SIGRAV lecture 4: Test of the Equivalence principle

2/19/25

Vietri, Italy, 2/16/2024

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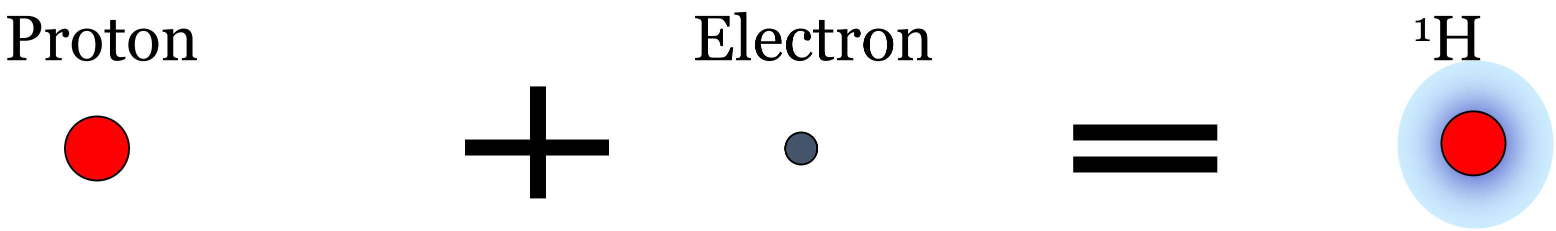
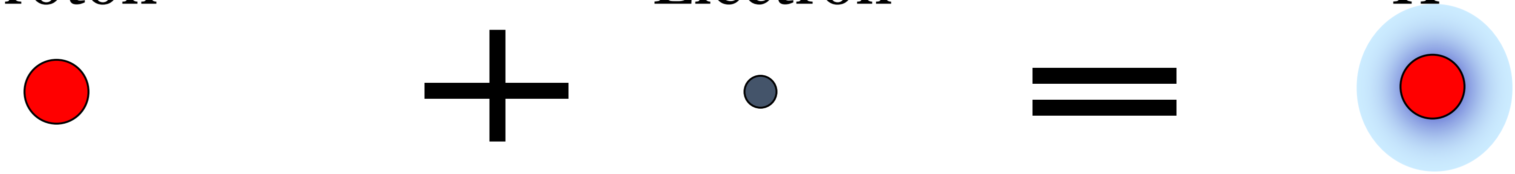
Mass does not add up



Mass does not add up

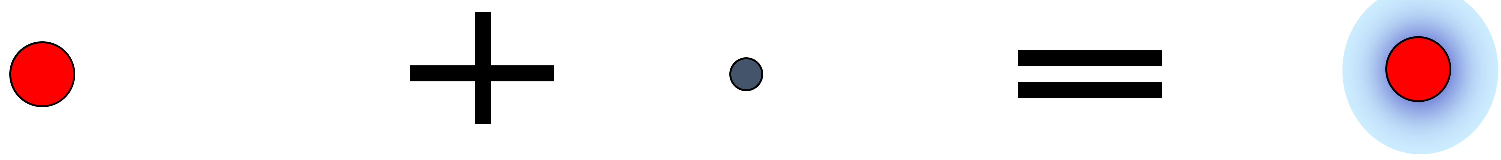
Proton + Electron = ^{1}H



938 272 013 eV/c² + 510 999 eV/c² ≠ 938 782 999 eV/c²

Mass does not add up

Proton + Electron = ^{1}H



938 272 013 eV/c² + 510 999 eV/c² ≠ 938 782 999 eV/c²

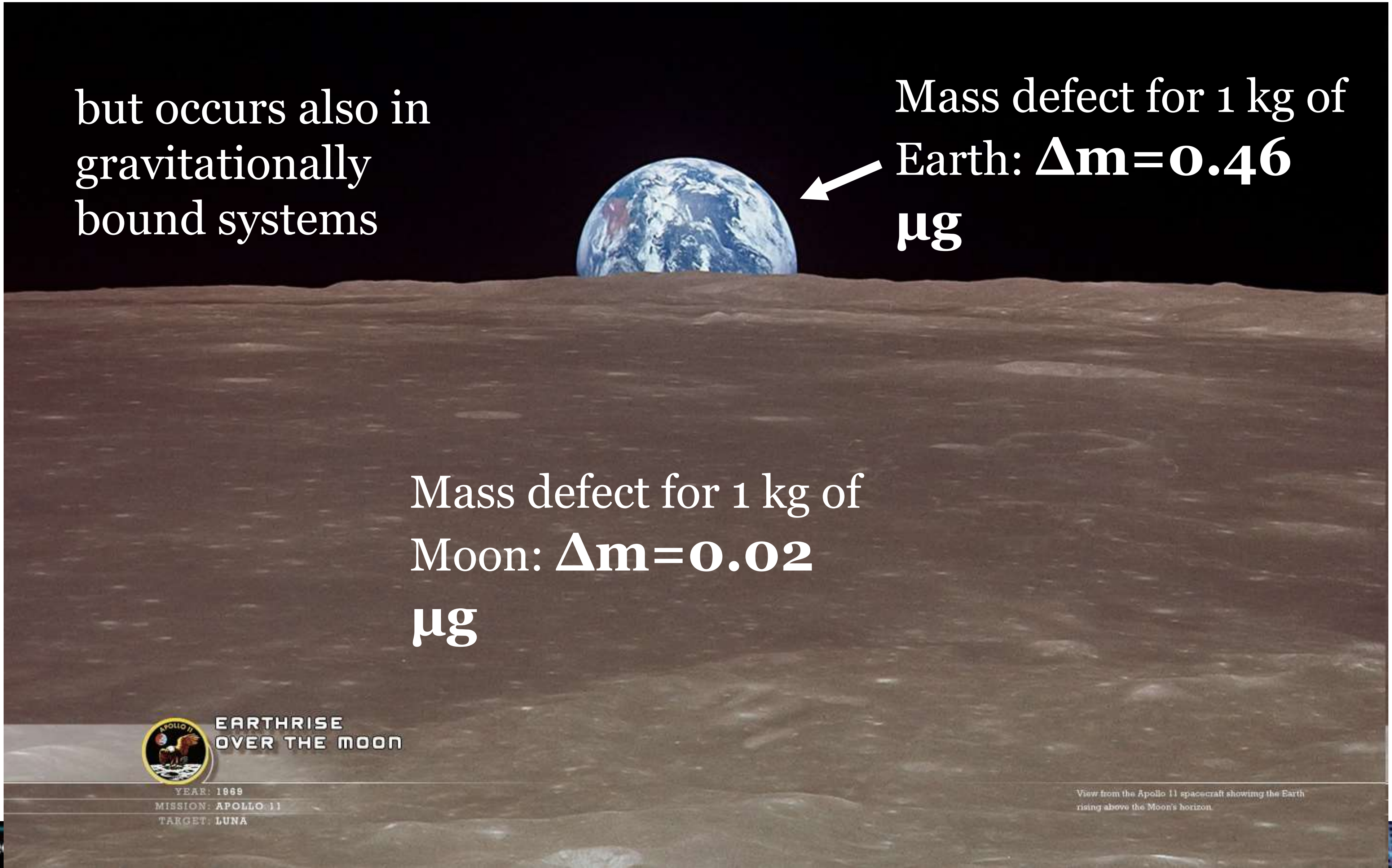
938 272 013 eV /c² + 510 999 eV/c² = 938 783 012 eV/c²

13.6 eV/c²

Mass defect = binding Energy

Binding Energy = $\Delta E_{pot} - \Delta E_{kin}$

The non-linearity is rather small



but occurs also in gravitationally bound systems

Mass defect for 1 kg of Earth: **Δm=0.46 μg**

Mass defect for 1 kg of Moon: **Δm=0.02 μg**

The mass of an object

$$m = \sum m_c + \sum Ekin/c^2 - \sum Epot/c^2$$

Newton's Principia (1689)

Newton's 2nd Law

$F_i = m_i a$

Gravitational Law

$F_G = G\, m_{g1}\, m_{g2} / r^2$

Equivalence Principle (EP): $m_i = m_g$

Weak Equivalence Principle:

Gravitational binding energy is excluded.

Strong Equivalence Principle:

Includes all 4 fundamental interactions.

Searching for New Interactions

Good model for a new interaction: $V(r) = k\, Q_1 Q_2 \frac{1}{r}\, e^{-r/\lambda}$

$$V_{grav}(r) = G\, m_1 m_2 \frac{1}{r}$$

$$V(r) = \alpha G \left(\frac{q_1}{\mu_1}\right)\left(\frac{q_2}{\mu_2}\right) \frac{m_1 m_2}{r}\, e^{-r/\lambda}$$

Strength relative to gravity

Source

Test mass

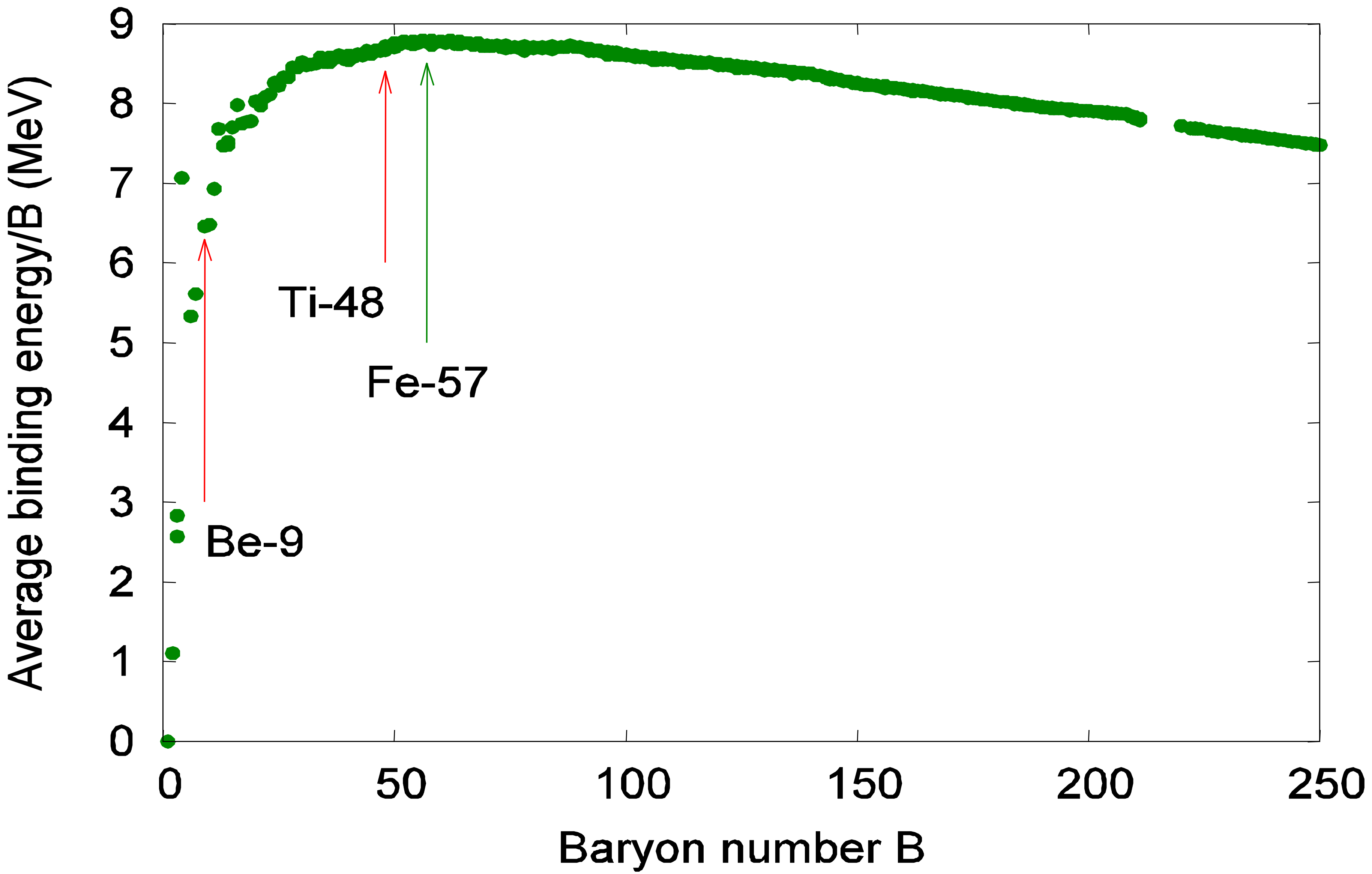
Interaction range

Assumed charge	Beryllium q/μ	Titanium q/μ	Difference
Baryon number **B**	0.99868	1.001077	-2.429×10^{-3}

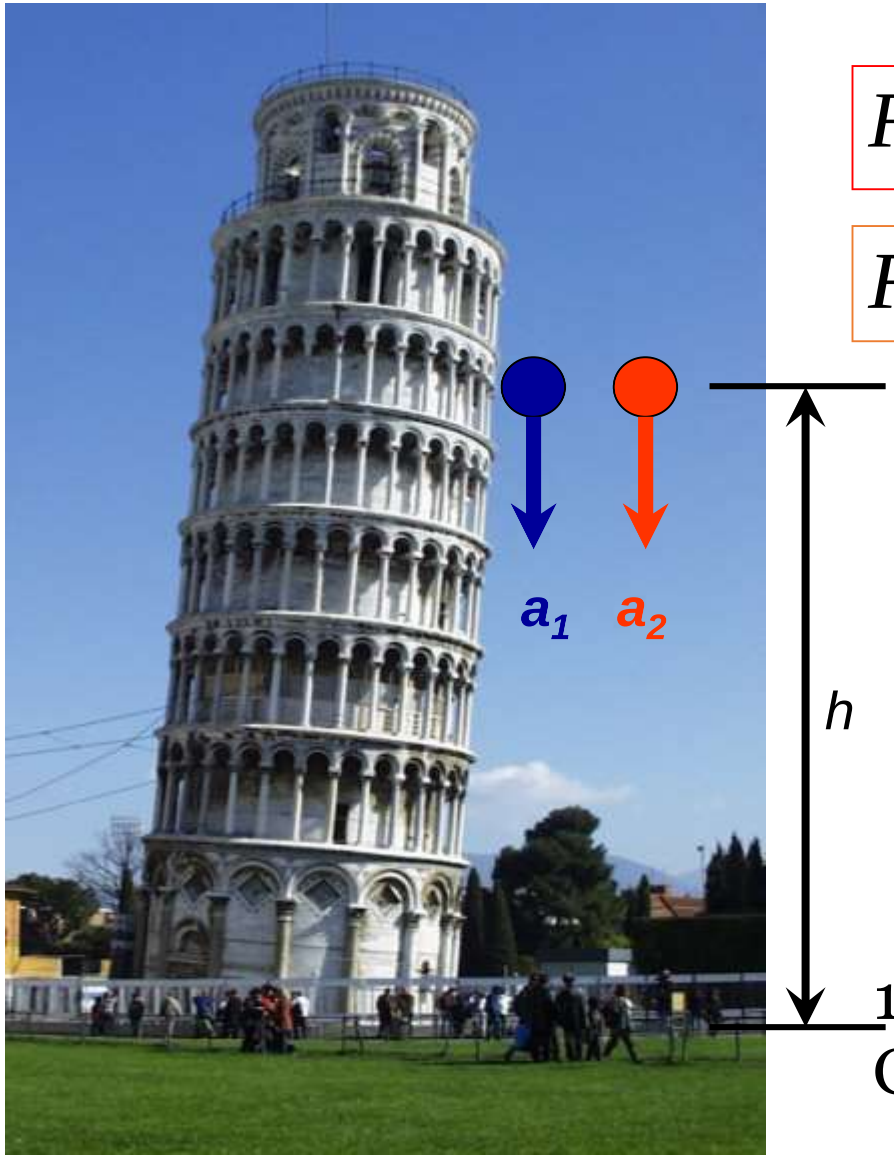
q/μ

	Mass (u)	q=B	q/μ
${}^{1}_{1}p$	1.007 3	1	0.992 8
${}^{1}_{0}n$	1.008 7	1	0.991 4
${}^{9}_{4}Be$	9.101 2	9	0.998 7
${}^{48}_{22}Ti$	47.947 9	48	1.001 1

B/μ varies, because



1st Tests of the Equivalence Principle



$$F = m_G g$$

$$F = m_I a$$

$$a = \frac{m_G}{m_I} g$$

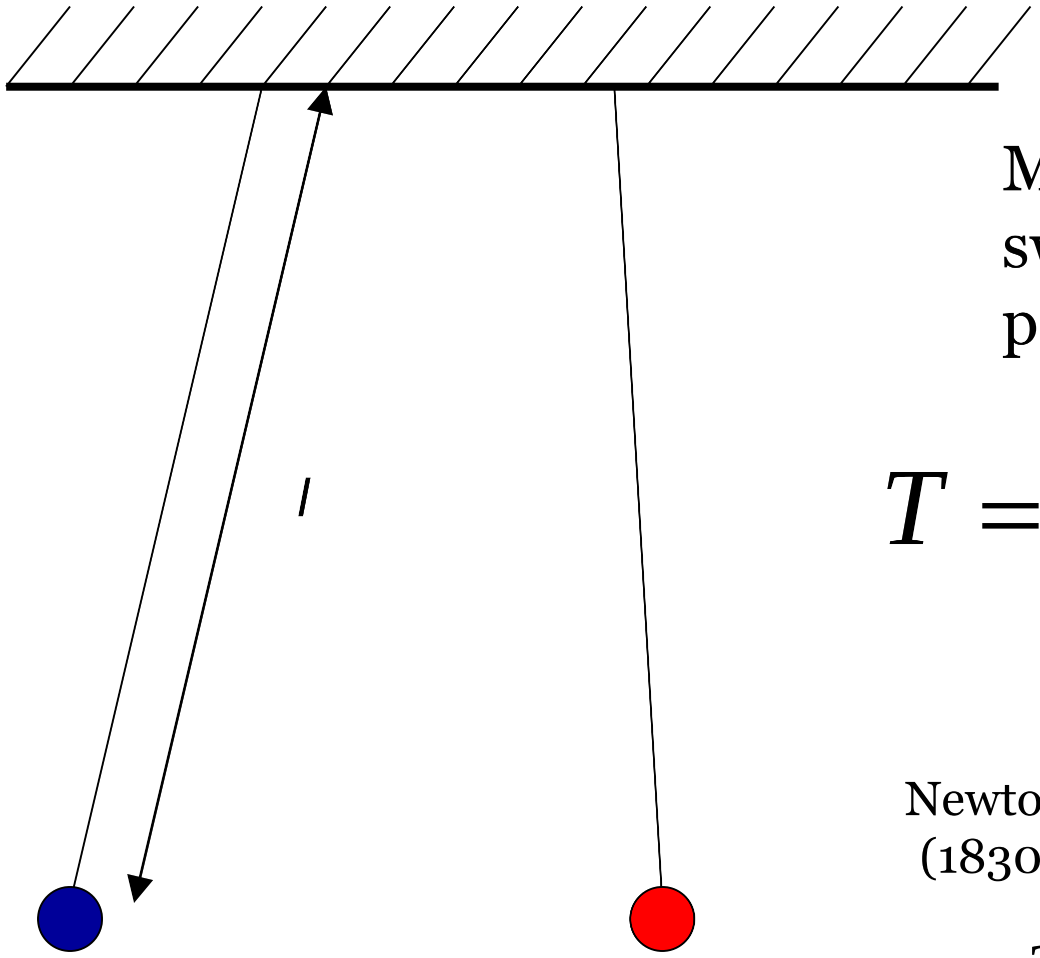
Time t to fall from h:

$$t = \sqrt{\frac{2h}{{}^{m_G}\!/_{m_I}\, g}}$$

1600 Galileo:

$$\eta = \frac{a_1 - a_2}{\tfrac{1}{2}(a_1 + a_2)} \approx 0.1$$

2nd Generation Tests



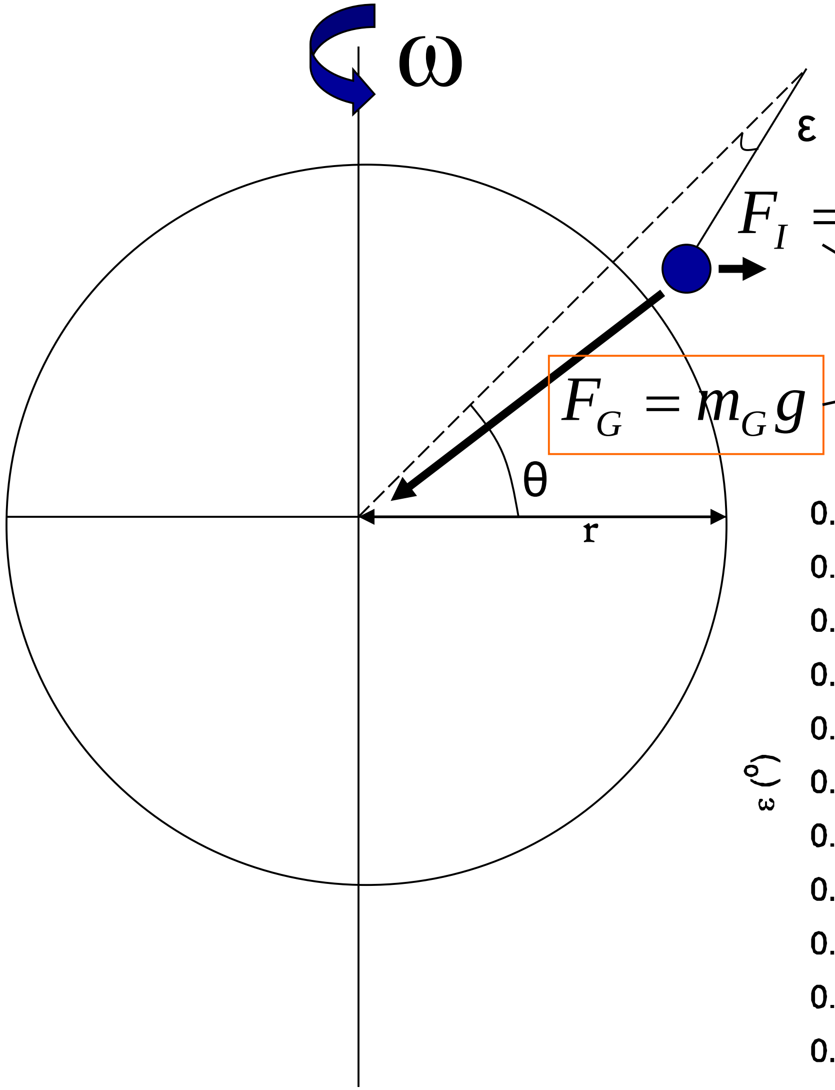
Measurement of the swing periods of pendula:

$$T = 2\pi \sqrt{\frac{L}{g}\frac{m_I}{m_G}}$$

Newton (1686), Bessel (1830), Potter (1923)

$$\eta \approx 2x10^{-5}$$

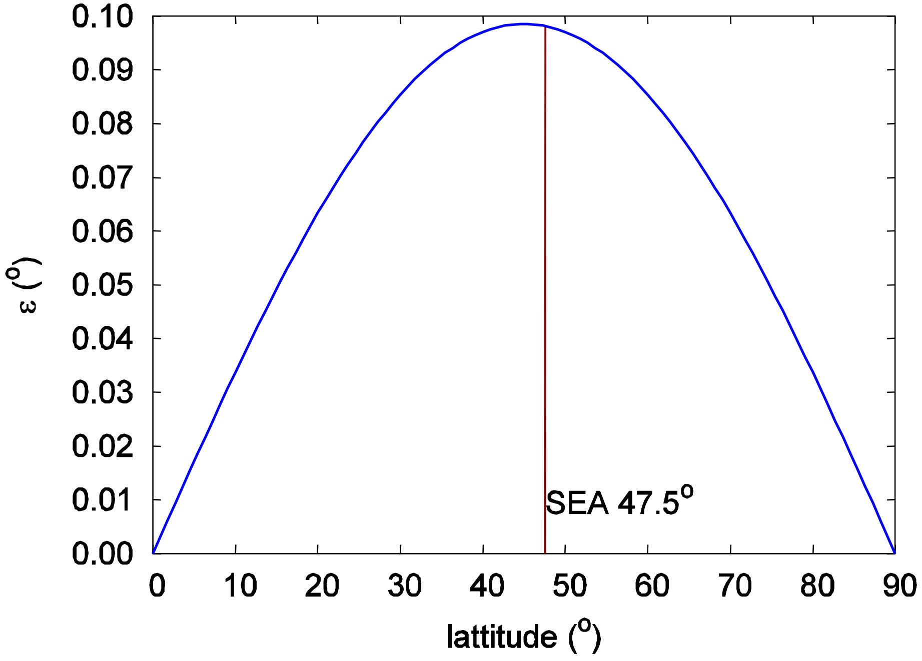
Eötvös Experiments



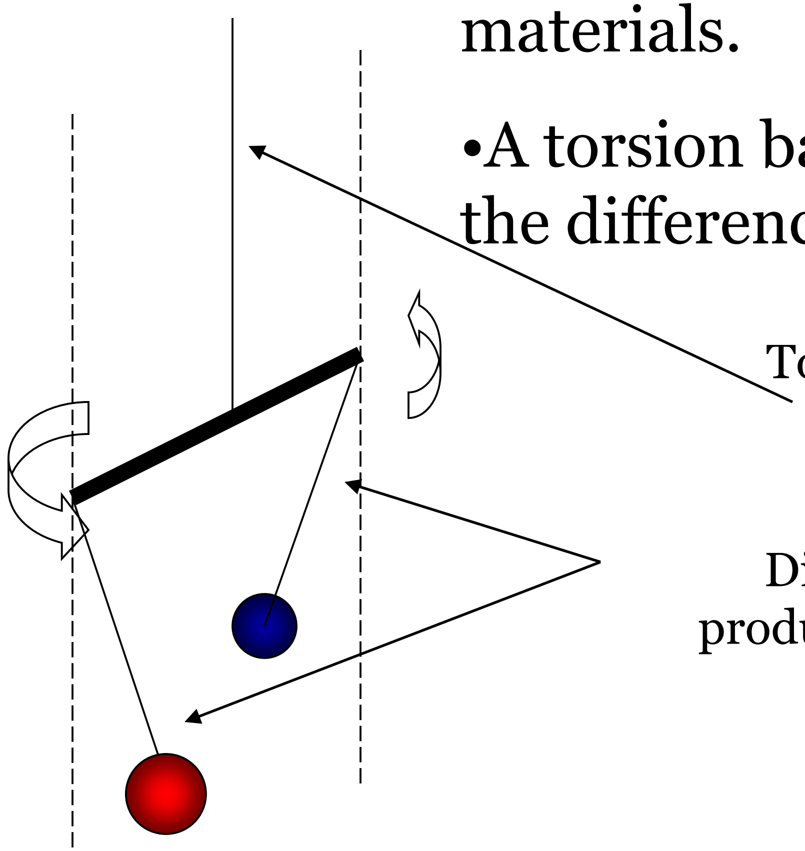
$$F_I = m_I \omega^2 r \cos\theta$$

$$F_G = m_G g$$

$$\varepsilon = \frac{m_I}{m_G} \frac{\omega^2 r}{2g} \sin(2\theta)$$



The torsion balance



- A violation of the EP would yield to different plumb-line for different materials.
- A torsion balance can be used to measure the difference in plumb-lines:

Torsion fiber hangs like the average plumb line.

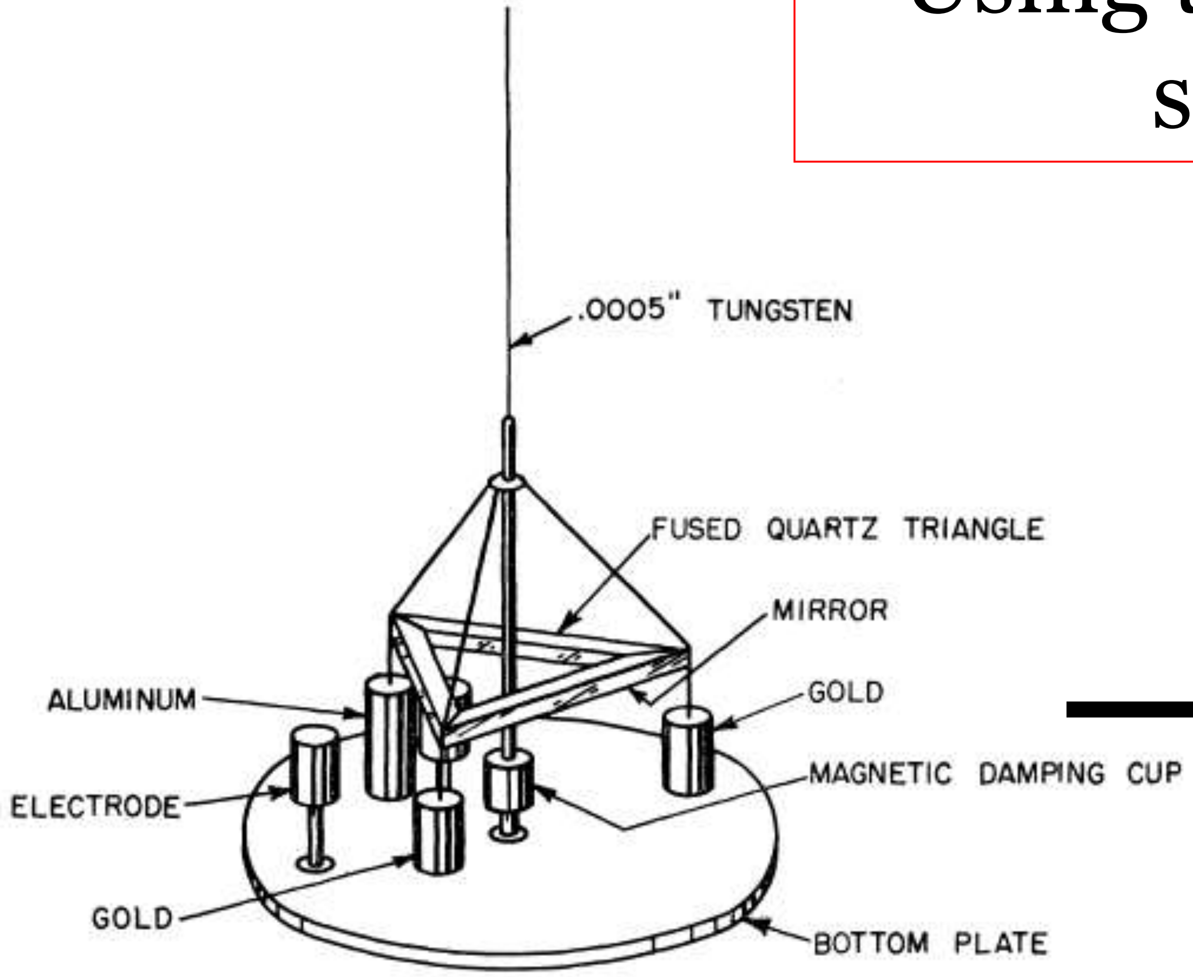
Difference in plumb lines produces a torque on the beam.

-> twist in the fiber

Eötvös (1922) $\eta \approx 5x10^{-9}$

Dicke's idea

Using the Sun as a source

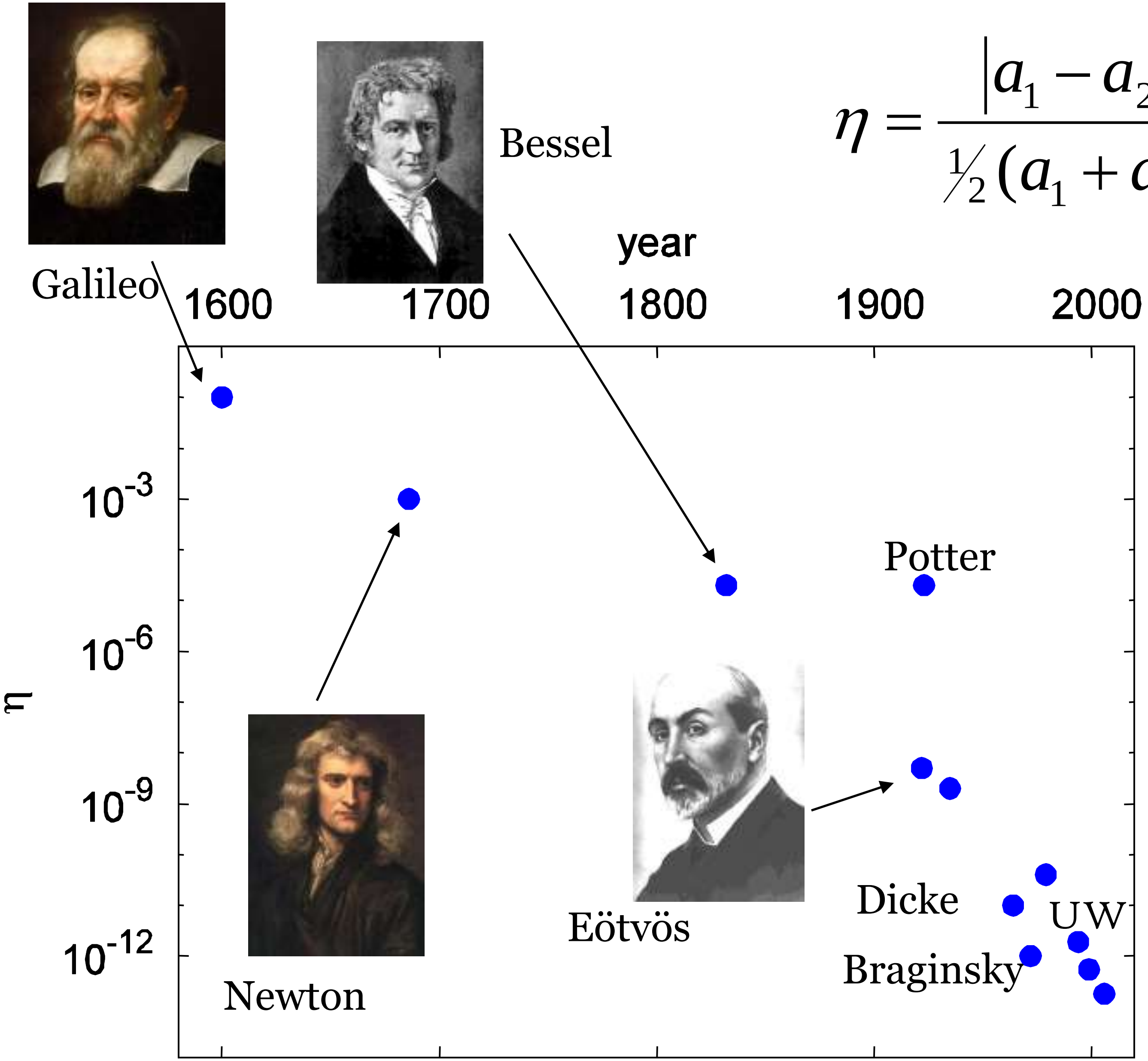


→ Sun

$$\eta \approx 1x10^{-11}$$

Historical overview

$$\eta = \frac{|a_1 - a_2|}{\frac{1}{2}(a_1 + a_2)}$$



Type of experiment

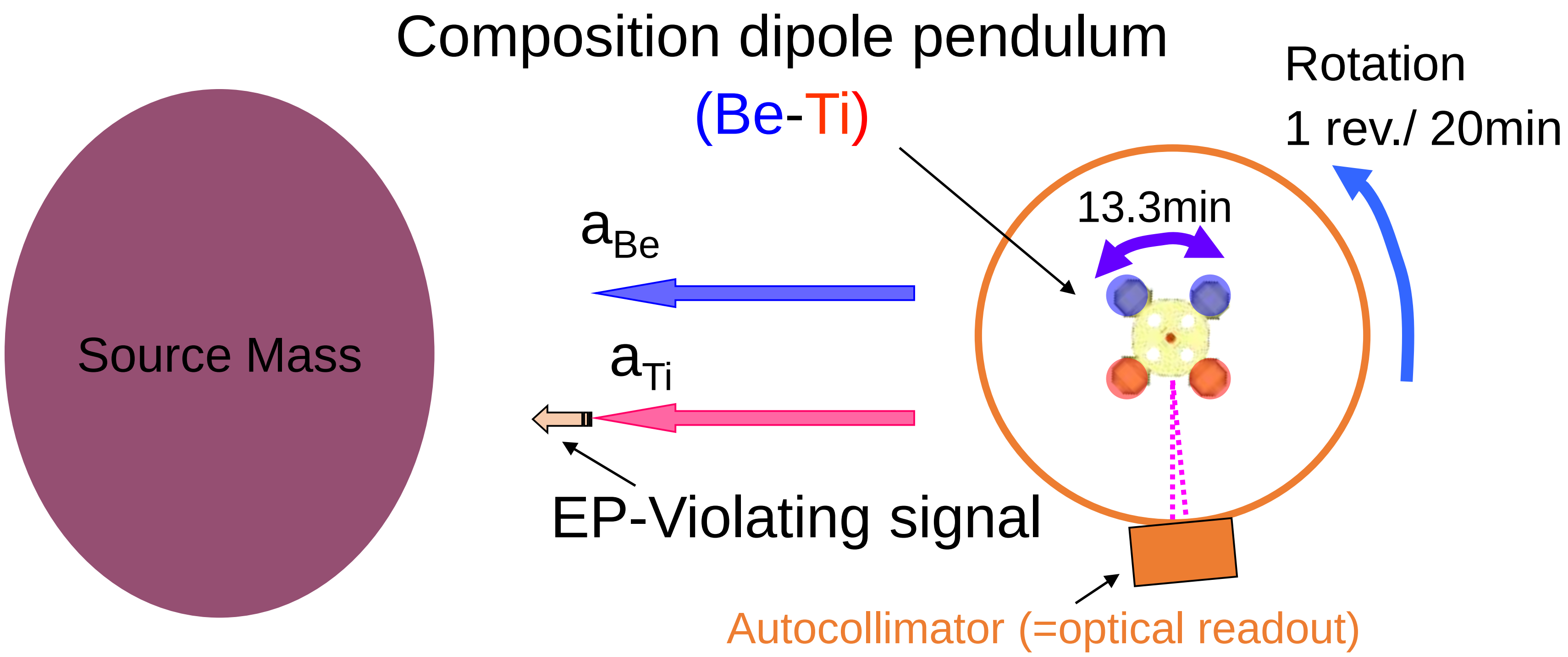
drop

pendula

torsion balance

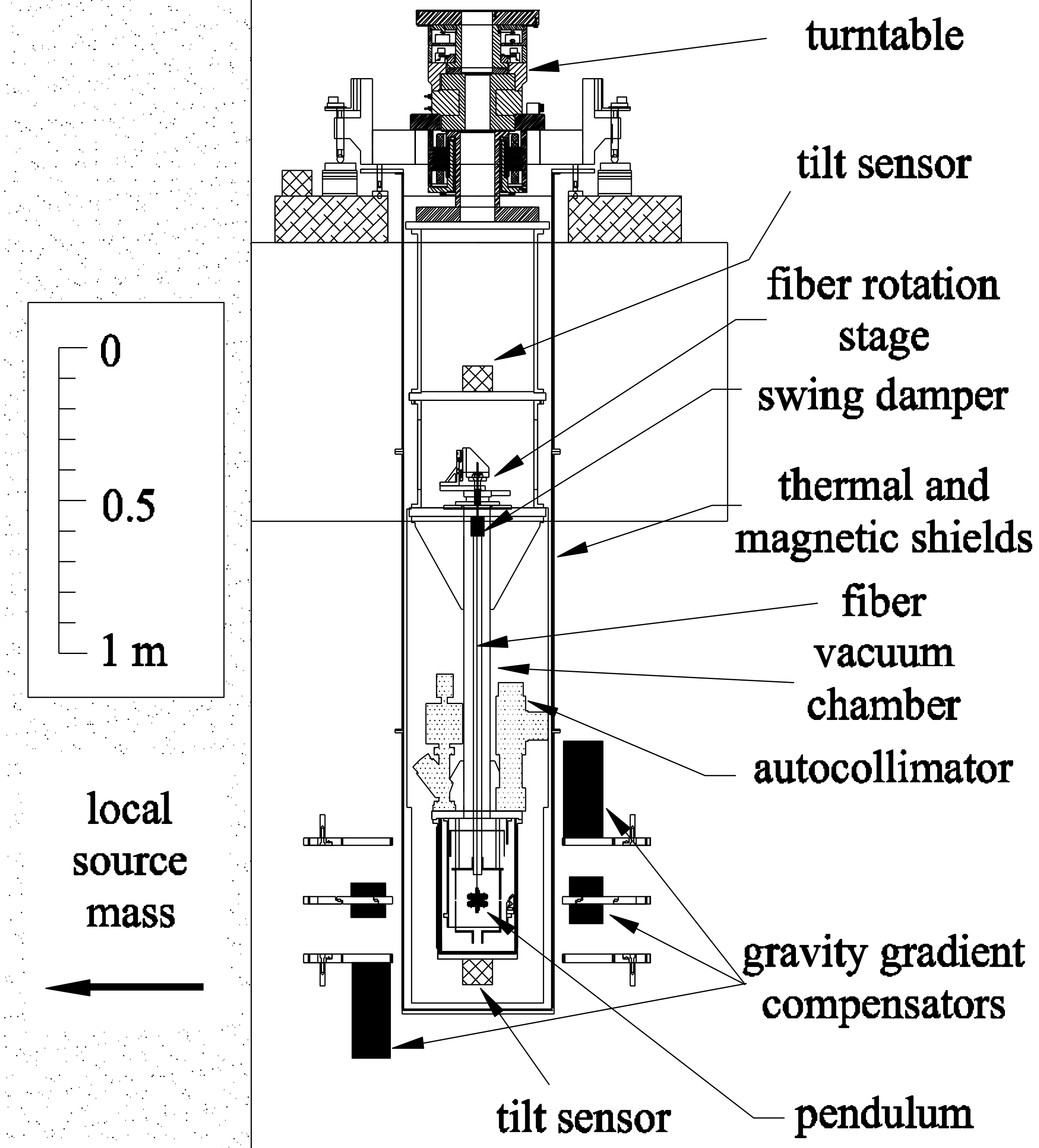
modulated torsion balance

Principle of our experiment

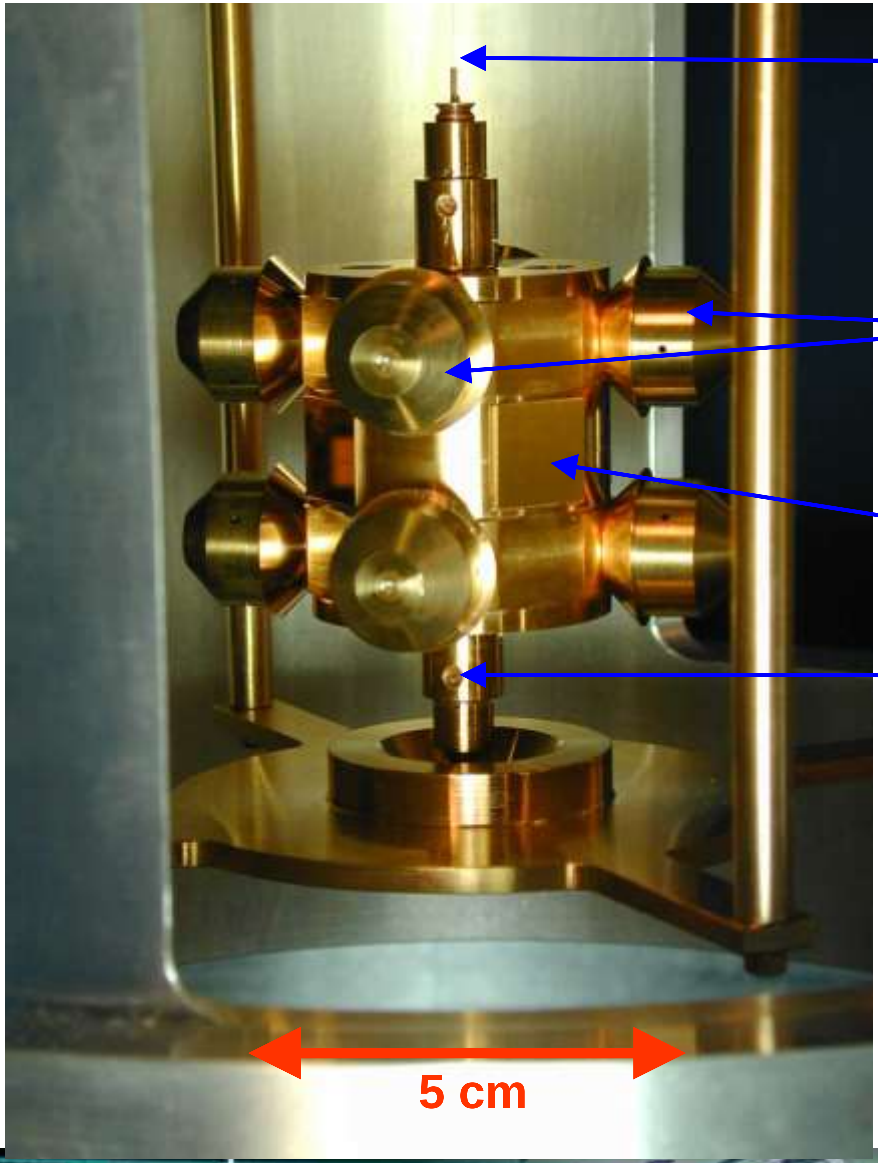


source mass	λ (m)
local masses (hill)	$1 - 10^4$
entire earth	$10^6 - 10^7$
Sun	$10^{11} - \infty$
Milky Way (incl. DM)	$10^{20} - \infty$

modulated (Sun, Milky Way)


turntable
tilt sensor
fiber rotation stage
swing damper
thermal and magnetic shields
fiber
vacuum chamber
autocollimator
gravity gradient compensators
pendulum
tilt sensor
0
0.5
1 m
local source mass

EP torsion pendulum



20 μm diameter tungsten fiber
(length: 108 cm)
K=2.36 nNm

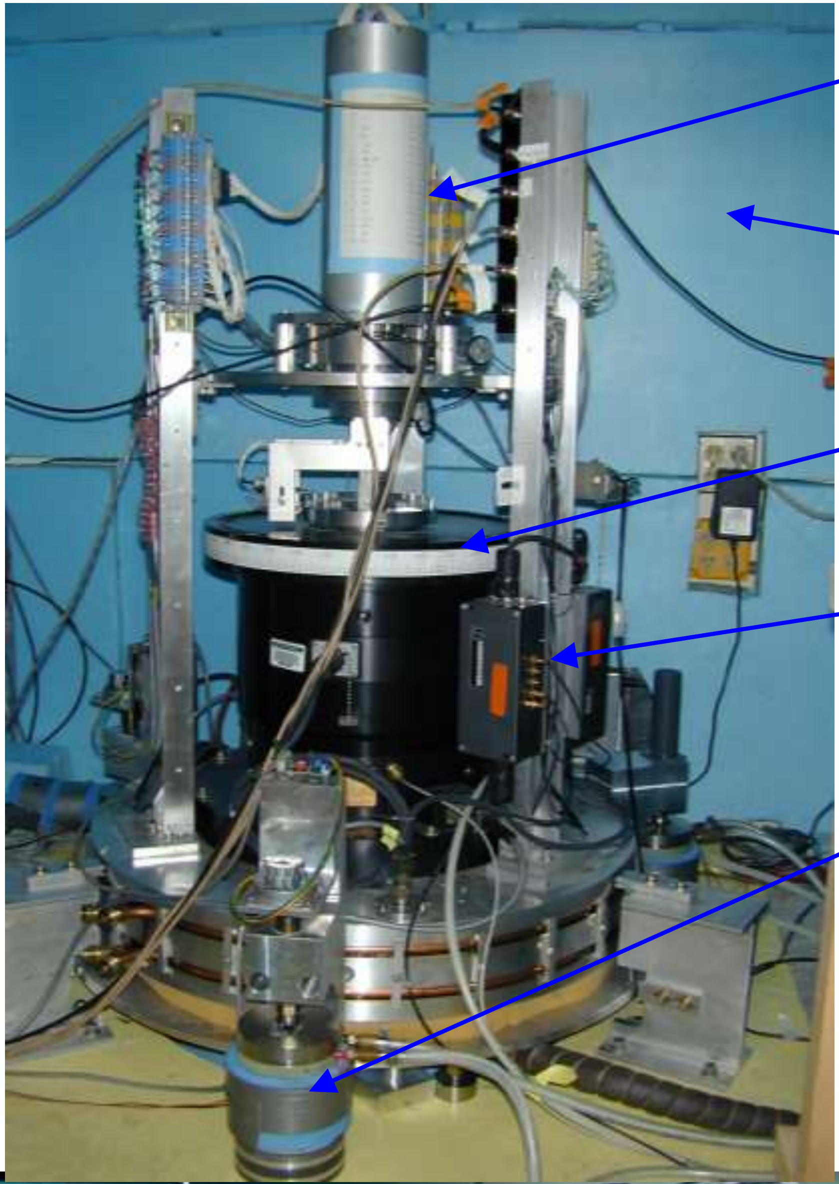
8 test masses (4 Be & 4 Ti)
4.84 g each (within 0.1 mg)
(can be removed)

4 mirrors

tuning screws for adjusting
tiny asymmetries

torsional frequency:	1.261 mHz
quality factor:	4000
decay time:	11d 6.5 hrs
machining tolerance:	5 μm
total mass :	70 g

The upper part of the apparatus



feedthrough for
electric signals

thermal insulation

air bearing turntable

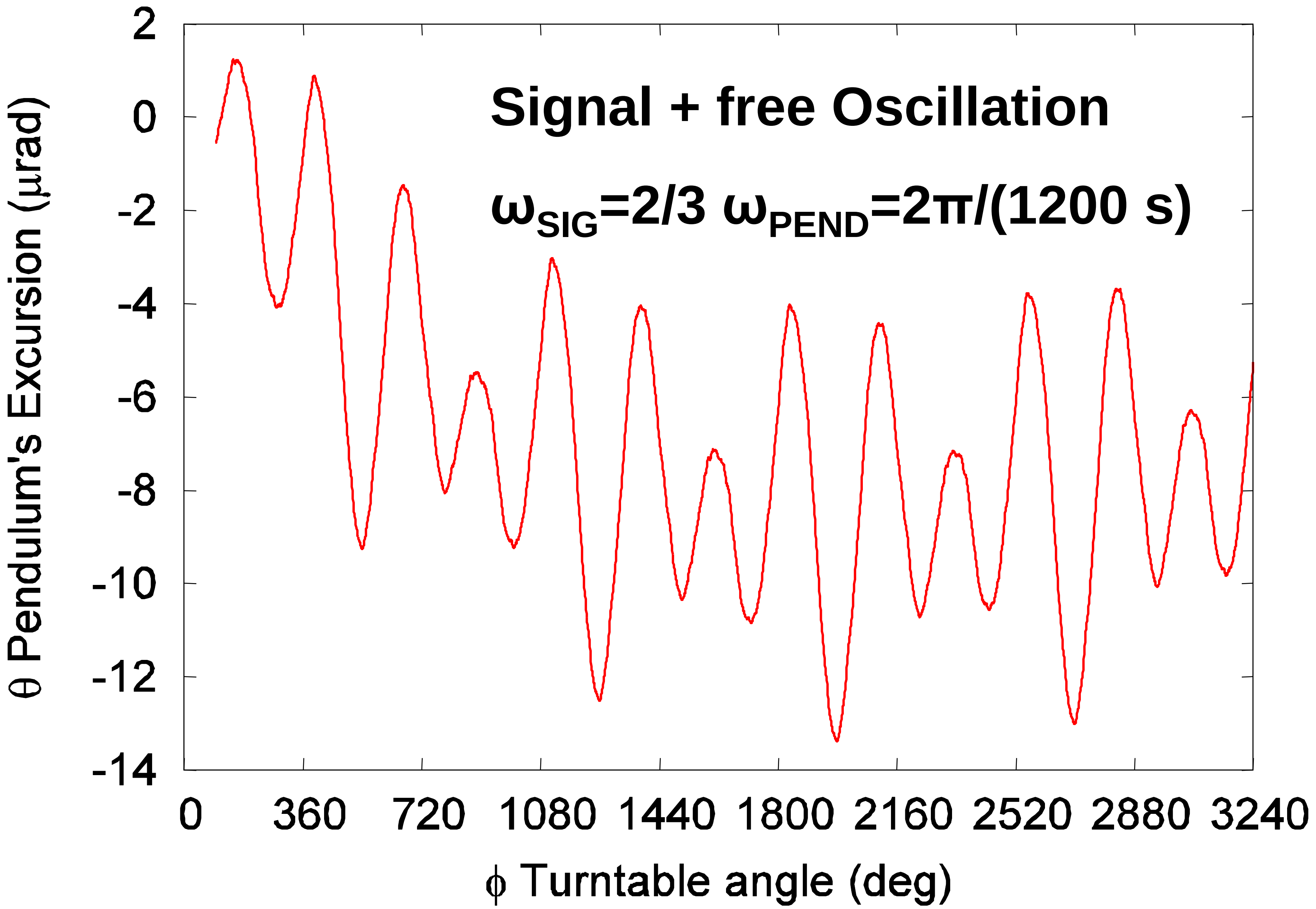
electronics of angle encoder

thermal expansion feet

to level turntable

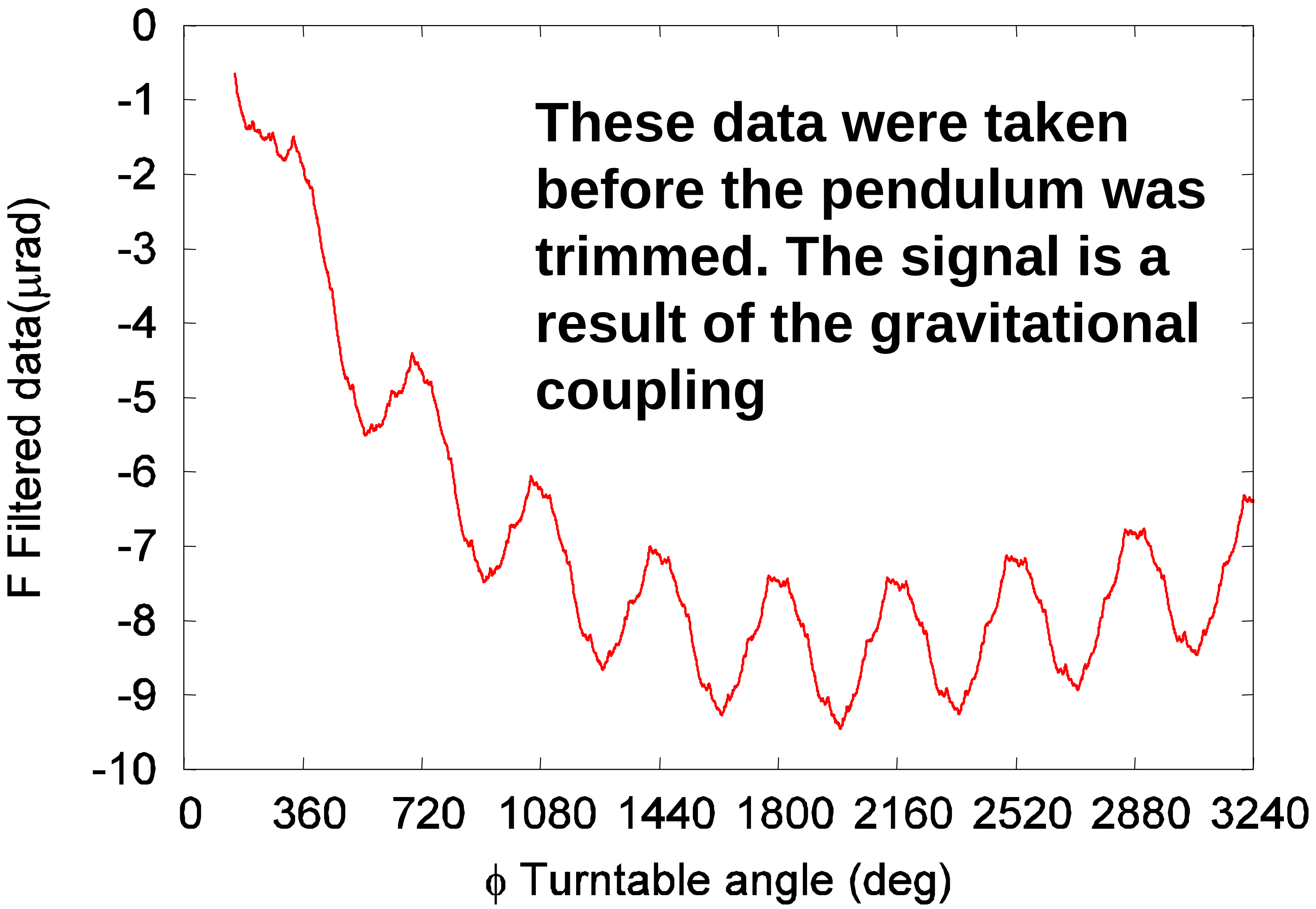
The lower part of the apparatus

Raw-data

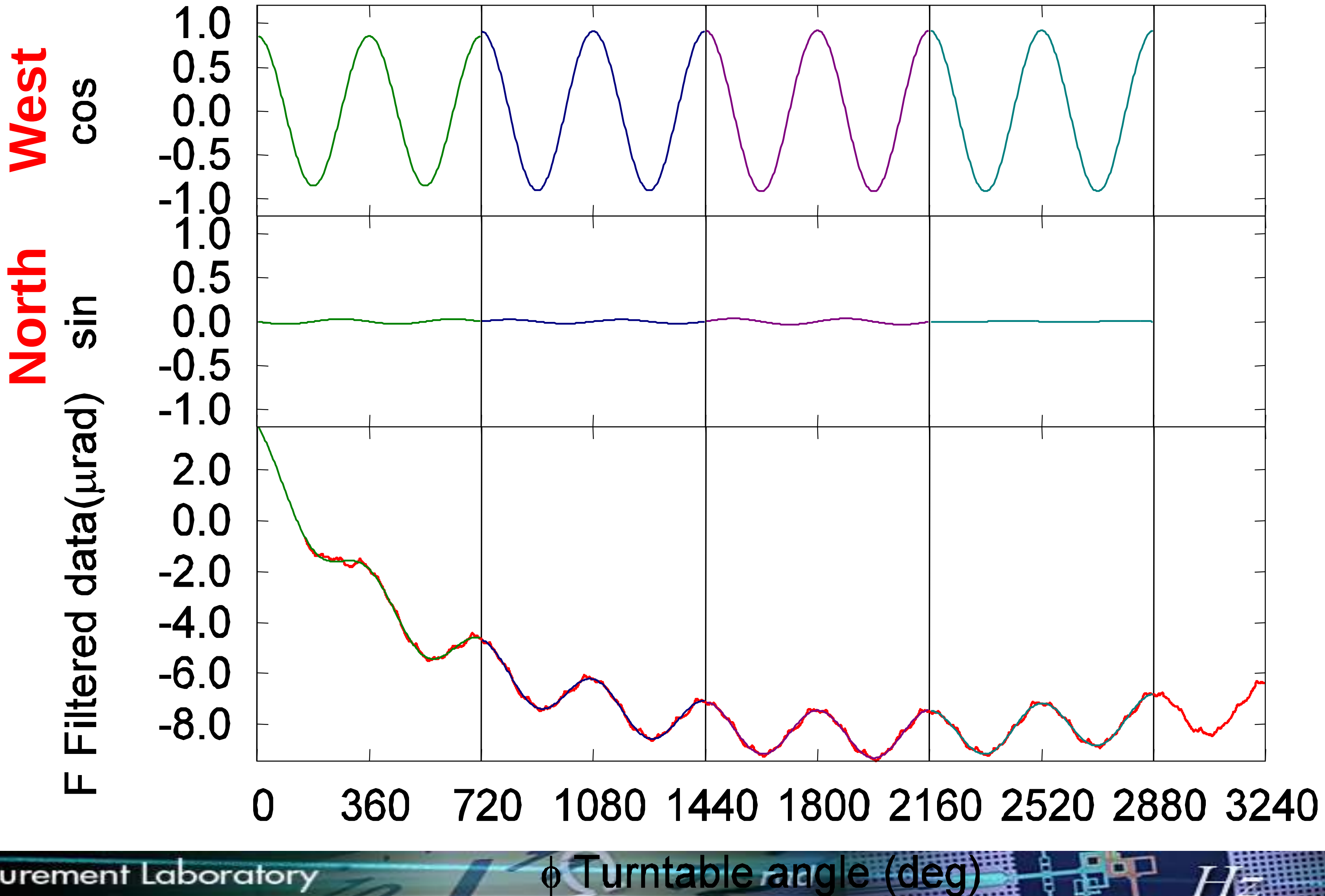


Signal + free Oscillation
$\omega_{SIG}=2/3\ \omega_{PEND}=2\pi/(1200\ s)$
θ Pendulum's Excursion (μrad)
2
0
-2
-4
-6
-8
-10
-12
-14
0
360
720
1080
1440
1800
2160
2520
2880
3240
ϕ Turntable angle (deg)

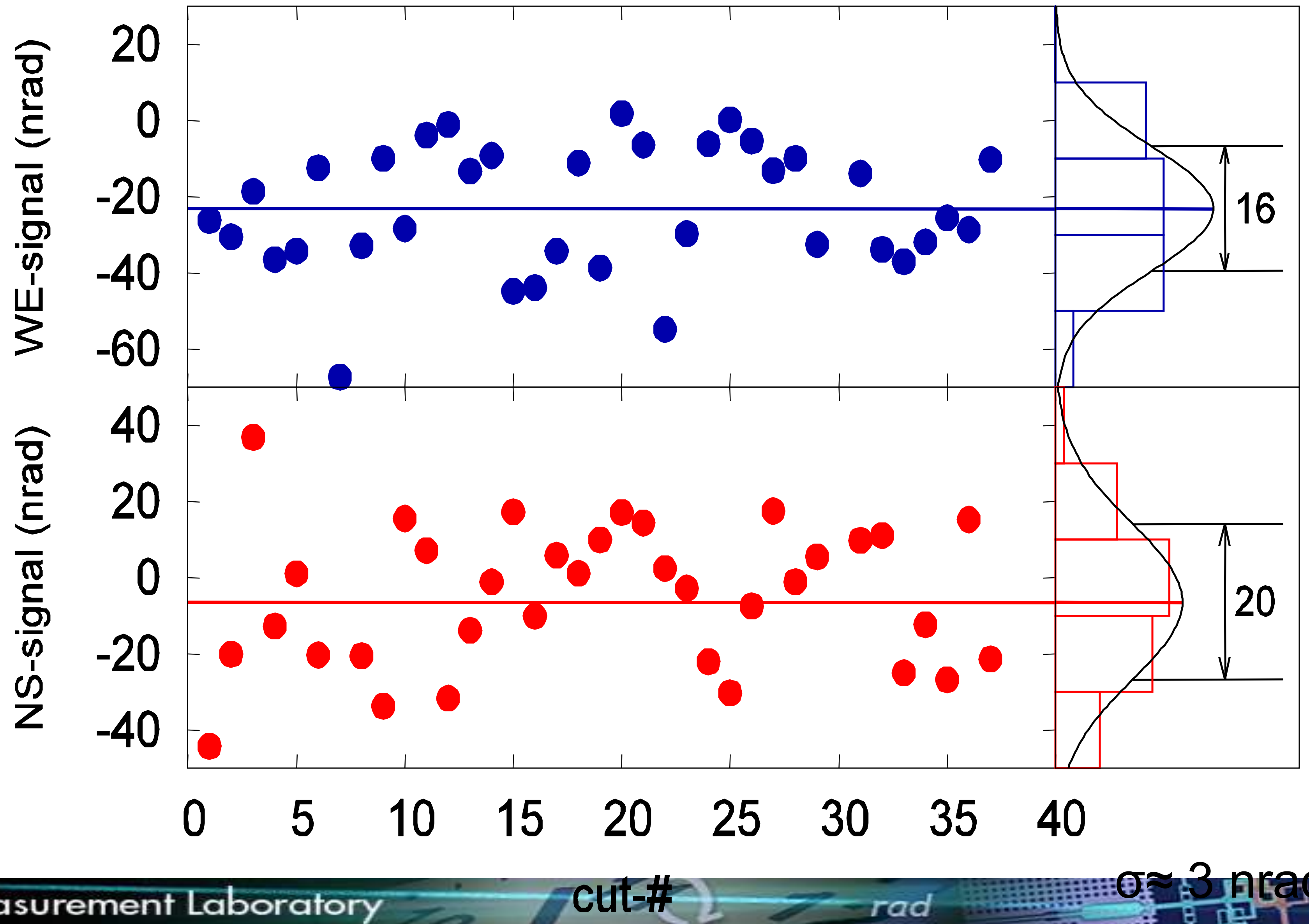
Filter: $F(t)=\theta(t-T/4)+\theta(t+T/4)$



Data reduction

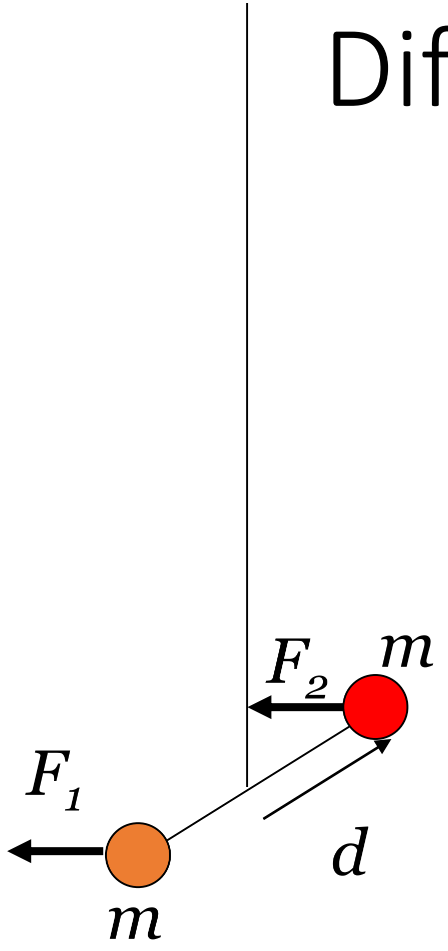


A complete day of data



σ≈ 3 nrad √day

Differential acceleration



Deflection from equilibrium: φ

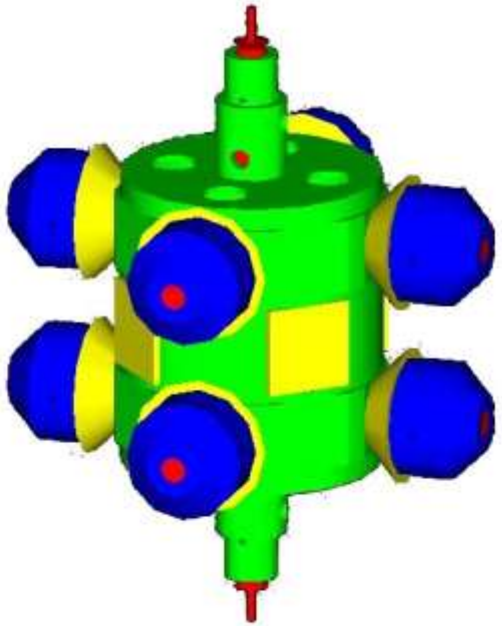
is caused by a torque: $\tau = \kappa\varphi$

$$\tau = F_1 d - F_2 d$$

$$\tau = ma_1 d - ma_2 d = md(a_1 - a_2) = md\Delta a$$

$$\Delta a = \frac{\kappa}{md}\varphi$$

Some numbers



κ	2.36×10^{-9} Nm
m	4.84×10^{-3} kg
d	1.9×10^{-2} m

$$\Delta a = \frac{\kappa}{4md}\varphi$$

6.41×10^{-15} m/s² / nrad

Can be measured with
An uncertainty of
3 nrad per day

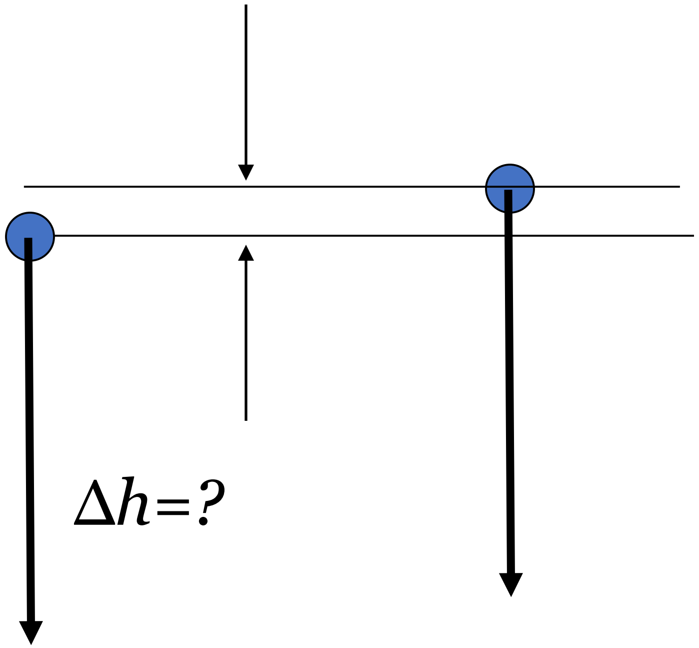
Δa can be measured to
20 fm/s²

A feel for numbers

φ=3 nrad

Δx=?

Δa=20 fm/s²



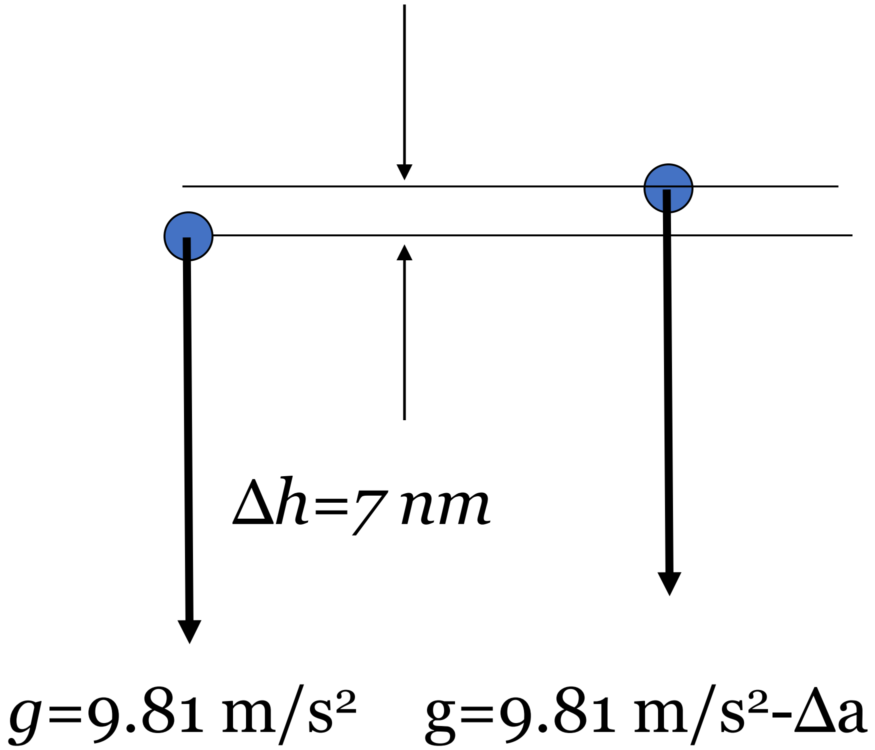
g=9.81 m/s²   g=9.81 m/s²-Δa

A feel for numbers

φ=3 nrad

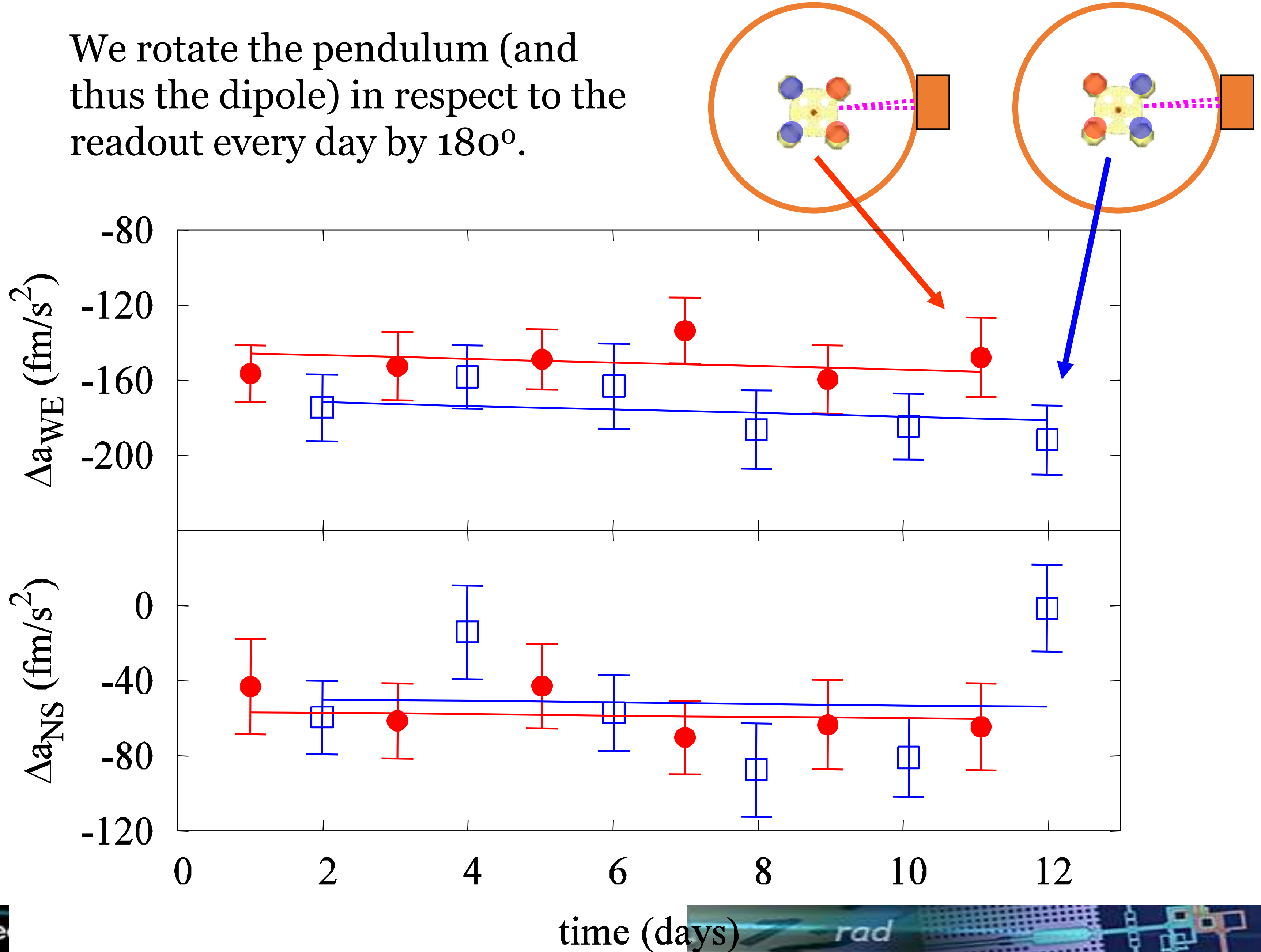
Δx=3 mm

Δa=20 fm/s^2



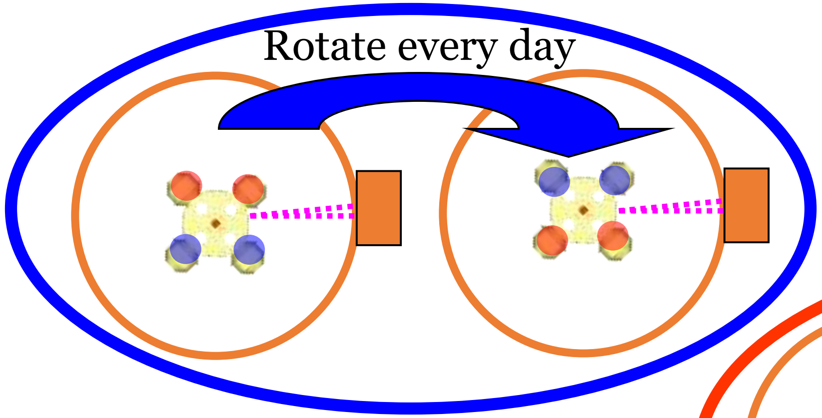
Data taking sequence (1/3)

We rotate the pendulum (and thus the dipole) in respect to the readout every day by 180º.

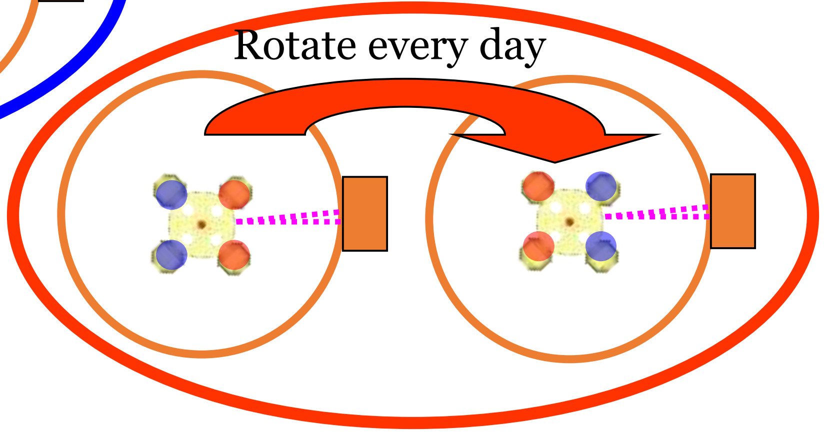


Data taking sequence (2/3)

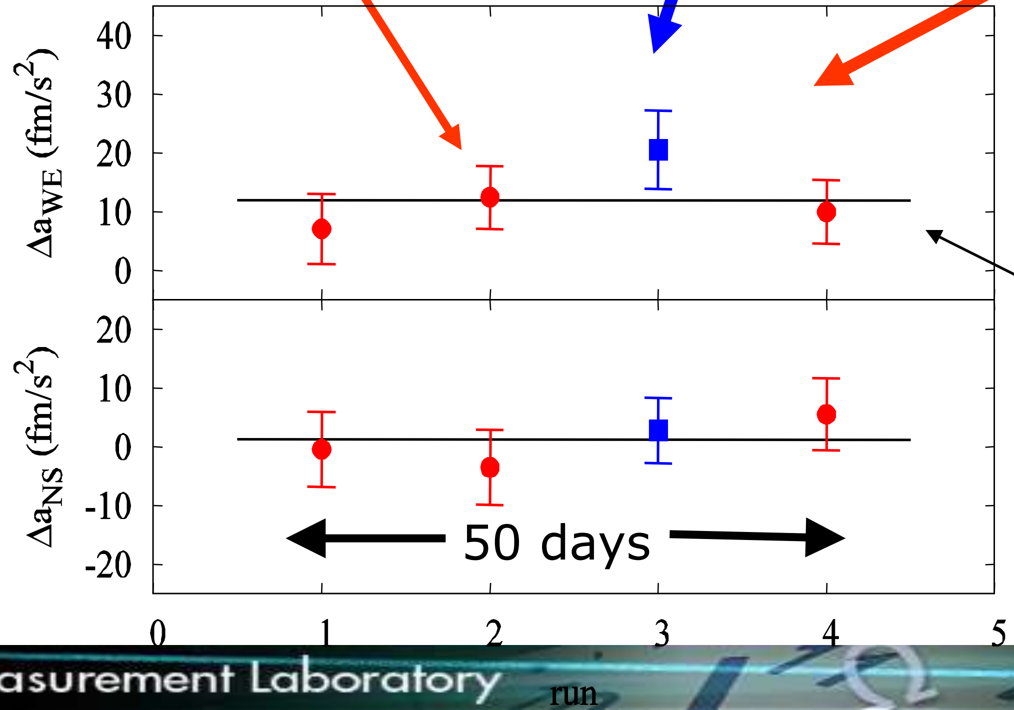
The 12 days of data from the previous slide are condensed into a single point.



Dipole is **orthogonal** to readout.



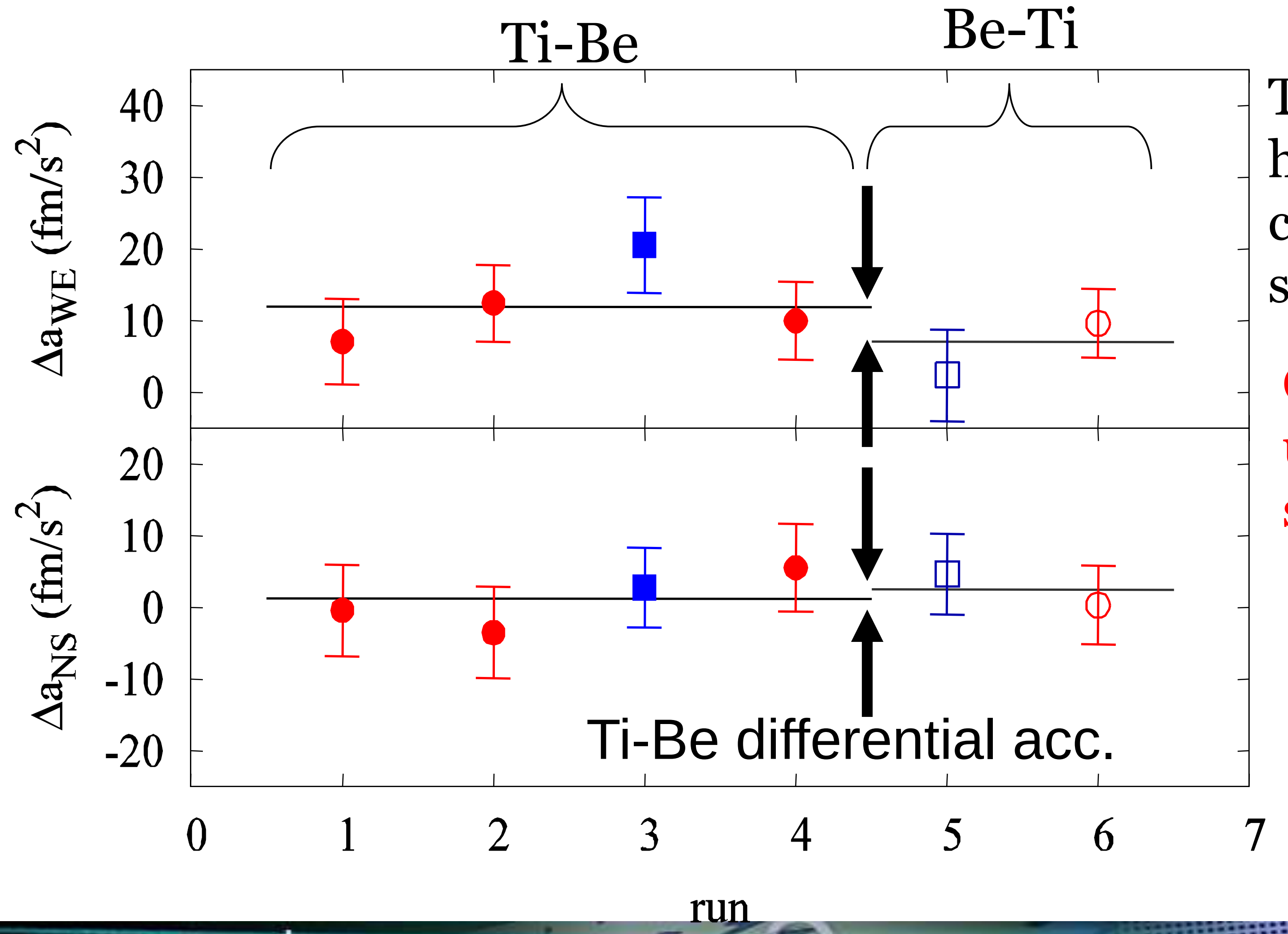
Dipole is (anti-) **parallel** to readout.



Resolved signal in WE-direction!

Data taking sequence (3/3)

After a 2 months of data taking and systematic checks we physically invert the dipole on the pendulum and put it back into the apparatus.



These data points have been corrected for systematic effects.

Only statistical uncertainties shown.

Corrected result

	$\Delta a_{N,Be\text{-}TI}$ (10^{-15} m/s^2)	$\Delta a_{W,Be\text{-}TI}$ (10^{-15} m/s^2)
as measured	3.3 ± 2.5	-2.4 ± 2.4
Due to gravity gradients	1.6 ± 0.2	0.3 ± 1.7
Tilt induced	1.2 ± 0.6	-0.2 ± 0.7
Temperature gradients	0 ± 1.7	0 ± 1.7
Magnetic coupling	0 ± 0.3	0 ± 0.3
Corrected	**0.6 ± 3.1**	**-2.5 ± 3.5**

Gravity gradients (1/4)

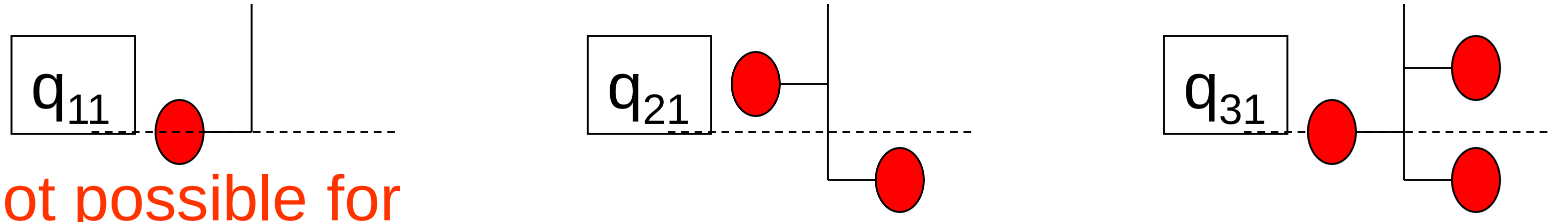
Gravitational potential energy between the pendulum and the source masses is given by

$$w = -4\pi G \sum_{l=0}^{\infty} \frac{1}{2l+1} \sum_{m=-l}^{l} Q_{lm} q_{lm} e^{-im\phi}$$.

$$Q_{lm} = \int d^3 r' \rho_{source}(\vec{r}')\, r'^{-(l+1)}\, Y_{lm}(\hat{r}')$$ gravity gradient field

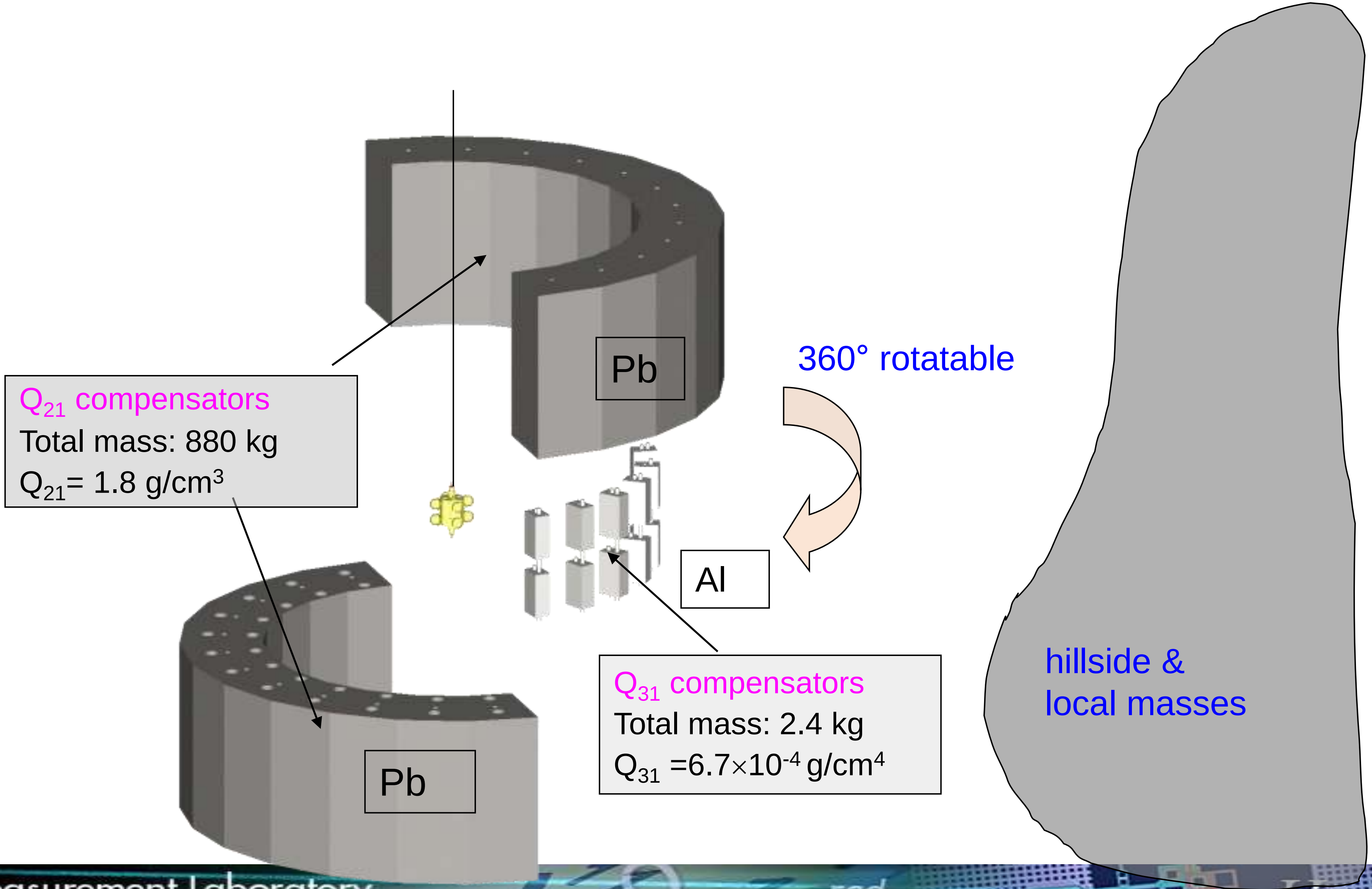
$$q_{lm} = \int d^3 r\, \rho_{pend}(\vec{r})\, r^{l}\, Y_{lm}^{*}(\hat{r})$$ gravity multipole moment

Torque for 1ω (m=1) signal: $\tau = 8\pi G \frac{1}{2l+1} |q_{l1}||Q_{l1}|$



not possible for
a torsion pendulum

Gravity gradients (2/4)

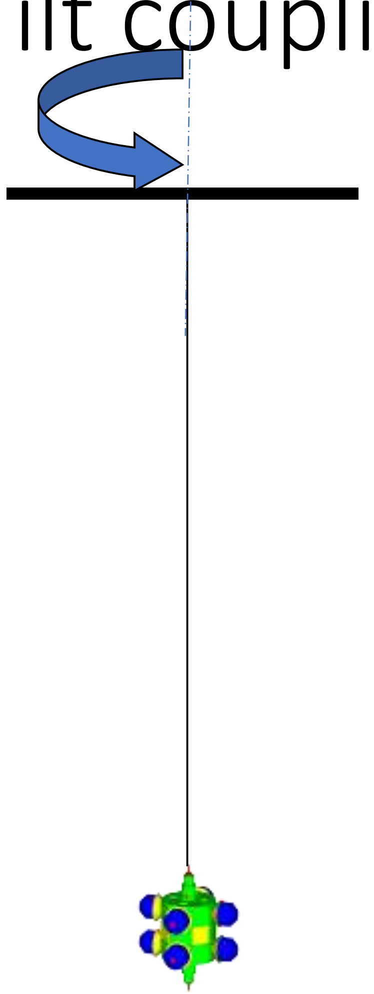


Gravity gradients (3/4)

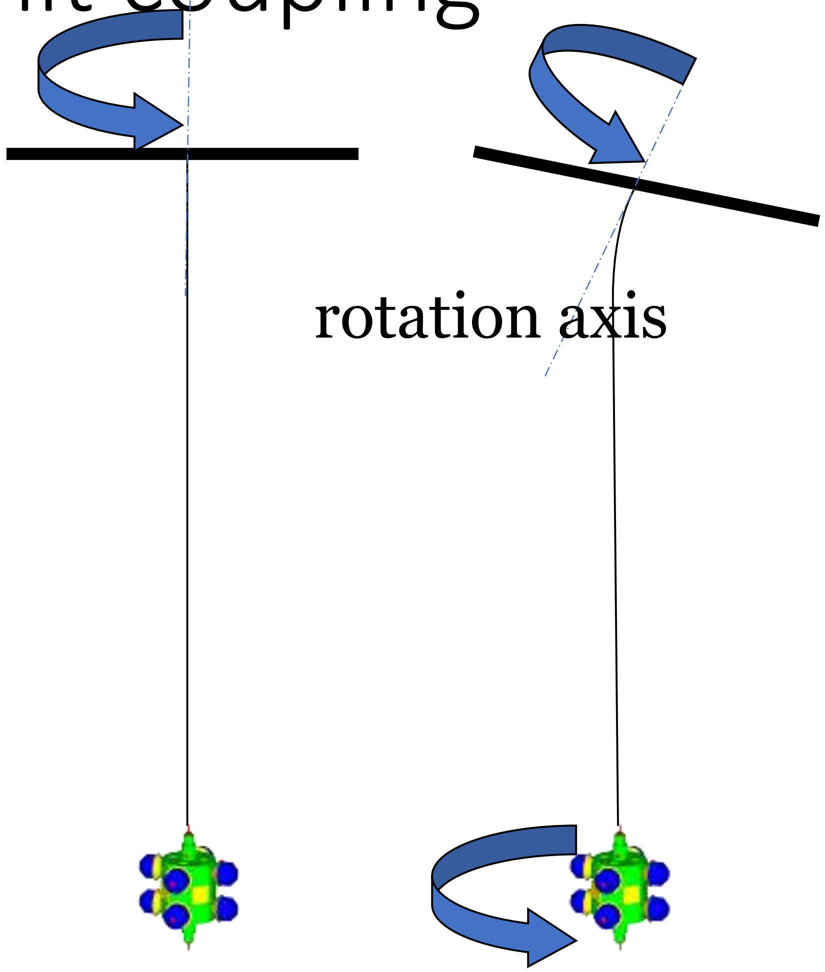
q_{41} configuration

q_{21} configuration installed

Tilt coupling

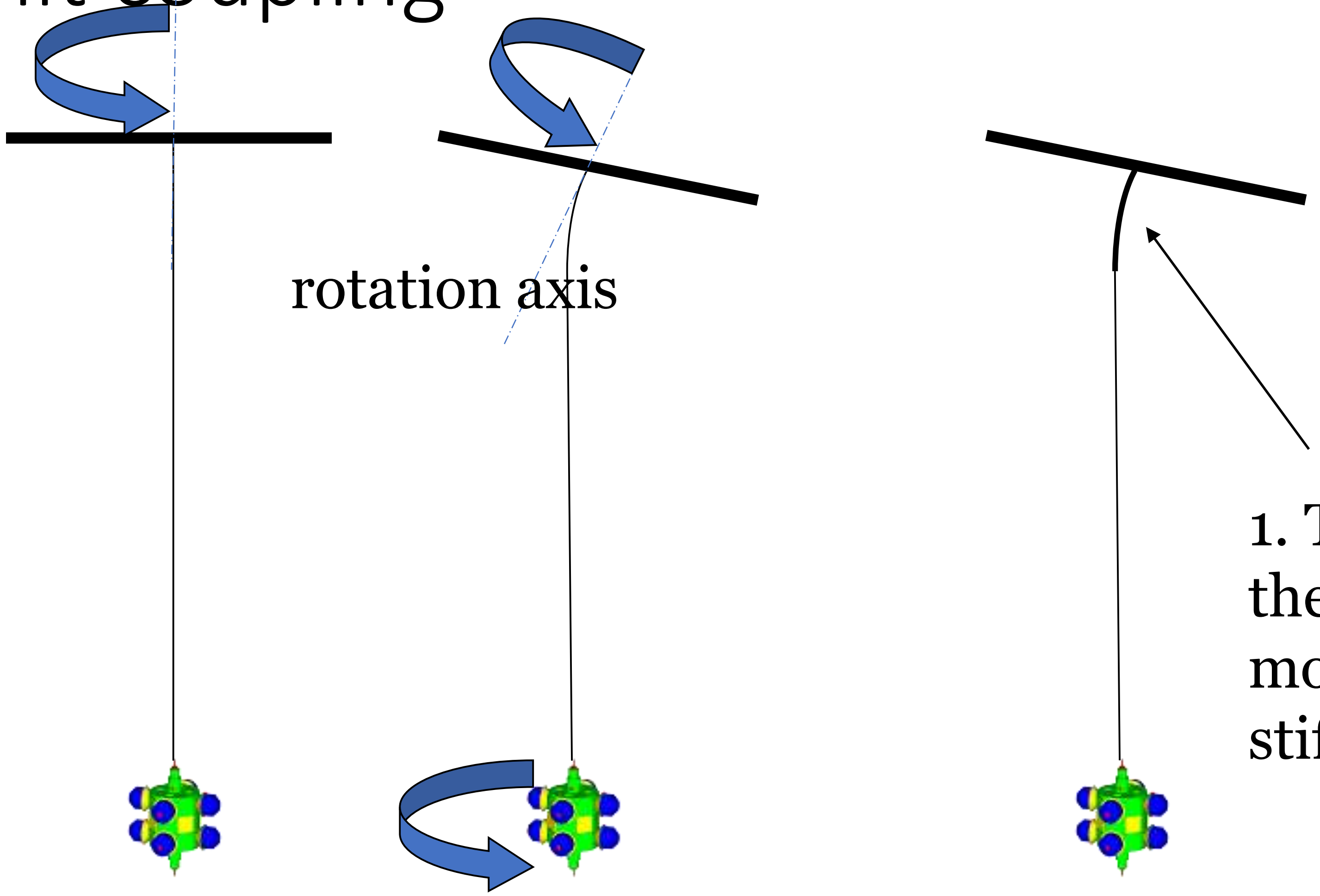


Tilt coupling



Tilt of the suspension point
+ anisotropies of the fiber
→ will rotate

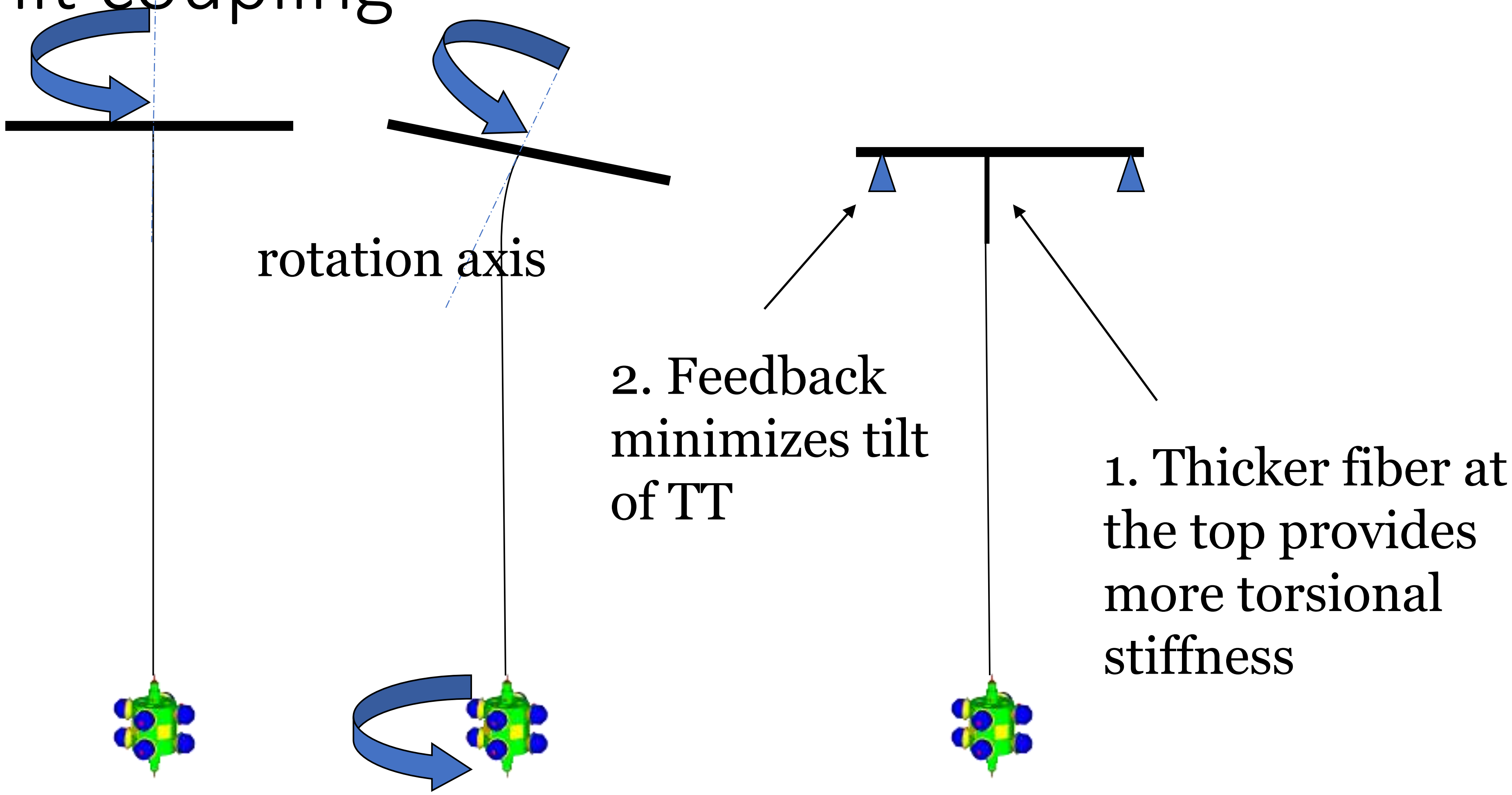
Tilt coupling



1. Thicker fiber at the top provides more torsional stiffness

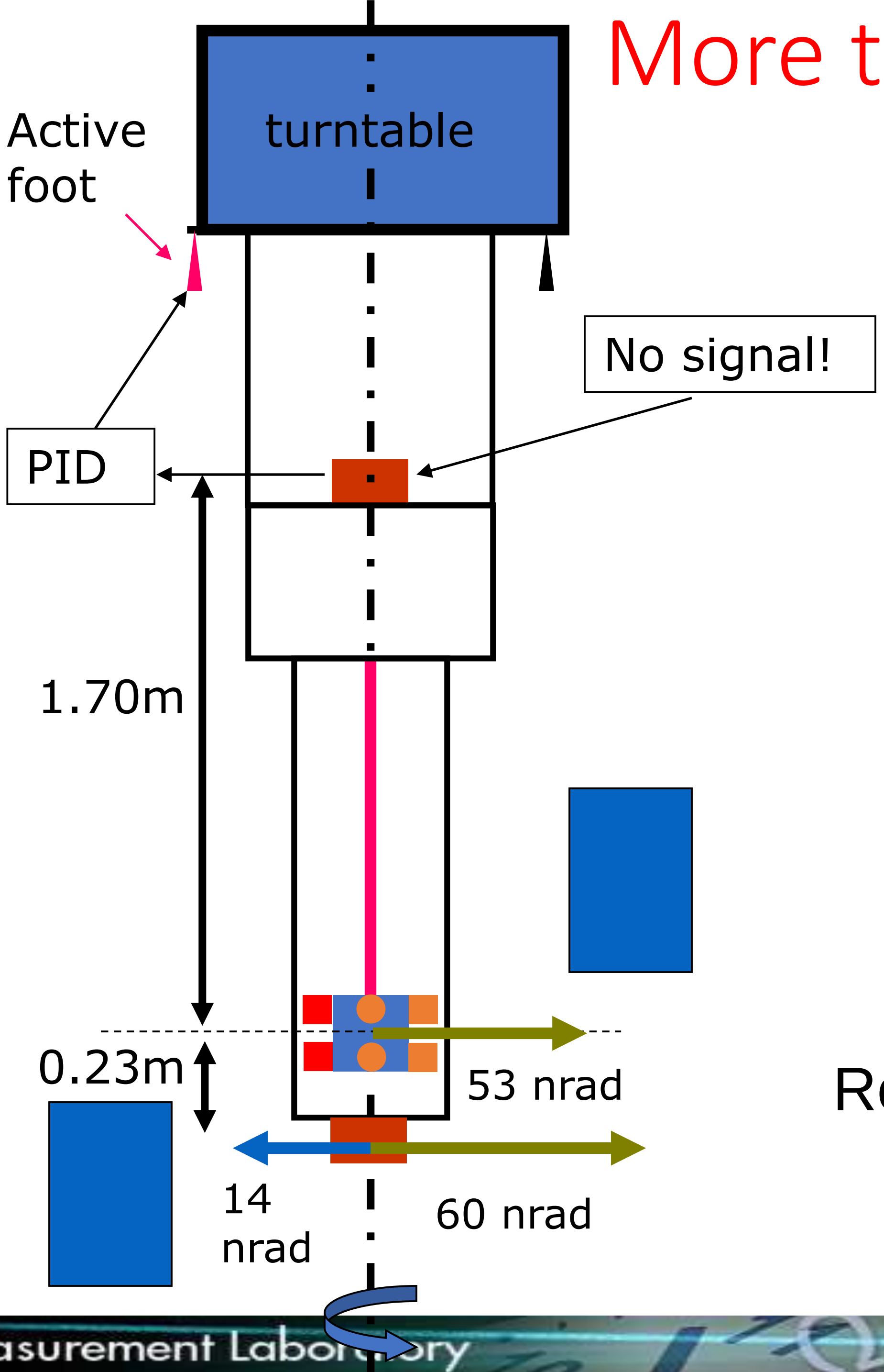
Tilt of the suspension point
+ anisotropies of the fiber
will rotate

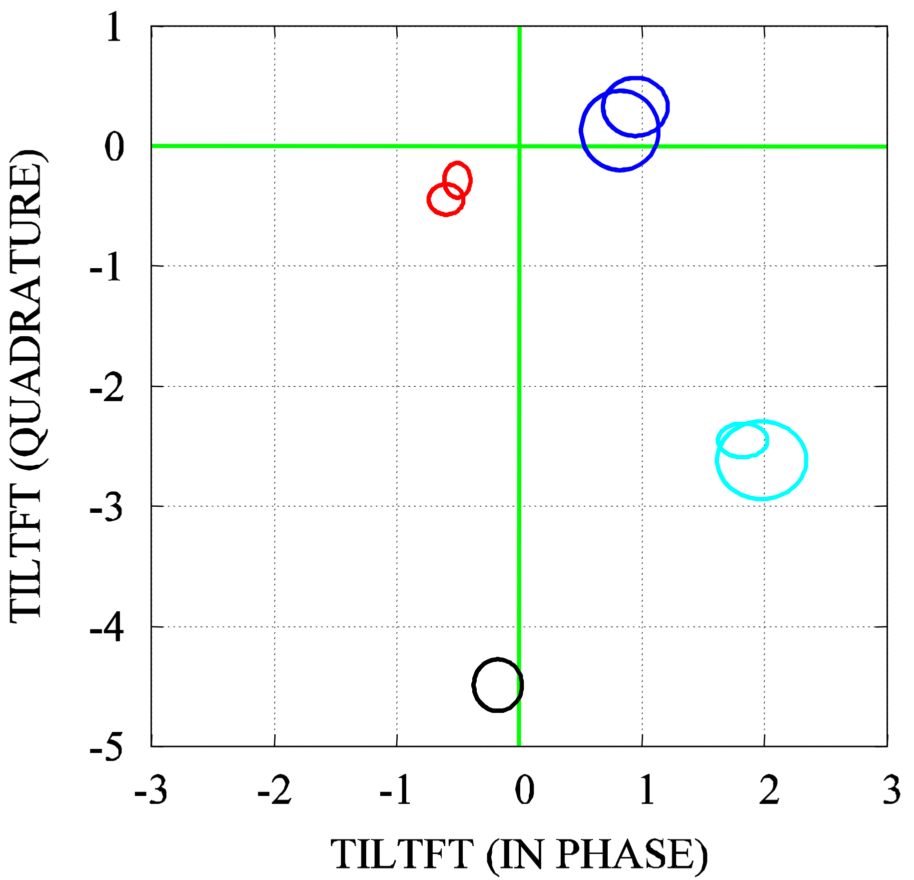
Tilt coupling



Tilt of the suspension point
+ anisotropies of the fiber
→ will rotate

More tilt

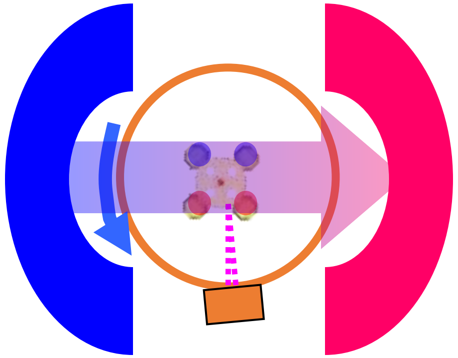




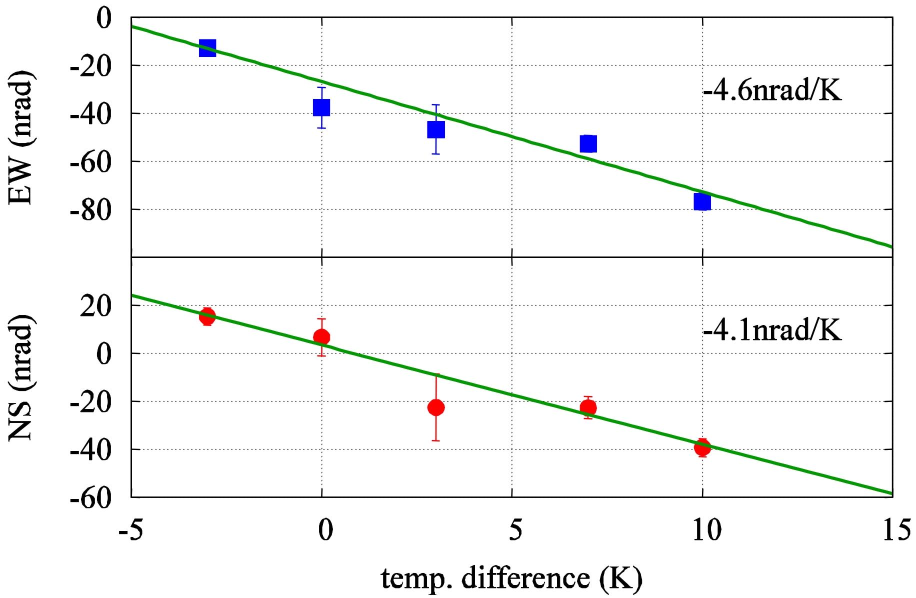
Remaining tilt uncertainty:

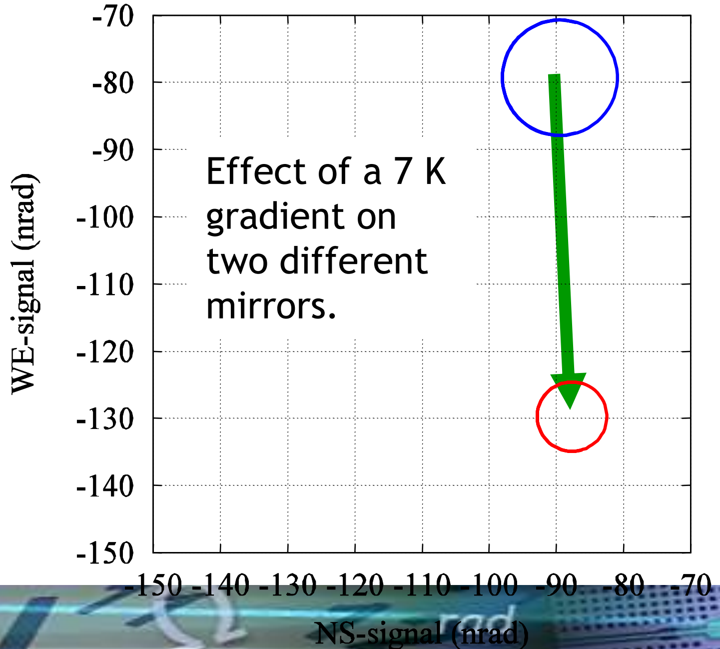
0.6 fm/s^2 N
0.7 fm/s^2 W

Thermal



Effect on the applied gradient on the signal (measurement was done on one mirror):





50 nrad Signal

ΔT =7K

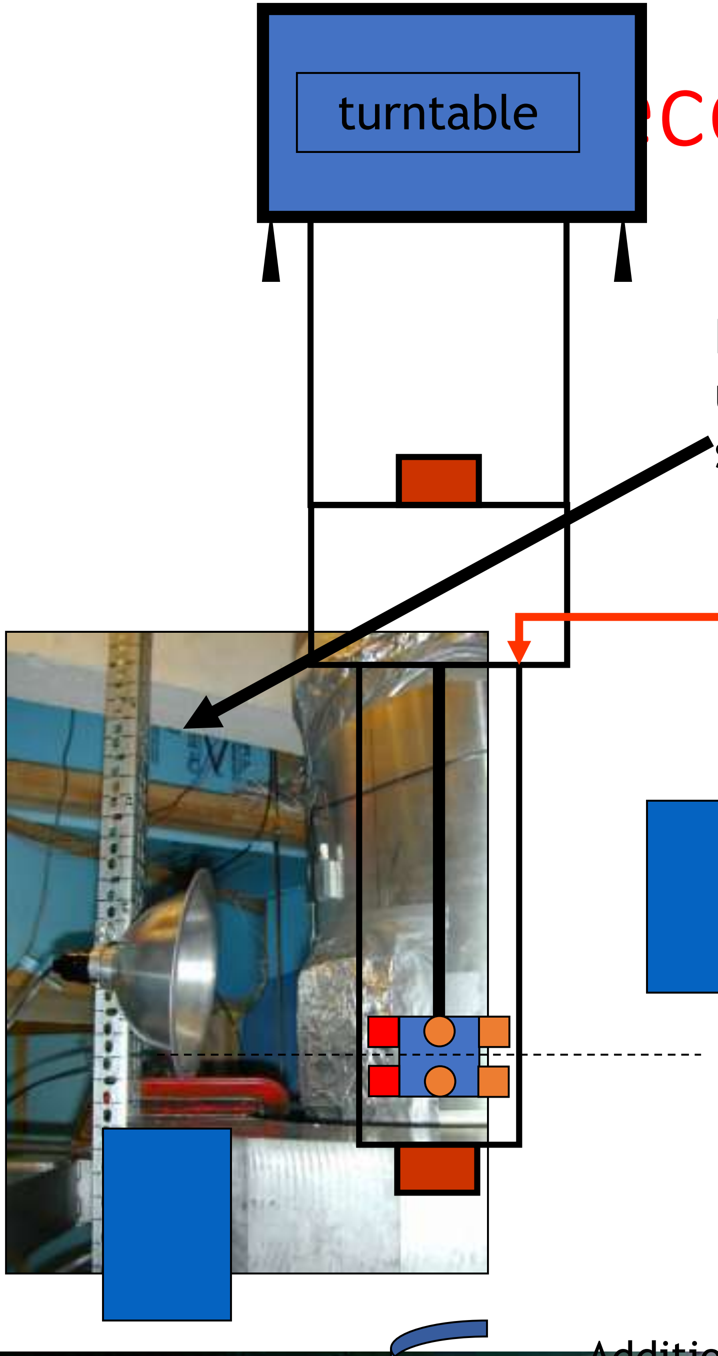
During data taking ΔT≈0.053 K

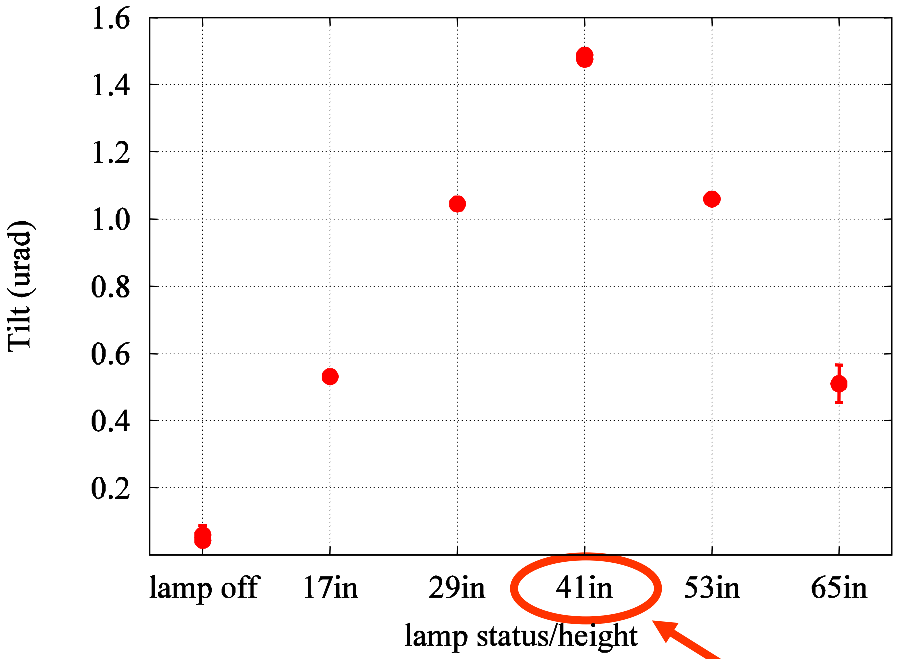
⇒ 0.38 nrad

1.7 fm/s² N

1.7 fm/s² W

Recent improvements





Highest sensitivity to the lamp

Additional insulation (@ 41 in) reduced the temperature feedthrough.

Interpreting our result

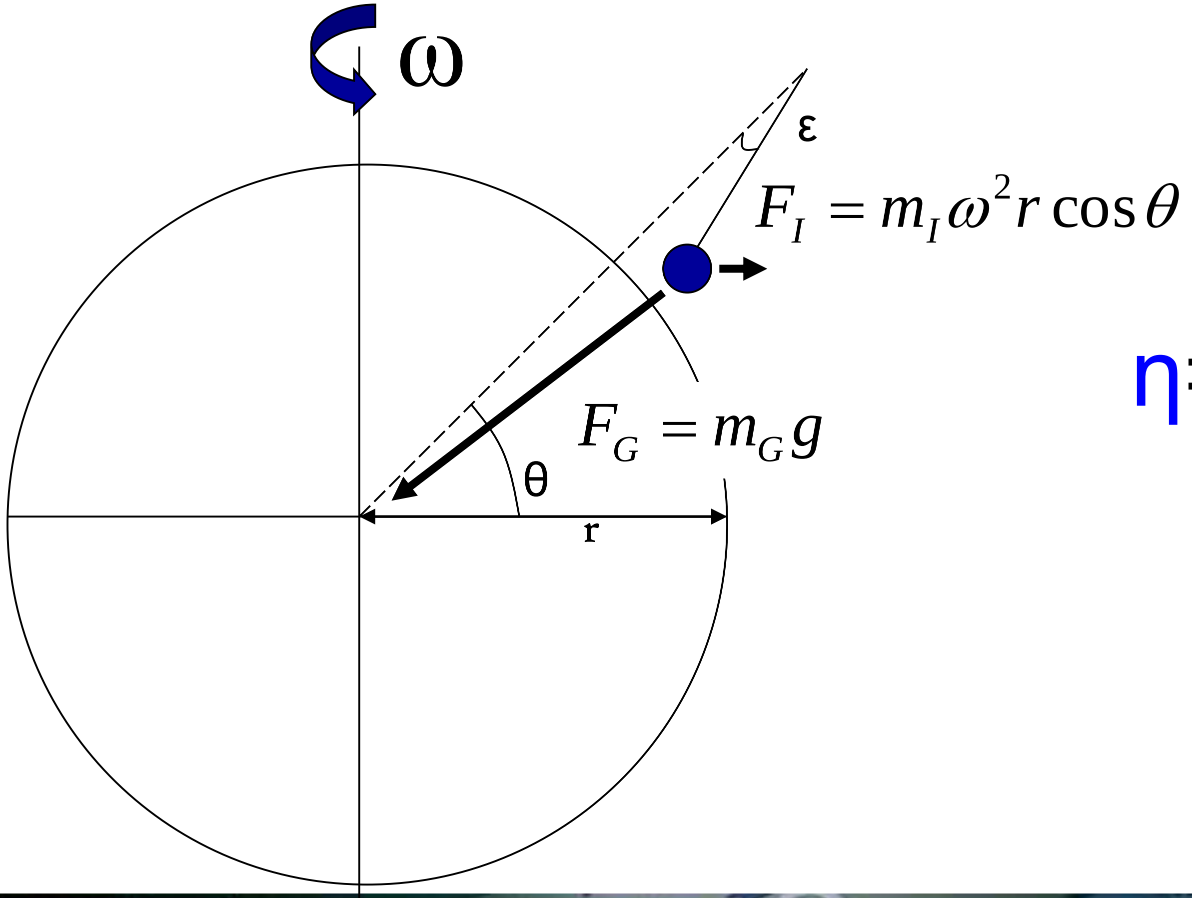
North: $a_{Be}-a_{Ti} = (0.6 \pm 3.1) \times 10^{-15}$ m/s^2

West: $a_{Be}-a_{Ti} = (-2.5 \pm 3.5) \times 10^{-15}$ m/s^2

Interpreting our result

North: a_{Be}-a_{Ti}= (0.6 ± 3.1) x 10^{-15} m/s²

West: a_{Be}-a_{Ti}= (-2.5 ± 3.5) x 10^{-15} m/s²

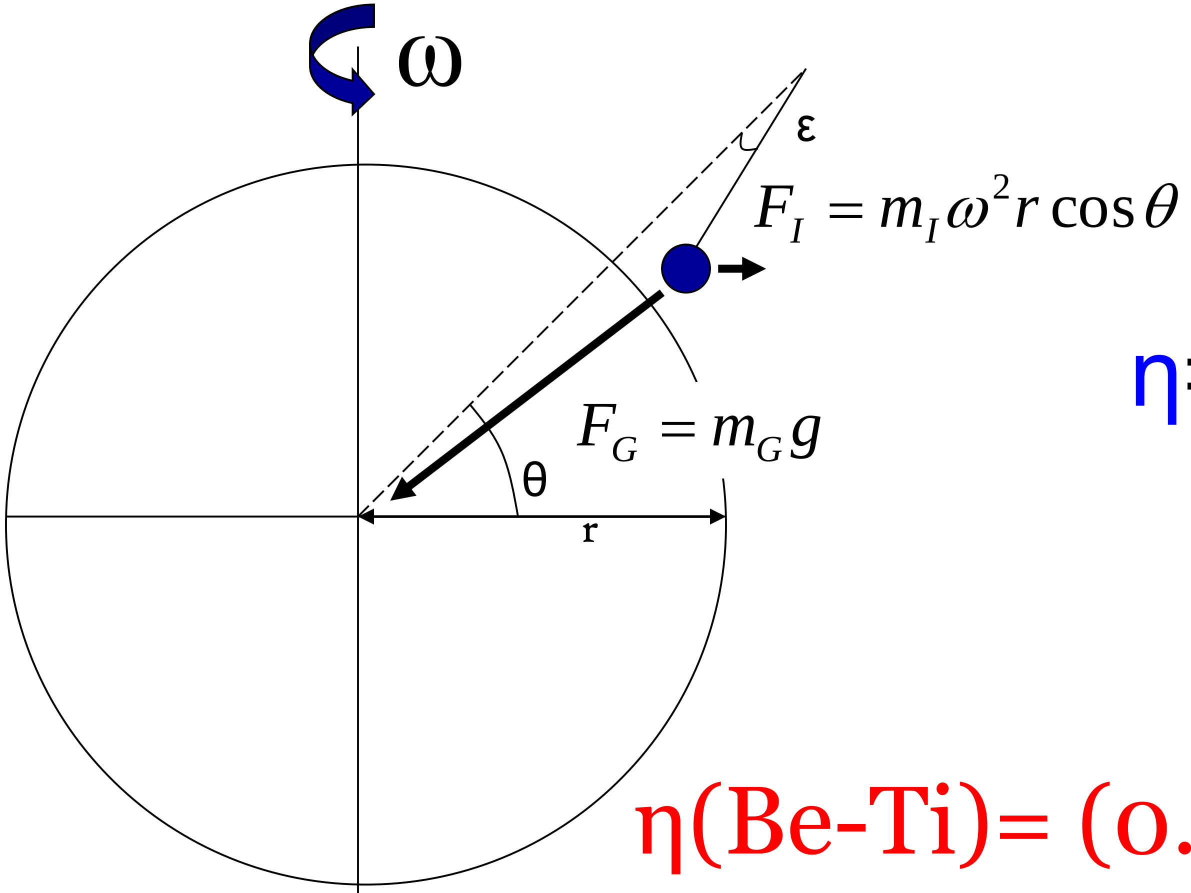


$$\eta = \frac{|a_1 - a_2|}{\frac{1}{2}|a_1 + a_2|}$$

Interpreting our result

North: $a_{Be}-a_{Ti} = (0.6 \pm 3.1) \times 10^{-15}\ m/s^2$

West: $a_{Be}-a_{Ti} = (-2.5 \pm 3.5) \times 10^{-15}\ m/s^2$

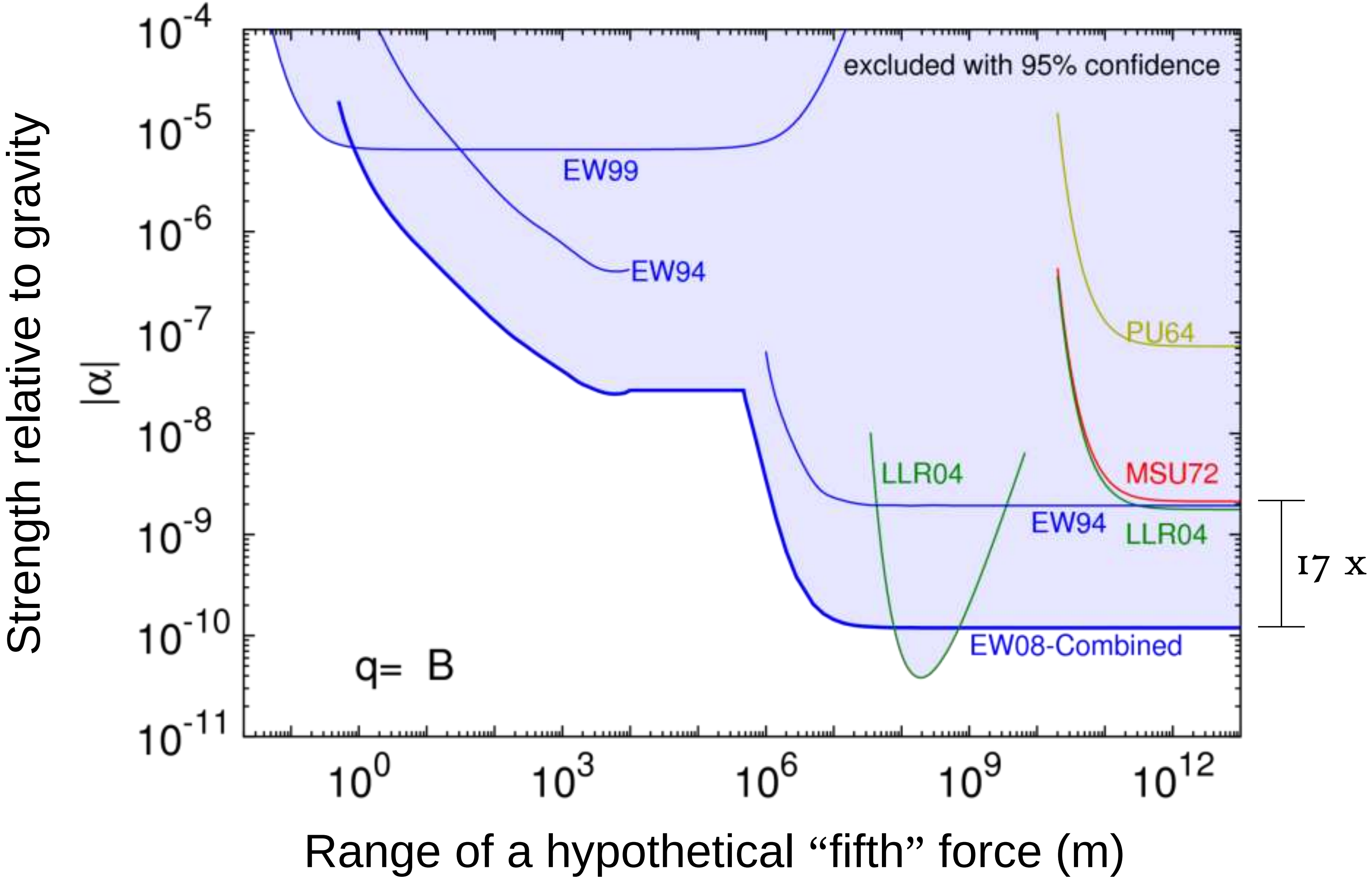


$$\eta = \frac{|a_1 - a_2|}{\frac{1}{2}|a_1 + a_2|}$$

$\eta(Be\text{-}Ti) = (0.3 \pm 1.8) \times 10^{-13}$

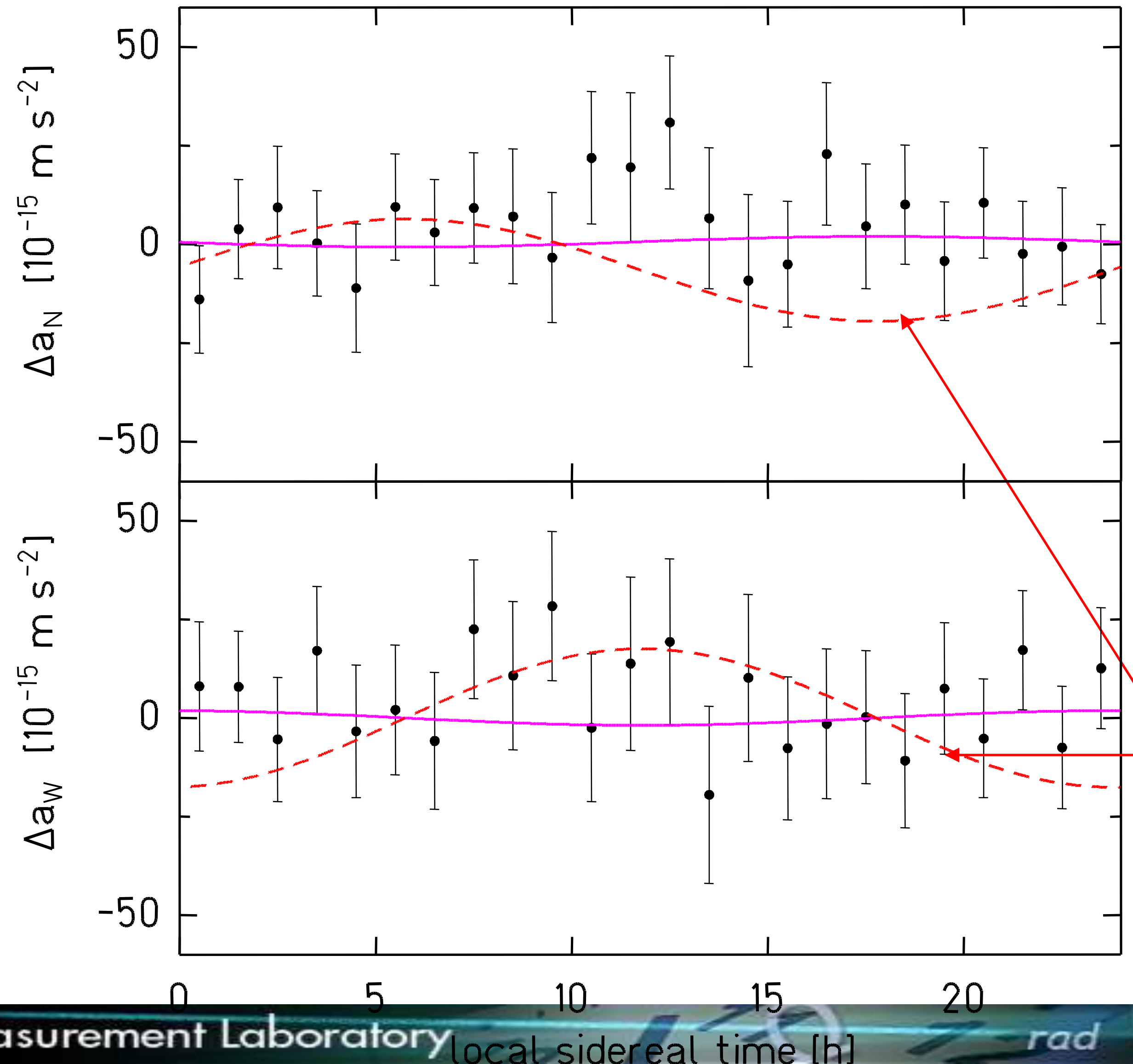
Results

Charge: B



Acceleration to the center of our galaxy

1825 h of data taken over 220 days



Differential acceleration to the center of the galaxy:

$(-2.1 \pm 3.1) \times 10^{-15}$ m/s^2

In quadrature:

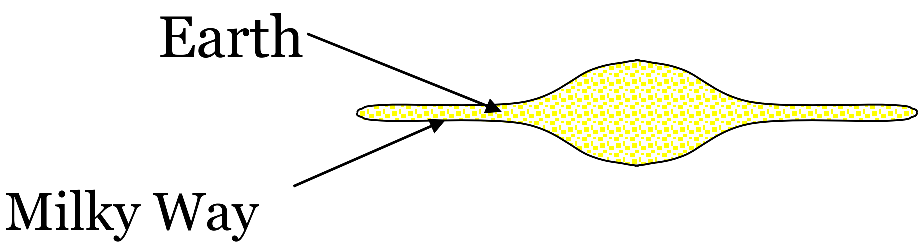
$(2.7 \pm 3.1) \times 10^{-15}$ m/s^2

Hypothetical signal:

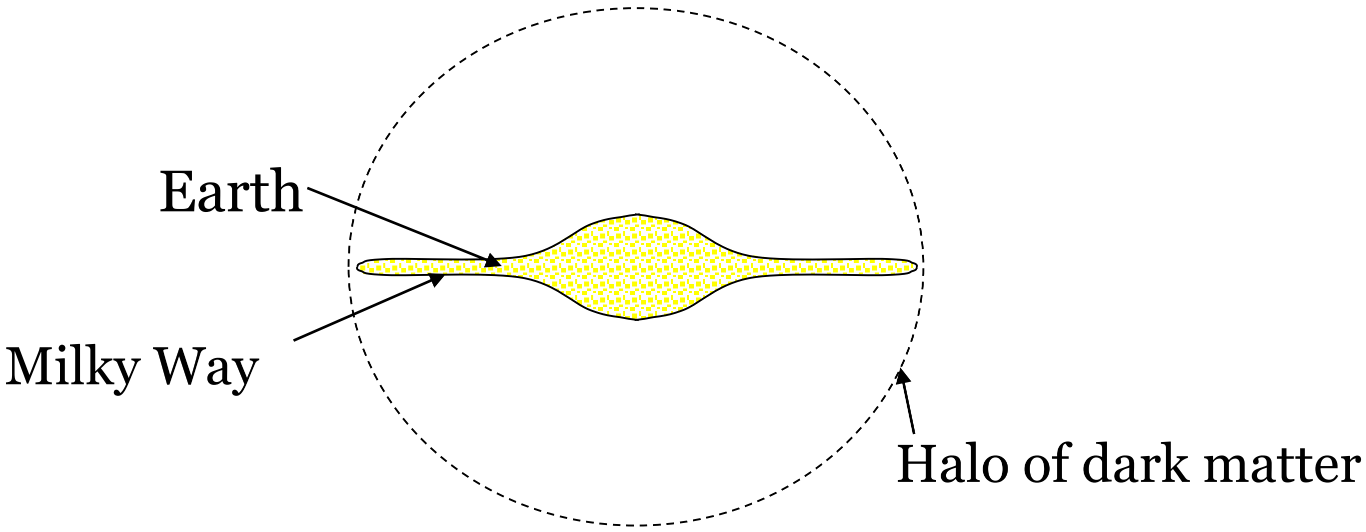
(7 times larger)

20×10^{-15} m/s^2

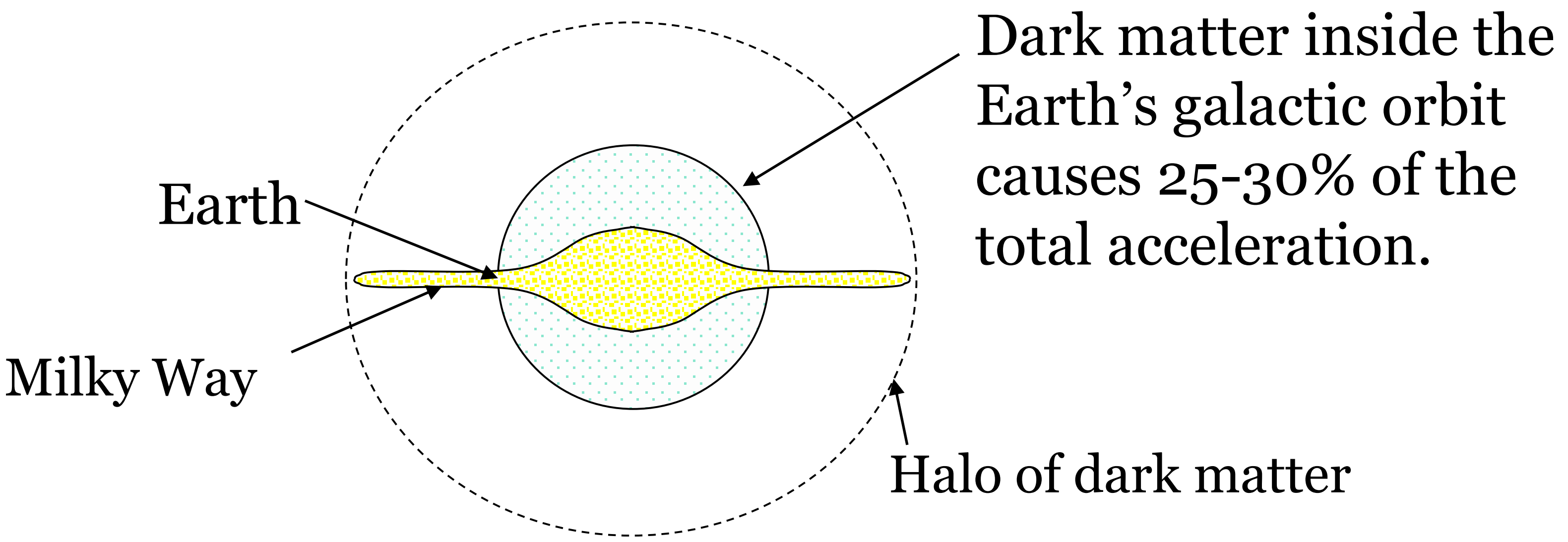
Galactic dark matter



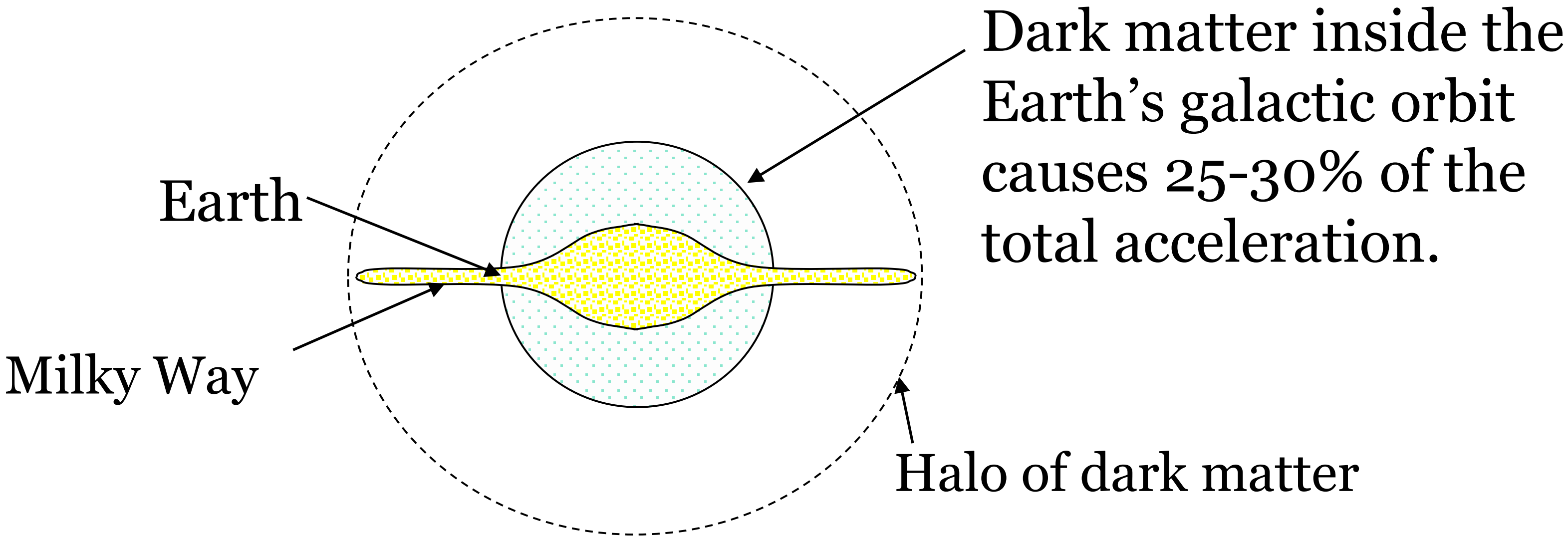
Galactic dark matter



Galactic dark matter



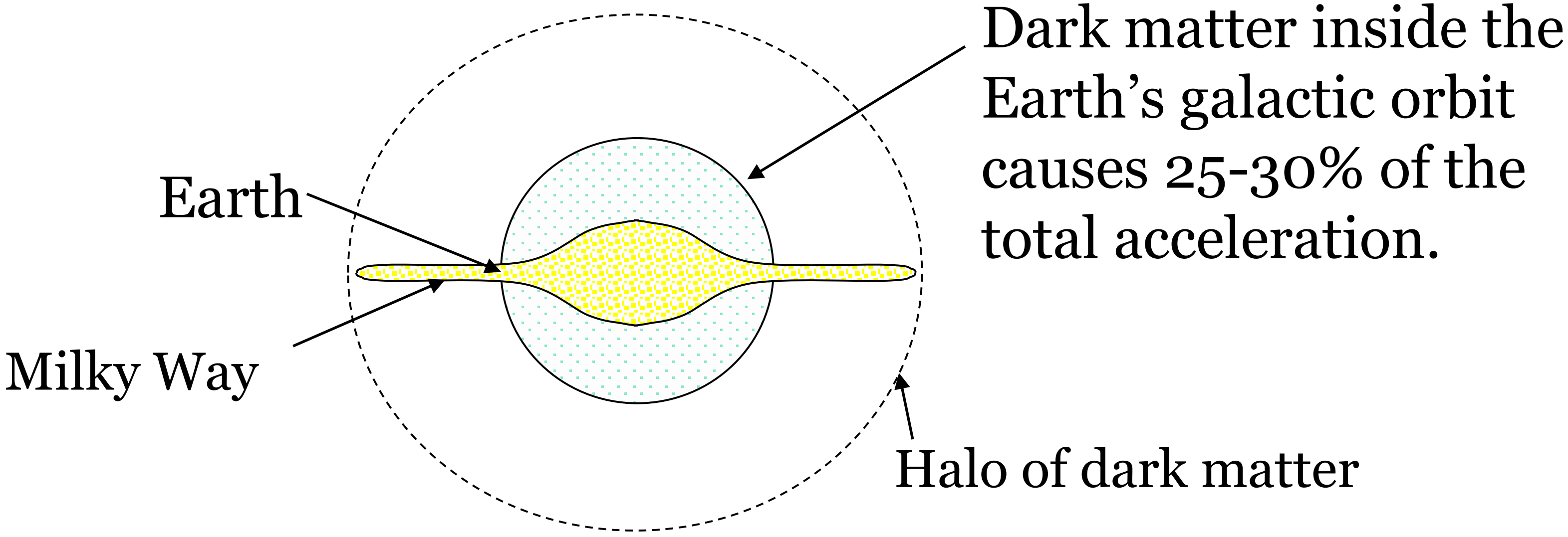
Galactic dark matter



Our acceleration towards the galactic center is:

$a_{gal} = a_{dark} + a_{ordinary} = 1.9 \times 10^{-10}\ m/s^2$ => $a_{dark} = 5 \times 10^{-11}\ m/s^2$

Galactic dark matter



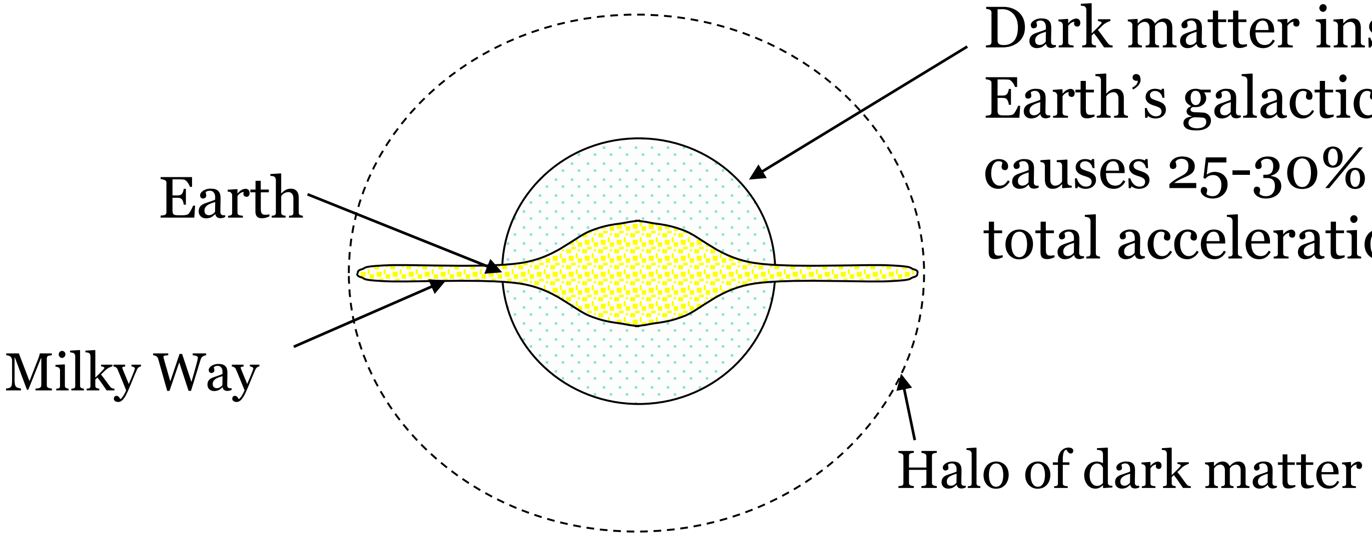
Our acceleration towards the galactic center is:

$a_{gal} = a_{dark} + a_{ordinary} = 1.9 \times 10^{-10}$ m/s² => $a_{dark} = 5 \times 10^{-11}$ m/s²

Measured differential acceleration towards the galactic center is:

$\Delta a_{gal} = (-2.1 \pm 3.1) \times 10^{-15}$ m/s² => $\eta_{dark} = |\Delta a_{gal}|/a_{dark} = (-4 \pm 7) \times 10^{-5}$

Galactic dark matter



Dark matter inside the Earth's galactic orbit causes 25-30% of the total acceleration.

Our acceleration towards the galactic center is:

$$a_{gal} = a_{dark} + a_{ordinary} = 1.9 \times 10^{-10}\ m/s^2 \Rightarrow a_{dark} = 5 \times 10^{-11}\ m/s^2$$

Measured differential acceleration towards the galactic center is:

$$\Delta a_{gal} = (-2.1 \pm 3.1) \times 10^{-15}\ m/s^2 \Rightarrow \eta_{dark} = |\Delta a_{gal}|/a_{dark} = (-4 \pm 7) \times 10^{-5}$$

The acceleration of Be and Ti towards dark matter does not differ by more than 150 ppm (with 95 % confidence).

Summary

- Test of the equivalence principle with a rotating torsion balance.
- Principle of the measurement
- Main systematic effects
- Results
  - Earth fixed (North): a_{Be}-a_{Ti}=(0.6 ± 3.1) x 10^{-15} m/s^2.
  - η=(0.3 ± 1.8) x 10^{-13}.
  - Towards Galaxy: a_{Be}-a_{Ti}=(-2.1 ± 3.1) x 10^{-15} m/s^2.
  - η_{DM}=(-4 ± 7) x 10^{-5}.

  - 10x improved limits on a long range interaction.