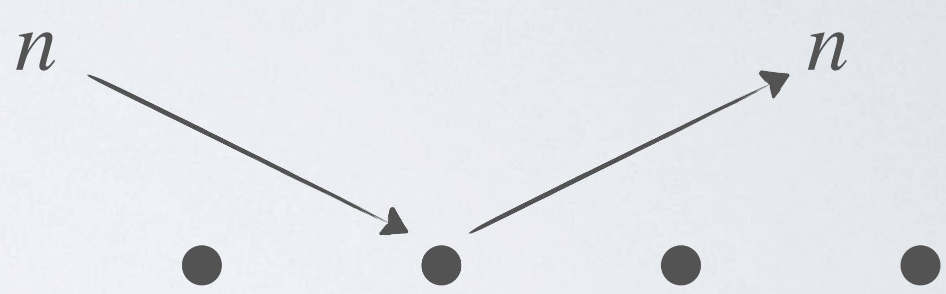

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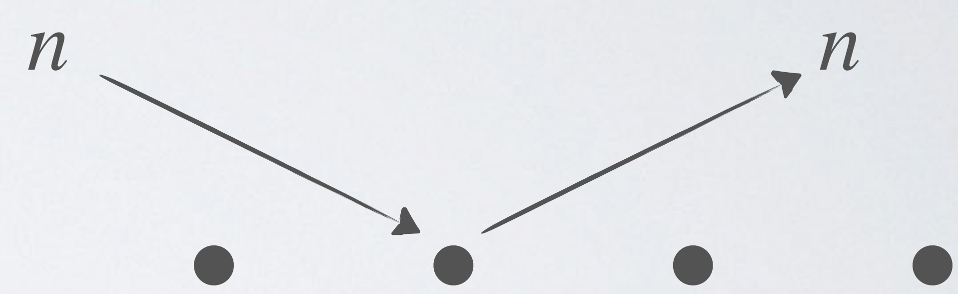

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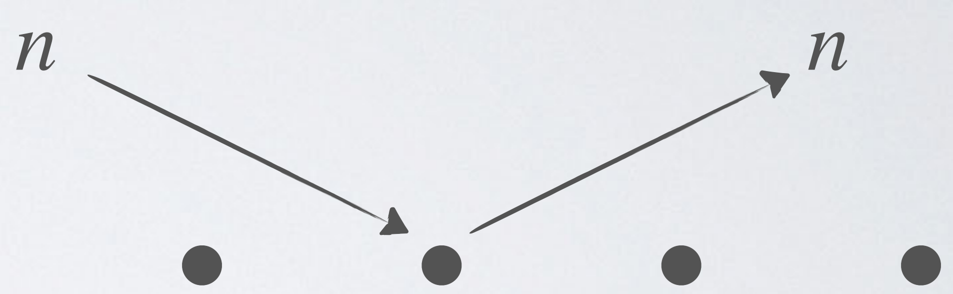

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- Is there a way of re-packaging all the nuclear response into the structure factor? [photoemission literature is vast!]

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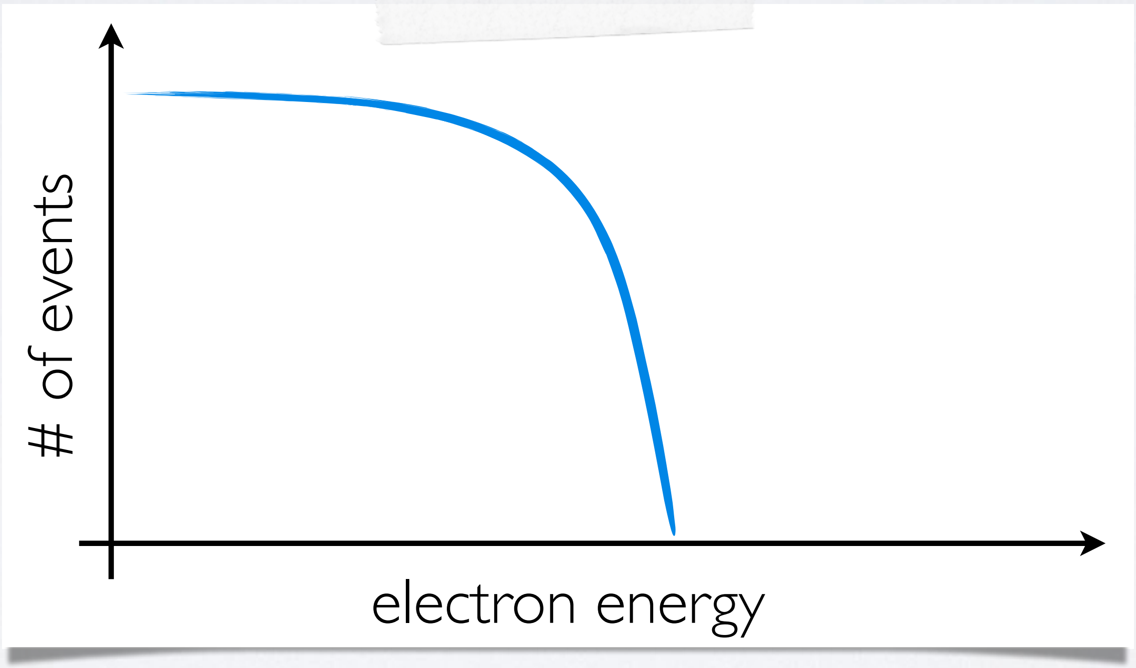

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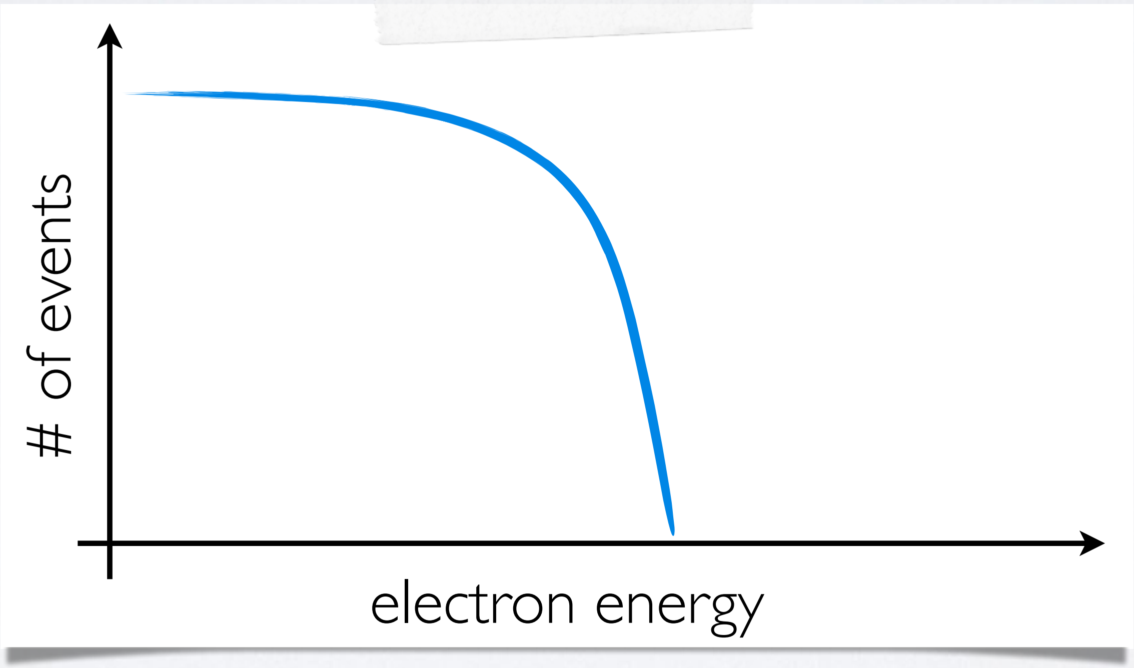

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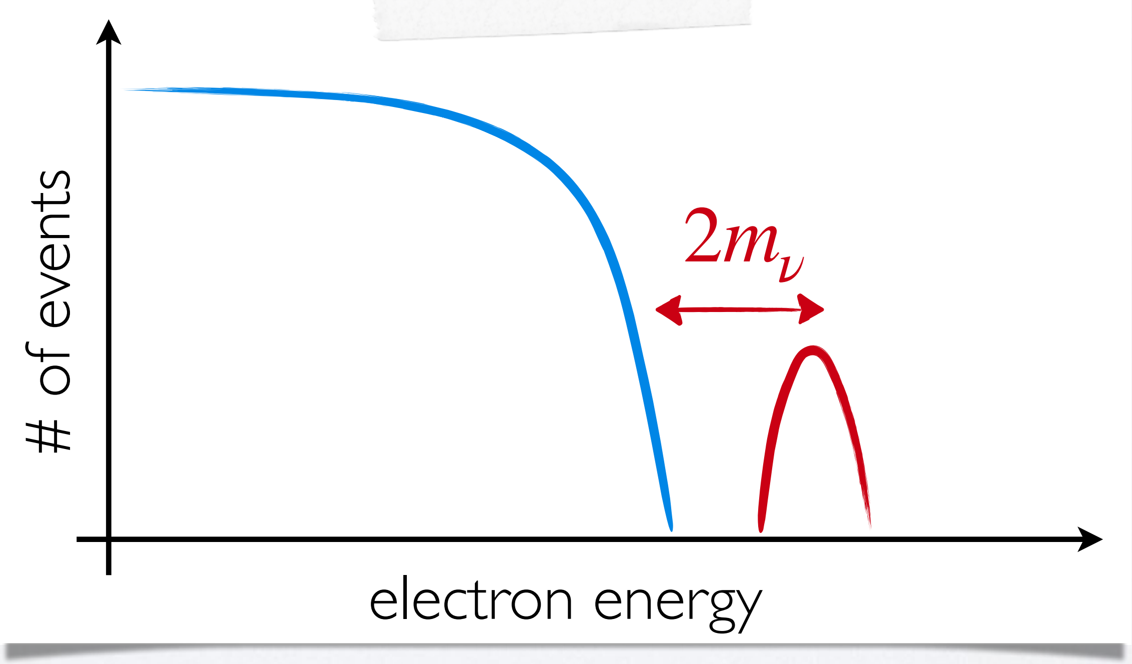
$$\nu_e + {}^3\text{H} \rightarrow {}^3\text{He}^+ + e^-$$



CνB IN A NUTSHELL

- PTOLEMY also aims at hunting for the cosmic neutrino background

β-decay part of the spectrum

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Absorption part of the spectrum

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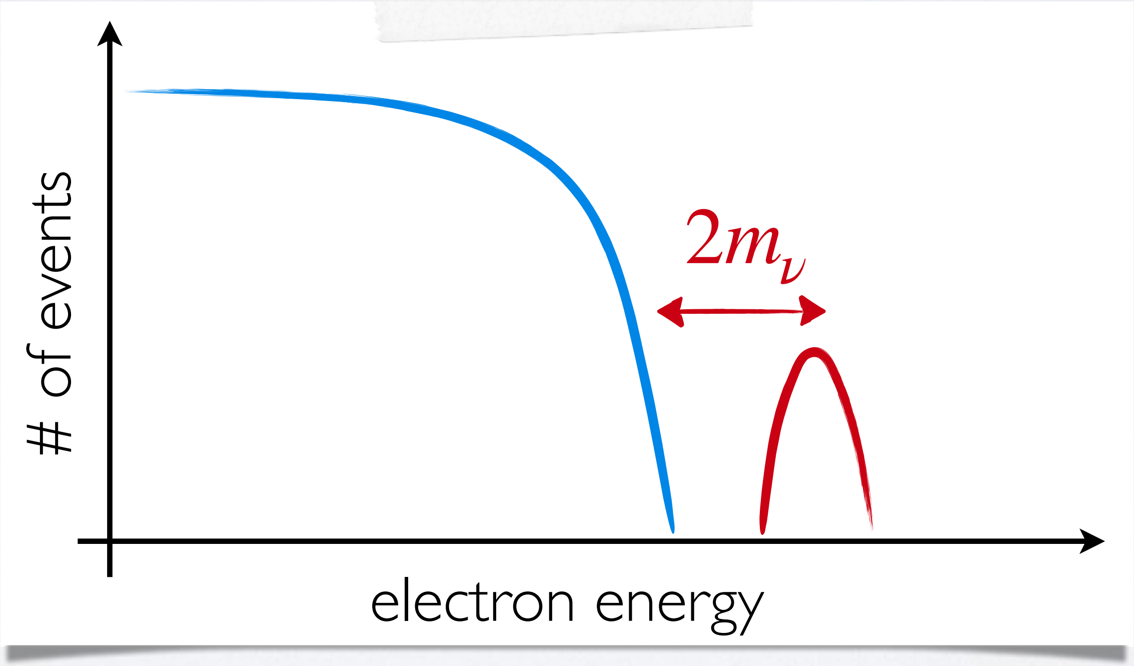

- Crucial to be able to distinguish the peak from the "background" coming from the β-decay

QUANTUM DISTORTION

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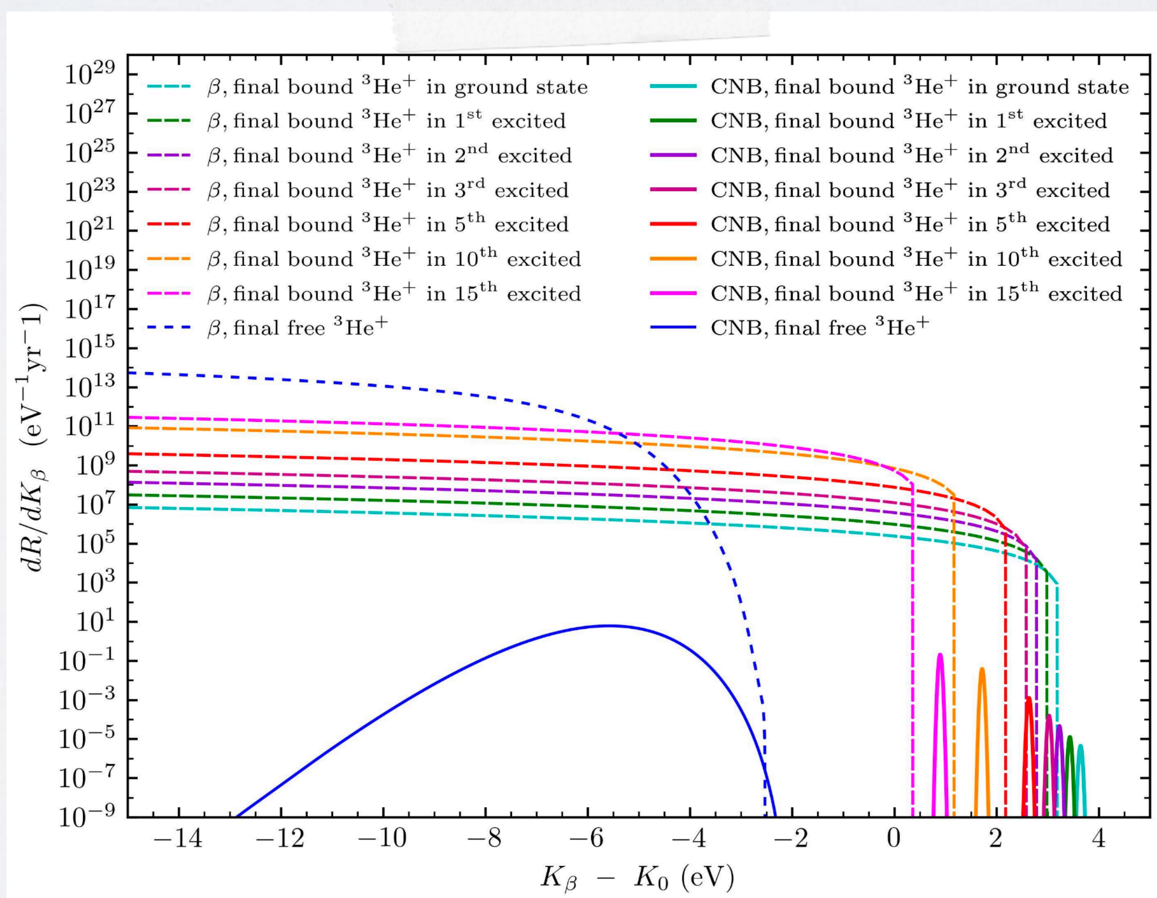

[A. Casale – Master's thesis]

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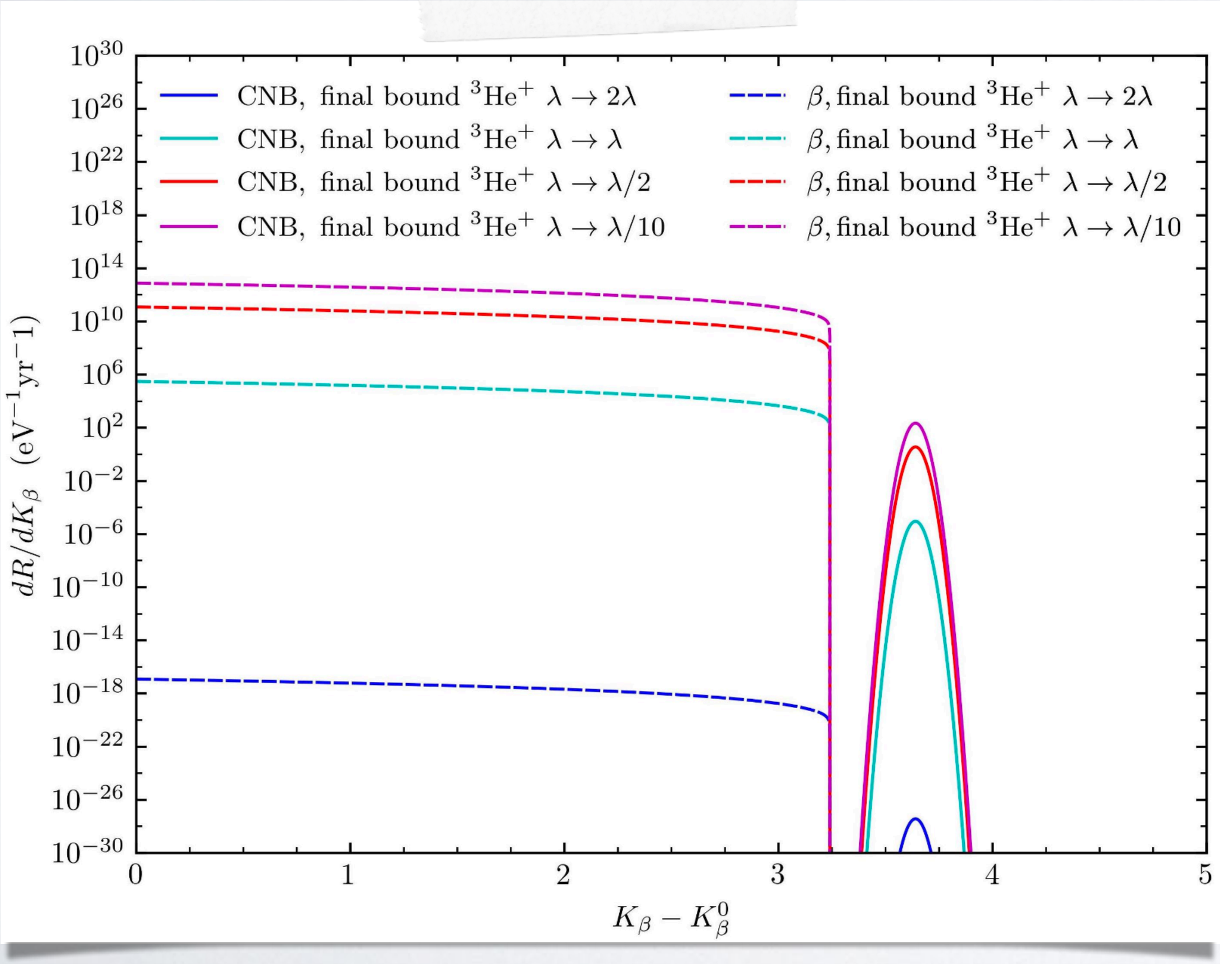
Localize tritium more

right-most peak gets enhanced



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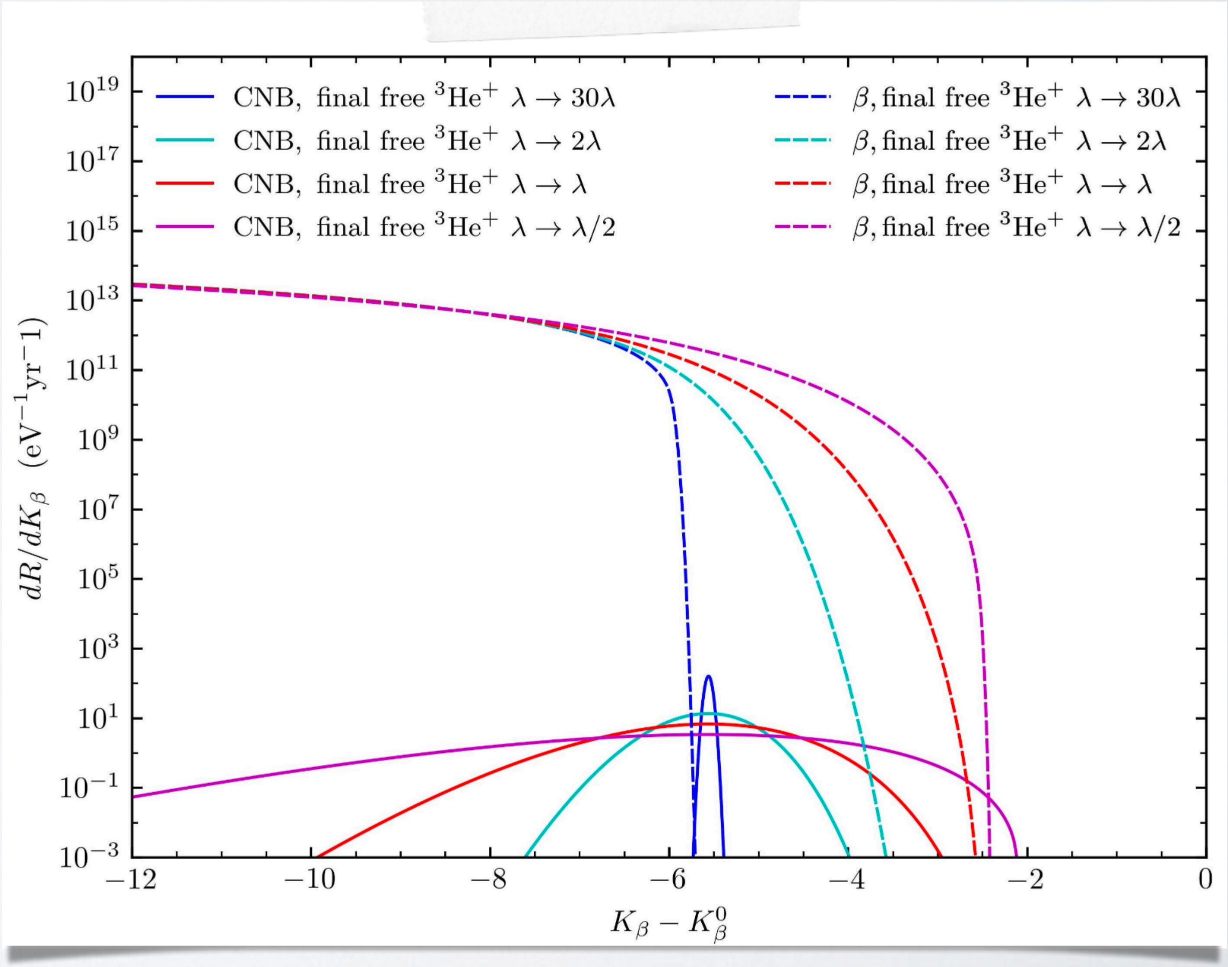

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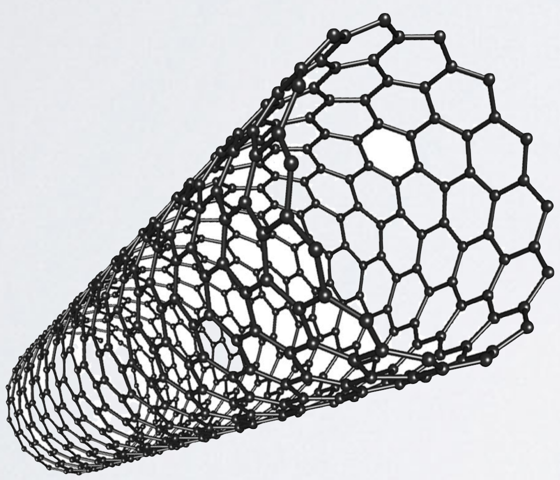

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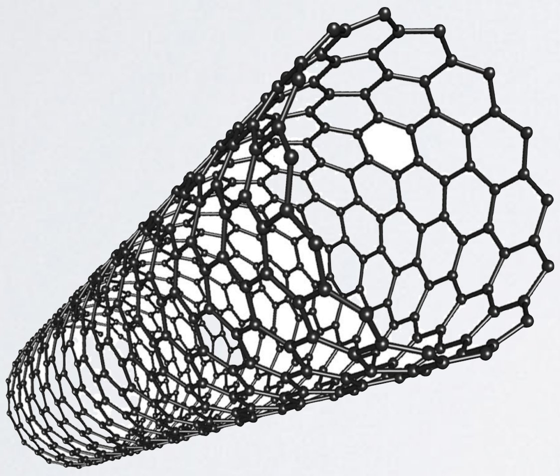

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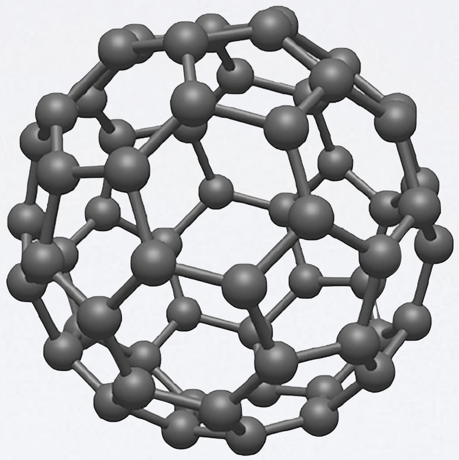

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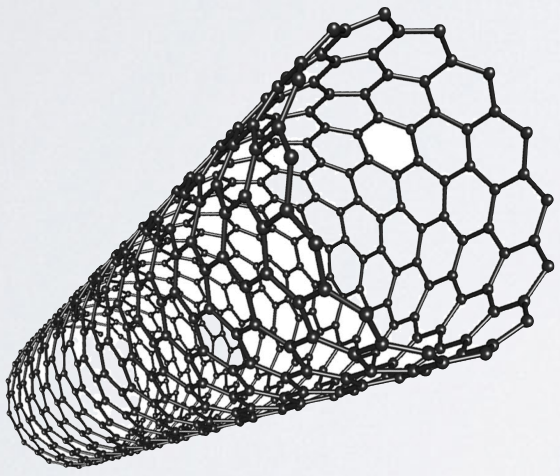

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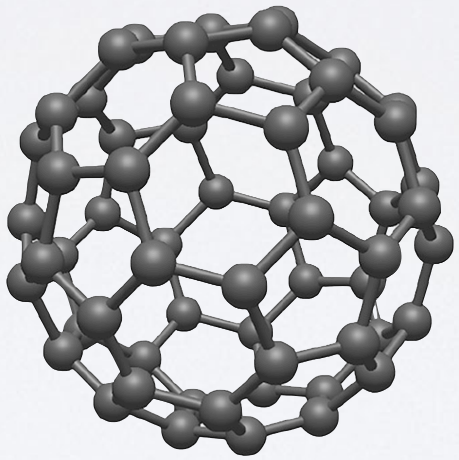

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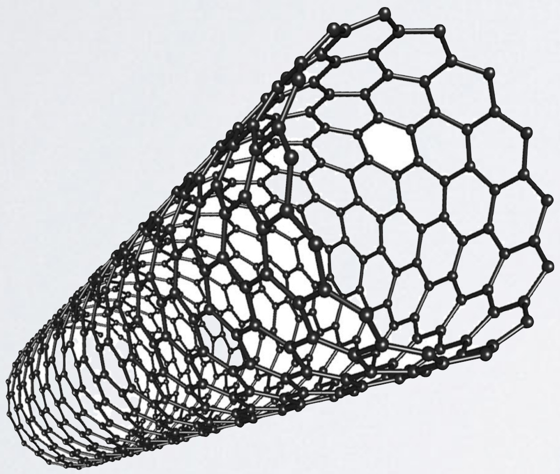

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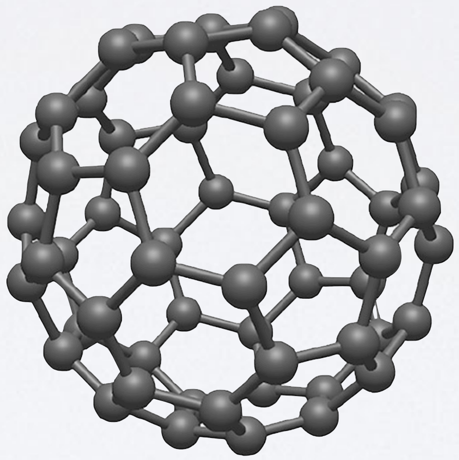

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…what else?

- We need to be creative and get to work!

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Thanks for your attention!