Binary BHs inspirals in dark matter cusps. Effect of DM dynamical friction.

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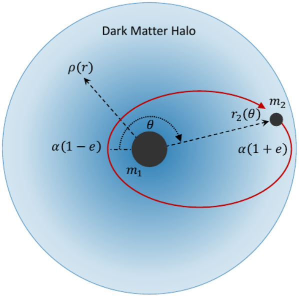
Context

- The presence of DM spikes around massive BHs would impact the dynamics of inspiralling IMRIs and EMRIs, which would be imprinted in the phase evolution of GWs emitted by the system
- Gravitational waves (GWs) are a promising avenue for constraining and perhaps even detecting Dark Matter (DM)
- DM dephasing is a promising signature for future GW observatories, detecting dephased signals and correctly inferring their properties will require accurate waveforms. This in turn requires accurate modelling of the dynamics of binaries embedded in DM spikes.

Context

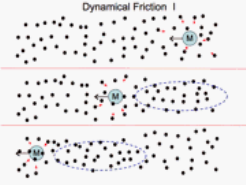


We consider a BH with mass M_2 orbiting a larger BH of mass M_1 embedded in a DM spike with density

$$\rho_{\rm DM}(r) = \rho_6 \left(\frac{r}{r_6}\right)^{-\gamma_{\rm sp}} \left(1 + \frac{r}{r_t}\right)^{-\alpha}$$

where $r_6 = 10^{-6}\,\mathrm{pc}$ is a reference radius; ρ_6 specifies the normalization of the DM spike; and $\gamma_{\rm sp}$ is the inner density slope

Dynamical friction



- DF is an important physical phenomenon, with consequences in both stellar dynamics and plasma physics.
- Slowing-down of a test particle of mass M, (or charge q) moving at v_{T} in a background of field particles with mean number density n and velocity distribution $f(\mathrm{v}_{\mathrm{F}})$, due to the cumulative effect of multiple $1/r^2$ interactions.

Dynamical friction: the classical case

Chandrasekhar (1943) estimation of the DF in stellar dynamics is:

$$\frac{d\mathbf{v}_{\rm T}}{dt} = -4\pi G^2 nm(M+m)\log\Lambda\frac{\Xi(v_{\rm T})}{v_{\rm T}^3}\mathbf{v}_{\rm T},$$

where

$$\Xi(v_{\rm T}) \equiv 4\pi\int_0^{v_{\rm T}} f(v_{\rm F})v_{\rm F}^2 dv_{\rm F} \qquad (1)$$

is the fractional velocity volume function and $\log\Lambda$ is the so-called Coulomb logarithm of $b_{\rm max}/b_{\rm min}$. For $M \gg m$

$$\frac{d\mathbf{v}_{\rm T}}{dt} = -4\pi G^2 M\rho\log\Lambda\frac{\Xi(v_{\rm T})}{v_{\rm T}^3}\mathbf{v}_{\rm T},$$

where $\mu = Mm/(M+m)$ is the reduced mass.

Dynamical friction: Special relativistic case

Following the classical derivation and making use of relativistic composition of velocities instead, one has (Chiari & Di Cintio 2023):

$$\frac{d\mathrm{v}_{\mathrm{T}}}{dt} = -4\pi G^2 \frac{(M+m)^2}{M} \rho \gamma_{v_{\mathrm{T}}} \log \Lambda \int \frac{\gamma_{v_{\mathrm{F}}} \mathcal{V} f(\mathbf{v}_{\mathrm{F}})(\mathrm{v}_{\mathrm{T}} - \mathrm{v}_{\mathrm{F}})}{V^4} d^3\mathbf{v}_{\mathrm{F}}$$

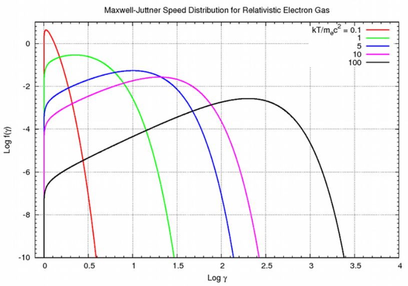
where

$$\mathcal{V} \equiv V\left(1 - \mathbf{v}_{\mathrm{T}} \cdot \mathbf{v}_{\mathrm{F}}/c^2\right) = V\left(1 - v_{\mathrm{T}} v_{\mathrm{F}} \cos\phi/c^2\right); \quad V = ||\mathbf{v}_{\mathrm{T}} - \mathbf{v}_{\mathrm{F}}||.$$

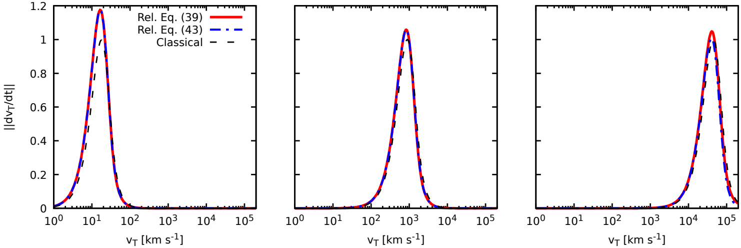
Dynamical friction: Special relativistic case

When considering a thermalized relativistic gas, a natural choice is the Maxwell-Jüttner distribution

$$f(v_{\mathrm{F}}) = \frac{\gamma_{v_{\mathrm{F}}}^5 v_{\mathrm{F}}^2}{c^3 \Theta \mathcal{K}_2(\Theta^{-1})} \exp\left(-\frac{\gamma_{v_{\mathrm{F}}}}{\Theta}\right). \tag{2}$$



Dynamical friction: Special relativistic case



Expressions differ significantly around σ.

Assuming relativistic velocities and using in lieu of $1/r^2$ the 1PN term

$$\mathbf{a}_{1\mathrm{PN}} = -\frac{G(M+m)}{r^2}\frac{\mathbf{r}}{r} + \frac{G(M+m)}{c^2 r^2}\left\{\left[(4+2\eta)\frac{G(M+m)}{r} + \right.\right.$$
$$\left.\left. - (1+3\eta)V^2 + \frac{3}{2}\eta\,\dot{\mathbf{r}}^2\right]\frac{\mathbf{r}}{r} + (4-2\eta)\dot{\mathbf{r}}\,\mathbf{V}\right\}.$$

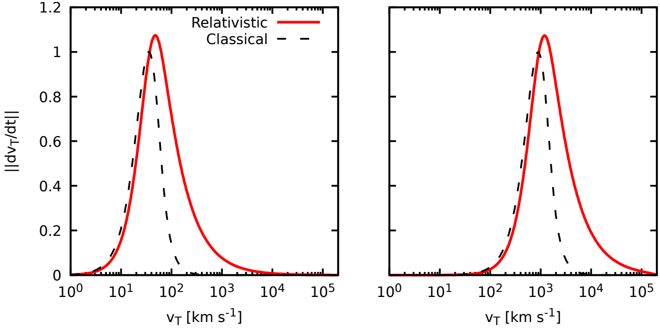
Dynamical friction: Post-Newtonian treatment

The dynamical friction accounting for both relativistic velocities and 1PN corrections becomes

$$\frac{dv_{\mathrm{T}\|}}{dt} = -4\pi G^2 (M+m)\rho \frac{\log\Lambda}{\gamma_{v_\mathrm{T}} v_\mathrm{T}} \int \frac{f'(\mathbf{w}_\mathrm{F})}{\left[1 + \frac{4mMw_\mathrm{F}^2}{(M+m)^2c^2}\right] w_\mathrm{F}^3}$$

$$\left\{ \frac{\mathrm{v}_\mathrm{T}\cdot\mathbf{w}_\mathrm{F}}{\gamma_{v_\mathrm{T}}^2}\left[1 + \frac{(9Mm+m^2)w_\mathrm{F}^2}{2(M+m)^2c^2}\right] + \frac{m^2(\mathrm{v}_\mathrm{T}\wedge\mathbf{w}_\mathrm{F})^2}{(M+m)^2c^2} \right\} d^3\mathbf{w}_\mathrm{F},$$

Dynamical friction: Post-Newtonian treatment



Single particle dynamics

The motion of the secondary BH M_2 under the effect of the central potential of the primary $\Phi_{BH,1}$ and the discreteness effects due to the DM density is given

$$\frac{d^2\mathbf{r}}{dt^2} = -\nabla\Phi_{BH,1}(\mathbf{r}) - \eta\frac{d\mathbf{r}}{dt} + F_W$$

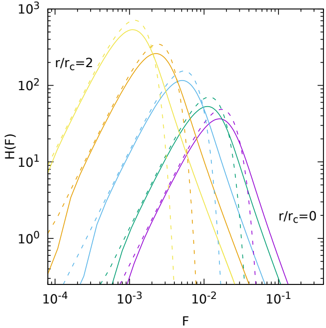
where η is either due to baryons only (in MOND) or to baryons and DM (in Newton) and the fluctuating force F_W is a fluctuating force per unit mass.

- for $M_2 \sim m$: $\eta; F \neq 0$
- for $m \gg M_2$ $\eta = 0$ and $F \neq 0$
- for $M_2 \gg m$: $\eta \neq 0$ and $F = 0$

Numerical methods: Langevin equation

We assume two different distributions of local random kicks. 3D Gaussian and Holtsmark (1911) distribution (Chandrasekhar & von Neumann 1942,1943):

$$H(F) = \frac{2}{\pi F}\int_0^\infty \exp\left[-\alpha(\xi/F)^{3/2}\right]\xi\sin(\xi)\mathrm{d}\xi; \quad \alpha = \frac{4}{15}(2\pi G m_*)^{3/2} n_*$$



Fat tailed distribution. For large forces (small mean inter-particle distance) $\tilde{H}(F) \sim 2\pi n_*(Gm_*)^{3/2}F^{-5/2}$

Numerical methods: Langevin equation

Stochastic ODEs, like Langevin equations, are not an easy computational task. We adopt the robust quasi-symplectic Mannella (2004) scheme:

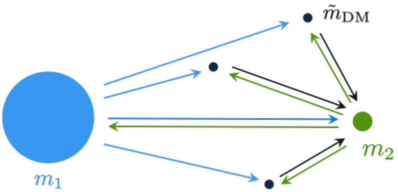
$$
\begin{aligned}
x(t+\Delta t/2) &= x(t) + \frac{\Delta t}{2} v(t) \\
v(t+\Delta t) &= c_2 \left[c_1 v(t) + \Delta t \nabla \Phi(x') + d_1 \tilde{F}(x') \right] \\
x(t+\Delta t) &= x(t+\Delta t/2) + \frac{\Delta t}{2} v(t+\Delta t).
\end{aligned}
$$

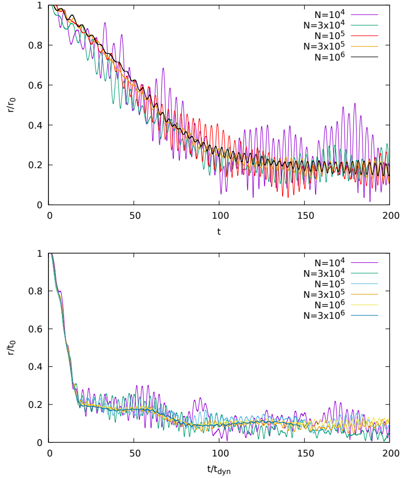
where:

$$
c_1 = 1 - \frac{\eta \Delta t}{2}; \quad c_2 = \frac{1}{1+\eta\Delta t/2}; \quad d_1 = \sqrt{2\zeta\eta\Delta t}.
$$

For $\eta; \zeta = 0$ becomes the standard second order *leapfrog*.

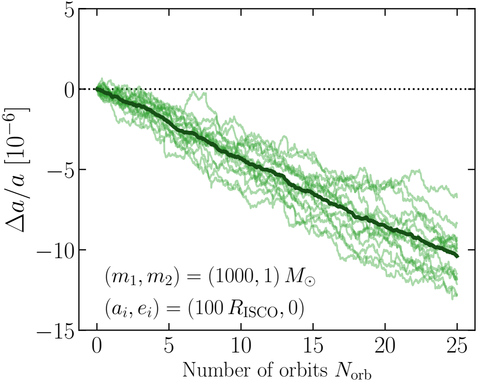
One can compute the dynamics of M_2 around M_1 with in a standard N-body code, where the interaction with N DM (macro) particles is evaluated only with M_1 and M_2





N=10^4
N=3x10^4
N=10^5
N=3x10^5
N=10^6
r/r0
t
N=10^4
N=3x10^4
N=10^5
N=3x10^5
N=10^6
N=3x10^6
r/t0
t/tdyn

Evolution of the mass density



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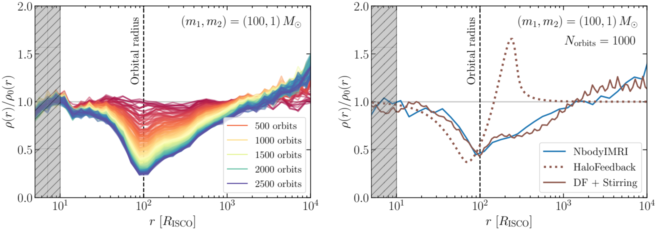
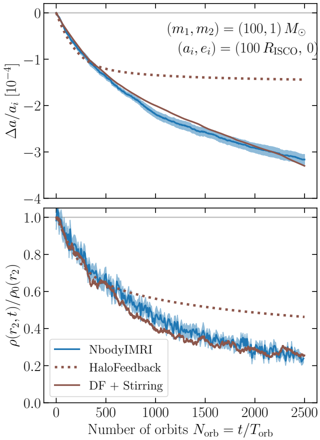
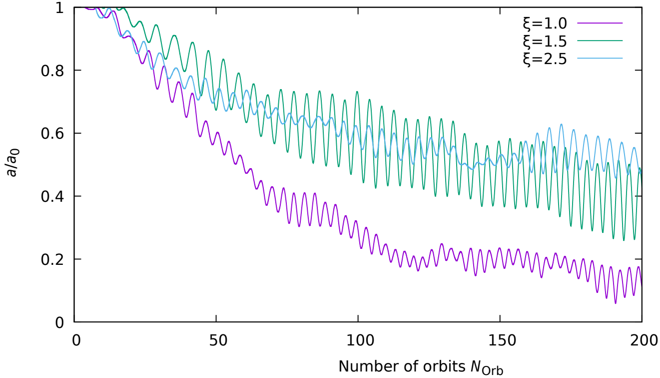


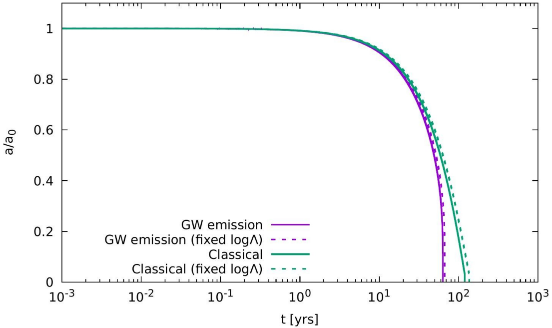
Evolution of the mass density



Effect of the velocity anisotropy



Effect of the GW emission



Summary and outlook

- DM dynamical slightly enhances the orbital decay with respect to GW emission only
- The efficiency of DF depends strongly on DM distribution function
- The nature of DM also possibly affects DF, relativistic treatment on the way.