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Luca Tagliacozzo

S. Carignano (BSC), C. Ramos (UB) arXiv:2307.11649
M. Piani (evolutionQ) J. Surace (ICFO) arXiv:1810.01231 PRB 2019
M. Frias-Perez (MPQ) MC. Bañuls (MPQ) arXiv:2308.04291

On temporal entropies
Tensor networks algorithms and their cost for out-of-equilibrium dynamics

Plan de Recuperación, Transformación y Resiliencia

PGC2018-095862-B-C22
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TED2021-130552B-C22

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN

Can you break the entanglement barrier in 1D?

Yes if you are interested in local equilibration by trading long range correlations with mixture

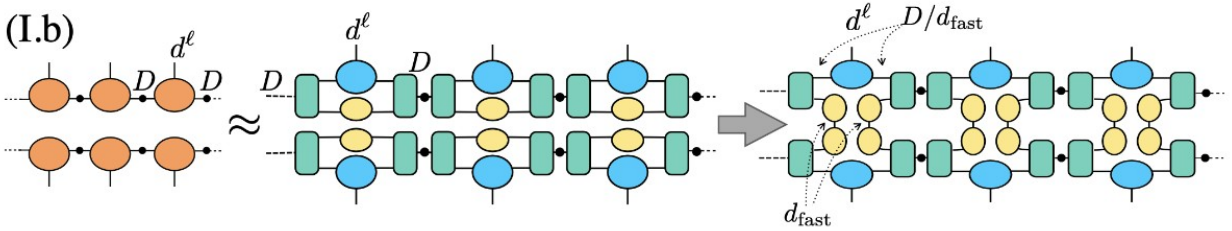
M Frias-Perez LT and MC Bañuls , arXiv:2308.04291 (PRL)

- Not by simple space-time duality and causal cones

S. Carignano C. Ramos L.T.
arXiv:2307.11649 (PR?)

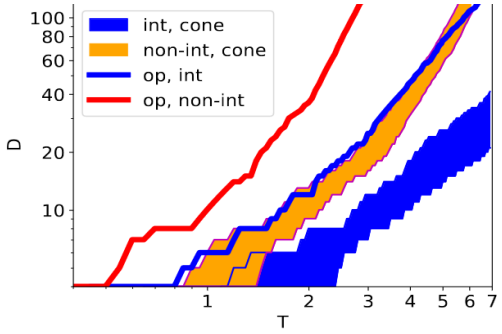
- TN algorithm for local equilibration with finite computational cost

M Frias-Perez LT and MC Bañuls , arXiv:2308.04291



- Upper bound the scaling of the temporal entanglement

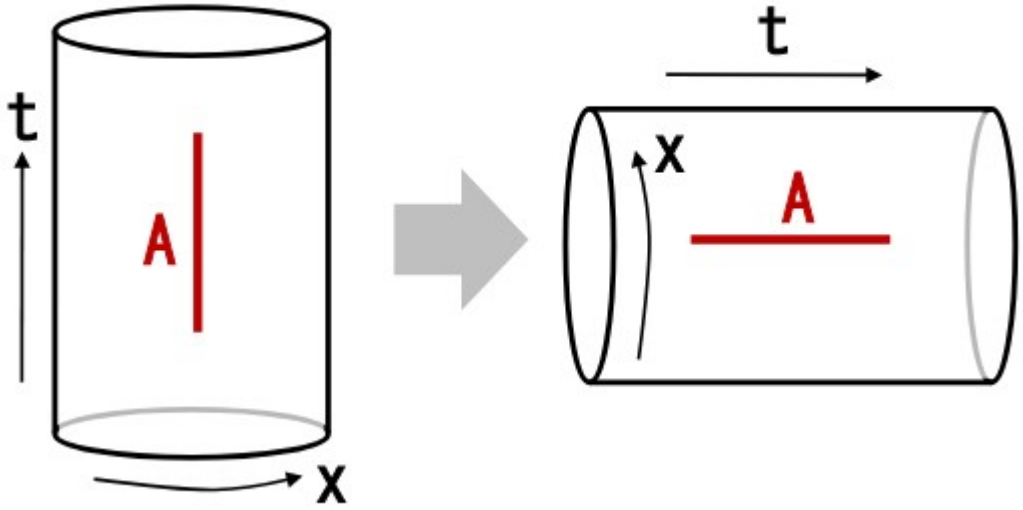
S. Carignano C. Ramos L.T arXiv:2307.11649



Temporal entropies & CFTs

- New quantifier of complexity of the out of equilibrium dynamics

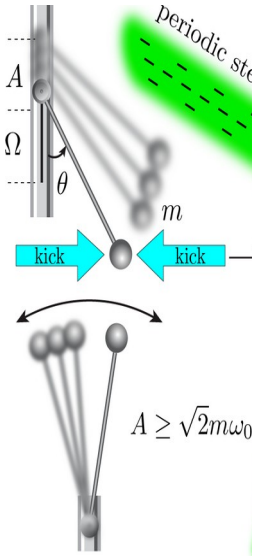
S. Carignano, L.T arXiv:2401.XXXX



Classical:

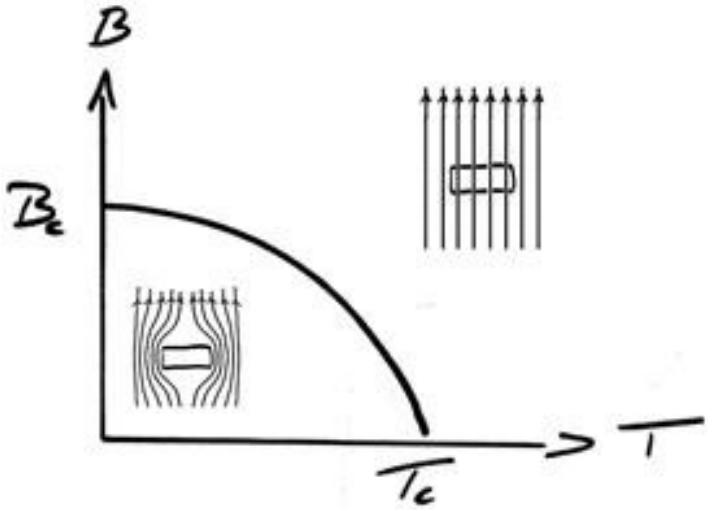
Kapitsa pendulum

Bukov

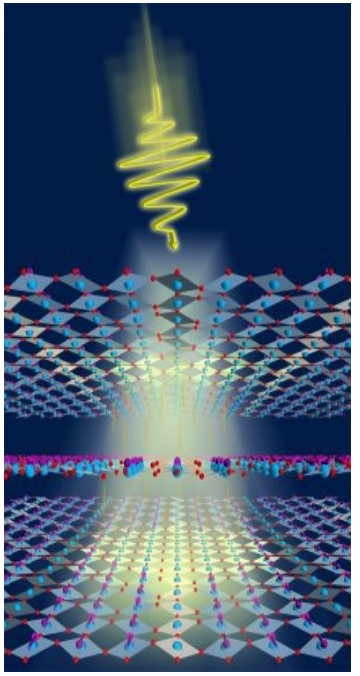


Quantum:

Light induced superconductor



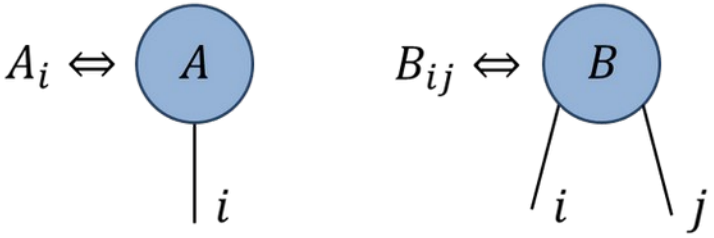
Cavallieri

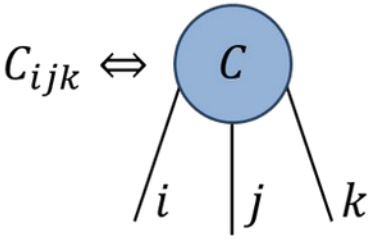


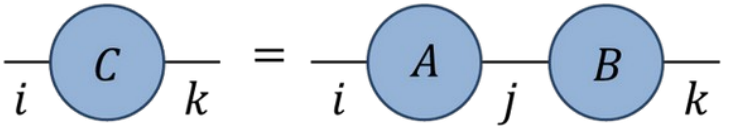
Tensor Networks

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{m1} & \cdots & B_{mn} \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} \begin{bmatrix} C_{111} & \cdots & C_{1n1} \\ \vdots & \ddots & \vdots \\ C_{m11} & \cdots & C_{mn1} \end{bmatrix}^{l}_{2} \end{bmatrix}^{l}_{3}$$

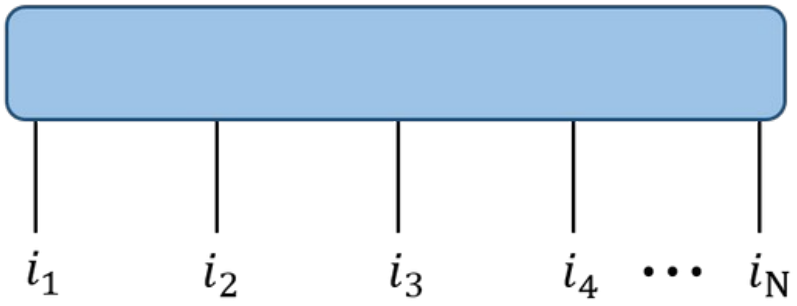




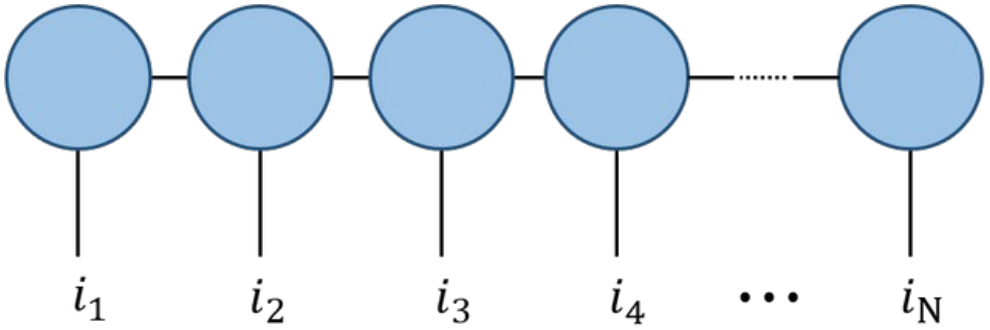


$$\Updownarrow$$

$$C_{ik} = \sum_j A_{ij} B_{jk}$$



$\Rightarrow$



Entanglement barrier

$$H = \sum_i h_i$$

$$U(T) = \exp\left(-iHT\right)$$

$$O(T) = U^\dagger(T) O_i U(T)$$

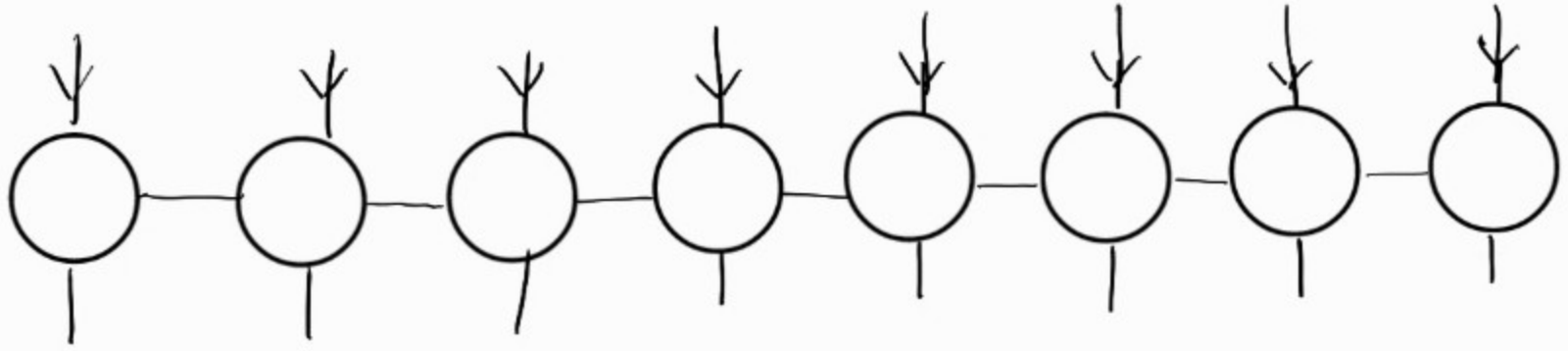
$$\langle \psi_0 | O(T) | \psi_0 \rangle$$

$$U(T) = \exp(-iHT)$$

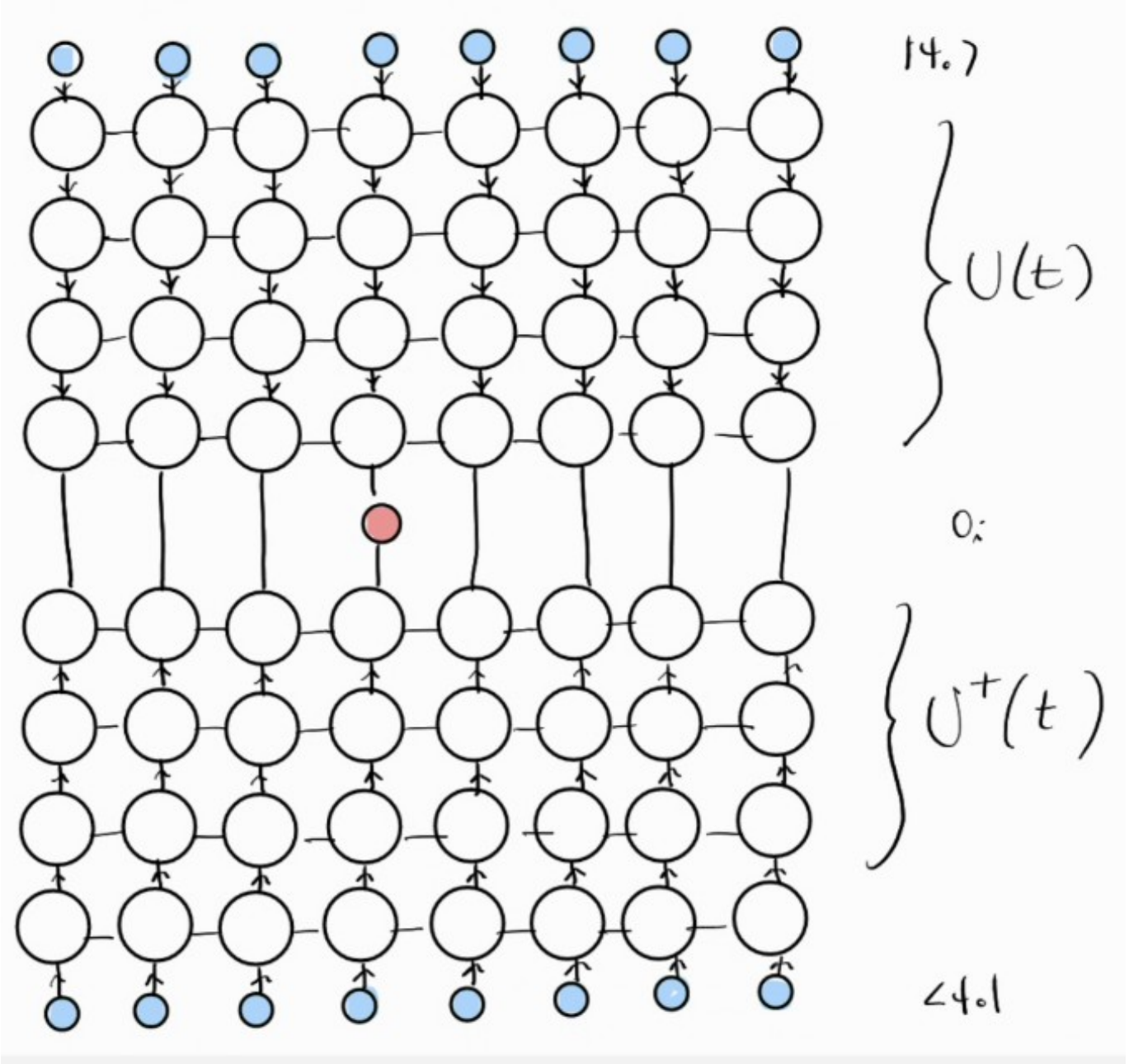
$$\delta t = t/N \ll 1$$

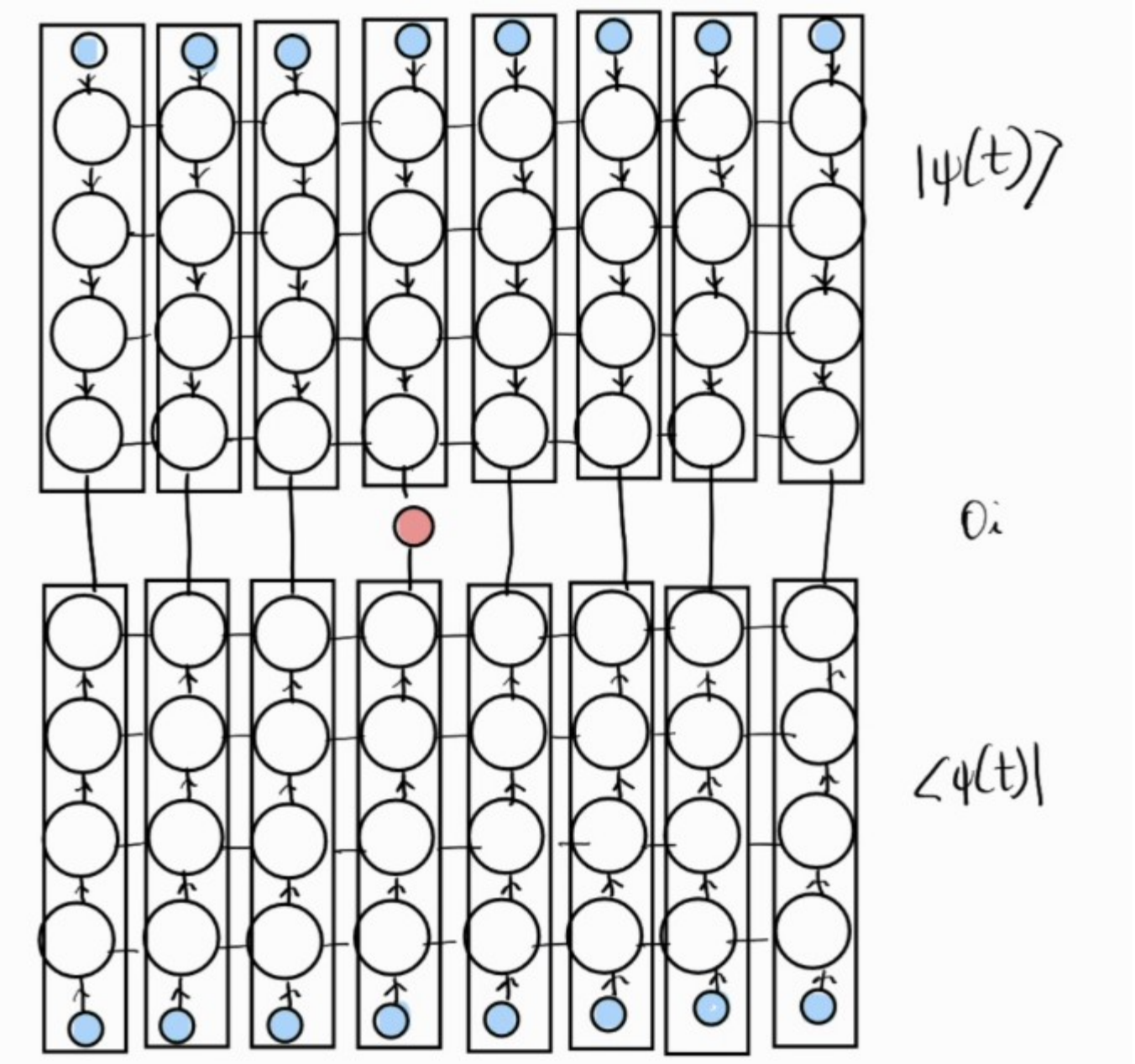
$$U(T) \simeq U(\delta t)^N$$

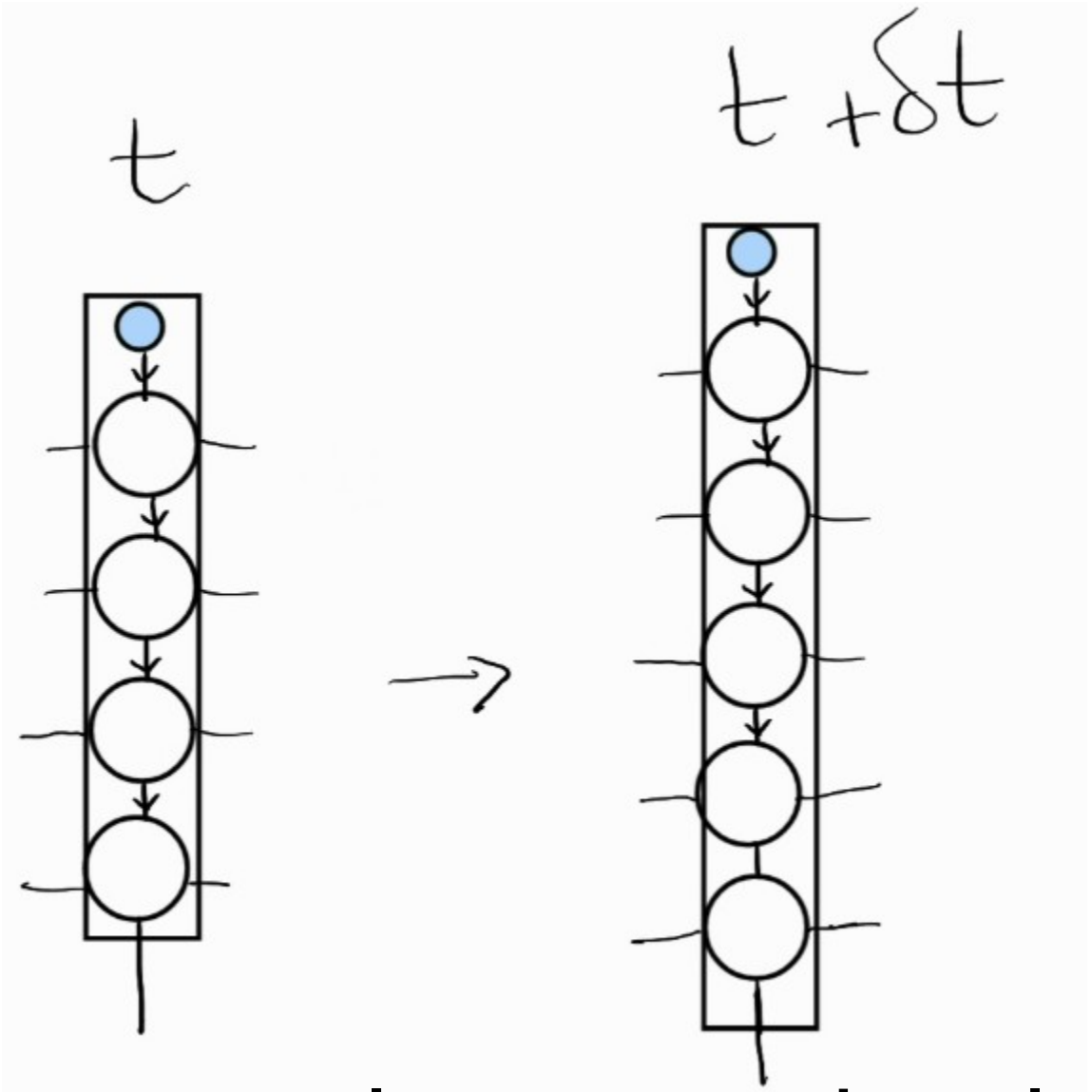
$U(\delta t) =$



$$\langle \psi_0 | O(T) | \psi_0 \rangle$$

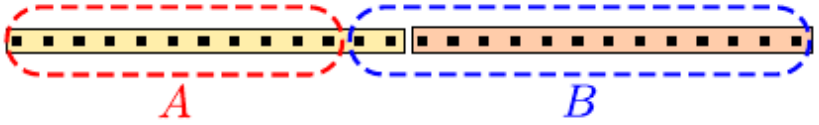



|ψ(t)⟩
Oi
⟨ψ(t)|



What is the actual computational cost?

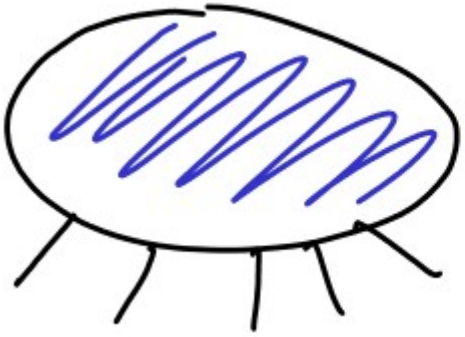
What is Entanglement

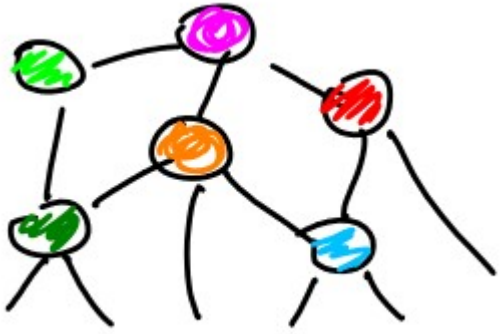


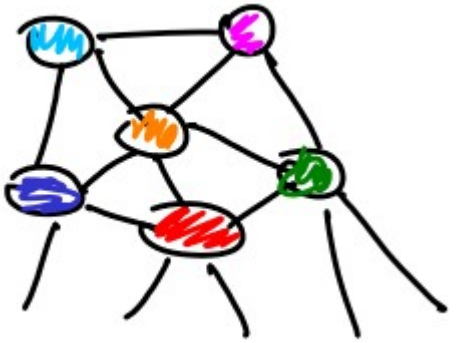
$$|\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

Tensor Networks and Product States

$$|\psi\rangle_A \otimes |\psi\rangle_B$$



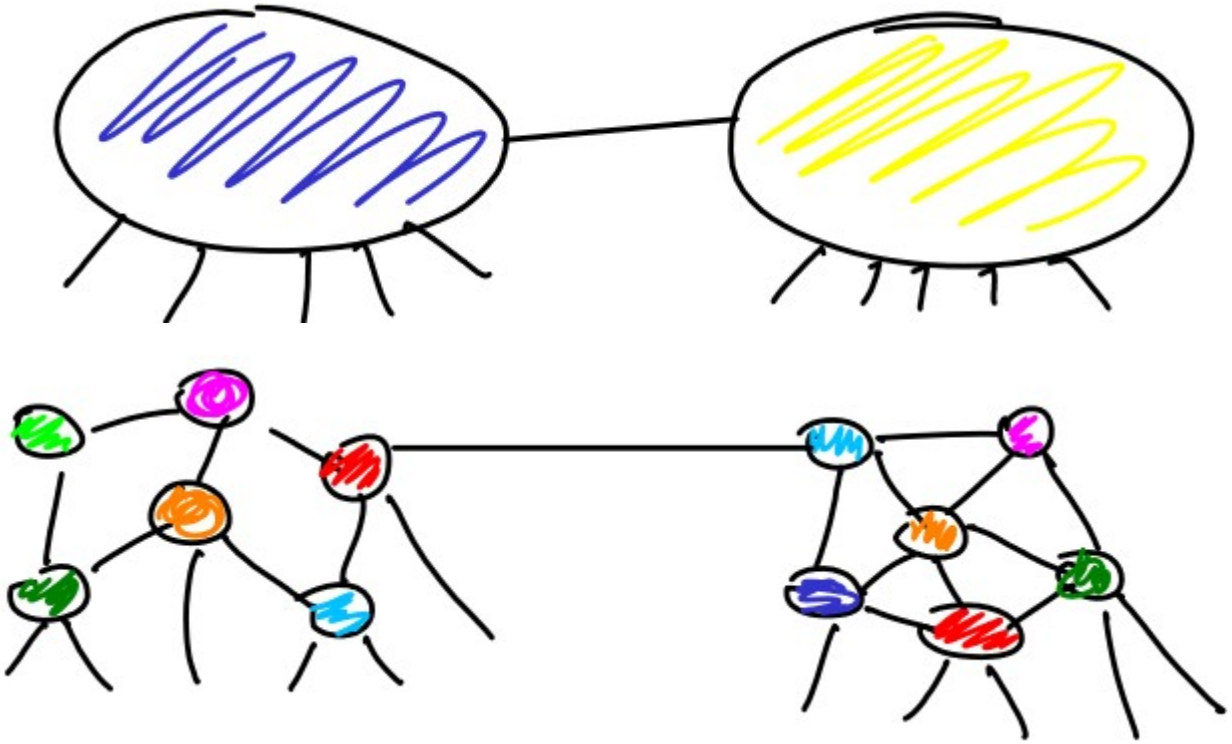


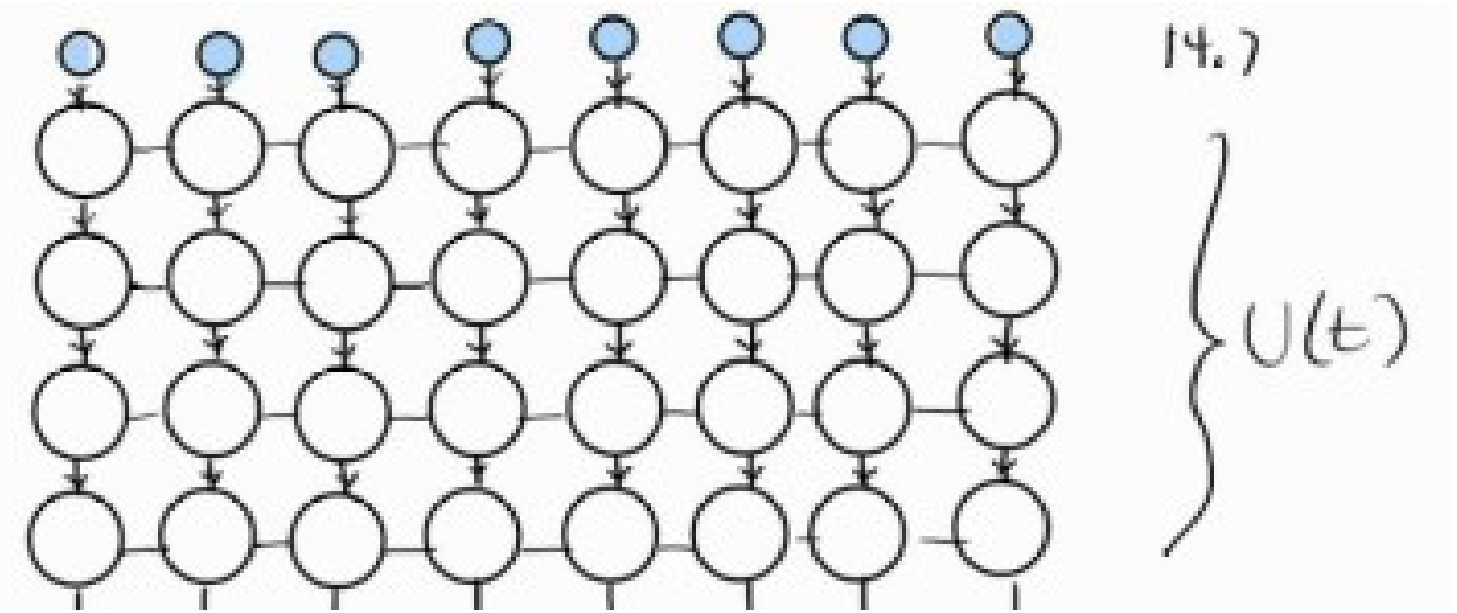


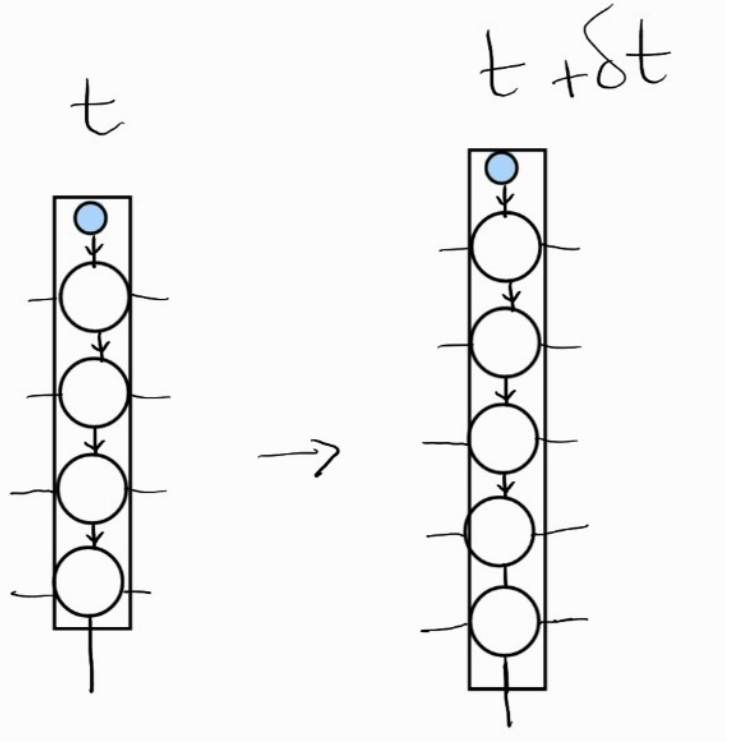
Tensor network entangled state

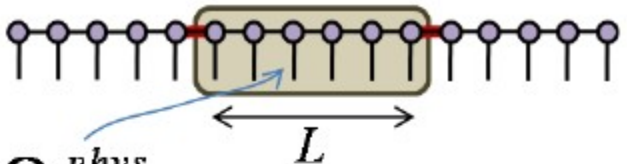
$$|\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

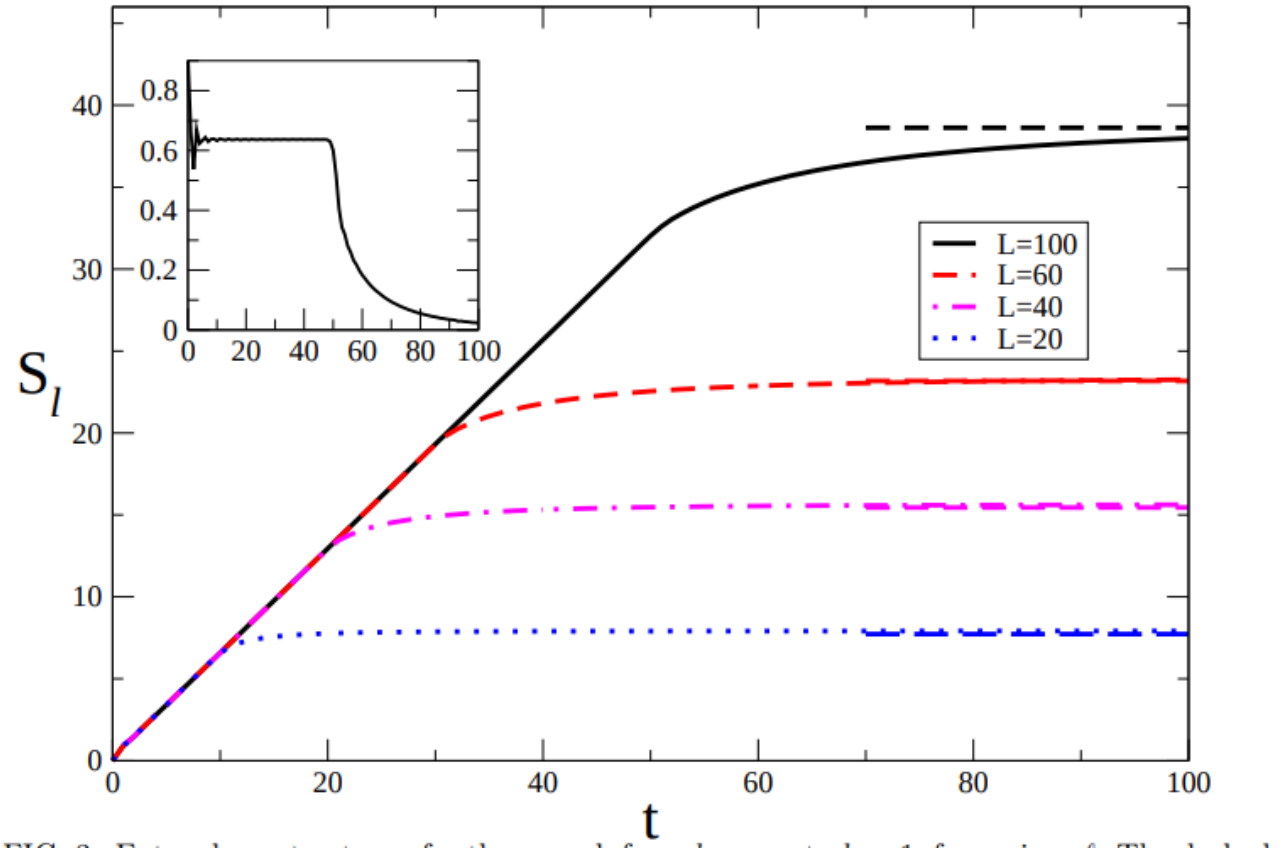
$$|\psi\rangle_{AB} = \sum_i c^i |\psi_i\rangle_A \otimes |\psi_i\rangle_B$$




14.)
U(t)


t
t +δt





from Calabrese Cardy 2005

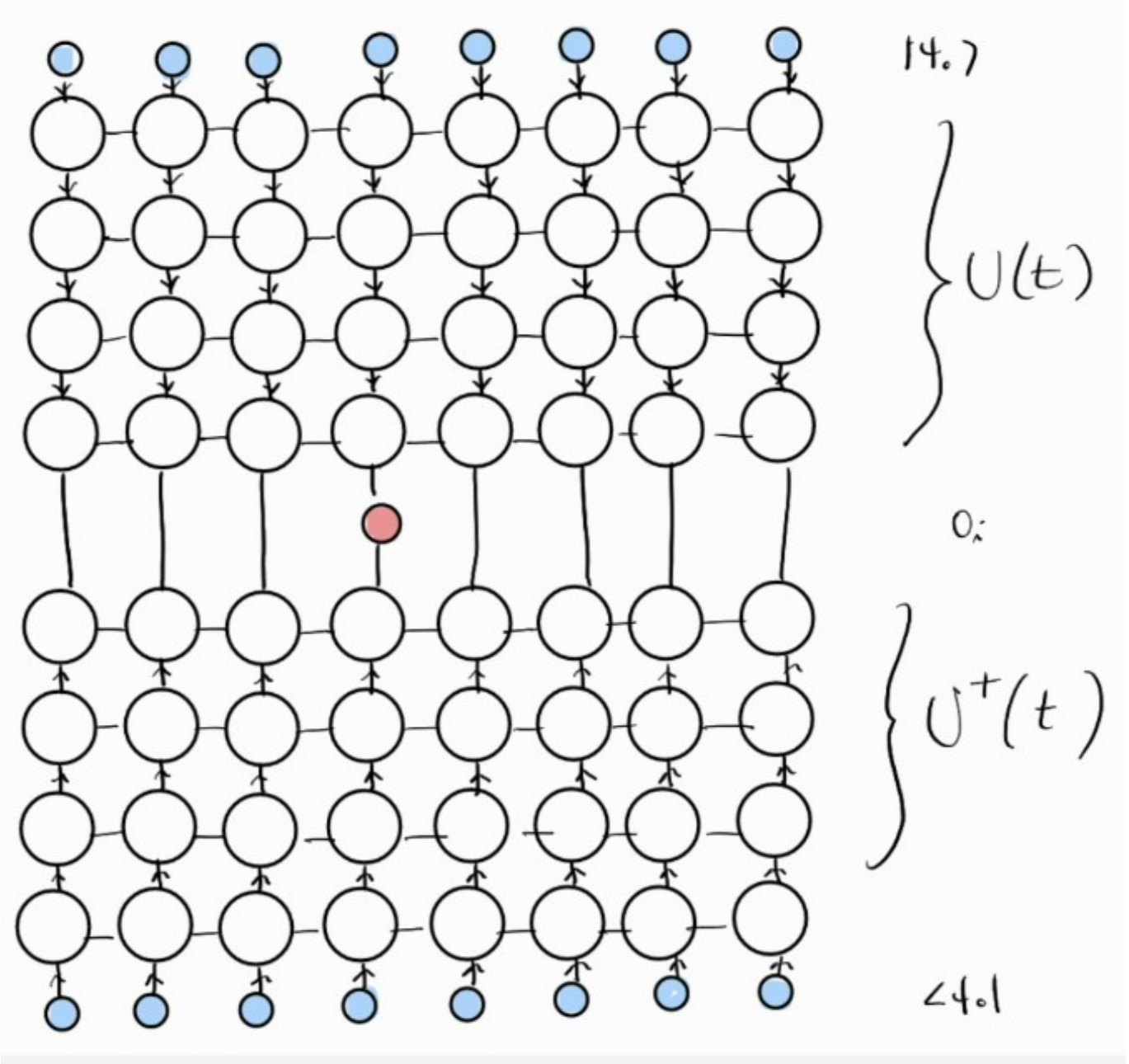
$$S \leq n_{AB} \log(\chi)$$

$$\chi^{n_{AB}} \geq \exp S \propto \exp(t)$$

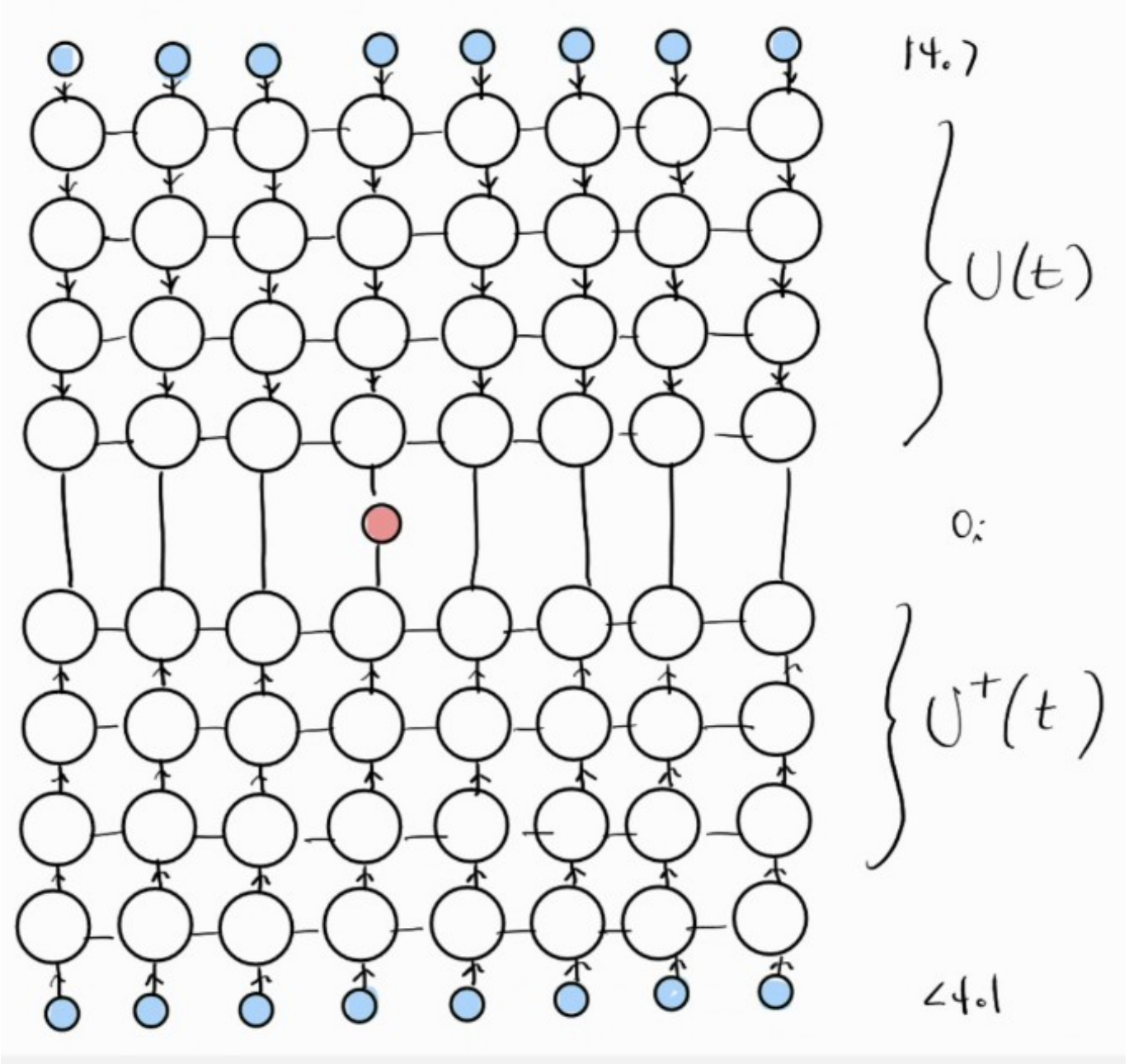
On temporal entanglement

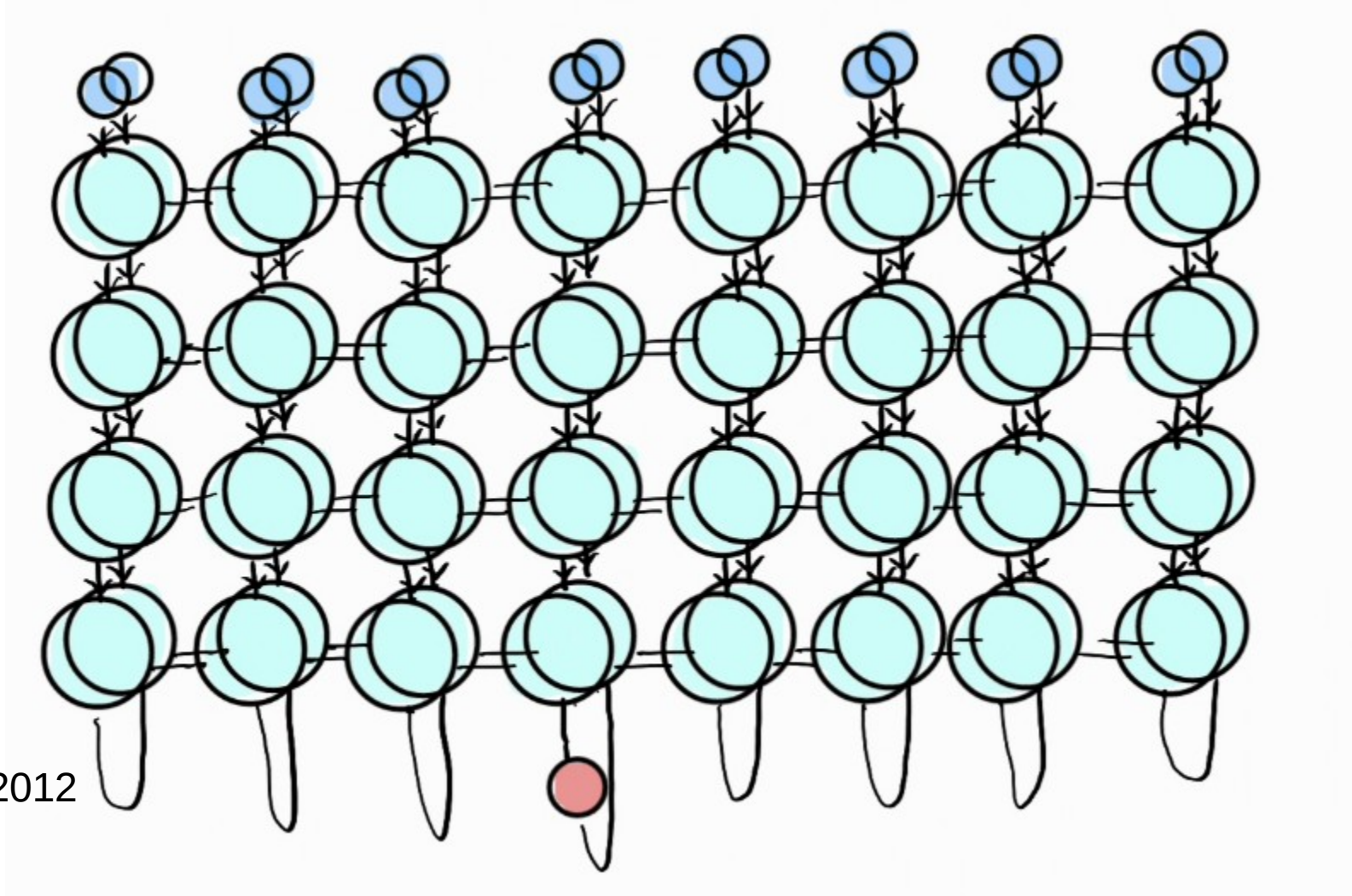
with S. Carignano arXiv:2401.xxxx

$$\langle \psi_0 | O(T) | \psi_0 \rangle$$

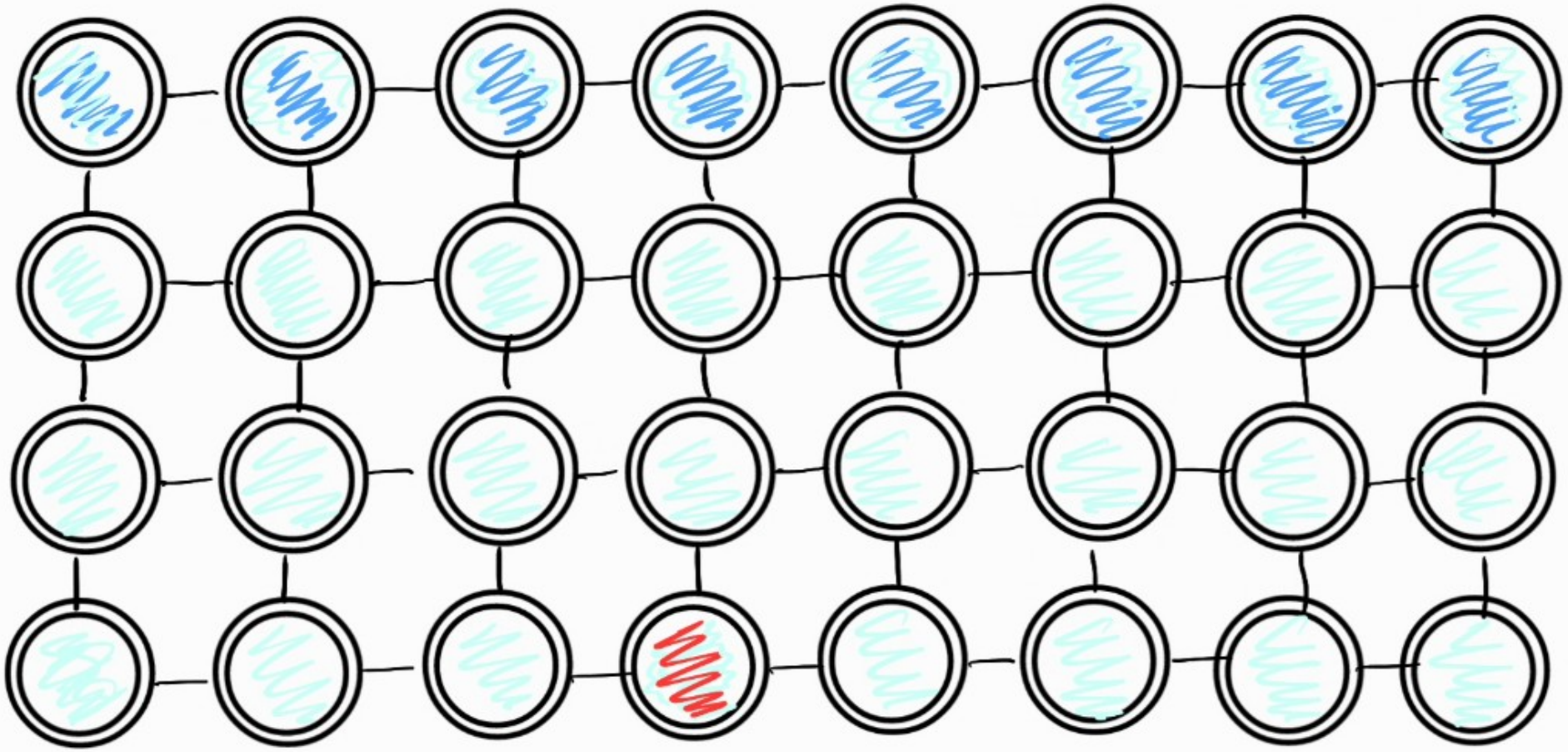


$$\langle \psi_0 | O(T) | \psi_0 \rangle$$

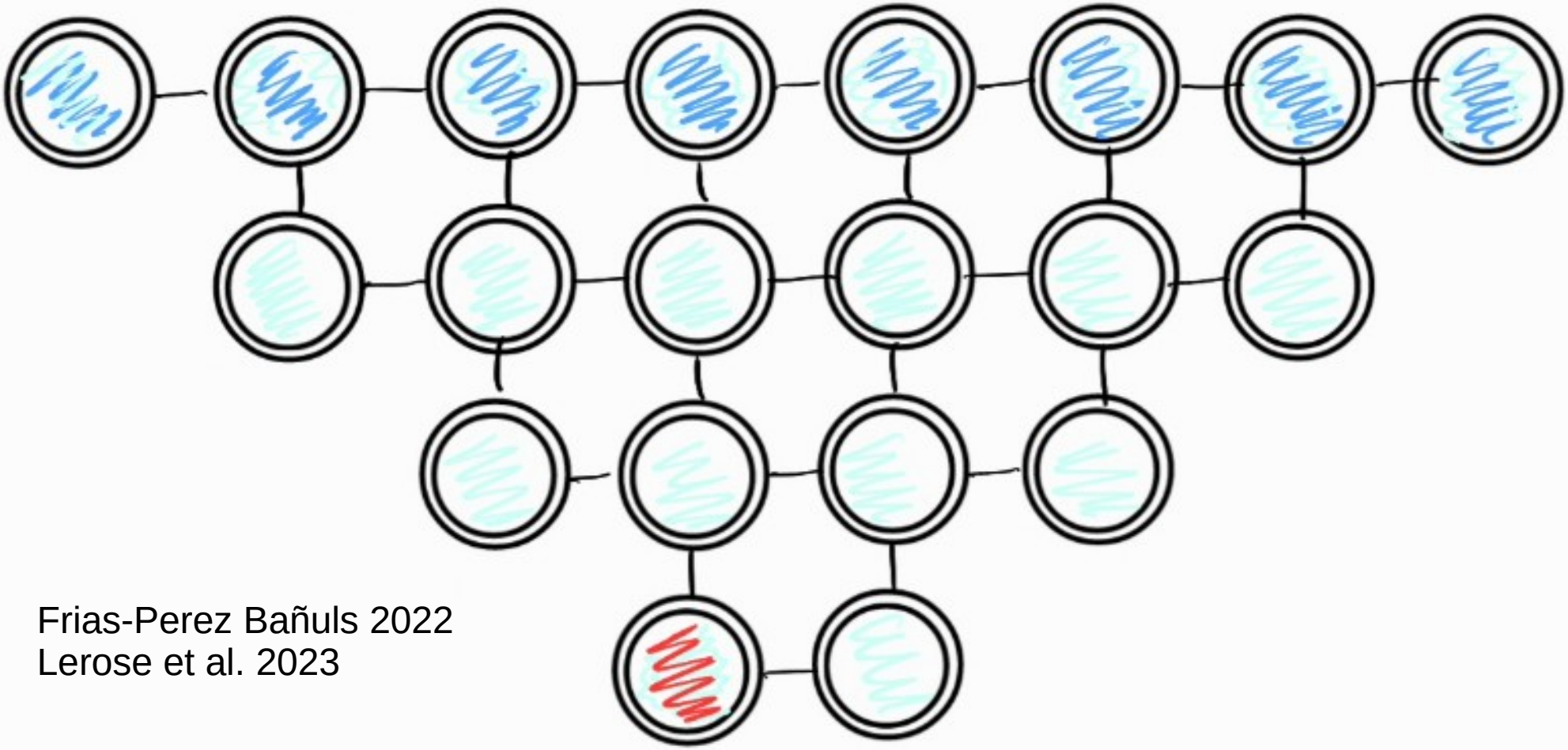




Bañuls et al. 2009
Muller Hermes et al 2012
Hastings et al. 2015
Tirrito et al (LT) 2018
Lerose et al 2021
Giudice et al 2021….

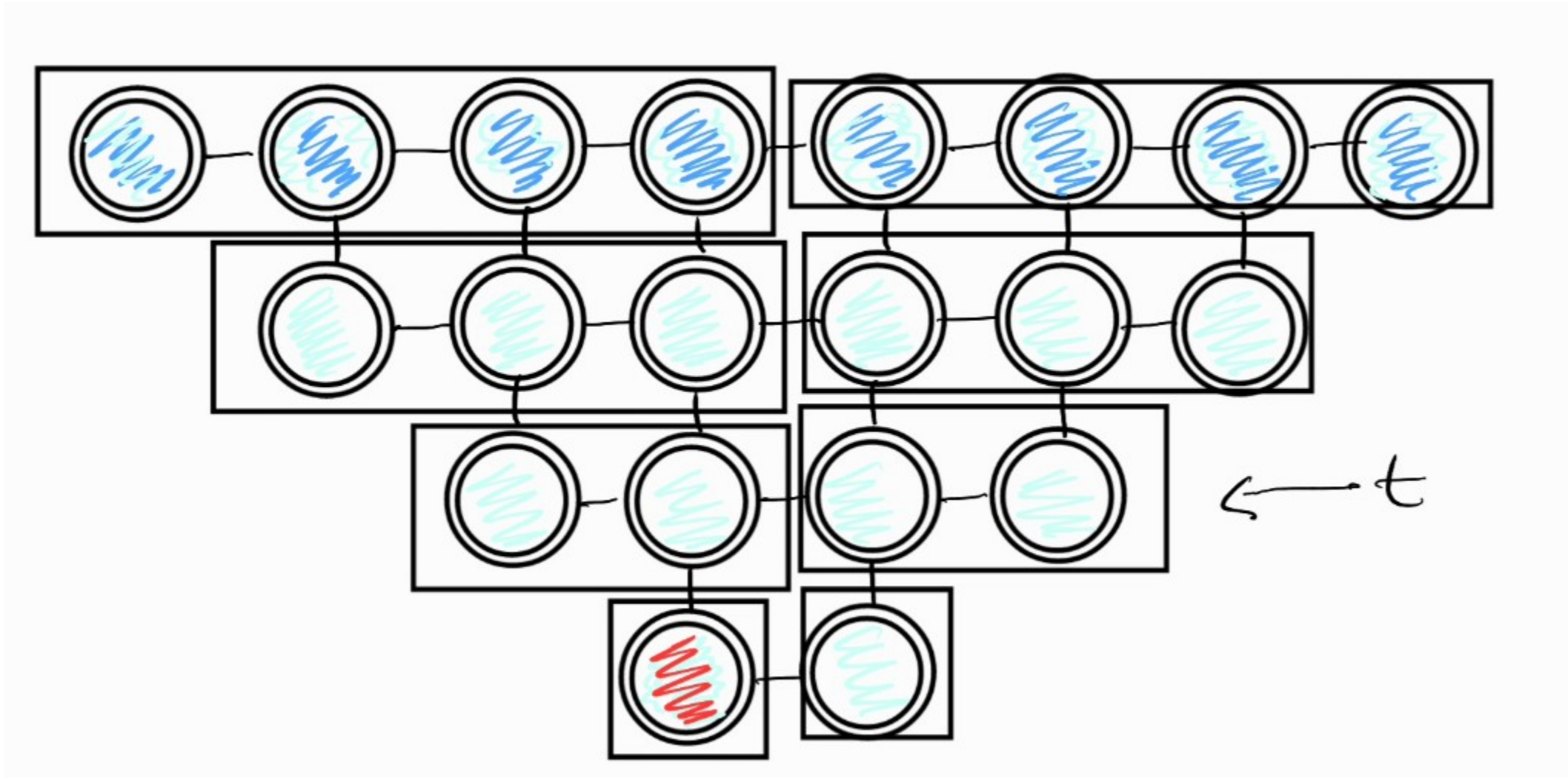


$$\langle 4_0 | U^{\dagger}(t)\, O_i\, U(t) | 4_0 \rangle$$



Frias-Perez Bañuls 2022
Lerose et al. 2023

$$\langle \psi_0 | \, U^{\dagger}(t) \, O_i \, U(t) \, | \psi_0 \rangle$$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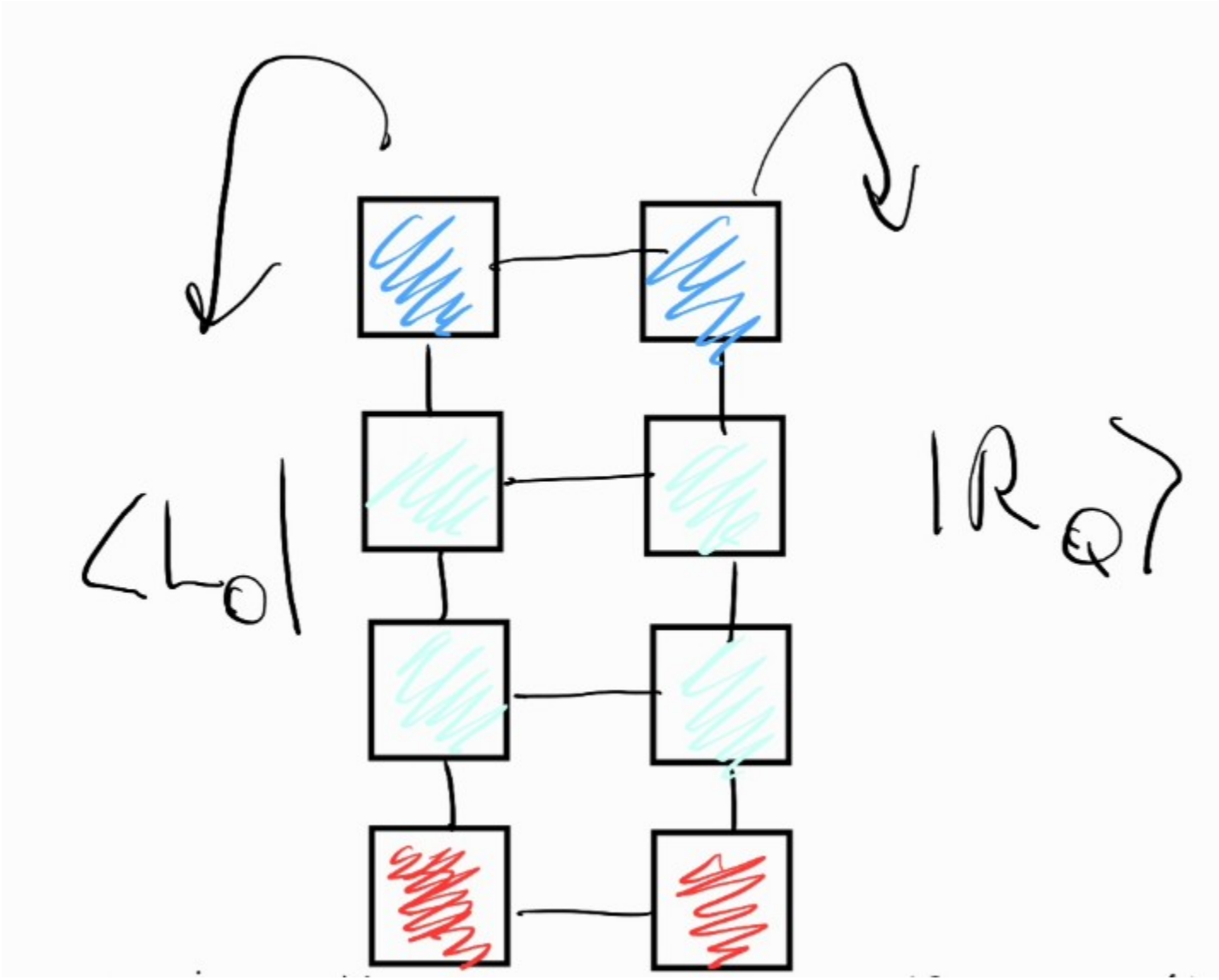
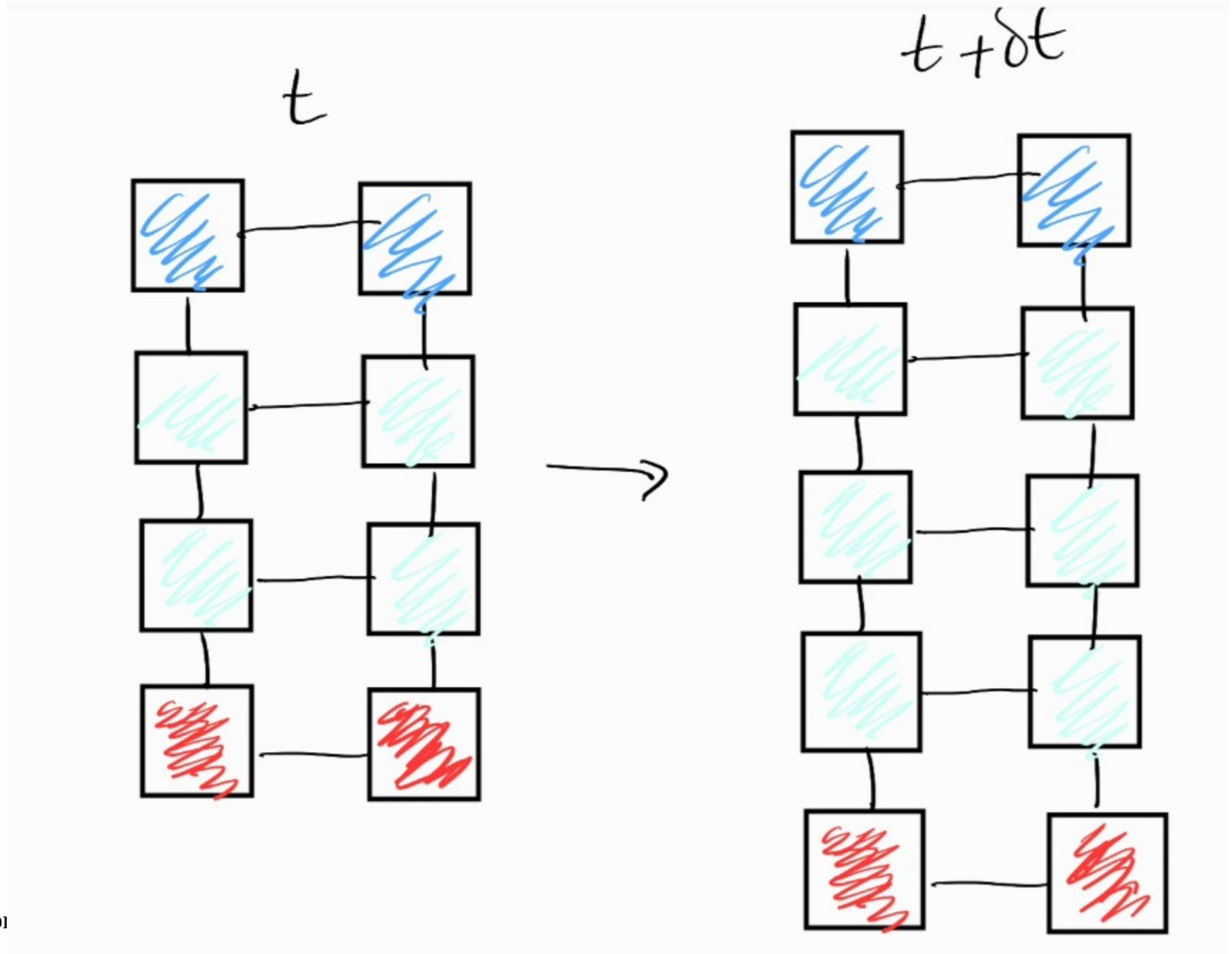


temporal MPS, tMPS



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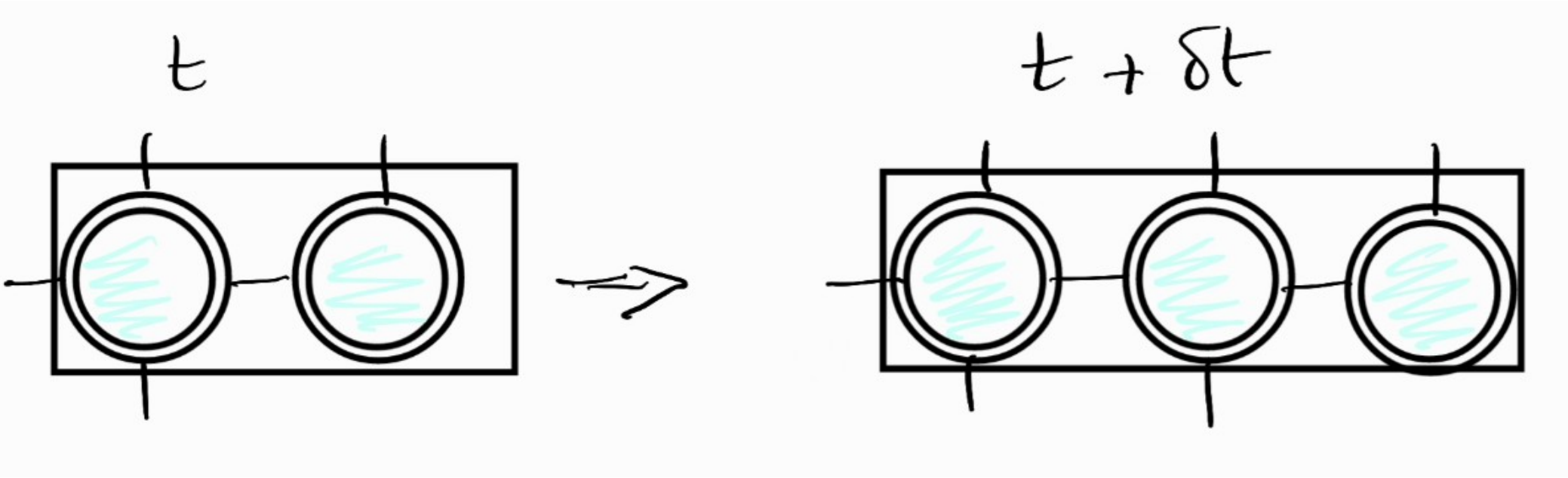
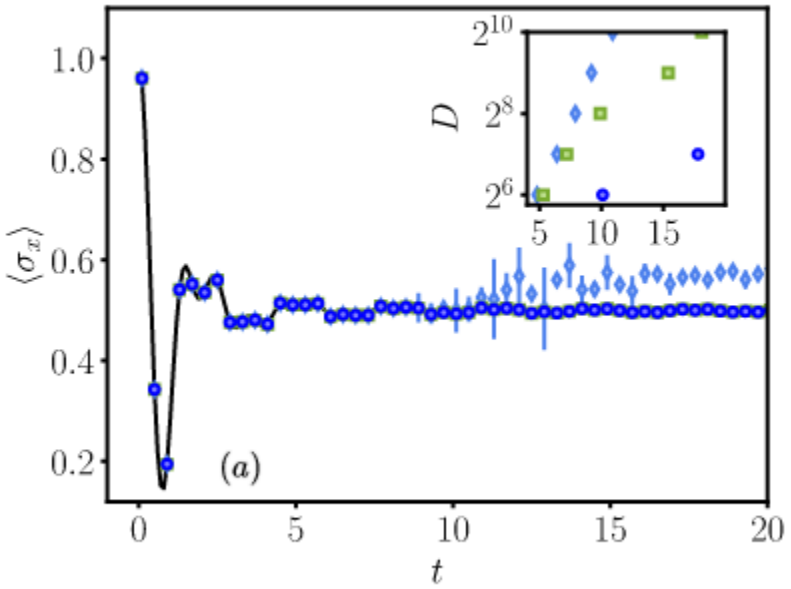
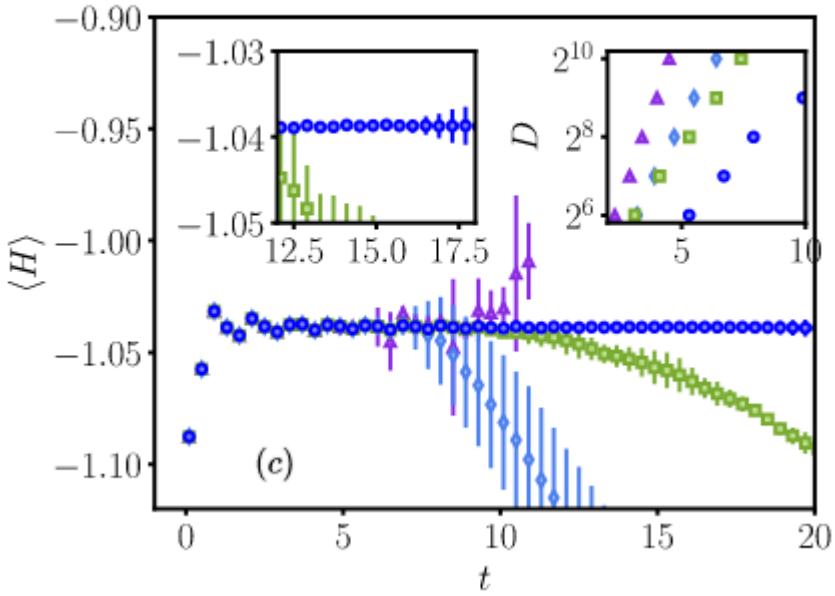


What is the actual computational cost?

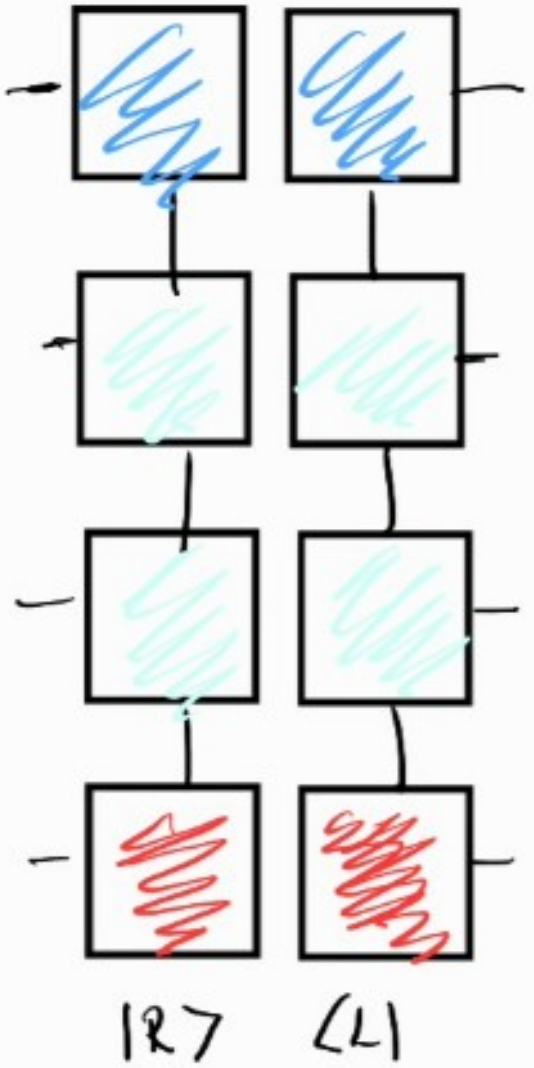




from Frias-Perez Bañuls 2022

Transition Matrix

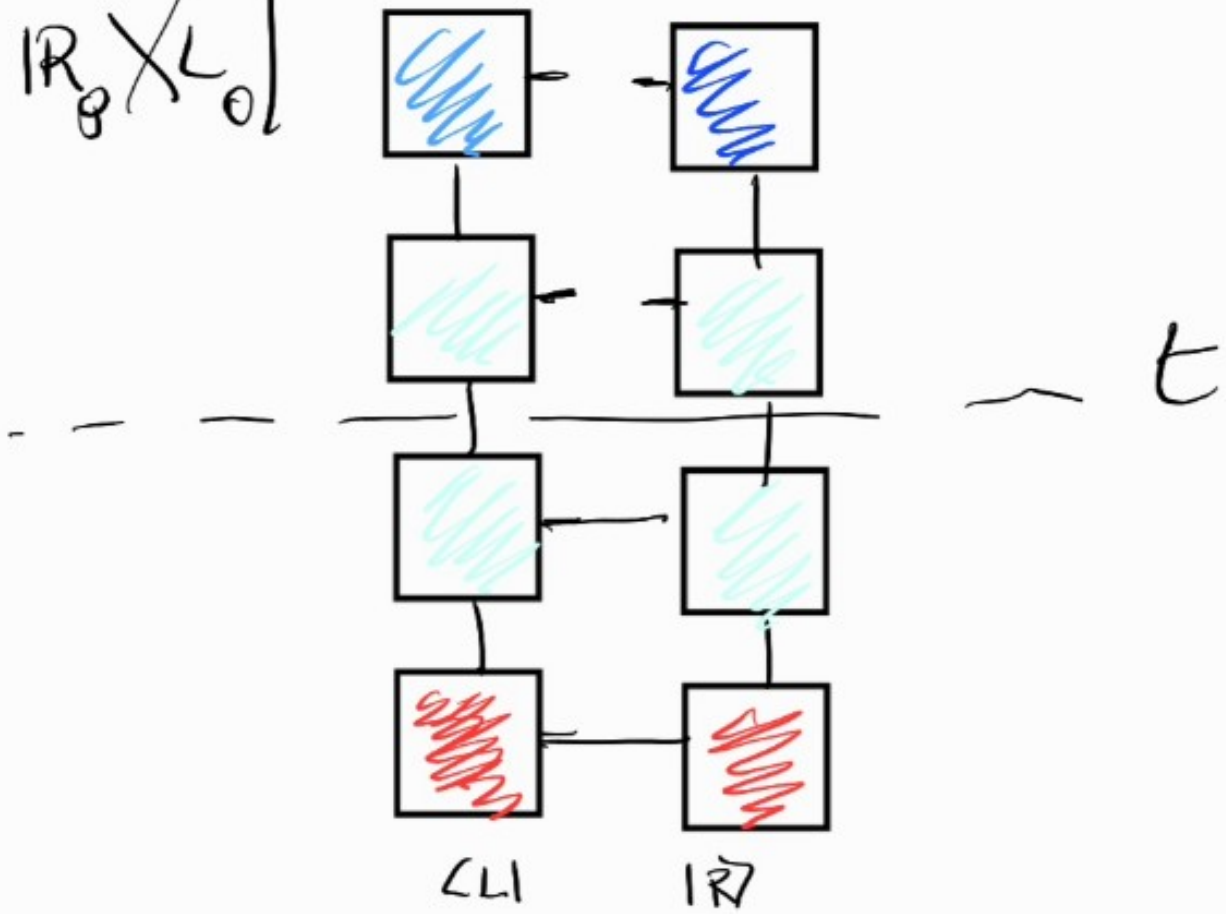
$$\tau_{|R_{Pitt} X L_{OC}|} =$$



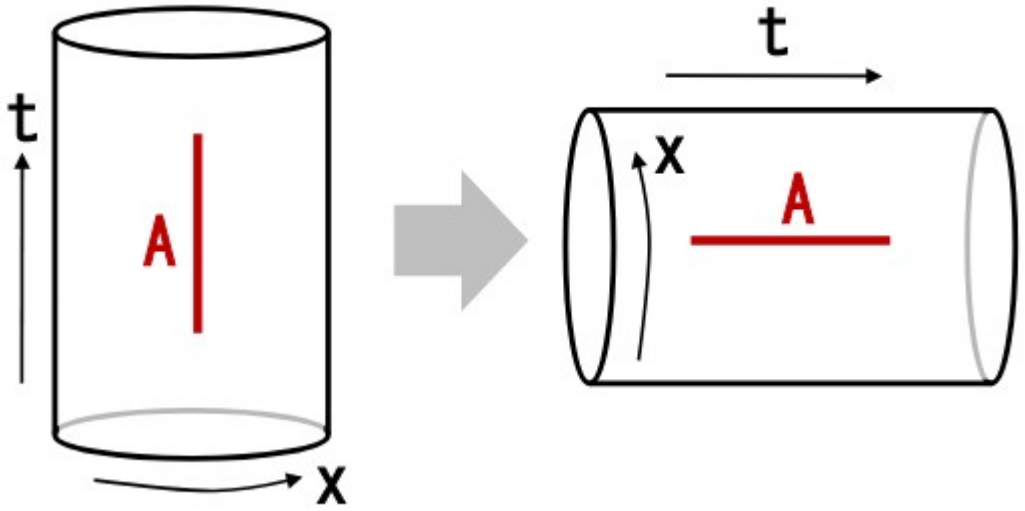
To

Generalized entropies

$$\tau^t = \mathrm{tr}_{(T-t)}\, \tau\, |R_0\rangle\langle L_0|$$



CFT predictions for temporal entropies

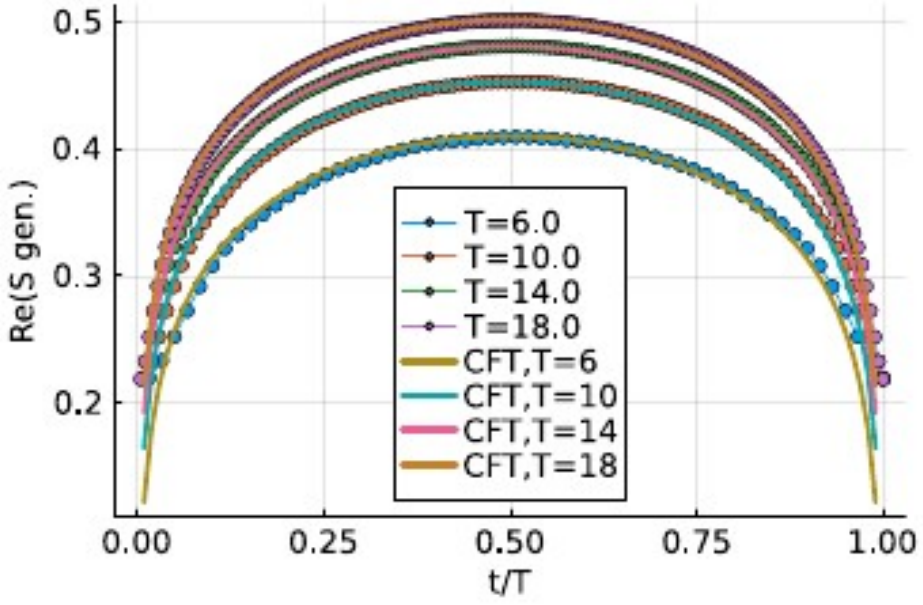


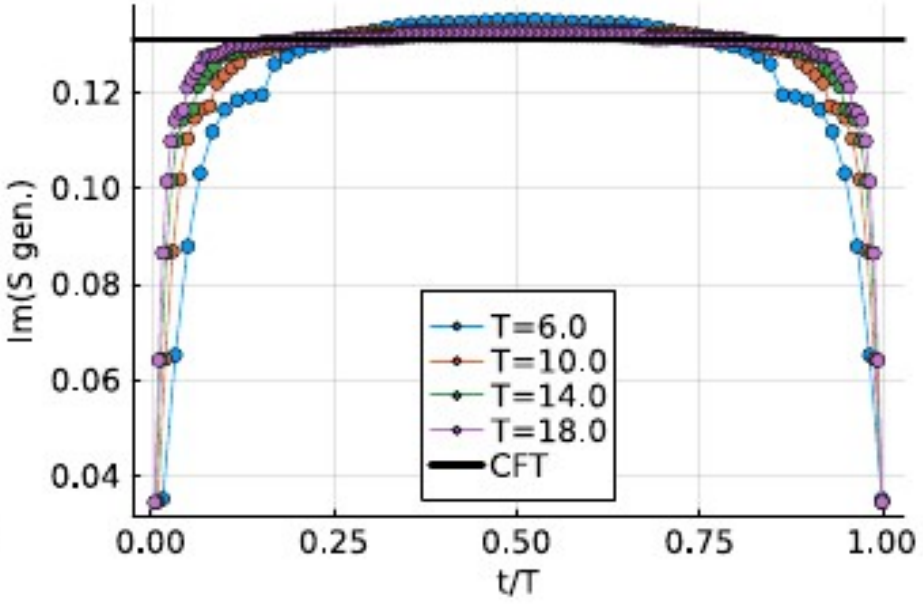
$$S_A = \frac{c_{\mathrm{AdS}}}{3} \log\left(\frac{T_0}{\epsilon}\right) + \frac{i\pi c_{\mathrm{AdS}}}{6}.$$

Doi et al (Takayanagi) PRL2023

Numerical results, Ising

$$S_{gen} = s_0 + \frac{i\pi c}{6} + \frac{c}{3}\log\left[\frac{2T}{\pi}\sin\left(\frac{\pi t}{T}\right)\right] \qquad H(g) = -\sum\left[\sigma_x^i\sigma_x^{i+1} + g\sigma_z^i\right],$$

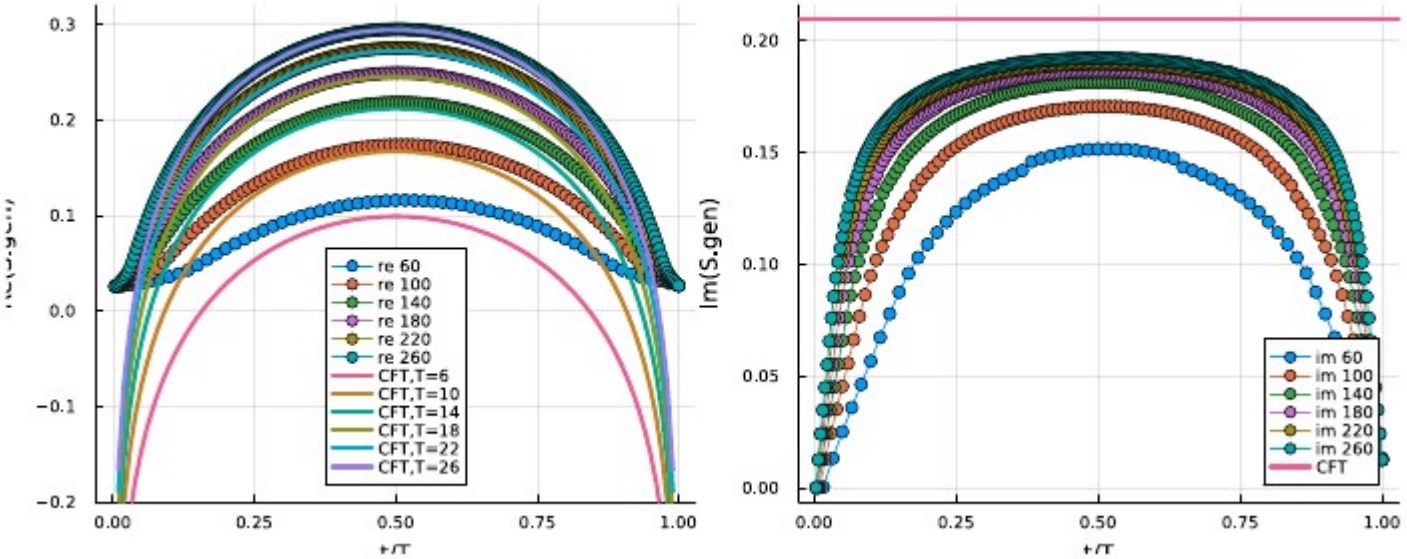




Numerical results, 3 states Potts

$$H(f) = -\sum_i \left[\left(\sigma_i \sigma_{i+1}^\dagger + \sigma_i^\dagger \sigma_{i+1} \right) + f\left(\tau_i + \tau_i^\dagger \right) \right],$$

$$\sigma = \sum_{s=0,1,2} |s\rangle \langle s+1| \qquad \tau = \sum_{s=0,1,2} \omega^s |s\rangle \langle s|, \; \omega = e^{i2\pi/3},$$



- The entanglement barrier in 1D **can be circumvented**
- We can compute the real time evolution of 1D systems
- We have direct access to the temporal entanglement and can confirm the available CFT predictions and extend them to new scenarios.

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