Galileo Galilei Institute

Florence Theory Group Day

GGI - 25/03/2024

UNIVERSITÀ DEGLI STUDI FIRENZE

VARYING HUBBLE CONSTANT IN THE $f(R)$ MODIFIED GRAVITY

Speaker: Dr. Tiziano Schiavone

Università di Pisa

Ciências ULisboa

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Fondazione Angelo Della Riccia Postdoctoral fellowship for 2023

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

- STANDARD ΛCDM COSMOLOGICAL MODEL
  AND THE HUBBLE CONSTANT TENSION
- REDSHIFT BINNED ANALYSIS OF THE SNE IA PANTHEON SAMPLE
- DECREASING TREND OF THE HUBBLE CONSTANT WITH REDSHIFT
- THEORETICAL INTERPRETATION
  IN THE JORDAN FRAME OF $f(R)$ MODIFIED GRAVITY THEORIES
- CONCLUSIONS

On the Hubble Constant Tension in the SNe Ia Pantheon Sample

ApJ 912, 150 (2021)   arXiv:2103.02117

Authors: M. G. Dainotti, B. De Simone, **TS**, G. Montani, E. Rinaldi, G. Lambiase

On the Evolution of the Hubble Constant with the SNe Ia Pantheon Sample and Baryon Acoustic Oscillations: A Feasibility Study for GRB-Cosmology in 2030

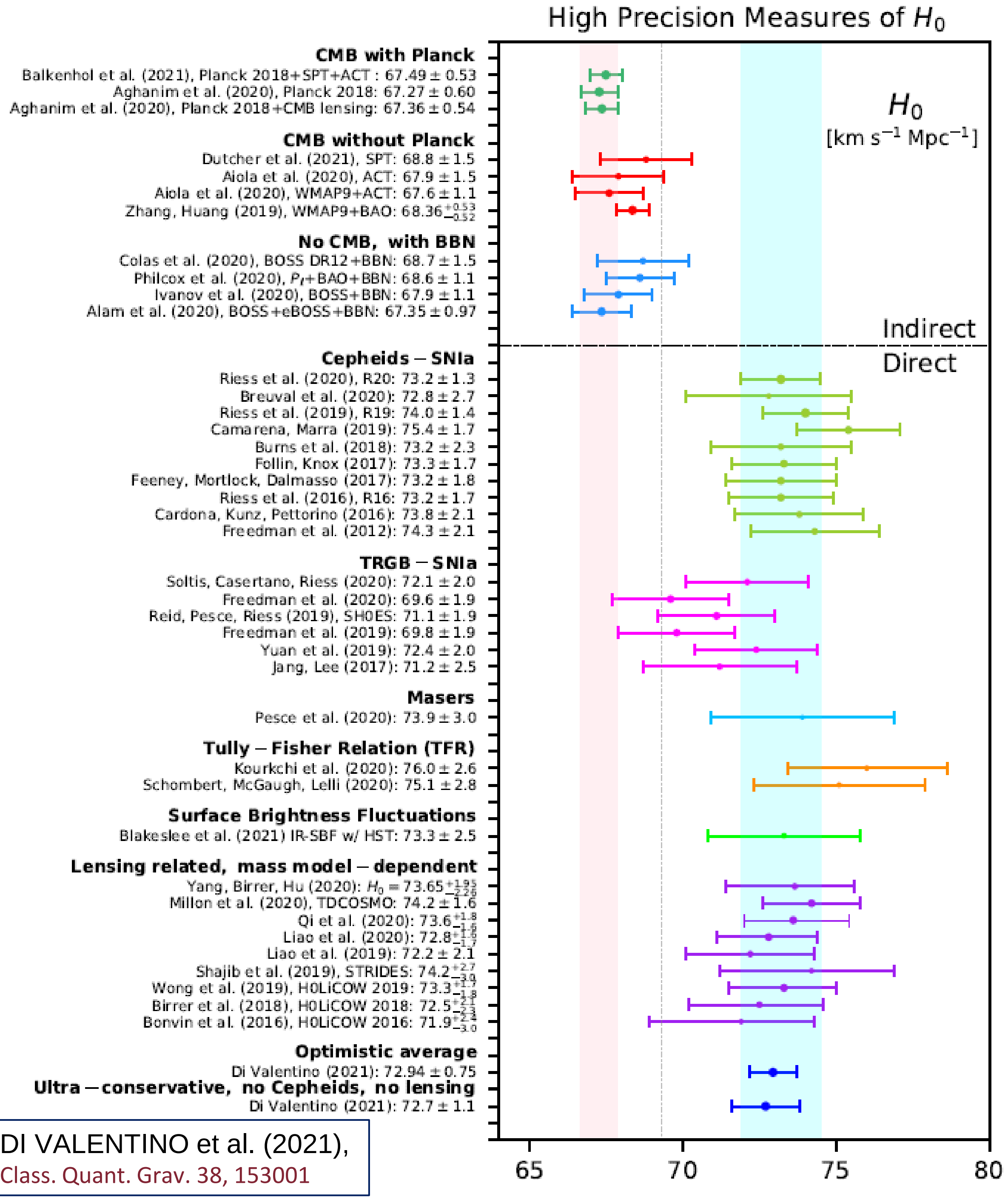
Galaxies, 10, 24 (2022)   arXiv:2201.09848

Authors: M. G. Dainotti, B. De Simone, **TS**, G. Montani, E. Rinaldi, G. Lambiase, M. Bogdan, S. Ugale

f(R) gravity in the Jordan Frame as a Paradigm for the Hubble Tension

MNRAS Letters, 522, L72-L77 (2023)   arXiv:2211.16737

Authors: **TS**, G. Montani, F. Bombacigno

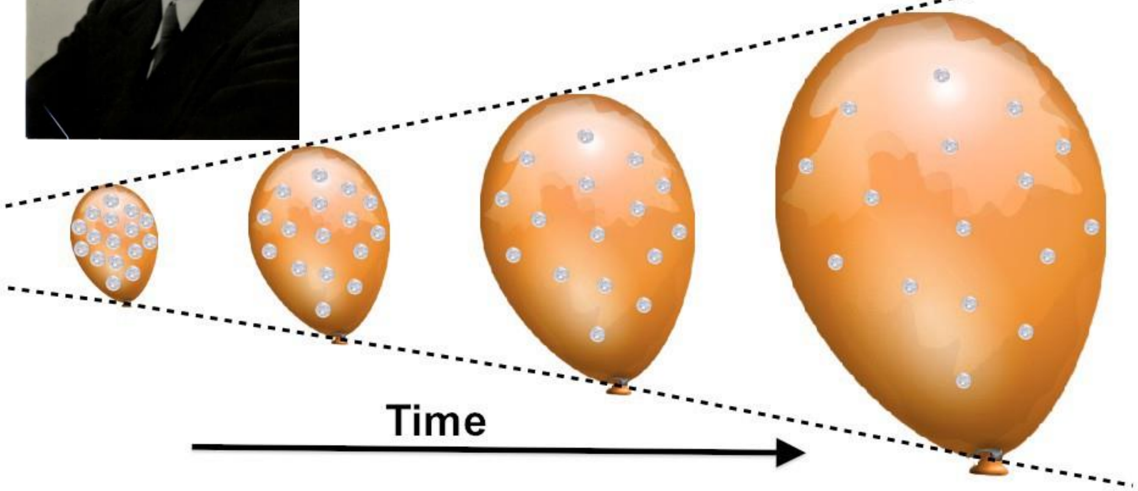


DI VALENTINO et al. (2021),
Class. Quant. Grav. 38, 153001

HUBBLE CONSTANT TENSION

Cosmic expansion and Hubble law

$$v_g = H_0\, d$$



Hubble constant definition

$$H_0 \equiv H(t = t_0) = H(z = 0)$$

Speaker
SCHIAVONE TIZIANO

PANTHEON SAMPLE BINNED ANALYSIS

Hubble constant tension within the SNe Ia redshift range?

1048 spectroscopically confirmed SNe Ia from different surveys
(PS1, SDSS, ESSENCE, SNLS, SCP, GOODS, CANDELS/CLASH)
Scolnic et al. (2018), ApJ 859, 101   Repository: https://github.com/dscolnic/Pantheon

$0.01 < z < 2.26$

- Equally populated subsamples of SNe Ia: 3, 4, 20, 40 redshift bins
- Statistical analysis for each redshift bin
  included statistical and systematic covariance matrices of Sne Ia
  χ^2 minimization, MCMC method
- Fixed $\Omega_{m0} = 0.298$ for the ΛCDM model [Scolnic et al. (2018), ApJ 859, 101]
- Extracting H_0 value for each bin
- Uniform priors: $60 < H_0 < 80\ km\ s^{-1}\ Mpc^{-1}$
- Test to check the values of H_0 in different redshift bins

Test: non-linear fit

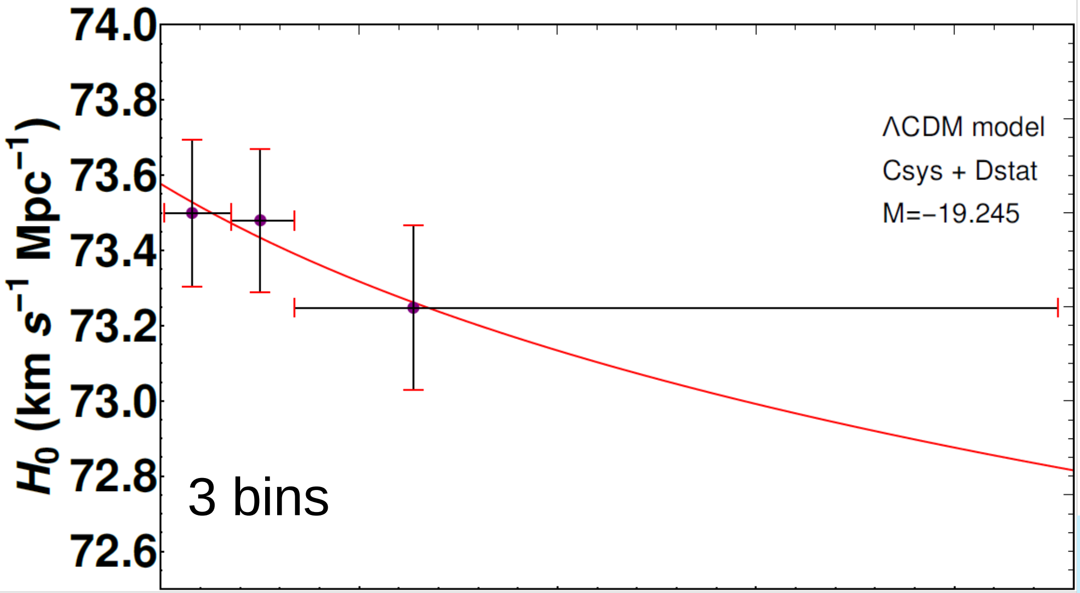
$$H_0(z) = \frac{\tilde{H}_0}{(1+z)^\alpha}$$

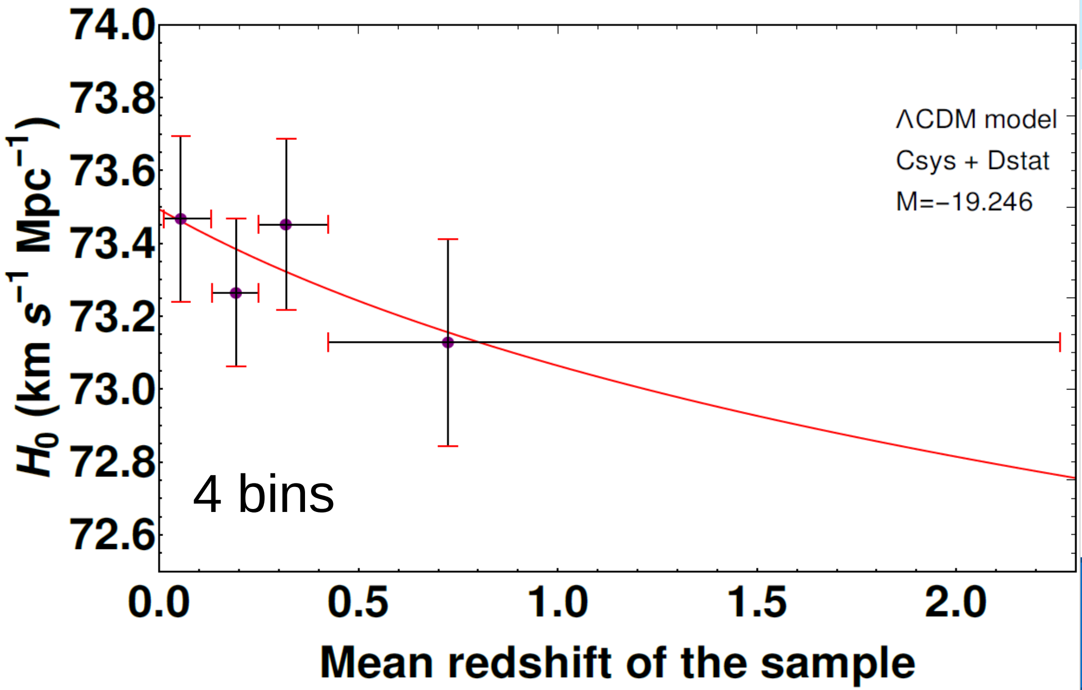
α: evolutionary parameter

$\tilde{H}_0 = H_0(z=0)$

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

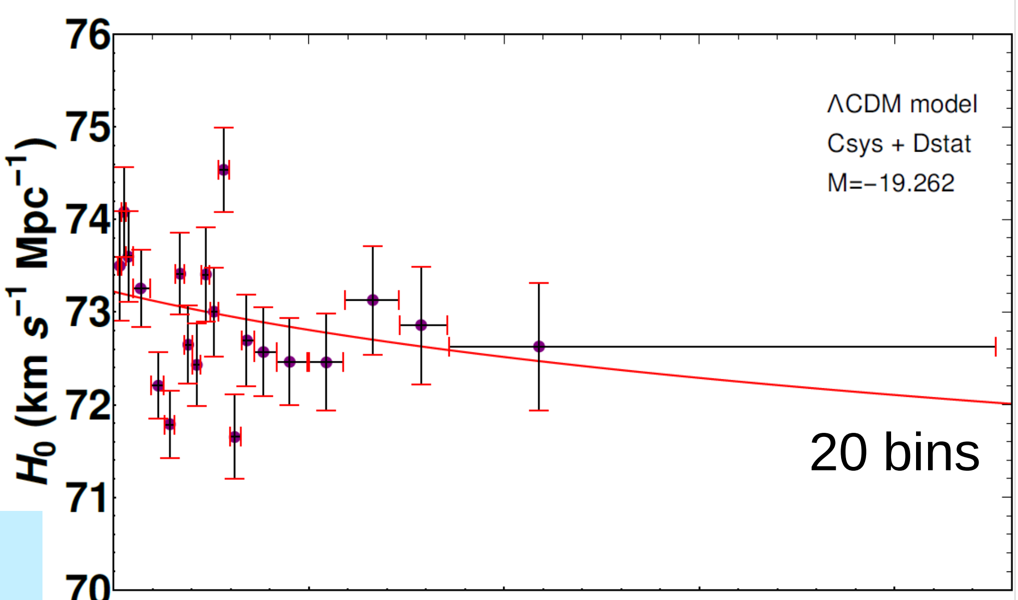
A NON-CONSTANT HUBBLE CONSTANT?

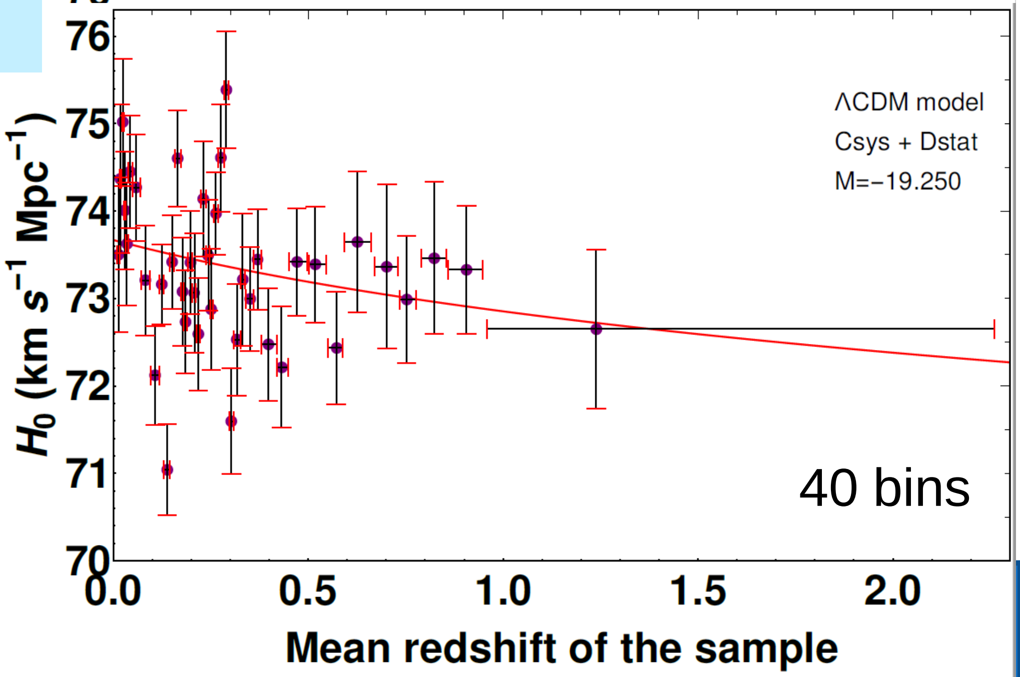




$$H_0(z) = \frac{\tilde{H}_0}{(1+z)^{\alpha}}$$

DAINOTTI, DE SIMONE, **SCHIAVONE** et al. (2021), ApJ 912, 150





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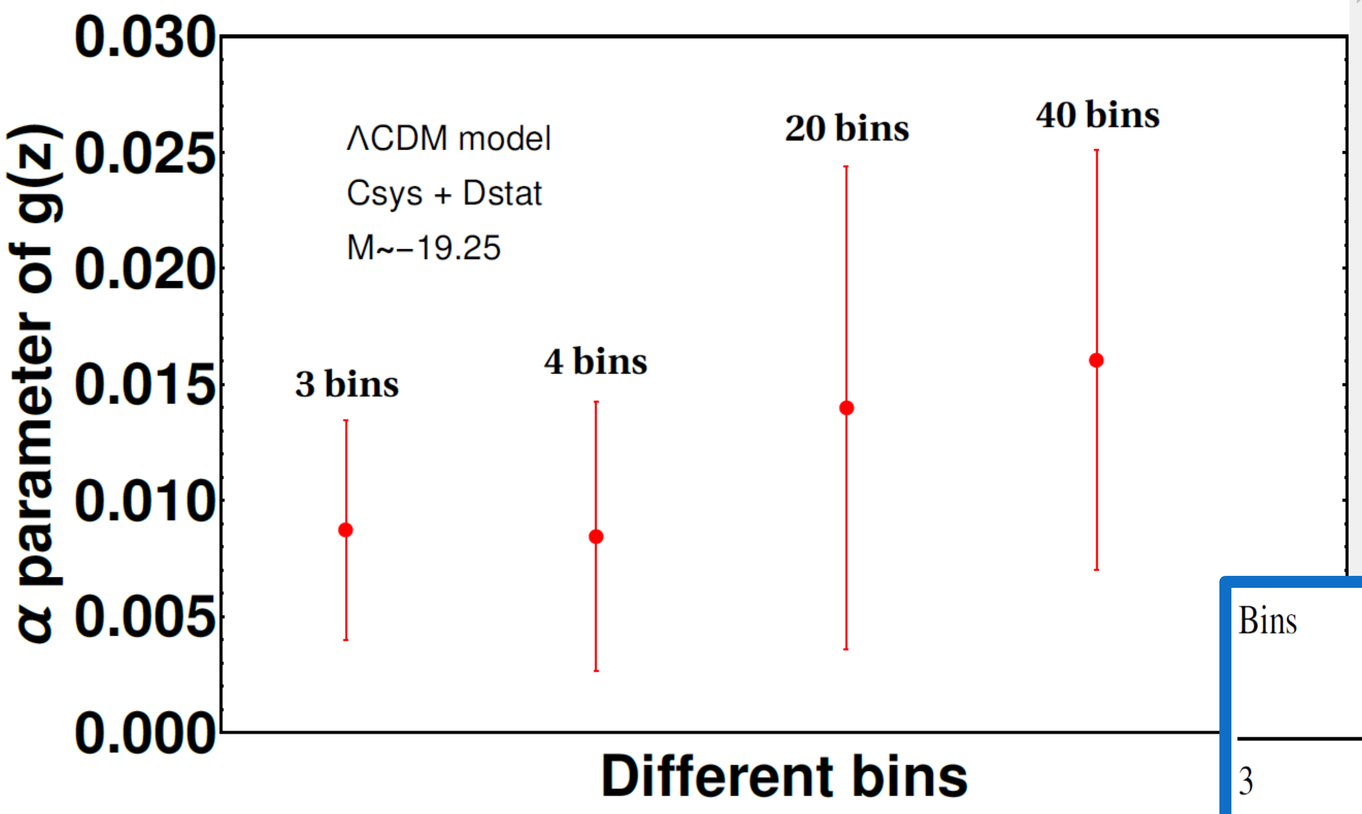


$$H_0(z) = \frac{\tilde{H}_0}{(1+z)^{\alpha}}$$

$\alpha = 0 \;\rightarrow\;$ no evolution

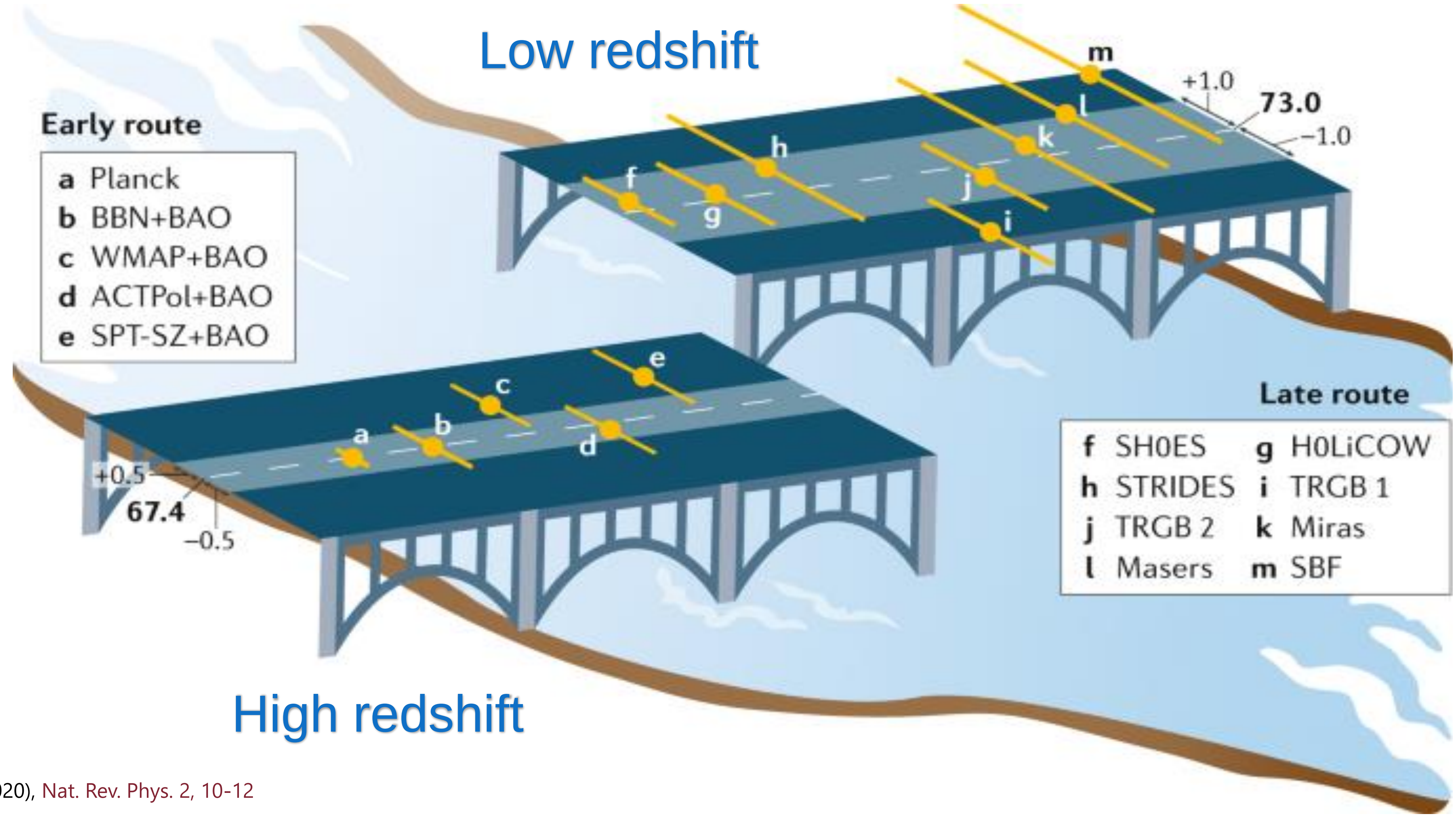
Error bars, 1 σ

Slight and unexpected evolution of $H_0(z)$

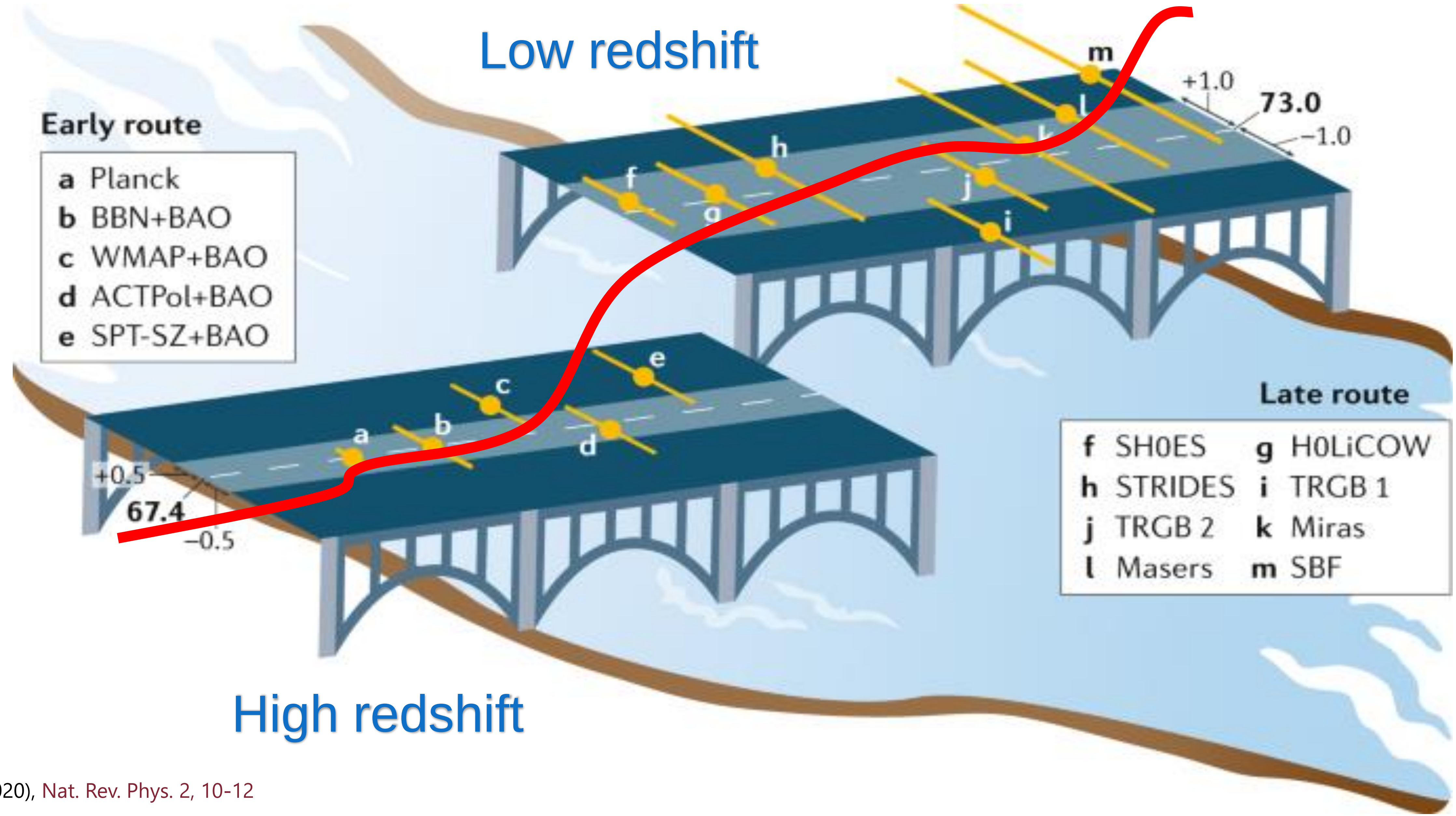
FIT RESULTS ΛCDM MODEL

Bins	$\tilde{H}_0$ (km s^{-1} Mpc^{-1})	α	$\frac{\alpha}{\sigma_\alpha}$	M
3	73.577 ± 0.106	0.009 ± 0.004	2.0	−19.245 ± 0.006
4	73.493 ± 0.144	0.008 ± 0.006	1.5	−19.246 ± 0.008
20	73.222 ± 0.262	0.014 ± 0.010	1.3	−19.262 ± 0.014
40	73.669 ± 0.223	0.016 ± 0.009	1.8	−19.250 ± 0.021

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150



RIESS (2020), Nat. Rev. Phys. 2, 10-12



RIESS (2020), Nat. Rev. Phys. 2, 10-12

EXTRAPOLATED VALUES AT HIGH REDSHIFTS

ΛCDM model

Bins	$H_0(z = 11.09)$ (km s^{-1} Mpc^{-1})	$H_0(z = 1100)$ (km s^{-1} Mpc^{-1})
3	72.000 ± 0.805	69.219 ± 2.159
4	71.962 ± 1.049	69.271 ± 2.815
20	70.712 ± 1.851	66.386 ± 4.843
40	70.778 ± 1.609	65.830 ± 4.170

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

$$H_0(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

Consistent in 1 σ with the Planck measurement from the CMB, at the redshift of the last scattering surface z=1100

$$H_0^{[CMB]} = (67.36 \pm 0.54)\ km\ s^{-1}\ Mpc^{-1}$$

PLANCK COLLABORATION
Planck 2018 result, VI: Cosmological parameters
A&A 641, A6 (2020).

POSSIBLE EXPLANATIONS OF $H_0(z)$

Astrophysical reasons or trouble with the Pantheon Sample

- ❑ Hidden redshift evolution of an astrophysical parameter of SNe Ia (stretch, metallicity, …)
- ❑ Astrophysical properties (host galaxies, selection effects)
- ❑ Biases not considered in the Pantheon Sample
- ❑ Systematics uncertainties of the sample

New Physics

- ❑ Late and/or early-time modification of gravity?

$f(R)$ MODIFIED GRAVITY in the JORDAN FRAME

- Extend GR to solve open problems in cosmology with extra degrees of freedom
- Geometrical modification of gravity theory
- Avoiding *ad hoc* components in the Universe, *e.g.* dark energy
- Generalized gravitational lagrangian $\mathcal{L}_g = f(R)$ R: Ricci scalar

- Dynamically equivalent action in the Jordan frame (JF), scalar-tensor theory
- The extra degree of freedom of $f(R)$ is replaced by a scalar field ϕ
- Non-minimal coupling between scalar field and metric

Scalar field

$$\phi = f'(R)$$

Potential

$$V(\phi) = R(\phi)\phi - f(R(\phi))$$

NOJIRI & ODINTSOV (2006), eConf C0602061, 06

SOTIRIOU & FARAONI (2010), Rev. Mod. Phys. 82, 451

$f(R)$ MODIFIED GRAVITY in the JORDAN FRAME

Dynamically equivalent action to $f(R)$ theories

Jordan Frame (JF)

$$S_g = \frac{1}{2\chi} \int_\Omega d^4x \sqrt{-g}\; [\phi\, R - V(\phi)]$$

Scalar field

$$\phi = f'(R)$$

Potential

$$V(\phi) = R(\phi)\phi - f(R(\phi))$$

For a flat FLRW metric:

Generalized Friedmann eq.

$$H^2 = \frac{\chi\,\rho}{3\,\phi} - H\,\frac{\dot\phi}{\phi} + \frac{V(\phi)}{6\,\phi}$$

Scalar field eq.

$$3\,\ddot\phi - 2\,V(\phi) + \phi\,\frac{dV}{d\phi} + 9\,H\,\dot\phi = \chi\,\rho$$

Generalized cosmic acc. eq.

$$\frac{\ddot a}{a} = -\frac{\chi\,\rho}{6\,\phi} + \frac{V(\phi)}{6\,\phi} + \frac{1}{6}\frac{dV}{d\phi} + H\,\frac{\dot\phi}{\phi}$$

χ: Einstein constant

$\dot{(\dots)} = \partial_t(\dots)$

$f(R)$ COSMOLOGY

- Extra degree of freedom in the parameterization, functional form of $f(R)$
- Mimic ΛCDM in the high-redshift regime, well-tested by the CMB
- Cosmic accelerated expansion with an effective cosmological constant
- Phenomenology of ΛCDM as a limiting case

Hu-Sawicki

$$f(R) = R - m^2 \frac{c_1 \left(\frac{R}{m^2}\right)^n}{c_2 \left(\frac{R}{m^2}\right)^n + 1}$$

c_1, c_2 parameters; $n > 0$ $\quad m^2 \equiv \frac{\chi \rho_{m0}}{3}$

Starobinski

$$f(R) = R - \mu R_c \left[1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right]$$

Tsujikawa

$$f(R) = R - \mu R_c \tanh\left(\frac{|R|}{R_c}\right)$$

$n, \mu, R_c > 0$

HU & SAWICKI (2007), Phys. Rev. D ,76, 064004

AMENDOLA & TSUJIKAWA (2010), Cambridge University Press

STAROBINSKi (2007), Jetp Lett. 86, 157

TSUJIKAWA (2008), Phys. Rev. D, 77, 023507

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

- Decreasing trend of $H_0(z)$ from binned analysis of SNe Ia + BAOs [1,2]
- Failure of $w_0 w_a$CDM model [3,4] and the $f(R)$ Hu-Sawicki proposal to explain $H_0(z)$ [1,2]
- Need for a new $f(R)$ model able to both mimic a dark energy component and provide a mechanism for an effective Hubble constant
- Non-minimally coupled scalar field plays a crucial role
- Goal: effective Hubble constant that evolves with z to match

$H_0^{[CMB]} = (67.36 \pm 0.54)$ km s^{-1} Mpc^{-1} and $H_0^{[loc]} = (73.04 \pm 1.04)$ km s^{-1} Mpc^{-1}

[1] DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

[2] DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2022), Galaxies 2022, 10, 24

[3] CHEVALLIER & POLARSKI (2001), Int. J. Mod. Phys. D 10, 213

[4] LINDER (2000), Phys. Rev. Lett. 90, 091301

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

Generalized Friedmann eq. in JF:

$$H^2 = \frac{1}{\phi - (1+z)\frac{d\phi}{dz}} \frac{\chi}{3}\left[\rho + \frac{V(\phi)}{2\chi}\right]$$

Approximation:
slight deviation
from ΛCDM

$$V(\phi) \equiv 2\chi\rho_\Lambda + g(\phi)$$

$$g(\phi) \ll V(\phi)$$

We obtain a form similar to flat ΛCDM but with an effective Hubble constant

$$H^{[\Lambda CDM]}(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}$$

$$H(z) \approx H_0^{\text{eff}}(z)\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}$$

$$H_0^{\text{eff}}(z) = \frac{H_0}{\sqrt{\phi - (1+z)\frac{d\phi}{dz}}}$$

SCHIAVONE, MONTANI, & BOMBACIGNO (2023), MNRAS Letters, 522, L72-L77

Solving the cosmological dynamics in the JF, we can reconstruct analytically the scalar field potential, then the f(R) model

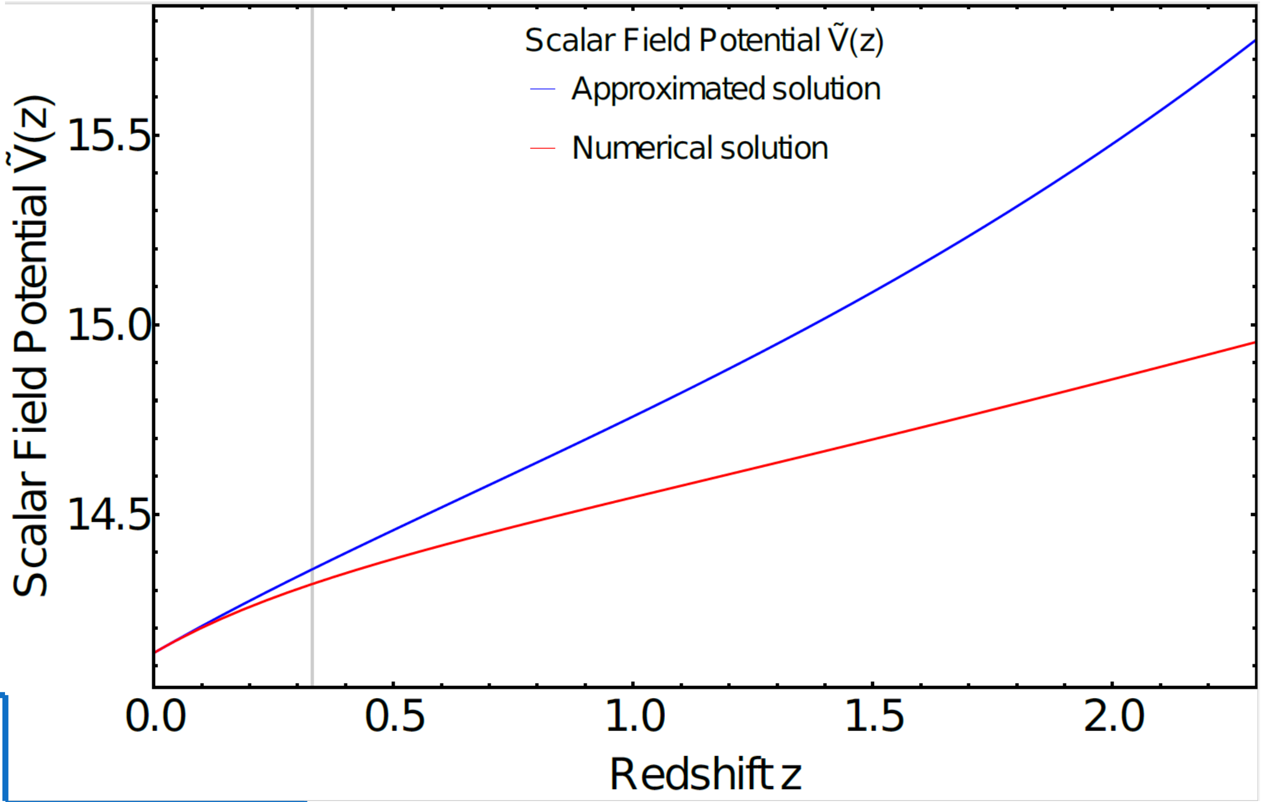
AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

Approximated analytical solution:

$$\phi(z) = K\,(1+z)^{2\alpha}$$

$$H_0^{\text{eff}}(z) = \frac{H_0}{\sqrt{K\,(1-2\,\alpha)}\,(1+z)^{\alpha}}$$

$$\tilde{V}(\phi) = \frac{V(\phi)}{m^2}$$ See the expression in *



Conditions to match $H_0^{[CMB]}$ and $H_0^{[loc]}$:

$\phi(0) = K = 1 - 10^{-7}$

$\alpha = 1.1 \times 10^{-2}$

$H_0 = 72.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

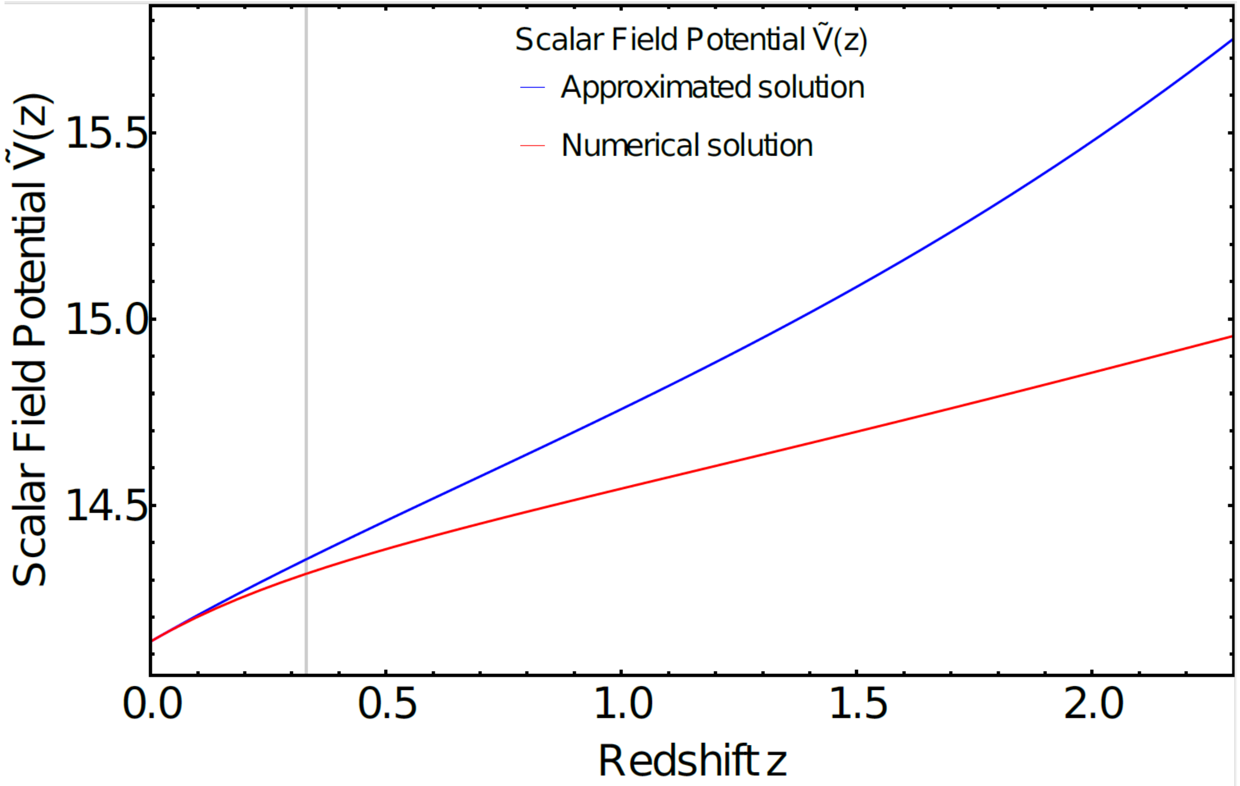
$\Omega_{m0} = 0.298$

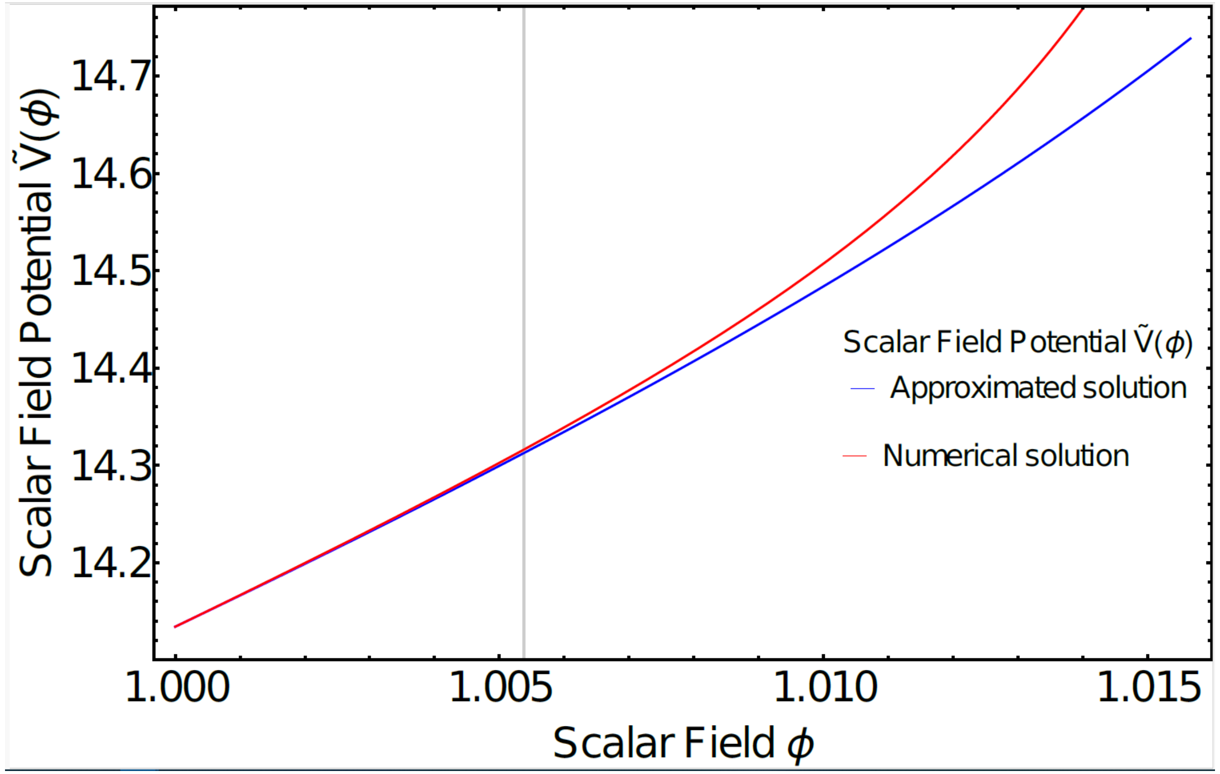
$\tilde{V}(\phi = K) = \tilde{V}(z=0) = 6\,\dfrac{1-\Omega_{m0}}{\Omega_{m0}}$

ΛCDM today

* **SCHIAVONE,** MONTANI, & BOMBACIGNO (2023), MNRAS Letters, 522, L72-L77

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY





Percentage variation of $\tilde{V} \sim 1.6\%$

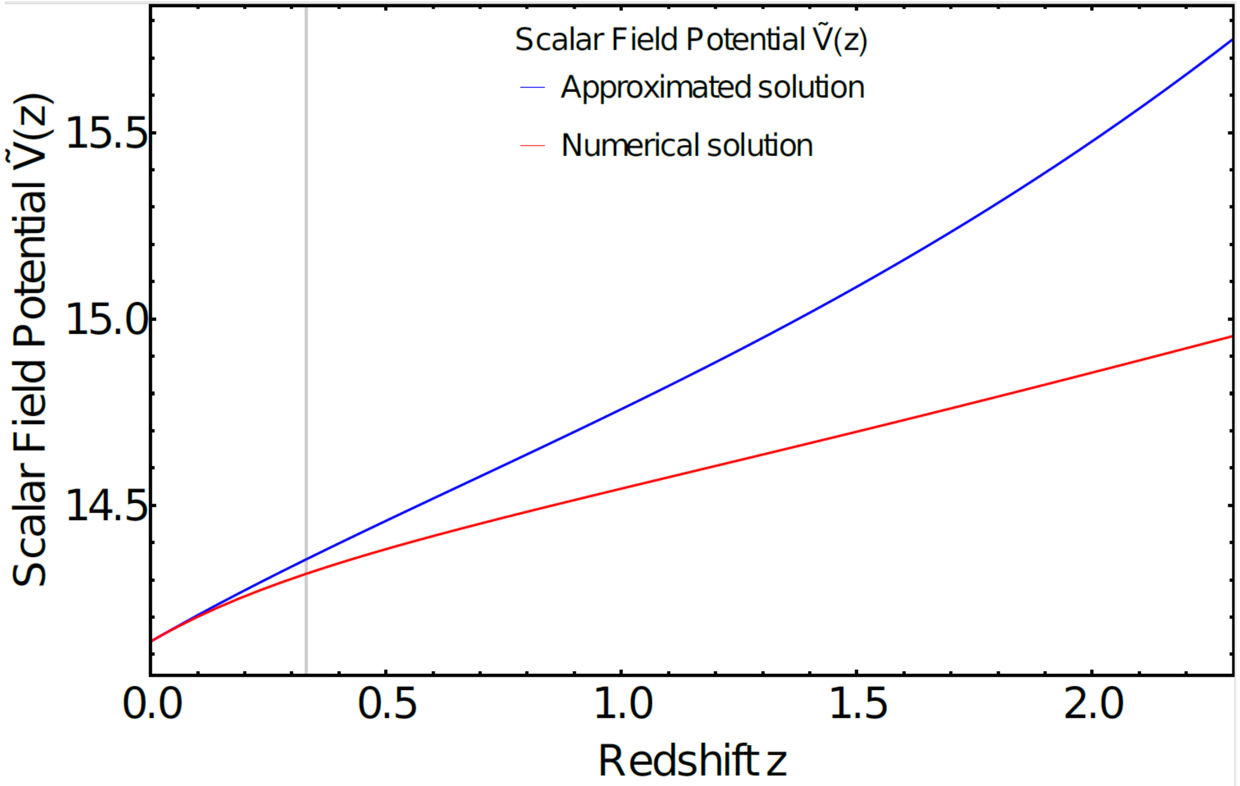
Nearly flat region occurs for $0 < z < 0.3$

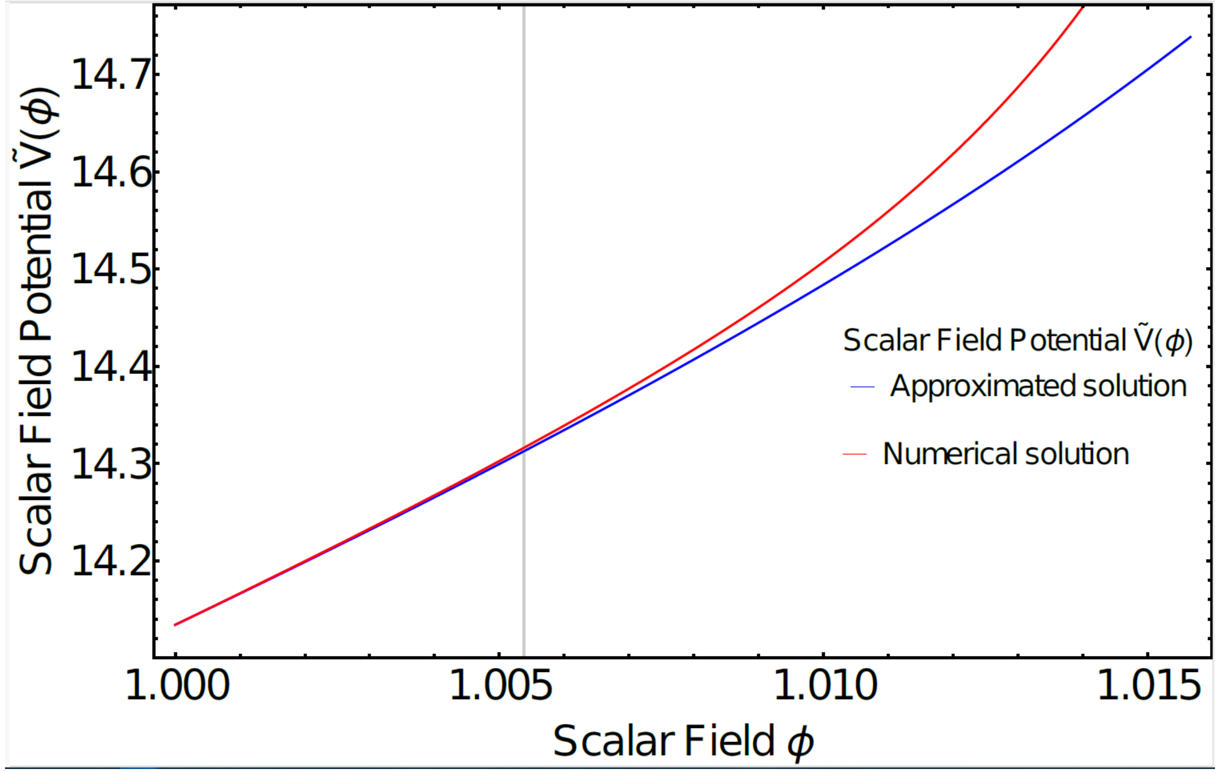
$$\tilde{V}(\phi) = \frac{V(\phi)}{m^2}$$

SCHIAVONE, MONTANI, & BOMBACIGNO (2023),

MNRAS Letters, 522, L72-L77

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY





$$f(R) \approx m^2 B_0 + B_1 R + B_2 \frac{R^2}{m^2}$$

Approximated solution for $z \ll 1$:
$f(R)$ – quadratic gravity

SCHIAVONE, MONTANI, & BOMBACIGNO (2023),
MNRAS Letters, 522, L72-L77

It can be shown that ΛCDM is recovered for $\alpha \rightarrow 0$ and $K \rightarrow 1$

SUMMARY

- Binned analysis of the SNe Ia Pantheon Sample (low redshifts)
- Unexpected evolution and decreasing trend of $H_0(z)$ in different bins and for various cosmological models
- A running Hubble constant allows a new interpretation of the tension: it could be no longer a discrepancy between local probes and Planck data, but an intrinsic evolutionary behavior of $H_0(z)$ as viewed in a modified gravity scenario
- New surveys in the future (Euclid, LSST, DESY, etc.) and other cosmological probes (Pantheon+, quasars, GRBs, etc.) to obtain better constraints on α
- Possible hints of new physics (modified gravity?)

Backup slides

LUMINOSITY DISTANCE d_L

d_L depends on the cosmological model.

For a flat geometry ($k = 0$):

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

$$H(z) = H_0\,E(z)$$

Ω_{i0}: cosmological density parameters

m: matter

r: radiation

- ΛCDM model

$$E(z) = \sqrt{\Omega_{m0}\,(1+z)^3 + \Omega_{r0}\,(1+z)^4 + \Omega_\Lambda}$$

- wCDM model

$$w = w(z)$$

$$E(z) = \sqrt{\Omega_{m0}\,(1+z)^3 + \Omega_{r0}\,(1+z)^4 + \Omega_{\mathrm{DE}0}\exp\left[3\int_0^z \frac{dz'}{1+z'}\,(1+w(z'))\right]}$$

- $w_0 w_a$CDM model

$$w(z) = w_0 + \frac{w_a\,z}{1+z}$$

CPL parameterization (Chevallier-Polarski-Linder)

CHEVALLIER & POLARSKI (2001), Int. J. Mod. Phys. D 10, 213

LINDER (2000), Phys. Rev. Lett. 90, 091301

DISTANCE MODULUS μ

From the underlying theoretical cosmological model:

$$\mu_{th} = 5\log_{10} d_L + 25$$

TRIPP (1998), A&A 331, 815

BETOULE et al. (2014), A&A 568, A22

SCOLNIC et al. (2018), ApJ 859, 101

From SNe Ia observations:

$$\mu_{obs} = m_B - M + \alpha\, x_1 - \beta\, c + \Delta M + \Delta B$$

Modified Tripp formula

- m_B: B-band apparent magnitude
- M: Absolute magnitude for $x_1 = 0 = c$
- x_1: Stretch parameter
- c: Color parameter
- ΔM: Host-galaxy mass correction
- ΔB: Bias correction

PANTHEON SAMPLE ANALYSIS

1048 spectroscopically confirmed SNe Ia from different surveys
(PS1, SDSS, ESSENCE, SNLS, SCP, GOODS, CANDELS/CLASH)

$$0.01 < z < 2.26$$

Statistical analysis:

$$\chi^2 = \Delta\mu^T C^{-1} \Delta\mu$$

where

$$\Delta\mu = \mu_{obs} - \mu_{th}$$

Theoretical distance modulus

$$\mu_{th} = 5 \log_{10} d_L + 25$$

$$C = D_{stat} + C_{sys}$$

Uncertainty matrix C (1048x1048)

Statistical matrix (diagonal matrix, 1048x1048)

Systematic covariance matrix (symmetric, 1048x1048)

Cobaya package in Python to minimize χ^2

Scolnic et al. (2018), ApJ 859, 101

Repository: https://github.com/dscolnic/Pantheon

PANTHEON SAMPLE ANALYSIS

$$C = D_{stat} + C_{sys}$$

D_{stat} Statistical matrix
(diagonal matrix, 1048x1048)
Includes distance errors σ^2 for each SNe

C_{sys} Systematic covariance matrix
(matrix, 1048x1048)
Includes N systematic (S_k) sources of errors

$$\sigma^2 = \sigma_N^2 + \sigma_{mass}^2 + \sigma_{\mu-z}^2 + \sigma_{lens}^2 + \sigma_{int}^2 + \sigma_{bias}^2$$

- σ_N^2 → Photometric error
- σ_{mass}^2 → Mass-step correction
- $\sigma_{\mu-z}^2$ → Peculiar velocity and redshift
- σ_{lens}^2 → Lensing
- σ_{int}^2 → Intrinsic scatter
- σ_{bias}^2 → Bias correction

$$C_{ij,sys} = \sum_{k=1}^{N} \frac{\partial \mu_i}{\partial S_k} \frac{\partial \mu_j}{\partial S_k} \sigma_{S_k}^2$$

S_k: systematics → $(m_B, x_1, c, m_B c, x_1 m_B, x_1 c)$

σ_{S_k}: systematic error

PANTHEON SAMPLE BINNED ANALYSIS

Hubble constant tension within the SNe Ia redshift range?

$$0.01 < z < 2.26$$

- Equally populated subsamples of SNe Ia: 3, 4, 20, 40 redshift bins
- Building submatrices C and subvector $\Delta\mu$, considering the redshift order of SNe
- Statistical analysis for each redshift bin, χ^2 minimization, MCMC method
- Uniform priors: $60 < H_0 < 80\ km\ s^{-1}\ Mpc^{-1}$
- Starting from the local value in the 1st bin: $H_0 = 73.5\ km\ s^{-1}\ Mpc^{-1}$
- Fixed $\Omega_{m0} = 0.298$ for the ΛCDM model
- Fixed $\Omega_{m0} = 0.308, \quad \mathrm{w}_0 = -1.009, \quad \mathrm{w}_\mathrm{a} = -0.129$ for the $w_0 w_a$CDM model
- Extracting H_0 value for each bin

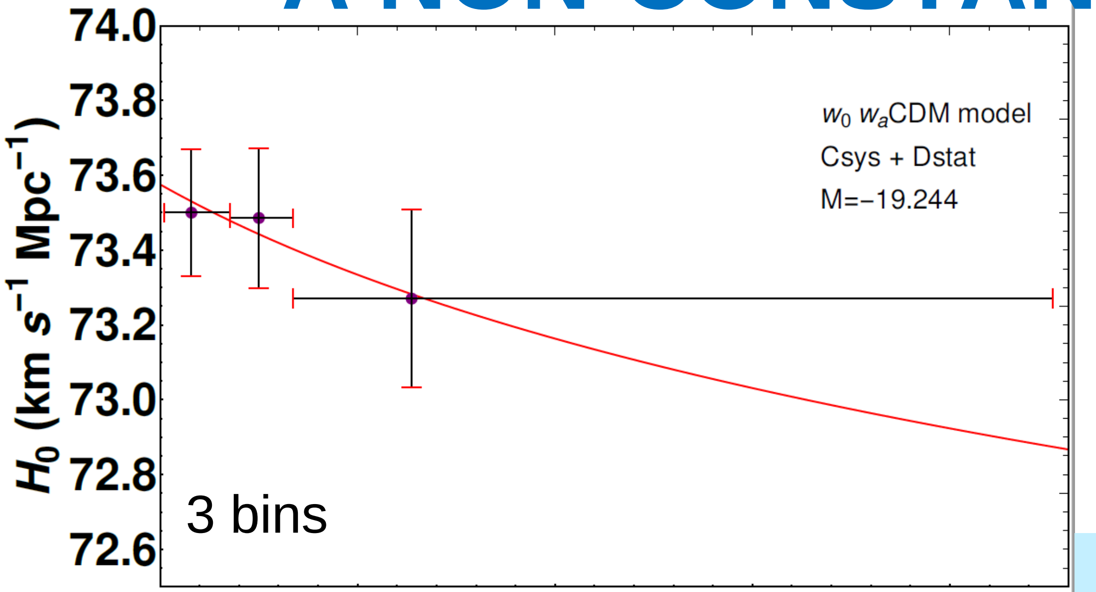
Test: non-linear fit

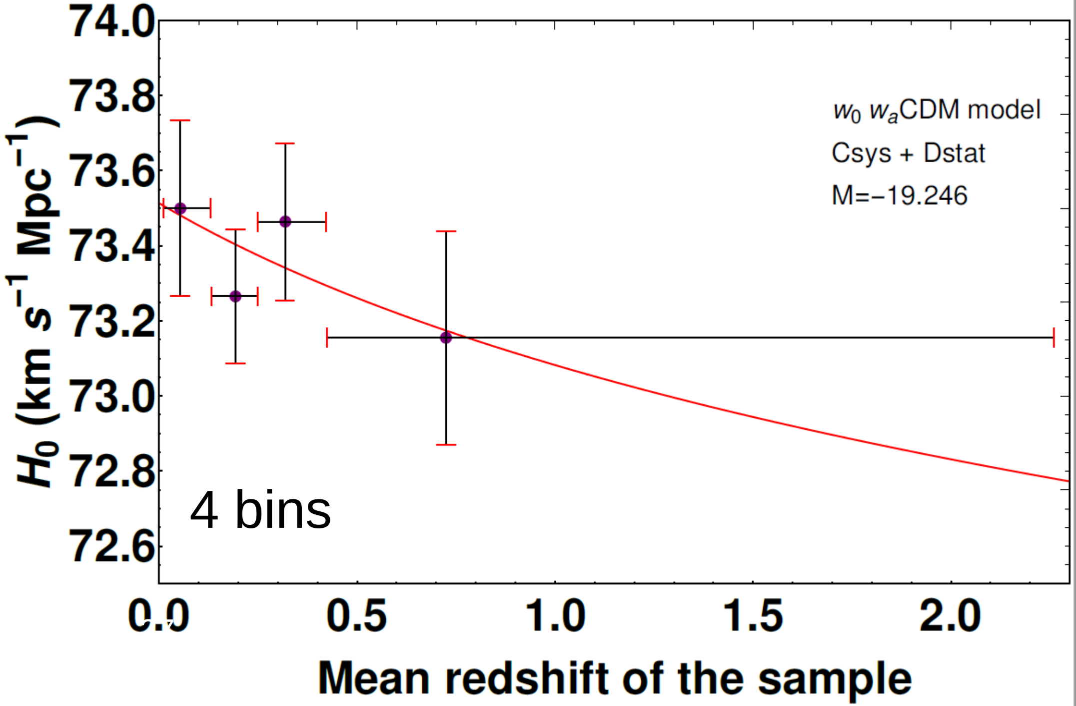
$$H_0(z) = \frac{\widetilde{H}_0}{(1+z)^\alpha}$$

α: evolutionary parameter

$\widetilde{H}_0 = H_0(z=0)$

A NON-CONSTANT HUBBLE CONSTANT?





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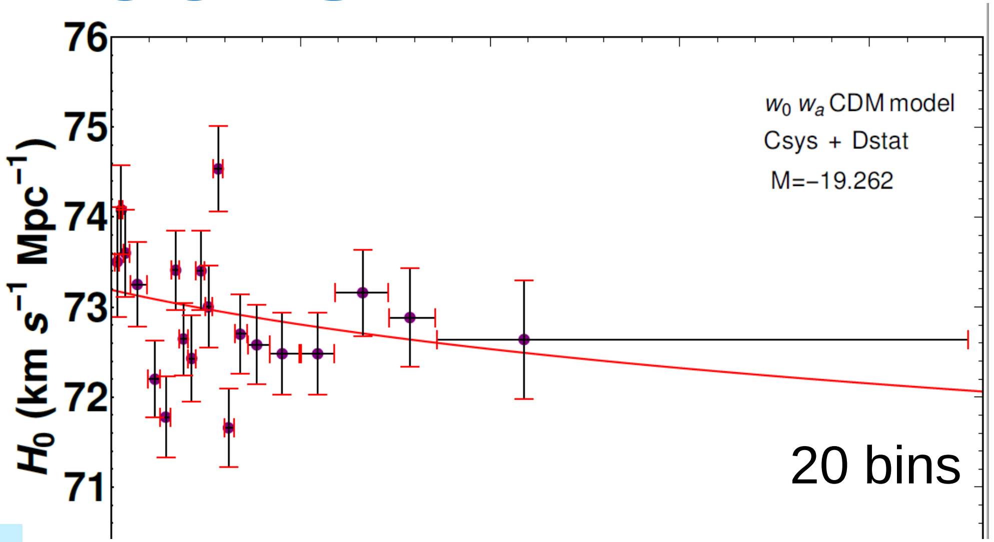
DAINOTTI,

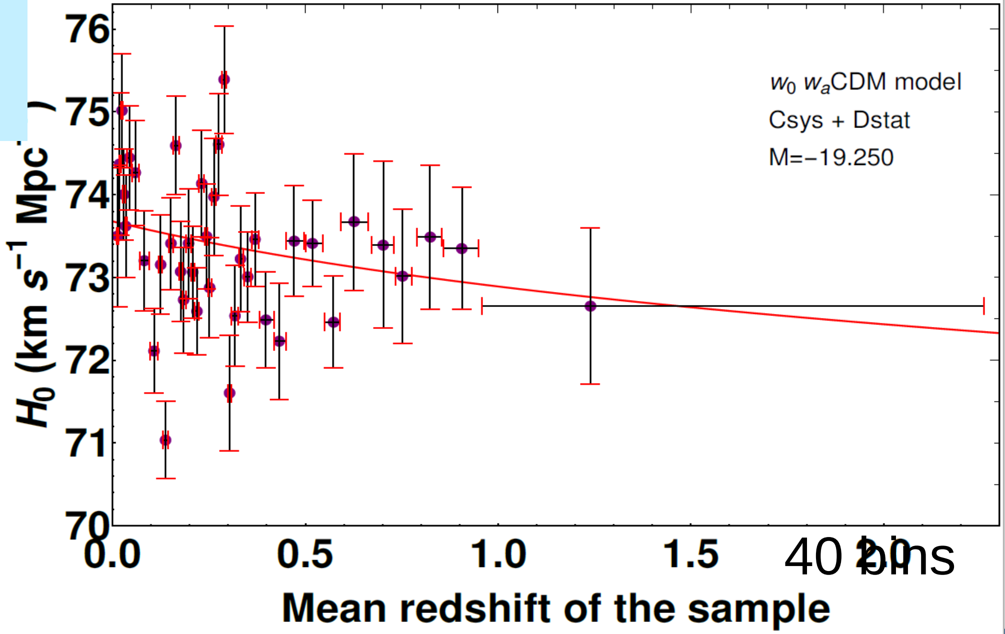
DE SIMONE,

SCHIAVONE

et al. (2021),

ApJ 912, 150





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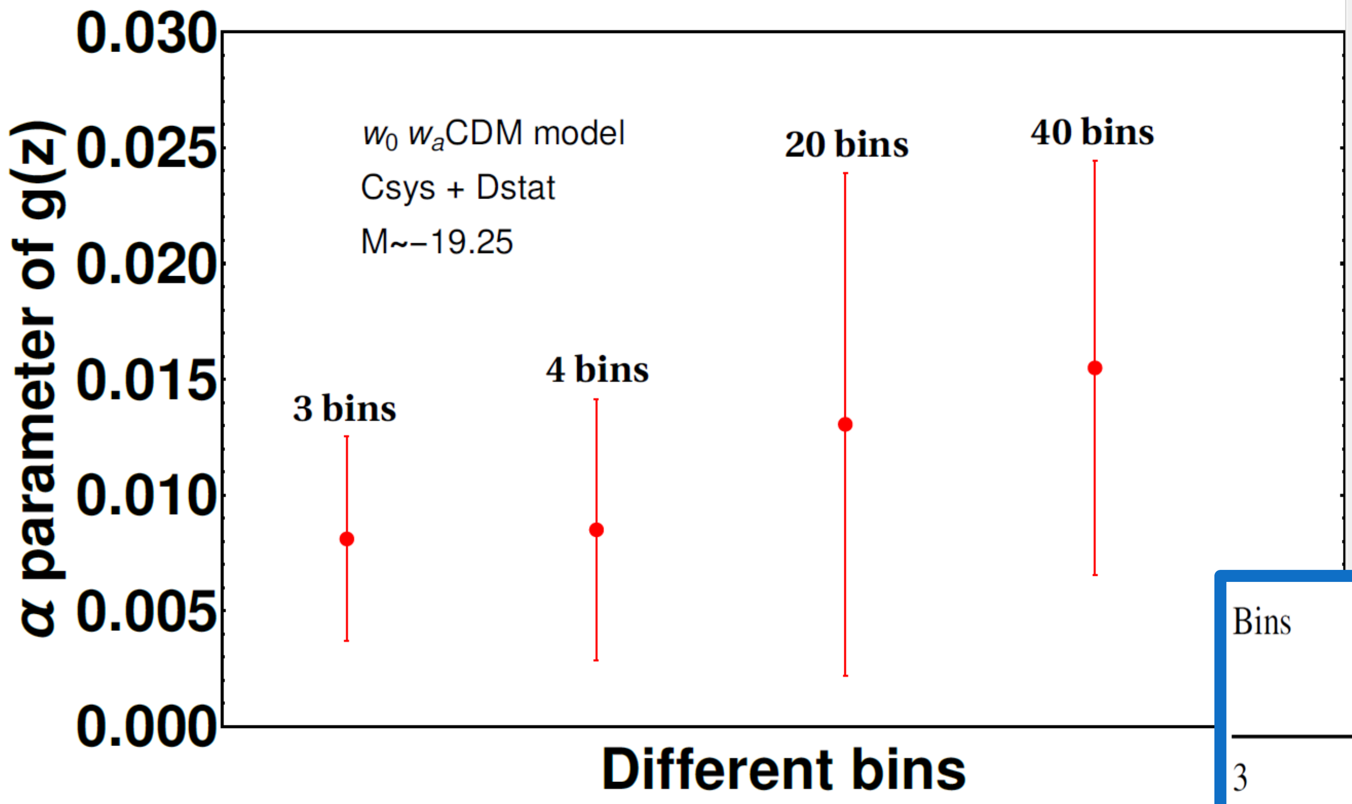

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$\alpha = 0 \;\rightarrow$ no evolution

Error bars, 1 σ

Slight and unexpected evolution of $H_0(z)$

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

FIT RESULTS $w_0 w_a$CDM MODEL

Bins	$\tilde{H}_0$ (km s^{-1} Mpc^{-1})	α	$\frac{\alpha}{\sigma_\alpha}$	M
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EXTRAPOLATED VALUES AT HIGH REDSHIFTS

$w_0 w_a$CDM model

Bins	$H_0(z=11.09)$ ($\mathrm{km\,s^{-1}\,Mpc^{-1}}$)	$H_0(z=1100)$ ($\mathrm{km\,s^{-1}\,Mpc^{-1}}$)
3	72.104 ± 0.766	69.516 ± 2.060
4	71.975 ± 1.020	69.272 ± 2.737
20	70.852 ± 1.937	66.804 ± 5.093
40	70.887 ± 1.595	66.103 ± 4.148

$$H_0(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

Consistent in 1 σ with the Planck measurement from the CMB, at the redshift of the last scattering surface z=1100

$$H_0^{[PLANCK]} = (67.4 \pm 0.5) km\, s^{-1}\, Mpc^{-1}$$

DAINOTTI, DE SIMONE, **SCHIAVONE,** et al. (2021), ApJ 912, 150

SNe + BAOs, BINNED ANALYSIS

- 3 redshift bins (≈ 350 SNe in each bin)
- Two free parameters in MCMC
  H_0 and Ω_{m0} for the ΛCDM model
  H_0 and w_a for the $w_0 w_a$CDM model
- $M = -19.35$ such that locally (in the first bin)
  $$H_0 = 70.0\ km\ s^{-1}\ Mpc^{-1}$$
  (conventional value of the PS release)
- New probes included, BAOs

- Gaussian priors:
  $\mu(H_0) = 70.393\ km\ s^{-1}\ Mpc^{-1}$
  $\sigma(H_0) = 2 * 1.079\ km\ s^{-1}\ Mpc^{-1}$
  $\mu(\Omega_{m0}) = 0.298 \qquad \sigma(\Omega_{m0}) = 2 * 0.022$
  [measurements from arXiv:1710.00845 in 2 σ]
  $\mu(w_a) = -0.129$
  $\sigma(w_a)$: $20\ \%$ deviation from its central value
- Fixed $w_0 = -0.905$ from arXiv:1710.00845

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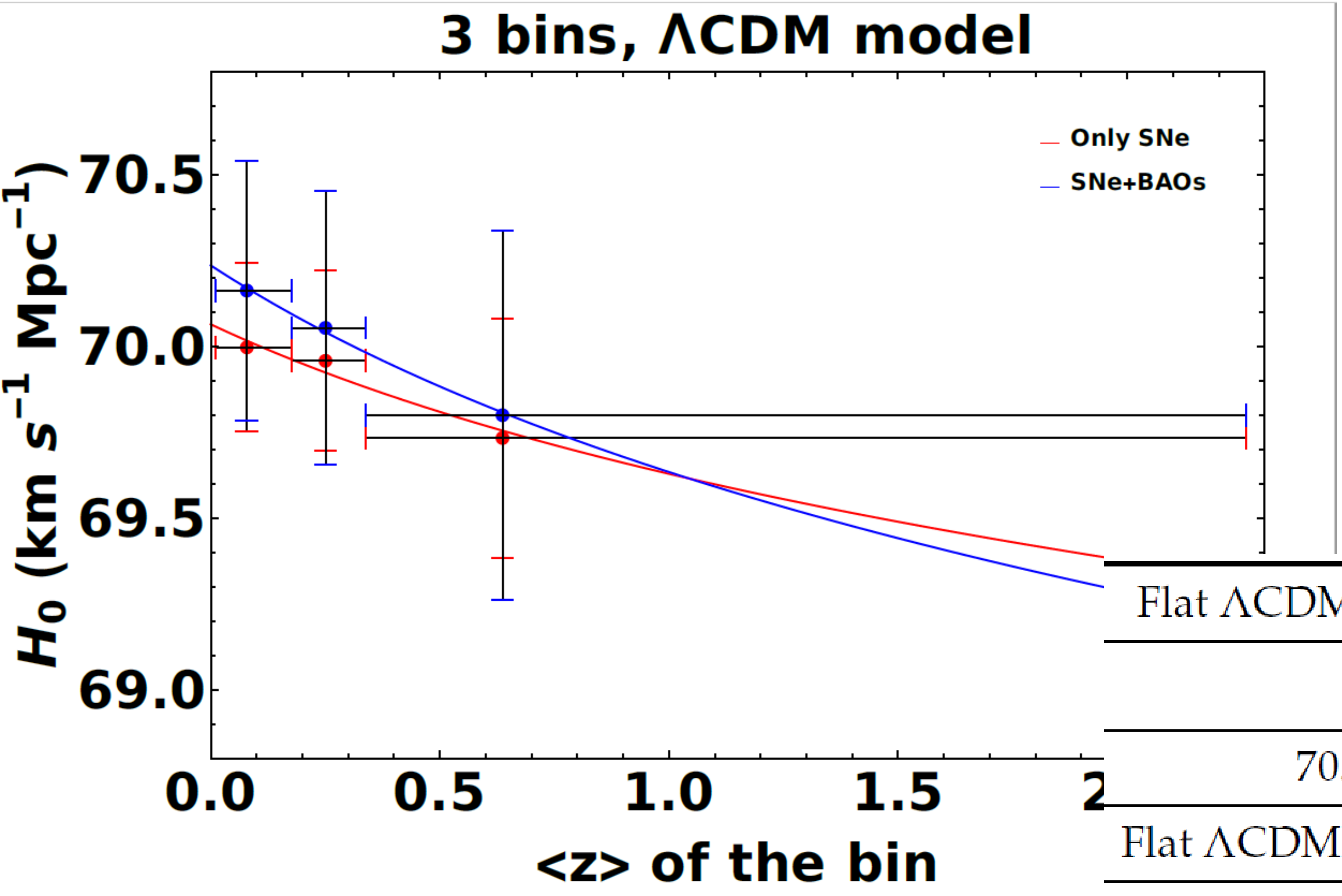
SNe + BAOs, BINNED ANALYSIS



$$H_0(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

$\alpha = 0 \rightarrow$ no evolution

Error bars, 1 σ

Flat ΛCDM model, without BAOs, varying H_0 and Ω_{0m}		
$\widetilde{H}_0$	α	α/σ_α
70.093 ± 0.102	0.009 ± 0.004	2.0
Flat ΛCDM model, including BAOs, varying H_0 and Ω_{0m}		
$\widetilde{H}_0$	α	α/σ_α
70.084 ± 0.148	0.008 ± 0.006	1.2

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GENERAL RELATIVITY

$$G_{\mu\nu} = \chi\, T_{\mu\nu}$$

Dark energy → modified source

$f(R)$ MODIFIED GRAVITY

$$G_{\mu\nu} = \chi\, T_{\mu\nu}$$

Geometrical modification of gravity theory

$\mathcal{L}_{EH} = R$ Einstein-Hilbert	Gravitational Lagrangian density	$\mathcal{L}_g = f(R)$ Extra degree of freedom
$S_{EH} = \frac{1}{2\chi}\int_\Omega d^4x\,\sqrt{-g}\,R$	Gravitational action	$S_g = \frac{1}{2\chi}\int_\Omega d^4x\,\sqrt{-g}\,f(R)$
$G_{\mu\nu} = \chi\, T_{\mu\nu}$	Gravitational field equations	$f'(R)\,R_{\mu\nu} - \frac{1}{2}\,f(R)\,g_{\mu\nu} + g_{\mu\nu}\,g^{\rho\sigma}\,\nabla_\rho\nabla_\sigma f'(R) - \nabla_\mu\nabla_\nu f'(R) = \chi\, T_{\mu\nu}$

NOJIRI & ODINTSOV (2006), eConf C0602061, 06
SOTIRIOU & FARAONI (2010), Rev. Mod. Phys. 82, 451

R: Ricci scalar

$$f'(R) \equiv \frac{df}{dR}$$

∇_μ: covariant derivative

$f(R)$ MODIFIED THEORIES OF GRAVITY

Geometrical modification of gravity theory

$$\mathcal{L}_g = f(R) = \mathrm{R} + \mathrm{F}(R)$$

Deviation from
Einstein-Hilbert theory

Modified gravitational field equations

$$f'(R)\, R_{\mu\nu} - \frac{1}{2}\, f(R)\, g_{\mu\nu} + g_{\mu\nu}\, g^{\rho\sigma}\, \nabla_\rho \nabla_\sigma f'(R) - \nabla_\mu \nabla_\nu f'(R) = \chi\, T_{\mu\nu}$$

$$G_{\mu\nu} = \chi\, \left(T_{\mu\nu} + T_{\mu\nu}^{[F]}\right)$$

Explicit modification of
Einstein-Hilbert equations

No-Einsteinian geometrical contribution can be recast as an effective source

$$T_{\mu\nu}^{[F]} = -\frac{1}{\chi}\left[F'(R)\, R_{\mu\nu} - \frac{1}{2}\, F(R)\, g_{\mu\nu} + g_{\mu\nu}\, g^{\rho\sigma}\, \nabla_\rho \nabla_\sigma F'(R) - \nabla_\mu \nabla_\nu F'(R)\right]$$

$f(R)$ HU-SAWICKI MODEL

Metric $f(R)$ formalism

$$n = 1 \qquad f(R) = R - m^2 \frac{c_1 \frac{R}{m^2}}{c_2 \frac{R}{m^2} + 1}$$

Jordan frame (equivalent scalar-tensor formalism)

$$V(\phi) = \frac{m^2}{c_2}\left[c_1 + 1 - \phi - 2\sqrt{c_1\,(1-\phi)}\right]$$

c_1, c_2 parameters $\qquad m^2 \equiv \frac{\chi\,\rho_{m0}}{3} = H_0^2\,\Omega_{m0}$

- Cosmological constant for $R \gg m^2$

  $f(R) \approx R - 2\Lambda_{eff}$ with $\Lambda_{eff} = \frac{c_1}{c_2} m^2$

- Constraining parameters, considering ΛCDM as a limiting case with $f(R) = R + F(R)$

  $\frac{c_1}{c_2} \approx 6\,\frac{\Omega_{0\Lambda}}{\Omega_{0m}}$ and $F_R(z=0) = \left(\frac{dF}{dR}\right)_{z=0} = -\frac{c_1}{c_2^2}\left[3\left(1 + 4\,\frac{\Omega_{0\Lambda}}{\Omega_{0m}}\right)\right]^{-2}$ with $|F_R(z=0)| \lesssim 10^{-7}$

Liu, T., Zhang, X., & Zhao, W., Phys. Lett. B, 777, 286 (2018)

LUMINOSITY DISTANCE IN $f(R)$ GRAVITY

$$d_L(z) = \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_{m0}\left[(1+z')^3 + y_H(z')\right]}}$$

(arXiv: 0705.1158)

Dimensionless variables

$$y_H = \frac{H^2}{m^2} - (1+z)^3$$

$$y_R = \frac{R}{m^2} - 3\,(1+z)^3$$

$y_H(z)$ encodes information of a specific $f(R)$ model.

Modified field eqs. can be solved numerically

in terms of y_H, y_R and their derivatives

Initial conditions: z_i

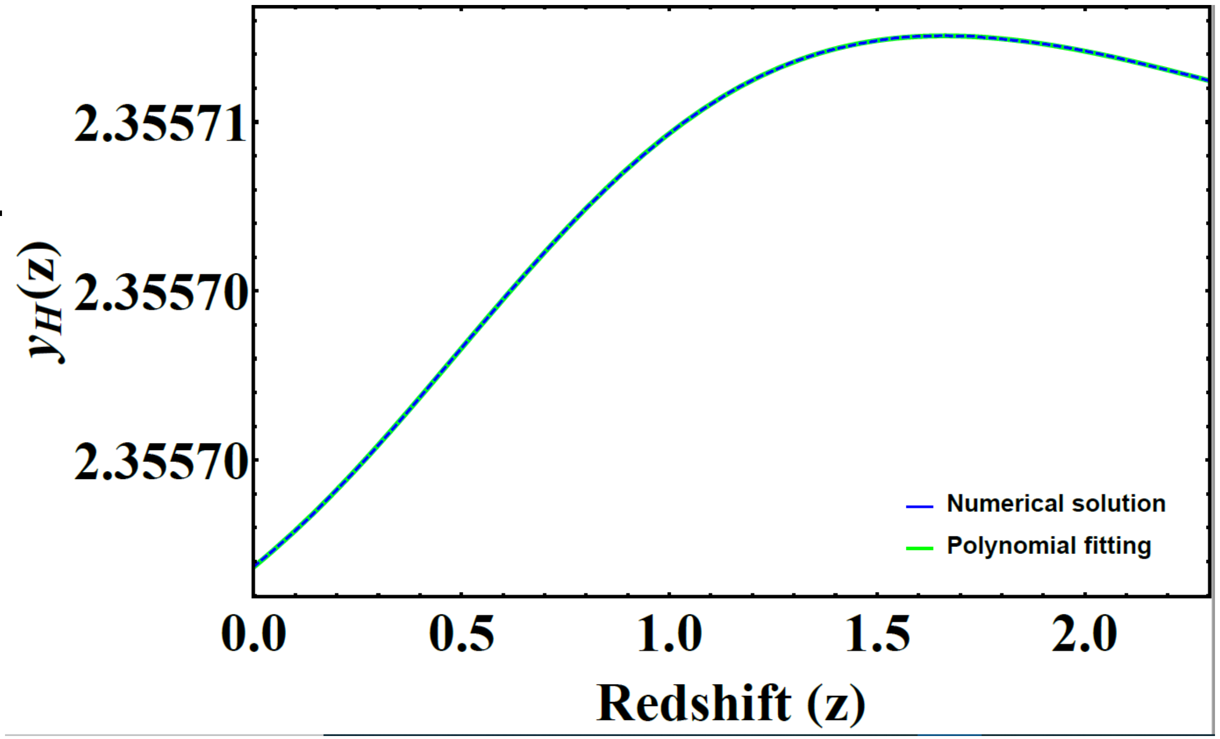
$$y_H(z_i) = \frac{\Omega_{\Lambda 0}}{\Omega_{m0}}$$

$$y_R(z_i) = 12\,\frac{\Omega_{\Lambda 0}}{\Omega_{m0}}$$

BINNING APPROACH WITH f(R) HU-SAWICKI MODEL

$$f(R) \equiv R + F(R) = R - m^2 \frac{c_1 \frac{R}{m^2}}{c_2 \frac{R}{m^2} + 1}$$

$$y_H = \frac{H^2}{m^2} - (1+z)^3$$



$y_H(z)$ encodes information of a specific $f(R)$ model.

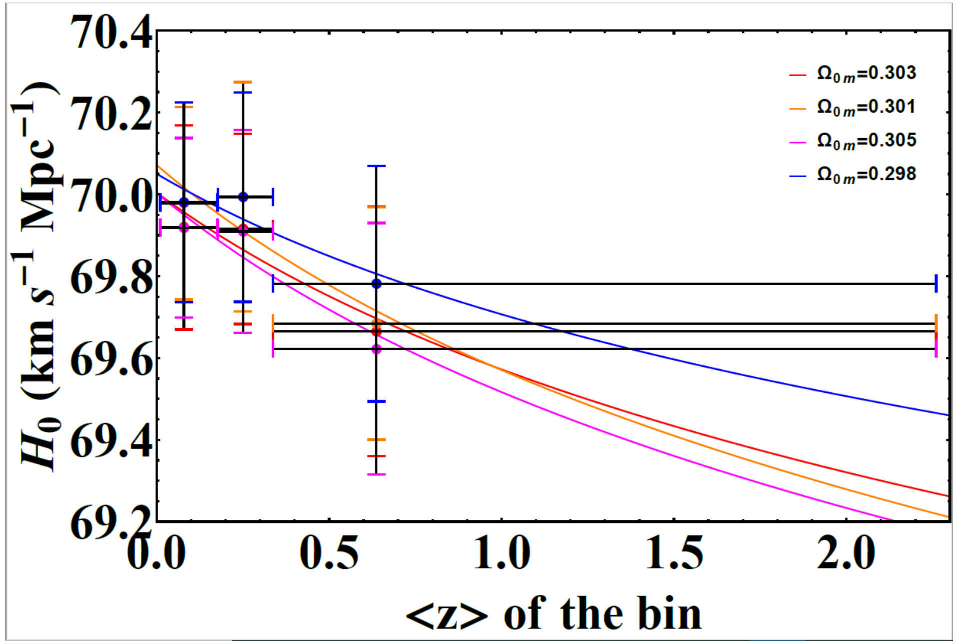
Modified field eqs. can be solved numerically

in terms of y_H, y_R and their derivatives

$$d_L(z) = \frac{(1+z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_{m0}\left[(1+z')^3 + y_H(z')\right]}}$$

$$\frac{c_1}{c_2} \approx 6\,\frac{\Omega_{\Lambda 0}}{\Omega_{m0}} \quad \text{and} \quad F_R(z=0) = \left(\frac{dF}{dR}\right)_{z=0} = -\frac{c_1}{c_2^2}\left[3\left(1 + 4\frac{\Omega_{\Lambda 0}}{\Omega_{m0}}\right)\right]^{-2} \quad \text{with} \quad |F_R(z=0)| \lesssim 10^{-7}$$

BINNING APPROACH WITH f(R) HU-SAWICKI MODEL



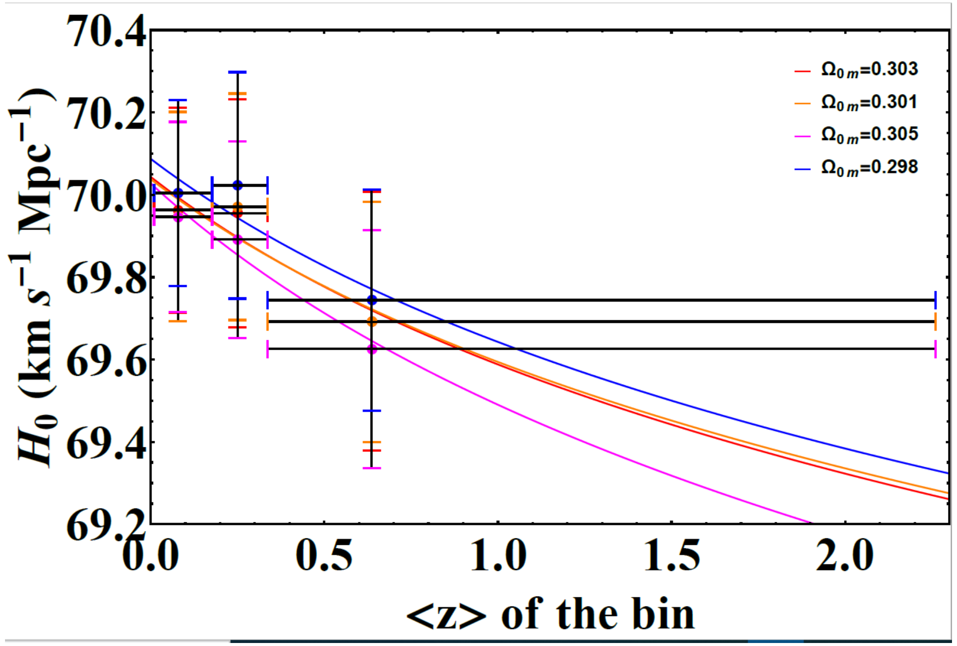


Figure 6. The Hubble constant versus redshift plots for the three bins of SNe Ia only, considering the Hu–Sawicki model. **Upper left panel.** The condition of $F_{R0} = -10^{-7}$ is applied to the case of SNe only, with the different values of $\Omega_{0m} = 0.301, 0.303, 0.305$. **Upper right panel.** The same of the upper left, but with the contribution of BAOs. **Lower left panel.** The SNe only case with the $F_{R0} = -10^{-4}$ condition, considering the different values of $\Omega_{0m} = 0.301, 0.303, 0.305$. **Lower right panel.** The same as the lower left, but with the contribution of BAOs. The orange color refers to $\Omega_{0m} = 0.301$, the red to $\Omega_{0m} = 0.303$, the magenta to $\Omega_{0m} = 0.305$, and the blue to $\Omega_{0m} = 0.298$.

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AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

Approximated analytical solution:

$$\phi(z) = K(1+z)^{2\alpha}$$

$$H_0^{\text{eff}}(z) = \frac{H_0}{\sqrt{K\,(1-2\,\alpha)\,(1+z)^{\alpha}}}$$

$\phi(0) = K = 1 - 10^{-7}$

$\alpha = 1.1 \times 10^{-2}$

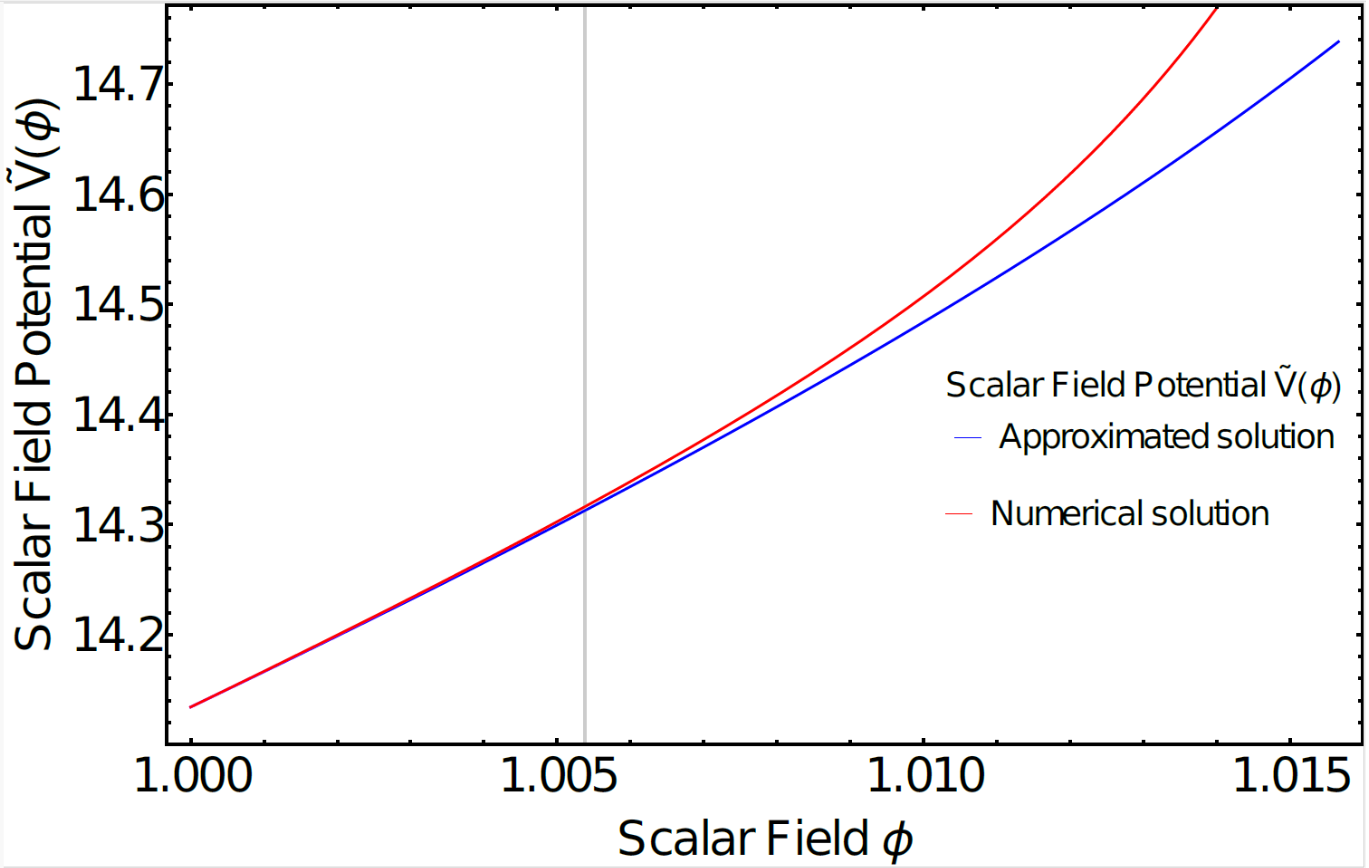
$H_0 = 72.2\ \text{km s}^{-1}\ \text{Mpc}^{-1}$

to match $H_0^{[CMB]}$ and $H_0^{[loc]}$

$\Omega_{m0} = 0.298$

$$\tilde{V}(\phi = K) = \tilde{V}(z=0) = 6\,\frac{1-\Omega_{m0}}{\Omega_{m0}}$$

ΛCDM toda



$$\tilde{V}(\phi) = \tilde{V}(\phi = K) + \frac{6\alpha}{1-\alpha}\left\{\frac{2+\alpha}{\alpha}\,\frac{1-\Omega_{m0}}{\Omega_{m0}}\ln\left(\frac{\phi}{K}\right) + \frac{1+2\alpha}{3}\left[\left(\frac{\phi}{K}\right)^{\frac{3}{2\alpha}} - 1\right]\right\}$$

SCHIAVONE, MONTANI, & BOMBACIGNO (2023), MNRAS Letters, 522, L72-L77

AN EFFECTIVE HUBBLE CONSTANT IN $f(R)$ GRAVITY

The low-redshift $f(R)$ profile

For $z \ll 1$:

$$\phi(z) \approx K(1 + 2\,\alpha\, z) + O(z^3)$$

$$\tilde{V}(\phi) \approx \tilde{V}(K) + A_1\,(\phi - K) + A_2(\phi - K)^2 + O[(\phi - K)^3]$$

where the dimensionless constant A_i and B_i are algebraically related to the values of α, Ω_{m0} and K

Relations in the Jordan frame

$$R = \frac{dV}{d\phi} \qquad V(\phi) = R(\phi)\phi - f(R(\phi))$$

$$f(R) \approx m^2\, B_0 + B_1\, R + B_2\, \frac{R^2}{m^2}$$

Approximated solution for $z \ll 1$: $f(R)$ – quadratic gravity

It can be shown that ΛCDM is recovered for $\alpha \to 0$ and $K \to 1$

SCHIAVONE, MONTANI, & BOMBACIGNO (2023), MNRAS Letters, 522, L72-L77