The quenched glueball spectrum from smeared spectral densities

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in collaboration with

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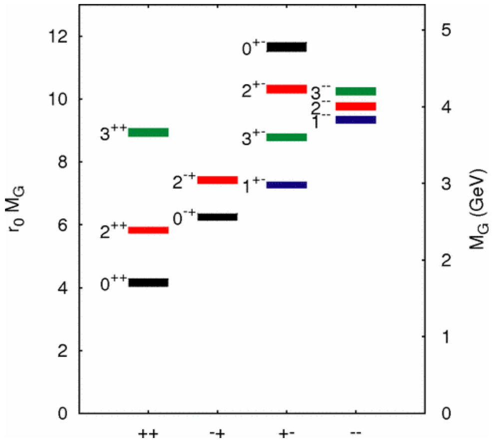
Glueball spectrum in pure $SU(3)$ Yang-Mills

- Gluballs are quarkless bound states predicted by QCD J^{PC}
- Calculation of glueball masses is important for helping experimental searches
- Lattice calculations (quenched/unquenched) are particulary useful in this regard



[Y. Chen *et al.* '06]

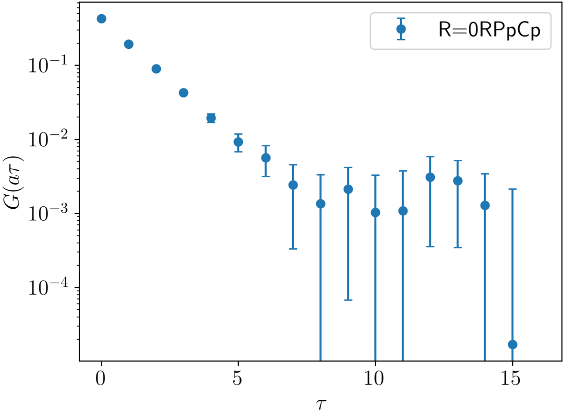
Glueballs on the lattice

Glueballs masses can be extracted from lattice correlation functions

$$G(a\tau) = \langle \Phi(a\tau)\Phi(0) \rangle_{\text{conn.}} = \sum_n |A_n|^2 e^{-a\tau\omega_n}$$

$A_n = \langle n|\Phi(0)|0\rangle$ $\rightarrow$ energy state overlap

Bad signal/noise ratio



$\beta = 5.8941$

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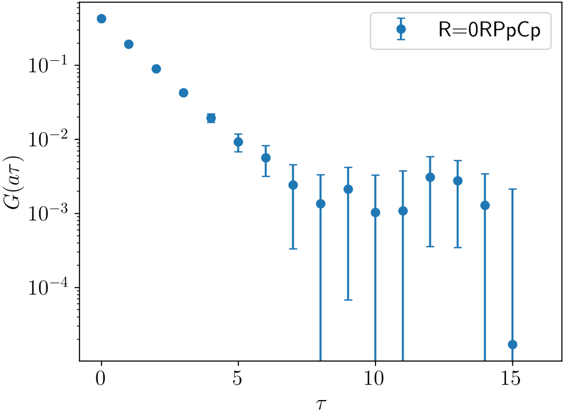
"We face an impasse: if t is small the estimated mass is not the true mass and if t is large the statistical error may be so large that nothing may be measured" G. Parisi



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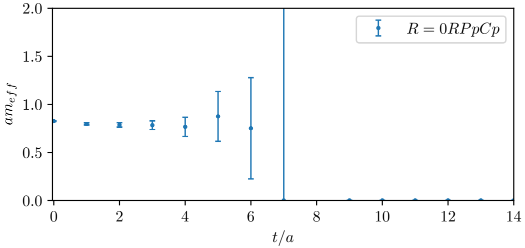
Variational method

The situation is improved considering a large set of operators with different smearing/blocking

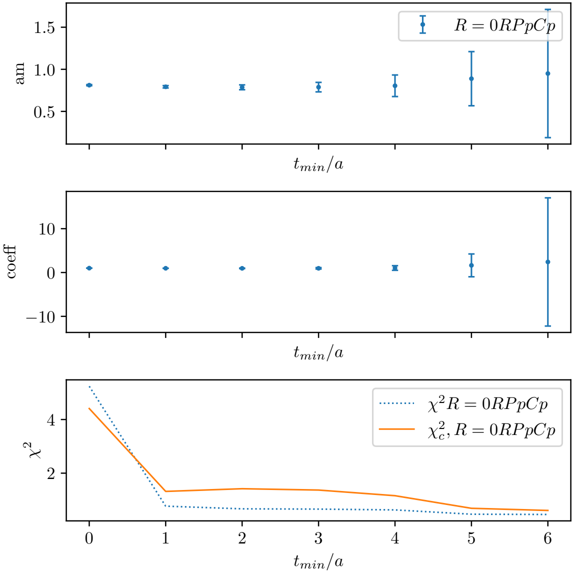
$$\sum_j C_{ij}(t_0)v_j = \sum_j \lambda_j(t_0)C_{ij}(0)v_j$$

$$am_{eff}(t_0) = \ln\left(\frac{v_i C_{ij}(t)v_j}{v_i C_{ij}(t-1)v_j}\right)$$

$$C_{ii}(a\tau) = |A_n|^2 \cosh(am_i\tau - \frac{N_L}{2})$$



- The "standard" method led to impressive results over the years
- Variational method help disentangle states
- However, effective mass plot could still be affected by excited states contribution
- Pratically can only use few lattice times



Can we use spectral functions?

A related idea appeared in [Pawlowski *et al.* '22]
Writing the Euclidean correlator in the Källen-Lehmann representation

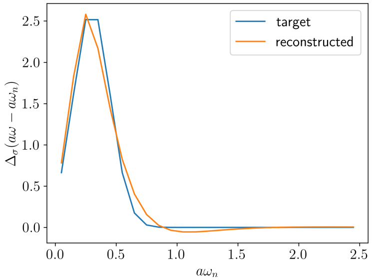
$$G(a\tau) = \int_{\omega_{\min}}^{\infty} d\omega \; \rho(\omega) e^{-a\omega\tau}$$

- ➜ For lattice correlators this leads to a **ill-posed inverse problem**
- ➜ Need a method to regularise the problem. Also, finite volume (L) means

$$\rho_L(\omega) = \sum_n \frac{|\langle n|\Phi(0)|0\rangle|^2}{2\omega_n(L)} \delta(\omega - \omega_n(L)).$$

HLT method [Hansen, Lupo, Tantalo '19]

We can use Backus-Gilbert regularisation to extract **smeared** spectral function from the lattice correlation function



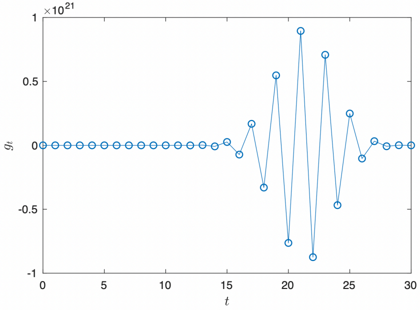
$$K(\omega;\boldsymbol{g}) = \sum_{\tau=1}^{\tau_{\max}} g_\tau(\sigma) e^{-a\tau\omega}$$

$$\rho_L^\sigma(\omega) = \int_0^\infty d\omega\ \rho_L(\omega)\Delta_\sigma(\omega - \omega_n(L)) \simeq a \sum_{\tau=1}^{\tau_{\max}} g_\tau(\sigma) G(a\tau).$$

Backus-Gilbert regularisation

Hansen, Lupo, Tantalo, **PRD**, 1903.06476

$$A_n[\boldsymbol{g}] = \int_{\omega_0}^{\infty} d\omega \; w_n(\omega) \Big| K(\omega; \boldsymbol{g}) - \Delta_\sigma(\omega - \omega_n(L)) \Big| .$$



[Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

Backus-Gilbert regularisation

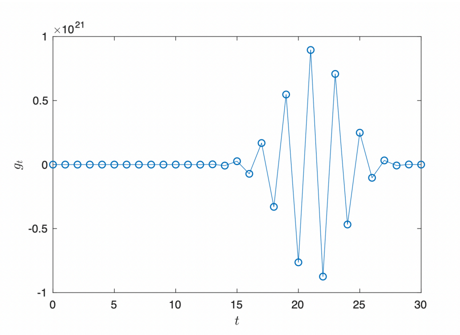
Hansen, Lupo, Tantalo **PRD**, 1903.06476

$$W_n[\boldsymbol{g}] = \frac{A_n[\boldsymbol{g}]}{A_n[\boldsymbol{0}]} + \lambda B[\boldsymbol{g}], \quad \left.\frac{\partial W_n[\boldsymbol{g}]}{\partial g_\tau}\right|_{g_\tau = \boldsymbol{g}^{\boldsymbol{p}}} = 0$$

$$A_n[\boldsymbol{g}] = \int_{\omega_0}^{\infty} d\omega\ w_n(\omega) \Big| K(\omega; \boldsymbol{g}) - \Delta_\sigma(\omega - \omega_n(L)) \Big|.$$

$$B[\boldsymbol{g}] = B_{\text{norm}} \sum_{\tau_1, \tau_2 = 1}^{\tau_{\max}} g_{\tau_1} g_{\tau_2} \operatorname{Cov}(\tau_1, \tau_2),$$

$$\boldsymbol{p} = (\alpha, \lambda, \omega_{\min}, \tau_{\max},)$$



[Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

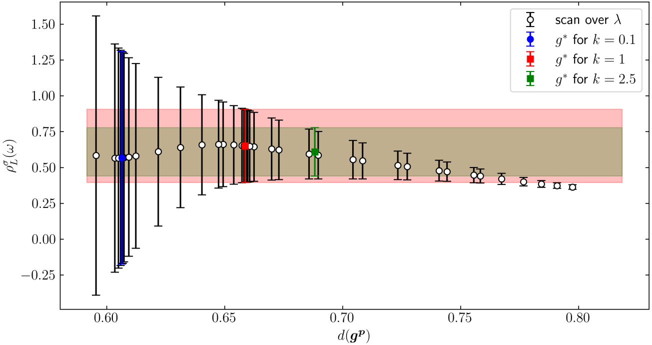
Stability analysis

- Method introduced in [Bulava *et al.* '21]
- Choose final result in statistically dominated region

$$\frac{A[\boldsymbol{g}^*]}{A[0]} = kB[\boldsymbol{g}^*]$$

- Final results need to be extrapolated

$$\rho(\omega) = \lim_{\sigma \to 0} \lim_{L \to \infty} \rho^\sigma_L(\omega)$$



$$d(\boldsymbol{g}^{\boldsymbol{p}}) = \sqrt{A[\boldsymbol{g}]/A[0]}$$

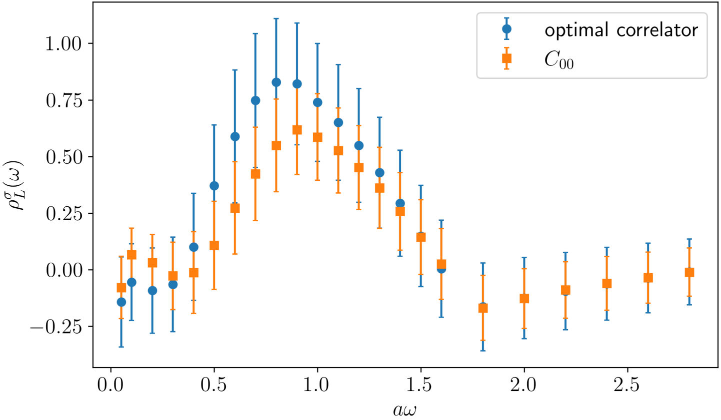
Ensembles details

We are currently at a very pleriminary stage and plan to soon include more values of β and other representations $A_1^{-+}, E^{++}, \ldots$

J^{PC}	β	$L^3 \times T$	N_{cnfg}
A_1^{++}	5.8941	$32^3 \times 32$	≈ 5000
A_1^{++}	6.0625	$32^3 \times 32$	15000

Glueball smeared spectral functions

Studying the spectral functions allows to check contributions to the optimal correlators in the variational method



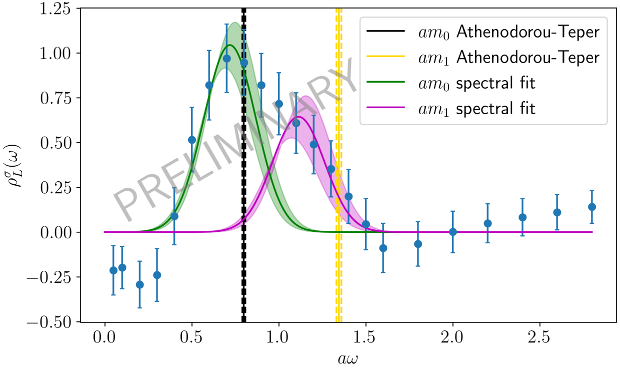
$\beta = 5.8941,\ \sigma = 0.15/a$

Fit of smeared spectral functions

[Athenodorou, Teper '20]

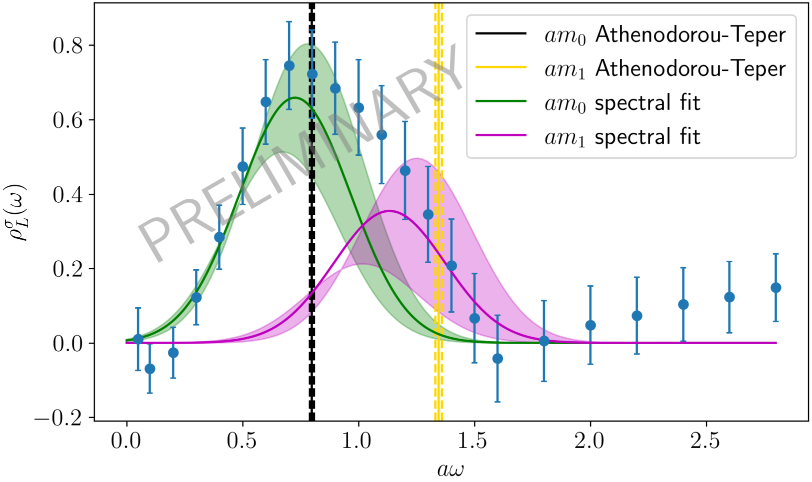
- Introduced in [Del Debbio, *et al.* '23]
- We can perform fit of spectral functions rather than correlators
- Minimise χ^2 function defined in terms of $\mathrm{Cov}[\rho^\sigma]$

$$f_k^\sigma(\omega) = \sum_k a_k \; e^{\frac{-(\omega-\omega_k)^2}{2\sigma^2}},$$



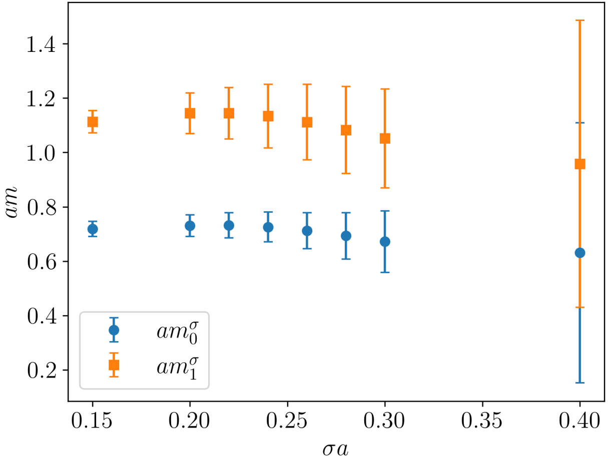
$\beta = 5.8941,\ \sigma = 0.15/a,\ \chi^2_{red} = 2.67$

- Extrapolation $\sigma \to 0$ crucial for accurate results



$\beta = 5.8941$, $\sigma = 0.24/a$, $\chi^2_{red} = 1.209$

We cannot yet extrapolate $\sigma \to 0$ but we can still check the σ dependence



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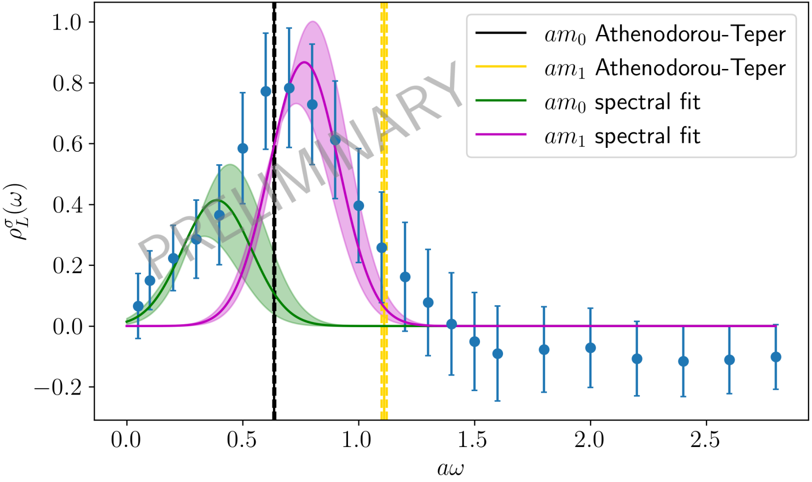

$$\beta = 6.0625,\ \sigma = 0.15/a,\ \chi^2_{red} = 1.33$$

What's next

- Accuracy of final results depends on correlator's precision:

  Increase statistics to match literature standard

  Use multi-level (?)

- Extend analysis to other representations $A_1^{-+}, E^{++}, \ldots$ and different volumes.

- Perform continuum limit $a \to 0$.

- Repeat study in un-quenched setting where glueballs are allowed to decay:

  Here the $\sigma \to 0$ will be a crucial step.

Take home points

- The standard method of extracting glueball masses is challenging and it might be affected by uncontrolled systematic errors.
- The investigation of the glueball spectrum using spectral densities allows to check carefully for systematic uncertainities.
- The limited precision of glueball correlators is a problem also in the spectral density picture.
- The preliminary results are encouraging and a full systematic study will appear (hopefully) soon!