Horizon symmetries, Hydrodynamics and Chaos

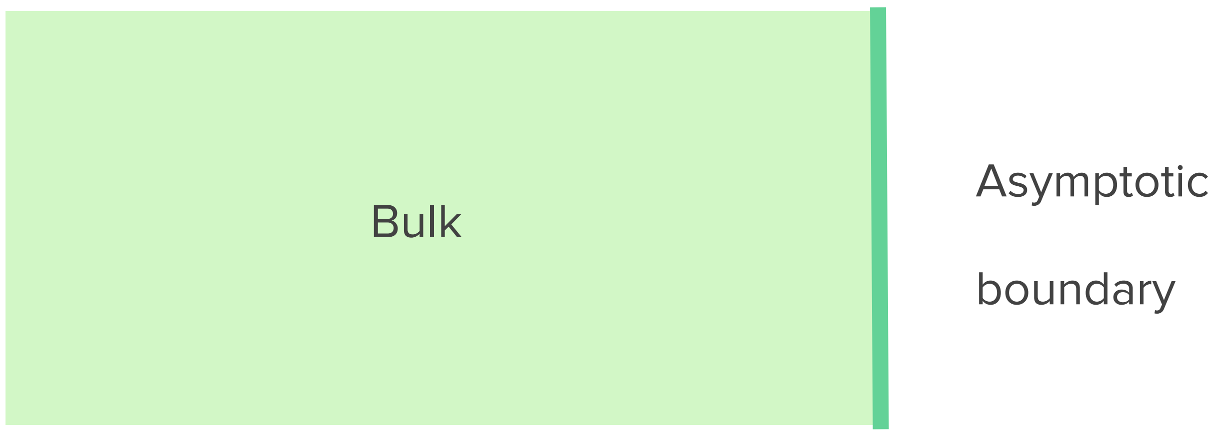
Natalia Pinzani Fokeeva - University of Florence

Theory Day - March 2024

with Hong Liu (MIT) and Maria Knysh (VUB) - to appear

Asymptotic symmetries

Local symmetries (diffs, gauge symm., ...) may become **global symmetries** of a theory in the presence of (asymptotic) **boundaries** → **Asymptotic symmetries**



- **Conformal group** in asymptotically AdS spacetimes [Brown, Hennaux 1986]
  - → AdS/CFT correspondence [Maldacena 1998]
- **BMS group** in asymptotically flat spacetimes [Bondi et al. 1962, 2 x Sachs 1962]
  - → Celestial Holography [Pasterski, Shao, Strominger, 2017]

Horizon symmetries

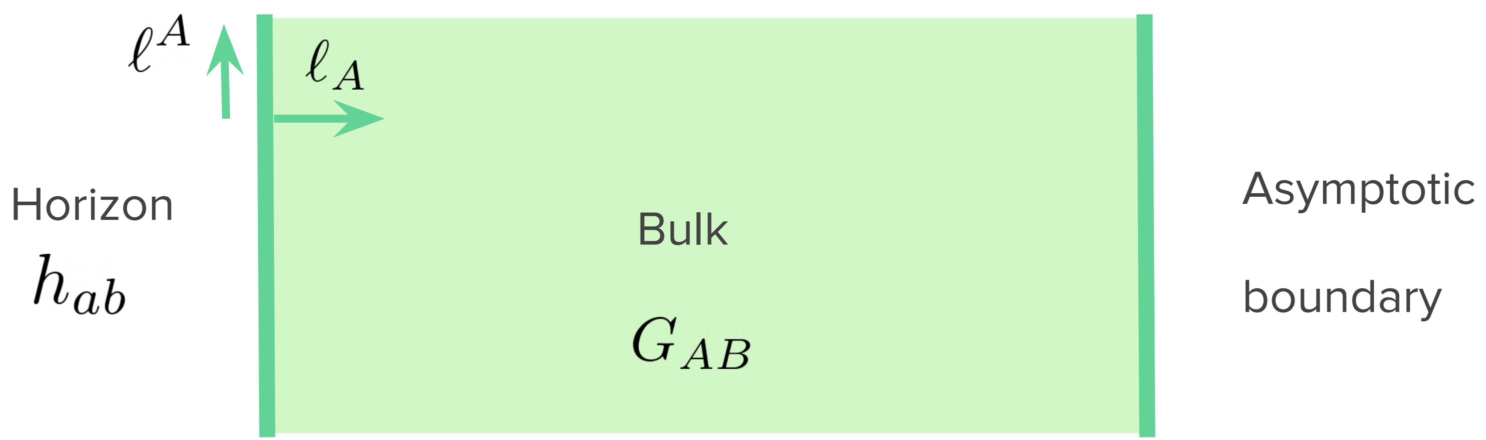
Black hole horizons can also be thought of as **boundaries**

→ **horizon symmetries** beyond isometries [Donnay et al. 2015 + older work]

- **Null surfaces** have infinite # of **symmetries** [Chandrasekaran et al 2018]
- Hawking's information loss paradox? [Hawking, Perry and Strominger 2016]
- What is the **interpretation** of these symmetries?

Horizon symmetries: setup

- Consider a Black hole with a horizon
- The induced metric $h_{ab}(\sigma) = \partial_a Y^A \partial_b Y^B G_{AB}(Y(\sigma))$



- The horizon is a null boundary → there is a vector such that $h_{ab}\hat{\ell}^a = 0$
- With the null vector $\ell^A \triangleq \partial_a Y^A \hat{\ell}^a$ $\quad \ell^A \ell^B G_{AB} \triangleq 0$ $\quad \ell_A = G_{AB}\ell^B$
- Non-affinity parameter $\kappa \ell^A \triangleq \ell^B \nabla_B \ell^A$

Horizon symmetries: definition

- Consider bulk diffs $x^A \to x^A + \chi^A \qquad G'_{AB} = G_{AB} + \pounds_\chi G_{AB}$
- χ^A is a **horizon preserving diffeomorphism** if
  a. The null manifold and the null vector remains null $h'_{ab}\hat{\ell}^a = 0$
  b. The non-affinity parameter is unchanged $\ell^A \nabla'_A \ell^B = \kappa \ell^B$

→ Horizon symmetry constraints

$$\nabla_C(\pounds_\chi G_{AB}\ell^A\ell^B) \triangleq 0 \qquad \pounds_\chi G_{AB}\ell^B \triangleq 0$$

- Diffs along the horizon; $\chi^A \ell_A \triangleq 0$

$$\sigma^0 \to \sigma^0 + \lambda(\vec{\sigma}) + \alpha(\vec{\sigma})e^{-\kappa\sigma^0}$$

$$\sigma^i \to \sigma^i + \zeta^i(\vec{\sigma})$$

Results

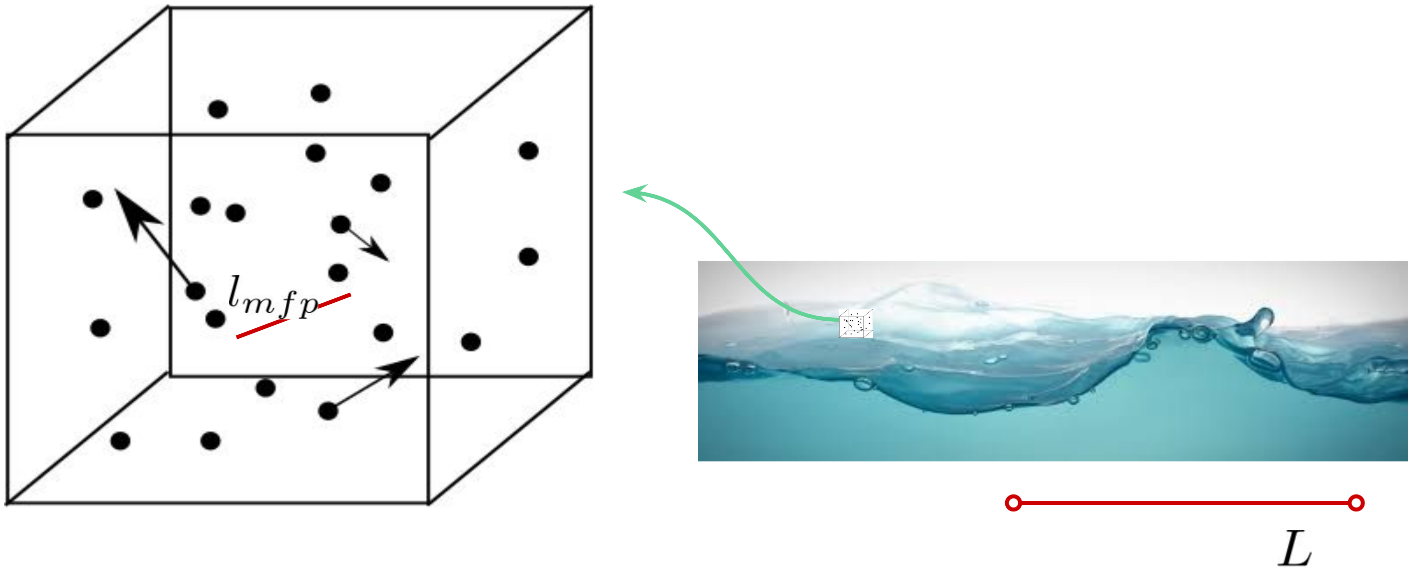
- Near horizon symmetries as **symmetries of hydrodynamics**
- **New** horizon symmetries
- New horizon symmetries as **symmetries of chaotic systems**

Results

- Near horizon symmetries as **symmetries of hydrodynamics**

Conventional hydrodynamic d.o.f.

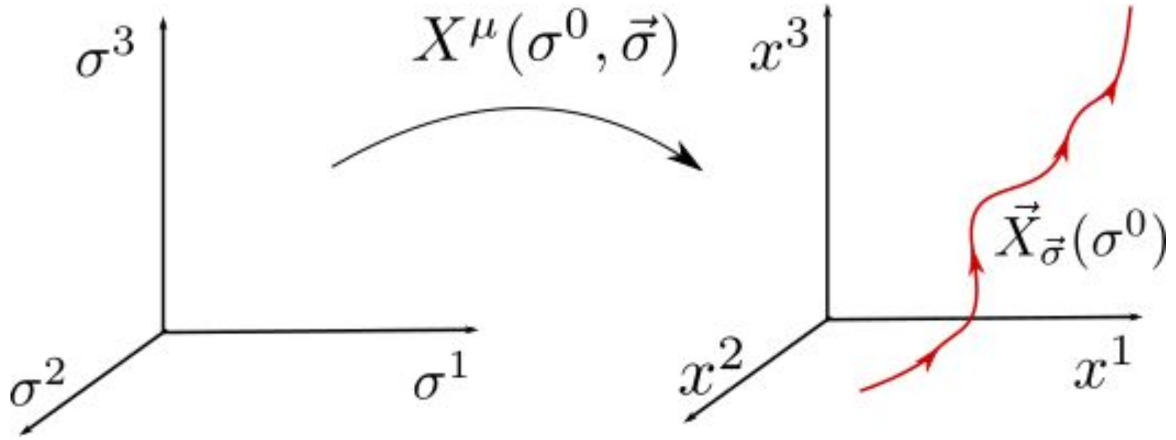
Fluids as a continuum of a coarse grained set of **fluid elements,** each at **local thermal equilibrium,** as long as $l_{mfp} \ll L$



- Few macroscopic (relativistic) variables: $u^{\mu}(t, \vec{x})$ $T(t, \vec{x})$
- Equations of motion are conservation equations: $\nabla_{\mu} T^{\mu\nu} = 0$

Modern hydrodynamic d.o.f.

Fluid dynamics d.o.f. are **maps** from the **fluid frame** to the **laboratory space**



Fluid (Lagrangian) **frame**

Laboratory (Eulerian) **frame**

- These are the convenient d.o.f. whose e.o.m. from a variational principle are the hydrodynamic equations [G. Herglotz (1911), A. H. Taub (1954), B. Carter (1973)]

$$0 = \frac{\delta S_{eff}}{\delta X^\mu} = \nabla_\nu T^{\mu\nu}$$

The redundancy

- Relation to the conventional variables:

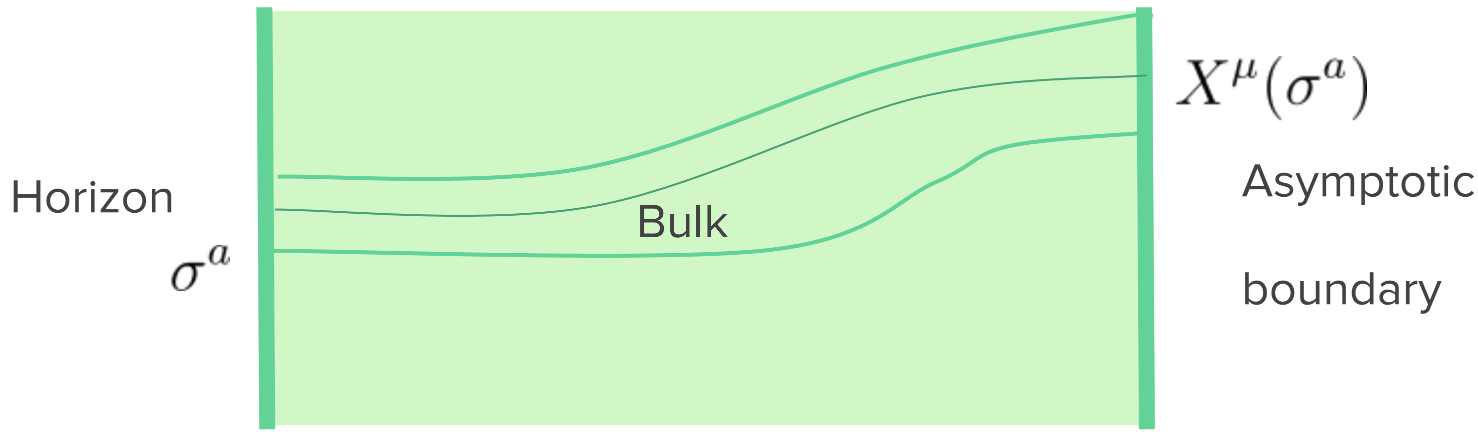
$$\frac{\partial X^\mu}{\partial \sigma^0} = \beta^\mu = \frac{u^\mu}{T}$$

- The following transformation leaves the definition of the thermal vector field invariant:

$$\sigma^0 \to \sigma^0 + \lambda(\vec{\sigma}), \qquad \sigma^i \to \sigma^i + \zeta^i(\vec{\sigma})$$

- This symmetry, together with other properties of S_{eff} based on the Schwinger-Keldysh path integral leads to a **consistent effective field theory for hydrodynamics** [Hong Liu, Paolo Glorioso et al 2015, Jensen, Marjieh, NPF, Yarom, 2017]

Fluid d.o.f. in gravity



Fluid (Lagrangian) **frame**

Laboratory (Eulerian) **frame**

- Fluid dynamics d.o.f. In gravity are **maps** (spacelike or null geodesics) **from the horizon to the boundary** [Nickel and Son 2010, Liu, Glorioso et al 2015, de Boer, Heller, NPF 2015]
- The velocity field is given by $\beta^\mu = \frac{\partial X^\mu}{\partial \sigma^a} \hat{\ell}^a$
- These maps are the correct hydrodynamic degrees of freedom as they reproduce the correct hydrodynamic effective action [Liu, Glorioso et al 2019, de Boer, Heller, NPF 2019]

Results

- **New** horizon symmetries

- New horizon symmetries as **symmetries of chaotic systems**

New horizon symmetries

- Considering diffs not aligned on the horizon $\chi^A \ell_A \neq 0$
- The horizon symmetry constraints imply

Postulated as symmetries of **chaotic systems**

$$\sigma^0 \to \sigma^0 + \lambda(\vec{\sigma}) + \alpha(\vec{\sigma}) e^{-\kappa\sigma^0} + \gamma(\vec{\sigma}) e^{\kappa\sigma^0}$$

$$\sigma^i \to \sigma^i + \zeta^i(\vec{\sigma}) + \#\partial_i\gamma(\vec{\sigma}) e^{\kappa\sigma^0}$$

Hydro symmetries

New prediction

New horizon symmetries

Leads to an **effective field theory for chaotic systems*** (certain out-of-time ordered correlators exhibit exponential growth) [Hong Liu, Hyunseok Lee, Mike Blake 2018]

$$-\langle [V(t), W]^2 \rangle \sim \frac{1}{N} e^{2\pi T t}$$

Postulated as symmetries of **chaotic systems**

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Hydro symmetries

New prediction

They give rise to a consistent field theory for hydrodynamics

* Maximally chaotic, no momentum conservation

Thank you!

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The equations of motion

For example, to write a Lorentz covariant $T^{\mu\nu}$ it is convenient to couple the theory to an external source $g_{\mu\nu}$ and require diff invariance

The pullback sources:
$$g_{ij} = \frac{\partial X^\mu}{\partial \sigma^i} \frac{\partial X^\nu}{\partial \sigma^j} g_{\mu\nu}(X(\sigma))$$

Sigma-model Lagrangian:
$$S_{eff} = \int d^d\sigma \sqrt{-g} \mathcal{L}(g_{ij})$$

The constitutive relations:
$$T^{ij} = \frac{2}{\sqrt{-g}} \frac{\delta S_{eff}}{\delta g_{ij}}$$

E.o.m.:
$$0 = \delta_X S_{eff} = \int d^d\sigma \sqrt{-g} (\nabla_i T^{ij}) \partial_j X^\mu g_{\mu\nu} \delta X^\nu$$

Fluid d.o.f. in gravity

- Start from any metric: $ds^2 = G_{AB} dx^A dx^B$
- Perform a diff to bring it to a specific form $ds^2 = \tilde{G}_{AB} d\tilde{x}^A d\tilde{x}^B$
- Set one end $\tilde{x}^A\big|_{bry} = (const, x^\mu)$
- At the other end $\tilde{x}^A\big|_{hor} = (\tau(x), \sigma^\mu(x))$

Gapped mode

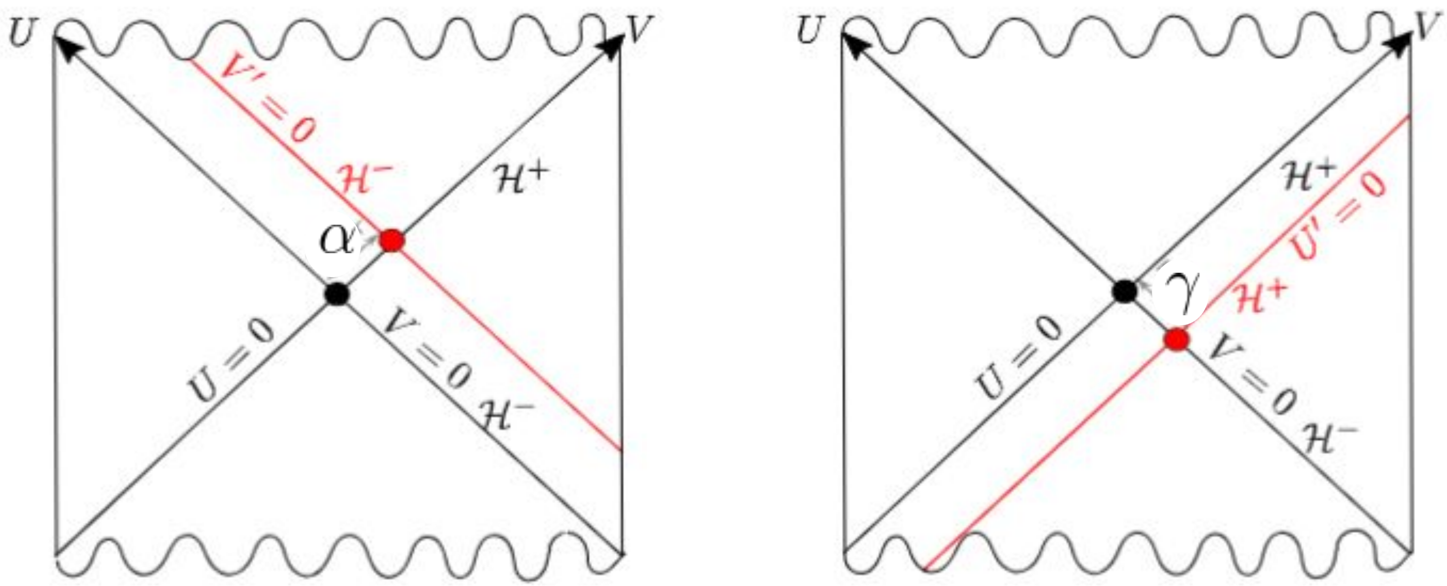
Hydro d.o.f.

- Equations for $\sigma^\mu(x)$ turn out to be Einstein's constraint equations that in turn correspond to the equations for fluid dynamics

Horizon symmetries: Shock waves

- In Kruzkal-Skezeres coordinates*: $V = e^{k(t+r^*)}, \quad U = -e^{k(t-r^*)}$

$$V \to V + \lambda(\vec{\sigma})V + \alpha(\vec{\sigma}) + \gamma(\vec{\sigma})V^2$$
$$U \to U - \lambda(\vec{\sigma})U + \gamma(\vec{\sigma})$$



* In a gauge where $G_{rr} = 0$

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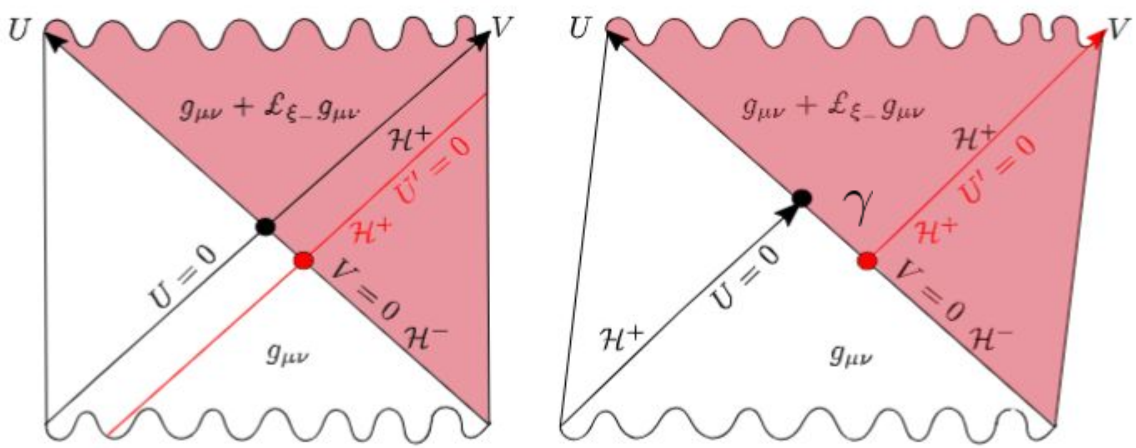


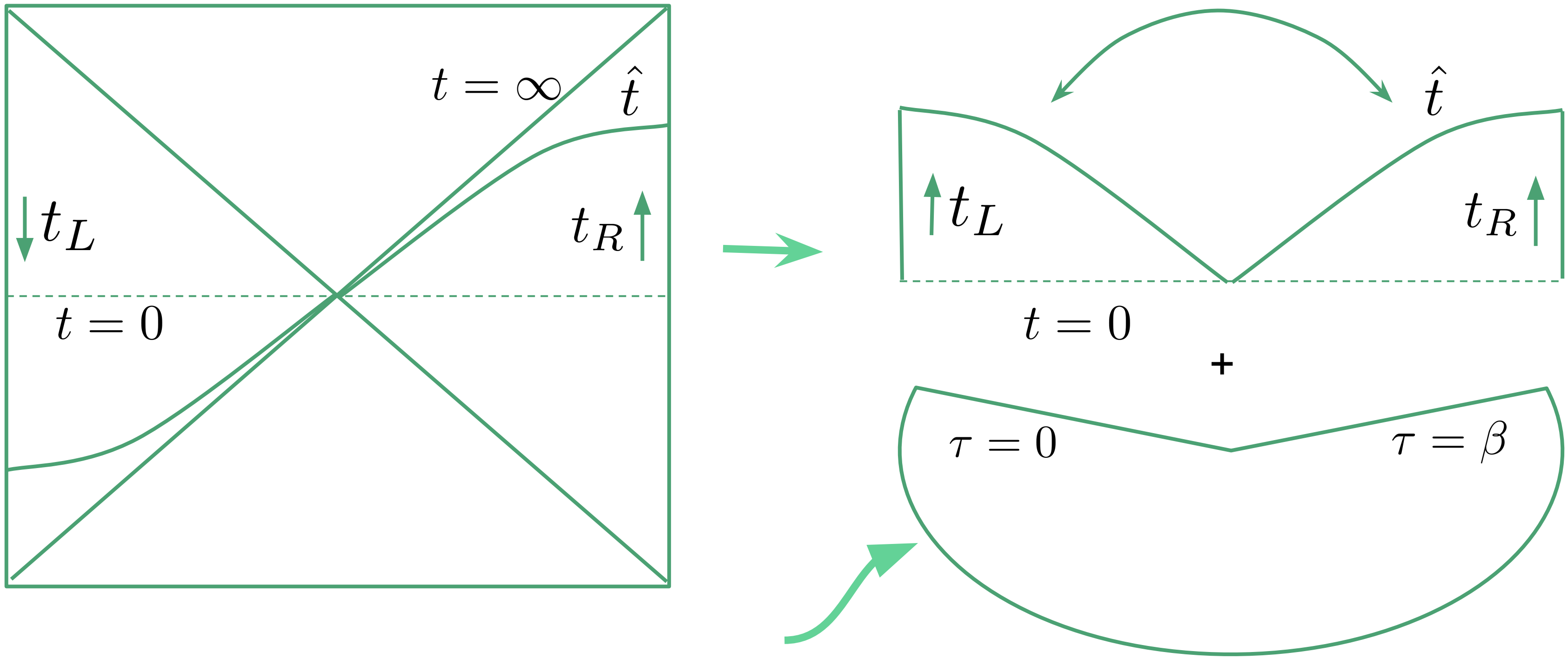
- The shift symmetry generates the RHS of a shock wave solution

* In a gauge where $G_{rr} = 0$

The gravity counterpart

Can be constructed using the real-time holography formalism

[K. Skenderis and B. Van Rees (2008)]



KMS symmetry (**3**)

[J. de Boer, M. P. Heller, N.P.F. 1812.06093]

Fluid d.o.f. in gravity: e.o.m.

- The action for the new metric is

$$S[G_{AB}] = S[\tilde{G}_{AB}, \tilde{x}^A]$$

- The variation of the action

$$\delta S = \frac{1}{2}\int d^D x \sqrt{-G} E^{MN} \delta G_{MN}$$

- Variation w.r.t. the dynamical d.o.f.

$$0 = \frac{\delta S}{\delta \tilde{x}^\mu} \longrightarrow E^{\mu r} = 0 \longrightarrow \nabla_\nu T^{\mu\nu} = 0$$