QCD in Florence: the legacy of Stefano Catani

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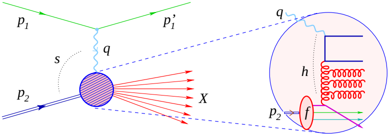
Fundamentals of QCD

- Stefano began his studies on QFT in Florence with Marcello Ciafaloni (supervisor)
- QCD was recognized as the right QFT of strong interactions
  - Asympt. freedom $\Longrightarrow$ Successful in inclusive hard processes
  - Very rich and difficult (strong, confinement, IR divergencies)
- Fundamental constituents not observable, hadrons are strongly interacting composite states
- in order to test QCD and to provide predictions one has to devise proper observables,
  - insensitive to NP dynamics
  - whose aspects can be controlled by perturbative calculations
  - free of IR divergences
- Stefano gave important contributions on both aspects:
  - to the definition of the so-called IR-safe observables
  - to precise PT calculations of many important processes

Factorization and Resummation

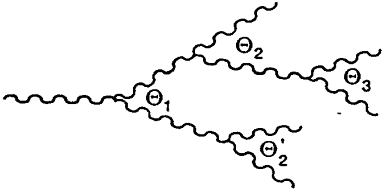
Keywords of perturbative QCD (in particular in Florence)

- Factorization: some observable can be decomposed $\sigma = f \otimes h$
  - non-perturbative but universal term f (PDFs)
  - perturbative process-dependent term h (hard cross sections)
- In the high-energy regime $s \gg Q^2$ the PT series is affected by large $\log(s/Q^2)$ so that $\alpha_s \log(s/Q^2) \sim 1$
- Resummation:
  - Compute leading log. terms $[\alpha_s \log(s/Q^2)]^n$ to all orders
  - Sum all those terms (evolution equations for f and h)



QCD coherence

- *[M. Ciafaloni '87]* discovered the phenomenon of QCD coherence: the double logarithmic contributions $[\alpha_s \log Q^2 \log s]^n$ to high-energy scattering stem from gluon emissions with angular ordering
- *[S. Catani, Fiorani, Marchesini '89]* studied in more detail MC's work leading to the celebrated CCFM equation, which is one of the keystone of high-energy PT QCD
- It improves the PDFs at small-x ($x = Q^2/s$) with modified branching kernels and form factors



$$\theta_1 < \theta_2 < \theta_3$$

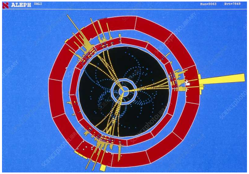
$$\theta_1 < \theta_2' < \cdots$$

High-energy factorization

- *[S. Catani, M. Ciafaloni, F. Hautmann '93]* proposed and proved a new factorization formula (high-energy factorization) $\sigma = F \otimes H$ where F is the "transverse-momentum dependent" gluon density.
- Here the high-energy logs can be computed and resummed to all orders in the (N)LLA, thus paving the way to realistic calculations at high energies.
- *[G. Camici, M. Ciafaloni '98]* resummed also the NL logs $\alpha_s^n \log^{n-1} s$

Jets

- Factorization and resummation are usually possible for inclusive processes, hadronization dynamics is not (so) important



- In many events partons leave their footprint in the form of jets
- The detailed structure of a jet is NP but the global structure of jets isn't
  it can provide much more information on QCD dynamics
- In order to be quantitative, a jet must be precisely defined
  [S. Catani, Y. Dokshitzer, B. Webber '93] proposed an algorithm for an IR-safe reconstruction of jets (kt-algorithm)
- Very practical both experimentally and theoretically
- In the last decade it is by far the most common algorithm used

Two-loop structure of QCD

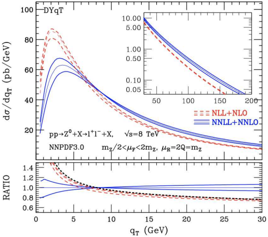
- Development of QCD naturally led to calculations at higher orders: QCD enters its precision era (1%)
- The presence of IR singularities poses conceptual and practical problems in the calculation of observables, whose difficulty increases with PT orders.
- *[S. Catani -98]* wrote a fundamental paper analysing the IR singularities of QCD in a generic two-loop calculation. Starting from his analysis, any NNLO calculation in QCD became possible.

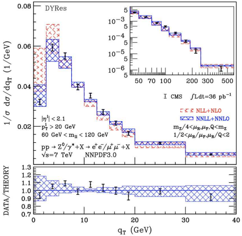
The two-loop coefficient subamplitude $\mathcal{M}_m^{(2)}$ has $1/\epsilon^4, 1/\epsilon^3, 1/\epsilon^2$ and $1/\epsilon$ poles. Because of the increased degree of singularities, it is not a priori guaranteed that all of them can be controlled by a *universal* factorization formula as in the one-loop case. The main result presented in this paper is that such a factorization formula does exist and can be written in the following form

$$\begin{aligned} |\mathcal{M}_m^{(2)}(\mu^2;\{p\})\rangle_{\rm RS} &= \boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\})\,|\mathcal{M}_m^{(1)}(\mu^2;\{p\})\rangle_{\rm RS} \\ &+ \boldsymbol{I}^{(2)}_{\rm RS}(\epsilon,\mu^2;\{p\})\,|\mathcal{M}_m^{(0)}(\mu^2;\{p\})\rangle_{\rm RS} + |\mathcal{M}_m^{(2){\rm fin}}(\mu^2;\{p\})\rangle_{\rm RS}\;. \quad (18) \end{aligned}$$

Transverse momentum resummation

- *[S. Catani, M. Grazzini et al. '02]* introduced transverse momentum resummation of heavy particles (heavy quarks, EW bosons, Higgs, . . .) produced at hadron colliders
- Perturbatively the cross section is more and more divergent when $q_T \to 0$, due to soft gluon emissions
- For $q_T \to 0$ they were able to show exponentiation in a vanishingly small cross section
- if $0 < q_T \ll m_H$ resummation has been performed at NNLO



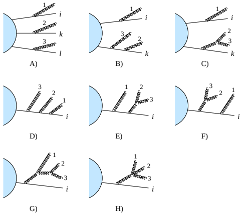


Soft currents

My work with Stefano: soft theorems
Given a generic QCD amplitude, if some external momenta $q_1, \cdots, q_n$ are much softer than the others $p_1, \cdots, p_k$ (hard momenta), the amplitude factorizes:

$$|M_{k+n}(p,q)\rangle = \hat{J}_k(p,q)\,|M_k(p)\rangle$$

- 1 soft gluon *[Bassetto, Ciafaloni, Marchesini '83]*
- 2 soft gluons: *[Catani, Grazzini '98]*
- 1 soft gluon at 1-loop: *[Catani, Grazzini '00]*
- 3 soft gluons: *[Catani, DC, Torrini '20]*
- soft $g\,q\,\bar{q}$: *[Catani, Cieri, DC, Coradeschi '23]*



Conclusions

- This was just a short and non-exhaustive list of Stefano's contributions to QCD
- Besides its scientific excellence,
  Stefano deserves esteem and respect for his personal qualities,
  having been a very friendly colleague,
  often available for discussions and help,
  modest with students and younger collaborators,
  many of which are researcher in Italy or abroad.
  I am sure they share my opinion and good memories