Florence Theory Group Day 25/03/2024

Applications of Quantum Channel Discrimination

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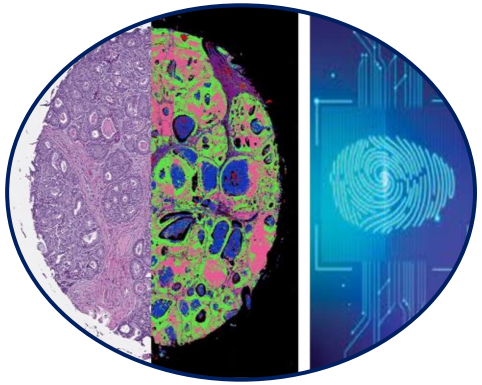
Quantum metrology and Hypothesis testing

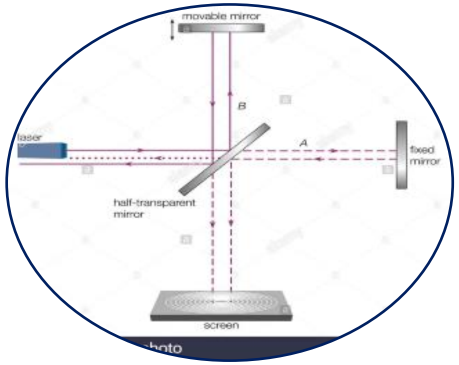
Imaging and pattern recognition



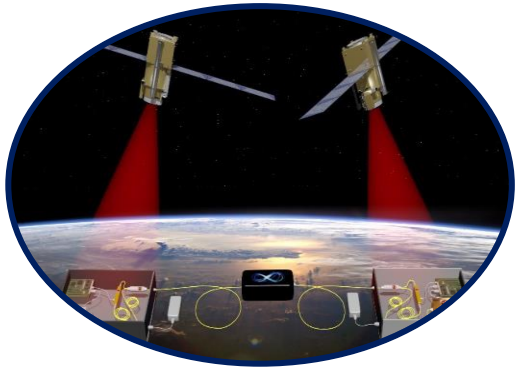
Optical memories readout

Beating the **classical limits** in the **estimation/discrimination** of a **parameter** exploiting **quantum resources**

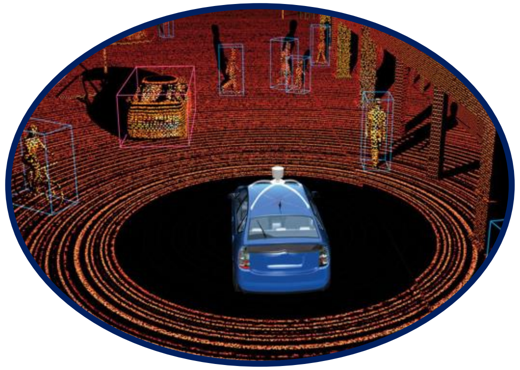
Interferometry



Characterization of quantum technologies

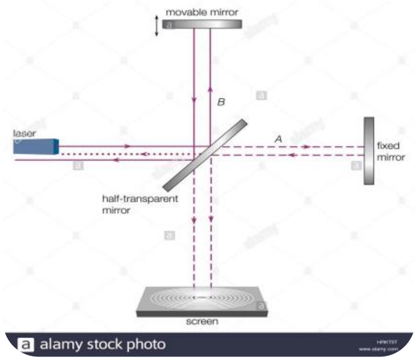


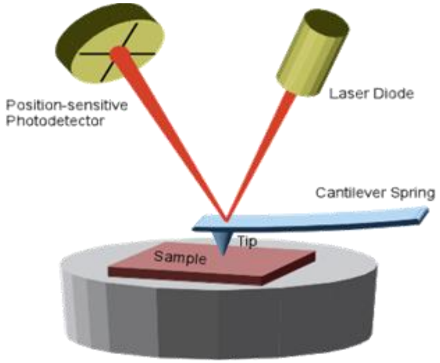
Target detection and ranging

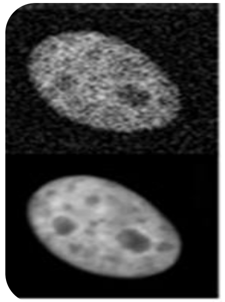


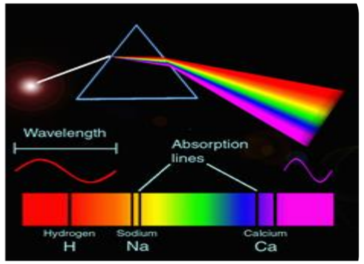
Classical limit: The Shot Noise Limit

Shot Noise Limit (SNL)/Standard Quantum Limit (SQL) comes form the discrete nature of the light and sets the lower bound to the sensitivity of conventional (classical) optical measurements









phase (interferometry)

$$\Delta\varphi_{\min} \approx \frac{1}{\sqrt{N}}$$

Displacement/spatial resolution

$$\Delta x_{\min} \approx \frac{X_R}{\sqrt{N}}$$

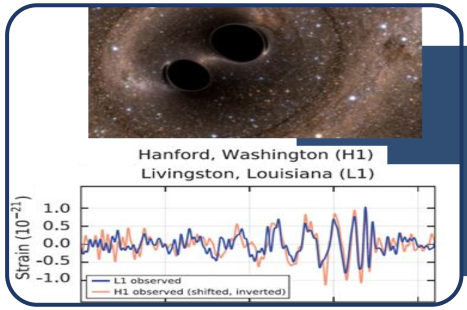
Loss/transmission

$$\Delta\alpha_{\min} \approx \frac{\sqrt{1-\alpha}}{\sqrt{N}}$$

$0 < \alpha < 1 =$ loss

Classical limit: The Shot Noise Limit

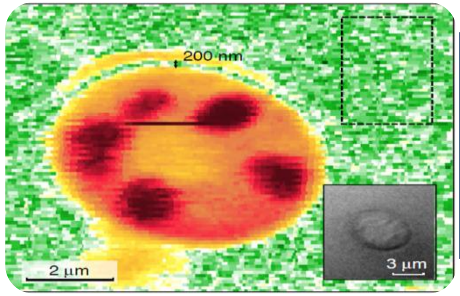
But increasing optical power is not always an option…



Super accurate measurements ->technical noise, quantum back action (GW detection)

$N = 10^{23}$ ph/s (100 kW)

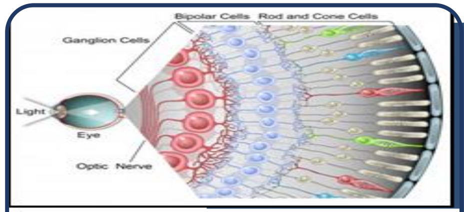
J. Aasi et al., Nat. Photonics 7, 613 (2013)



Probing delicate systems (biological samples, photosensitive chemicals)

$N < 10^{12}$ ph/s (< 1 µW at the sample)

Casacio et al. Nature, **594**, 201-205,(2021)



©Image courtesy of Jamie Simon, Salk Institute.

Investigate response of a system at few photons level (SPD, photo-transduction, photosynthesis…)

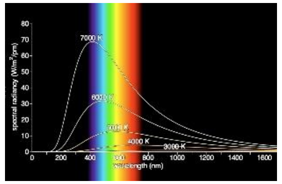
sensitivity threshld $N = 1\ ph!$

J.N. Tinsley *et al,* Nature Comm. 7:12172 (2016)

Classical limit: The Shot Noise Limit





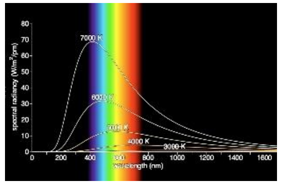


SNL coincides with the uncertainty statistical scaling of *N* independent repetitions (CENTRAL LIMIT THEOREM)

Classical limit: The Shot Noise Limit







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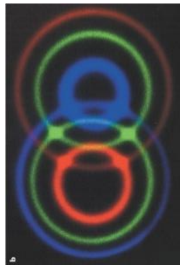
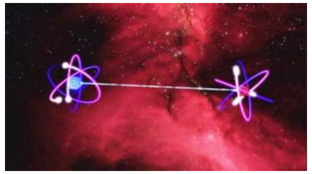
Quantum correlation: a degree of cooperation among particles which is not possible in the classical description



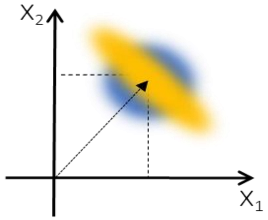
Extra information available in quantum states enables reducing the uncertainty

Overcoming the SNL: Quantum resources

- **Entanglement**

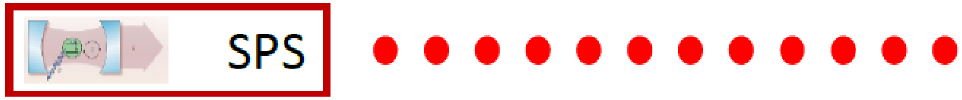


- **Squeezing**

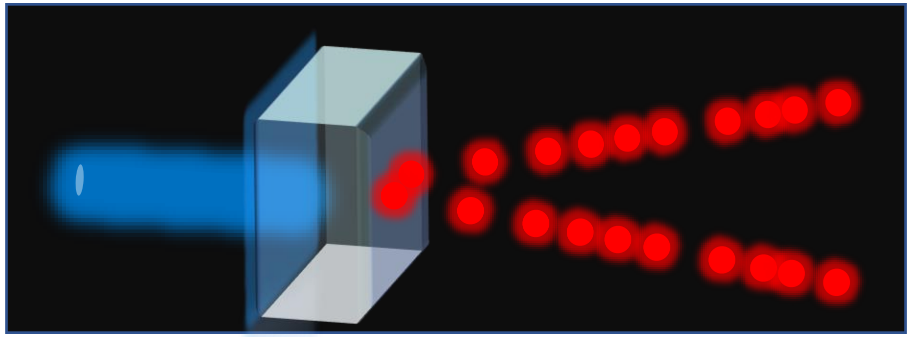


- **Sub-Poisson Light:**

  - **Single photon sources**



- **Photon number Non-Classical correlation**



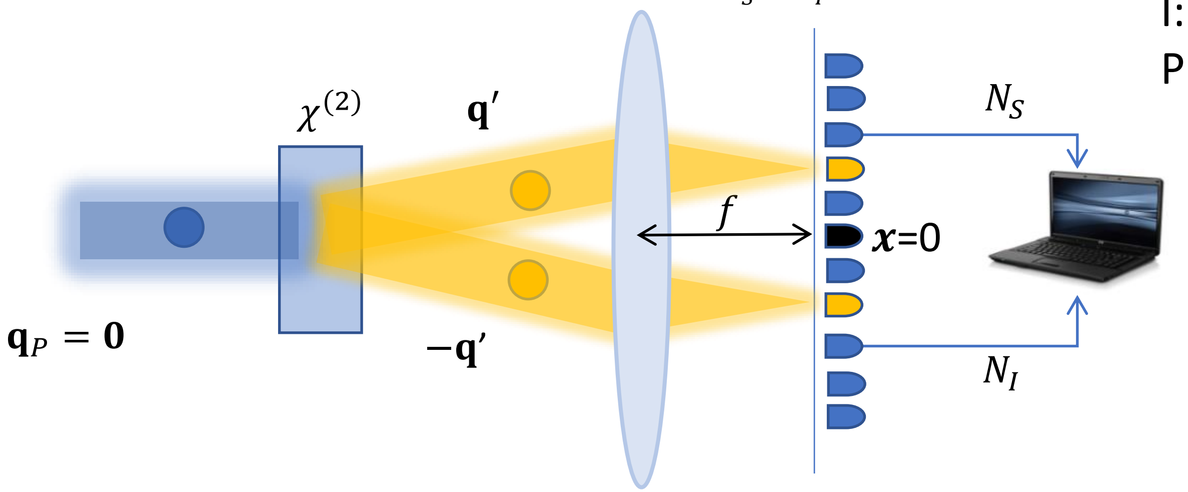
Non-Classical PN correlations: SPDC

Spontaneous Parametric Down Conversion (SPDC)

Energy and momentum conservation

$\omega_S + \omega_I = \omega_p$

$\boldsymbol{k}_P = \boldsymbol{k}_S + \boldsymbol{k}_I$



S: signal

I: Idler

P:pump

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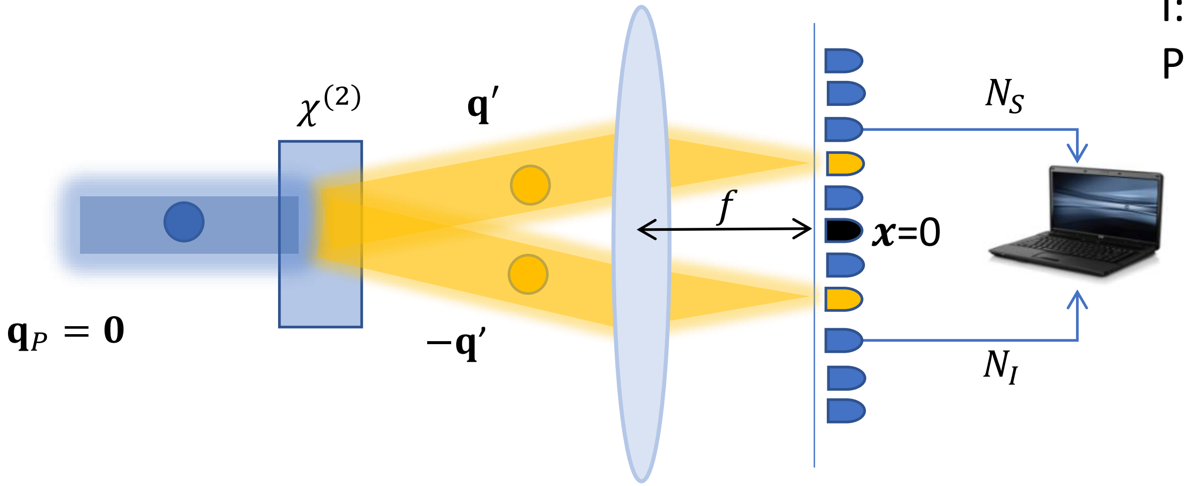

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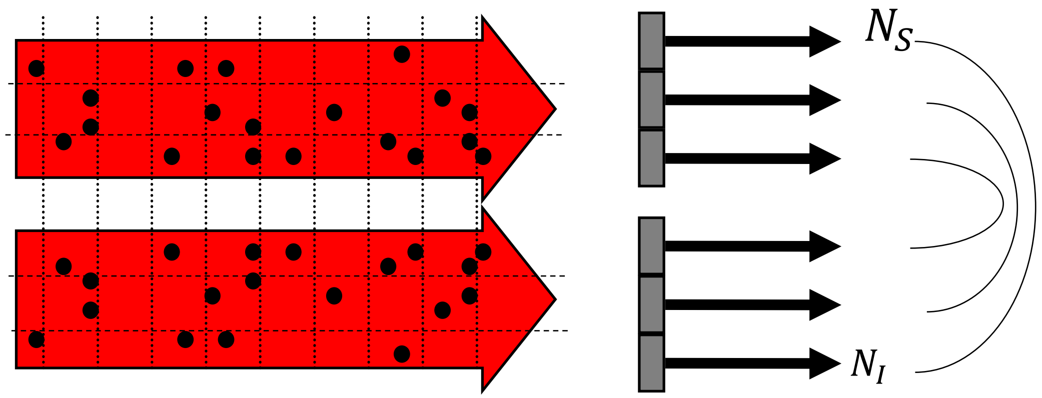
Strong **temporal** and **spatial** **photon number correlation**



Two-Mode Squeezed Vacuum (**TMSV**) state

$$|\mathrm{TMSV}\rangle_{S,I} \propto \sum_n \sqrt{\mathrm{P}_{\mu_0}(n)}|n\rangle_S|n\rangle_I.$$

$$\mathrm{P}_{\mu_0}(n) = \mu_0^n/(\mu_0+1)^{n+1}$$

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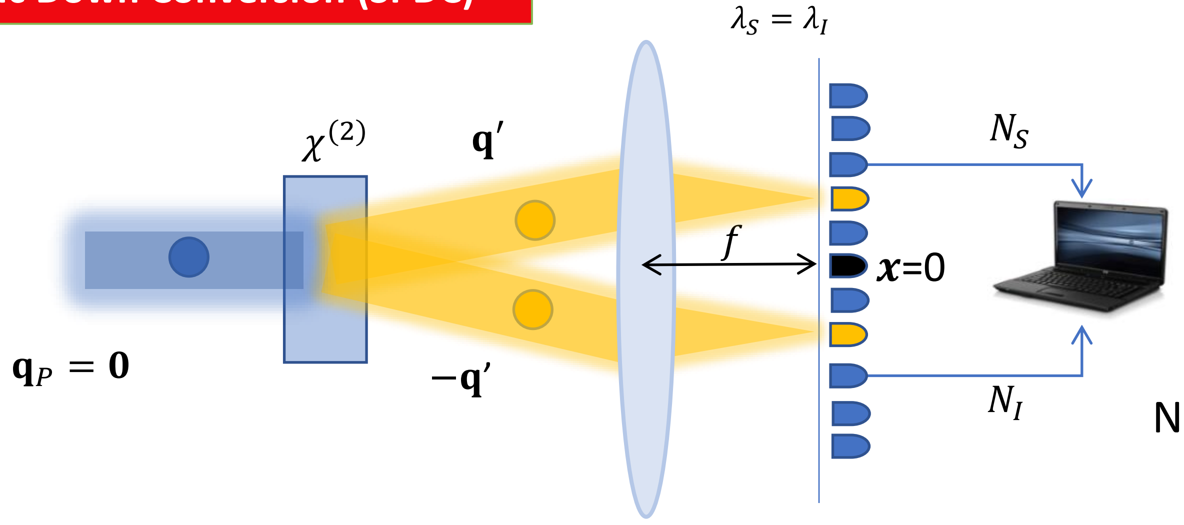
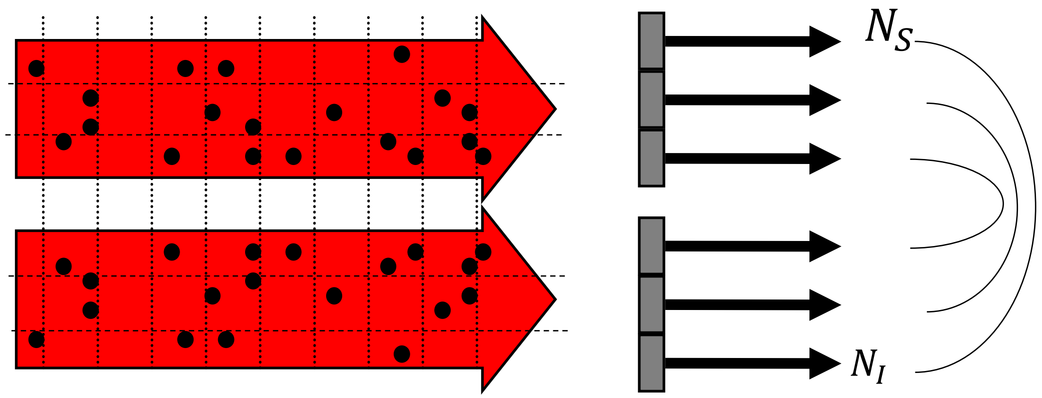


η:efficiency in detecting correlated photons

Noise Reduction Factor (NRF) $= \dfrac{Var(N_S - N_I)}{\langle N_S + N_I \rangle}$

Strong **temporal** and **spatial** **photon number correlation**



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$$\mathrm{P}_{\mu_0}(n) = \mu_0^n/(\mu_0+1)^{n+1}$$

$\mathrm{NRF} \geq 1$ For classical light

$0 \leq \mathrm{NRF} = 1 - \eta < 1$ For TMSV

NON-CLASSICAL

Quantum Hypothesis Testing

Binary state discrimination

$\mathcal{H}_0 \colon \rho = \rho_0$

$\mathcal{H}_1 \colon \rho = \rho_1$

Π

Helstrom formula

$$p_{err} = \frac{1}{2}(1 - ||\pi_1\rho_1 - \pi_0\rho_0||)$$

Quantum channel discrimination

$\sigma \longrightarrow \mathcal{E}_0 | \mathcal{E}_1 \longrightarrow$

$\rho_0 = \mathcal{E}_0(\sigma)$

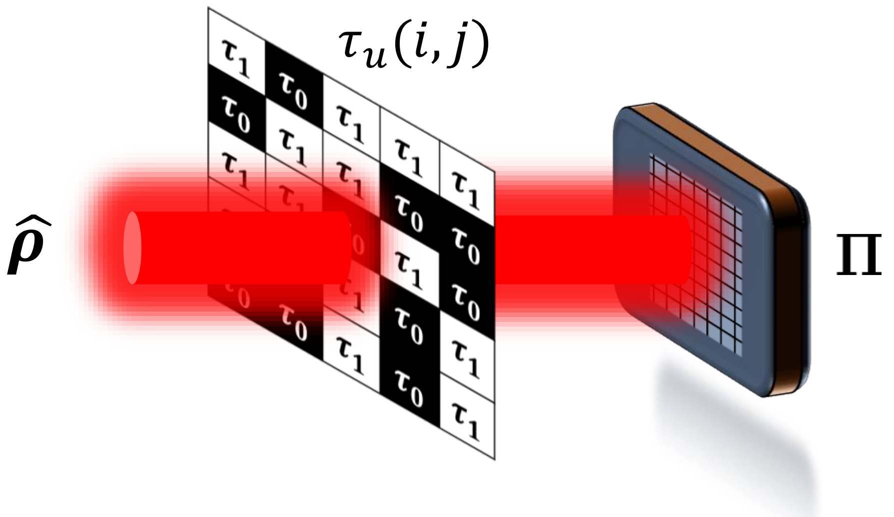
$\rho_1 = \mathcal{E}_1(\sigma)$

Discrimination of losses

$\mathcal{E}_{\tau_0}(\sigma)$ OR $\mathcal{E}_{\tau_1}(\sigma)$

Discrimination of bosonic losses

Discrimination of bosonic loss channels



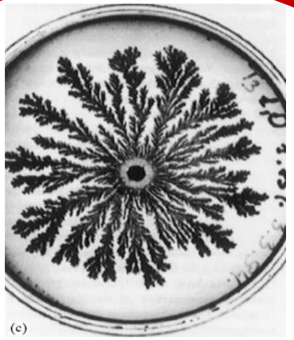
- Each cell (i,j) encodes a bit in two values of optical parameter τ_u (u =0,1)
- Loss channel discrimination

$v(i,j)$
Assigned bit value

$p_{err}(\hat{\rho}, \mathbf{\Pi})$
Error probability

Some Applications

Optical Memory



Spatial Pattern recognition

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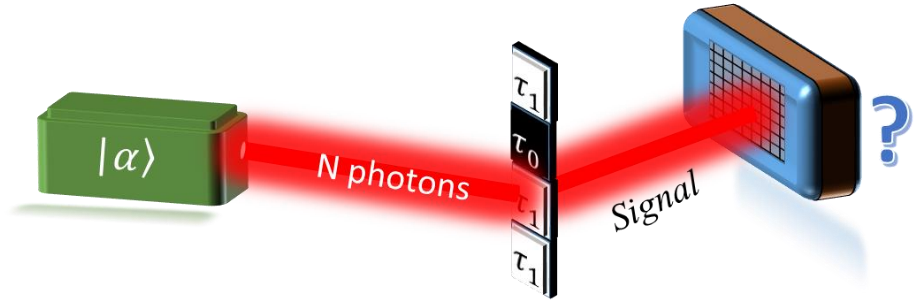
Quantum reading of a digital memory

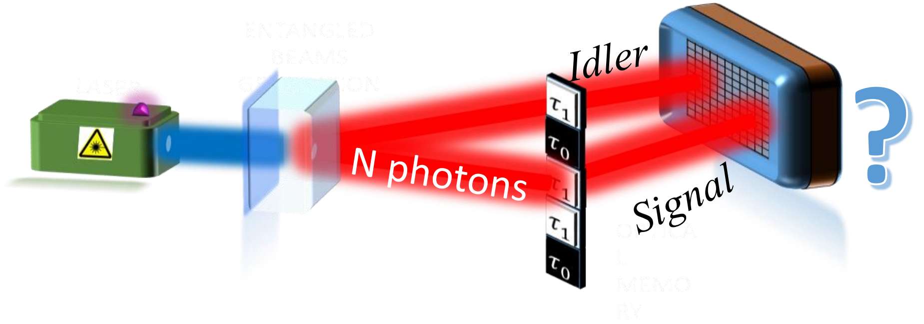
Optimal Classical bound



$$p_{err}^{cla} \geq \mathcal{C}(N, \tau_0, \tau_1) := \frac{1 - \sqrt{1 - e^{-N\left(\sqrt{\tau_1} - \sqrt{\tau_0}\right)^2}}}{2}$$

- Single coherent mode is the optimal classical transmitter $\otimes^M \rho_{SI} \rightarrow |\alpha\rangle_s$, $|\alpha|^2 = N$ (no ancillary modes needed)
- The optimal measurement is highly theoretical

A specific Quantum Transmitter



M replicas of Two Mode Squeezed Vacuum (TMSV)

$$|\mathrm{TMSV}\rangle_{S,I}^{\otimes M} \qquad |TMSV\rangle = \sum_{n=0}^{\infty} c_n |n\rangle_S |n\rangle_I$$

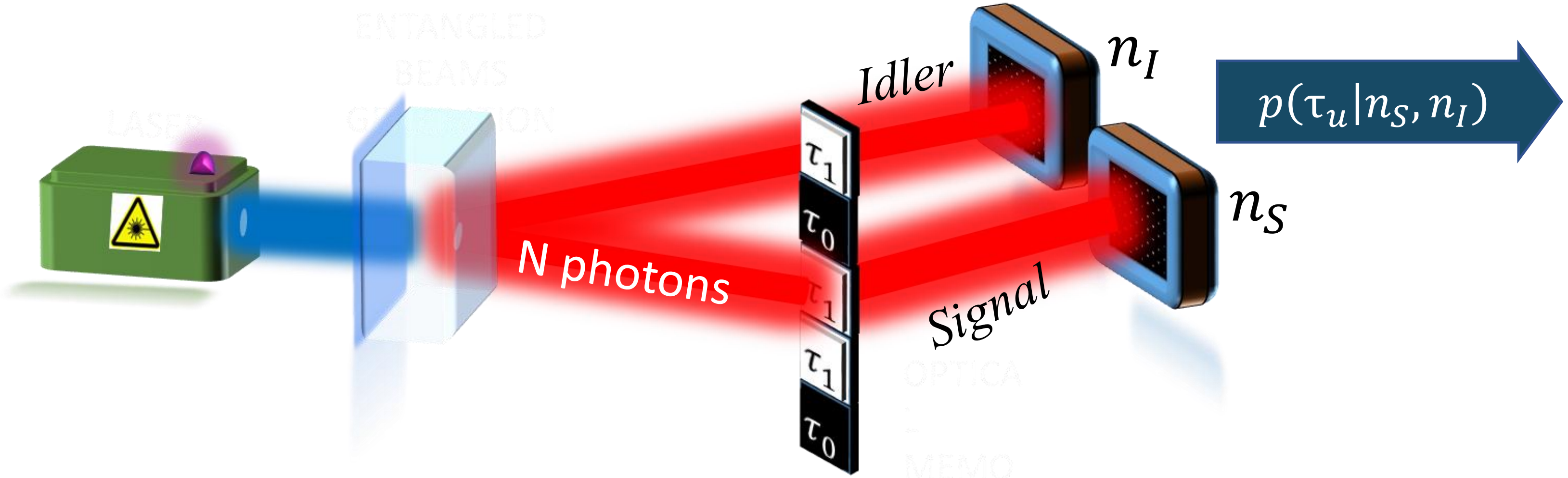
Given $N > N_{th}(\tau_0, \tau_1)$ There is $\overline{M}$

$$p_{err}^{TMSV} < \mathcal{C}(N, \tau_0, \tau_1)$$

Theoretical Quantum advantadge

Experimental quantum reading: Photon counting receiver

Practical Implementation: Photon Counting Receiver



M replicas of TMSV + **Local photon counting** + **Best Bayesian decision**

$|\mathrm{TMSV}\rangle_{S,I}^{\otimes M}$

n_S, n_I

$u = \mathrm{argmax}_u\, p(\tau_u|n_S, n_I)$

Performance is close to the one theoretically predicted for the optimal receiver

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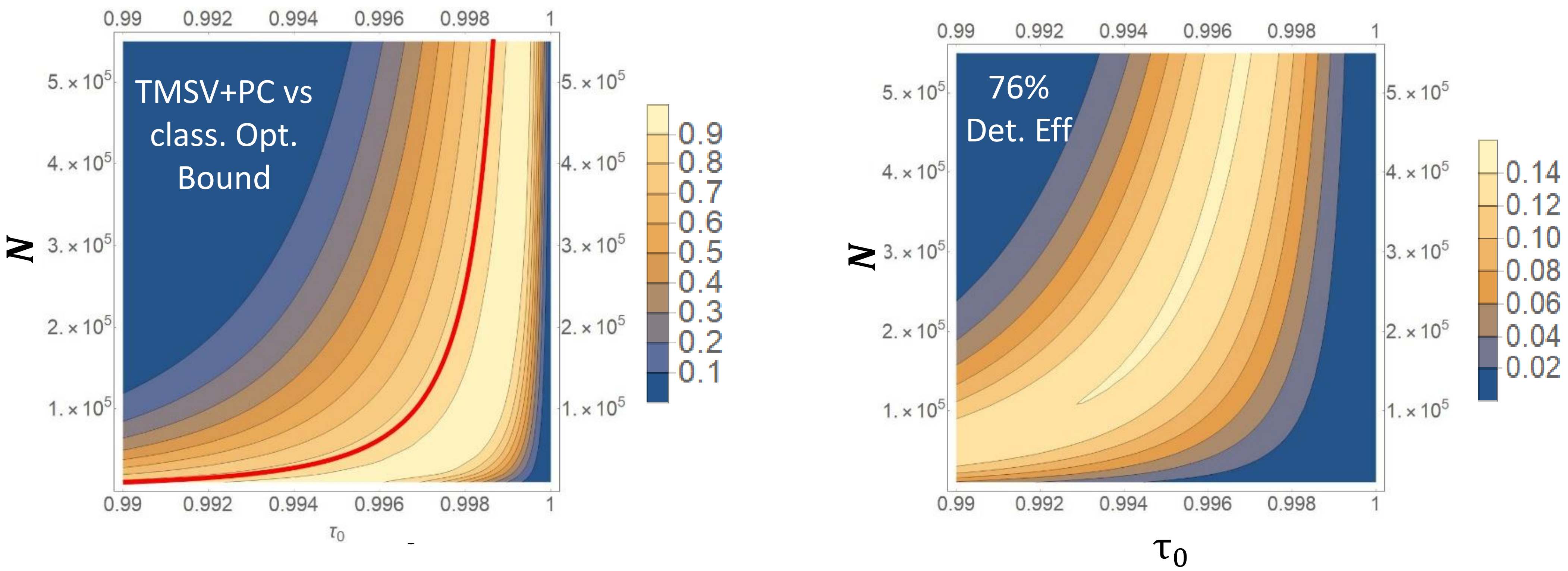
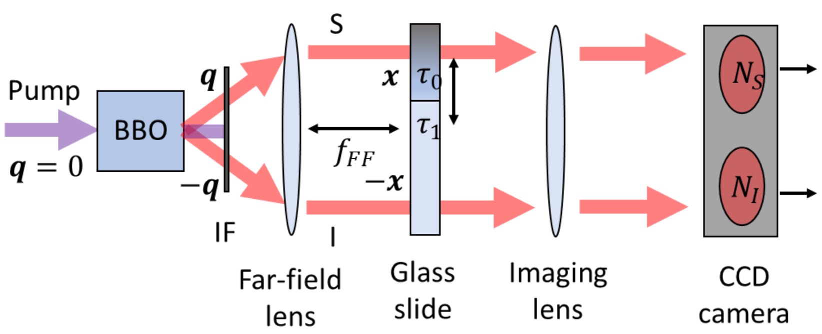
Experimental quantum reading: Results

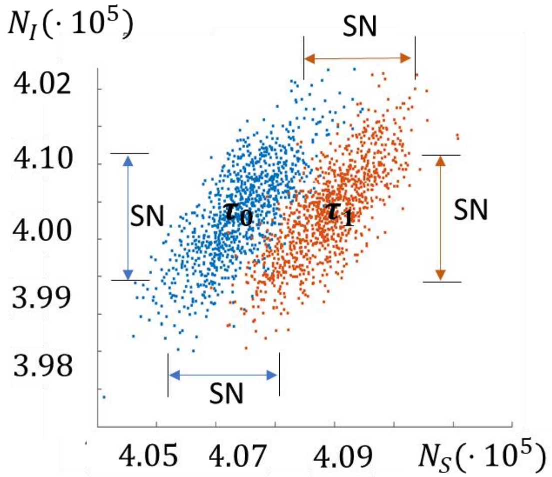
Theoretical Quantum Gain *(bits/cell) in function of transmissivity* $\boldsymbol{\tau_0}$ *mean number of photons* ***N***
(higher transmissivity is set to $\tau_1 = 1$*)*

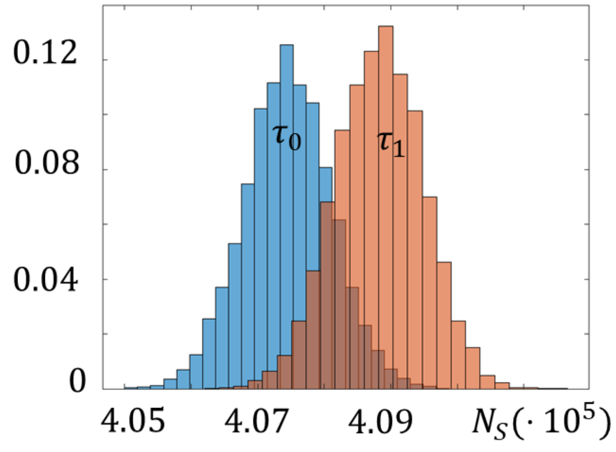


For certain region of the parameter space QR retrieves almost all the information while the best classical strategy completely fails!

Experimental quantum reading: Results



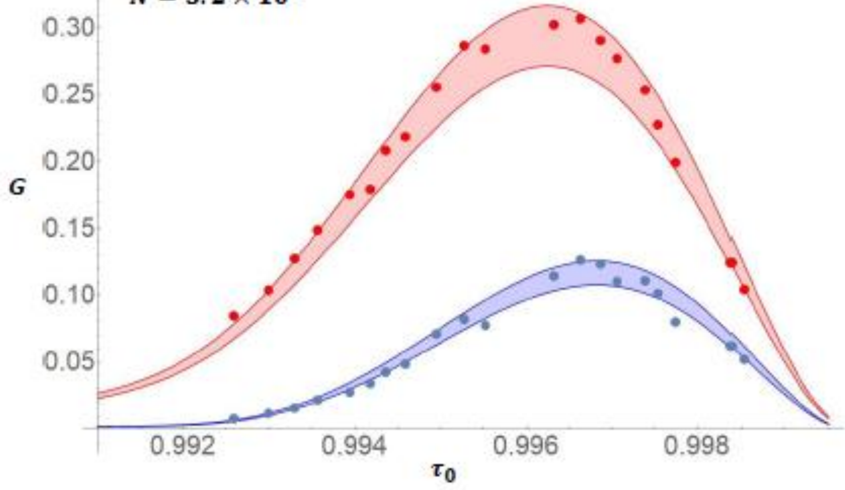




- Single measurement Time: 10 ms
- Spatio-temporal mode: $M \sim 10^{13}$
- Mean number of Photon: $N \sim 10^5$
- Transmittance (bit): $\tau_0 \sim 0.995, \tau_1 = 1$
- Detection efficiency: $\eta_S, \eta_I = 0.78$

$N/M << 1$
Poissonian marginal distribution (shot noise limited)

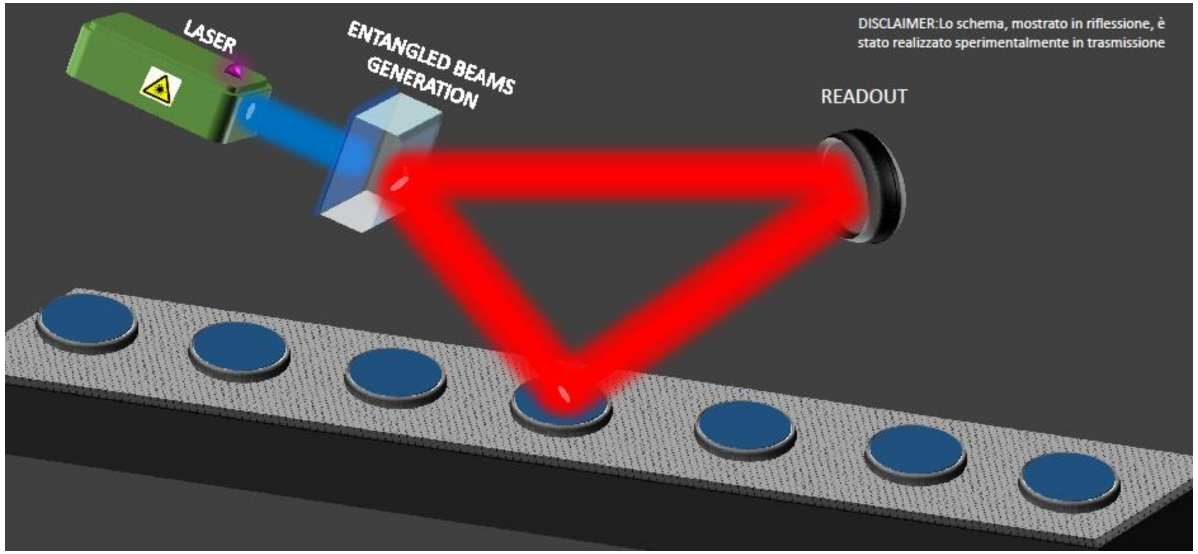
QUANTUM GAIN (BITS)

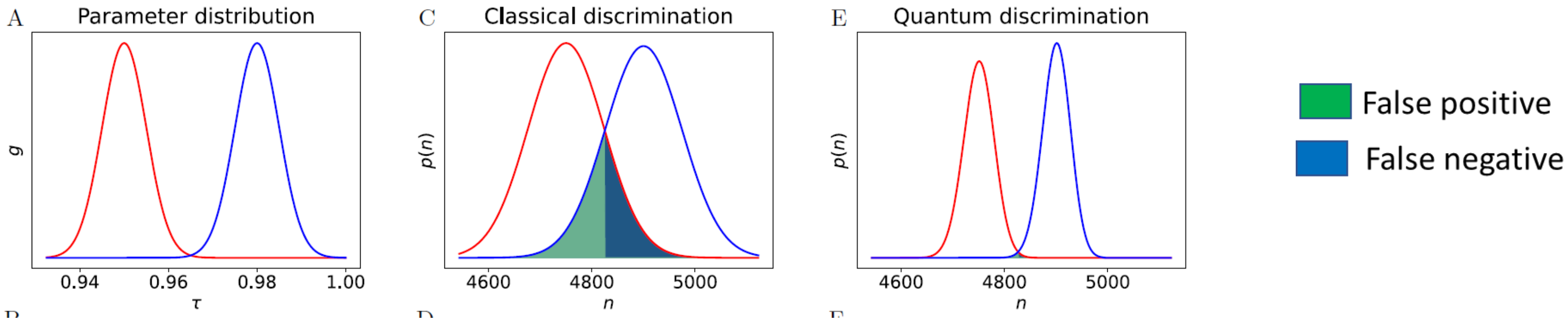


- **Gain up to 0.14 bit per cell compared to the optimal classical strategy (coherent state +unknown receiver)**
- **0.3 bit per cell compared to a the optimal classical strategy based on photon counting**

Distinguishing two distributions of losses

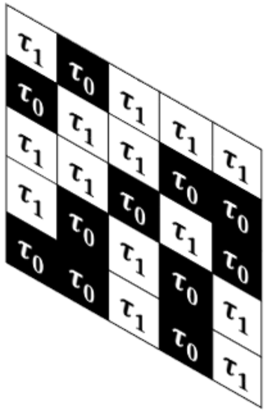
- **Production process monitoring (Quantum conformance test)**
- **Readout of imperfect memories**





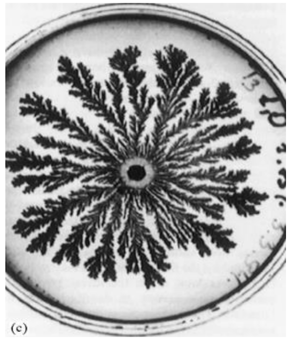
Quantum-enhanced Pattern recognition

From quantum reading/single cell task….

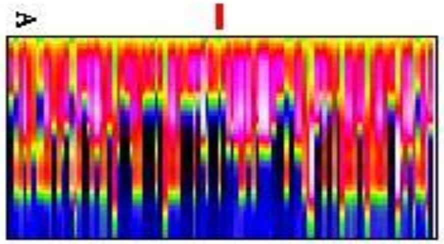


Optical Memory

…. To pattern recognition/multicell



Pattern recognition, e.g. biological structures (Petri Pattern)



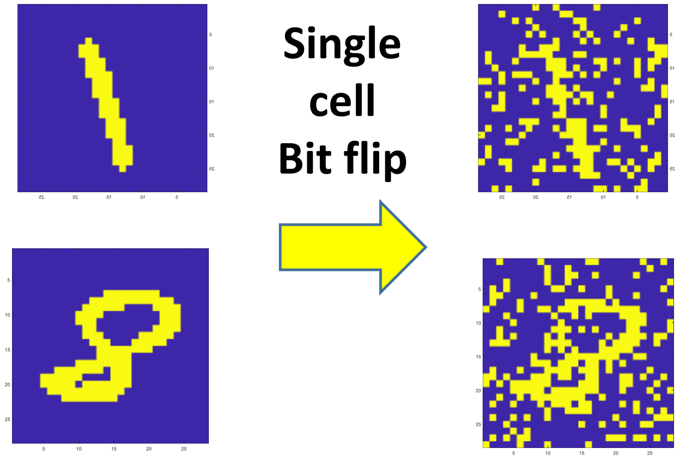
Fingerprints of a substance in spectroscopy

- The relevant information are global features (patterns) rather then just the sum of the individual cells → the task is the pattern classification/recognition

- Is the quantum advantage in sensing preserved when detecting global feature with complex post processing (e.g. machine learning)?

Quantum-enhanced Pattern recognition

Classification of handwritten digits



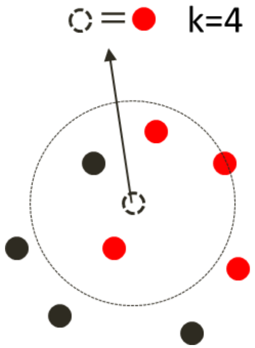
From pixel error to classification error:

$$p_{err}^{C} = \mathcal{M}_C(p_{err}^{px})$$

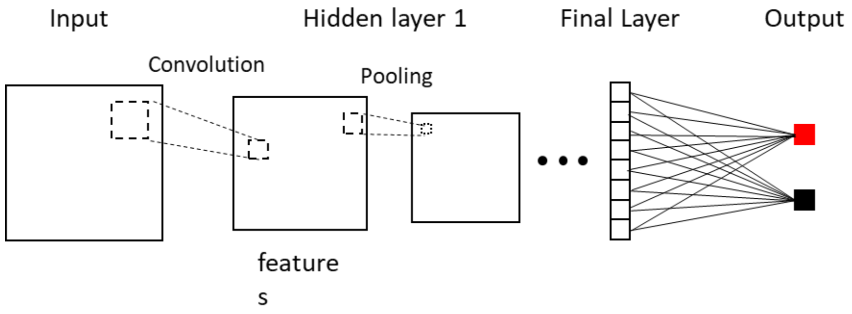
Classification performed with two classification algorithms:

Supervised Classification

k-Nearest Neighbor (K-NN)

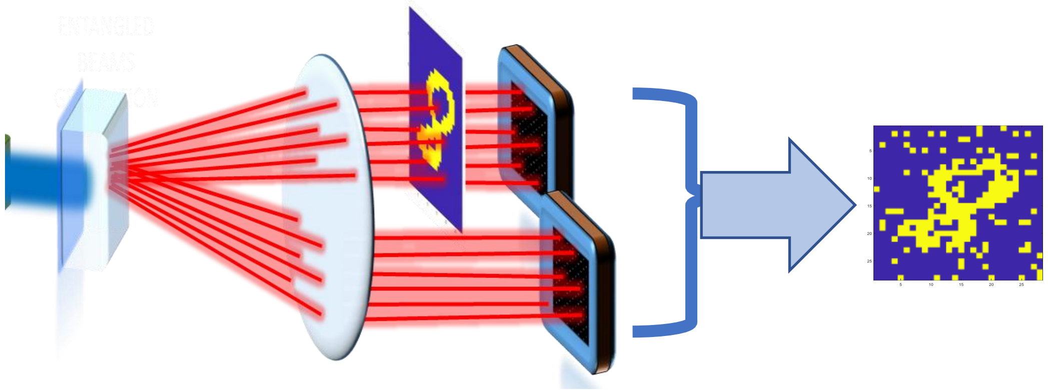


Convolutional Neural Network (CNN)



Quantum-enhanced Pattern recognition

- The photon count in each pixel is used to assign the value of the bit → the final image is binary (black and white)



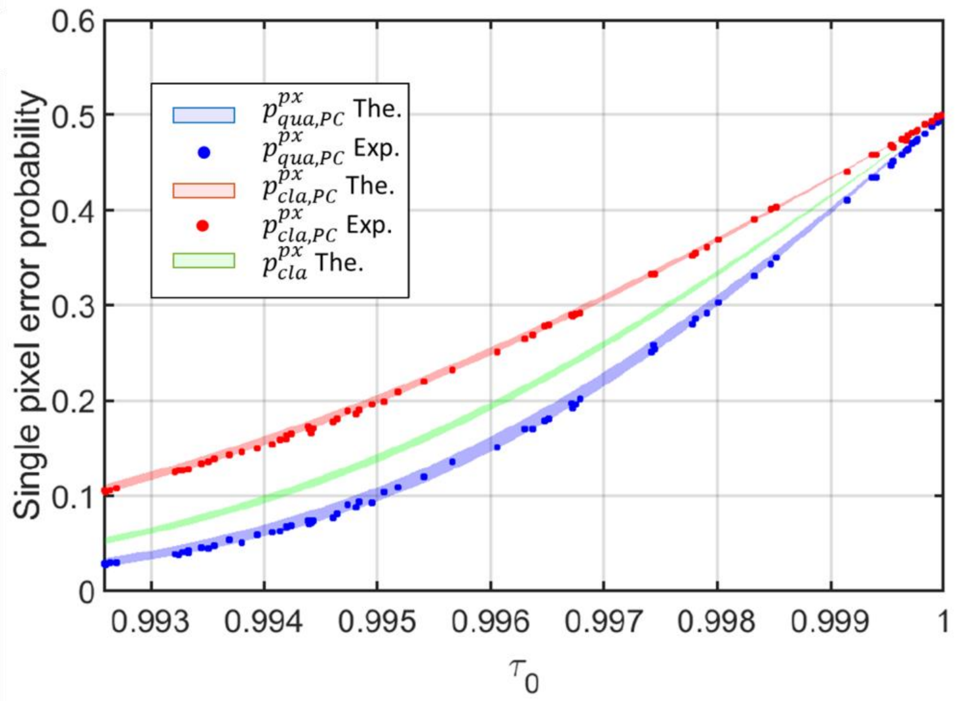
- Binary image are classified by the **supervised NN learning algorithm**

trained on a training set (60000 images)

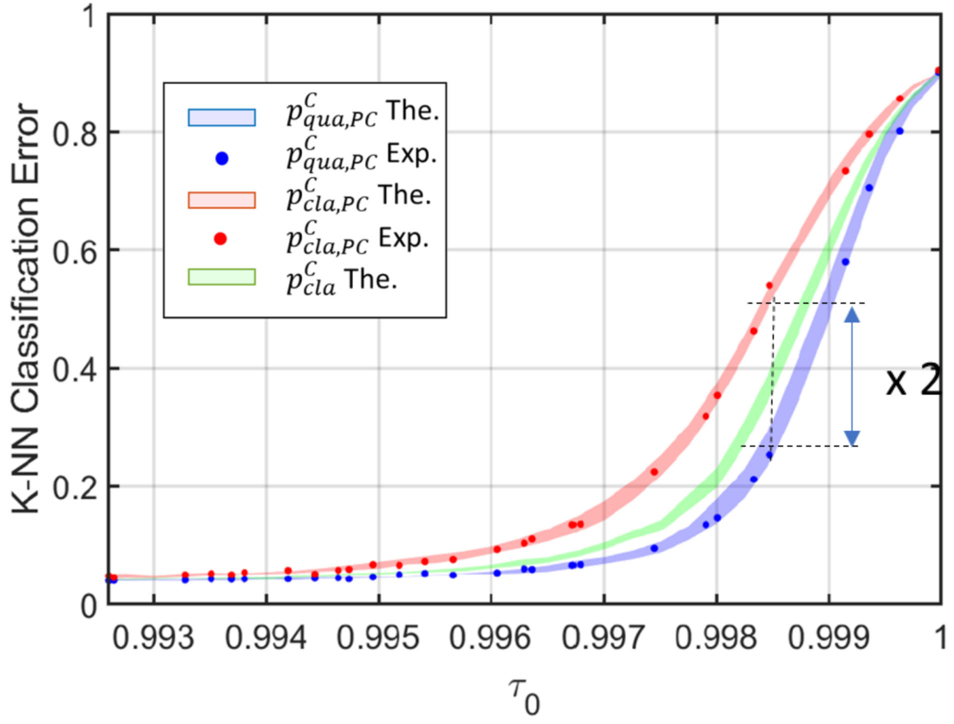
Classification Error determined

G.Ortolano *et al.,* Phys. Rev. Applied 20, 024072 (2023)

Discrimination of bosonic losses



K-Nearest Neighbor (k-NN) classifier



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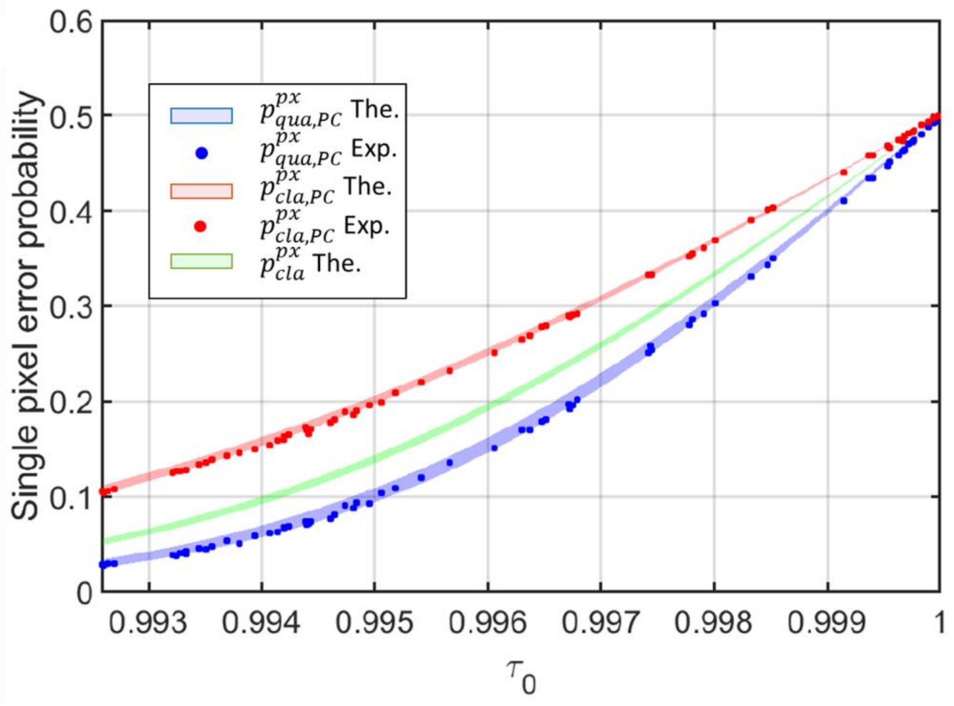

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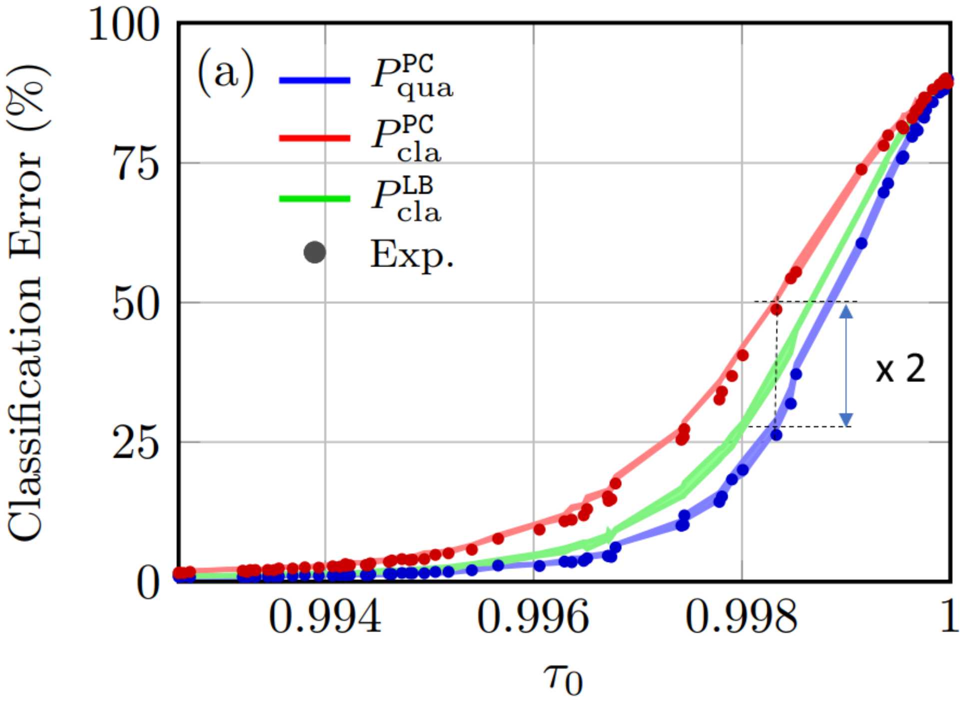

- The CNN classifier behaves very similarly to the K-NN algorithm confirming that the quantum advantage is sustained through sophisticated classical post-processing

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Conclusions

- Quantum resources can boost the performance over classical schemes in the optical domain
- Experimental quantum advantage in the readout of a digital memory and related protocols
- Scalability of the sensing advantage to complex tasks such as pattern recognition

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Thank You!