Hagedorn temperature in string and gauge theories

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Based on:

F. Bigazzi, **T. Canneti**, A. L. Cotrone, [arXiv:2210.09893 [hep-th]]
F. Bigazzi, **T. Canneti**, W. Mück, [arXiv:2306.00588 [hep-th]]
F. Bigazzi, **T. Canneti**, A. L. Cotrone, [arXiv:2306.17126 [hep-th]]
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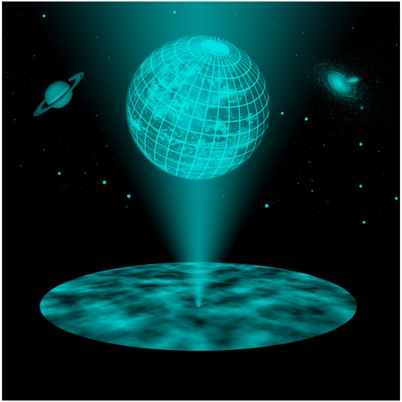
Holographic Correspondence [Maldacena '97, Witten '98, Gubser et al. '98]

theory of gravity
in $d+1$-dimension
(string theory)

$\overset{exact}{\equiv}$

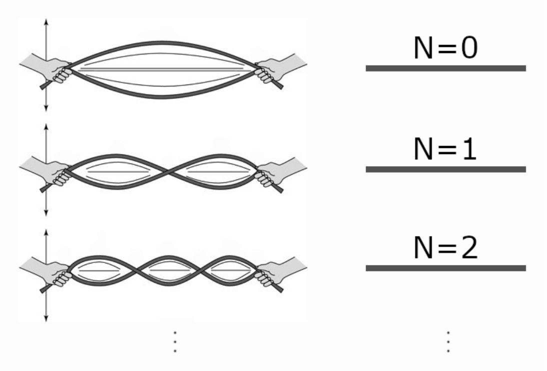
gauge theory
in d-dimension



- Keywords: holographic, weak/strong, conjecture, duality
- String theory and gauge theory share the very same properties

String theory

- Building block of nature: one-dimensional string (length scale l_s)
- Emergence of elementary particles/fundamental interactions
- String spectrum:



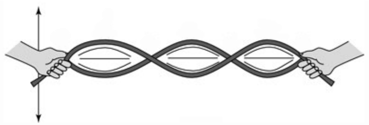
$$l_s^2\, M^2 \approx N$$

(mass-shell condition)

$\Downarrow$

states belonging to the same level N have the same mass M

Asymptotic density of states $d(N)$ [Huang et al. '70, Sundborg '85]



$$\hat{H} \approx \omega_0\, \hat{N}, \quad \hat{N} = \sum_{k=1}^{+\infty} k\, \hat{a}_k^\dagger\, \hat{a}_k$$

$$|\psi\rangle = (\hat{a}_1^\dagger)^{n_1} (\hat{a}_2^\dagger)^{n_2} \ldots (\hat{a}_k^\dagger)^{n_k} \ldots |0\rangle \;:\quad \hat{N}|\psi\rangle = N|\psi\rangle\,, \quad N = \sum_{k=1}^{+\infty} k\, n_k$$

N	$\|\psi\rangle$	$d(N)$
1	$\hat{a}_1^\dagger$	1
2	$\hat{a}_2^\dagger \quad (\hat{a}_1^\dagger)^2$	2
3	$\hat{a}_3^\dagger \quad \hat{a}_2^\dagger \hat{a}_1^\dagger \quad (\hat{a}_1^\dagger)^3$	3
4	$\hat{a}_4^\dagger \quad \hat{a}_3^\dagger \hat{a}_1^\dagger \quad (\hat{a}_2^\dagger)^2 \quad \hat{a}_2^\dagger (\hat{a}_1^\dagger)^2 \quad (\hat{a}_1^\dagger)^4$	5
$\vdots$	$\vdots$	$\vdots$

$$d(N) \overset{N \gg 1}{\approx} e^{\#\sqrt{N}}$$

[Hardy et al. '18]

The Hagedorn temperature [Hagedorn '68]

Partition function:

$$Z \sim \int \mathrm{d}E\, \rho(E)\, e^{-\beta E}$$

- $\beta = 1/T$ (inverse temperature)
- Mass-shell condition $\Rightarrow E^2 \sim N$ (mass $M \sim$ energy E)
- $\rho(E)\mathrm{d}E = d(N)\mathrm{d}N \Rightarrow \rho(E) \sim e^{\beta_H E}$

Definition

The Hagedorn temperature $T_H = 1/\beta_H$ is defined as temperature above which Z does not exist (divergent!)

Methods

- String in flat space: Z is well-known (end of the story)
- String in curved space: we cannot compute Z in general

The lightest state of the spectrum of a string winding once the thermal (compact) direction is tachyonic above T_H

$$M^2(|0\rangle; \beta \leq \beta_H) \leq 0$$

[Sathiapalan'87, Kogan '87, O'Brien et al. '87]

Idea

[Atick&Witten '88]: flat space computation relying just on the mass-shell condition

$\Rightarrow$ we extend this method to curved space for a large class of models

Example [Urbach '22, Ekhammar et al. '23, Bigazzi et al. '23]

String sitting at the center of $d+1$-dim Anti de Sitter (AdS_{d+1}) space

$$\mathrm{d}s^2 = R_{AdS}^2 \left(\cosh^2\!\rho \, \mathrm{d}t^2 + \mathrm{d}\rho^2 + \sinh^2\!\rho \, \mathrm{d}\Omega_{d-1} \right)$$

Deduce T_H by fixing $M^2 = 0$:

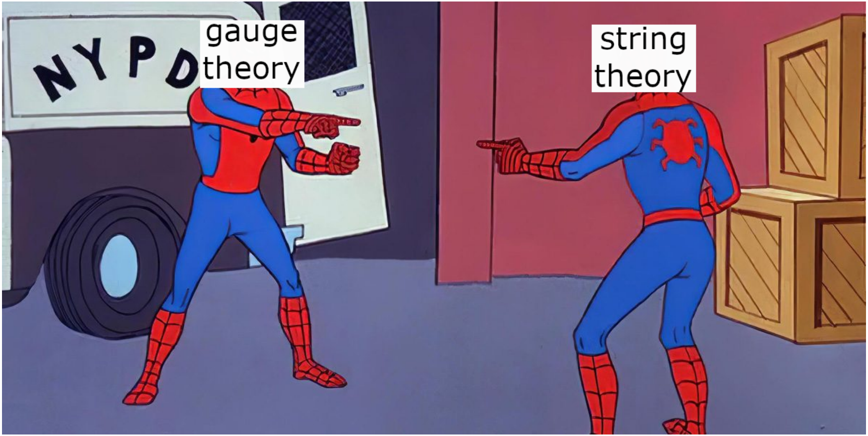
$$\boxed{T_H = \sqrt{\frac{g}{2\pi}} + \frac{d}{8\pi} + \frac{d^2 + d - 8d\log 2}{32\sqrt{2}\pi^{3/2}\sqrt{g}} + \frac{4d^3 + 7d^2 - 2d}{1024\pi^2 g} + \mathcal{O}(g^{-3/2}) \,,}$$

where $g = 1/4\pi l_s^2$.

Here, $R_{AdS} = 1$: to get physical result $T_H \mapsto T_H \, R_{AdS}$ and $l_s \mapsto l_s / R_{AdS}$

Holography

$$T_H^{\text{gauge theory}} \equiv T_H^{\text{string theory}}$$



Comparison with numerical results
$d = 4$: (Super) Yang-Mills theory on $S^1 \times S^3$

$$T_H = \sqrt{\frac{g}{2\pi}} + \frac{1}{2\pi} + \frac{5 - 8\log 2}{8\pi\sqrt{2\pi}\sqrt{g}} + \frac{45}{128\pi^2 g} + \mathcal{O}(g^{-3/2})$$
$$\approx 0.39894...\sqrt{g} + 0.15916... - \frac{0.00865...}{\sqrt{g}} + \frac{0.0356...}{g} + \mathcal{O}(g^{-3/2})$$

vs

$$T_H^{\text{[Ekhammar et al.'23]}} \approx (0.39894 \pm 0.00001)\sqrt{g} + (0.15916 \pm 0.00001) - \frac{(0.00865 \pm 0.00001)}{\sqrt{g}} + \frac{(0.0356 \pm 0.0001)}{g} + ...$$

That's an outstanding and non-trivial test of holography!

Conclusions

- T_H for (confining) gauge theories up to next-to-next-to-leading order (NNLO) or NNNLO in the string length
- Agreement with expectation in the literature and numerical analysis
- Some ambiguities fixed in an analytical way from the string

Thanks a lot for your attention!