Quantum Many Body Scars in a Lattice Gauge Theory

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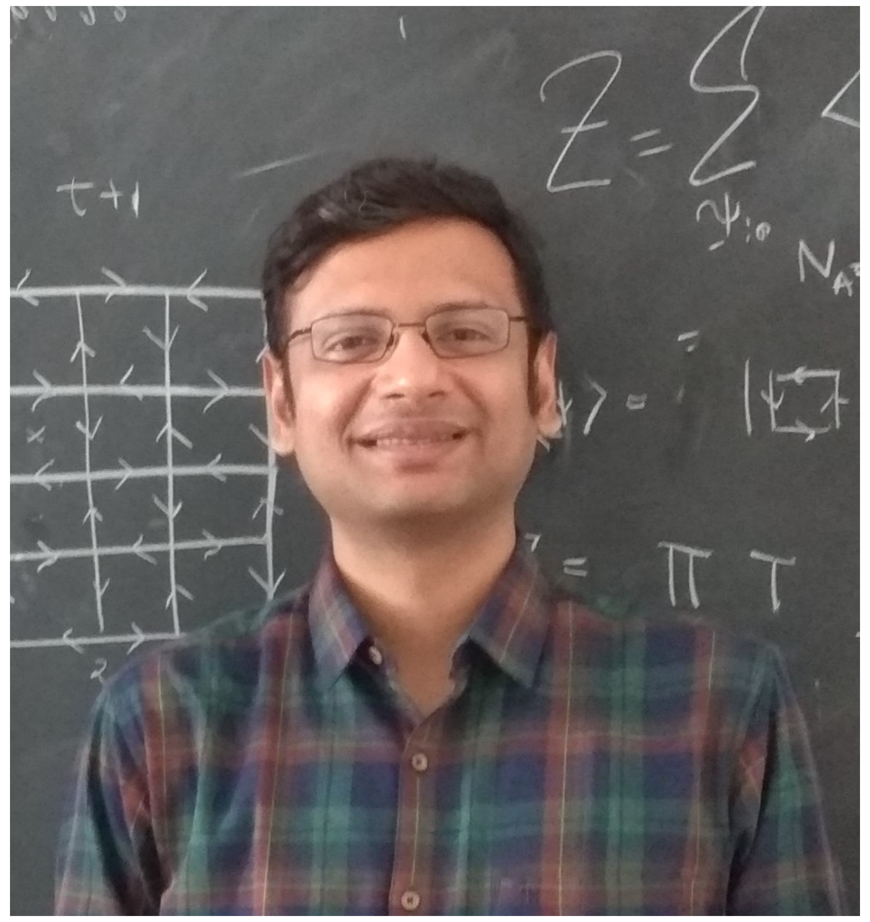
21/12/2023

Collaborators



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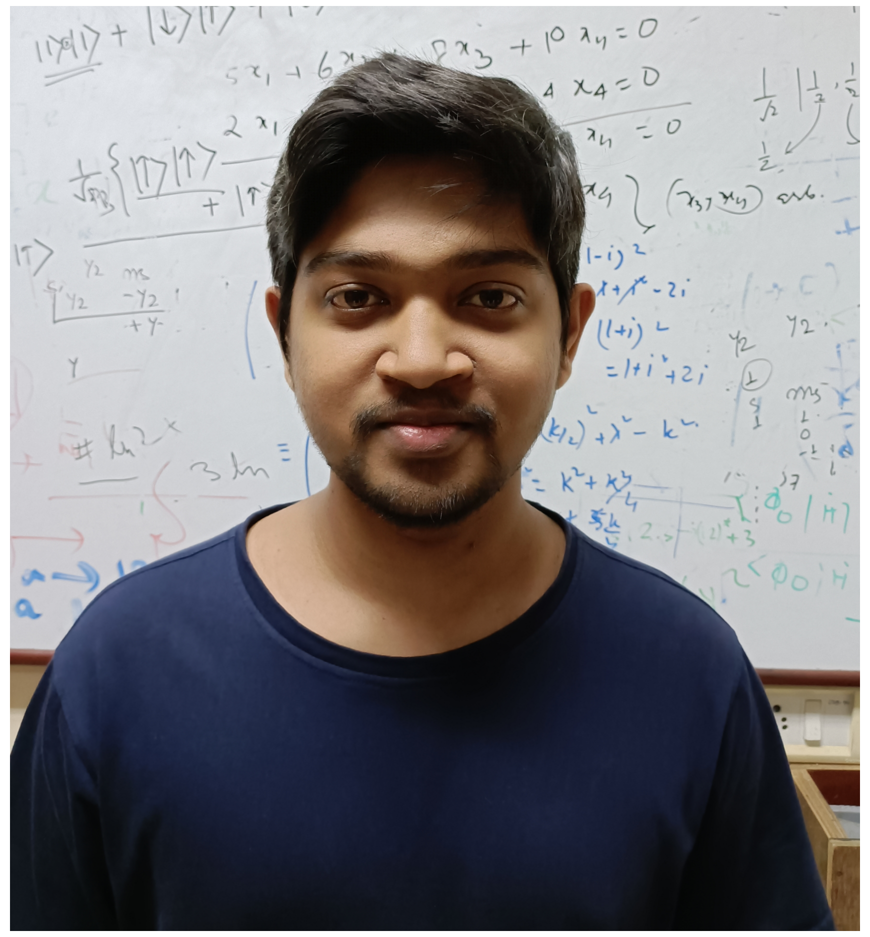


Indrajit Sau

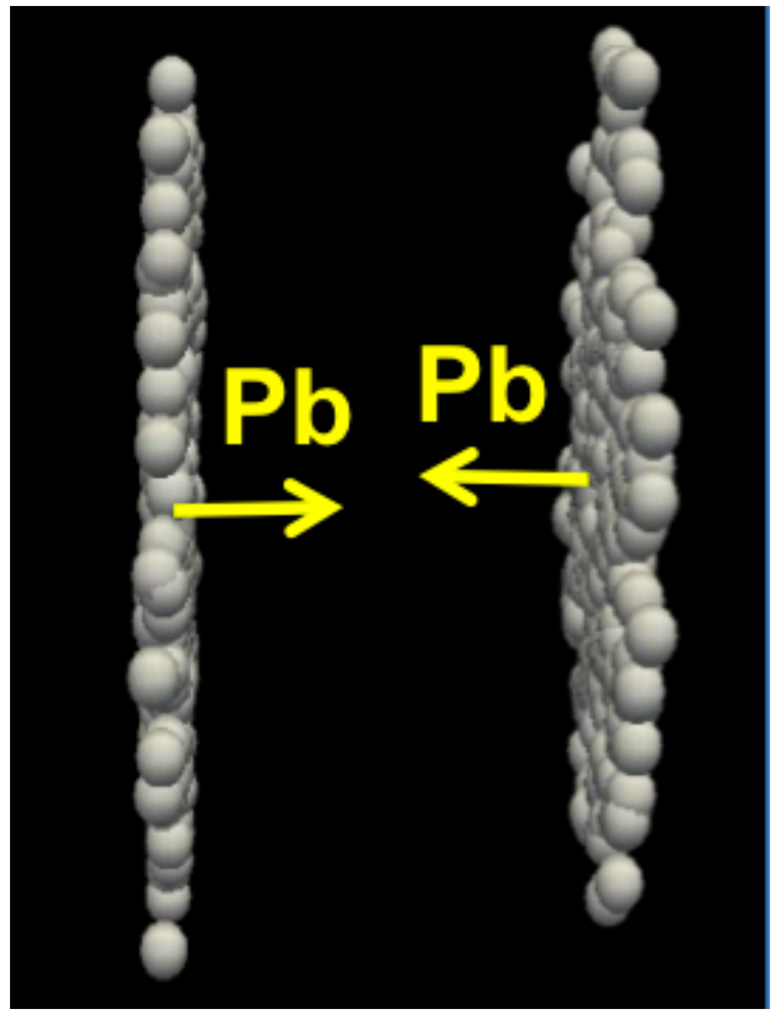
Homi Bhabha
National Institute

Sublattice scars and beyond in two-dimensional U(1) quantum link lattice gauge theories
Indrajit Sau, **PS**, Debasish Banerjee, and Arnab Sen arXiv:2311.06773, under review PRD

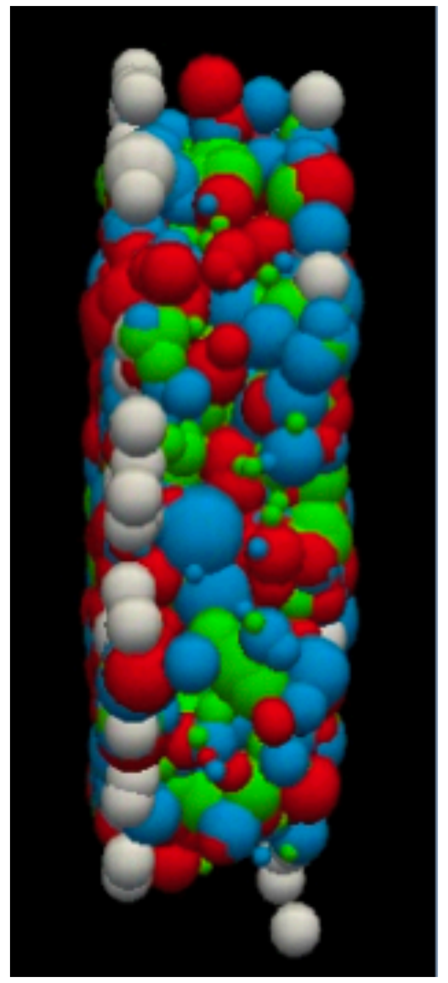
Thermalisation at colliders

Follow the real time evolution of particles at colliders

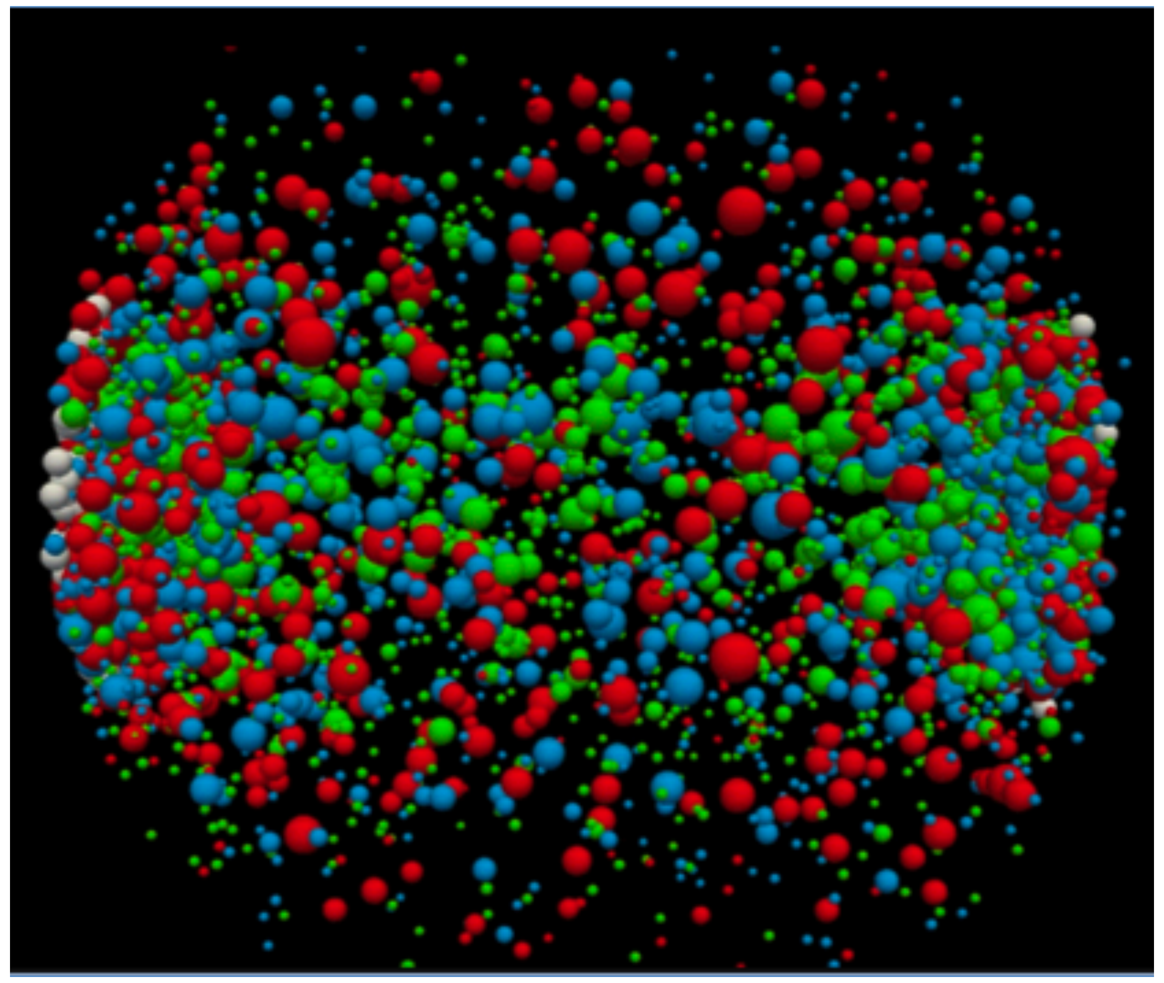
Pre-Reaction



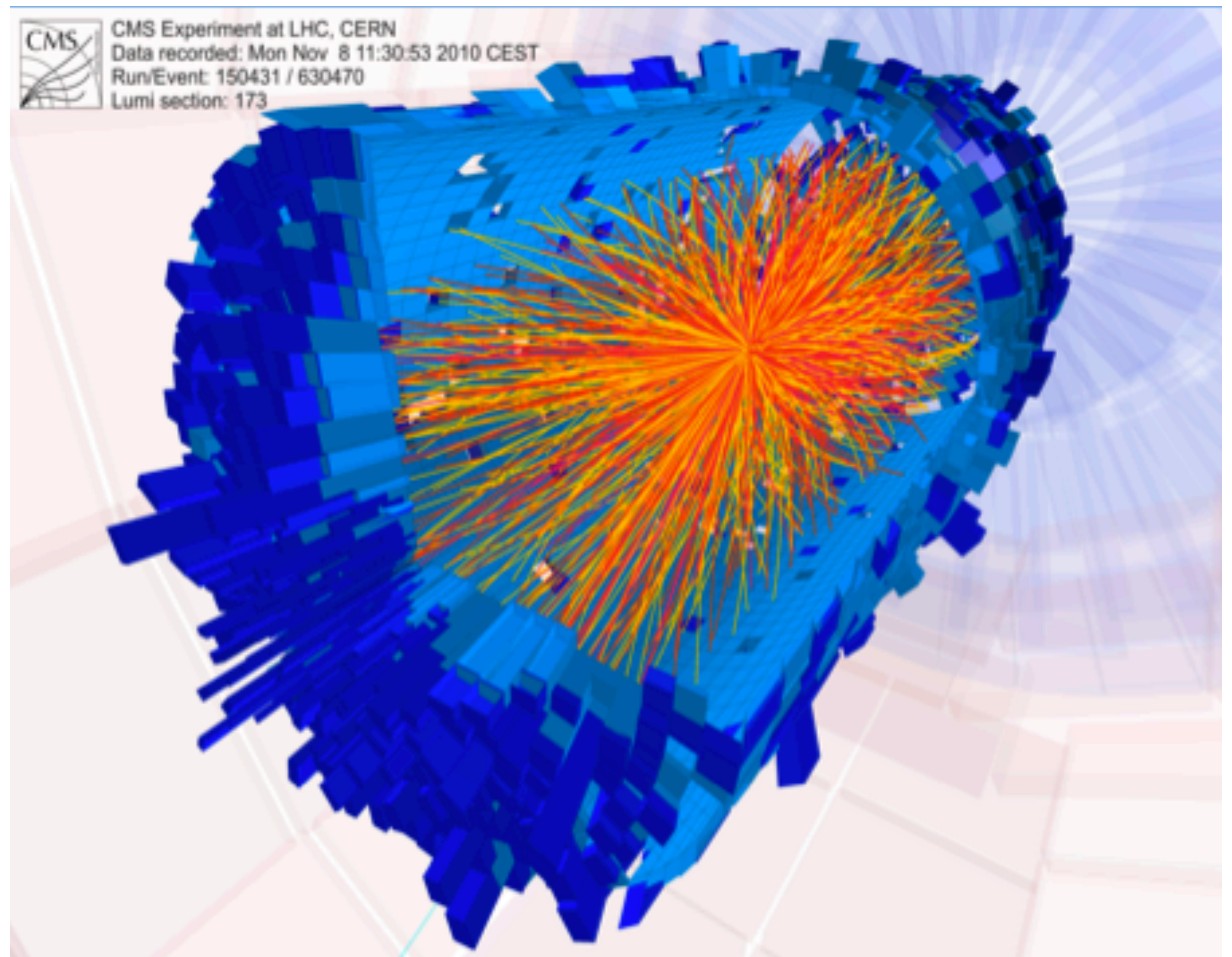
QGP



Hadronization



Detection



Time

Eigenstate Thermalization Hypothesis

- The local observable expectation value of a non-equilibrium initial state $|\psi>$ converge to the expectation value of a statistical mechanical system with the same energy

$$|\psi> \to \rho \sim e^{-\beta H}$$

- Eigenstates should look typical for their energy (eigenstate thermalisation)

Exceptions:

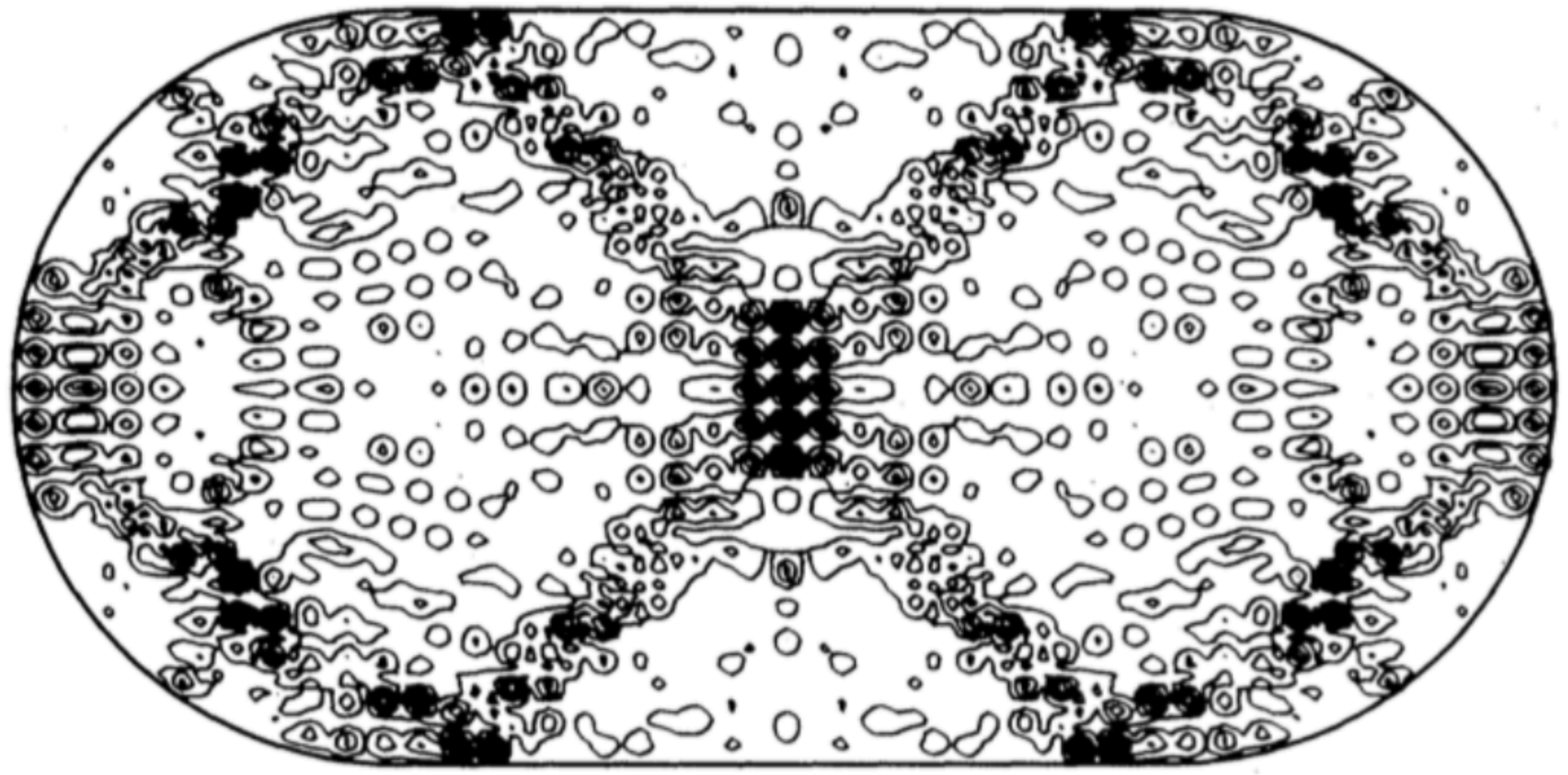
- Non Interacting systems
- Integrale systems
- Many body localised systems

} ⟶ **Generically fail to equilibrate**
Ergodicity is completely broken

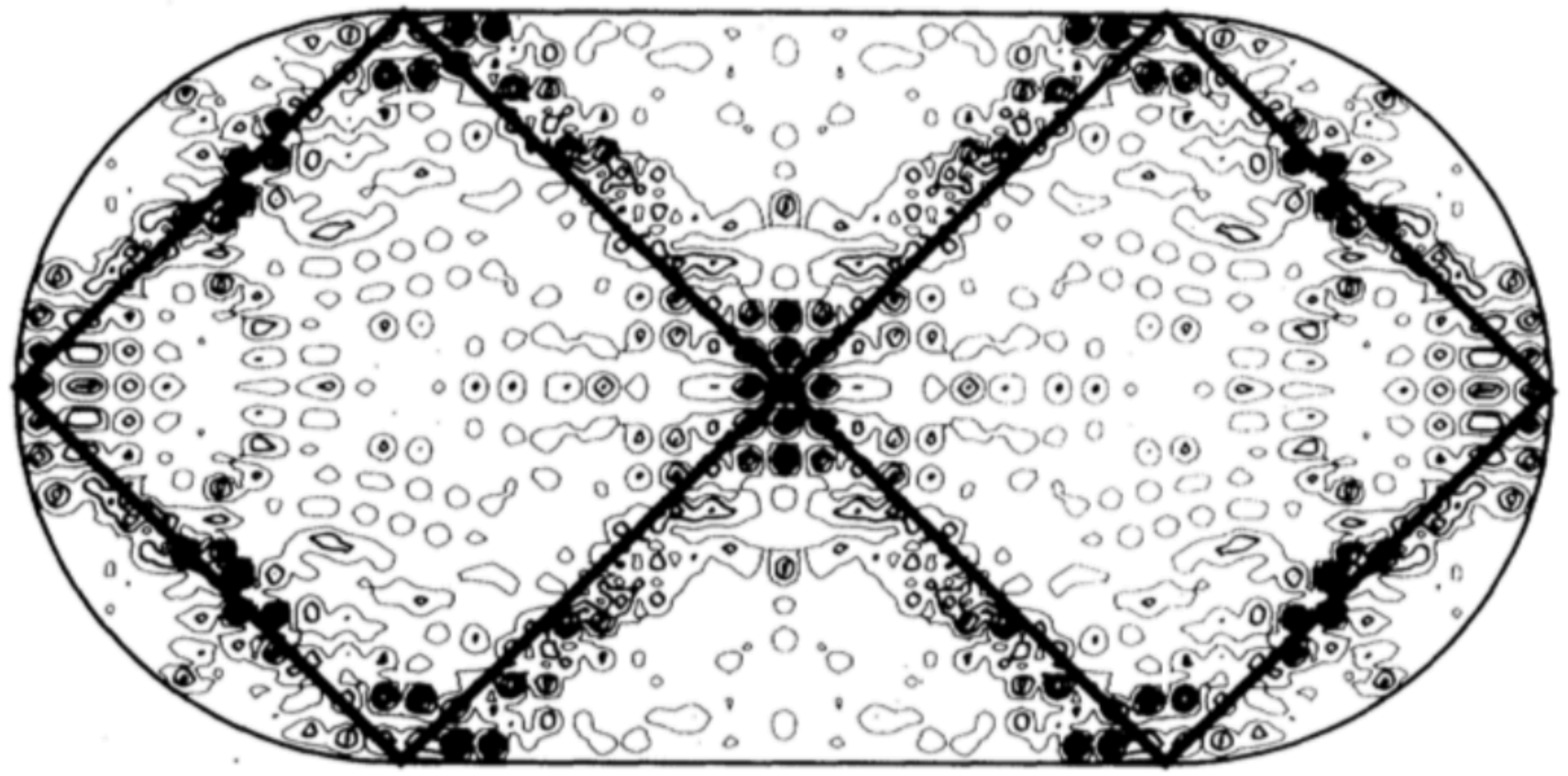
C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn & Z. Papić, *Nature Physics* (2018)14, 745–749

Quantum Many Body Scars

Short unstable periodic classical orbits leave an imprint
on the system's quantum dynamics and eigenstate properties



SCARRING WAVEFUNCTION



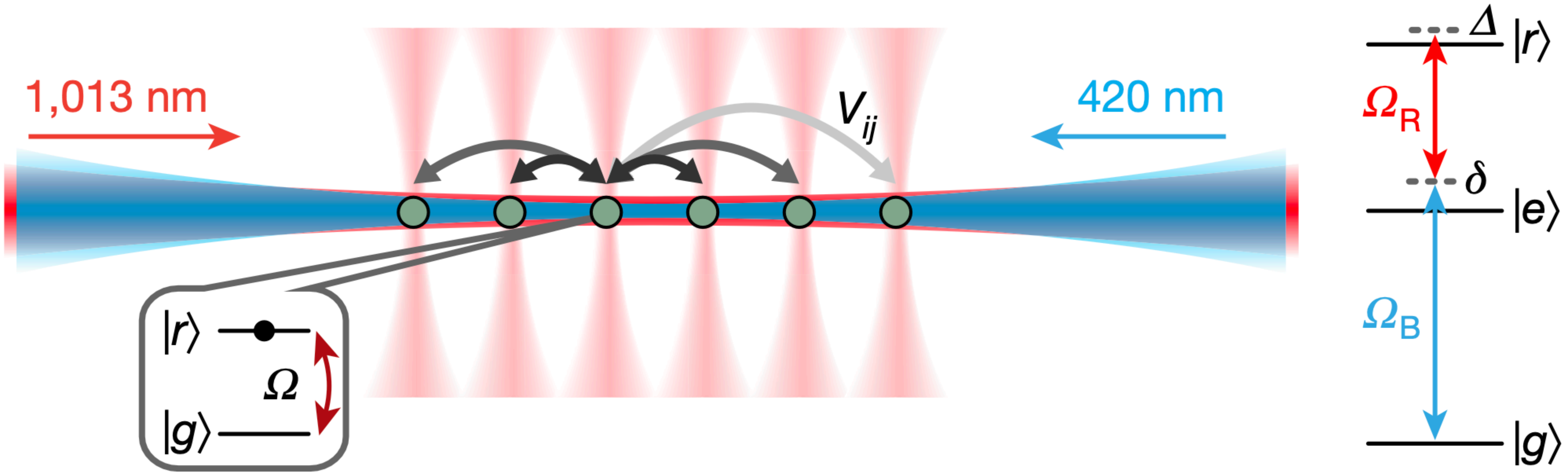
CLASSICALLY UNSTABLE PERIODIC ORBITS

- Unstable classical period orbits to be lost in the transition to quantum mechanics as the particle becomes "blurred"
- A model is quantum ergodic but not quantum unique ergodic:

Eigenstate thermalisation for all eigenstates vs. almost all eigenstates

E.J. Heller, PRL (1984) 53,1515

Rydberg Atoms experiment



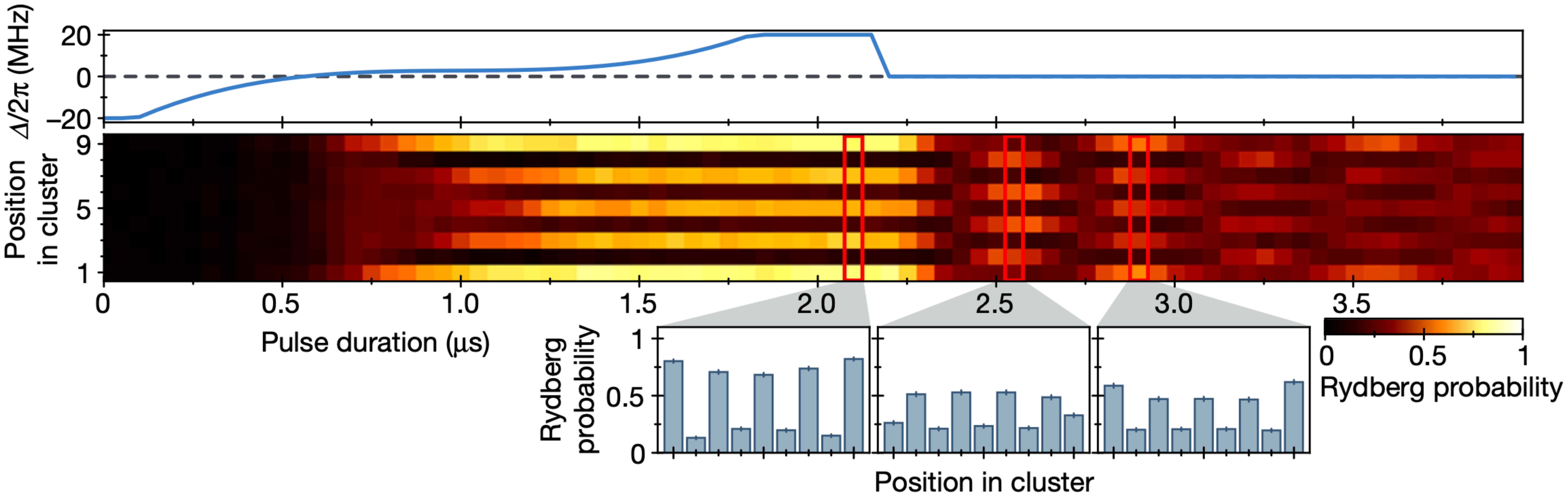
$$H = \sum P_i X_{i+1} P_{i+2}$$

$Xi = |\circ\rangle\langle\bullet| + |\bullet\rangle\langle\circ|$ creates or removes an excitation at a given site

$Pi = |\circ\rangle\langle\circ| = (1 - Zi)/2$ ensures nearby atoms not simultaneously in Rydberg state

H. Bernien et al. *Nature* 551, 579–584 (2017)

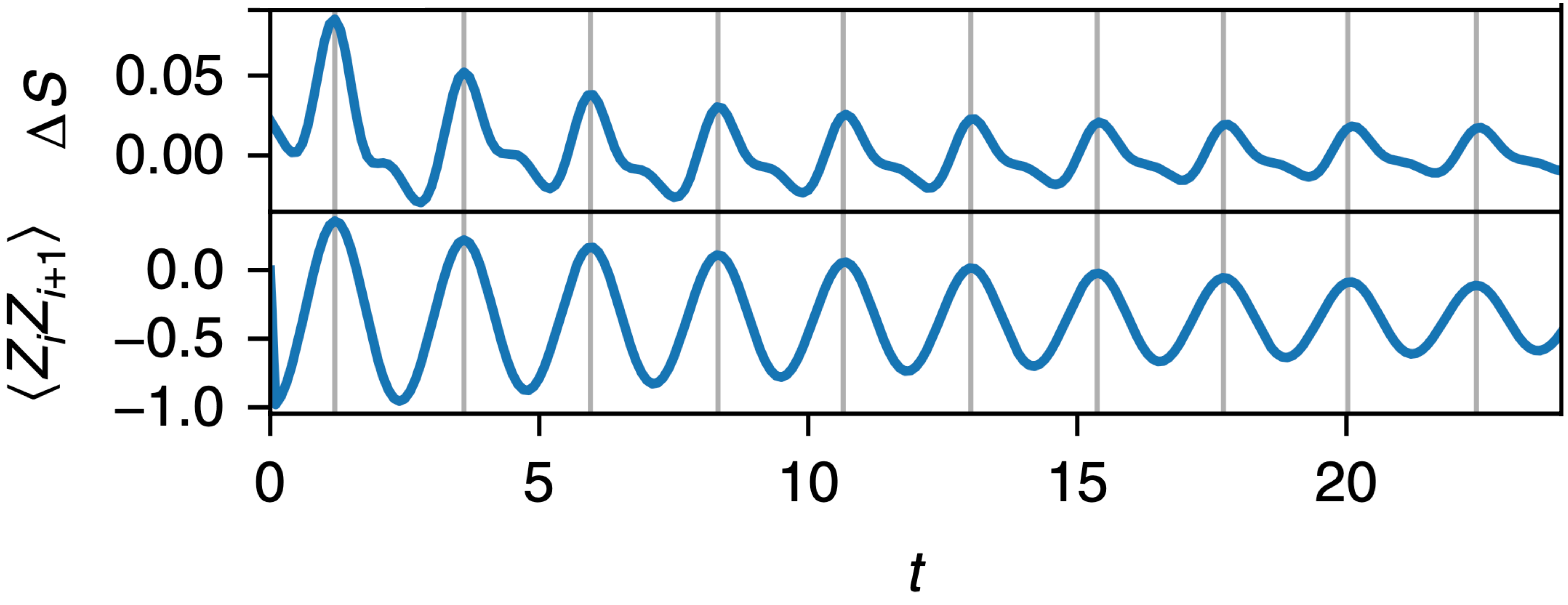
Observation



Revivals of the initial state in time evolution →lack of thermalisation

H. Bernien et al. *Nature* 551, 579–584 (2017)

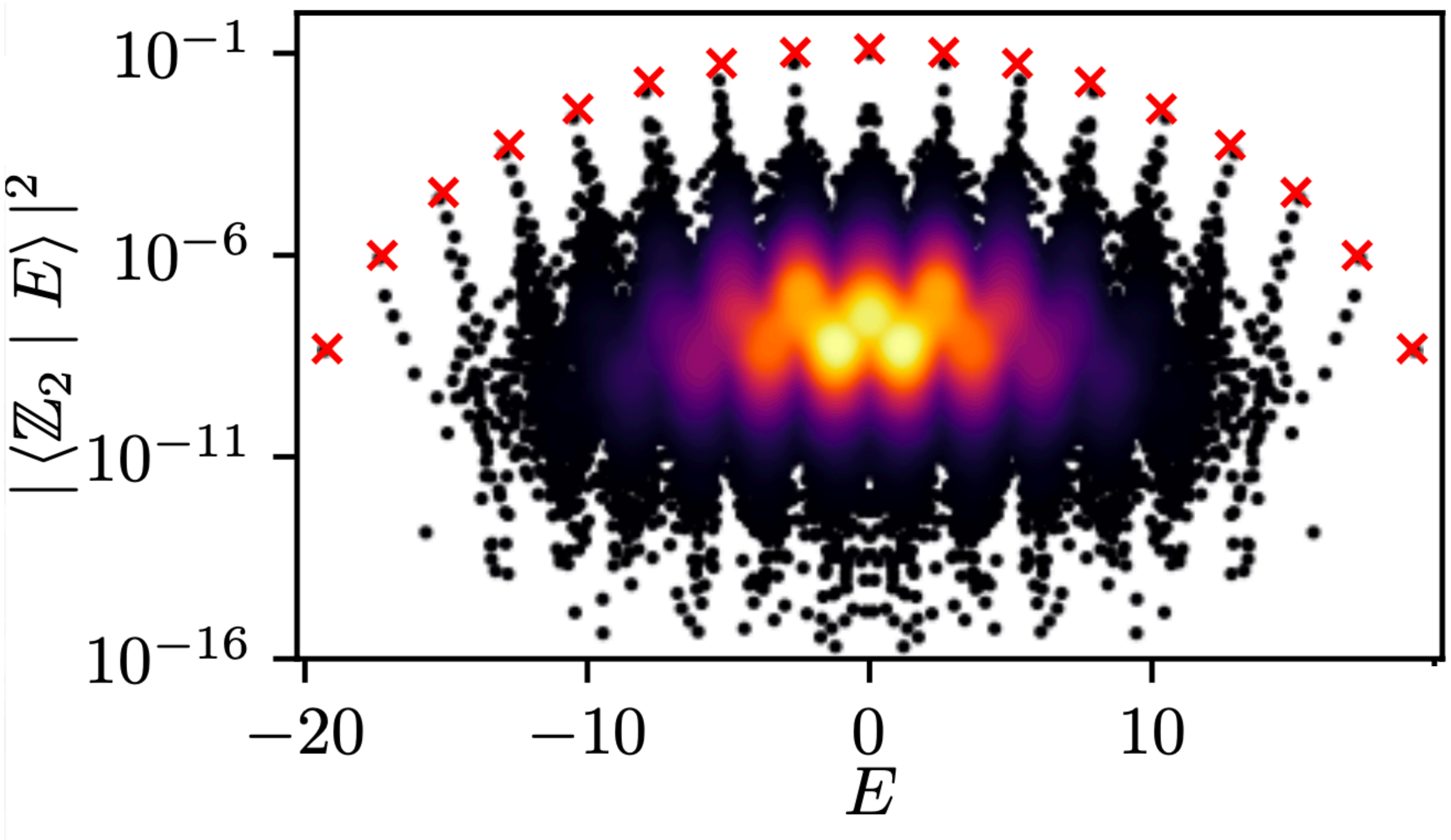
Revivals in local observables



Revivals are signature of non-ergodicity (lost of memory of the initial state)

C.J.Turner et al., *Nature Physics* (2018)14, 745–740

High overlap in the Spectrum



Quantum many body scars have abnormal overlap with Neel State

C. J. Turner et al. *Nature Physics* (2018) 14, 745–749

Quantum Many Body Scars

- High energy eigenstates that weakly break ergodicity of the system
- Show revivals in the Dynamics
- High overlap with typical states
- Low entanglement entropy
- A conjecture stated that those states arise from strong constrains
- QMBS are present in LGT too

Quantum Link Model

- Quantum link models extend LGT formulation using finite-dimensional Hilbert spaces
- For U(1) QLM with spin 1/2 is one of the simplest gauge theory in 2+1 D
- The Hamiltonian of the system is:

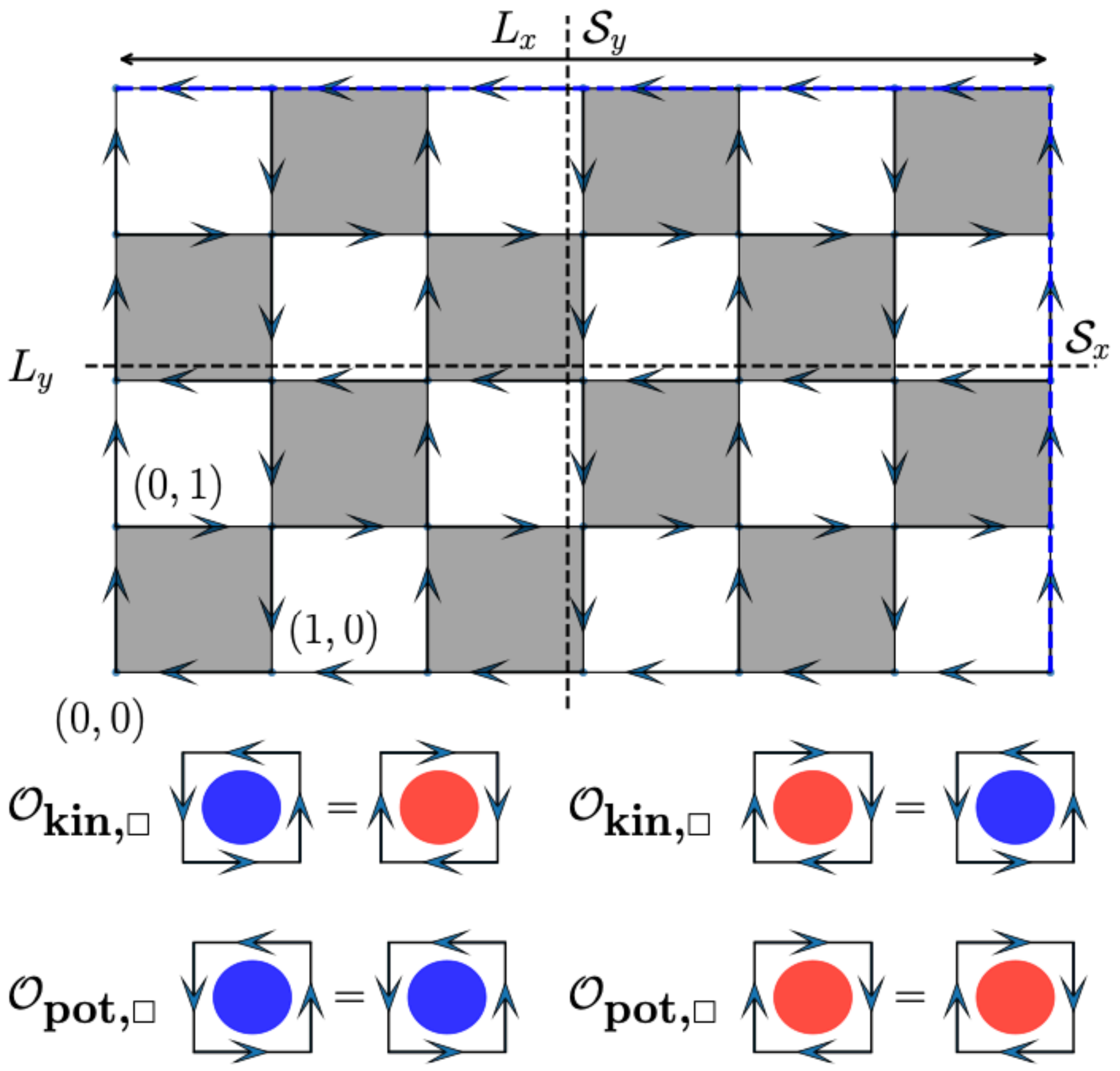
$$H = O_{kin} + \lambda O_{pot} = -\sum_{\square}(P_{\square} + P^{\dagger}_{\square}) + \lambda \sum_{\square}(P_{\square} + P^{\dagger}_{\square})^2$$

- We study the presence of quantum many body scars in the model

D. Banerjee and Arnab Sen, PRL (2021) 126, 220601

Quantum Link Model

- Smallest dimensional representation of a U(1) lattice gauge theory (1/2)
- Electric flux operator: $E_{r,\mu} = S^z$
- Gauge fields are raising (lowering) operators: $U_{r,\mu} = S^+, U^\dagger_{r,\mu} = S^-$
- Plaquette operator: $P_\square = U_{r,\mu} U_{r+\mu,\nu} U^\dagger_{r+\nu,\mu} U^\dagger_{r,\nu}$
- $O_{kin,\square} = P_\square + P^\dagger_\square$ flips a plaquette
- $O_{pot} = O^2_{kin}$ check if a plaquette is flippable
- $H_{RK} = \sum_\square O_{kin,\square} + \lambda \sum_\square O_{pot,\square}$



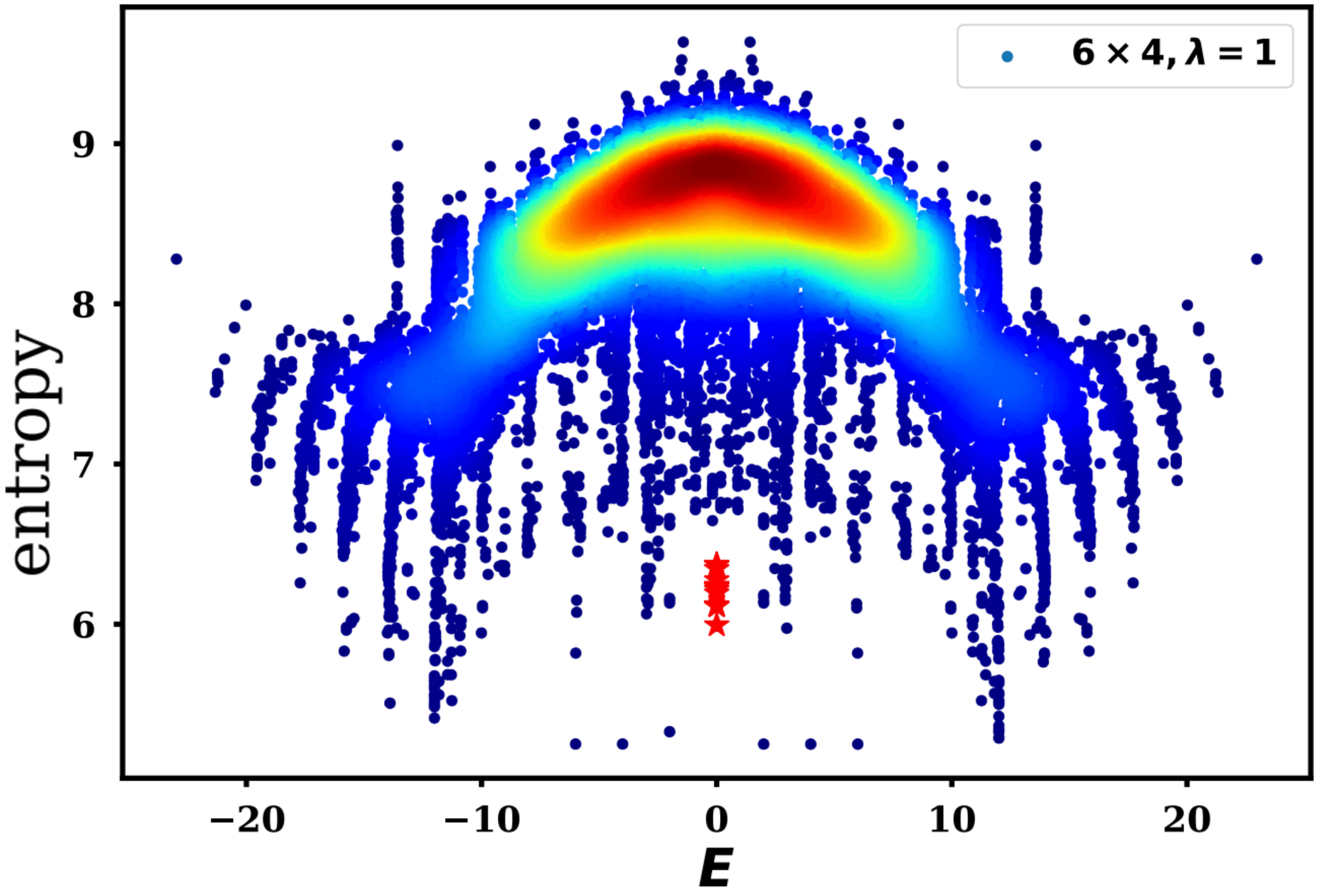
I.Sau, PS, D. Banerjee, and A.Sen, arXiv:2311.06773

Zero Modes in QLM

- Entropy of the systems

$$S = Tr[\rho \log(\rho)]$$

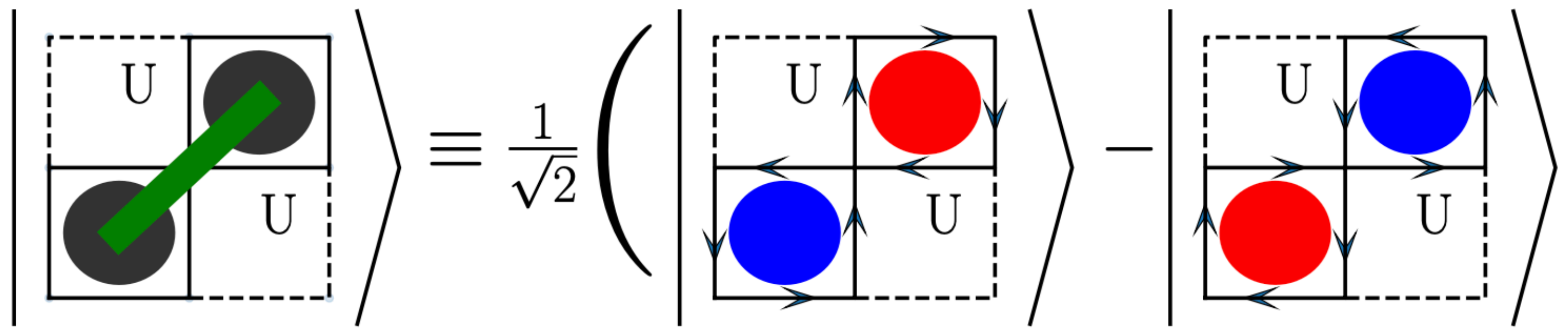
- H_{RK} has an exponentially large number of zero energy eigenstate for $\lambda = 0$
- Those are Quantum many body scars
- Lifted when $\lambda \neq 0$
- Zero modes still present

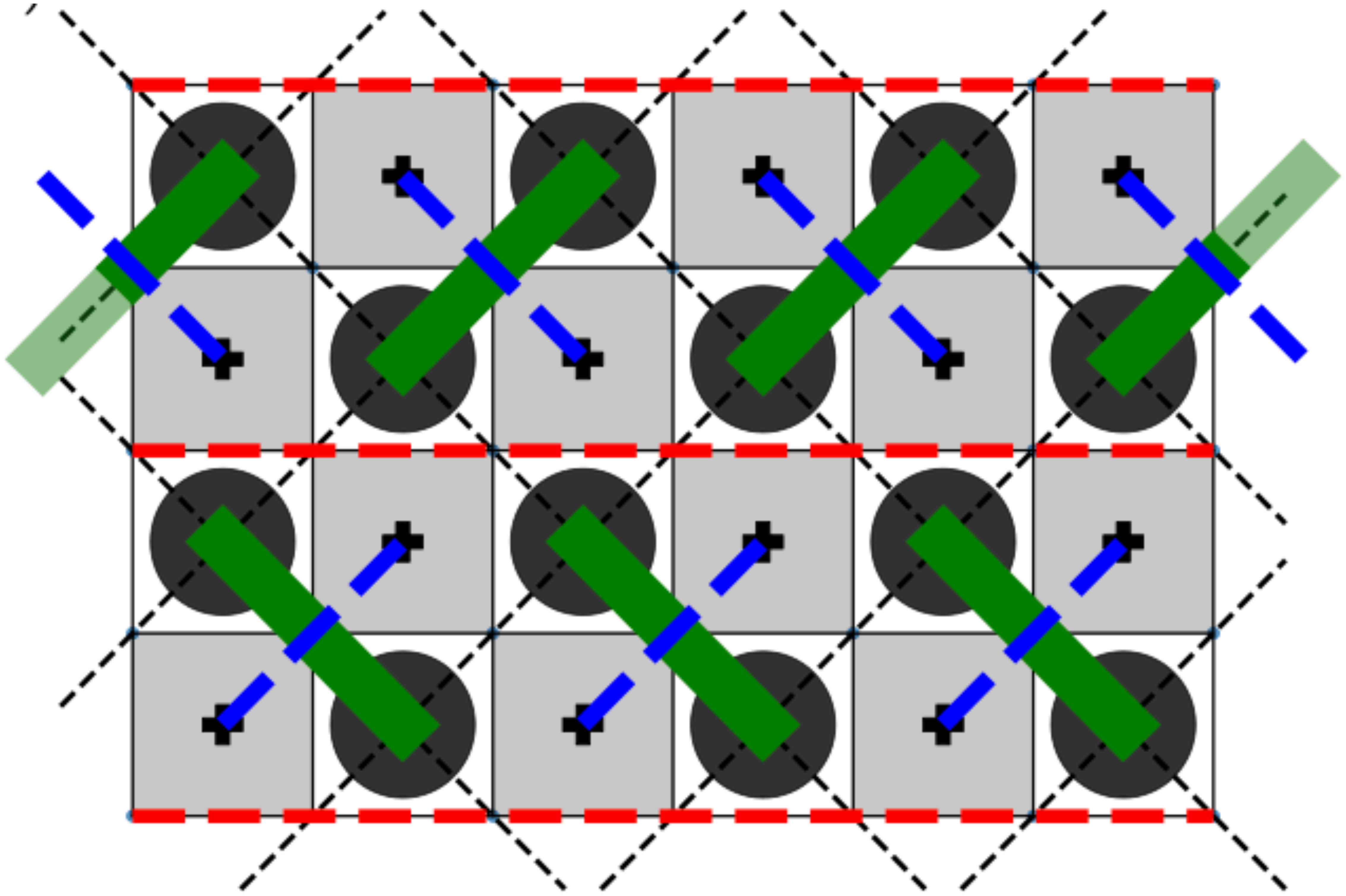


I.Sau, **PS**, D. Banerjee, and A.Sen, arXiv:2311.06773

Sublattice Scars and Singlet Description

- All scars in the model can be understood from a checker board pattern division
- Half the lattice has $< O_{pot,\square} > = 1$
- Half the lattice has $< O_{pot,\square} > = 0$
- Description of scars in a singlet fashion





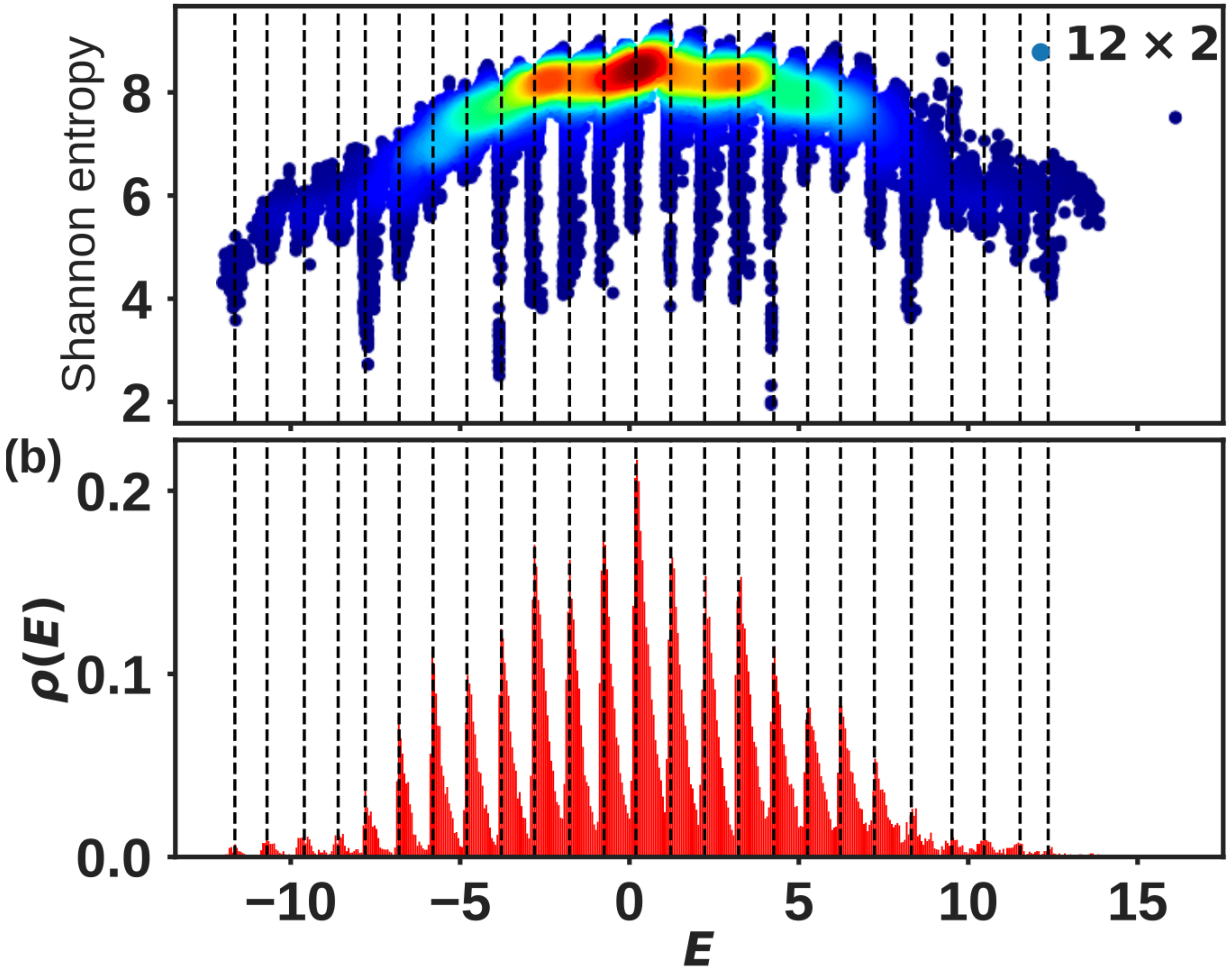
I.Sau, **PS**, D. Banerjee, and A.Sen, arXiv:2311.06773

Parent Hamiltonian for sub lattice scars

- QMBSare High energy mid spectrum states generally
- We consider the parent hamiltonian:

$$H_{LR} = \frac{1}{N_p} \sum_{\square_i, \square_j} O_{kin,\square_i} O_{kin,\square_j} + c \sum_{\square} (-1)^{(r_x + r_y)} O_{pot,\square}$$

- Ground states of H_{LR} are QMBS of the original model
- Spectrum show an extremely intricate structure
- Zero modes of H_{LR} are unchanged as a function of c



I.Sau, **PS**, D. Banerjee, and A.Sen, arXiv:2311.06773

Conclusions

Quantum Many Body Scars are Long lived quantum states

Present in various lattice gauge theories

We presented a Singlet description

Long range parent Hamiltonian

Outlooks

Extension to Non abelian case
A classification of Systems presenting QMBS?

U(1) Quantum link model Hamiltonian

QUANTUM LINK OPERATOR $U_{x,\mu} = C_{x,\mu} + iS_{x,\mu}$

PLAQUETTE OPERATOR $U_{\square} = U_{x,\mu}U_{x+\hat{\mu},\nu}U^{\dagger}_{x+\hat{\nu},\mu}U^{\dagger}_{x,\nu}$

HAMILTONIAN $\mathcal{H} = -J\sum_{\square}(U_{\square} + U^{\dagger}_{\square}) + \lambda(U_{\square} + U^{\dagger}_{\square})^2$

Rokhsar-Kivelson term

GAUGE OPERATOR $G_x = \sum_{\mu}\left(E_{x-\hat{\mu},\mu} - E_{x,\mu}\right)$

GAUGE INVARIANCE $[\mathcal{H}\,, G_x] = 0$

ZERO CHARGE CONDITION $G_x\,|\Psi\rangle = 0$

Spin representation

COMMUTATION RELATION

$$
\begin{aligned}
[E_{x,\mu}, U_{y,\nu}] &= U_{x,\mu}\delta_{\mu,\nu}\delta_{x,y} \\
[E_{x,\mu}, U^{\dagger}_{y,\nu}] &= -U^{\dagger}_{x,\mu}\delta_{\mu,\nu}\delta_{x,y} \\
[U_{x,\mu}, U^{\dagger}_{y,\nu}] &= 2E_{x,\mu}\delta_{\mu,\nu}\delta_{x,y}
\end{aligned}
$$

SU(2) REPRESENTATION

$$E_{x,\mu} = S^3_{x,\mu} \qquad C_{x,\mu} = S^1_{x,\mu} \qquad S_{x,\mu} = S^2_{x,\mu}$$

FINITE HILBERT SPACE

$$
\begin{aligned}
U_{x,\mu} &= S^x_{x,\mu} + iS^y_{x,\mu} = S^+_{x,\mu} \\
U^{\dagger}_{x,\mu} &= S^x_{x,\mu} - iS^y_{x,\mu} = S^-_{x,\mu}
\end{aligned}
$$

GAUGE OPERATOR

$$G_x = \sum_{\mu} \left(E_{x-\hat{\mu},\mu} - E_{x,\mu}\right) = \sum_{\mu} \left(S^3_{x-\hat{\mu},\mu} - S^3_{x,\mu}\right)$$

[S Chandrasekharan and U.-J Wiese, Quantum link models: A discrete approach to gauge theories. Nuclear Physics B, Volume 492, Issues 1–2, 12 May 1997, Pages 455-471]

Eigenstate Thermalisation Hypothesis (ETH)

- A small subsystem of an isolated, interacting quantum many-body system is described by a thermal density matrix after a long time evolution, irrespectively of the initial non-equilibrium state.
- The ETH states that individual eigenstates of quantum-ergodic systems act as thermal ensembles, thus the system's relaxation does not depend strongly on the initial conditions
- Some systems violate this paradigm of quantum ergodicity and exhibit a long-time behaviour dependent on the initial state.
- Examples of such non-ergodic systems include integrable systems, and many-body localized phases in the presence of quenched disorder
- In the single-particle case, quantum scars correspond to wavefunctions that concentrate in the vicinity of unstable periodic classical trajectories.
- scarred many-body bands give rise to a new universality class of quantum dynamics, opening up opportunities for the creation of novel states with long-lived coherence in systems that are now experimentally realizable.

Weak ergodicity breaking from quantum many-body scars C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn & Z. Papić
Nature Physics volume 14, pages745–749 (2018