Superfluid Dark Matter in galaxies

and how to probe it!

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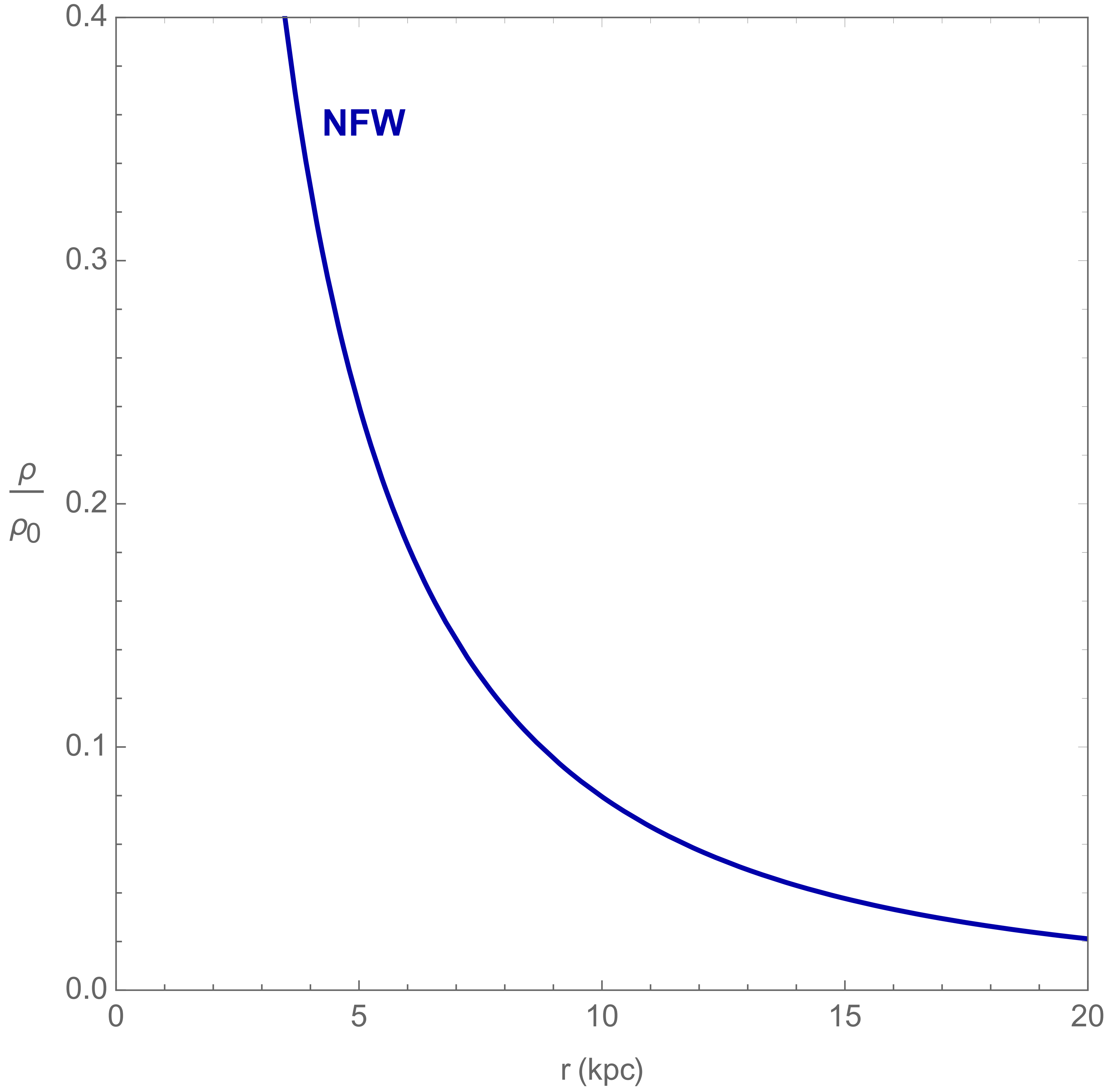
Florence Theory Group Day 2024
Galileo Galilei Institute

based on "L. Berezhiani, GC, V. De Luca, J. Khoury, arXiv:2311.07672"

Cold Dark Matter (*DM as a pressure-less fluid of massive particles*) works fine on Large and Cosmological scales, but presents several puzzles on small scales (~kpc) …



Prediction of CDM:

$$\rho_{\rm nfw} = \frac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

Navarro-Frenk-White density distribution

✔ - Universal density profile from simulations of collision-less dark

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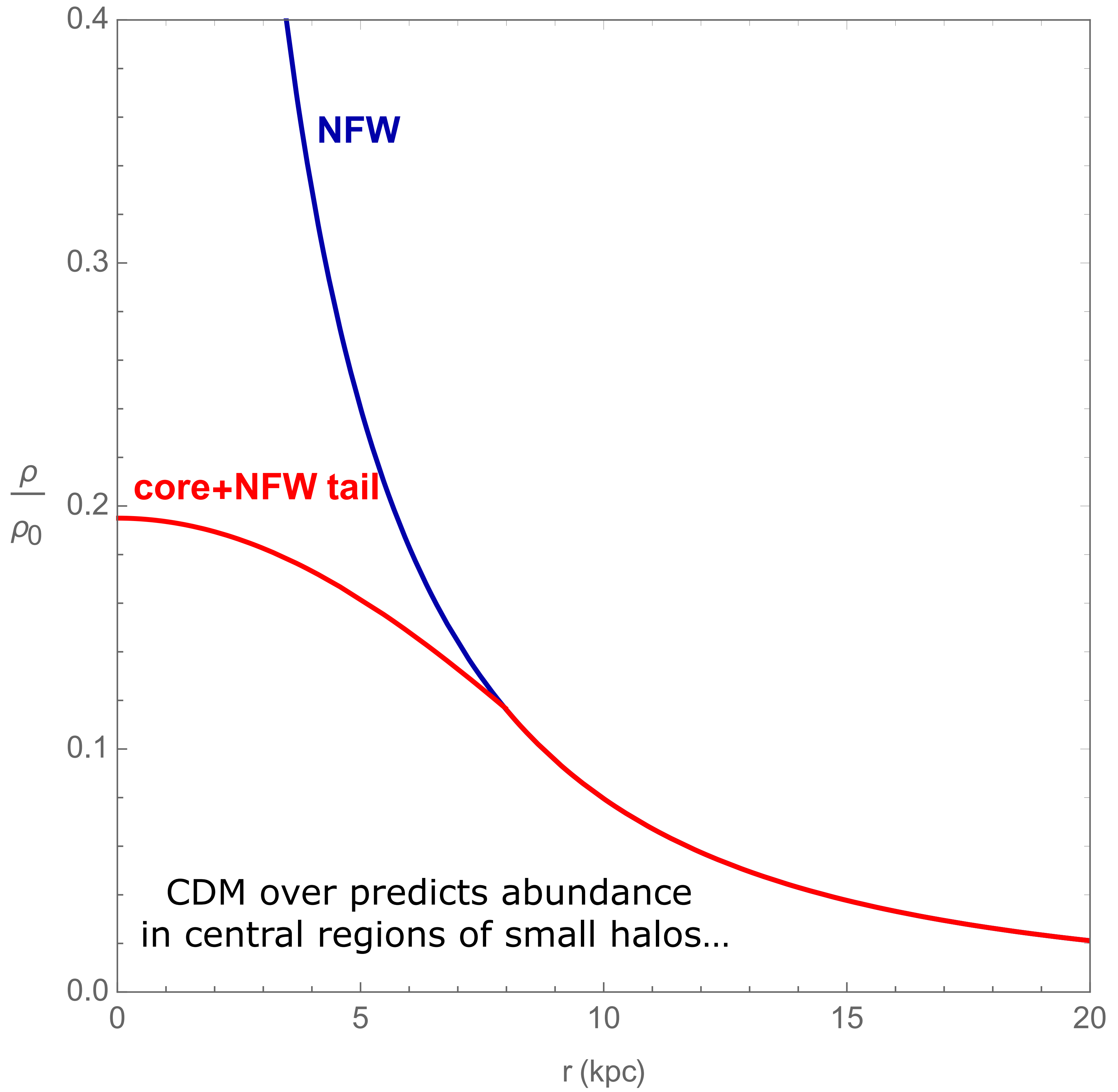

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✔ - Universal density profile from simulations of collision-less dark

✘ - For small halos (dwarf galaxies), this universal behavior is not observed…

The mismatch is known as the ***Core-Cusp problem***

Different ways of addressing it (and other open puzzles!)

1) Understand better the role of the **baryonic physics**…

2) New **properties of dark matter** beyond CDM!

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What if dark matter is in a ***Superfluid phase*** in halos?

What is a dark matter superfluid?

1) Condensate of self-interacting dark bosons in thermal equilibrium ($T<T_c$): **BEC + repulsive self-interactions**

2) **Phonons:** low energy degree of freedom of the condensate are sound waves and not single particles:

$$\underbrace{\omega^2 = c_s^2 k^2}_{\text{Collective waves}} + \underbrace{\frac{k^4}{4m^2},}_{\text{Single particles}} \qquad c_s^2 = \frac{\partial P}{\partial \rho}$$

(In general, more complicated spectrum E.g. rotons and maxons in He4)

3) **Landau's criterion for superfluidity**: Subsonic perturber (no accelerations) may not perturb the superfluid!

$\mathbf{v < c_s}$ → Cannot excite phonons! Kinematically prohibited.

Why a dark matter superfluid?

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda\phi^4}{4!}$$

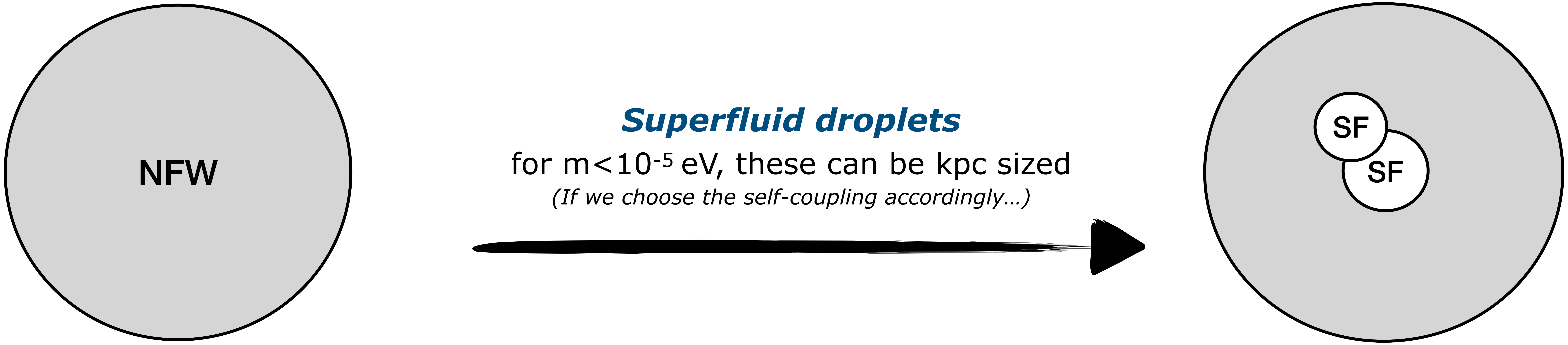
$$\left(c_s^2 = \frac{\lambda\rho}{8m^4}\right)$$

Ultralight scalar with ***repulsive*** self-interactions

✓ 1) If m< 1 eV, dark matter de Broglie volume is highly occupied in halos ***(degeneracy!)***

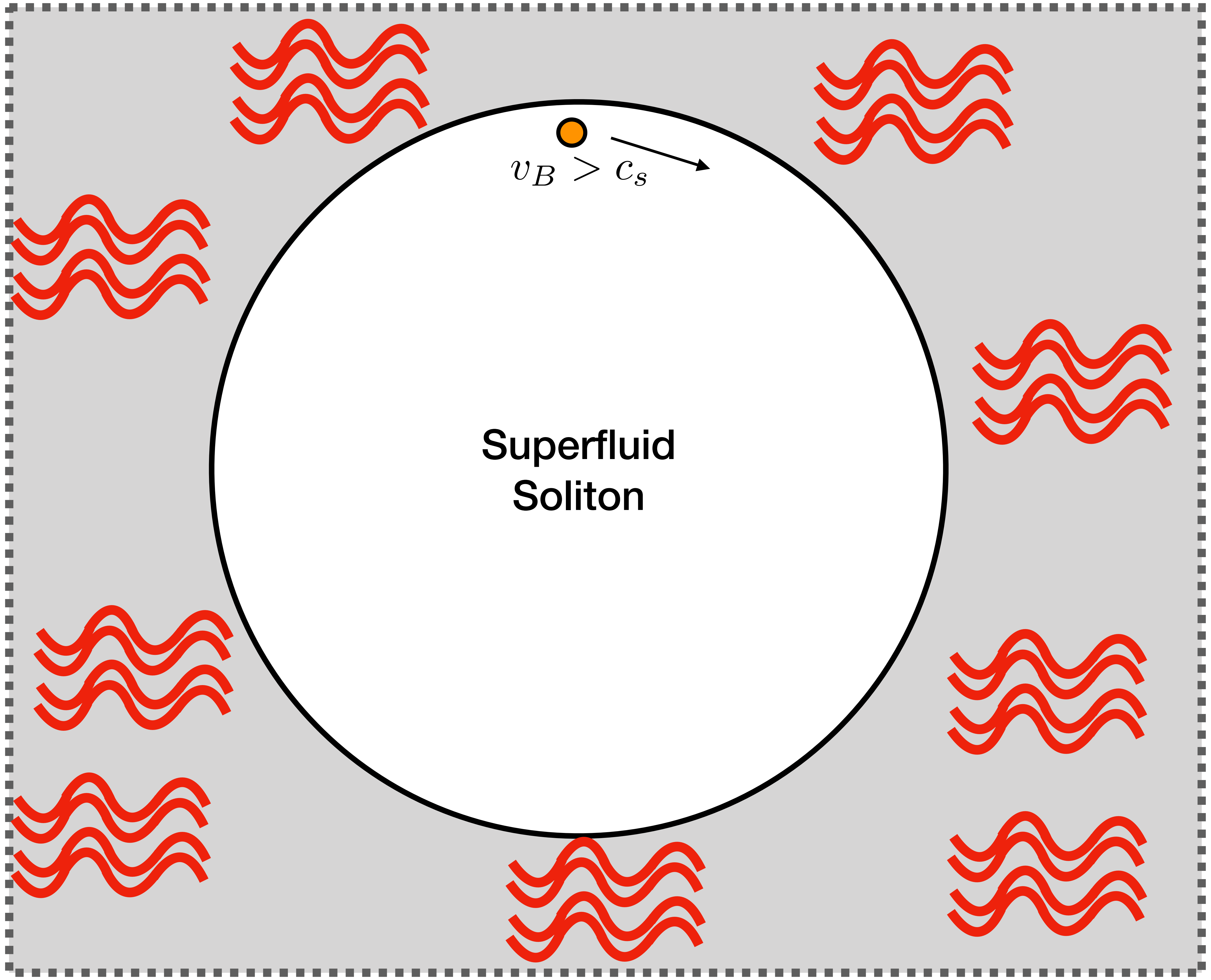
✓ 2) Efficient Self-interactions would thermalize dark matter ***(thermal equilibrium!)***

Implications: Formation the superfluid phase. The ground state pressure sustains self-gravitating quasi homogeneous configurations (non topological solitons)



Cold Dark Matter *Superfluid Dark Matter*

To probe the superfluid behavior of dark matter,
we need to perturb the system with subsonic probes ($v<c_s$)…



$$\left[\begin{array}{ll} \frac{1}{\rho_{\rm dm}(r)}\frac{dP}{dr} = -\frac{d\Phi(r)}{dr} & \textit{Hydrostatic Eq.} \\ P(r) = \frac{\lambda \rho_{\rm dm}(r)^2}{4m^4} & \textit{Equation of state} \\ \vec{\nabla}^2\Phi = 4\pi G \rho_{\rm dm}(r) & \textit{Poisson's equation} \end{array}\right.$$

$$\rho_{\rm dm}(r) = \rho_0 \frac{\sin(r/\ell)}{r/\ell}$$

Quasi-homogeneous in the centre,
possible solution to the core-cusp problem!

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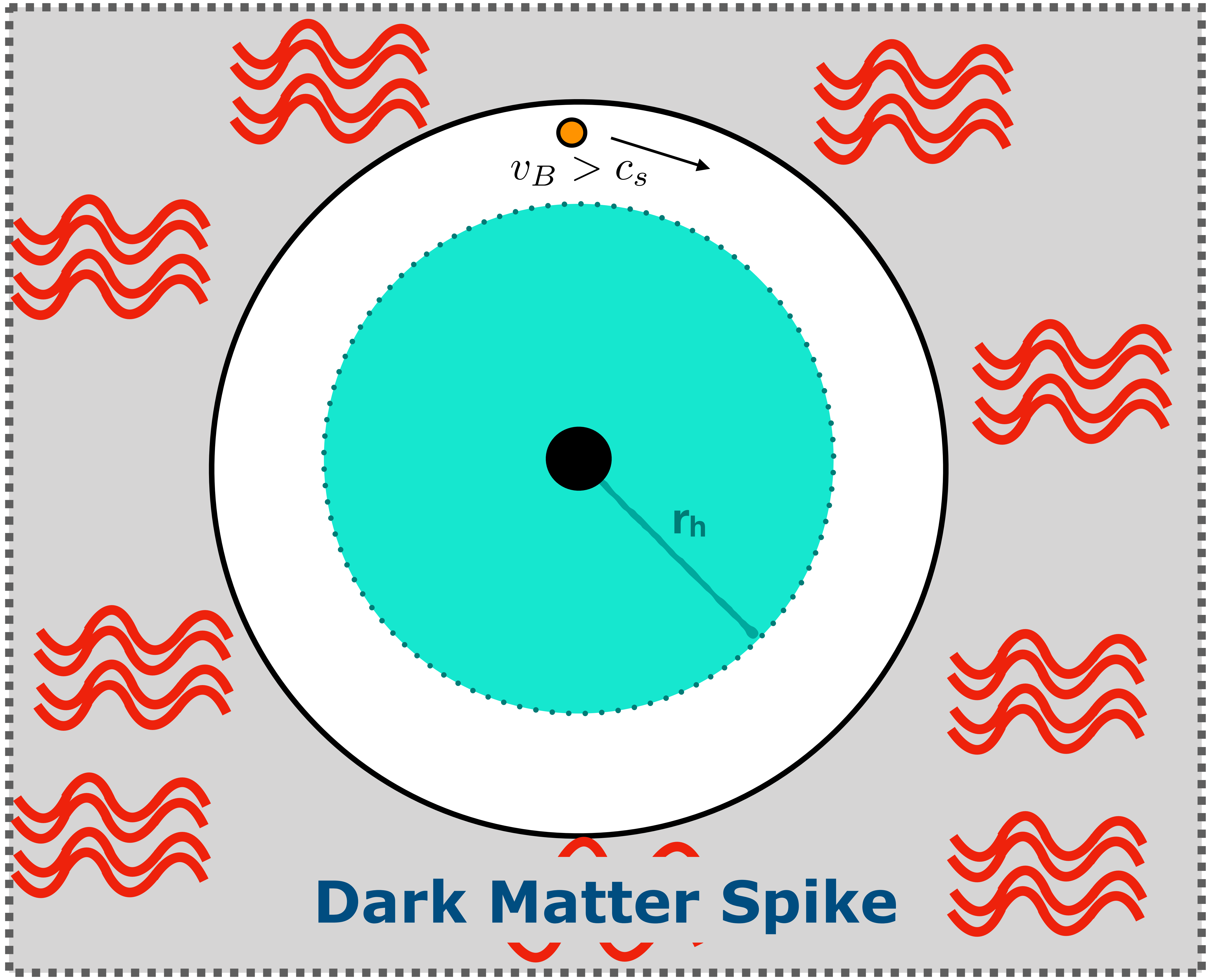
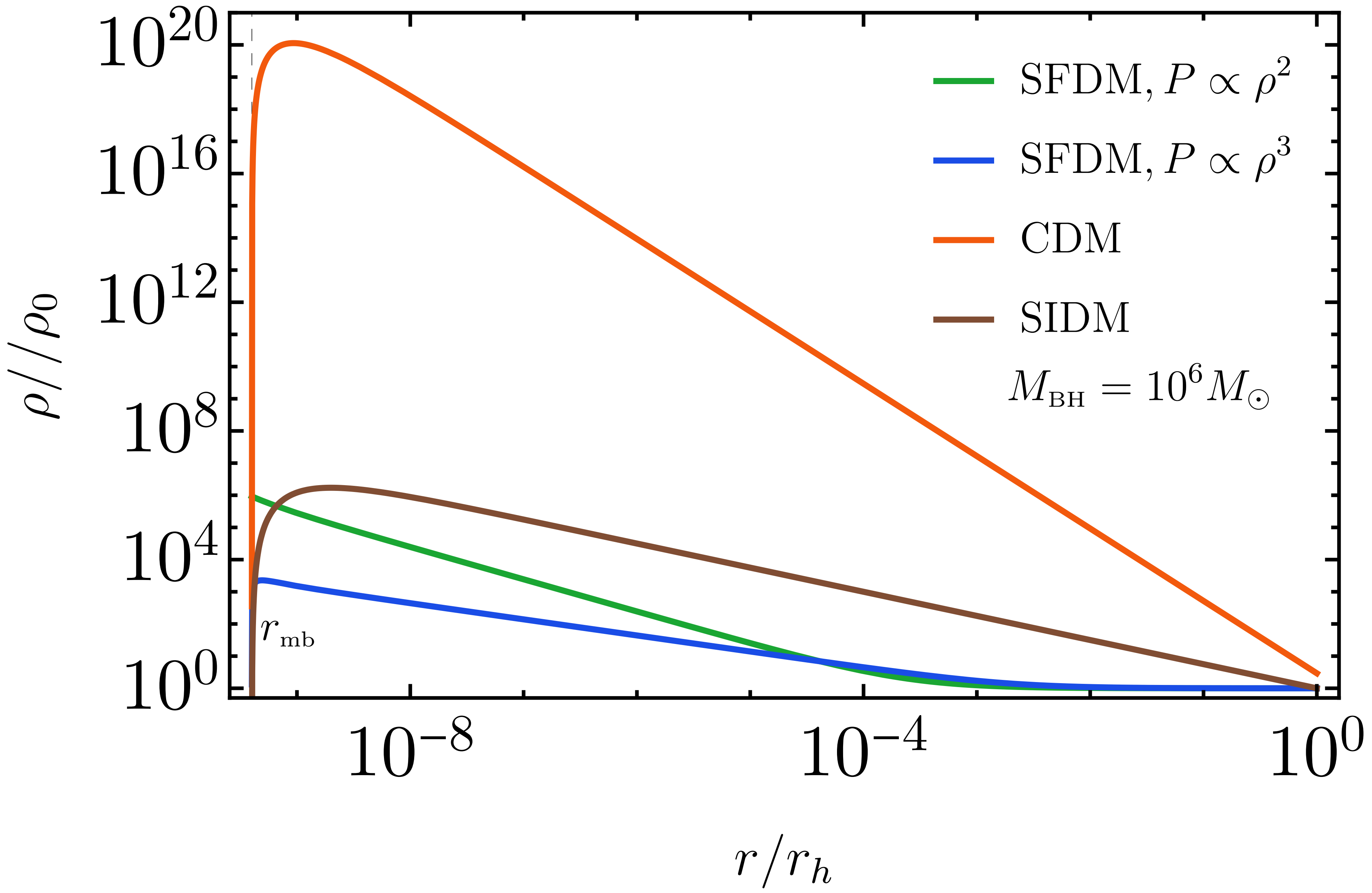


$$\frac{1}{\rho_{\rm dm}(r)}\frac{dP}{dr} = -\frac{d\Phi(r)}{dr}$$ *Hydrostatic Eq.*

$$P(r) = \frac{\lambda \rho_{\rm dm}(r)^2}{4m^4}$$ *Equation of state*

$$\vec{\nabla}^2\Phi = 4\pi G\left(\rho_{\rm dm}(r) + M_{\rm BH}\delta(r)\right)$$ *Poisson's equation*



(V. De Luca J. Khoury, 2023)

(Fig from L. Berezhiani, G.C., V. De Luca, J. Khoury, 2023)

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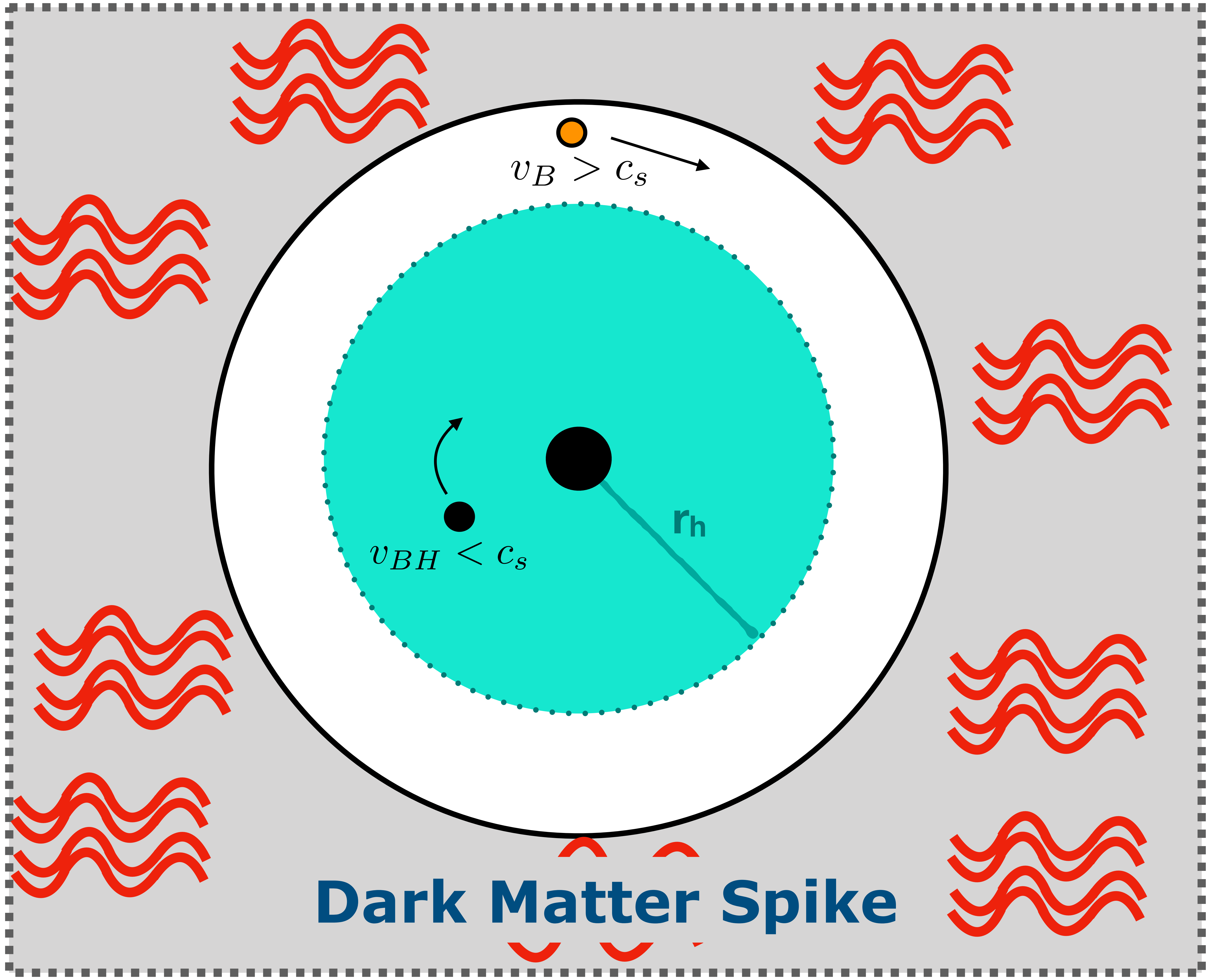
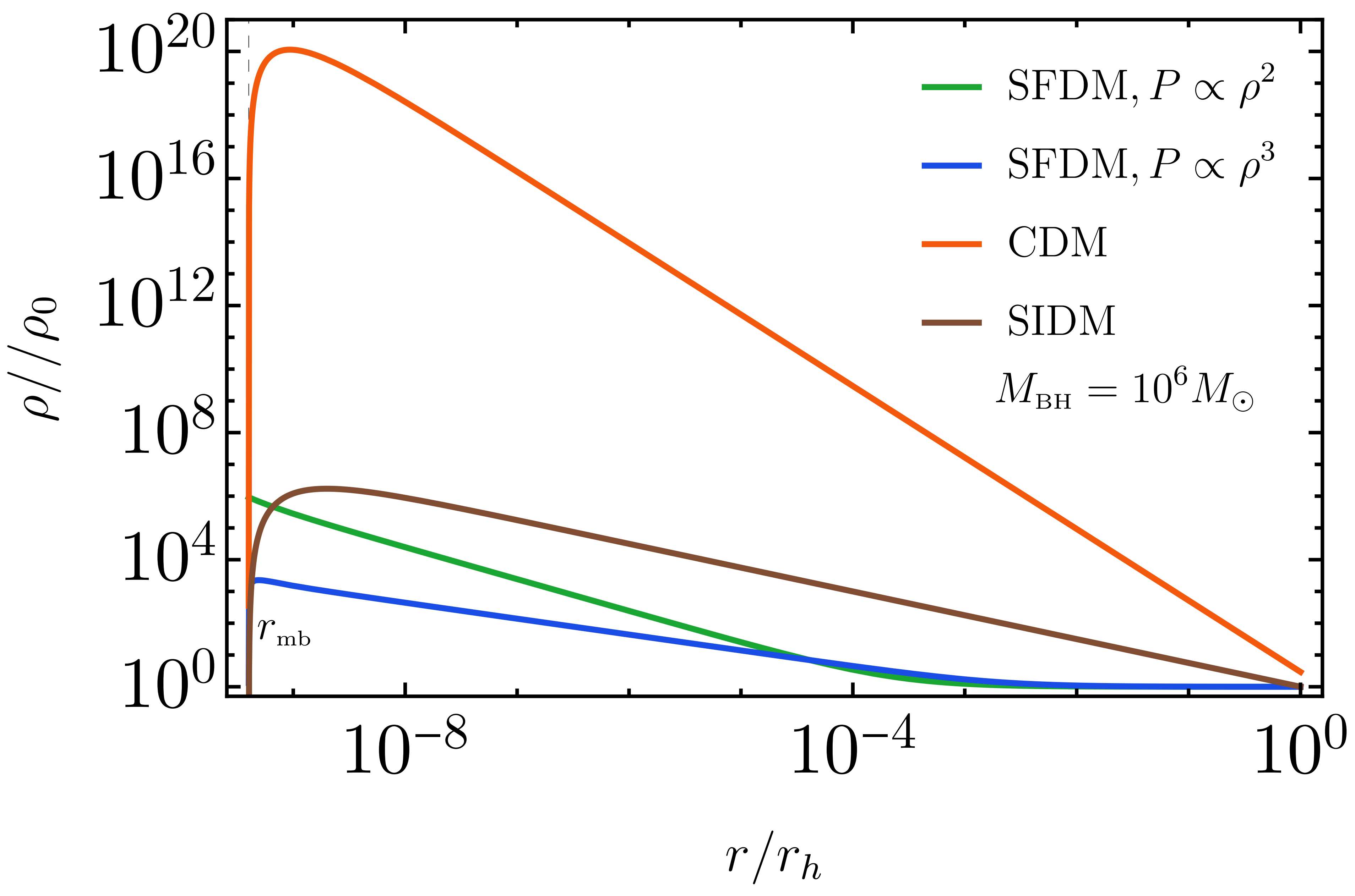


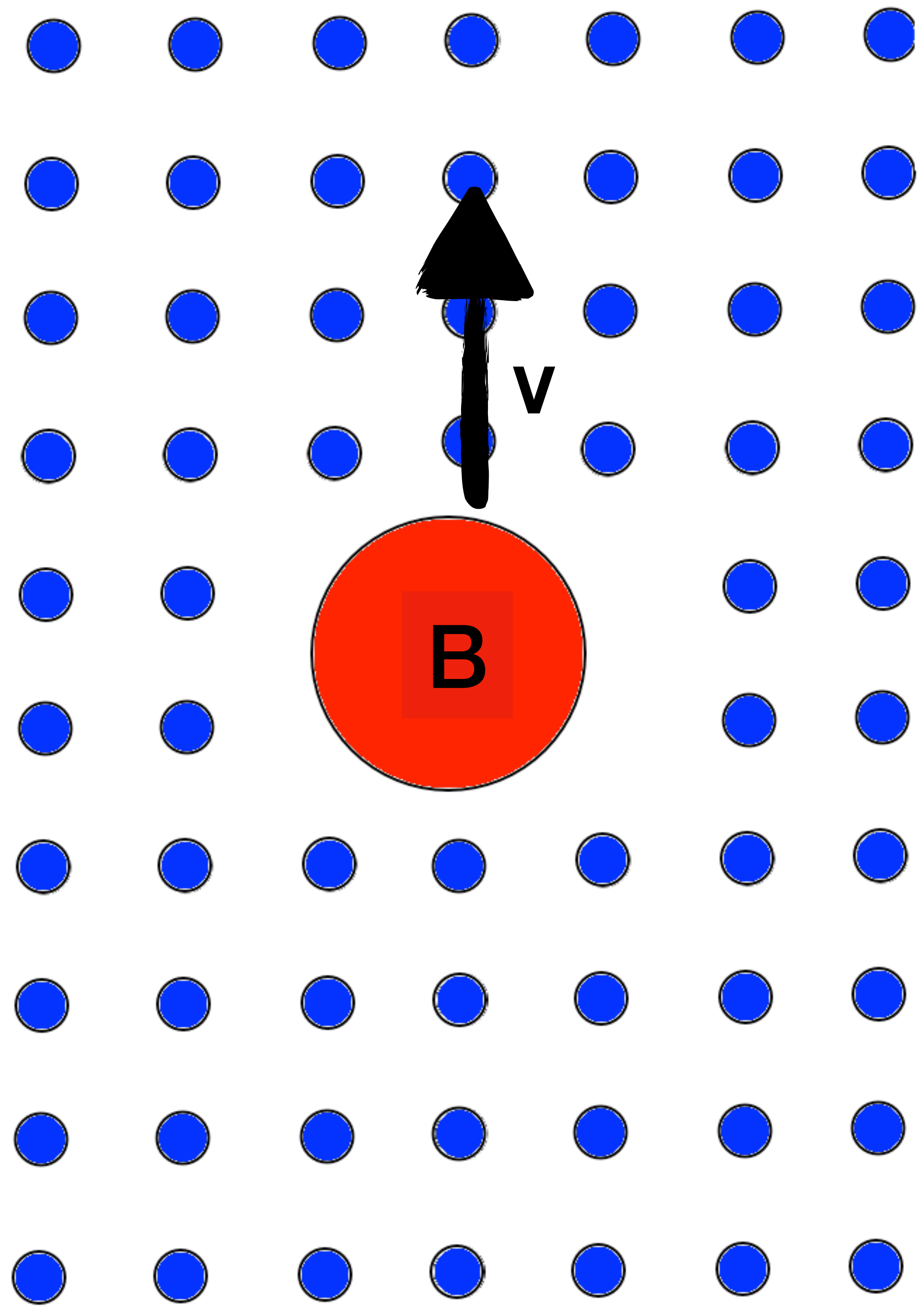


We specialize in extreme mass ratio inspirals (EMRIs): $\text{ratio}_{\text{masses}} < 10^{-5}$

(V. De Luca J. Khoury, 2023)

(Fig from L. Berezhiani, G.C., V. De Luca, J. Khoury, 2023)

Can we probe dark matter superfluidity? ***YES, with Dynamical friction in dark matter spikes***



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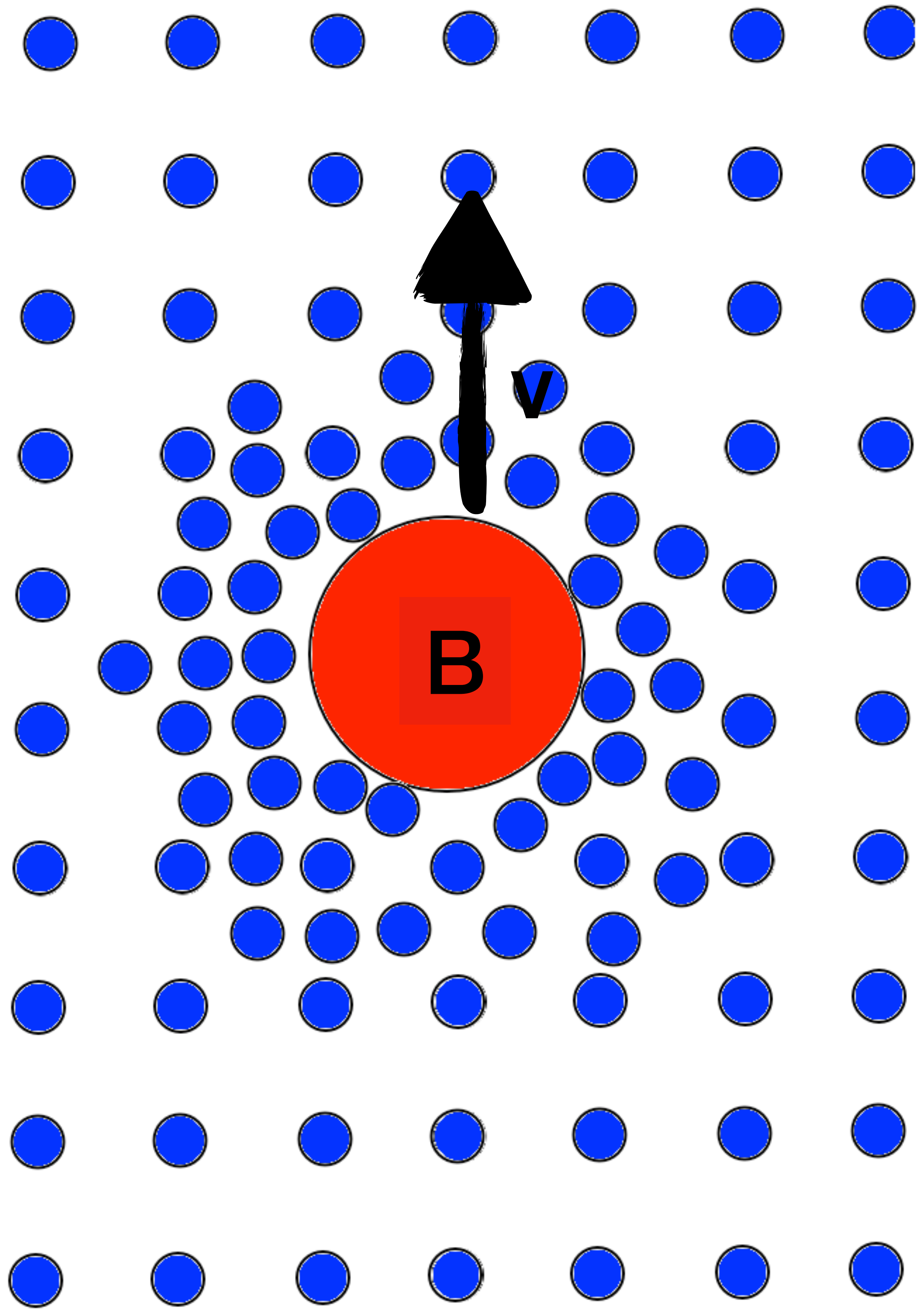

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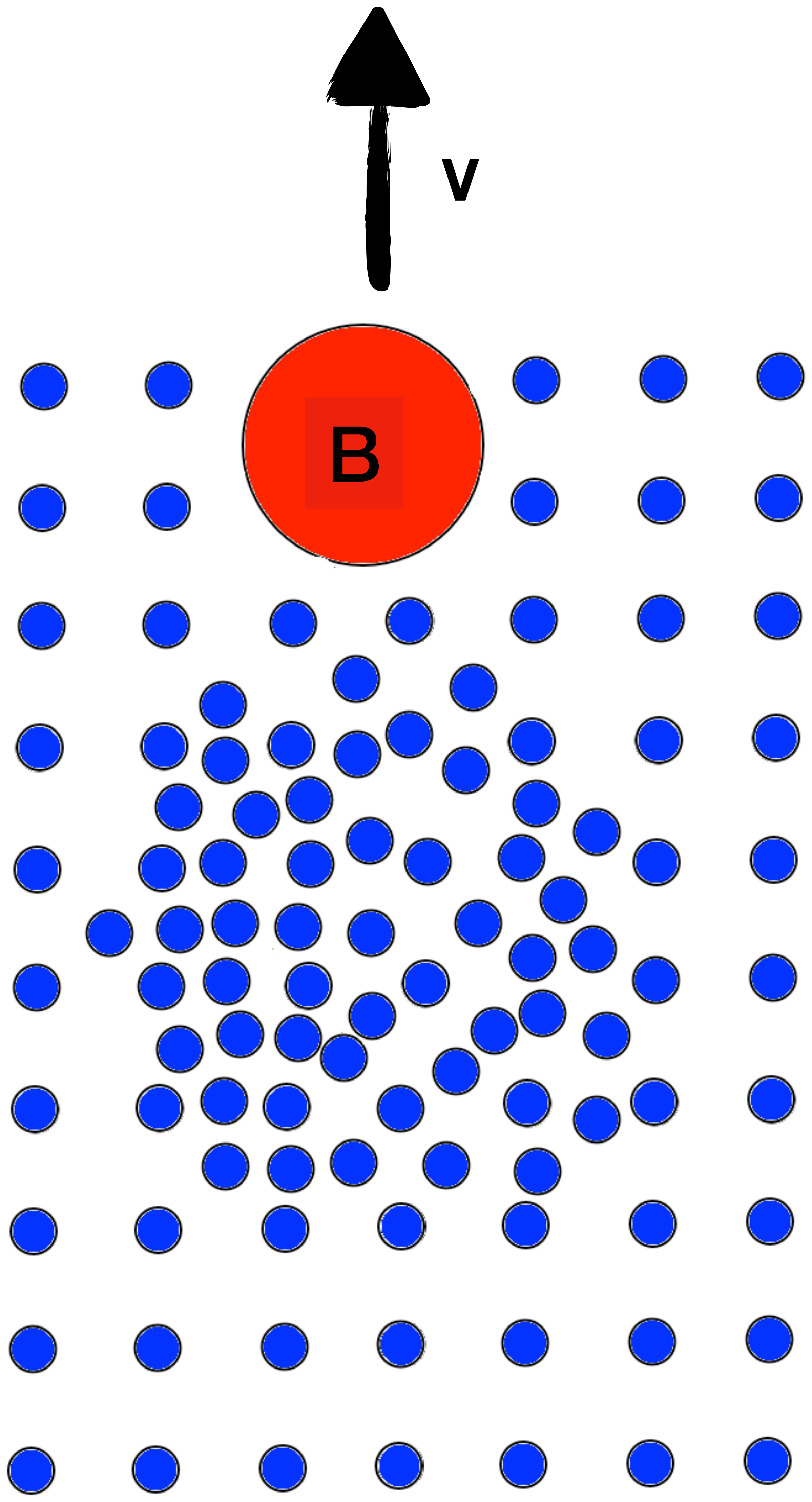

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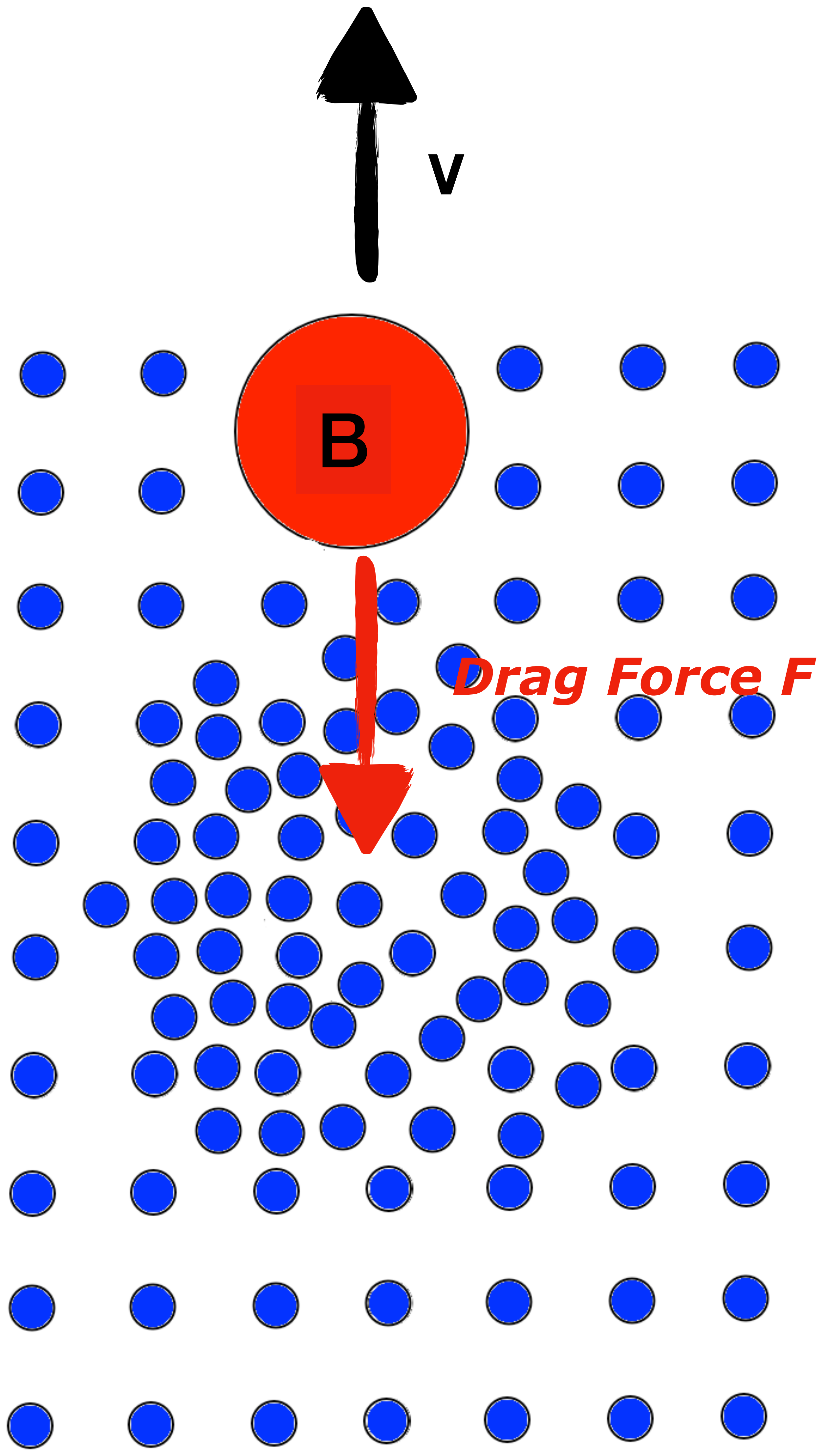

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Although there is only gravity that mediates interactions between baryons and DM the overdensity depends on the **properties of the medium!**

1) Collisionless gas of massive particles (S. Chandrasekhar, 1943)

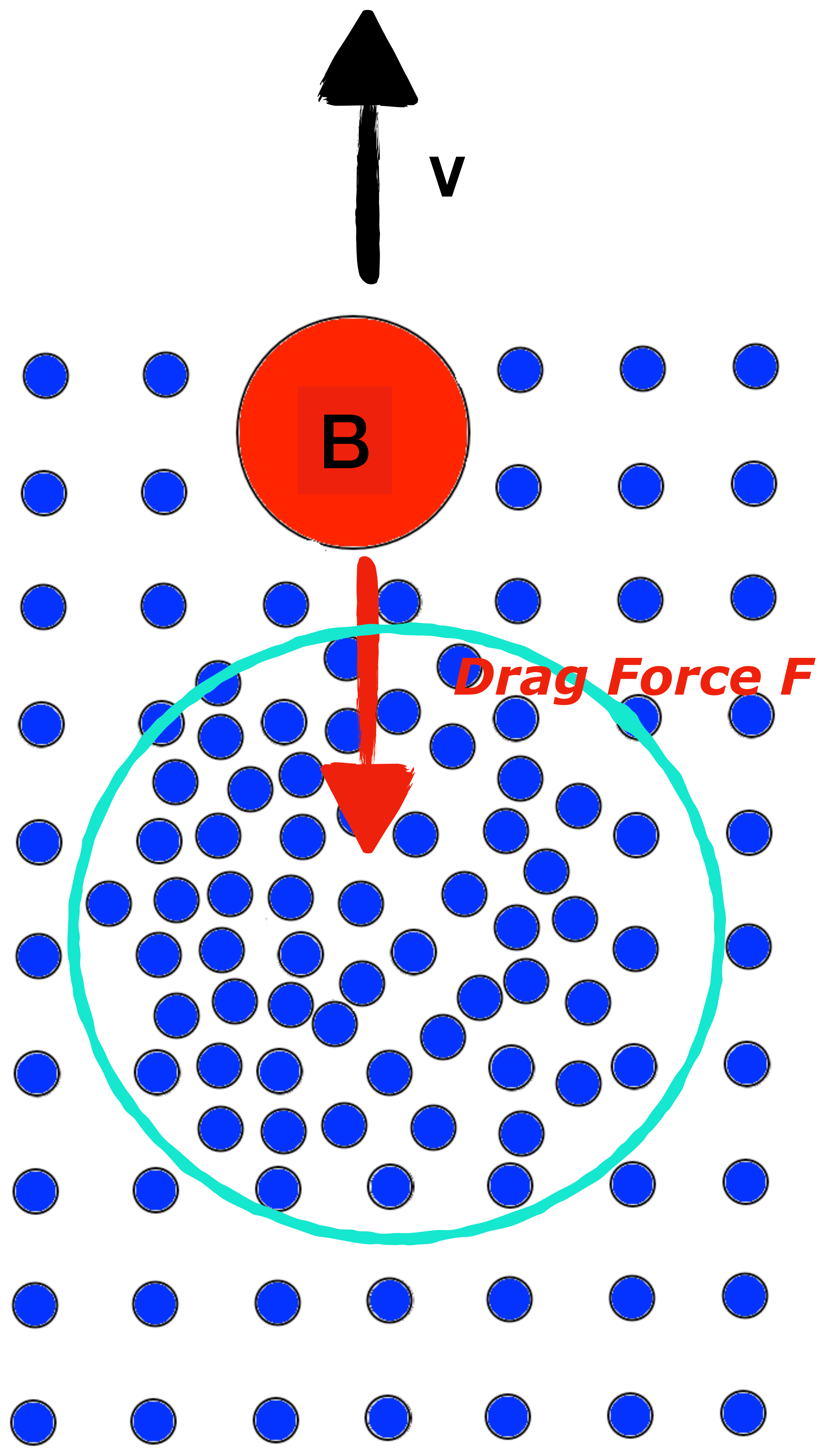
$$\vec{F}_{\rm CDM} = -\frac{4\pi G^2 M_B^2 \rho_0}{v_B^2} C(v_B, \sigma_v)$$

2) Superfluid medium (L. Berezhiani, B. Elder, J. Khoury, 2019)

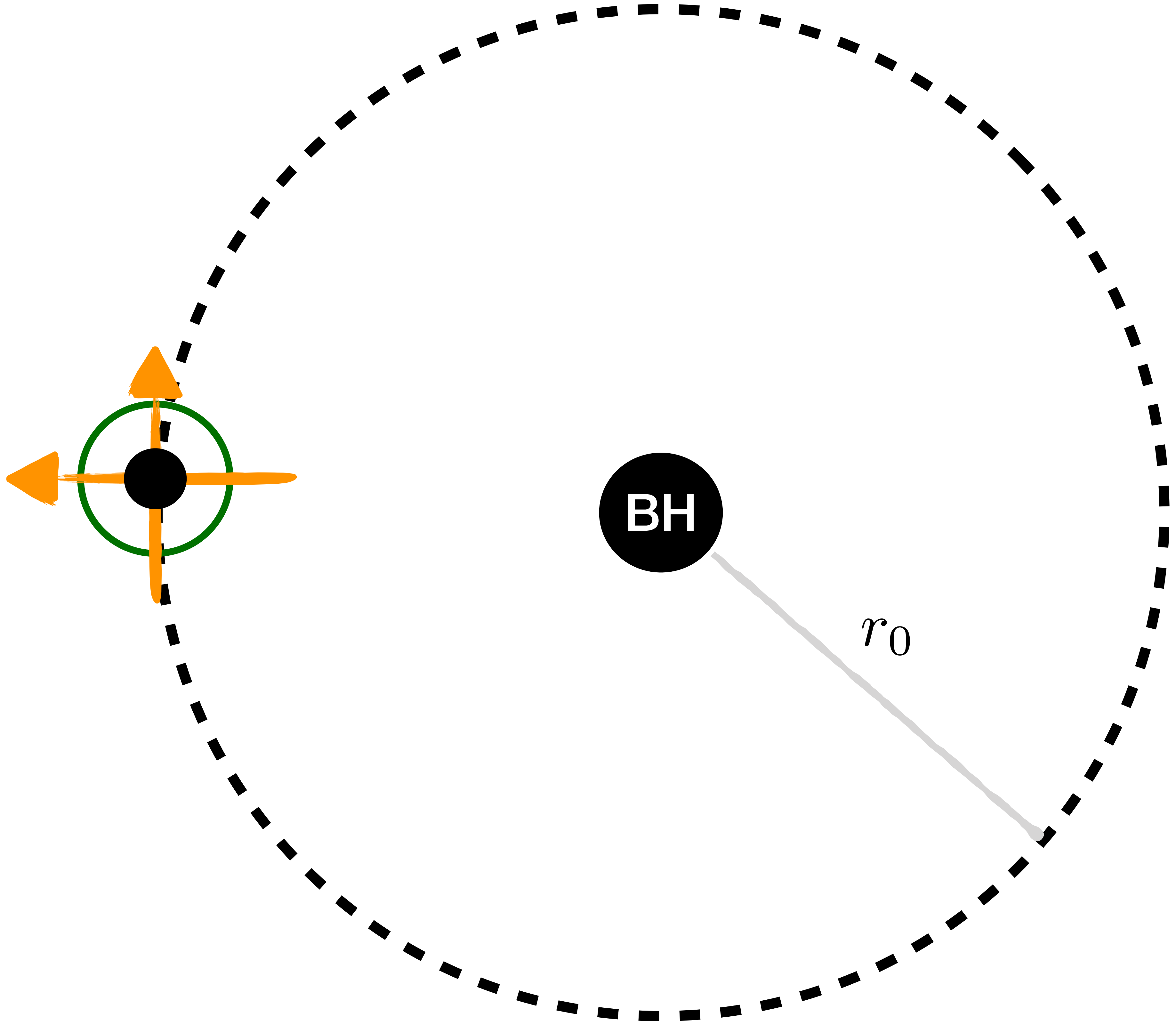
$$\vec{F}_{\rm SFDM} = 0 \qquad v < c_s$$

$$\vec{F}_{\rm SFDM} = -\frac{4\pi G^2 M^2 \rho_0}{v_B^2} \log(\beta^{-1}\sqrt{v^2/{c_s}^2 - 1 + \beta^2}) \qquad v > c_s$$

(β is a constant…)



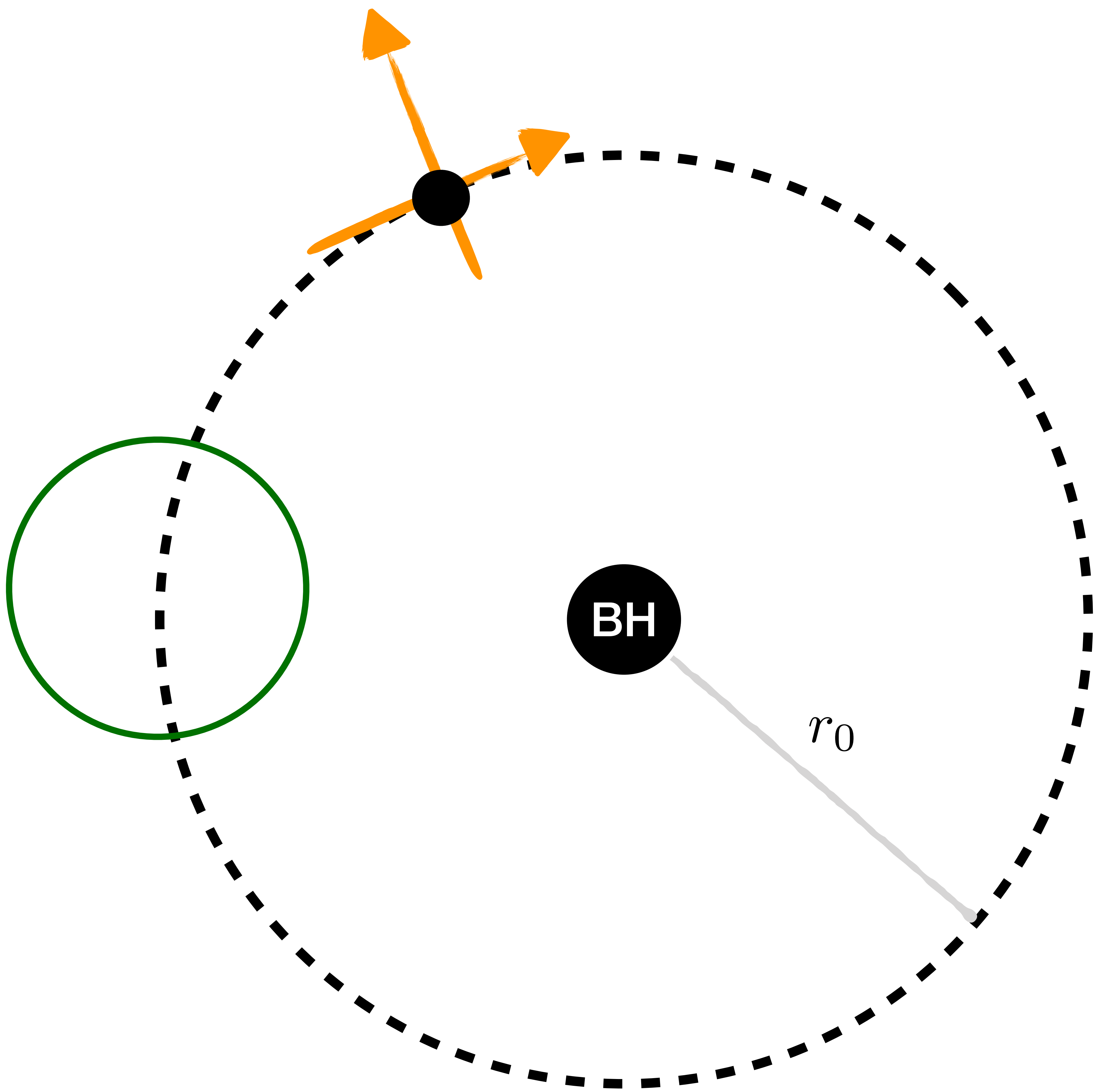
Subsonic case substantially different than CDM!
(All examples are for the linear motion…)



$$\mathcal{M} = \frac{\Omega r_0}{c_s}$$ *Mach Number*

(L. Berezhiani, G.C., V. De Luca, J. Khoury, 2023)

Formalism for circular orbits from 'V. Desjacques., A. Nusser., R. Bühler, 2022'



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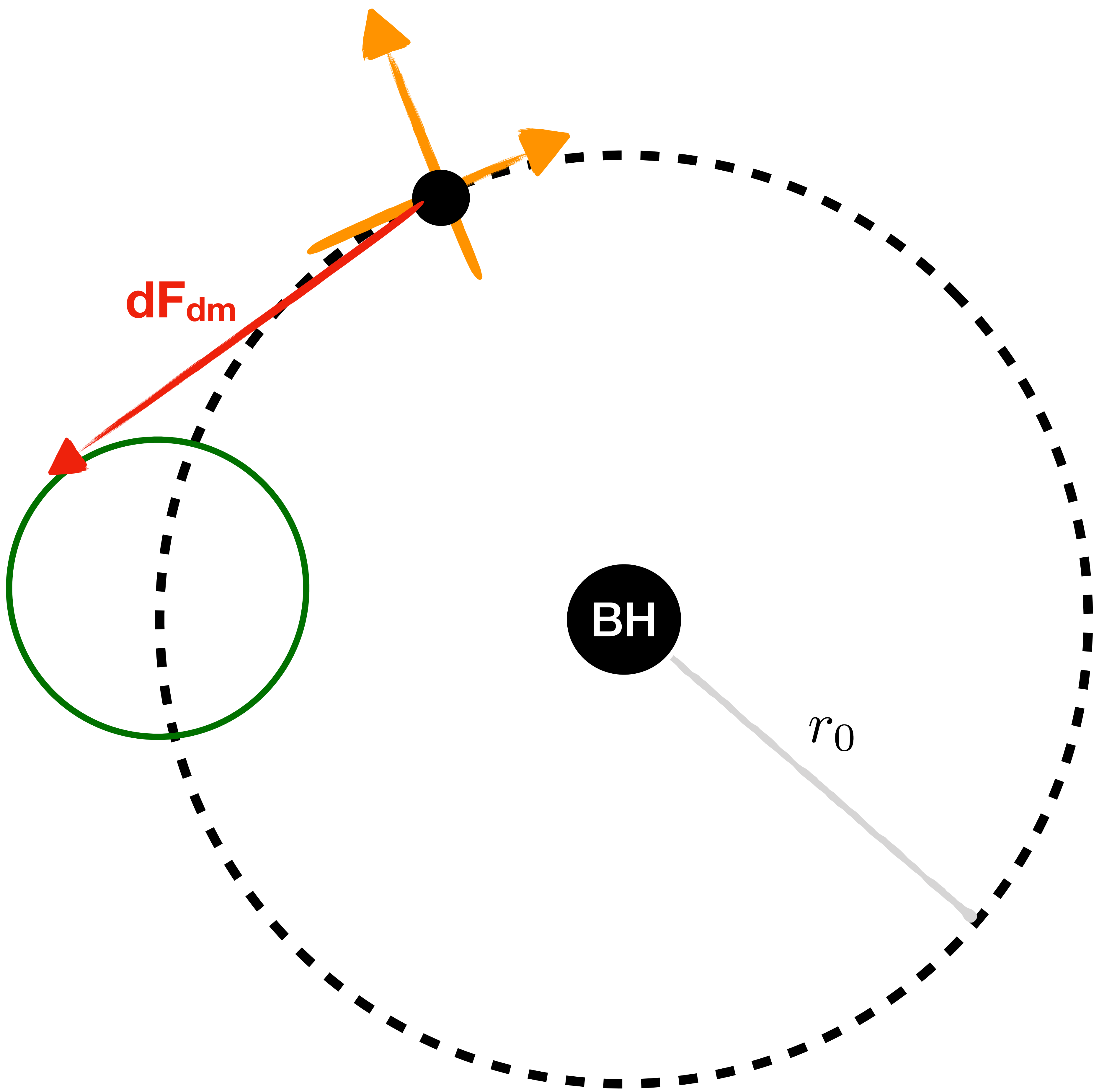

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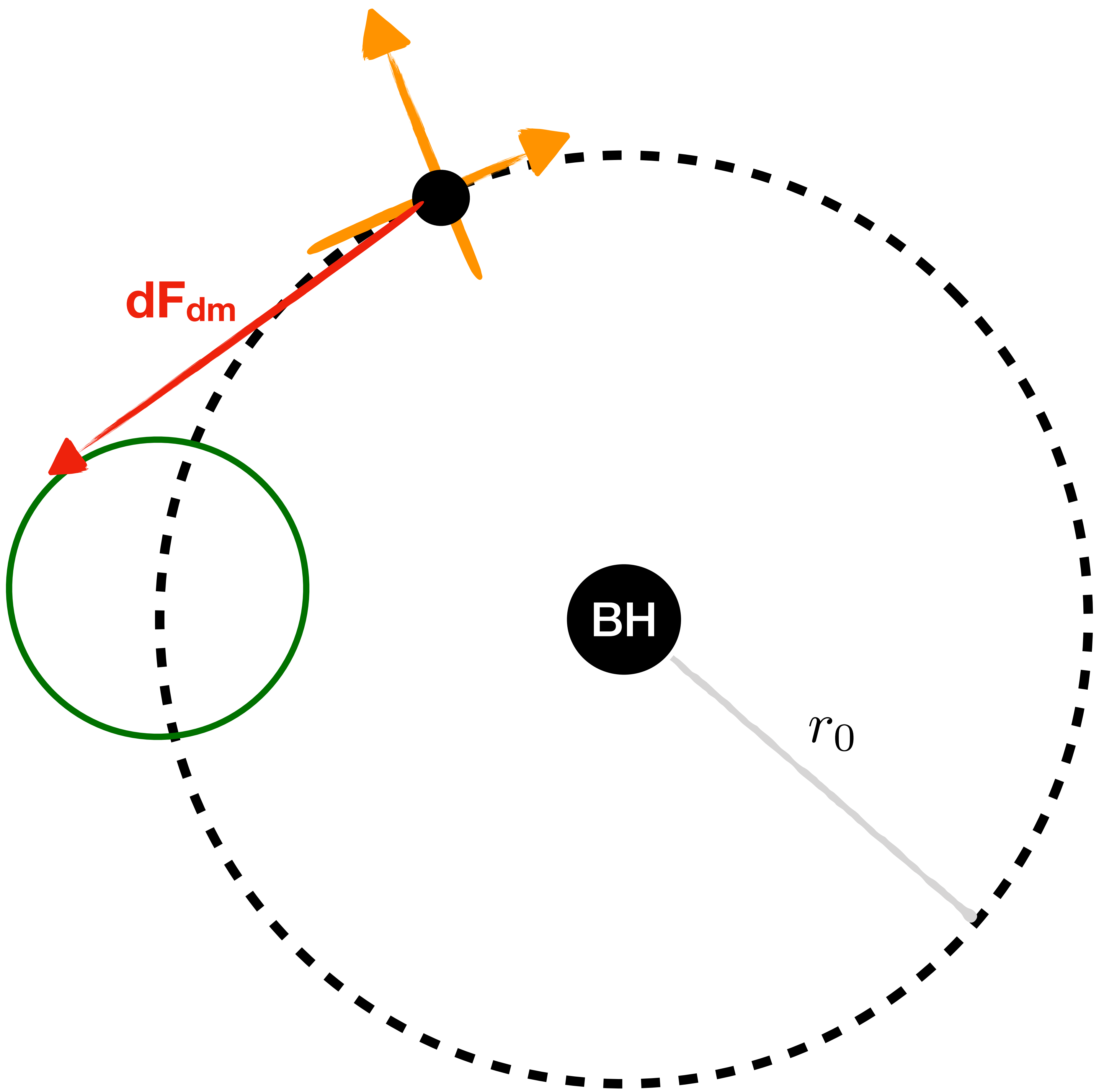

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$$\rho(r,t) = \rho_0\,(1+\alpha(r,t))$$

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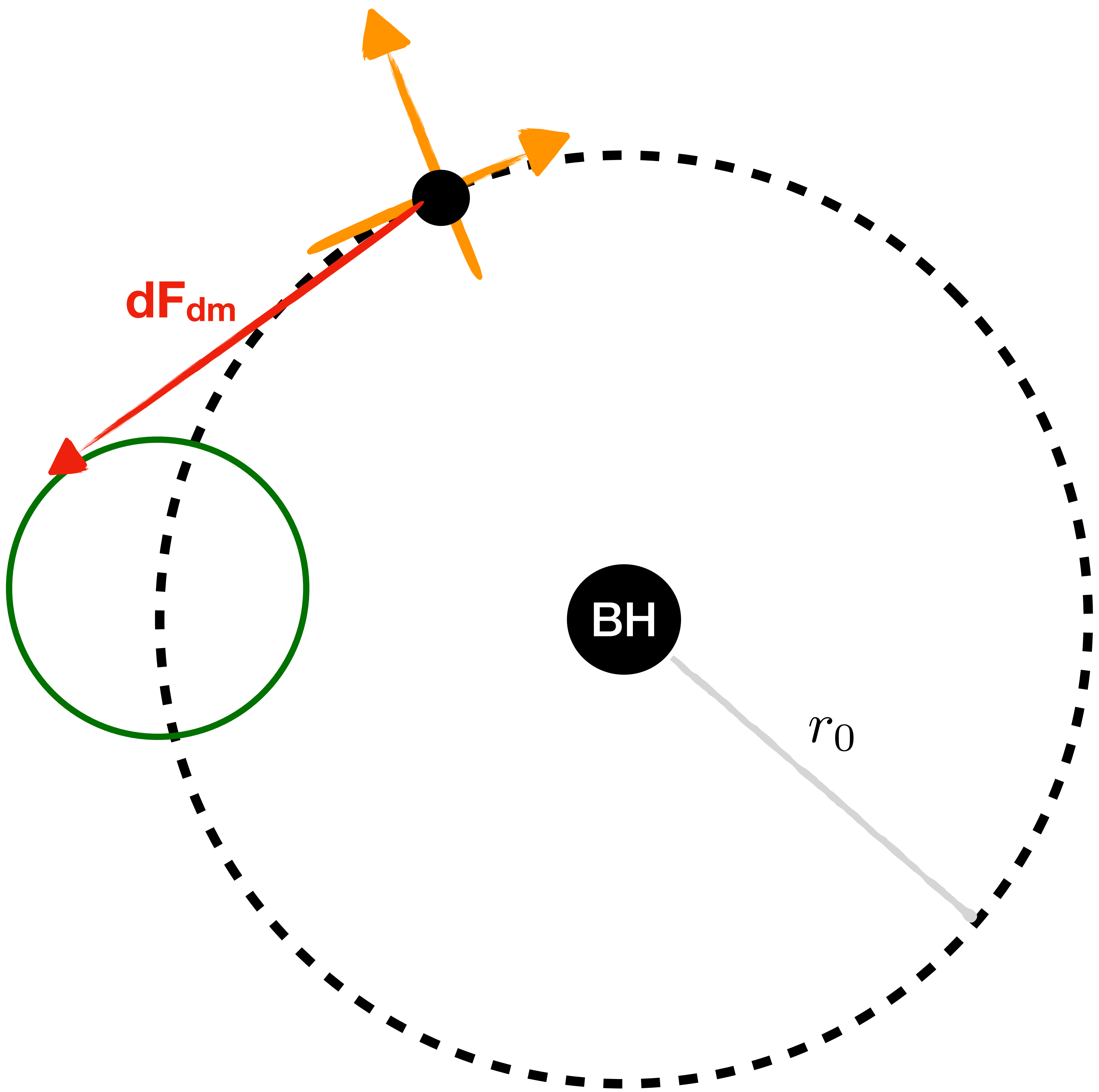

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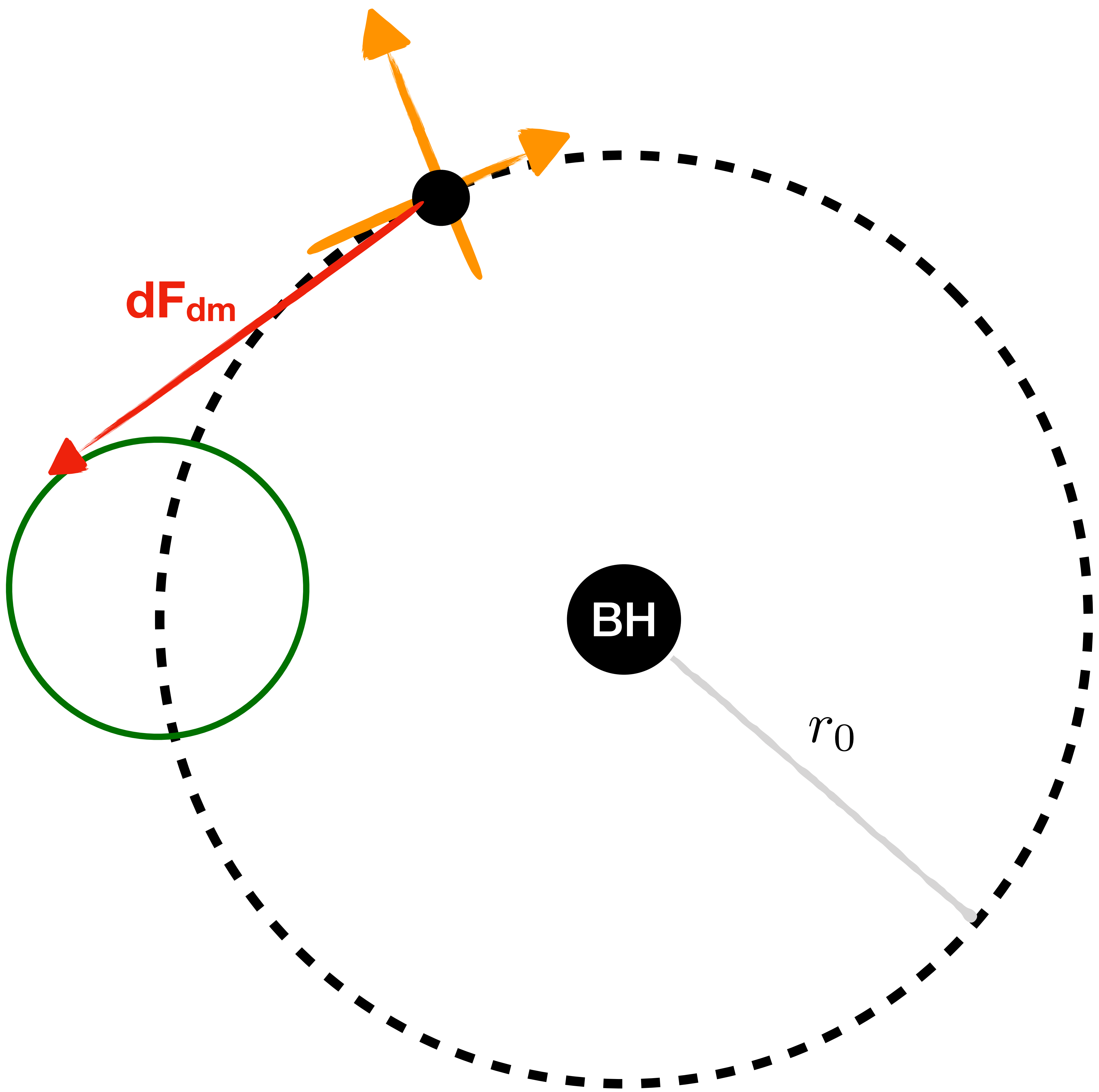

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3) Evaluate the force between the perturber and α(r,t). ***Circular orbits!***

*(similar to evaluation of Retarded Potential, **different mediator**)*

$$\vec{F}_{\rm DF} = (4\pi GM)^2 \rho_0 \int_0^\infty d\tau \int \frac{d^4k}{k^2} \frac{i\vec{k}}{c_s^2 k^2 + \frac{k^4}{4m^2} - (\omega + i\epsilon)^2} e^{-i\omega\tau + i\vec{k}\cdot(\vec{r}-\vec{r}')}$$

Phonon dispersion relation

(L. Berezhiani, G.C., V. De Luca, J. Khoury, 2023)

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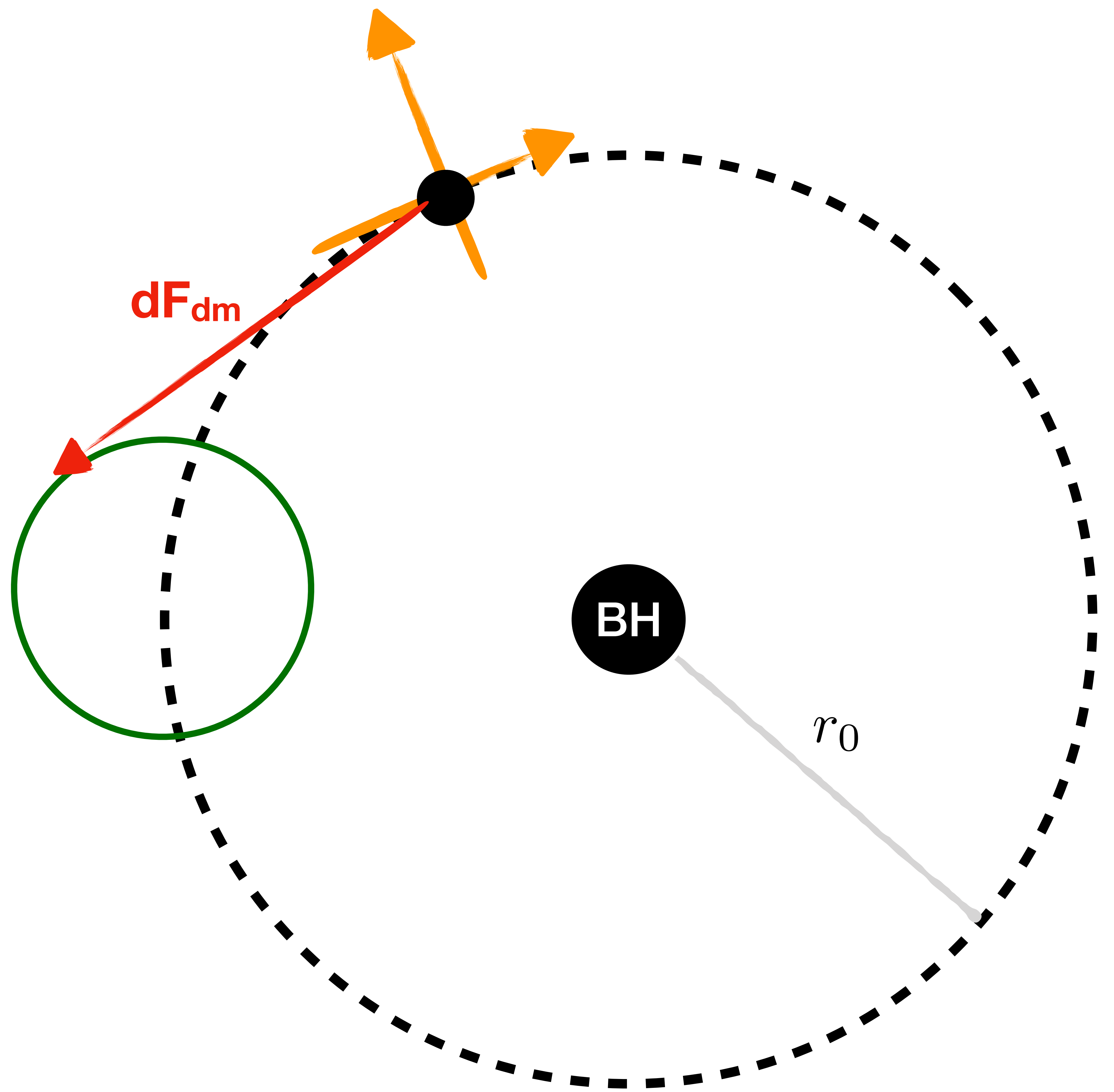

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Phonon dispersion relation

4) There is a general solution expressed as a sum over multipoles. For subsonic motion (M<1) and mostly sound waves perturbations ($r_0^{-1} < mc_s$):

$$(F_{\rm dm})_r = 0$$

$$(F_{\rm dm})_\phi = -\frac{4\pi G^2 M^2 \rho_0}{v^2} \times (\mathrm{arctanh}\mathcal{M} - \mathcal{M})$$

Collisionless gas — *Superfluid+circular motion Smaller than unity!*

(L. Berezhiani, G.C., V. De Luca, J. Khoury, 2023)
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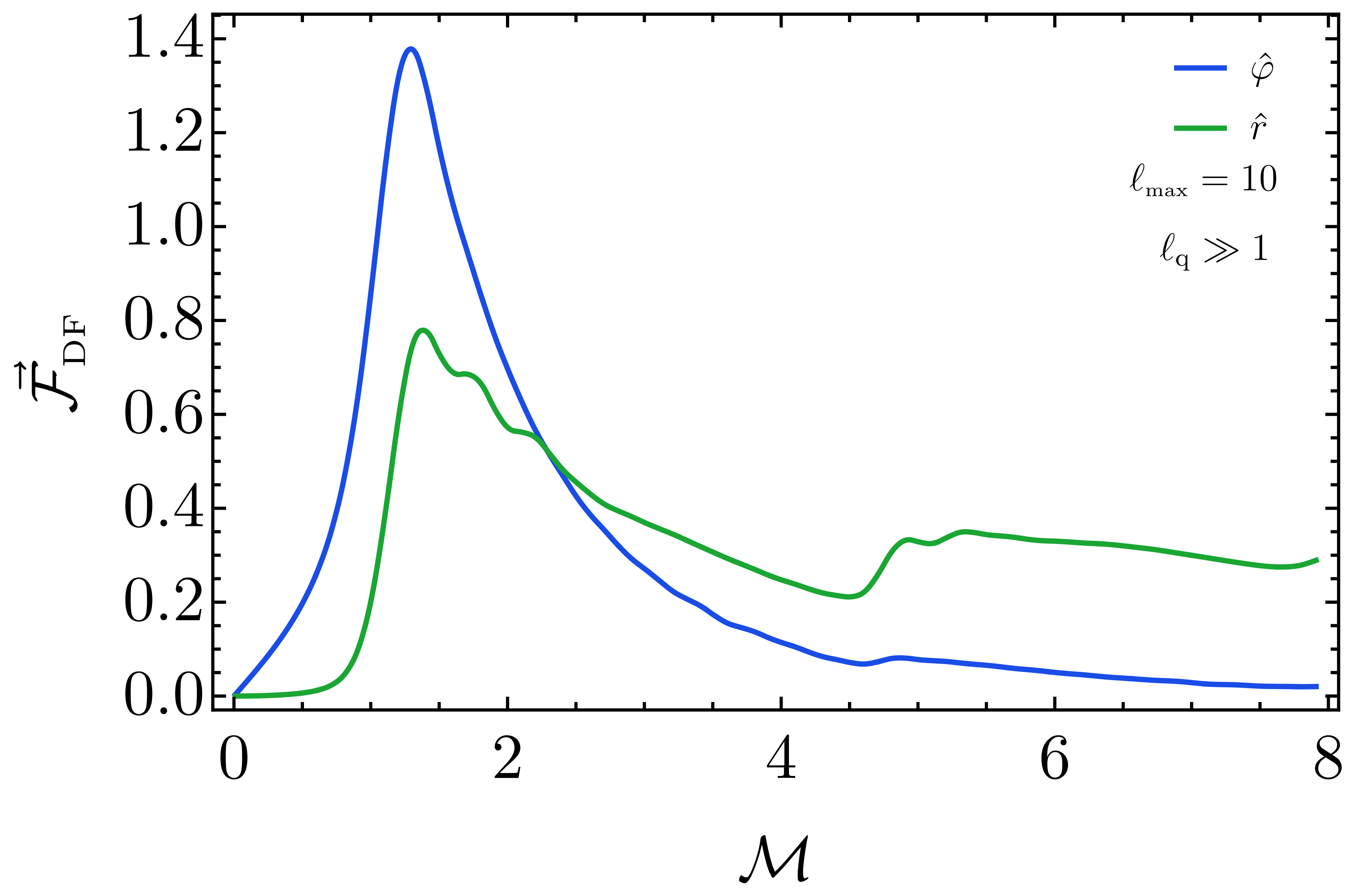


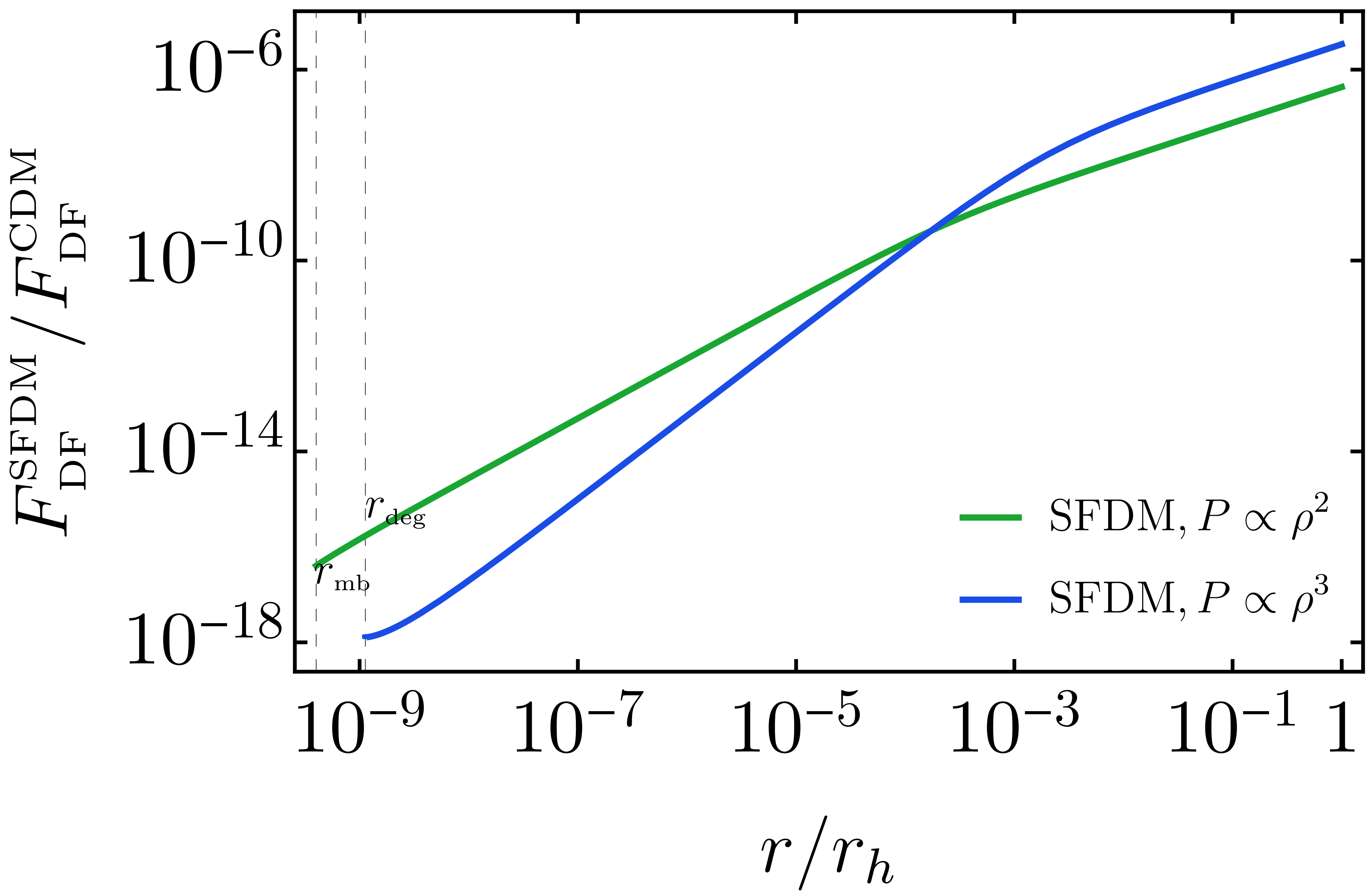
$$\vec{F}_{\rm DF} = -\frac{4\pi G^2 M^2 \rho}{c_s^2}\vec{\mathcal{F}}_{\rm DF}$$

1) Dynamical friction for sound waves perturbations (r_0^{-1}<mc_s)

A) ***Zero radial friction for subsonic motion***

B) ***Bumps****: the perturber renters its own wake*

C) ***Homogeneous medium***



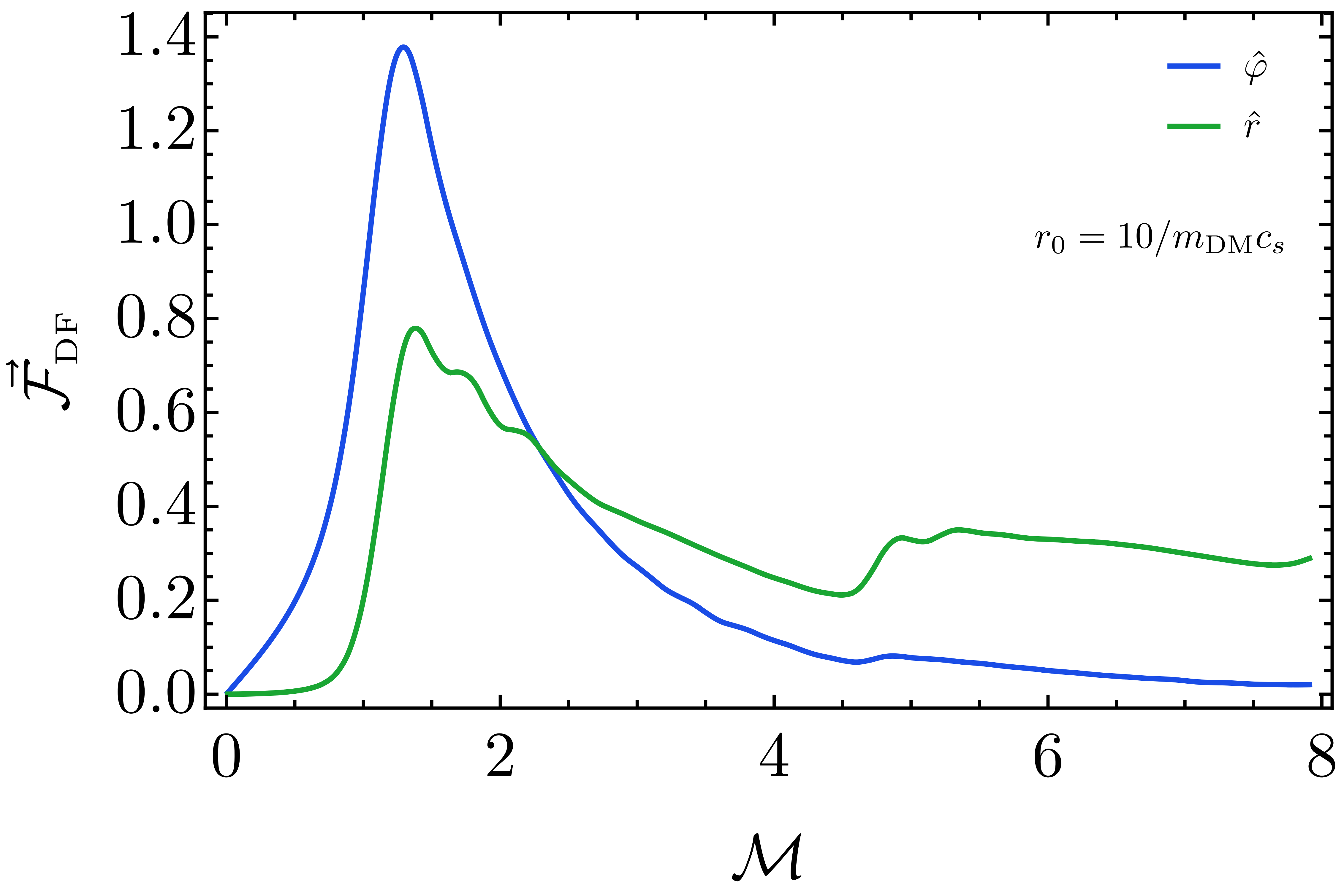
2) If we extend non homogeneous medium. Suppression of Dynamical friction (compared to CDM) due to

C) ***Phonons***

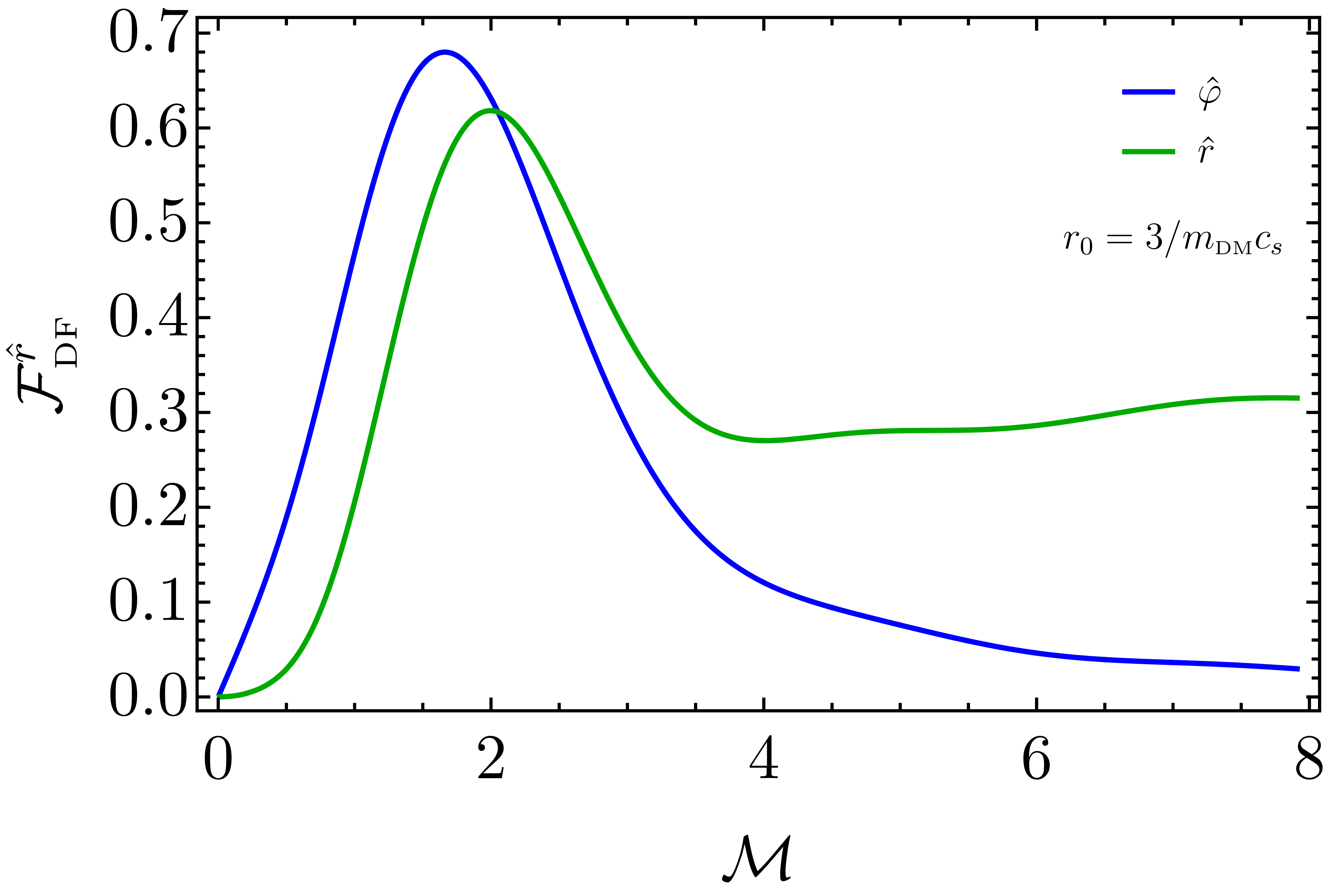
D) different ***density distribution of the spike***

(L. Berezhiani, G.C., V. De Luca, J. Khoury, 2023)

1) Dynamical friction when perturbations are mostly sound waves ($r_0^{-1} < mc_s$)



2) Dynamical friction when perturbations are in between sound waves and single particles ($r_0^{-1} \simeq mc_s$)



(L. Berezhiani, G.C., V. De Luca, J. Khoury, 2023)

3) Comparison of GW-emission and Dynamical friction for EMRIs

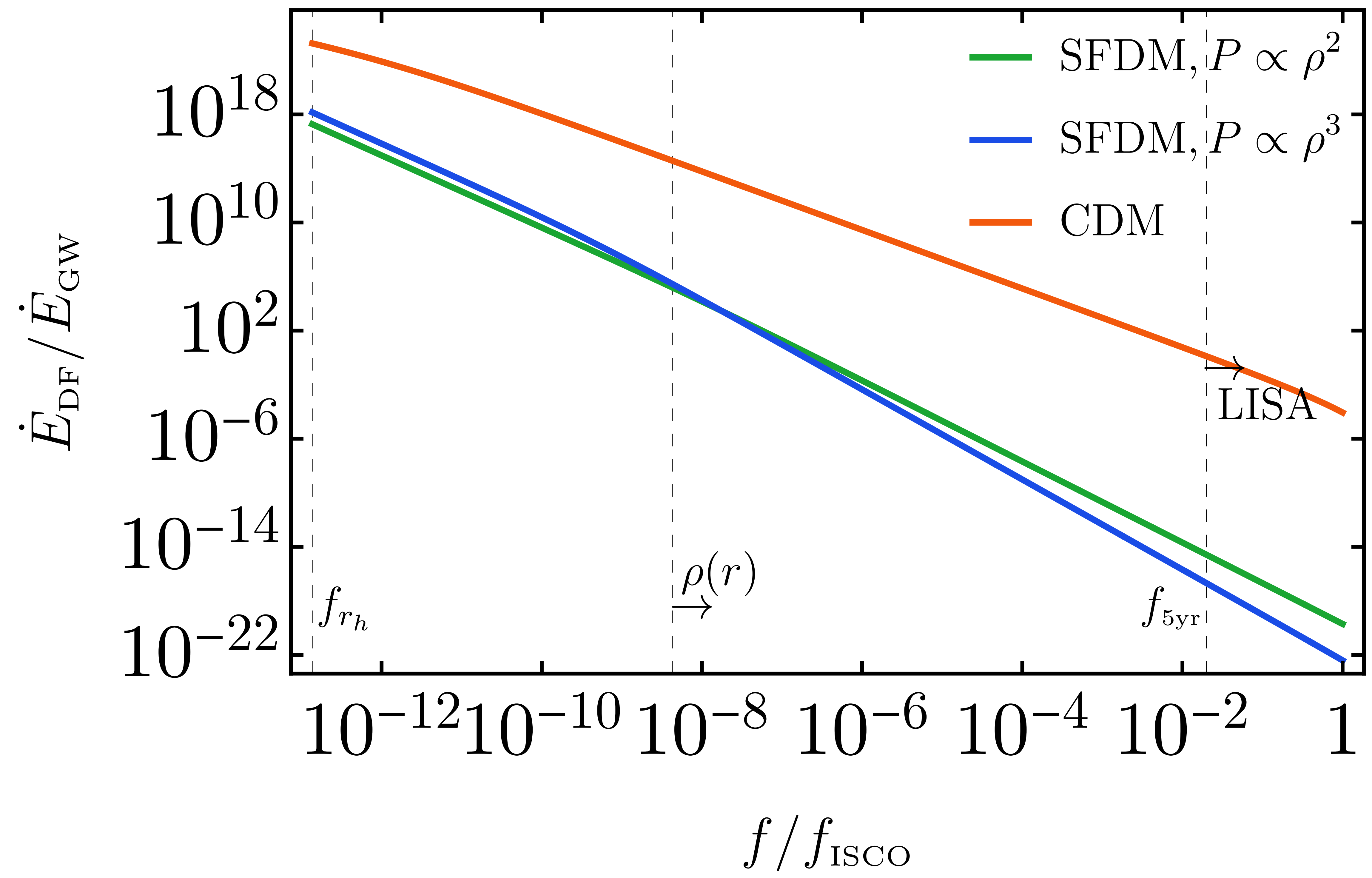
$$\left(\Delta\mathcal{N}_{\rm cycles} = \frac{\mathcal{N}_{\rm cycles}|_{\rm GW\,+\,DF}}{\mathcal{N}_{\rm cycles}|_{\rm GW}} \right)$$

A) GW-emission dominates at ***low radii*** *both for CDM and SFDM.*

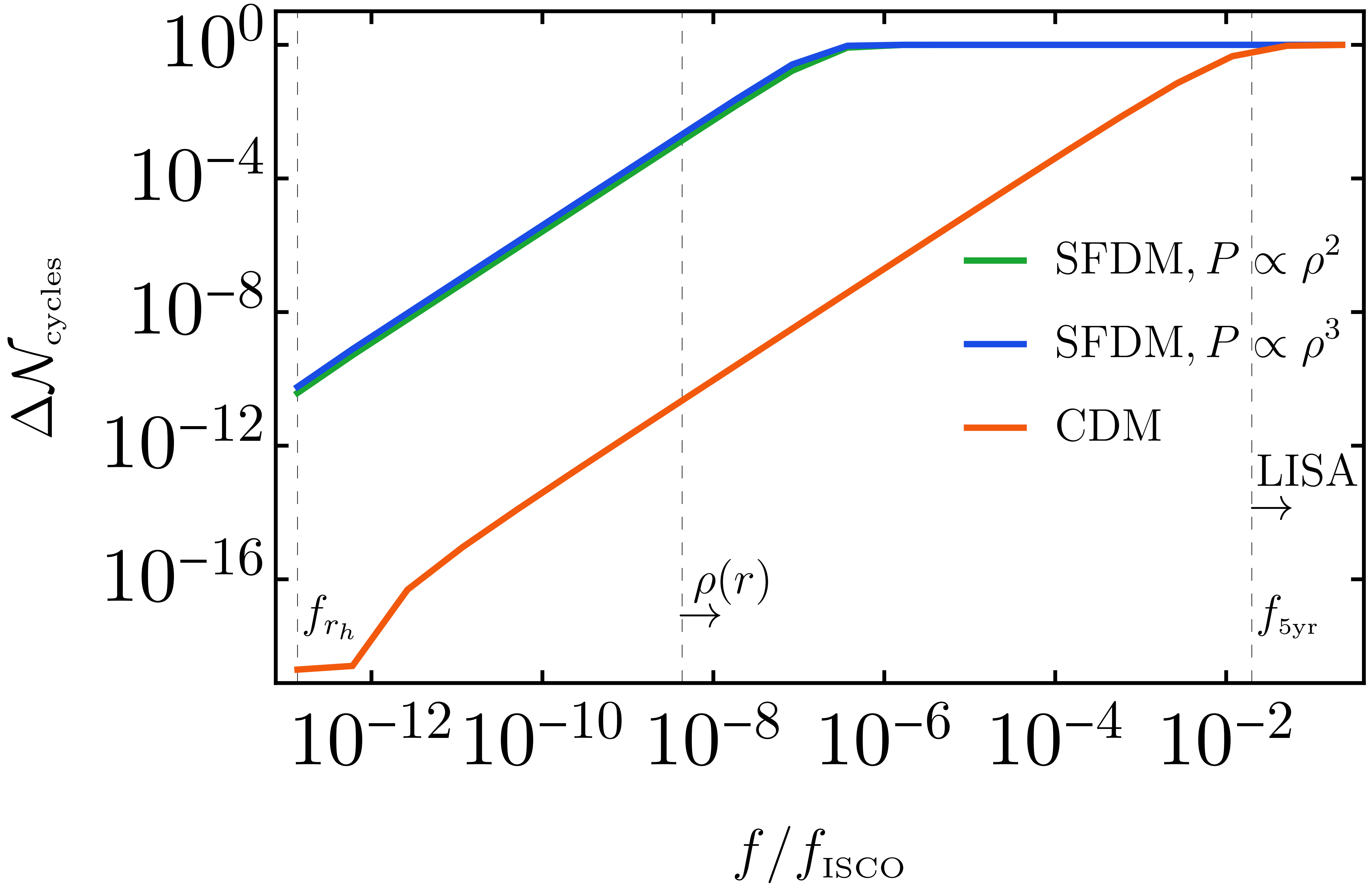
B) Compared to CDM, the transition takes place at larger radii for SFDM

C) The transition can be ***probed by LISA*** *for CDM. Not for SFDM!*

Energy loss: DF vs GW-emission



GW-dephasing



(L. Berezhiani, G.C., V. De Luca, J. Khoury, 2023)

Conclusions

1) Dark Matter superfluidity, in which ***sub-eV*** bosons with ***repulsive*** self-interactions condense at typical galactic density, is a proposal that can ameliorate the open questions that affect DM in galaxies.

2) Perturbers moving in the dark matter superfluid would experience a ***suppressed*** dynamical friction than CDM. It can lead to different observational signatures!

3) This difference is particularly noticeable in ***dark matter spikes***. It leads to a different evolution of EMRIs. In contrast to CDM, the suppression of Dynamical friction gives a GW dephasing which is ***not detectable with LISA.***