CAPA

Universidad Zaragoza
1542

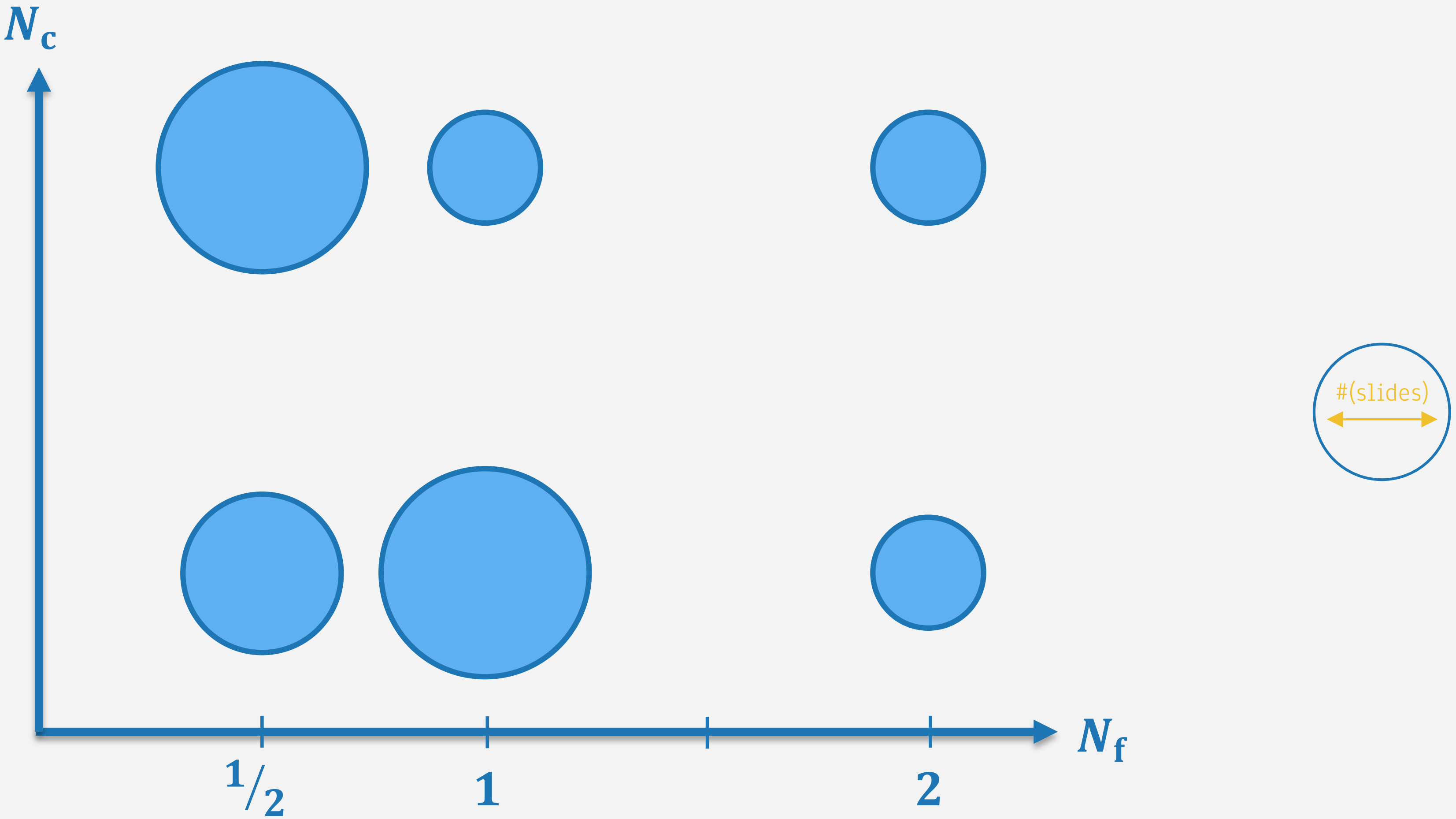
Turin Lattice Meeting 2023
December 22nd, 2023

GAUGE THEORIES WITH ADJOINT MATTER

Recent results and zoology from the lattice

Pietro Butti

OVERVIEW



I: SIMULATING ADJOINT FERMIONS

LARGE-N_c SIMULATIONS

TEK models w/ adjoint fermions

$SU(N_c{\sim}10^2\backslash10^3)$ on 1^4 lattice with **twisted BC**

$$S = bN_c \sum_{\mu\neq\nu} \mathrm{tr}\left(\mathbb{I} - z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right) + \bar{\psi} D_w \psi$$

$$z_{\mu\nu} = e^{\frac{2\pi i k}{\sqrt{N_c}}\epsilon_{\mu\nu}}$$

(N_c, k)
`(121,3), (169,5),`
`(289,5), (361,5),`
`(529,7), (841,9),`

"Inject" (random) momenta $U_\mu \to e^{ip_\mu} U_\mu$ with $p_\mu = \frac{2\pi}{\sqrt{N_c}} m_\mu + p_\mu^{(\mathrm{lat})}$

$$C_{AB}^{ij}(q_0) = \frac{1}{|\Lambda_p|} \sum_{p\in\Lambda_p} \left\langle \mathrm{Tr}\left(\mathbb{O}_A^i D_w^{-1}(\vec{p}, p_0 + q_0) \mathbb{O}_B^j D_w^{-1}(p)\right)\right\rangle$$

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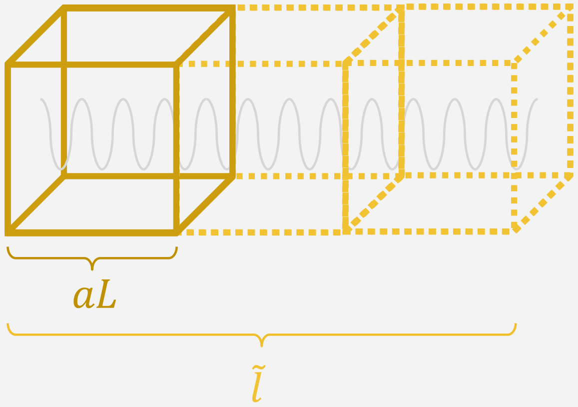
At **finite $\boldsymbol{N_c}$** (and $\boldsymbol{V}$) w/ twisted BC,
the IR cutoff length is
$$\tilde{l} = aL\sqrt{N_c}$$



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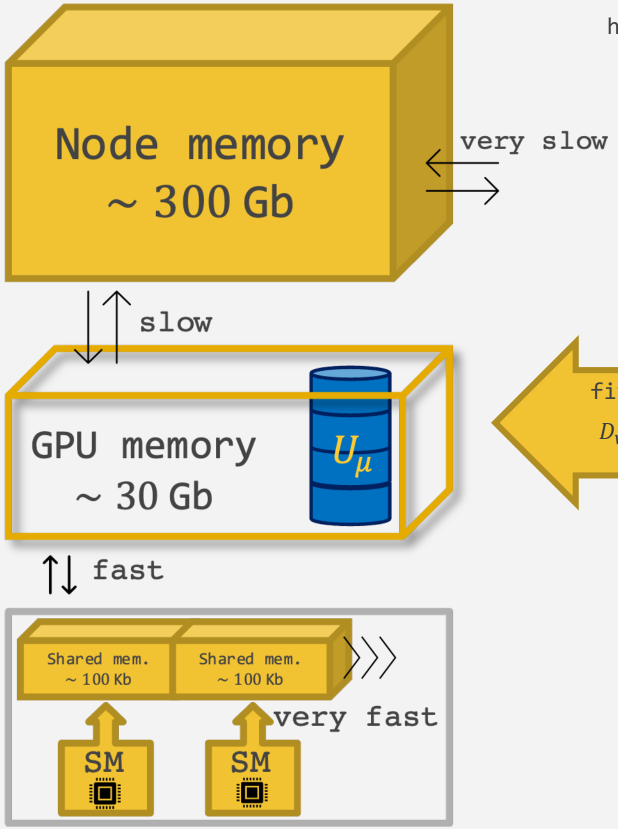


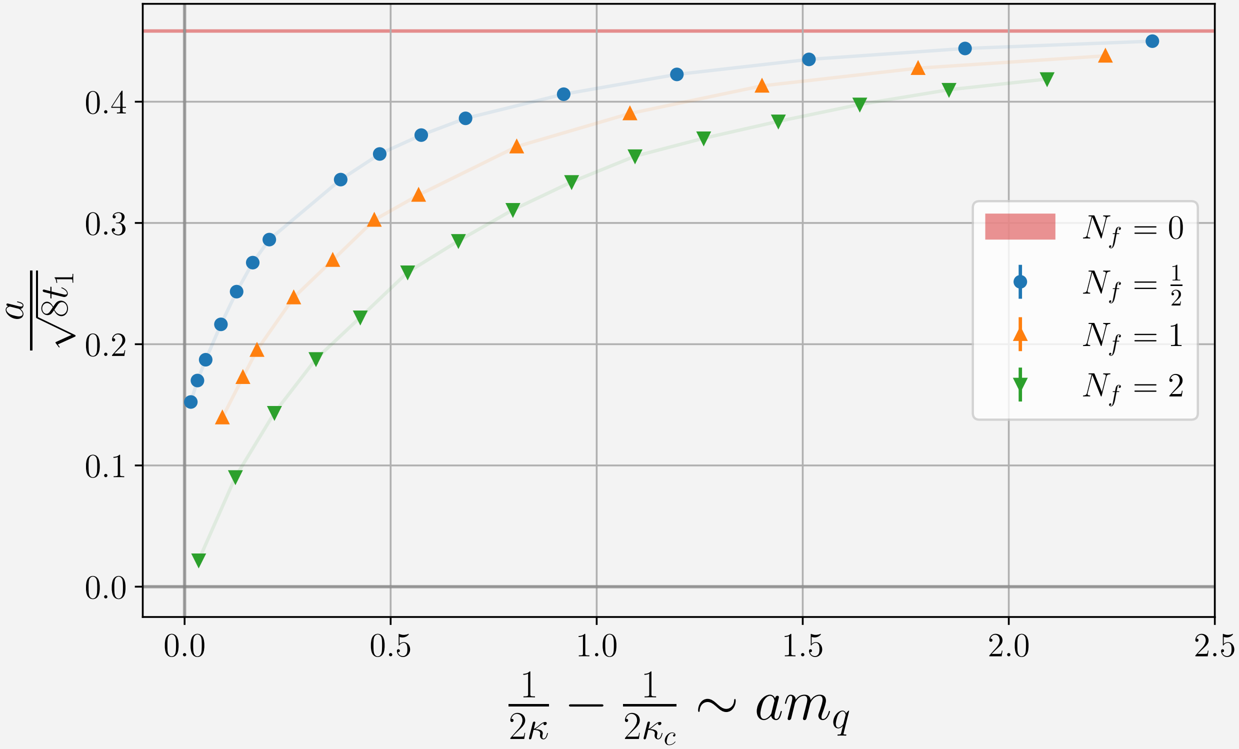
fit confs inside GPU memory

$$D_w \psi = \psi - \kappa \sum_{\pm\mu} (\mathbb{I} - \gamma_\mu) U_\mu \psi U_\mu^\dagger$$

https://gitlab.com/OneSite.jl

2: N_f FLAVORS OF ADJOINT FERMIONS

A BIRD-EYE PERSPECTIVE



$N_{\mathrm{f}} = \frac{1}{2}$: SUSY ON THE LATTICE

$\mathcal{N} = 1$ SUSY Yang-Mills

$$\mathcal{L} = -\frac{1}{2}\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\lambda^t(i\mathcal{C}\not{D})\lambda - \frac{m_{\tilde{g}}}{2}\lambda^t\lambda$$

gluons

gluinos
(adjoint Majorana fm)

soft SUSY breaking

- SUSY is broken on the lattice ($a \neq 0$, $m_{\tilde{g}} \neq 0$), but

Tune $m_{\tilde{g}} \to 0$ to trigger SUSY in the continuum ($a \to 0$)

[Curci, Veneziano] - Nucl. Phys. B 292 (1987)

- Chiral symmetry is broken as

$$U(1)_A \xrightarrow{\text{anomaly}} Z_{2N_c} \xrightarrow{\langle\bar{\lambda}\lambda\rangle} Z_2$$

... no “adjoint-π” in the spectrum!

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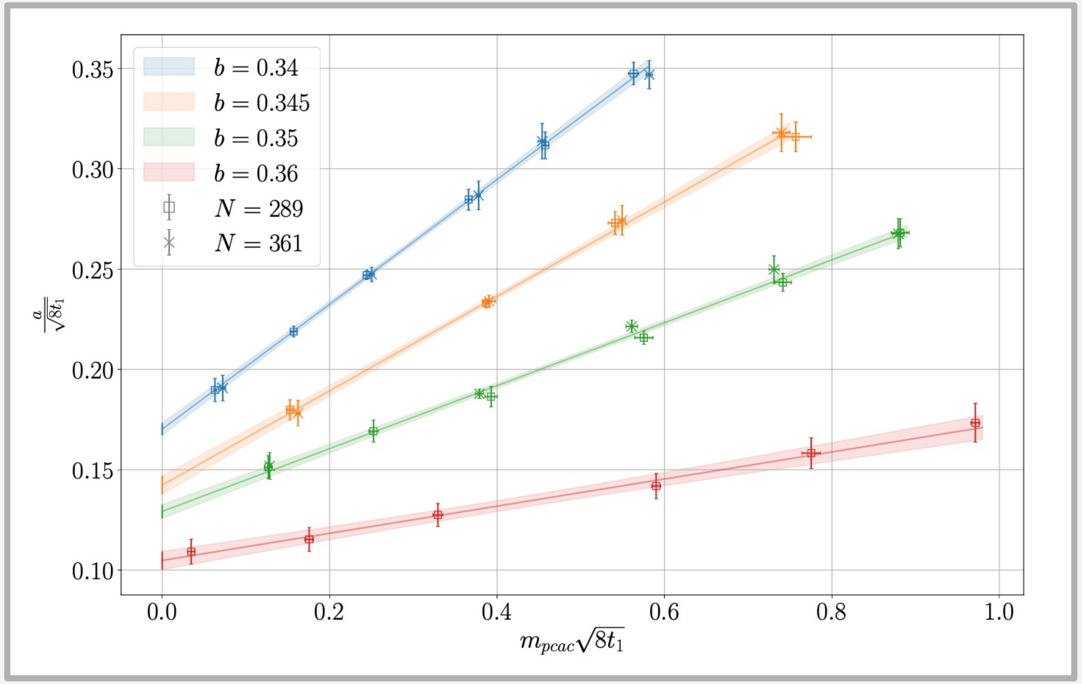
…how to trigger SUSY

Large-N_c



[PB, García Pérez, González-Arroyo, Ishikawa, Okawa] - JHEP 07 (2022) 074 [`2205.03166`]

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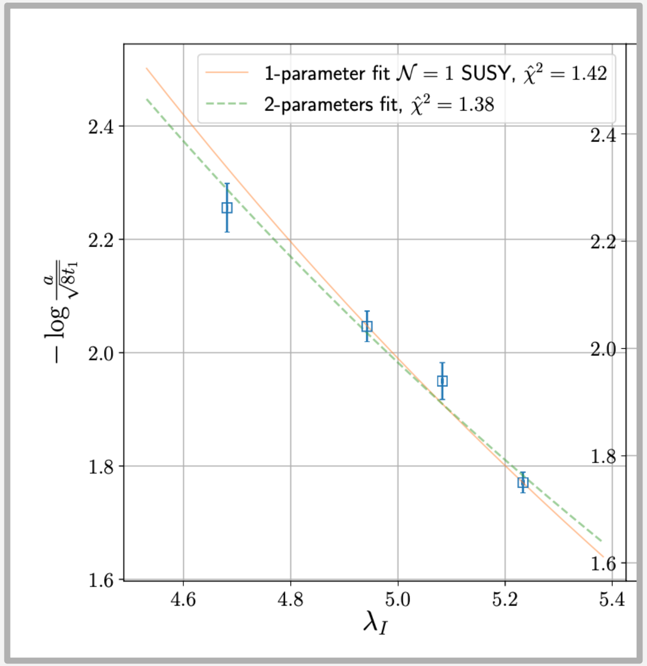
…SUSY, is it you?

Fit $-\log a\Lambda = \frac{1}{2b_0\lambda} + \frac{b_1}{2b_0^2}\log b_0\lambda$



Large-N_c

$N_\mathrm{f} = 0.3(2)$ and $(4\pi^2)b_0 = 9.8(8)$

[PB, García Pérez, González-Arroyo, Ishikawa, Okawa] - JHEP 2022, 07, 074 [2205.03166]

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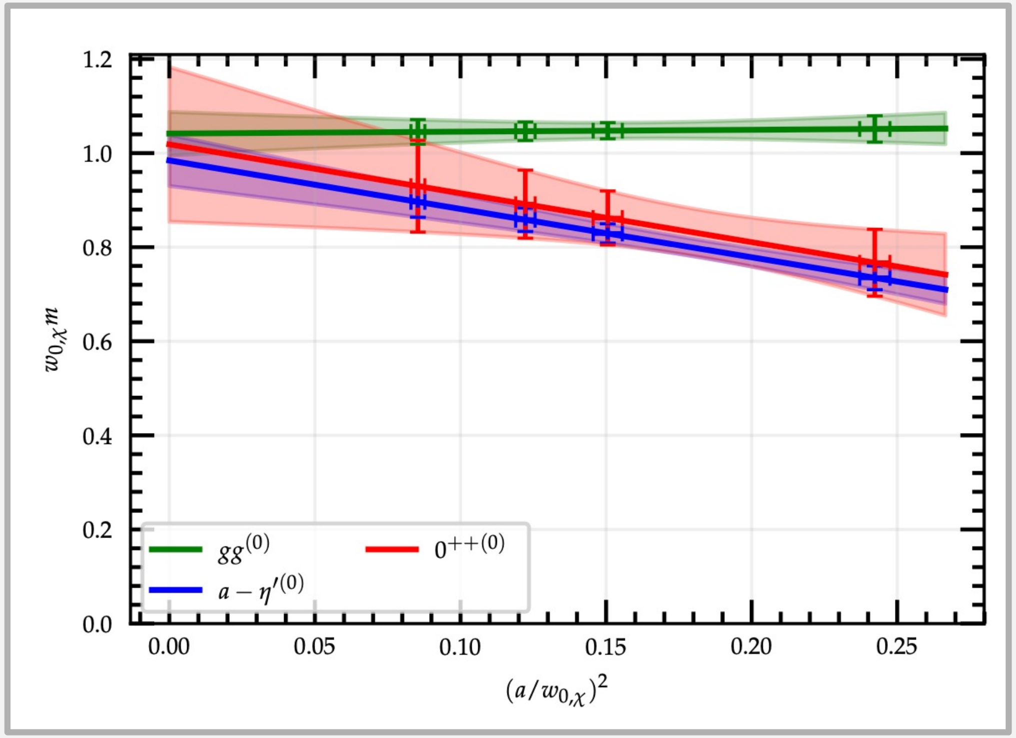
$N_c = 3$



[Ali, Bergner, Gerber, Montvay, Münster, Piemonte, Scior] – PRL 122 (2019) [1902.11127]

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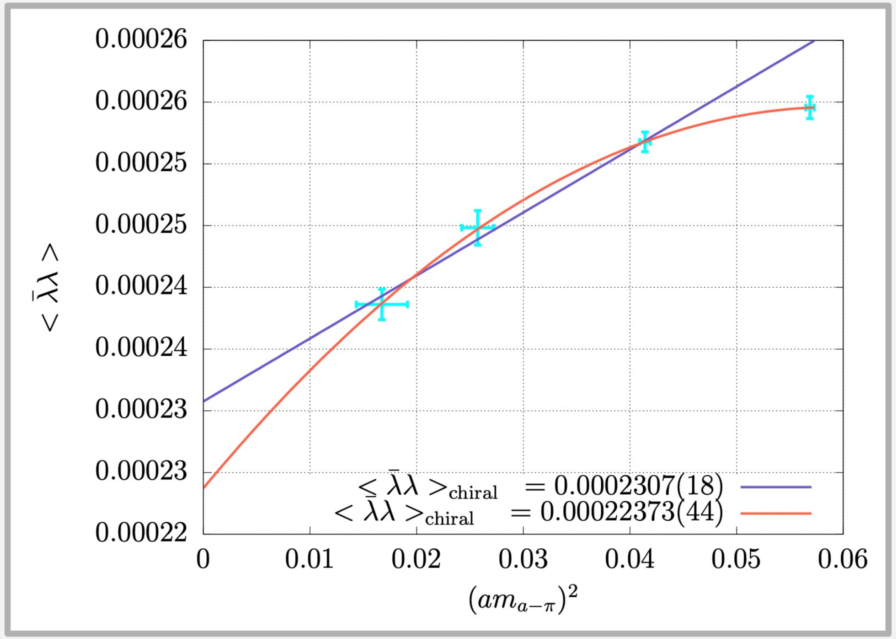

[Bergner, López, Piemonte] – Phys. Rev. D 100 (2019) 07 [1902.08469]

SEMI-CONFORMALITY ON THE LATTICE

Why (semi) conformal theories?

BSM requirements from EW precision data:

- Walking coupling (slow-running)
- Big mass anomalous dimension γ^*

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$$\frac{\mathrm{d}\lambda}{\mathrm{d}\log\mu} = \beta(\lambda) \sim -b_0\lambda^2 - b_1\lambda^3 - \cdots$$

with

$$b_0 = \frac{11 - 4N_\mathrm{f}}{3(4\pi)^3}$$

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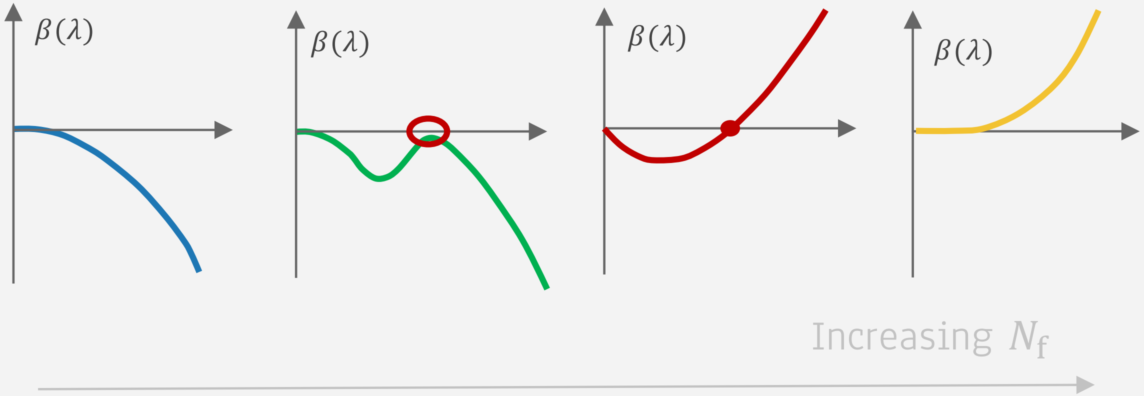
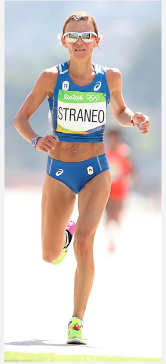
...but what about beyond PT?





SEMI-CONFORMALITY ON THE LATTICE

$SU(289)$

Signals for conformality in $N_\mathrm{f} = 1, 2$

Look at the eigenvalue spectrum of the Dirac operator

- Chirally broken scenario

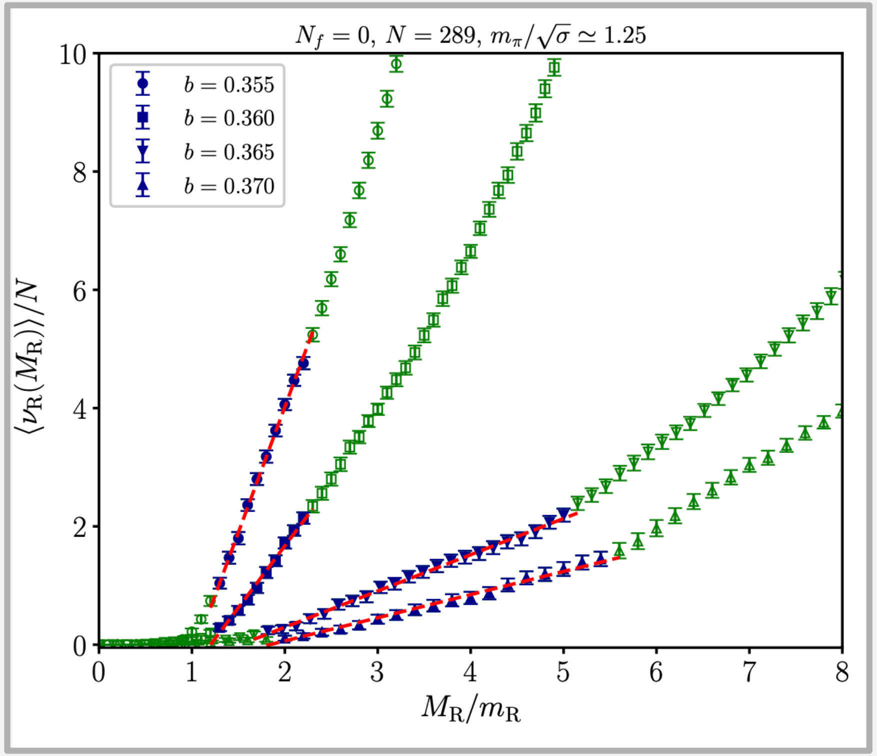
Low-modes condensate giving a non-vanishing value in the origin [Banks, Casher] – Nucl. Phys. B 169 (1980)

$$\lim_{\lambda\to 0}\lim_{m\to 0}\lim_{V\to\infty}\rho(\lambda,m)\propto\langle\bar\psi\psi\rangle$$

Linear rise of the mode number [Giusti, Lüscher] – JHEP 03 (2009)

$$\langle\nu(\Omega)\rangle = \#[\text{eigenv.} < \Omega] \propto \frac{2}{\pi}\langle\bar\psi\psi\rangle\Omega$$

$$\frac{\langle\bar\psi\psi\rangle}{Z_P N_c} = \frac{\pi}{2V}\sqrt{1-\left(\frac{m_r}{M_r}\right)^2}\,\frac{\text{slope}}{Z_A m_{PCAC}}$$



Large-N_c

[Bonanno, PB, García Pérez, González-Arroyo, Ishikawa, Okawa] – JHEP 12 (2023) 034 [2309.15540]

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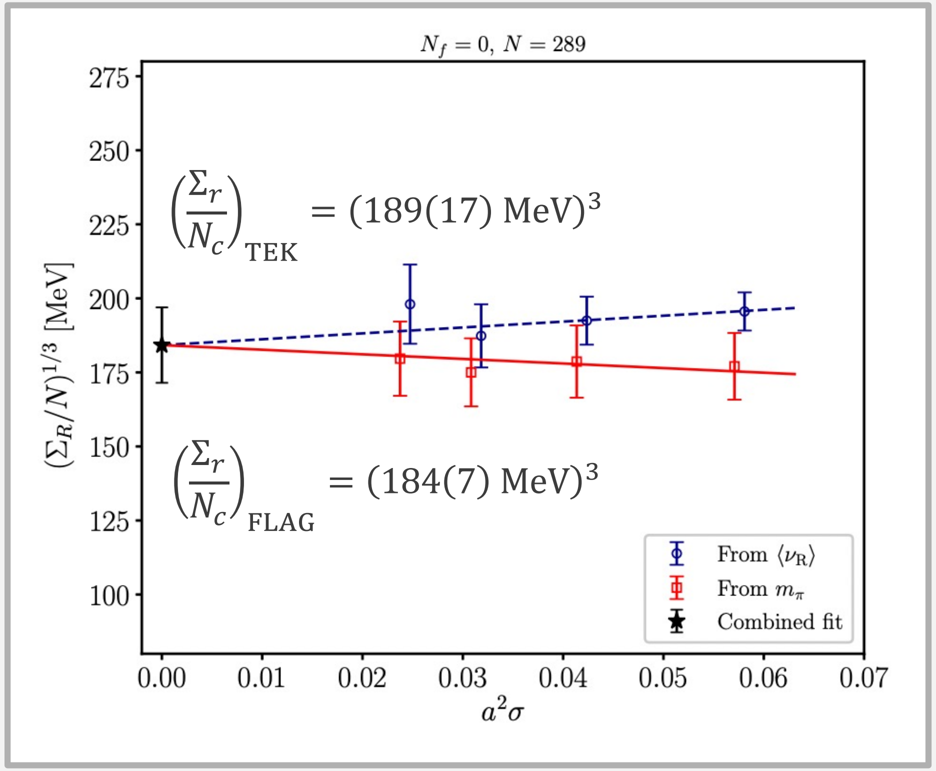

Large-N_c

[Bonanno, PB, García Pérez, González-Arroyo, Ishikawa, Okawa] – JHEP 12 (2023) 034 [2309.15540]

SEMI-CONFORMALITY ON THE LATTICE

$SU(2) + N_f = 2$

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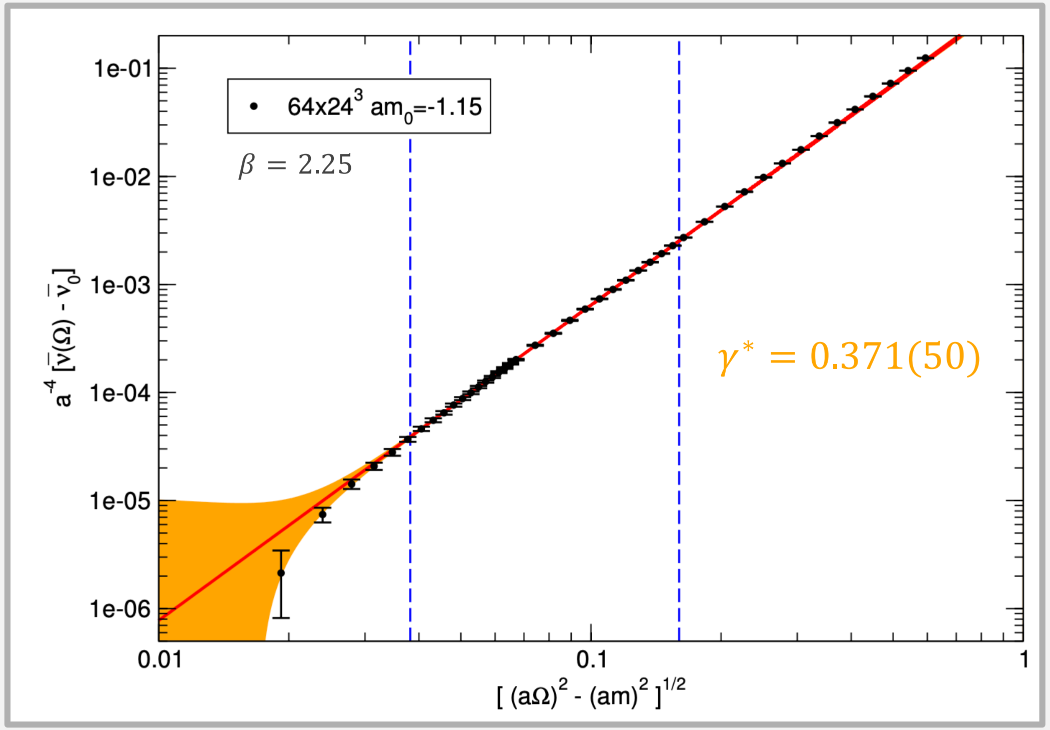
- **Conformal scenario**

Close to IRFP, RG equations give the behavior of the spectral density. On the lattice

$$a^{-4}\nu(\Omega) \simeq a^{-4}\nu_0 + A[(a\Omega)^2 - (am)^2]^{\frac{2}{1+\gamma^*}}$$

$\propto M_{PS}^4$

$\propto Z_A m_{\mathrm{PCAC}}$



[Patella] – Phys. Rev. D 86 (2012) [1204.4432]

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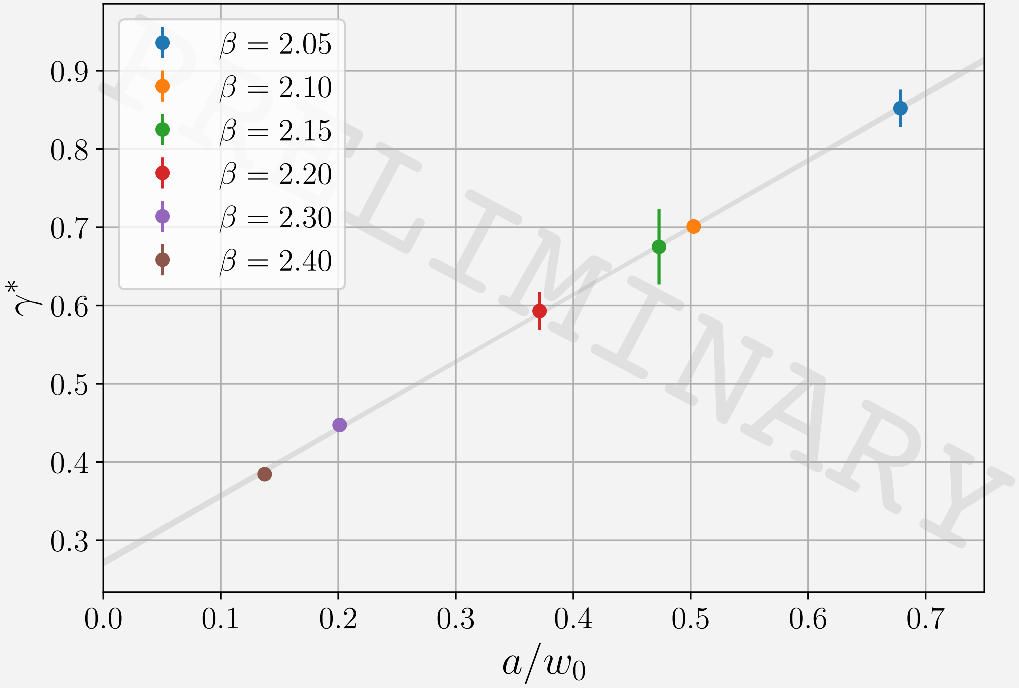

[Athenodorou, Bennett, Bergner, PB, Lucini] – in preparation

SEMI-CONFORMALITY ON THE LATTICE

Signals for conformality in $\boldsymbol{N_\mathrm{f} = 1, 2}$

An IR fixed point features *scale-invariance*

- χ-symmetry cannot break
- Universal exponent for power law-behavior of correlators at large distances

$$LM \sim L m_0^{\frac{1}{1+\gamma^*}}$$

Observed:

- "Would-be pseudo NG mode" (2^+) is not the lightest state in the spectrum (0^+)

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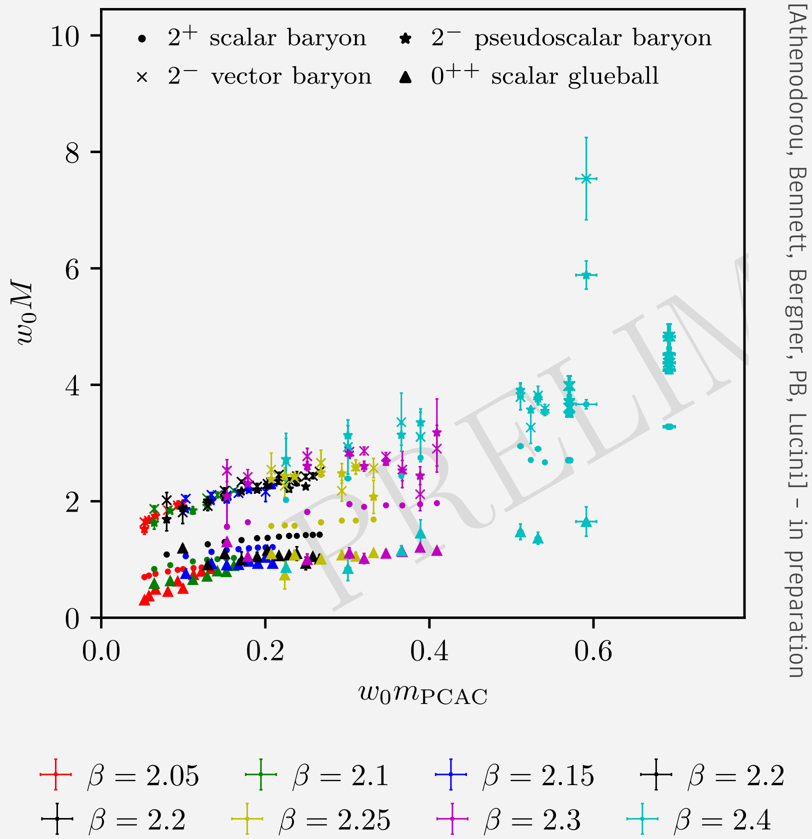

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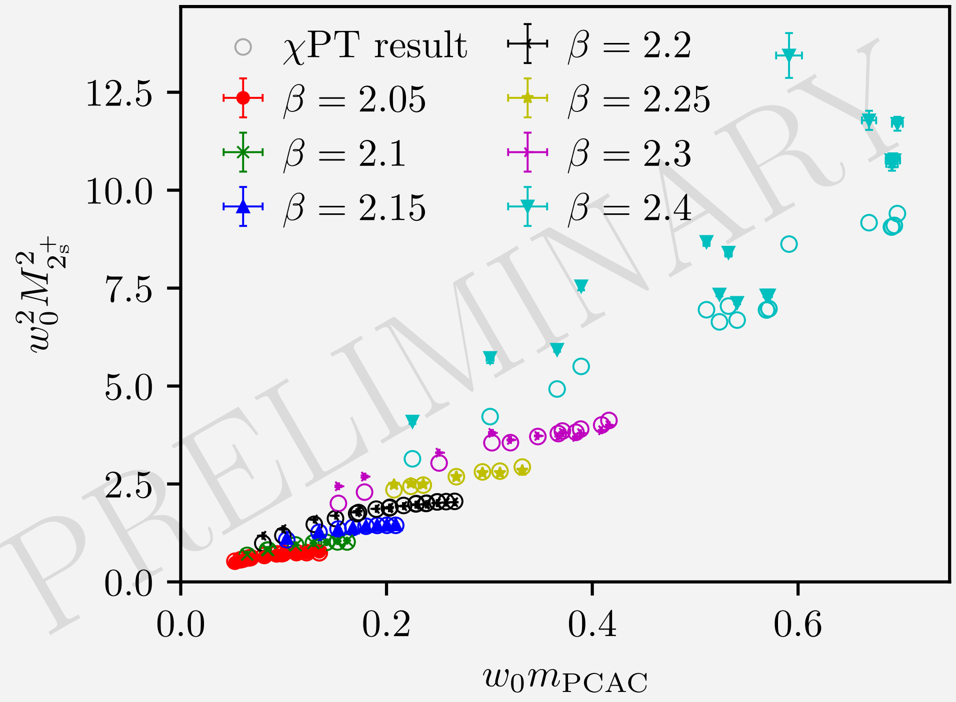
- Chiral PT does not describe well would-be pseudo NG mode

$$M_{2^+} = 2Bw_0 m_{\mathrm{PCAC}}(1 + Lw_0 m_{\mathrm{PCAC}} +$$
$$+ D_1 w_0 m_{\mathrm{PCAC}} \log(D_2 w_0 m_{\mathrm{PCAC}}) +$$
$$+ W_1 a m_{\mathrm{PCAC}} + W_2(a^2/w_0^2)$$



[Athenodorou, Bennett, Bergner, PB, Lucini] – in preparation

CONCLUSION

$N_\mathrm{f} = \frac{1}{2}$

- SUSY restored in chiral+cont. limit
- Confining theory
- Mass-degenerate multiplet
- Gluino condensate formation

- β-function @ large-N

[PB, García Pérez, González-Arroyo, Ishikawa, Okawa] – JHEP 2022, 07, 074 [2205.03166]

- Low-lying spectrum

[Ali, Bergner, Gerber, Montvay, Münster, Piemonte, Scior] – PRL 122 (2019) [1902.11127]

- Gluino condensate

[Bergner, López, Piemonte] – Phys. Rev. D 100 (2019) 07 [1902.08469]

$N_\mathrm{f} = 1$

"Near-conformal" behaviour,
strong lattice artefacts

- γ^* depends strongly on β
- The lightest state is not pseudo-NG
- Chiral PT does not describe data

[Athenodorou, Bennett, Bergner, PB, Lucini] – in preparation

- Signal for χ-SB from overlap fermions

[Bergner, Lopez, Piemonte, Soler Calero] – Phys. Rev. D 106 (2022) 9

$N_\mathrm{f} = 2$

Seemingly conformal theory,
with an IR fixed point

- Small anomalous dimension

[Patella] – Phys. Rev. D 86 (2012) [1204.4432]
[García Pérez, González-Arroyo, Keegan, Okawa] – JHEP 08 (2015) 034

- IRFP from running coupling

[Rantaharju, Rantalaiho, Rummukainen, Tuominen] – Phys. Rev. D 93 (2016) 9

BACKUP SLIDES

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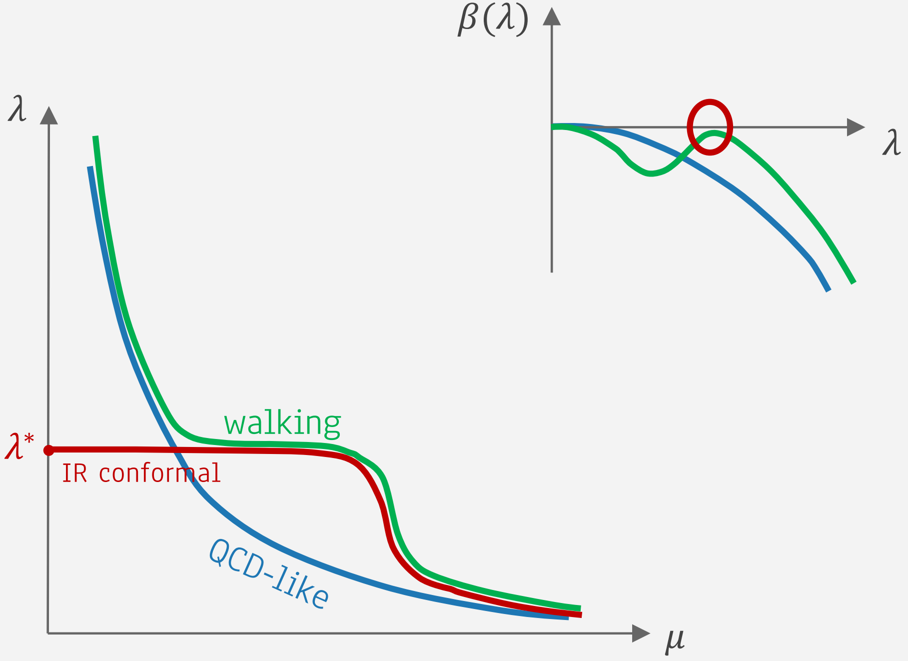

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Signals for conformality in $\boldsymbol{N_f = 1, 2}$

running

$SU(2) + N_f = 1$

LARGE-N_c SIMULATIONS

Asymptotic scaling with TEK

- Simulate $\boldsymbol{SU(841)}$ on a single-site with twisted BC

$$S = bN_c \sum_{\mu\neq\nu} \mathrm{tr}\left(\mathbb{1} - z_{\mu\nu}U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right)$$

- Set the scale with the (improved) Wilson flow (or $\sqrt{\sigma}$)

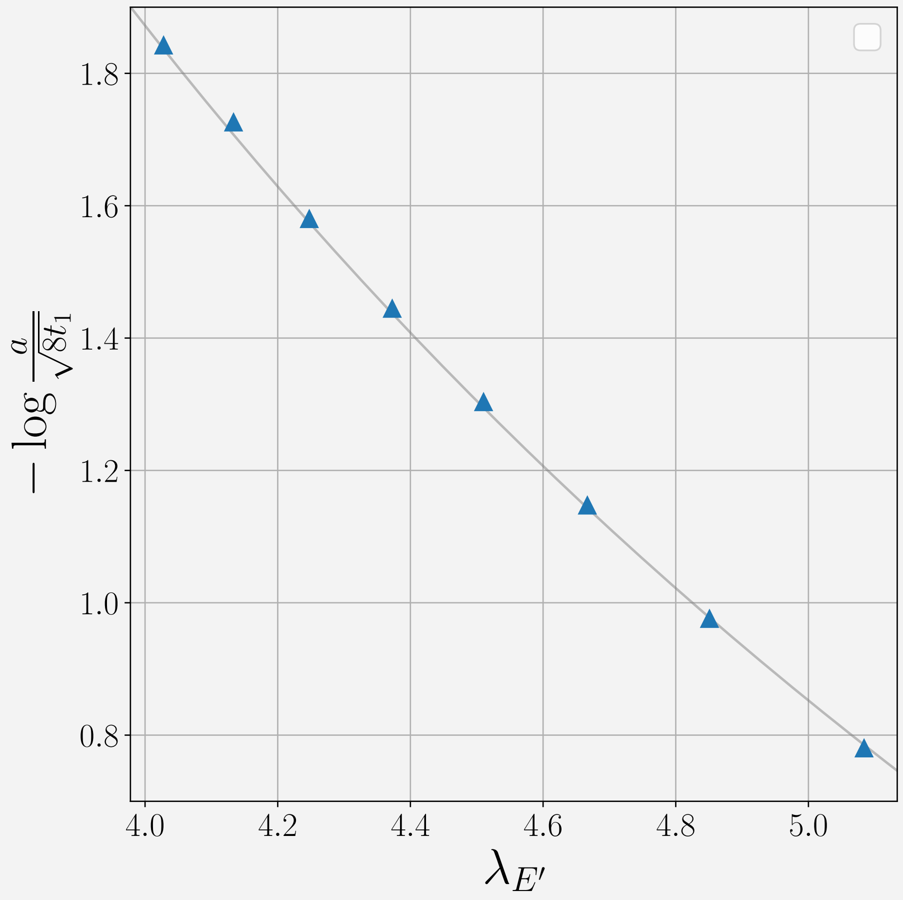
$$\frac{\mathcal{N}(c(t))}{N_c}\langle t^2 E(t)\rangle\Big|_{t=t_1} = 0.05$$

- Integrate the perturbative β-function at $\mathcal{O}(\lambda^4)$

$(\lambda_{\text{lat}} = g^2 N_c)$

$$-\log\frac{a}{\sqrt{8t_1}} = \log\Lambda_s\sqrt{8t_1} + \frac{1}{2b_0\lambda_s} + \frac{b_1}{2b_0^2}\log(b_0\lambda_s) + \frac{c_1^{(s)}}{2b_0}\lambda_s$$

- Choose an improved scheme, fit, and convert to $\overline{\text{MS}}$



[PB, González-Arroyo] – in preparation + [2311.18696]

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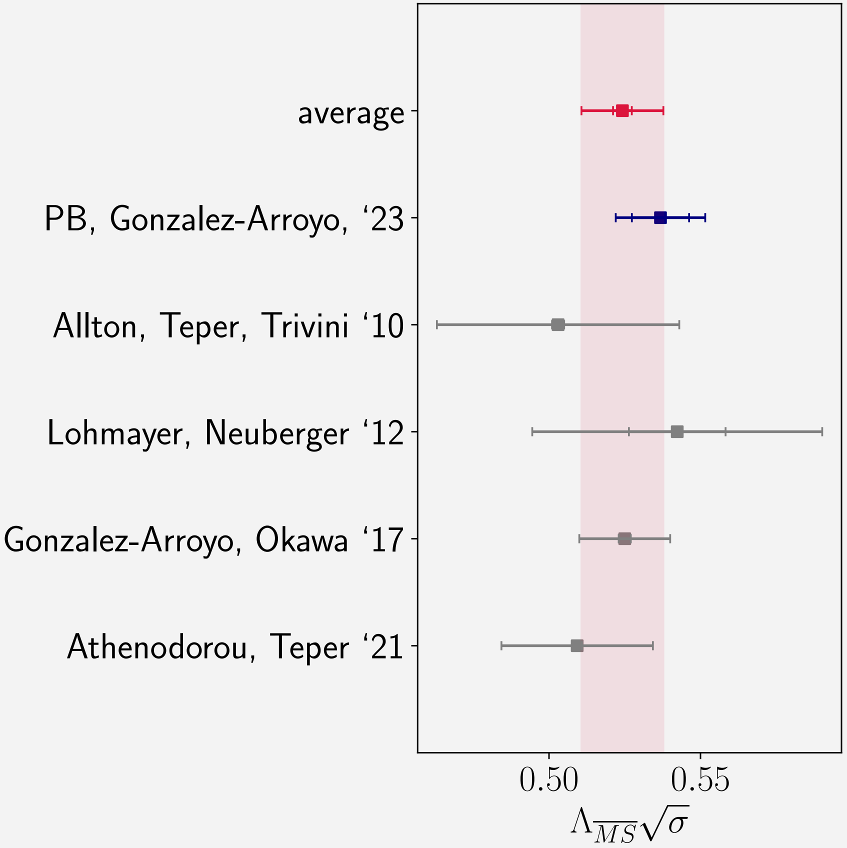
$(\lambda_{\text{lat}} = g^2 N_c)$

$$-\log \frac{a}{\sqrt{8t_1}} = \log \Lambda_s \sqrt{8t_1} + \frac{1}{2b_0\lambda_s} + \frac{b_1}{2b_0^2}\log(b_0\lambda_s) + \frac{c_1^{(s)}}{2b_0}\lambda_s$$

- Choose an improved scheme, fit, and convert to $\overline{\mathrm{MS}}$



[PB, González-Arroyo] – in preparation + [2311.18696]

LARGE-N_c SIMULATIONS

The chiral condensate with TEK

Condensation of low modes of the massless Dirac operator

[Banks, Casher] – Nucl. Phys. B 169 (1980)

$$\lim_{\lambda\to 0}\lim_{m\to 0}\lim_{V\to\infty}\rho(\lambda,m)\propto\langle\bar\psi\psi\rangle$$

Implies linear rise of the **mode number** at small mass

$$\langle\nu(M)\rangle = V\int_{-\Lambda}^{\Lambda}\rho(\lambda,m)\,\mathrm{d}\lambda = \frac{2}{\pi}\langle\bar\psi\psi\rangle\Lambda$$

with $\Lambda^2 = M^2 - m^2$

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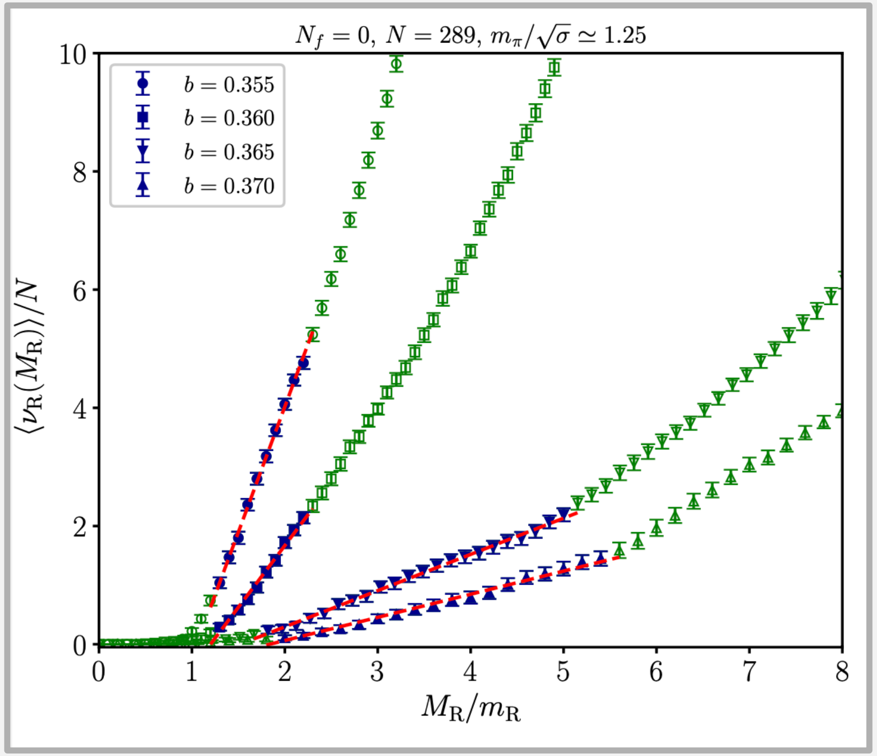
- Fit the slope s_r and compute [García, González-A., Okawa] – JHEP 04 2021

$$\frac{\langle\bar\psi\psi\rangle}{Z_P N_c} = \frac{\pi}{2V}\sqrt{1-\left(\frac{m_r}{M_r}\right)^2}\,\frac{s_r}{Z_A m_{PCAC}}$$

- Extrapolate at vanishing pion mass + “continuum” limit



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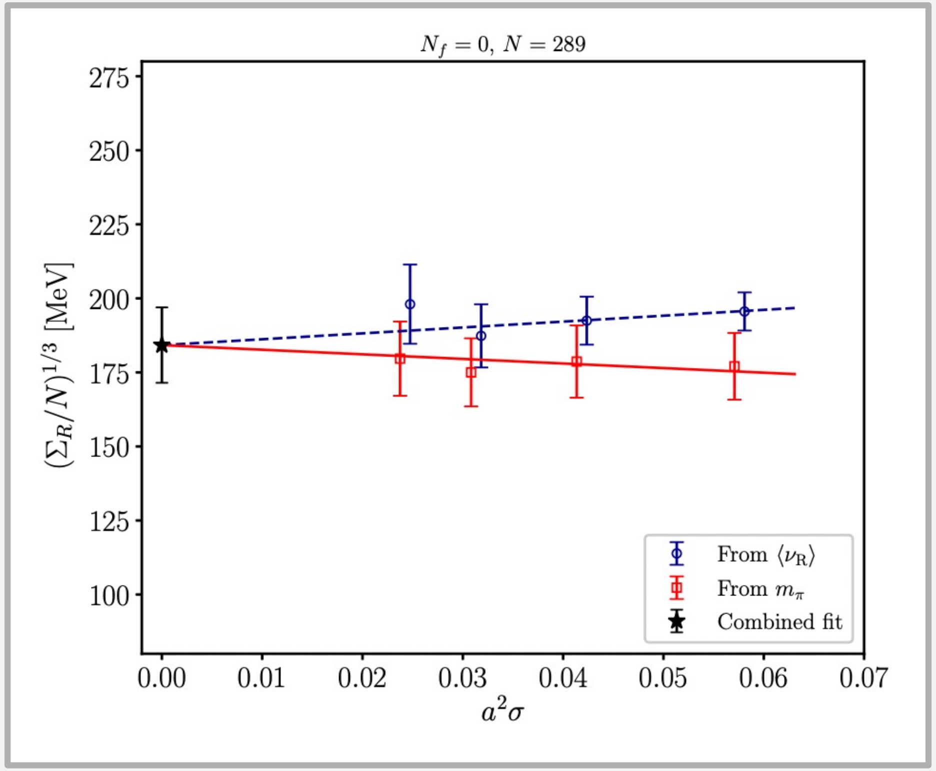

$$\left(\frac{\Sigma_r}{N_c}\right)_{\mathrm{TEK}} = (189(17)\ \mathrm{MeV})^3 \qquad \left(\frac{\Sigma_r}{N_c}\right)_{\mathrm{FLAG}} = (184(7)\ \mathrm{MeV})^3$$