

SIGRAV school

2/19/25

Vietri, Italy, 2/16/2024

S. Schlamminger

National Institute of Standards and Technology

- 1998 -2002 PhD at the University of Zurich:
 - on the determination of G
- 2003- 2010 post-doc at the University of Washington
 - Equivalence principle
 - Noise measurements/ patch fields and such for LIGO
 - Inverse square law test
- 2011 – 2017 physicist National Institute of Standards and Technology
 - Built a Kibble balance that ultimately led to the revision of the SI
- 2018 – 2019 Prof. at the University of Applied Science in Regensburg
- 2019 – today National Institute of Standards and Technology
 - Table top Kibble balances
 - Measurement of G
 - Impedance metrology

About the four lectures

1. Traceability of mass and force measurements.

- The SI,
- how well can we measure a Newton

2. Source masses

- What do you have to pay attention to

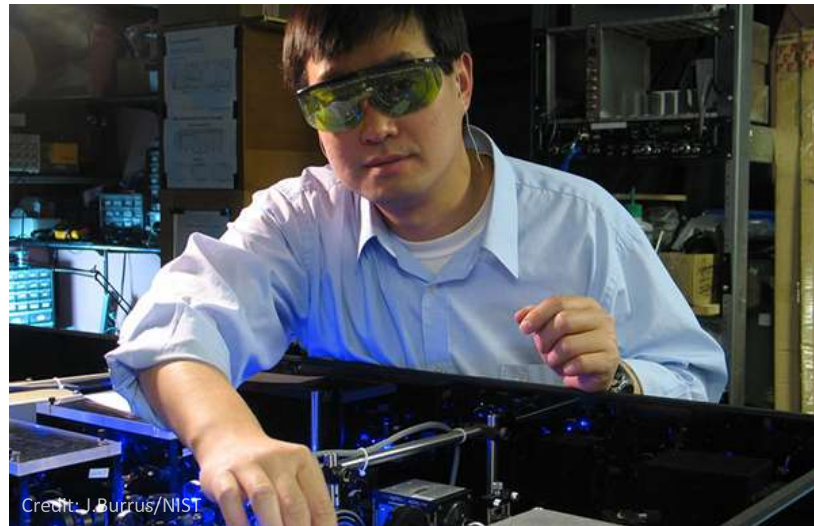
3. Thermal noise and inelastic effects

- How they come about and
- How they limit the measurement

4. Practical tips and experiences

- big G measurements
- EP measurements
- Inverse square law measurements

To promote U.S. innovation and industrial competitiveness by advancing **measurement science, standards, and technology** in ways that enhance economic security and improve our quality of life



Measurements essential to commerce, trade, and innovation

Federal role
established in the
U.S. Constitution



NIST AT A GLANCE

Industry's National Laboratory



3,400+
FEDERAL
EMPLOYEES



5
NOBEL PRIZES



2 CAMPUSES
GAITHERSBURG, MD [HQ]
BOULDER, CO



3,500+
ASSOCIATES



10
COLLABORATIVE
INSTITUTES



400+
BUSINESSES USING
NIST FACILITIES



16
NATL OFFICE FOR
MANUFACTURING
INSTITUTES

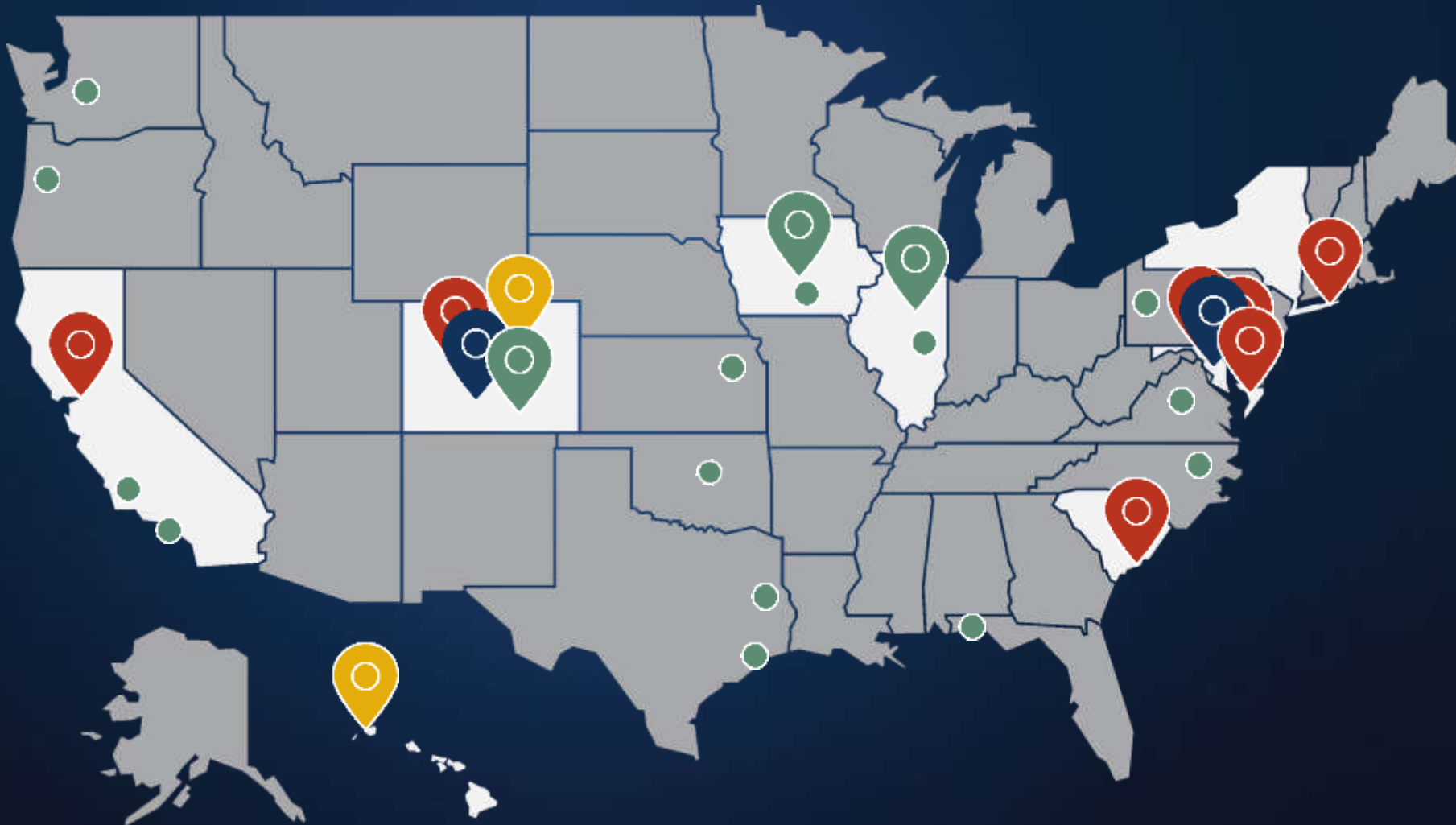


51
MANUFACTURING
EXTENSION
PARTNERSHIP CENTERS



U.S. BALDRIGE
PERFORMANCE
EXCELLENCE PROGRAM

NIST Joint Institute and Center Locations



Gaithersburg, MD
Boulder, CO



Joint Institutes and Centers



National Cybersecurity Center of Excellence

Institute for Bioscience & Biotechnology Research

Joint Institute for Quantum Computer Science

Joint Quantum Institute

JILA

Hollings Marine Laboratory

Brookhaven National Laboratory

Joint Initiative for Metrology in Biology

Atomic Clock Signal Stations



NIST Kauai HI WWVH

NIST Ft. Collins CO WWV

NIST Centers of Excellence



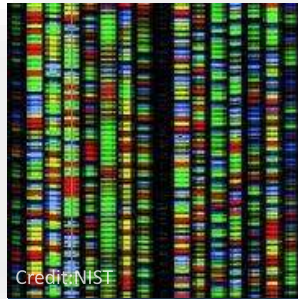
Forensic Science

Disaster Resilience

Advanced Materials

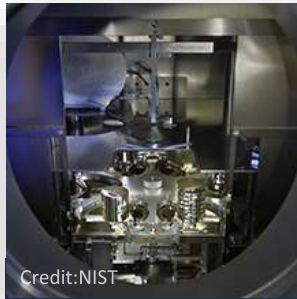
NIST Laboratory Programs

NIST



Credit: NIST

**Material
Measurement
Laboratory**



Credit: NIST

**Physical
Measurement
Laboratory**



Credit: Shutterstock/
Dmitry Kalinovsky

**Engineering
Laboratory**



Credit: Shutterstock

**Information
Technology
Laboratory**

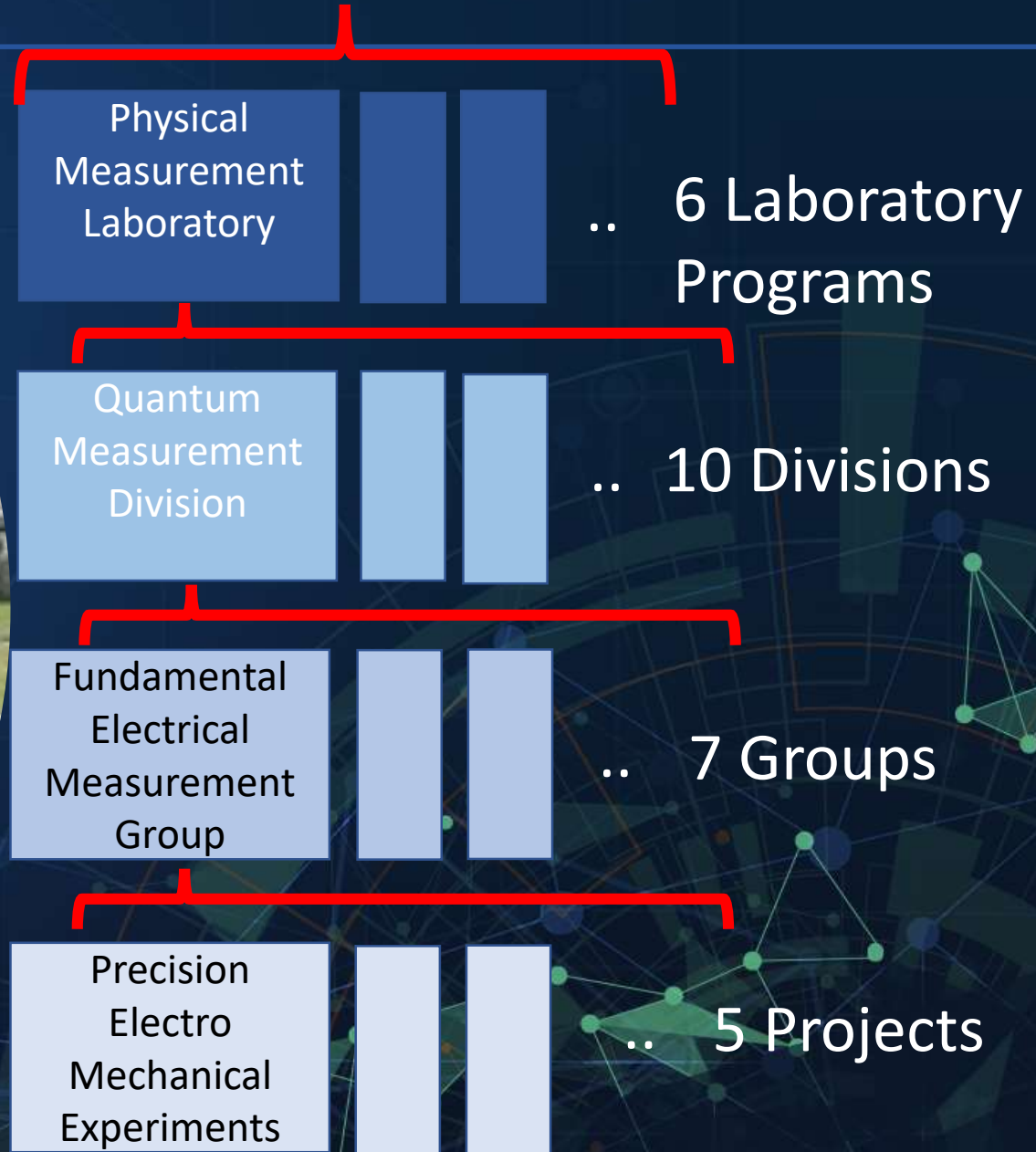


Credit: Shutterstock/italianastro

**Communication
Technology
Laboratory**



**NIST Center
for Neutron
Research**



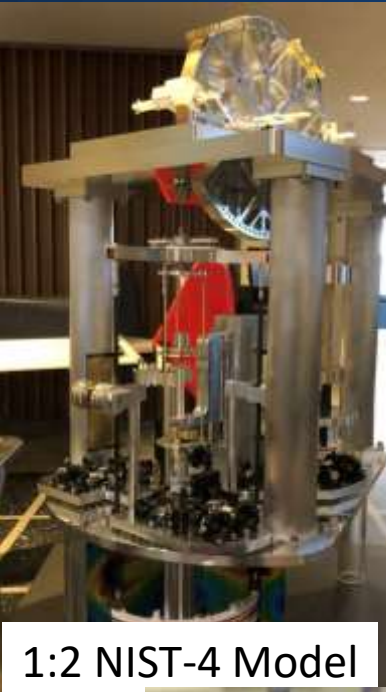


NIST's PREME* Team

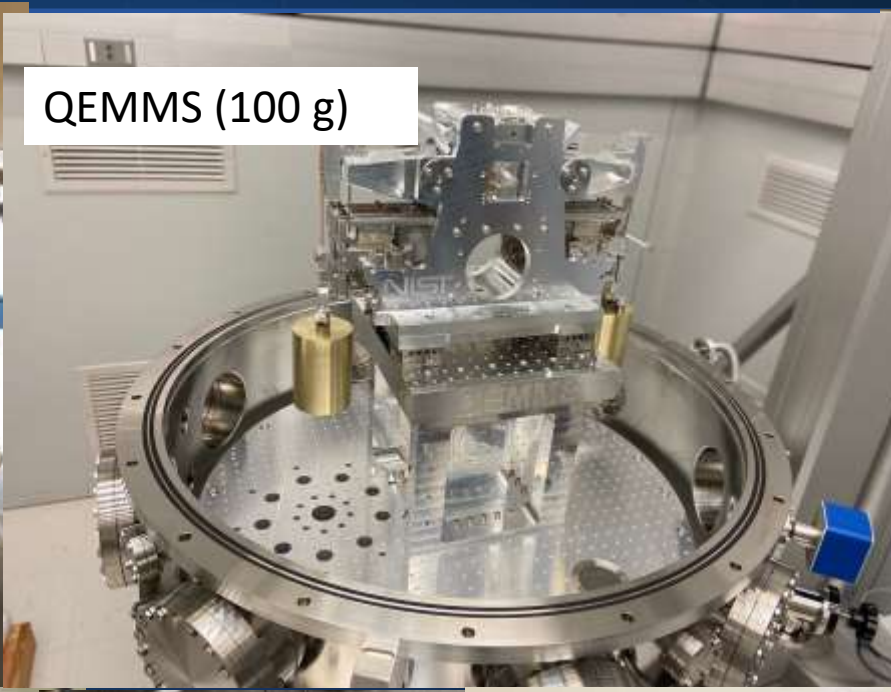
* Precision Electro-Mechanical Experiments.



NIST-4 (1 kg)



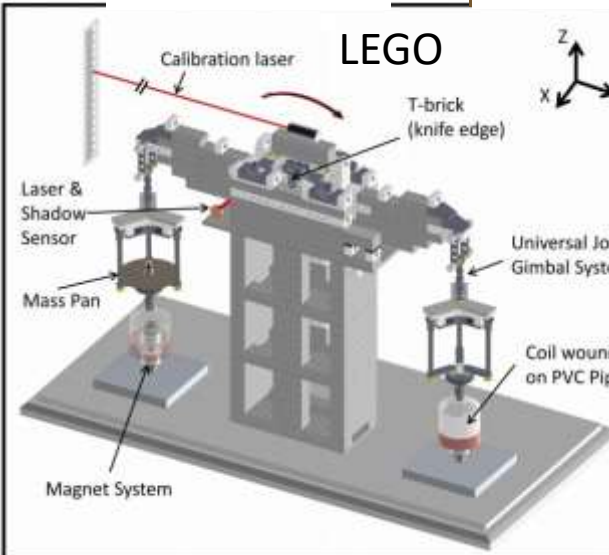
1:2 NIST-4 Model



QEMMS (100 g)



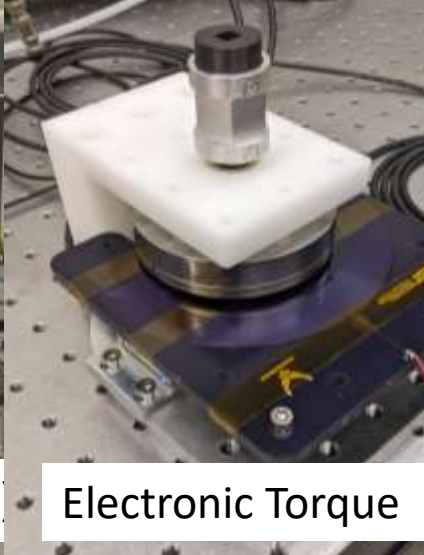
Measuring G



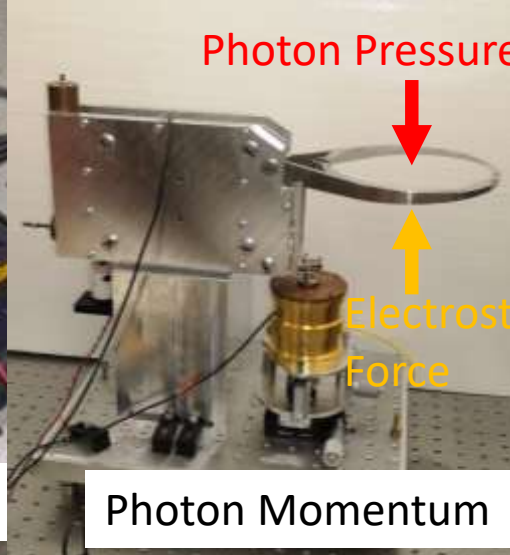
LEGO



KIBB-g1 (10 g)



Electronic Torque



Photon Momentum



Measurement process

Quantity

Unit

$$l = 1.2 \text{ m}$$

Numerical value

Detailed description: This diagram illustrates the components of a measurement. The equation $l = 1.2 \text{ m}$ is shown. A blue arrow labeled 'Quantity' points to the variable l . A green arrow labeled 'Unit' points to the unit 'm'. A red arrow labeled 'Numerical value' points to the number '1.2'.

$$l = \{l\}_m [\text{m}]$$

$$l = 4 \text{ ft}$$



System of units
(before 2019)

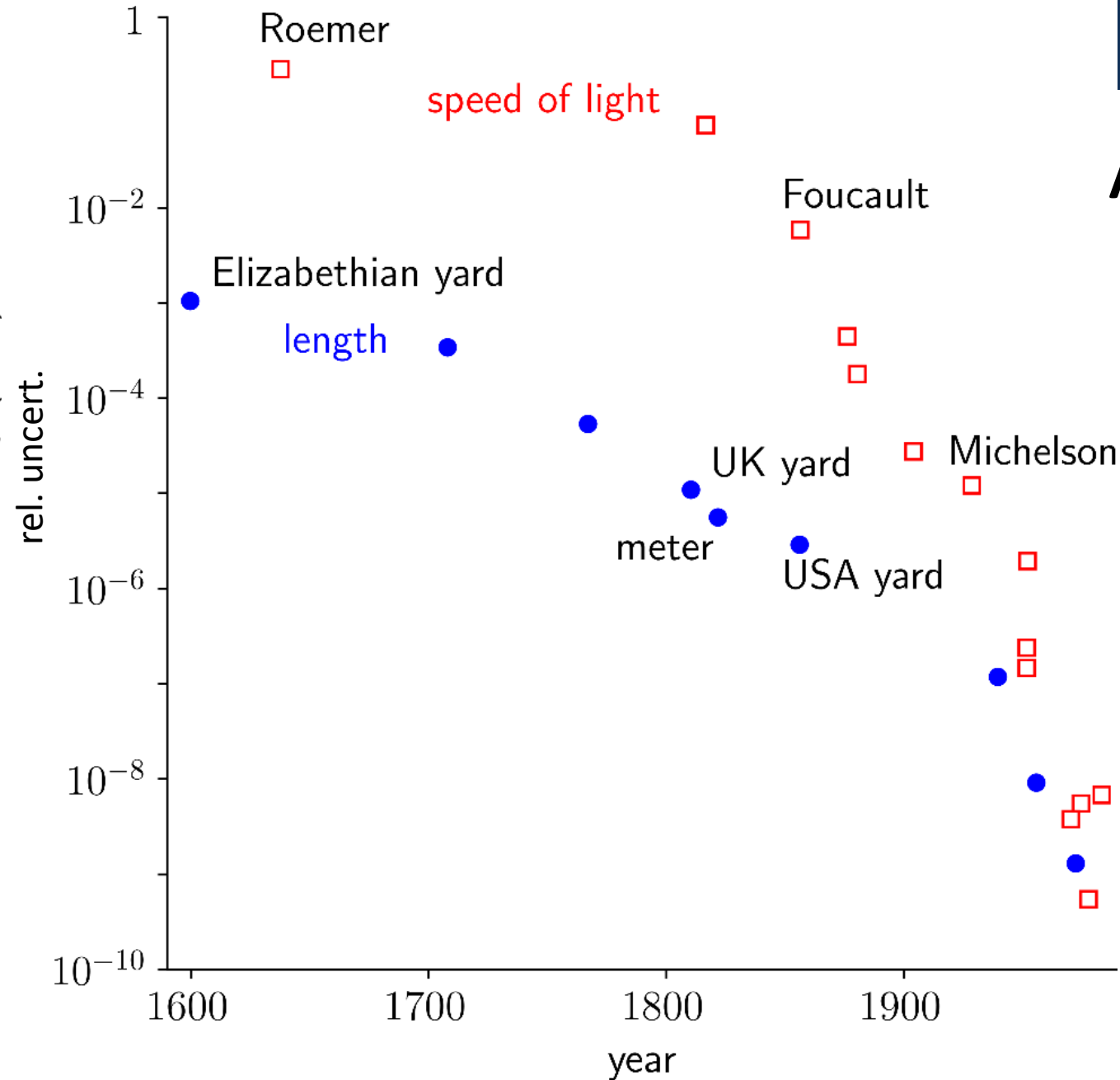
A top-down view of a kitchen counter covered in flour. In the top left, a white digital kitchen scale is visible. In the top right, three metal spoons are scattered. In the center, a large, rounded mound of flour sits. In the bottom center, a clear glass is partially filled with flour. The background is a light-colored, textured surface.

am·pere

That constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newtons per metre of length.



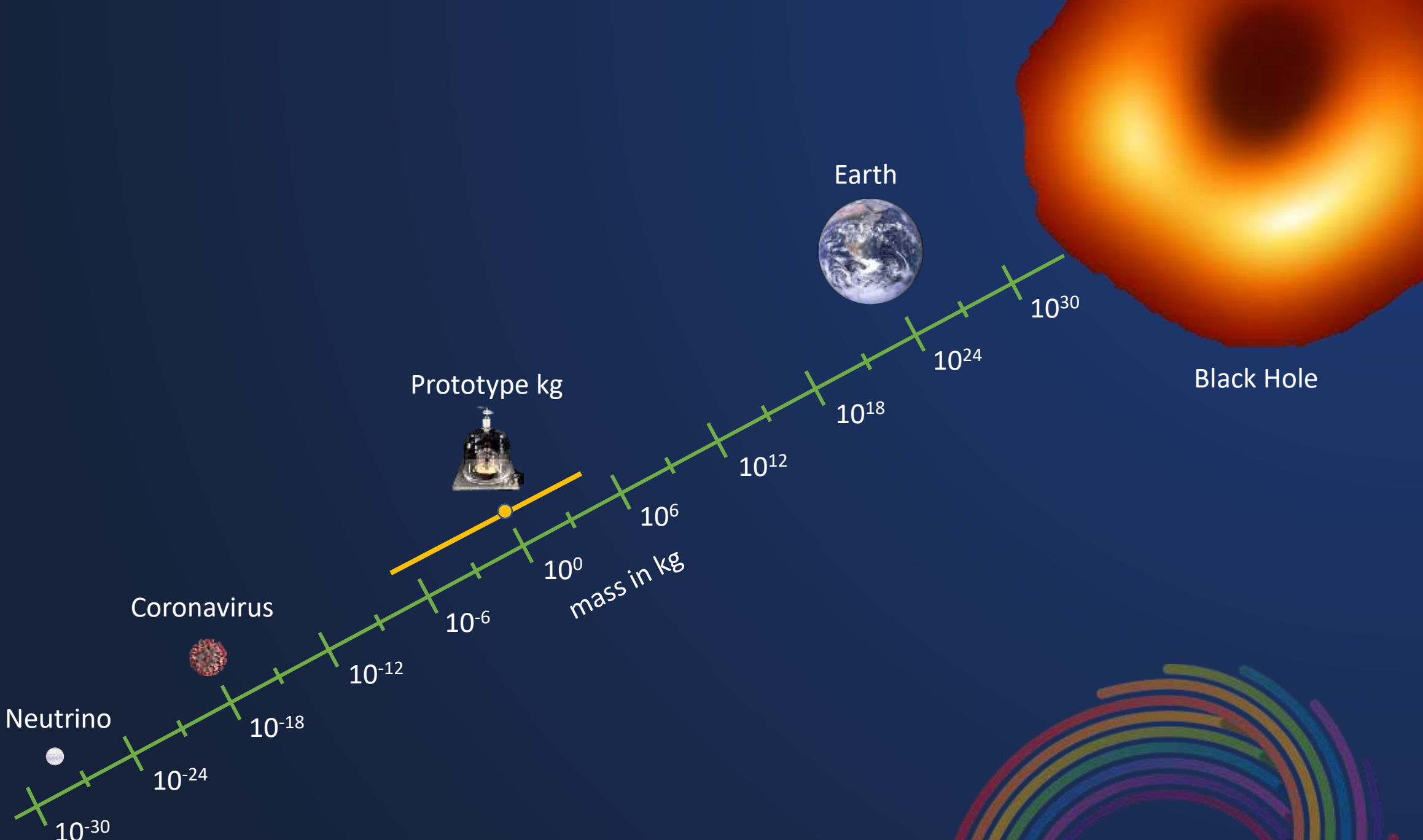
A unit system is dynamic



are:

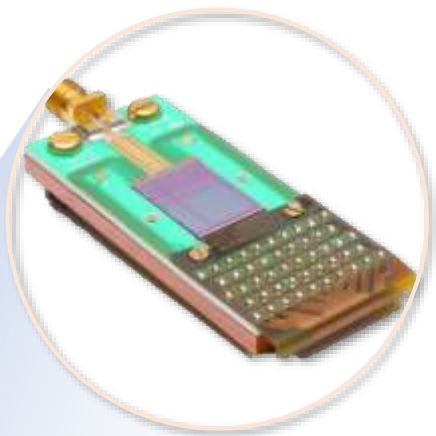
- scale invariant
- independent of space & time



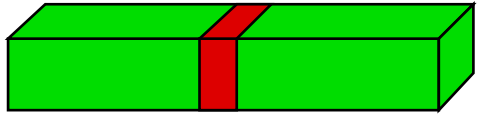




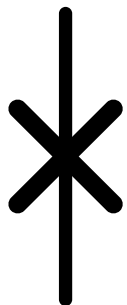
Josephson Voltage Standard



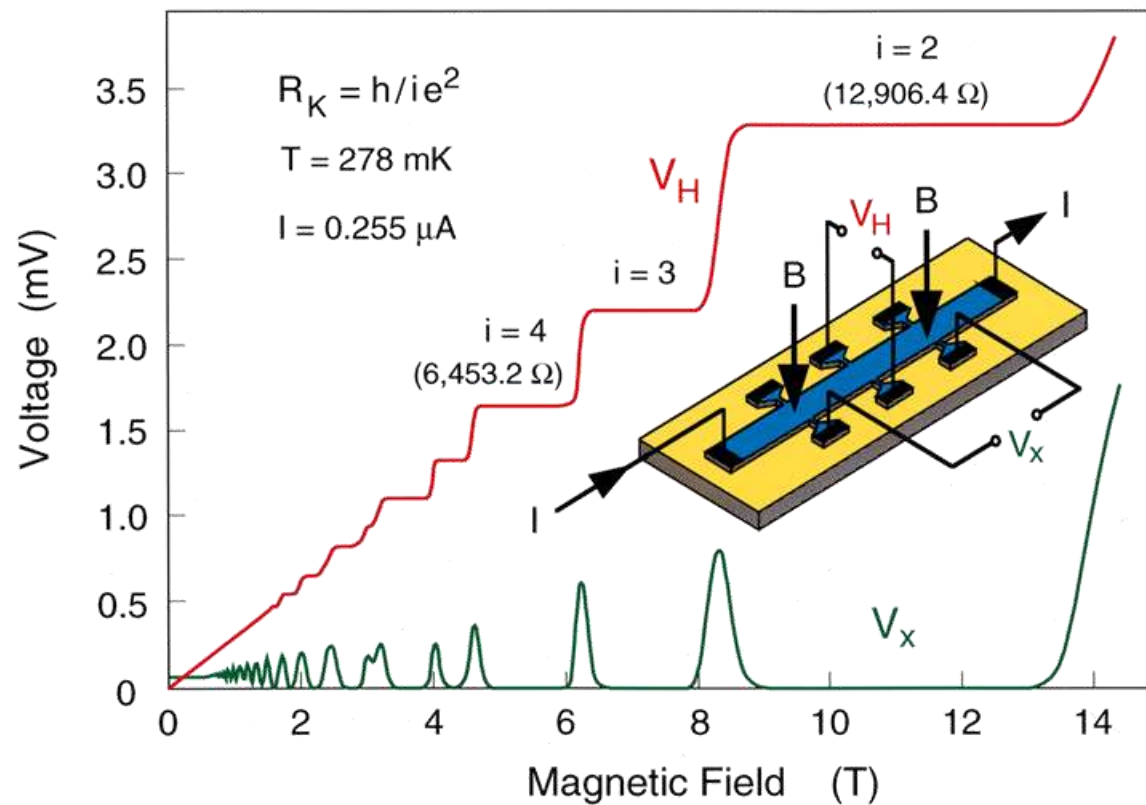
Quantum Hall Resistor

$$\Psi_1 = A_1 e^{i\theta_1} \quad \Psi_2 = A_2 e^{i\theta_2}$$


$$\phi = \theta_2 - \theta_1$$



$$V = \frac{h}{2e} f$$



$$R_H = \frac{V_H}{I} = \frac{B}{eN_s}$$

$$N_s = \left(\frac{eB}{h} \right) i$$

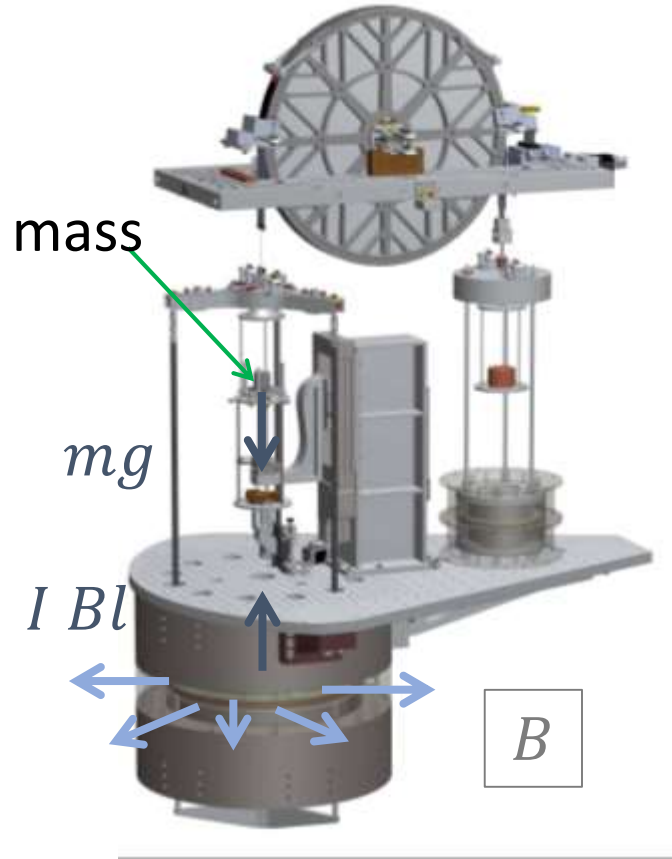
$$R_H = \left(\frac{h}{ie^2} \right)$$

$$P_{el} = V^2 R^{-1}$$



Watt balance – Kibble balance

Force mode



$$mg = I Bl$$

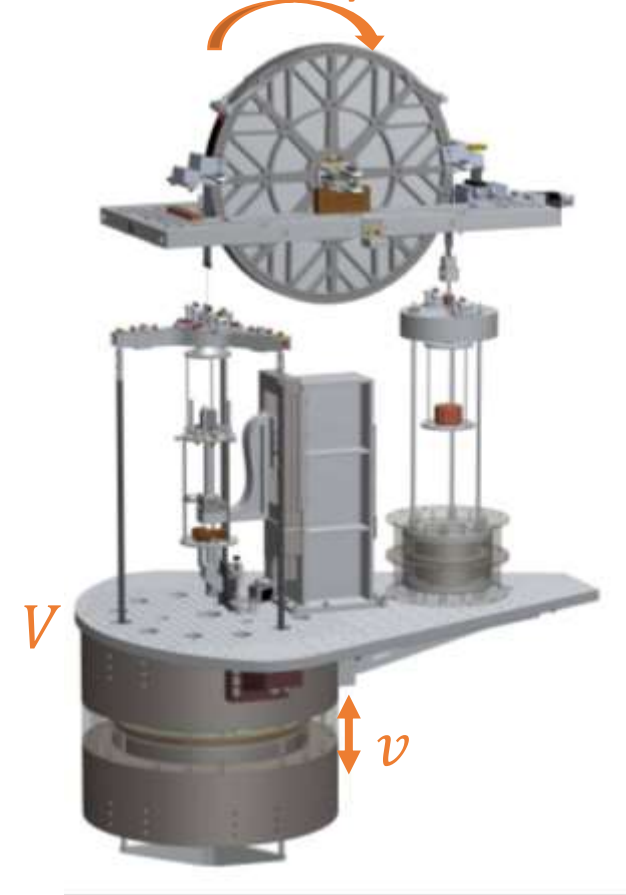
$$U = v Bl$$

$$\frac{mg}{U} = \frac{I}{v}$$

$$mgv = UI$$

$$mgv = \frac{n ni}{4} f_1 f_2 h$$

Velocity mode



balance wheel



mass pan



coil

motor



balance wheel

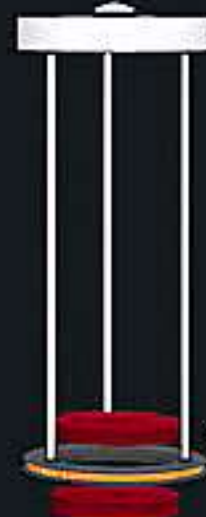


mass pan

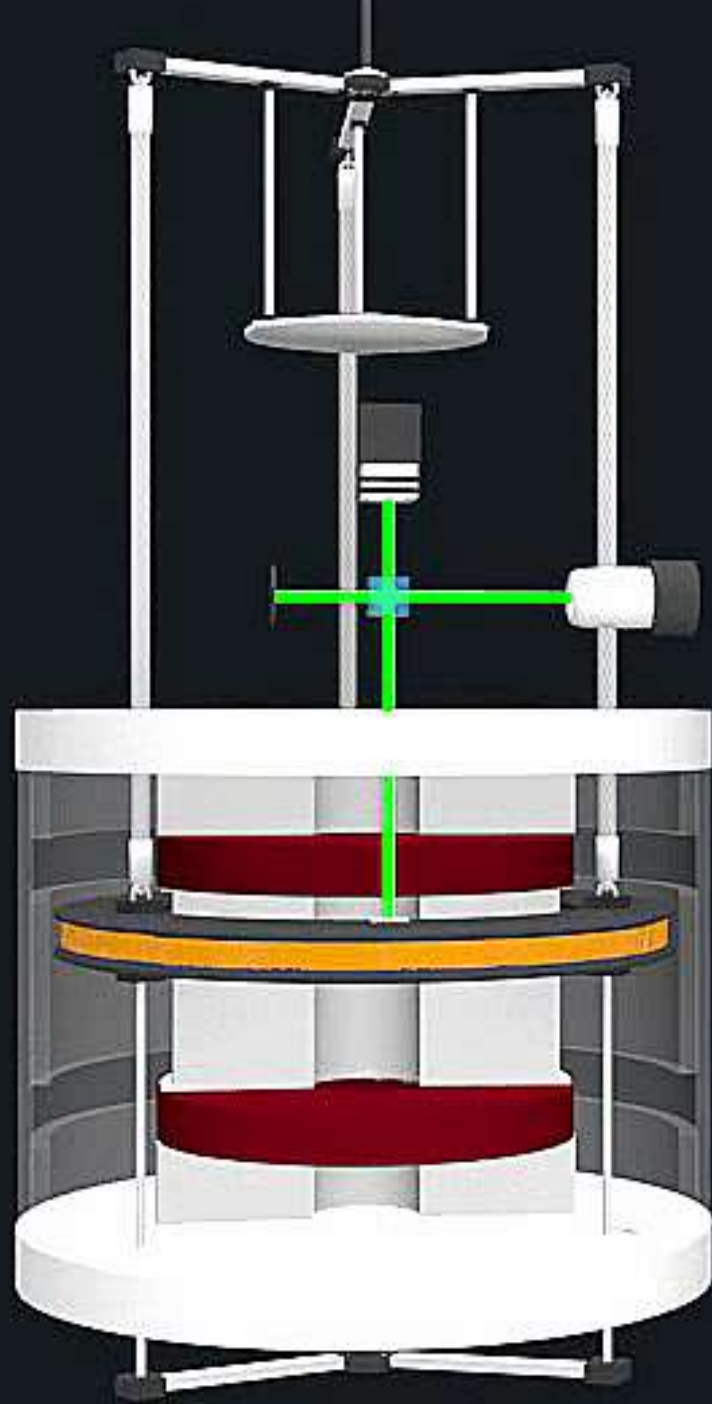


coil

motor

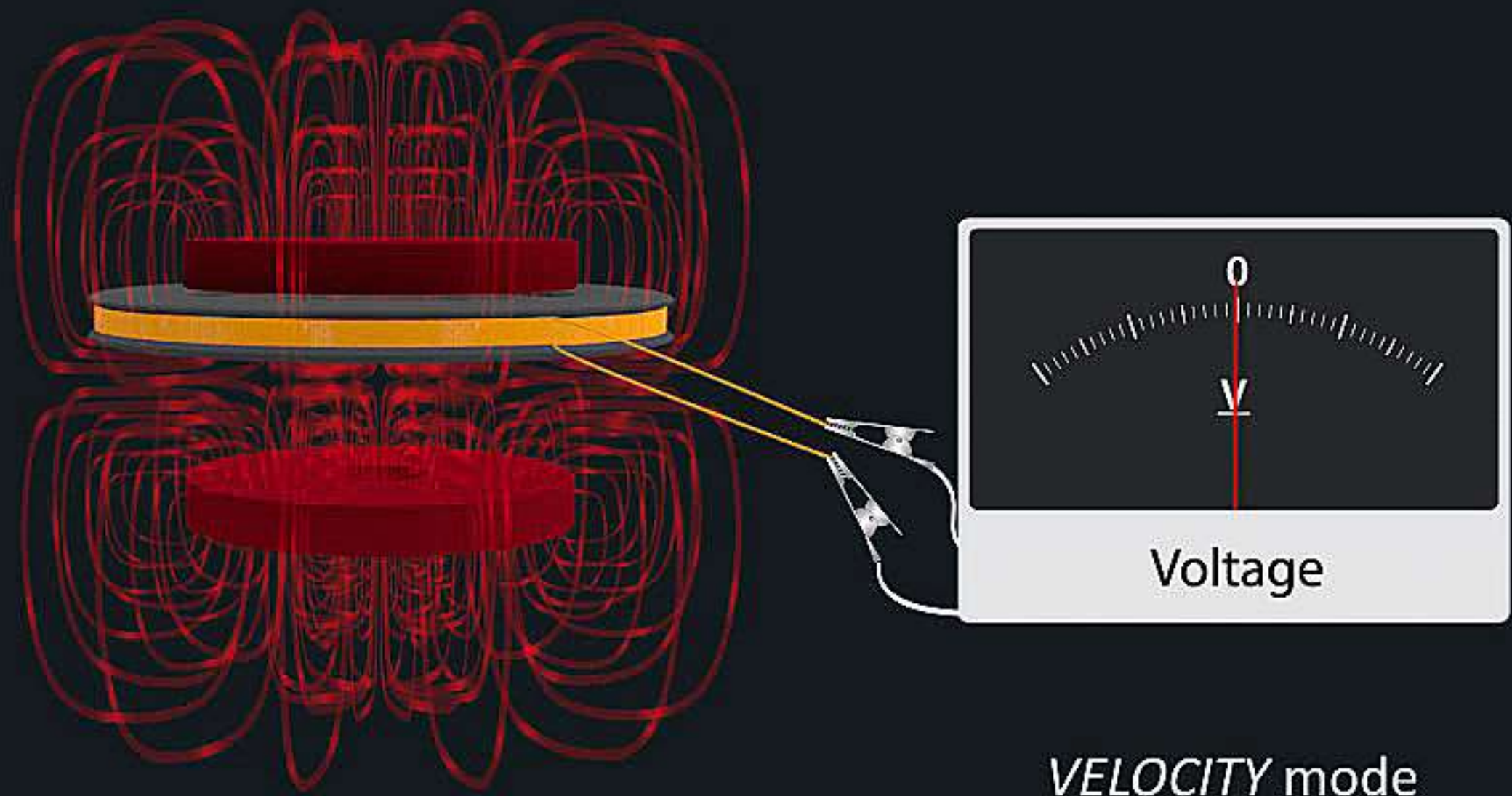


VELOCITY mode



Flow

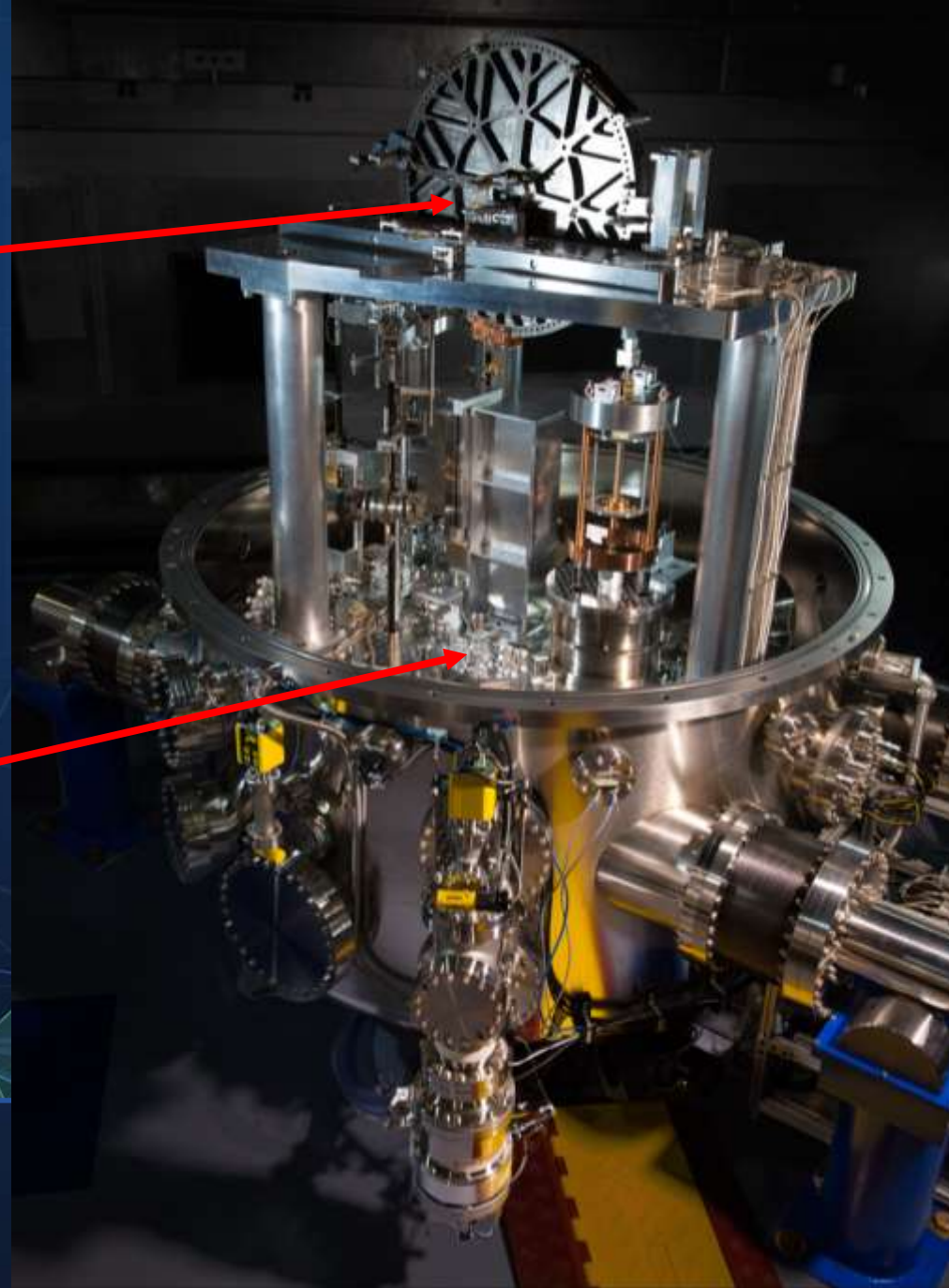
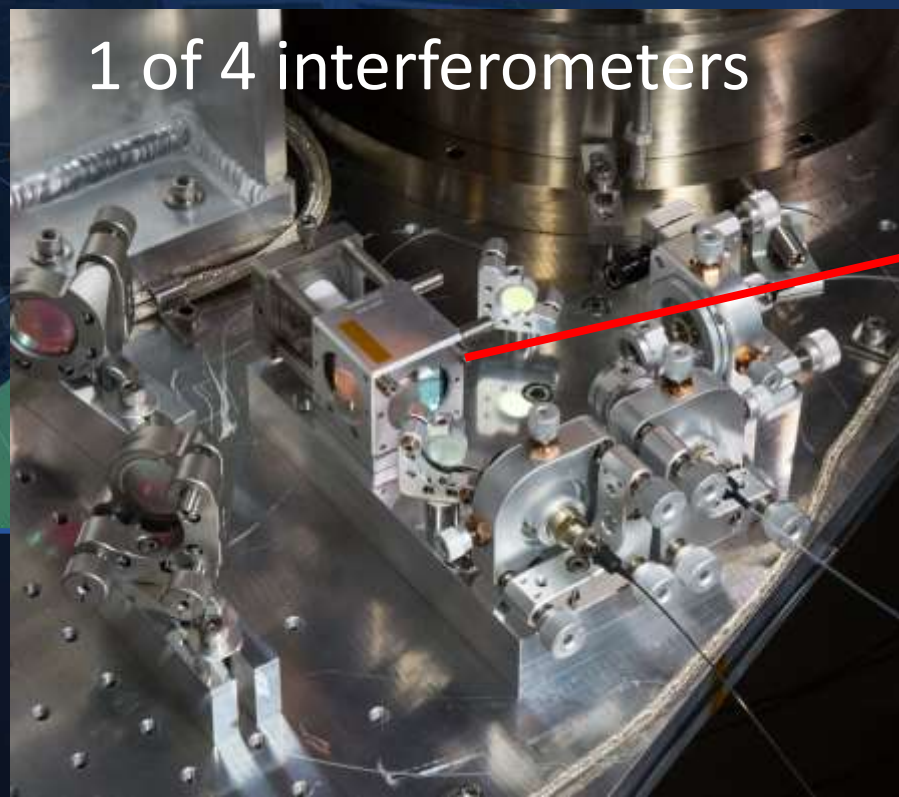


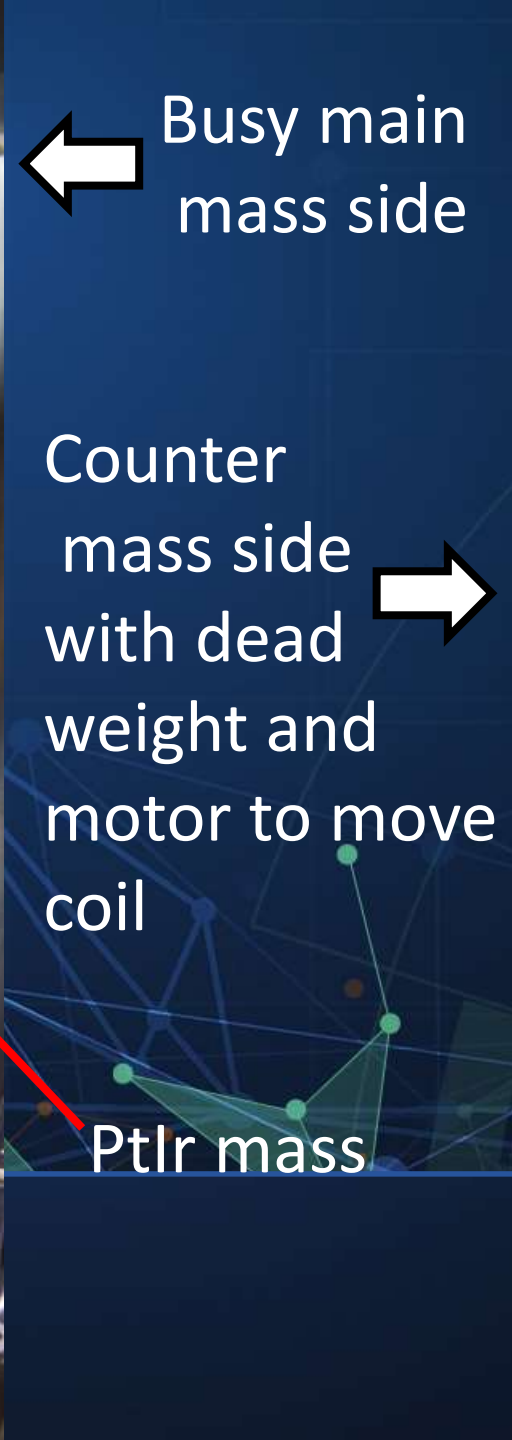
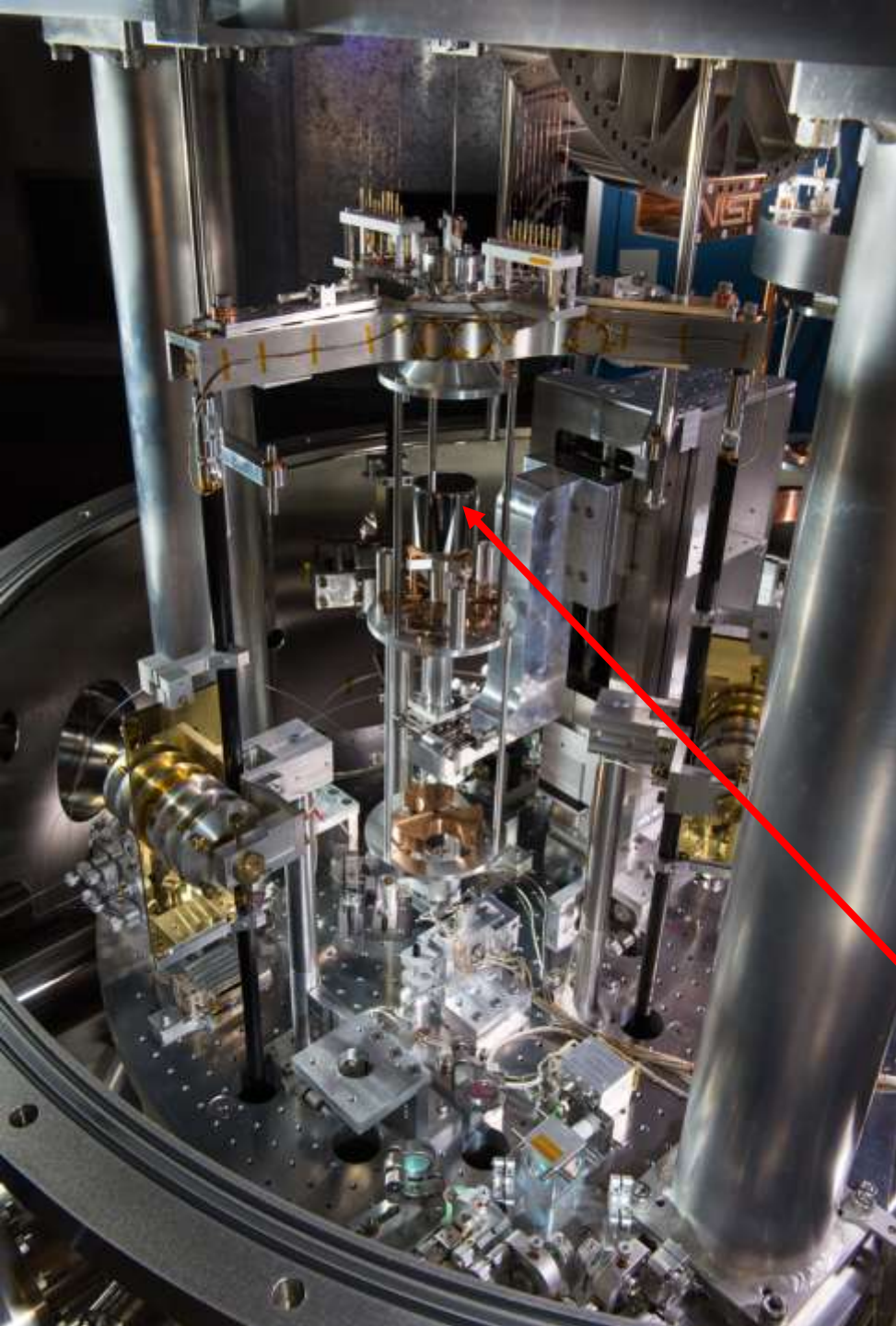


knife edge as a pivot



1 of 4 interferometers

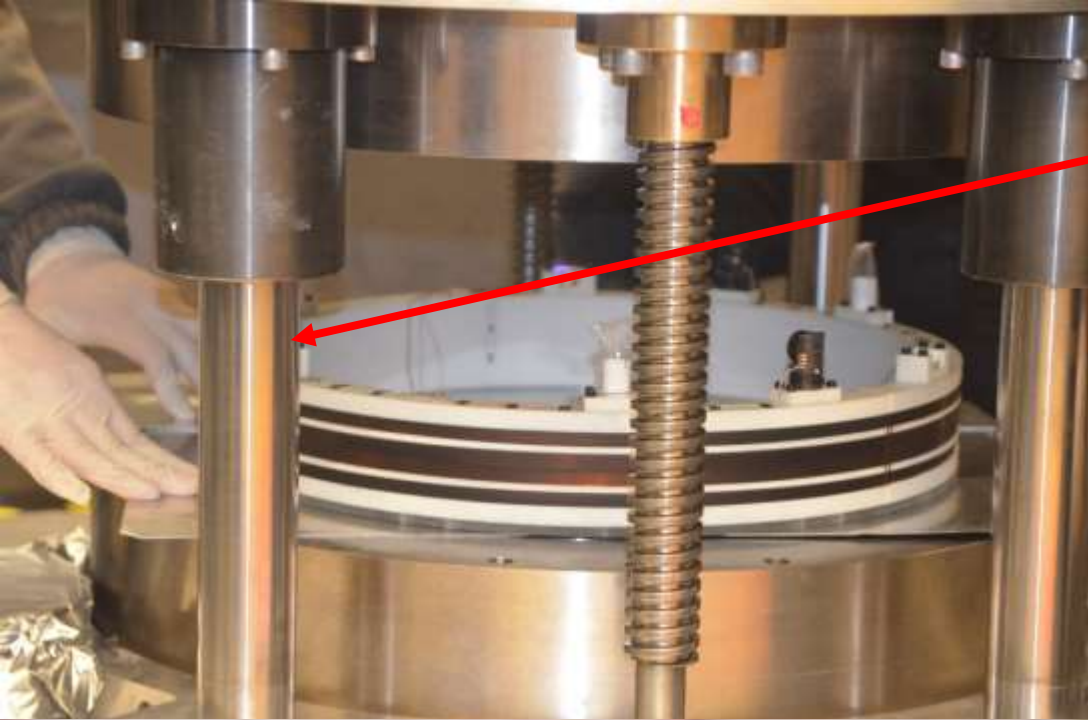




← Busy main mass side

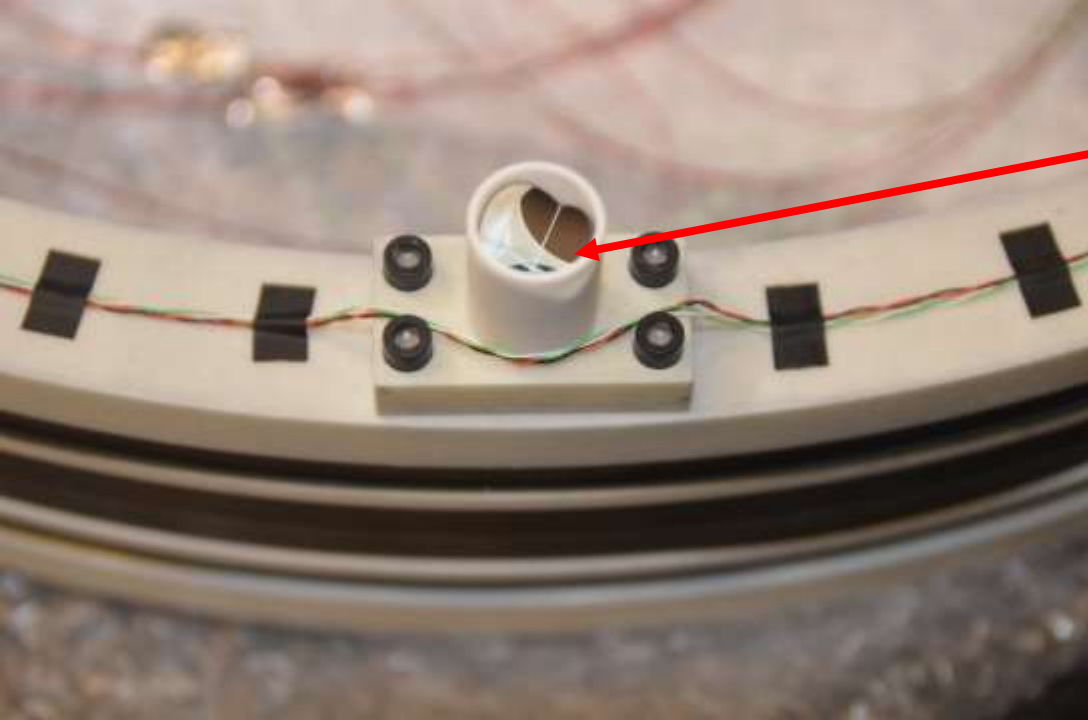
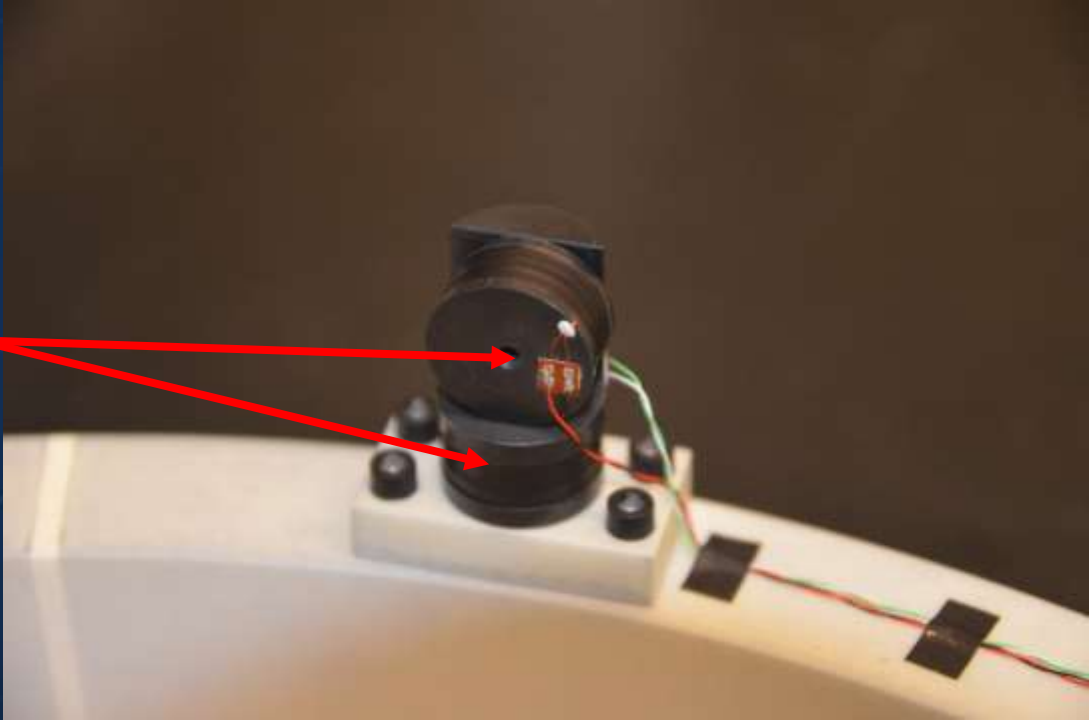
Counter mass side with dead weight and motor to move coil →

← PtIr mass



Installation
of the coil

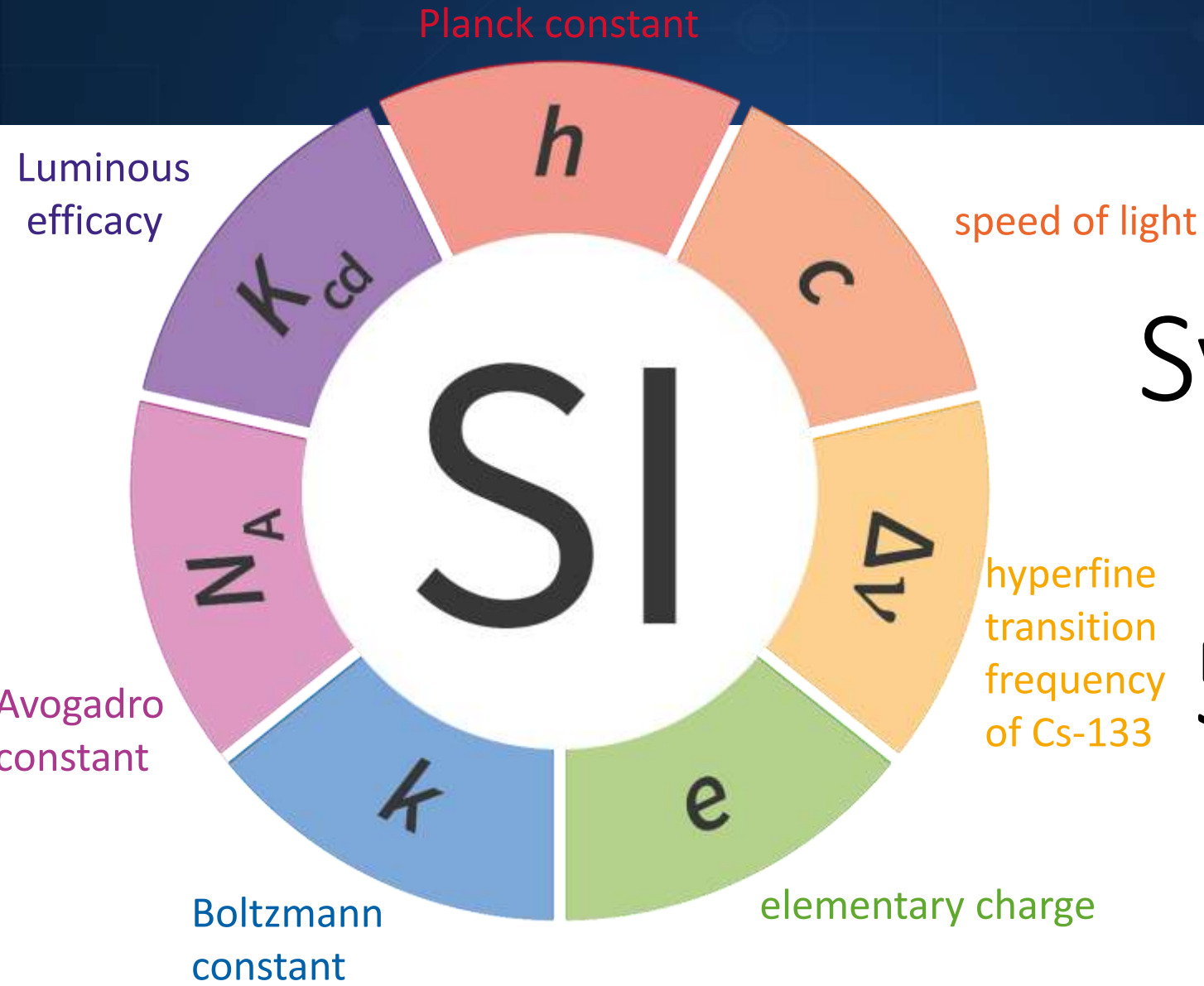
2 of 6 coils
for x/y and
 Θ_x/Θ_y
damping



1 of 3
corner
cubes for
inter-
ferometer



Coil inside
air gap



System of units
since
5/20/2019

A Tale of g and his brother

$$g = 9.797\,724\,52(4) \frac{\text{m}}{\text{s}^2}$$

$$G = 6.674\,30(15) \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}$$

Why?

measure an acceleration

measure a force

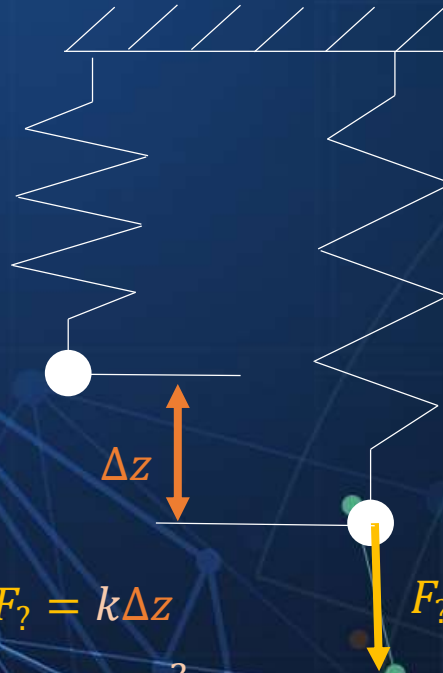
How to measure an unknown force

Measure the effect of the force

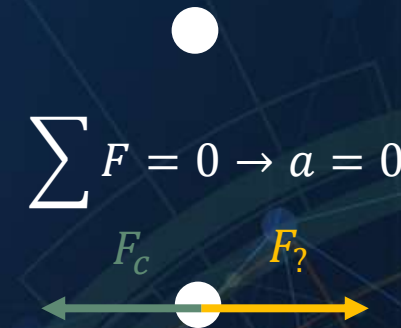


$$F_? = m a$$

measure a and infer $F_?$



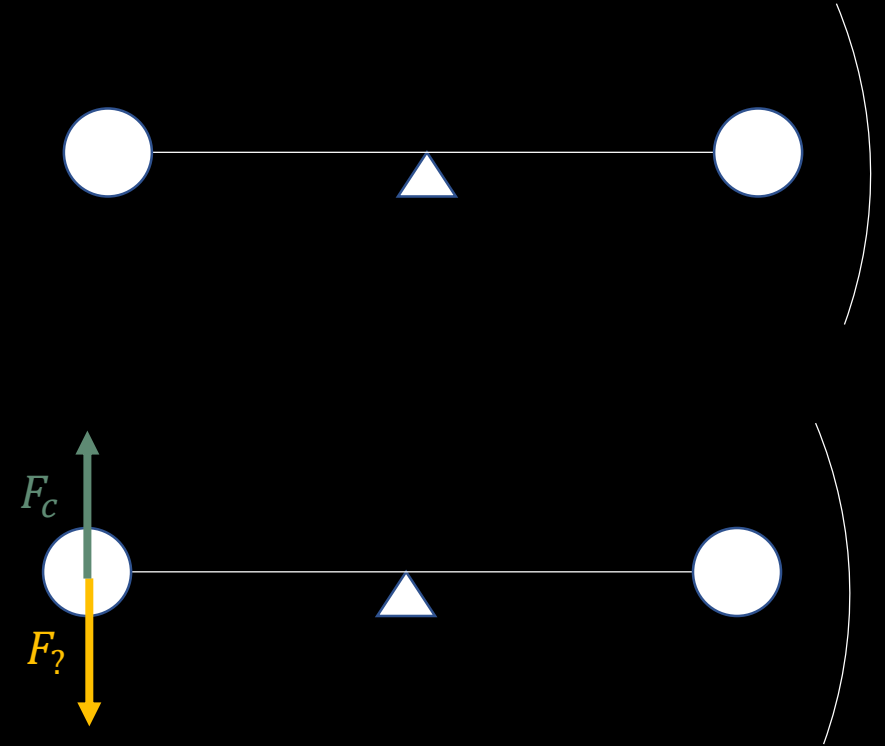
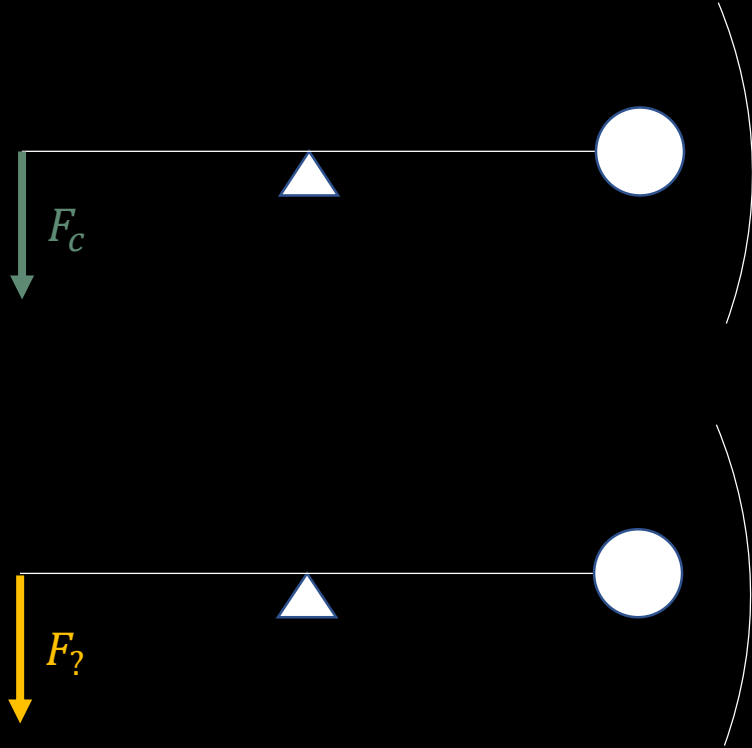
Compensate the force with a known force F_c



$$\sum F = 0 \rightarrow a = 0$$

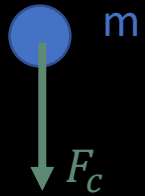
$$a = 0 \rightarrow F_? = F_c$$

Successive or simultaneous F_c



Three options for F_c

gravitational



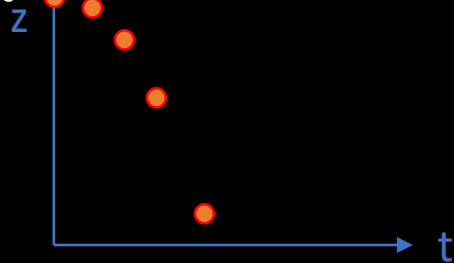
$$F_c = mg$$

calibration phase

determine m

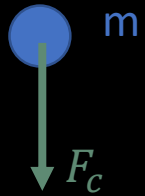
g can be looked up for $\frac{\sigma_g}{g} > 2 \times 10^{-6}$

For smaller uncertainty => measurement



Three options for F_c

gravitational



$$F_c = mg$$

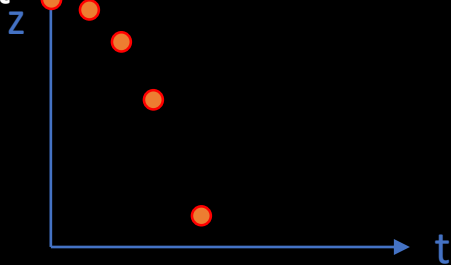
calibration phase

determine m

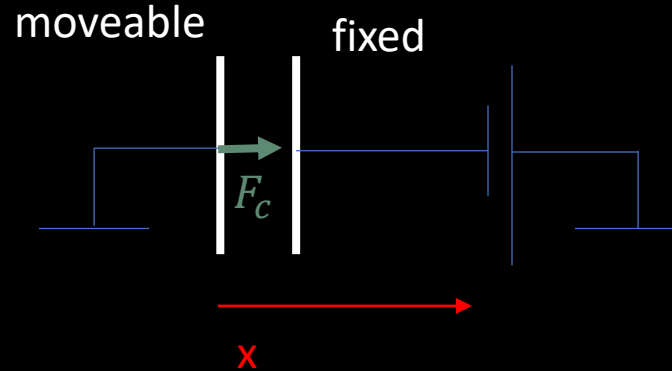
g can be looked up for $\frac{\sigma_g}{g} > 2 \times 10^{-6}$

For smaller uncertainty =>

measurement



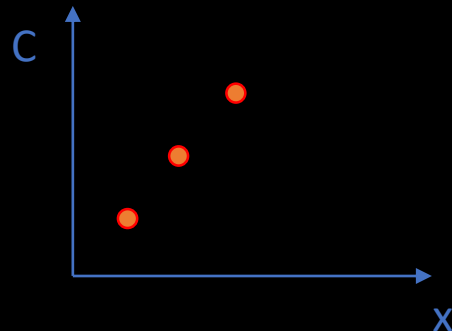
electrostatic



$$F_c = \frac{1}{2} V^2 \frac{dC}{dx}$$

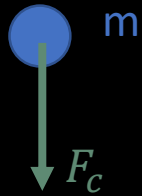
calibration phase

measure $C(x) \rightarrow dC/dx$



Three options for F_c

gravitational



$$F_c = mg$$

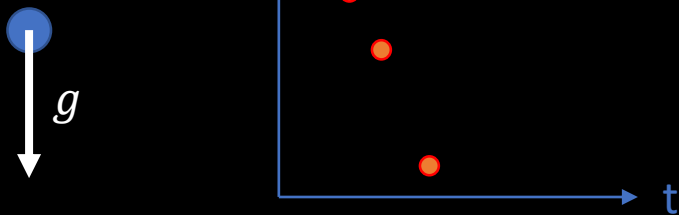
calibration phase

determine m

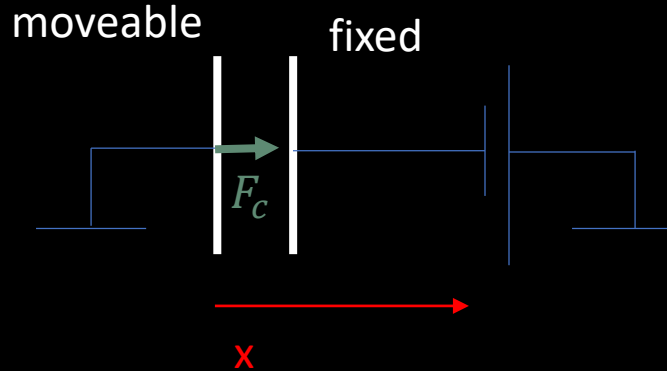
g can be looked up for $\frac{\sigma_g}{g} > 2 \times 10^{-6}$

For smaller uncertainty =>

measurement



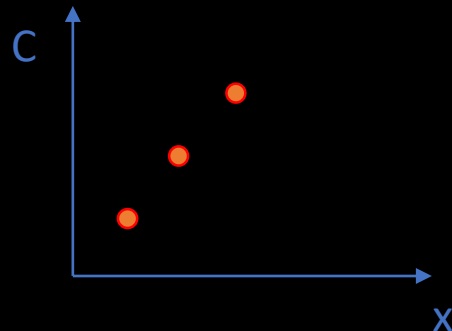
electrostatic



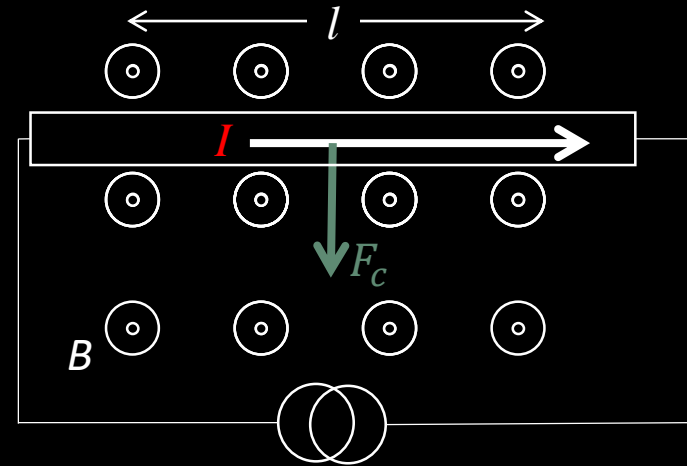
$$F_c = \frac{1}{2} V^2 \frac{dC}{dx}$$

calibration phase

measure $C(x) \rightarrow dC/dx$

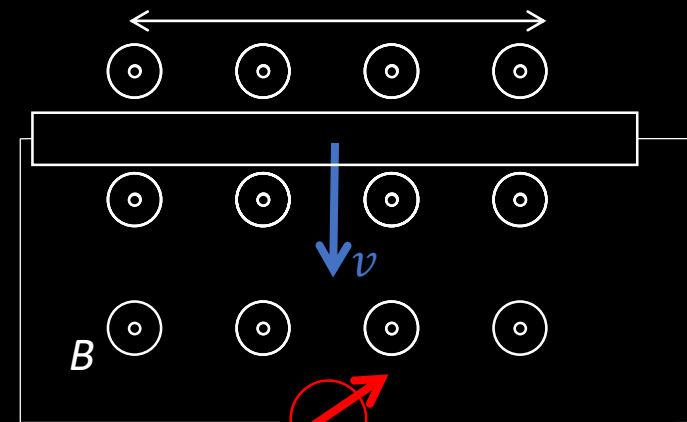


electromagnetic



$$F_c = IBl$$

calibration phase



$$U = vBl$$

Traceability

gravitational

works for forces

$$10^{-5}N \leq F_c \leq 4.5 \times 10^6N$$



Photo: J. Lee/NIST

electrostatic

Is traceable to fundamental constants via calculable capacitor or AC QHR

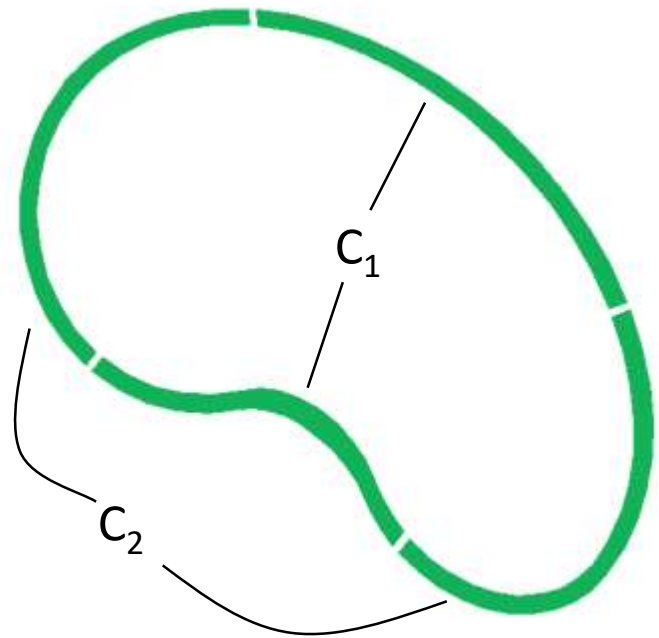


electromagnetic

Is traceable to fundamental constants via Quantum Hall Effect and Josephson effect.

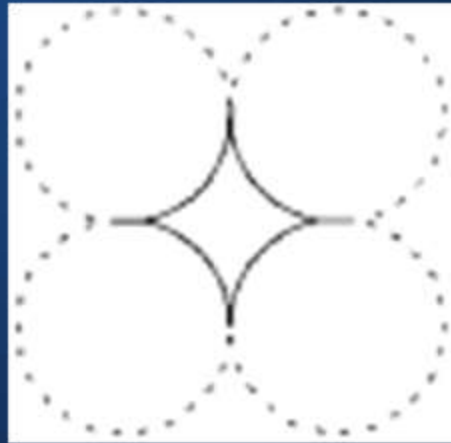


Calculable Capacitor?

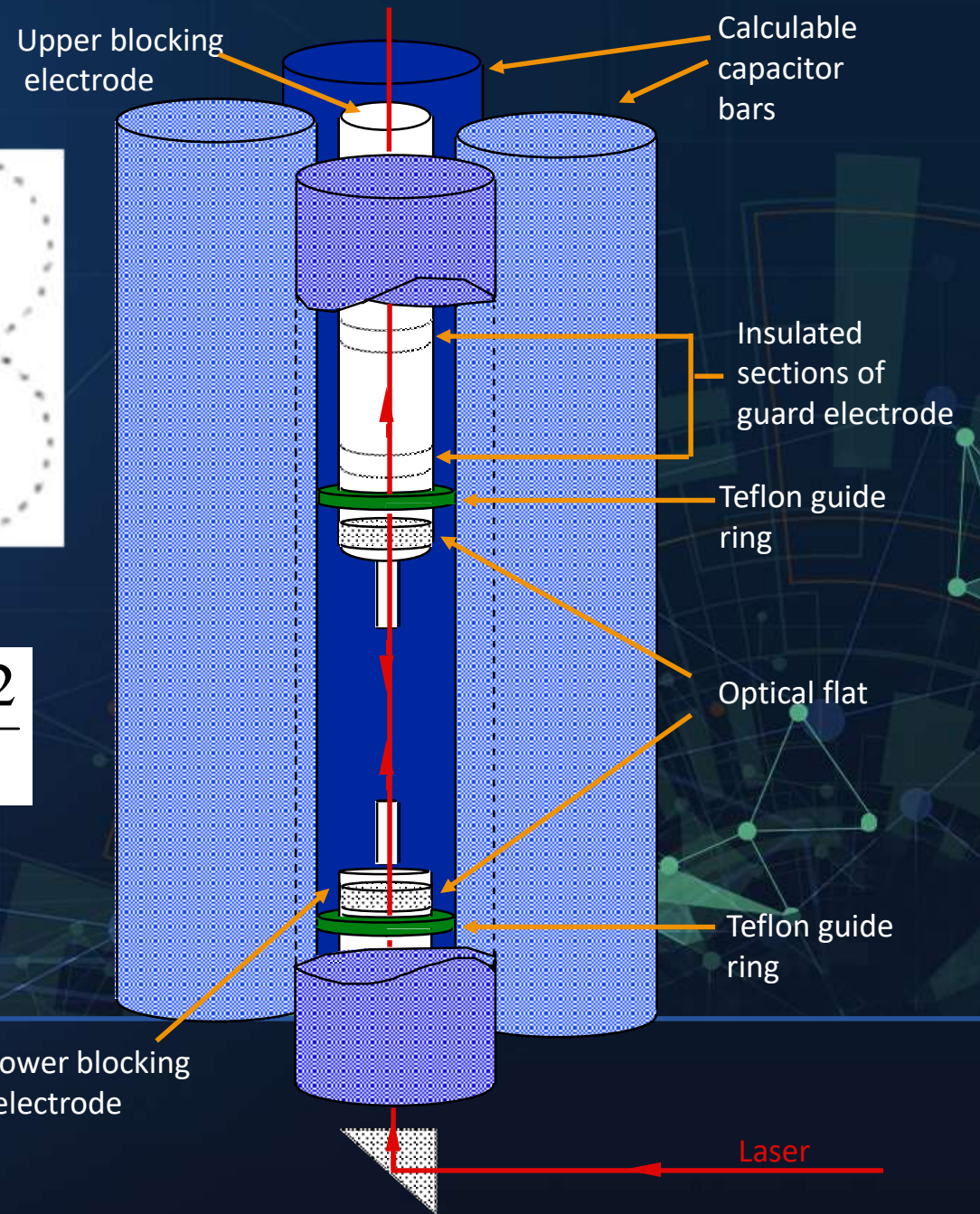


$$\exp\left(\frac{-\pi C_1}{\epsilon_0 L}\right) + \exp\left(\frac{-\pi C_2}{\epsilon_0 L}\right) = 1$$

A. Thomson, D. Lampard, Nature 177, 88 (1956).



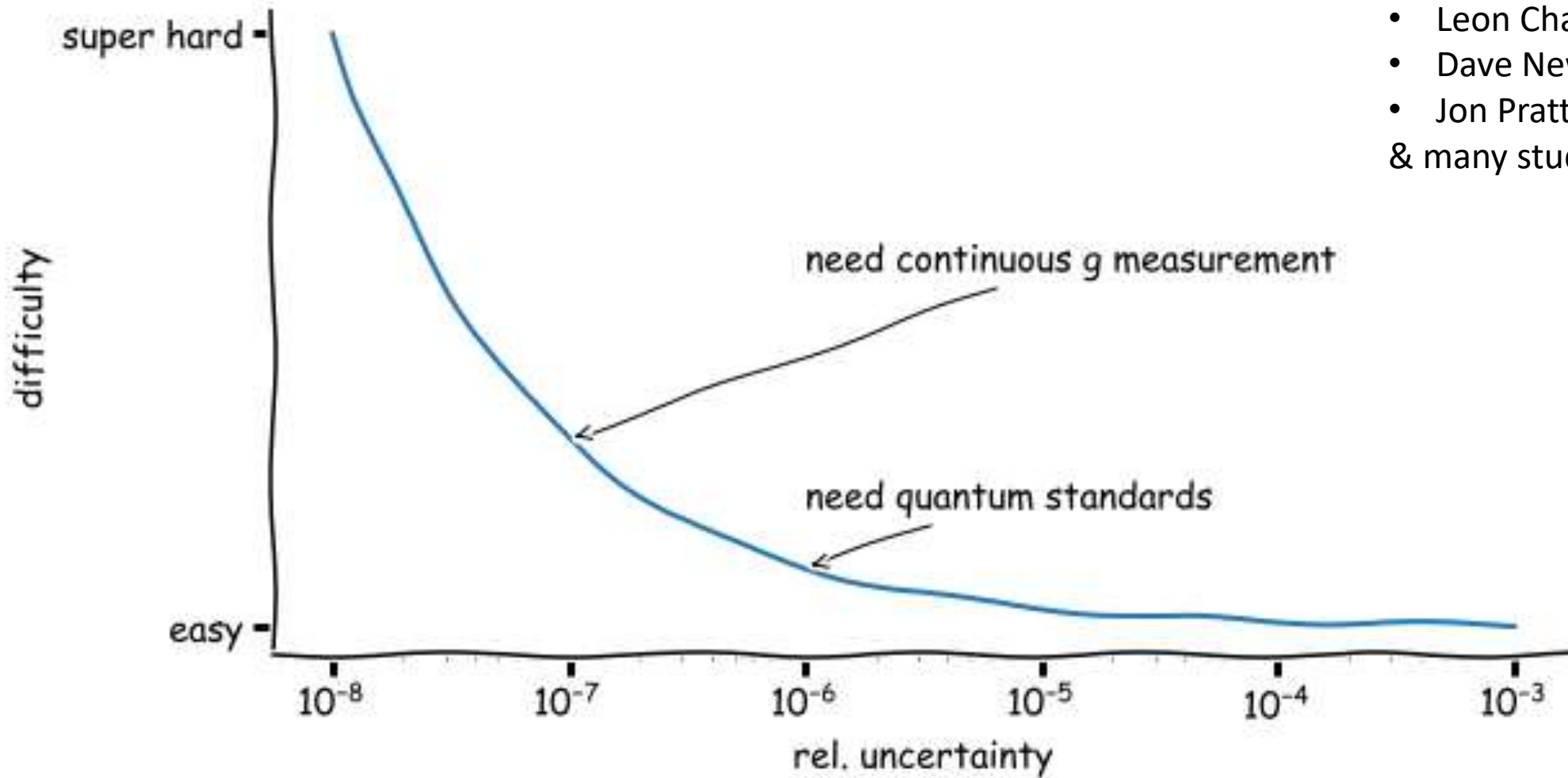
$$\frac{C_o}{L} = \epsilon_0 \frac{\ln 2}{\pi}$$

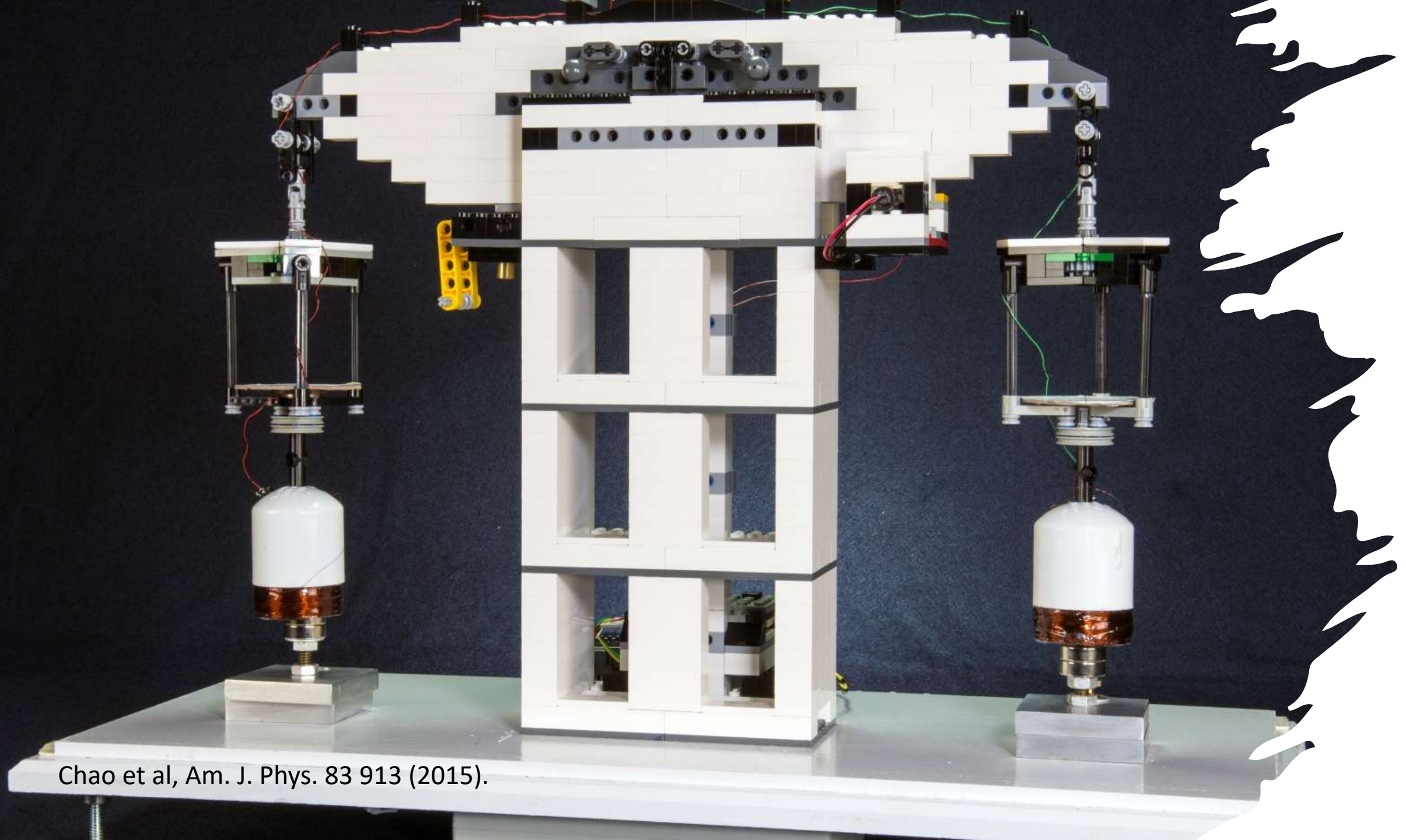


Level of difficulty building a KB

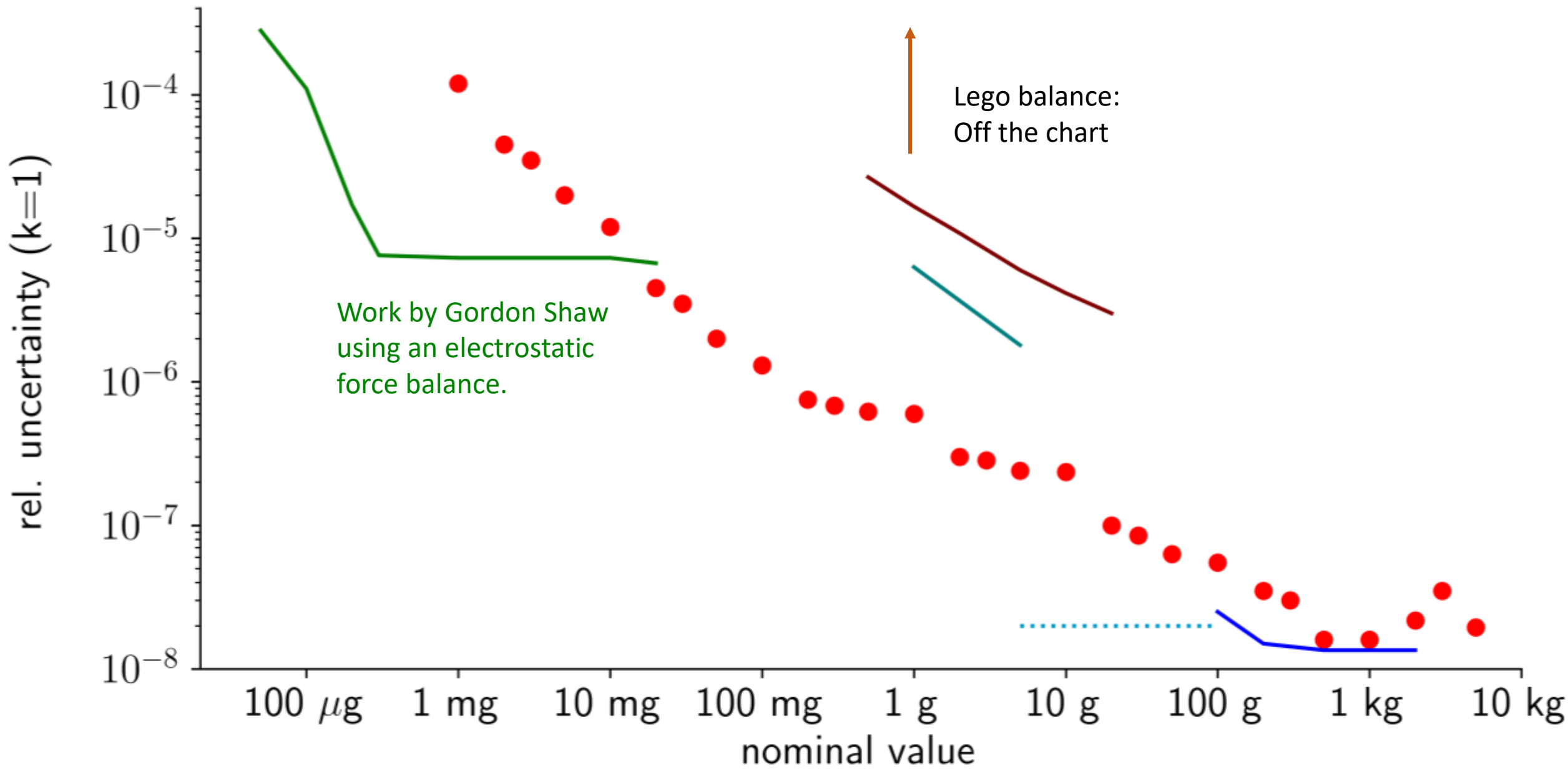
Kibble balance work with

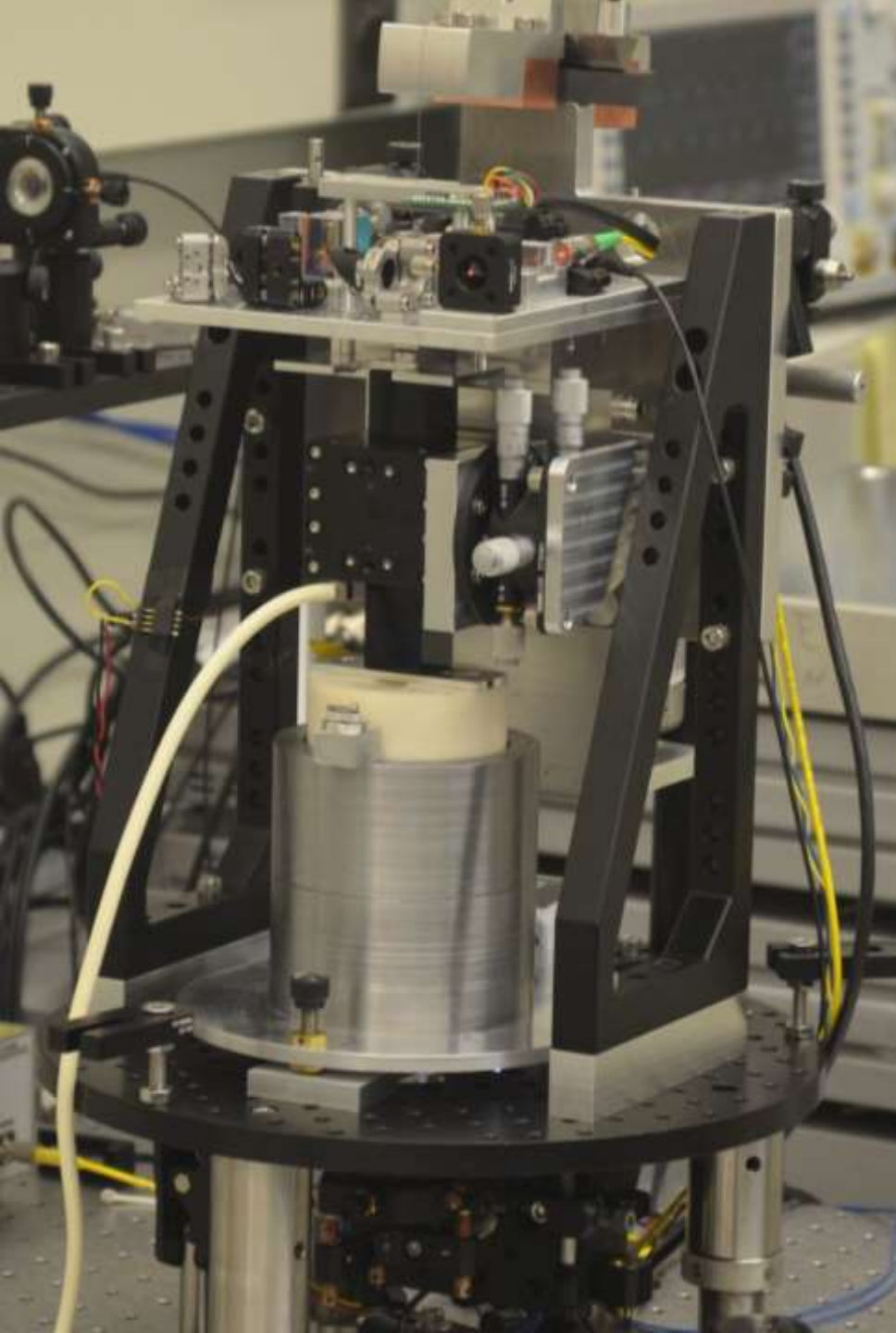
- Darine Haddad
 - Frank Seifert
 - Leon Chao
 - Dave Newell
 - Jon Pratt
- & many students



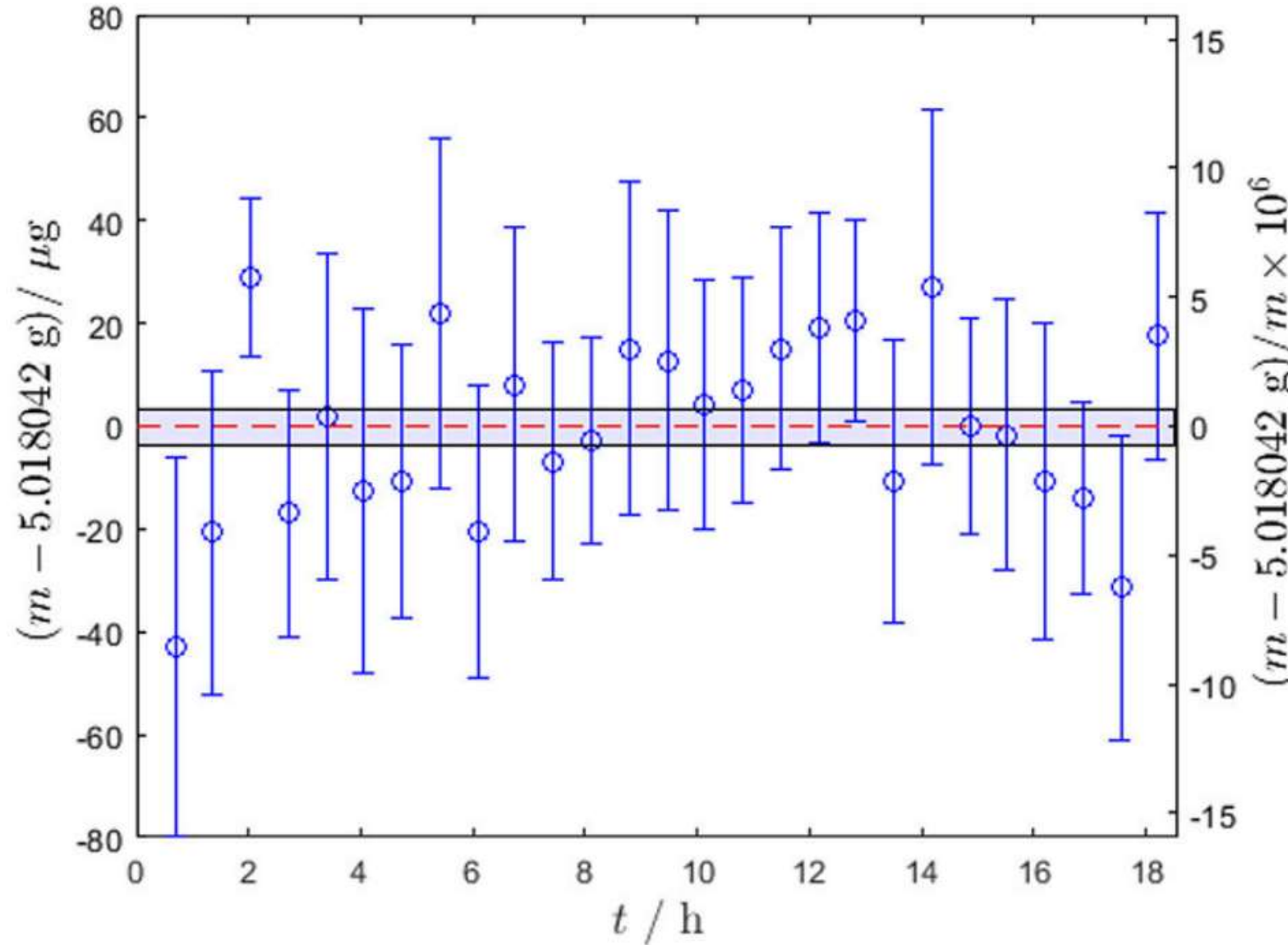


Chao et al, Am. J. Phys. 83 913 (2015).

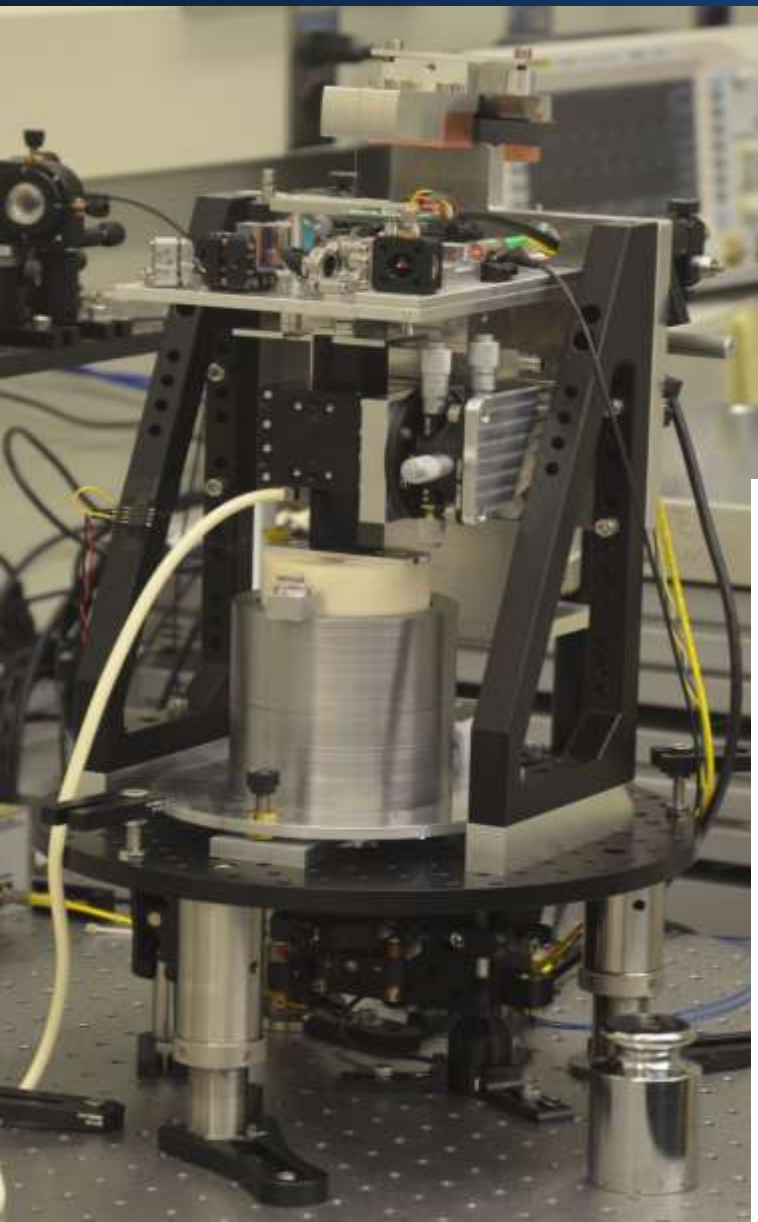




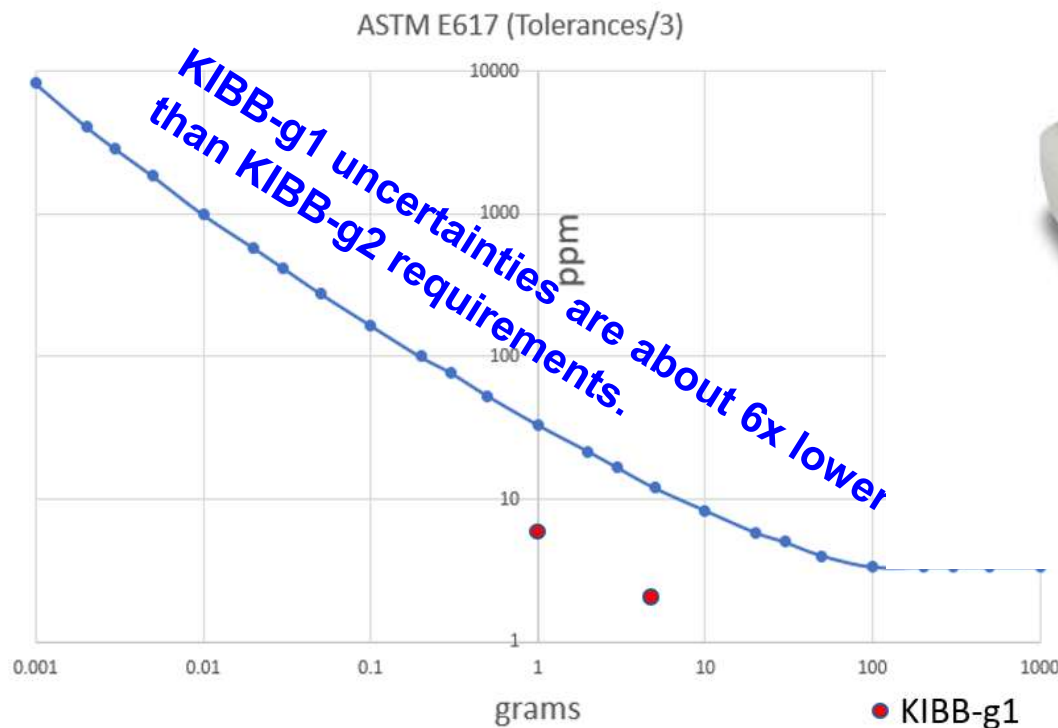
for $1g \frac{\sigma_m}{m} = 1.8 \times 10^{-6}$ for $5g \frac{\sigma_m}{m} = 6.3 \times 10^{-6}$



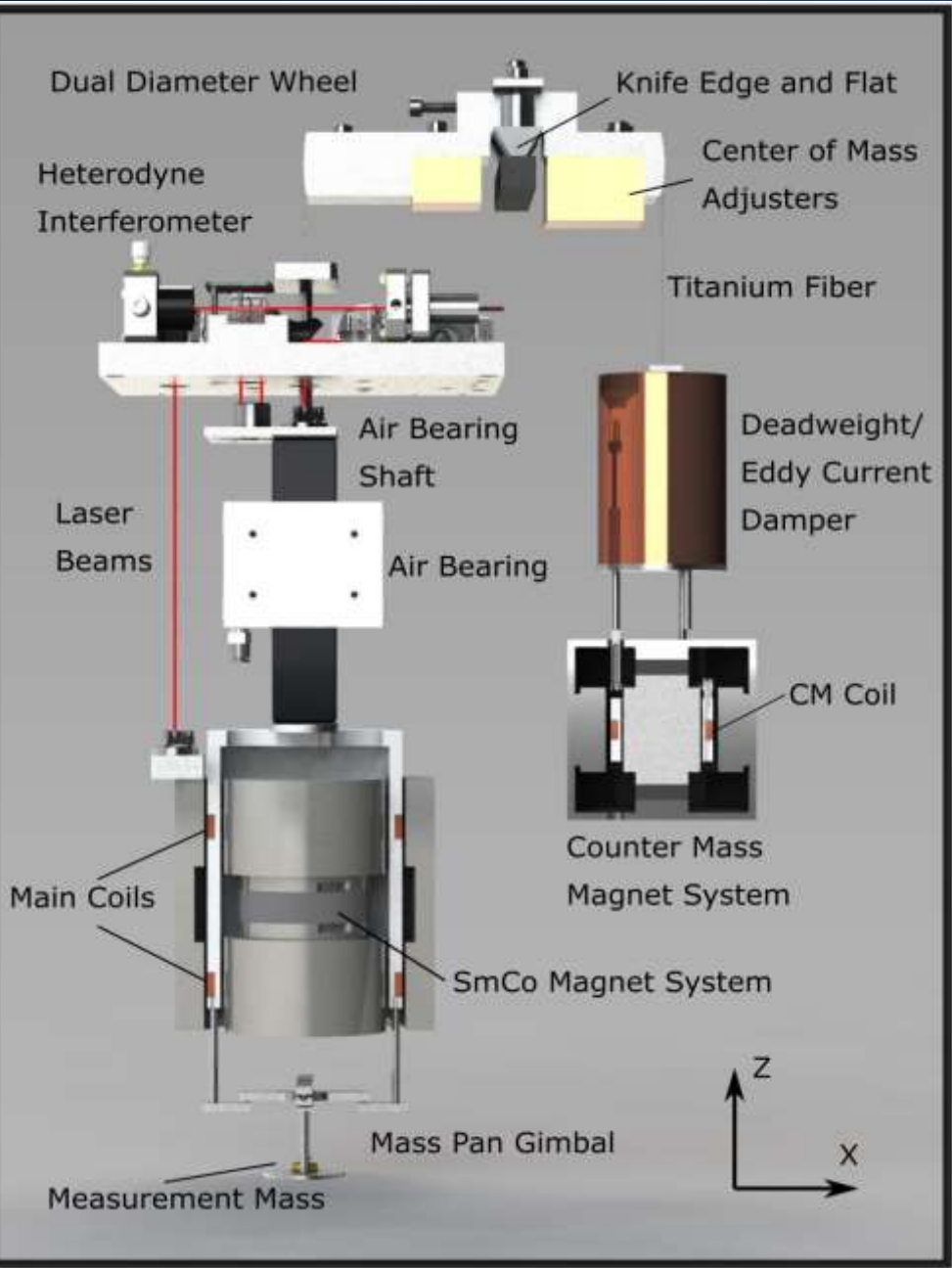
Next Generation Tabletop Balance



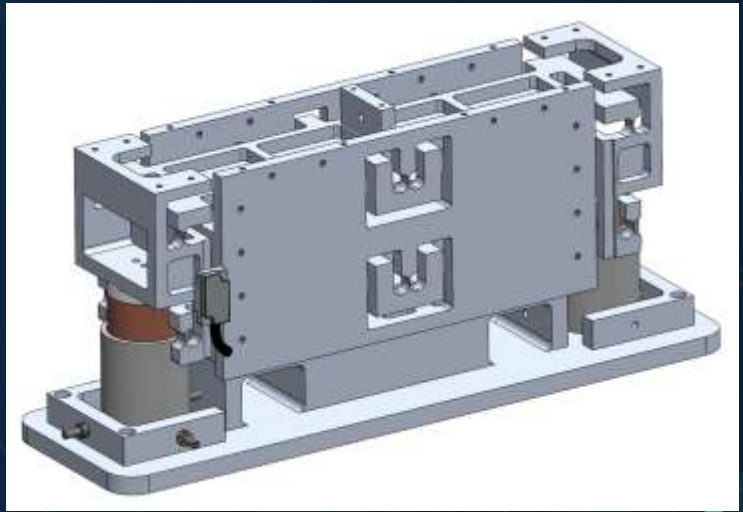
KIBB-g2 SOW: To design and construct two identical pushbutton tabletop Kibble balance systems capable of directly realizing [500 mg – 20 g] mass artifacts with uncertainties less than 1/3 the Class 3 Tolerance as defined in ASTM E617 within three calendar years.



12"



7"

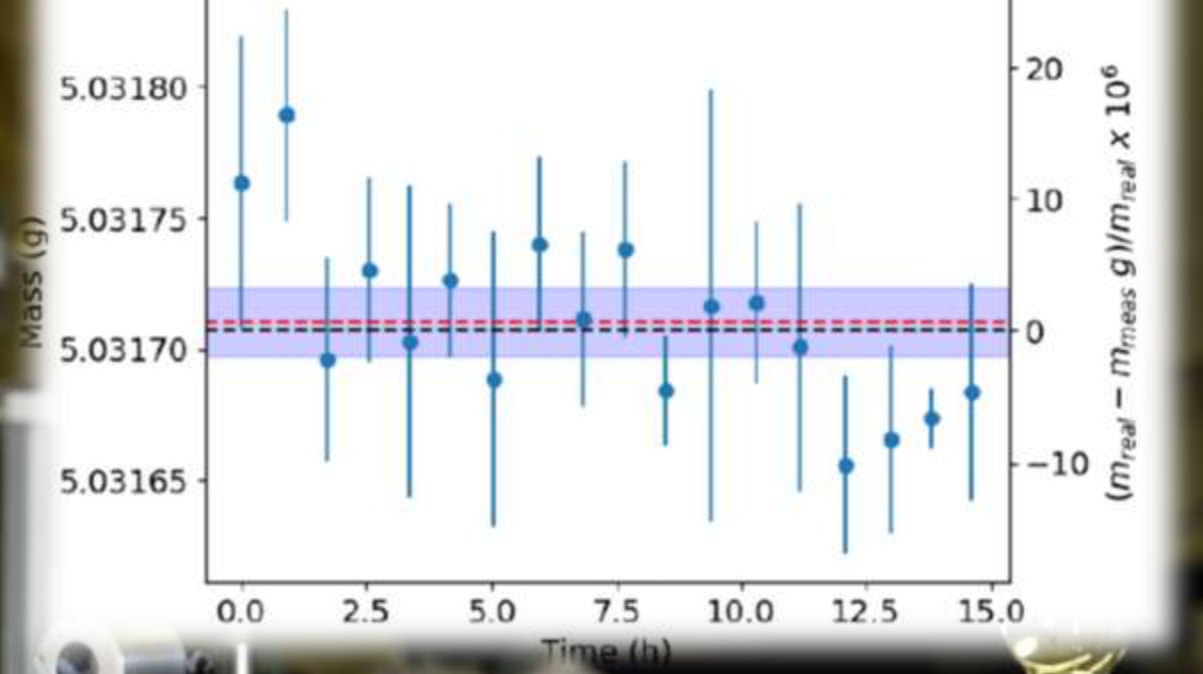
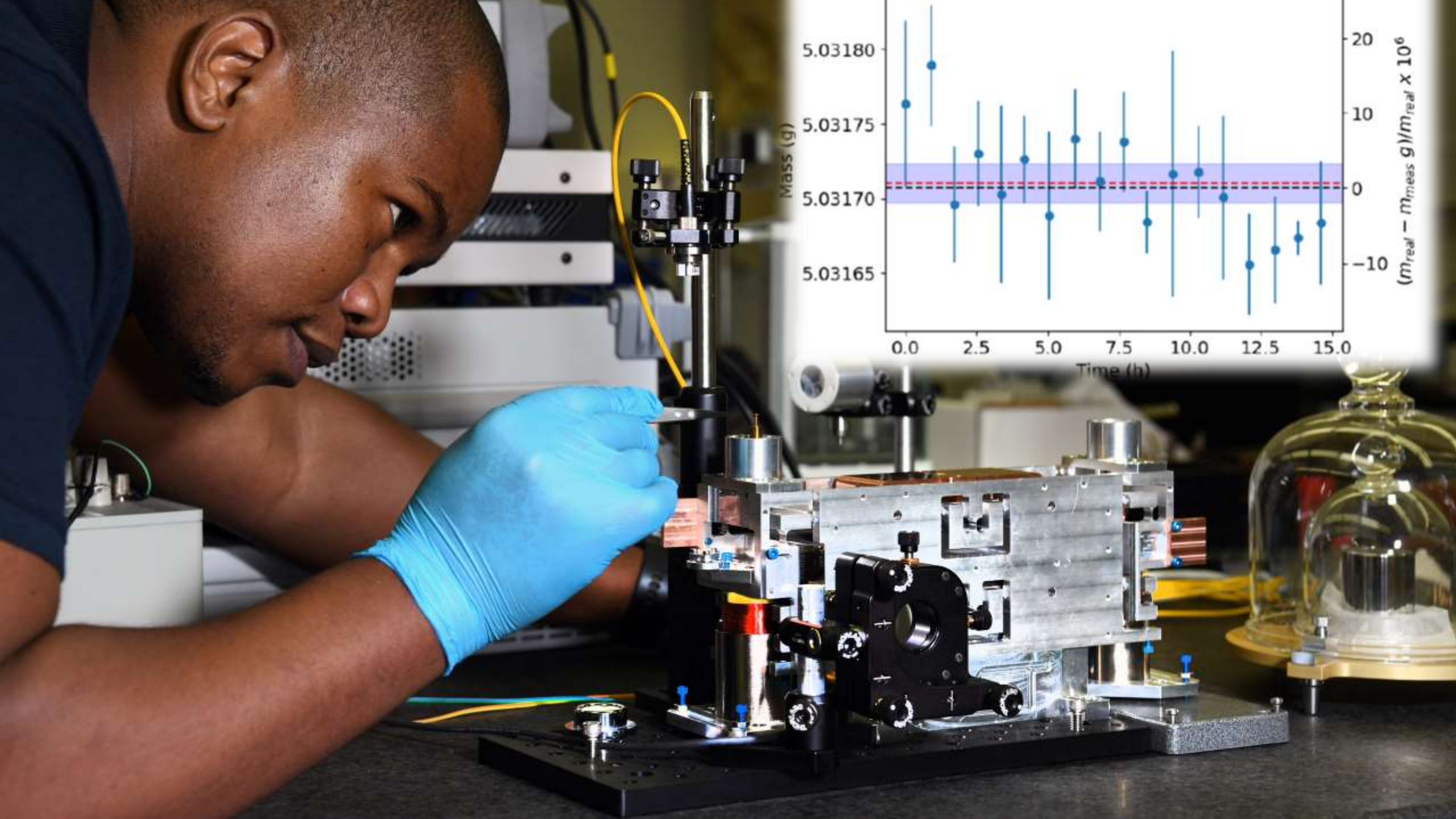


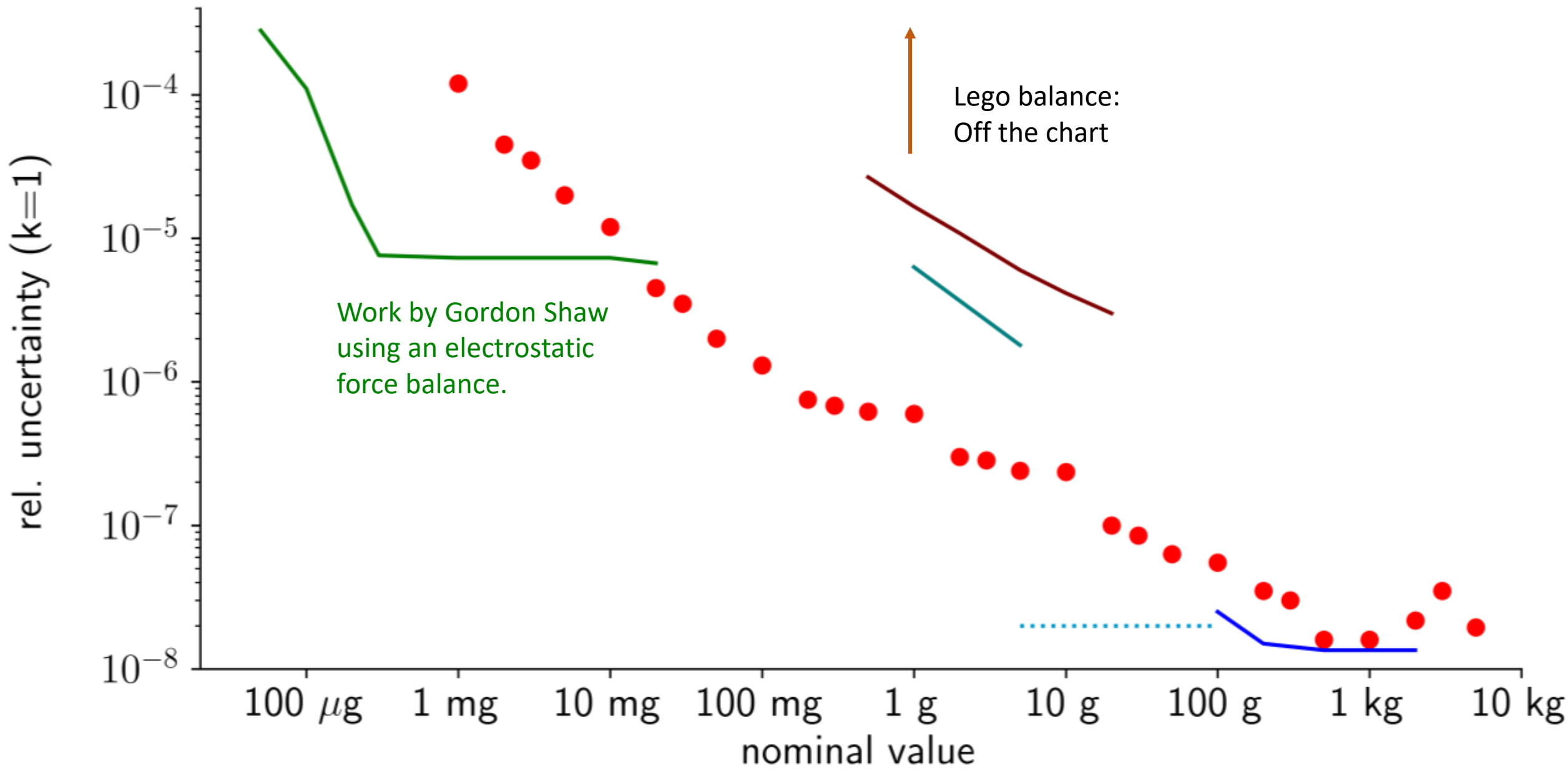
9"

Flexure based design.

Work with John Dragonov, Leon Chao, Darine Haddad

24"





QEMMS

- Quantum Electro-Mechanical Metrology Suite (QEMMS): a NMI in a room featuring a Kibble balance
- Measurement uncertainty $< 2 \mu\text{g}$ for masses $< 100 \text{ g}$
- Design and construction in 5 years (2019-2024)



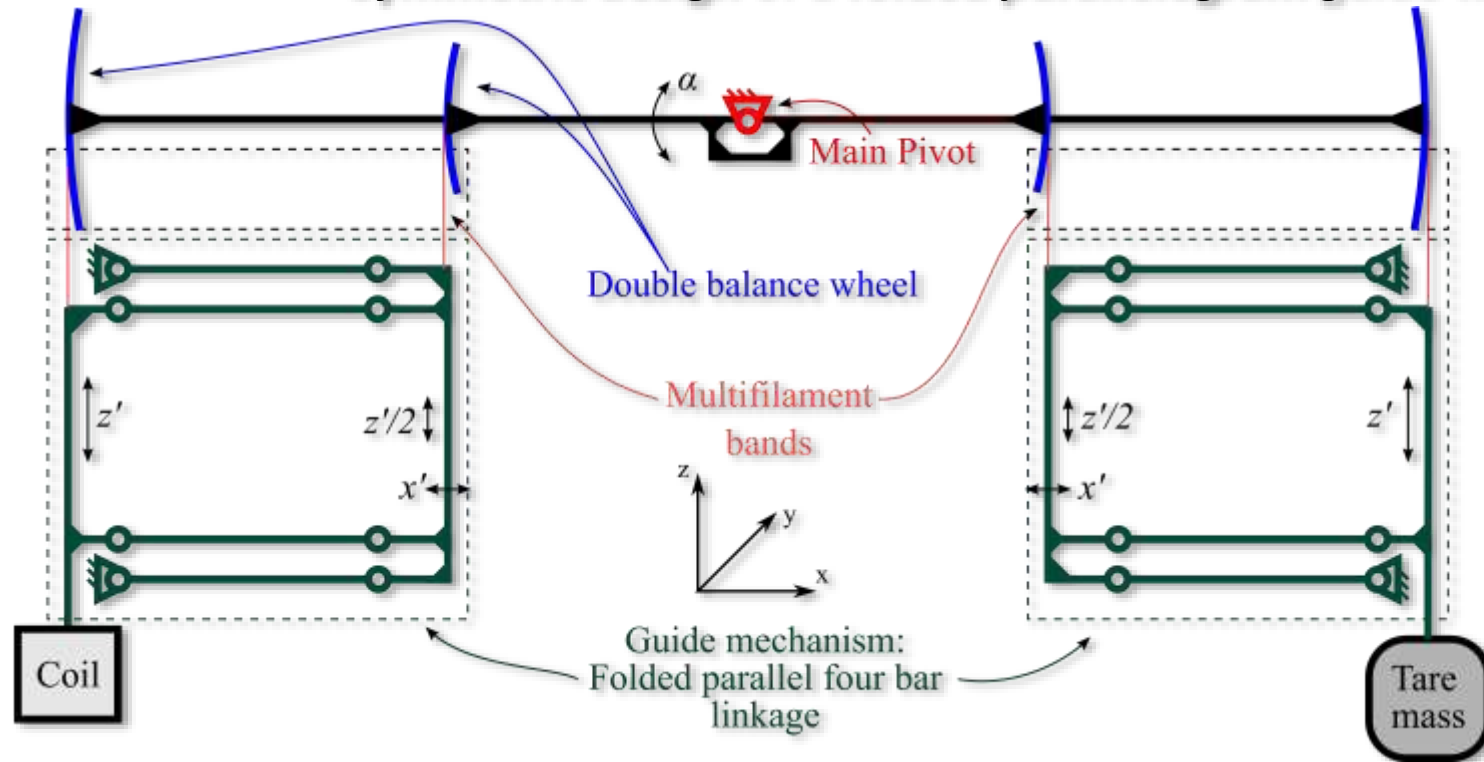
Graphene
Quantum Hall
Resistor



Cryo-cooled PJVS

Kinematic Chain

Symmetric design of a folded parallelogram guide with external linkage



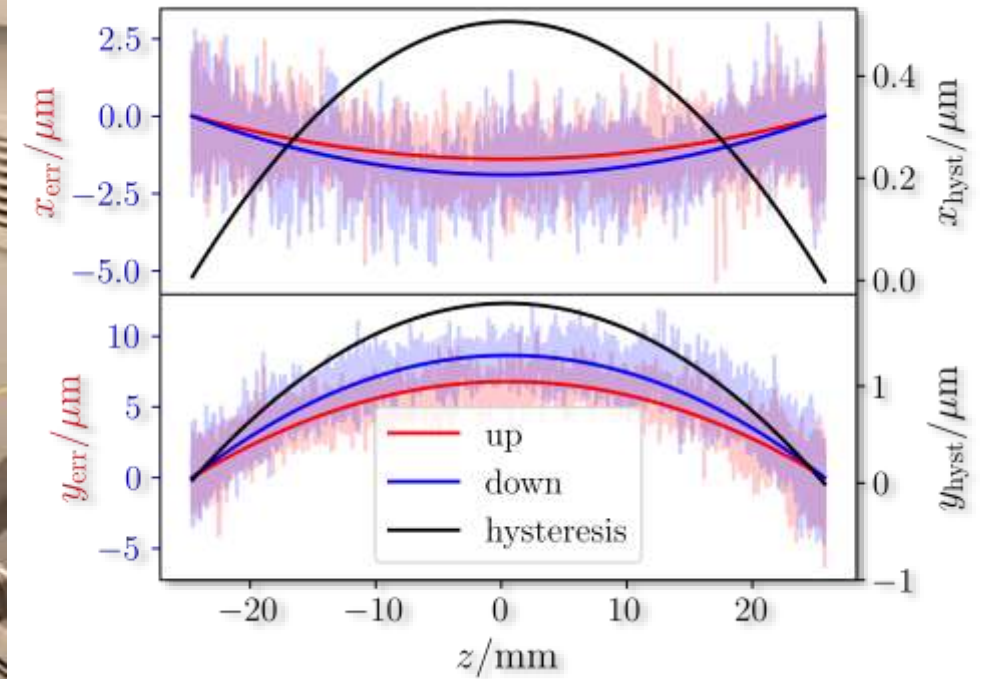
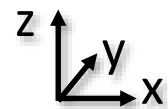
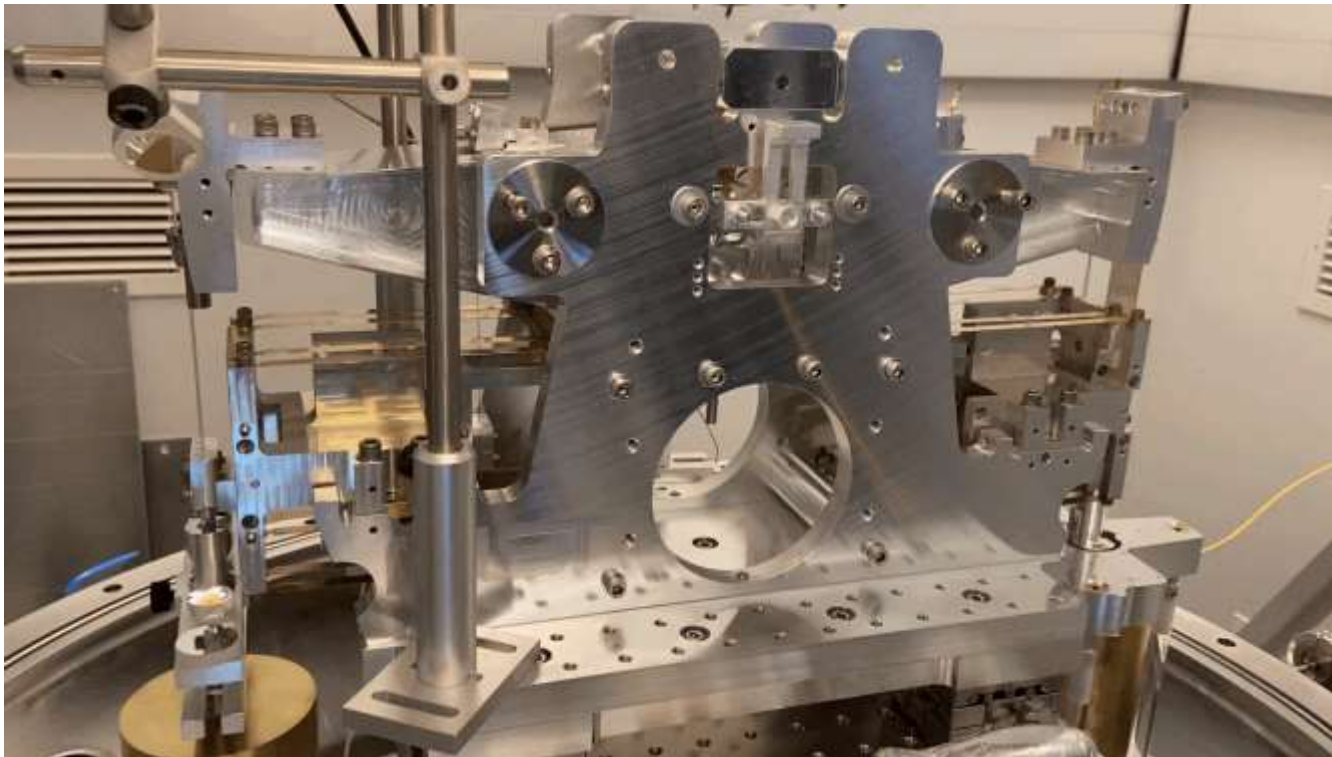
Total suspended mass: **15 kg**

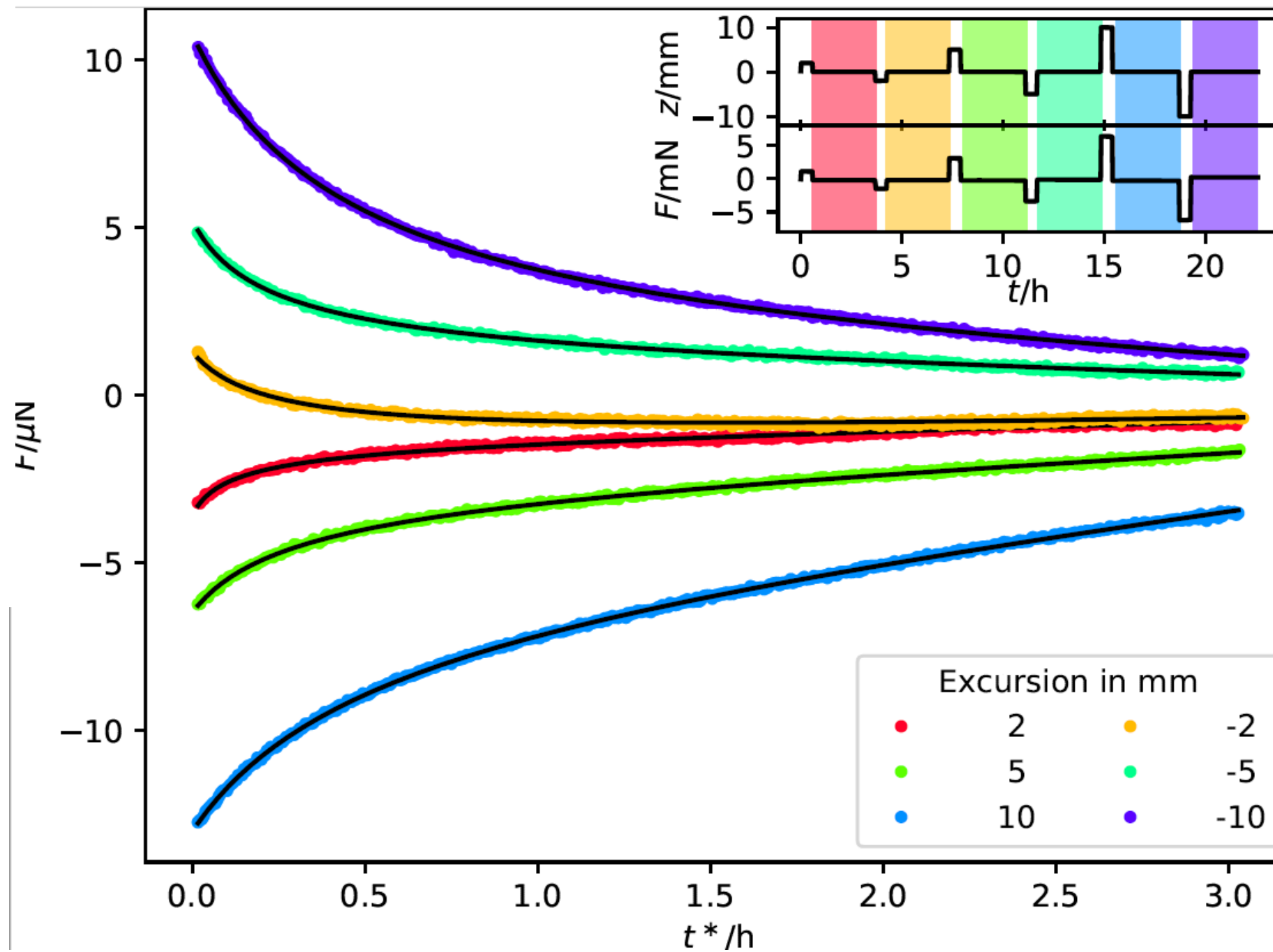
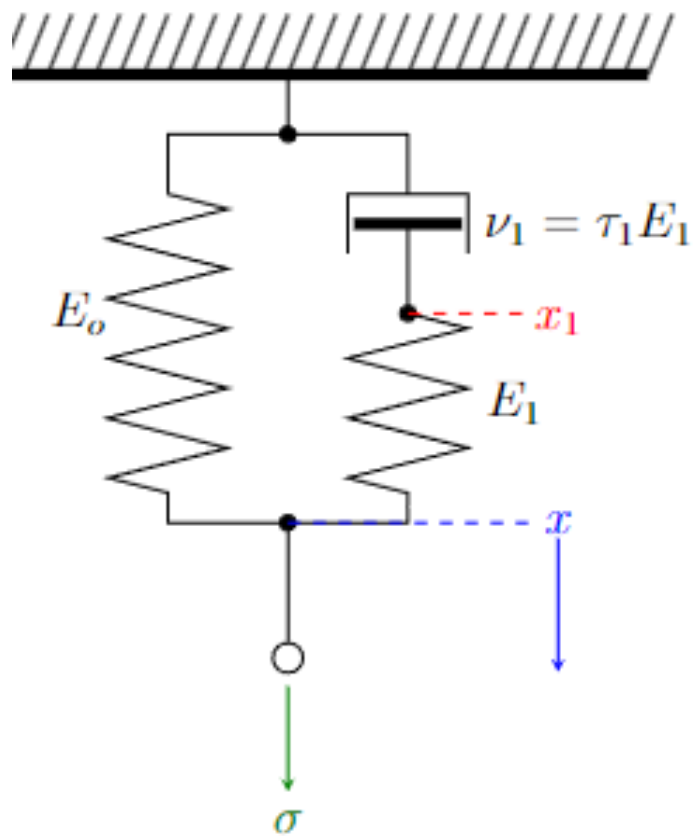
$z' = \pm 30 \text{ mm}$

$\alpha \approx \pm 7^\circ$

Work with Lorenz Keck, Frank Seifert, Darine Haddad

Kinematic Analysis





Kibble Principle in Rotational Frames

Linear

Rotational

Self-Calibration Mode

$$\mathbf{V} = \mathbf{BLv}$$

Velocity Mode

$$\mathbf{V} = \mathbf{B}(\varphi)\mathbf{Lr}\dot{\varphi}$$

Spin Mode

Measurement Mode

$$\mathbf{mg} = \mathbf{BLI}$$

Force Mode

$$\boldsymbol{\tau} = \mathbf{B}(\varphi)\mathbf{LrI}$$

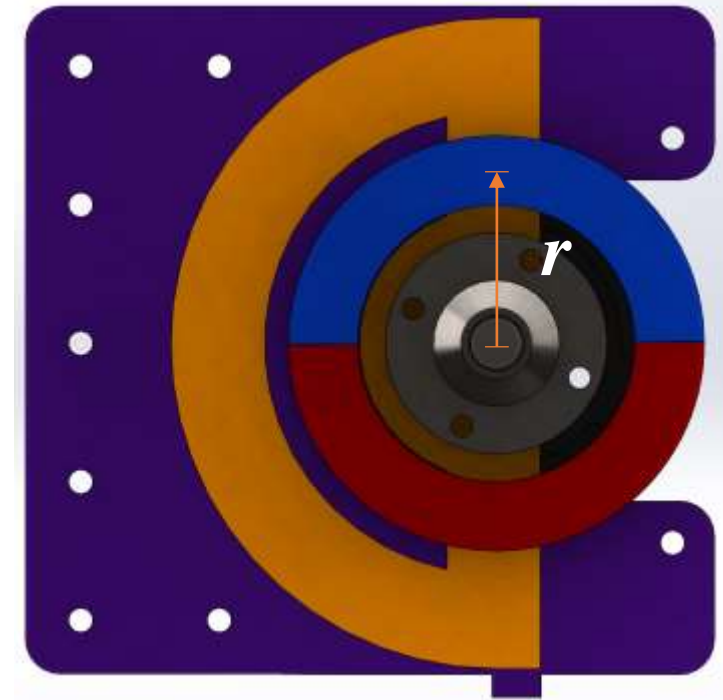
Torque Mode

\mathbf{BL} is common in both equations

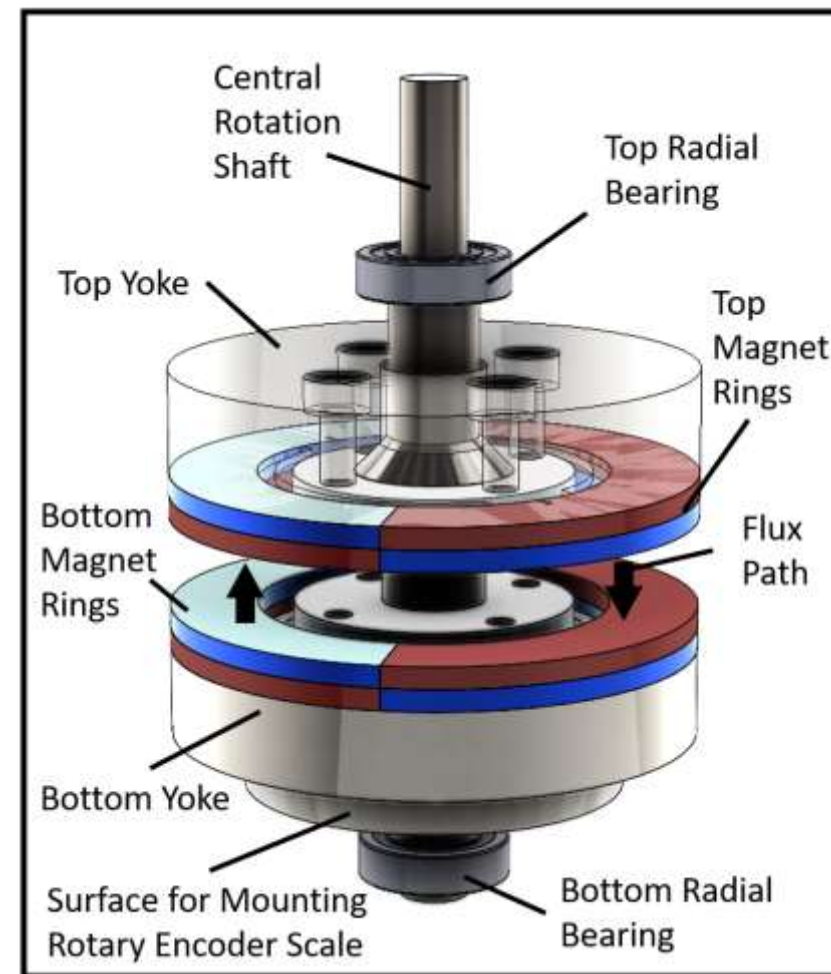
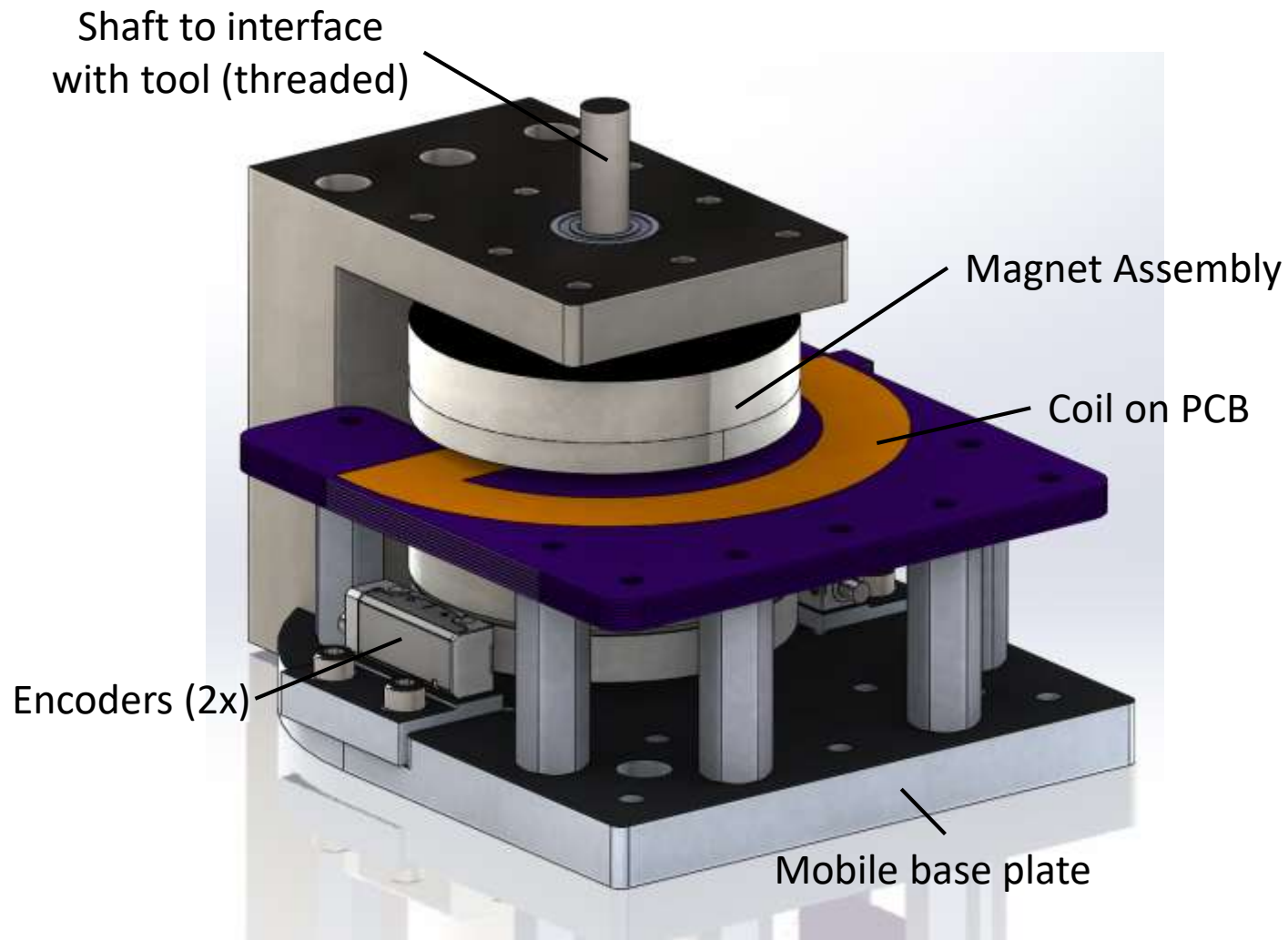
$\mathbf{B}(\varphi)\mathbf{Lr}$ is common in both equations

$$\mathbf{mg} = \mathbf{I}(\mathbf{V} / \mathbf{v})$$

$$\boldsymbol{\tau} = \mathbf{I} \frac{\mathbf{V}}{\dot{\varphi}}$$

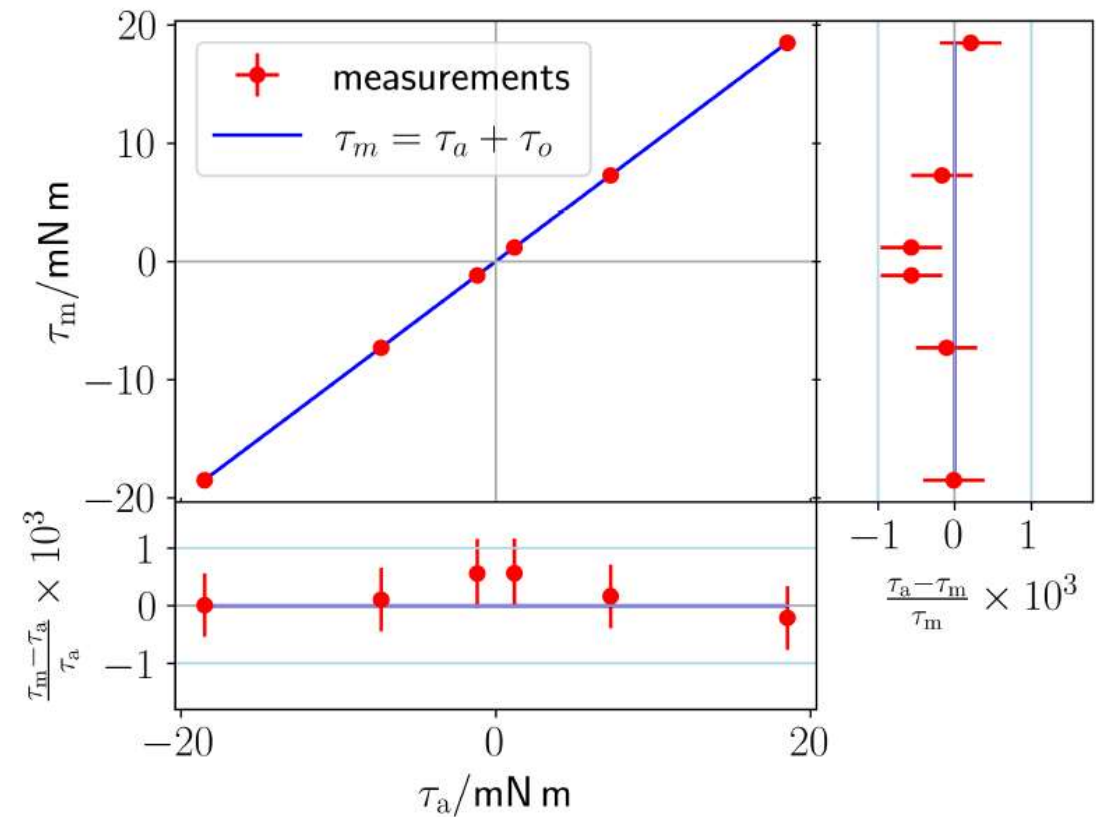
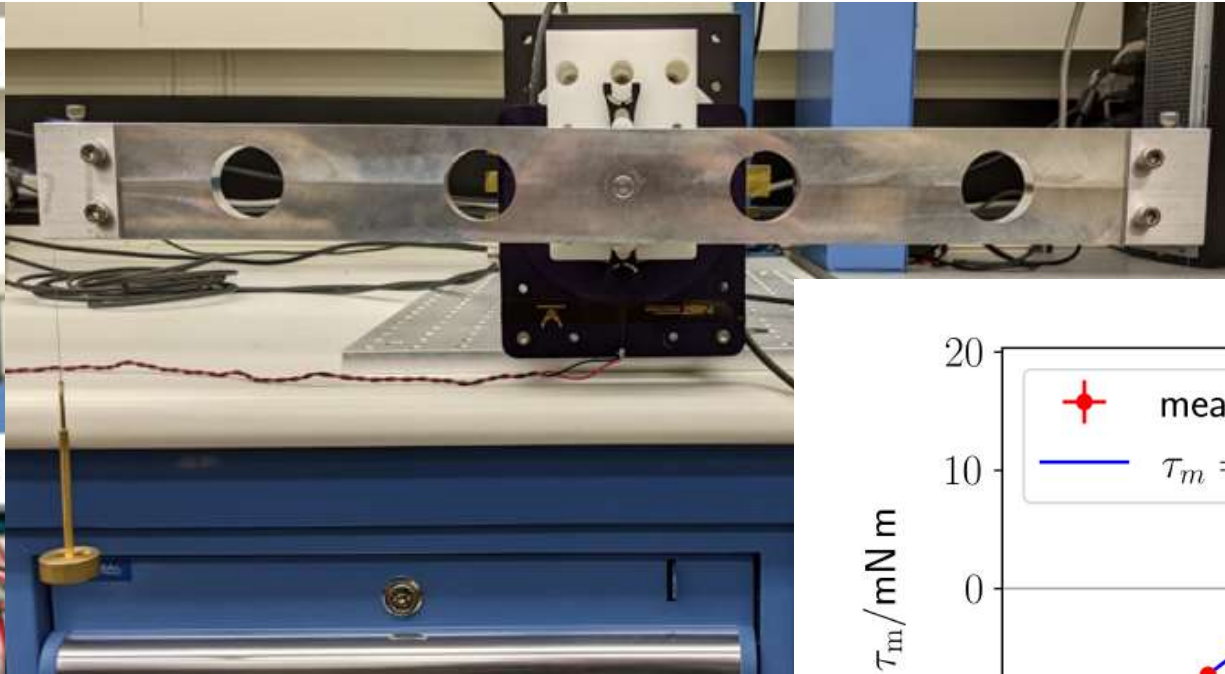
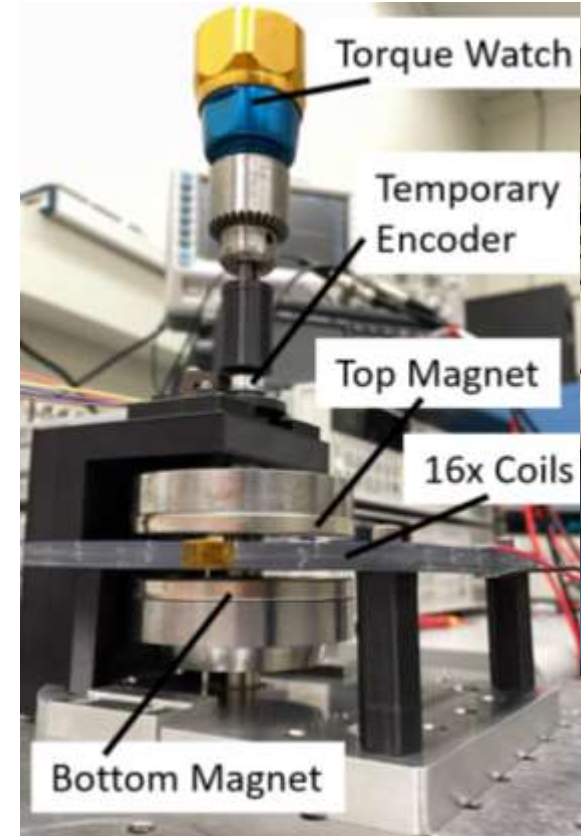


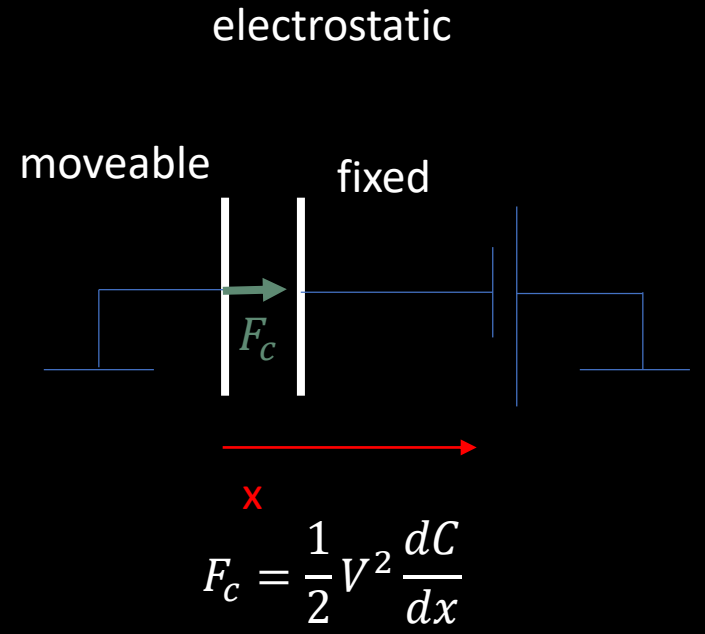
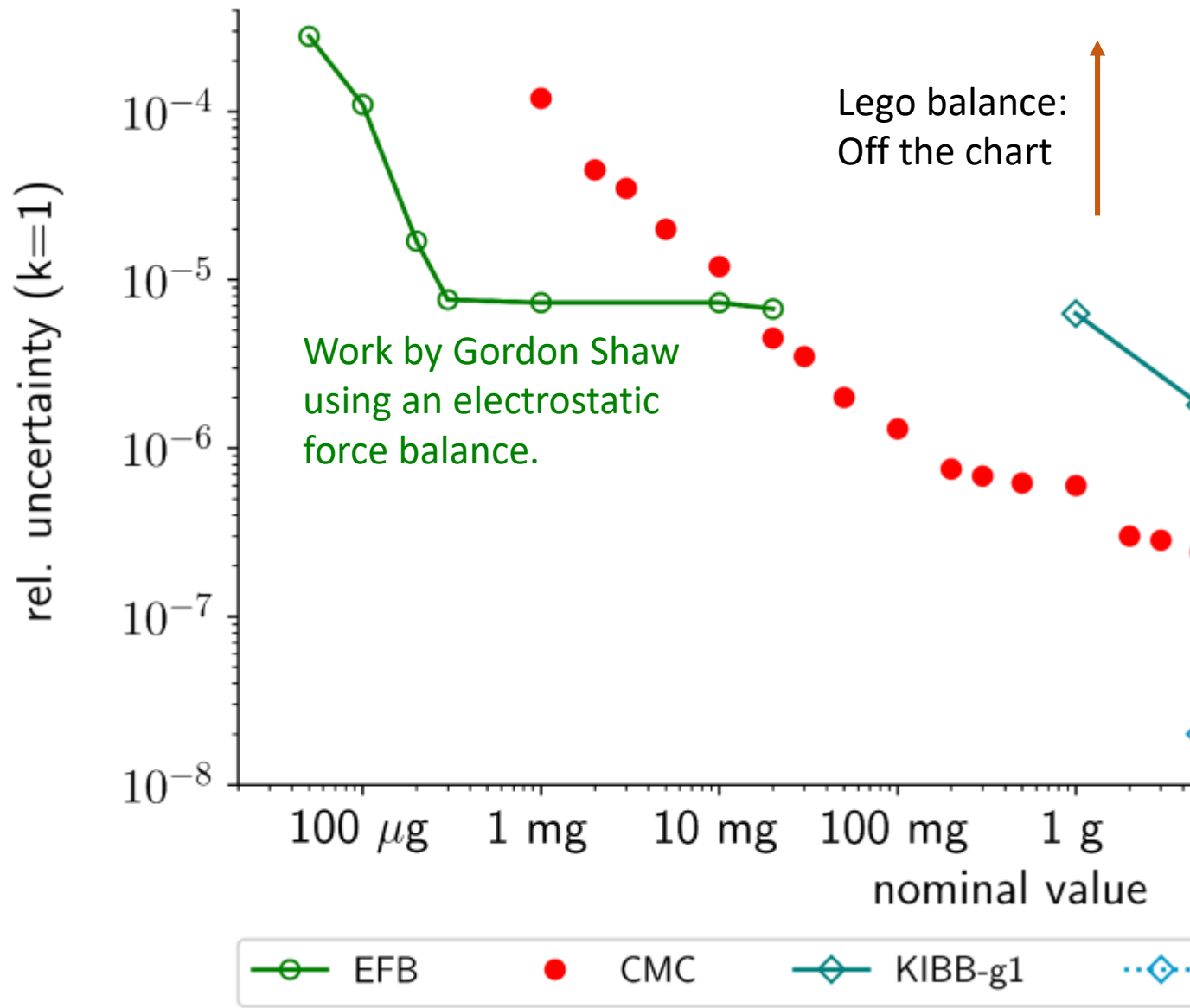
Electronic NIST Torque Realizer (ENTR)



Magnet Assembly Detail

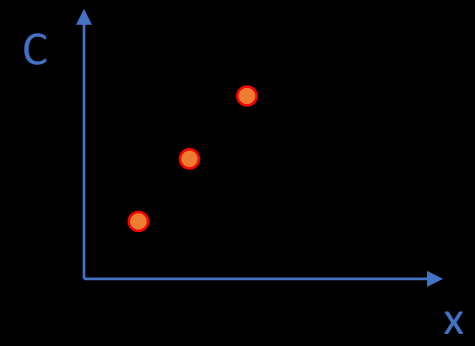
ENTR Prototype





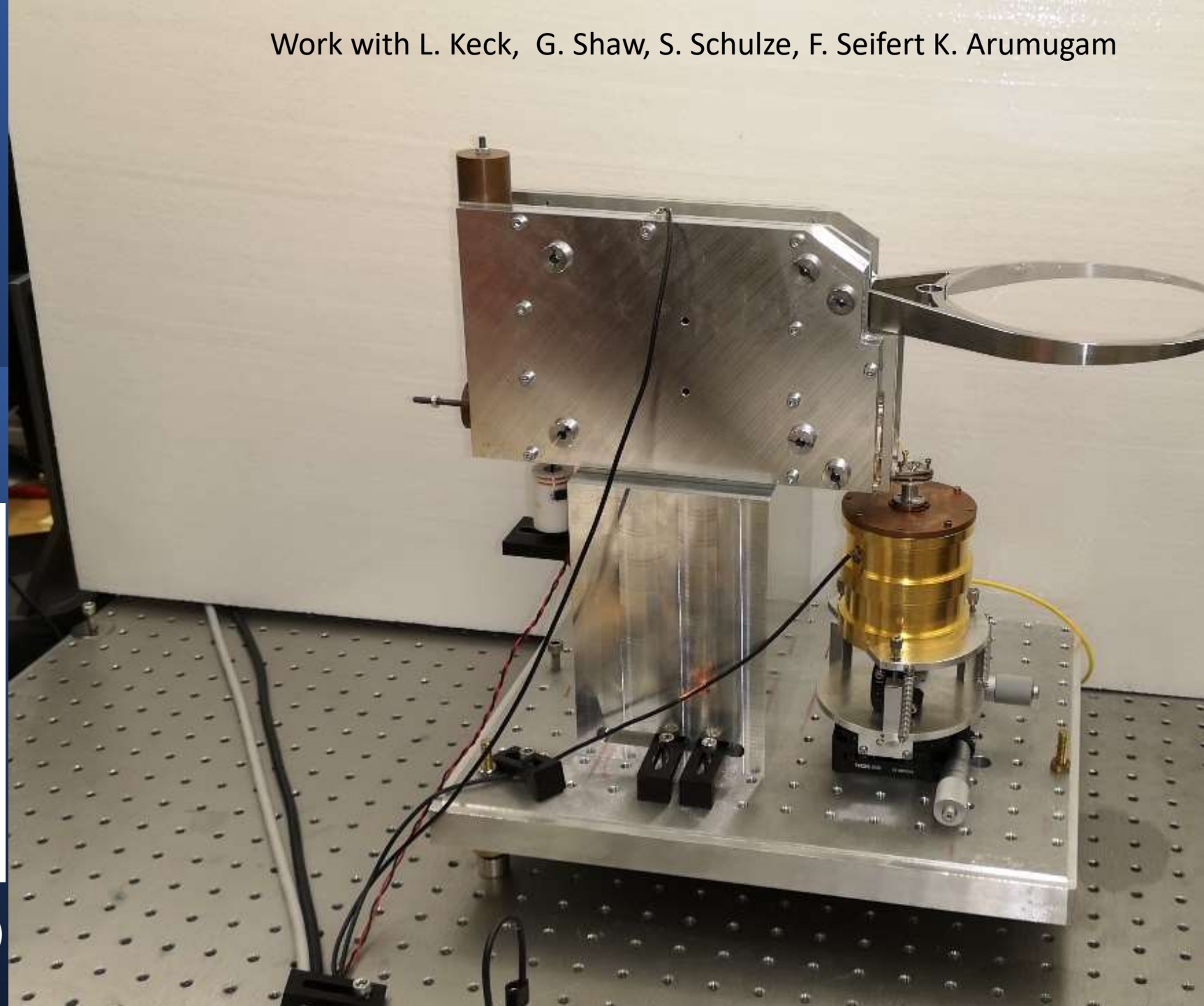
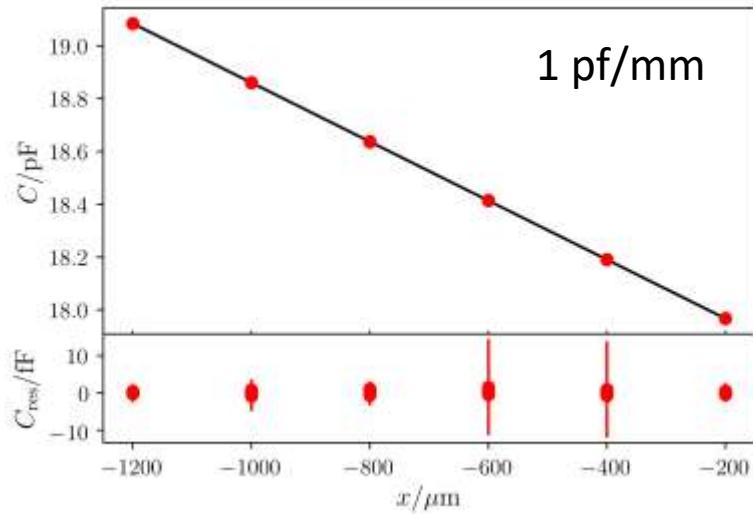
calibration phase

measure $C(x) \rightarrow dC/dx$

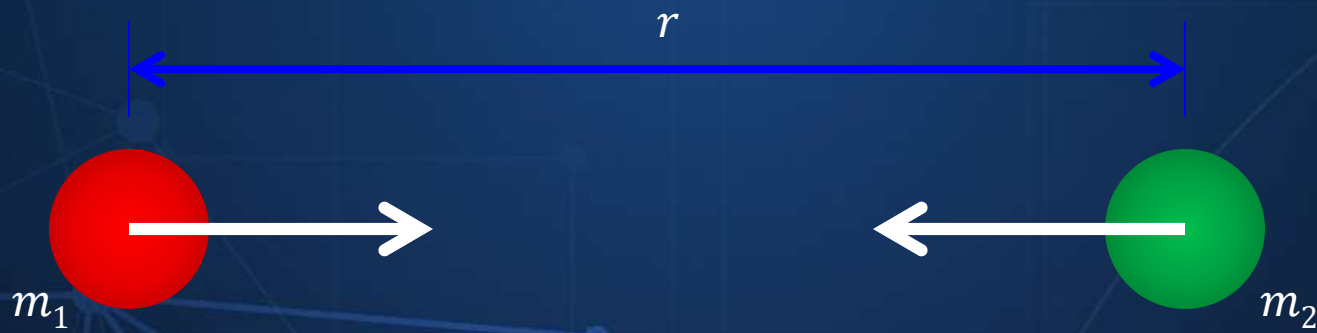


Photonic pressure balance

100 kW Laser produces in reflection $667 \mu\text{N}$.



Torque measurements to determine G

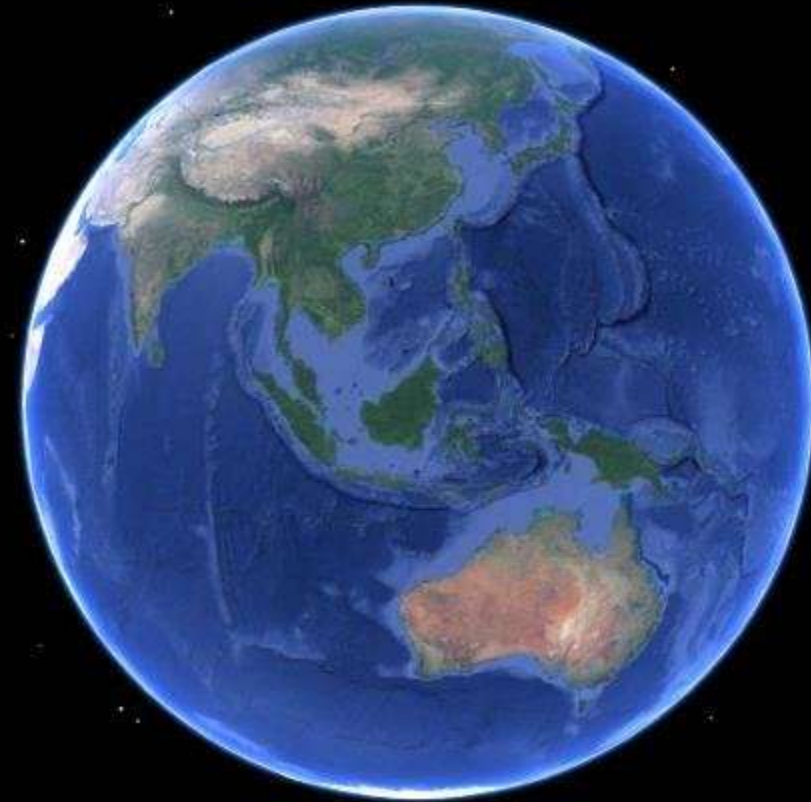


$$F = G \frac{m_1 m_2}{r^2}$$

$$G \approx 6.67 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$$

How big would a steel cable have to be to keep Earth going around the sun to replace gravity?

1.5×10^{11} m



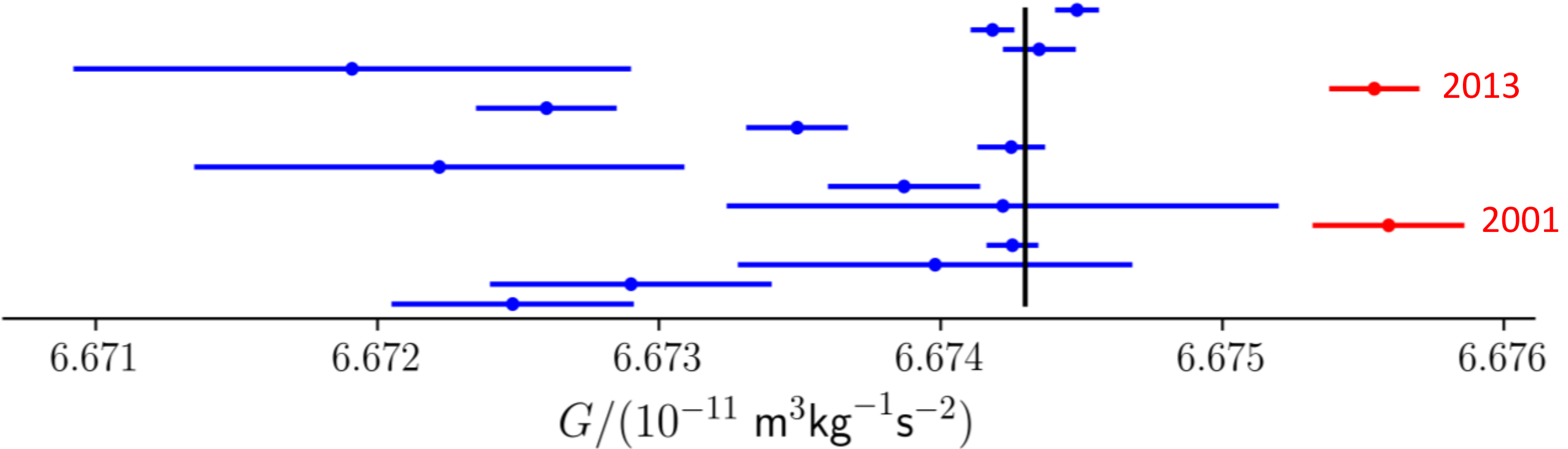
How big would a steel cable have to be to keep Earth going around the sun to replace gravity?

1.5×10^{11} m

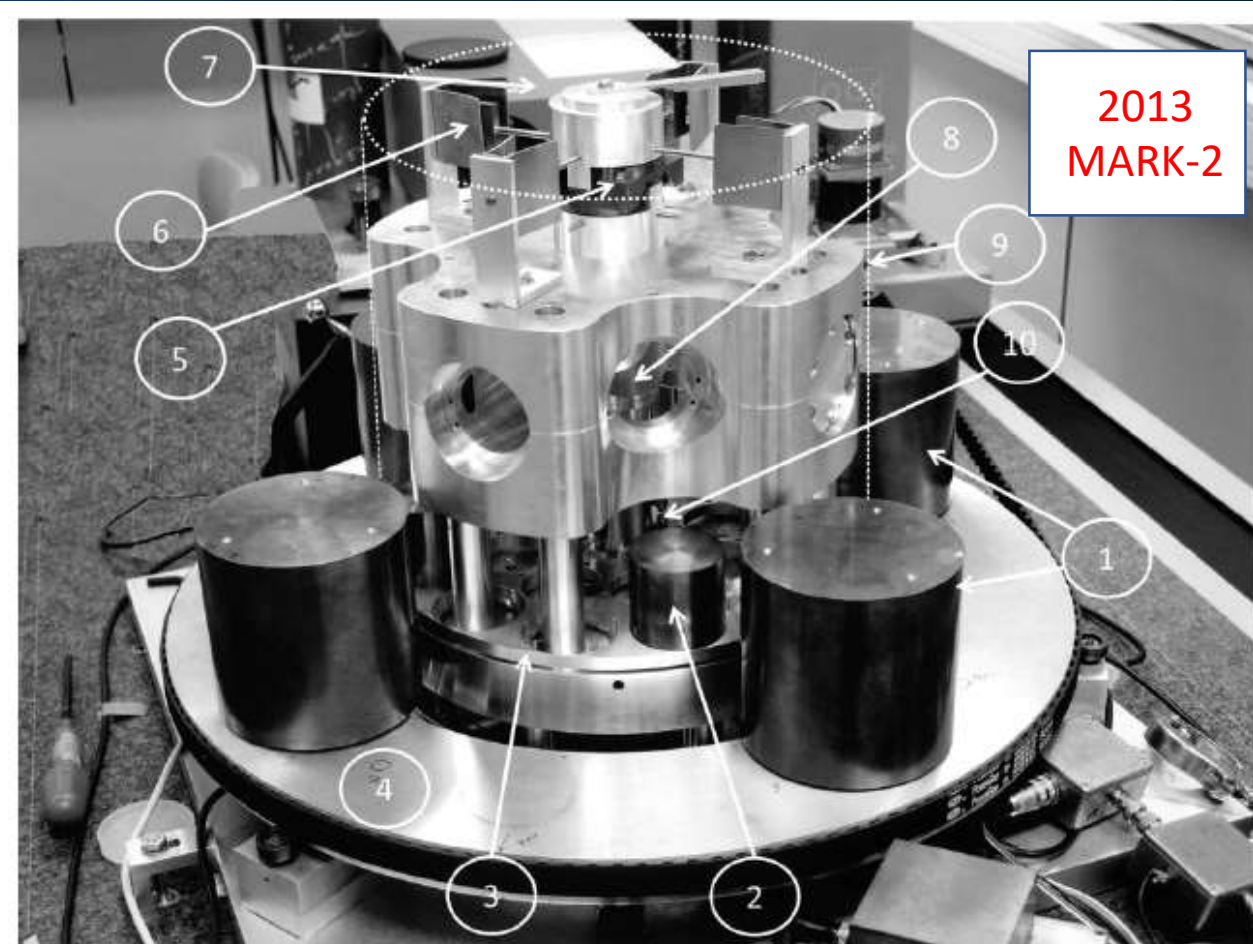
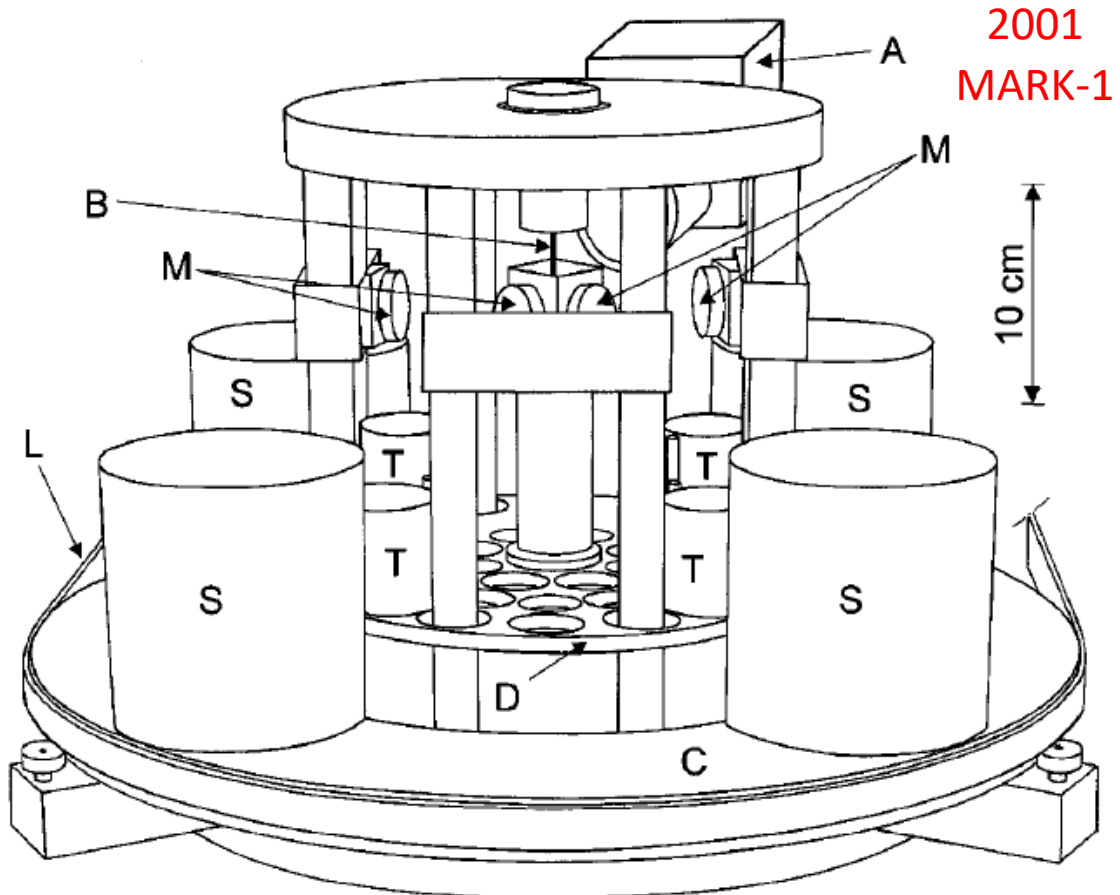
Steel cable $\frac{2}{3} D_{\text{Earth}}$



The story

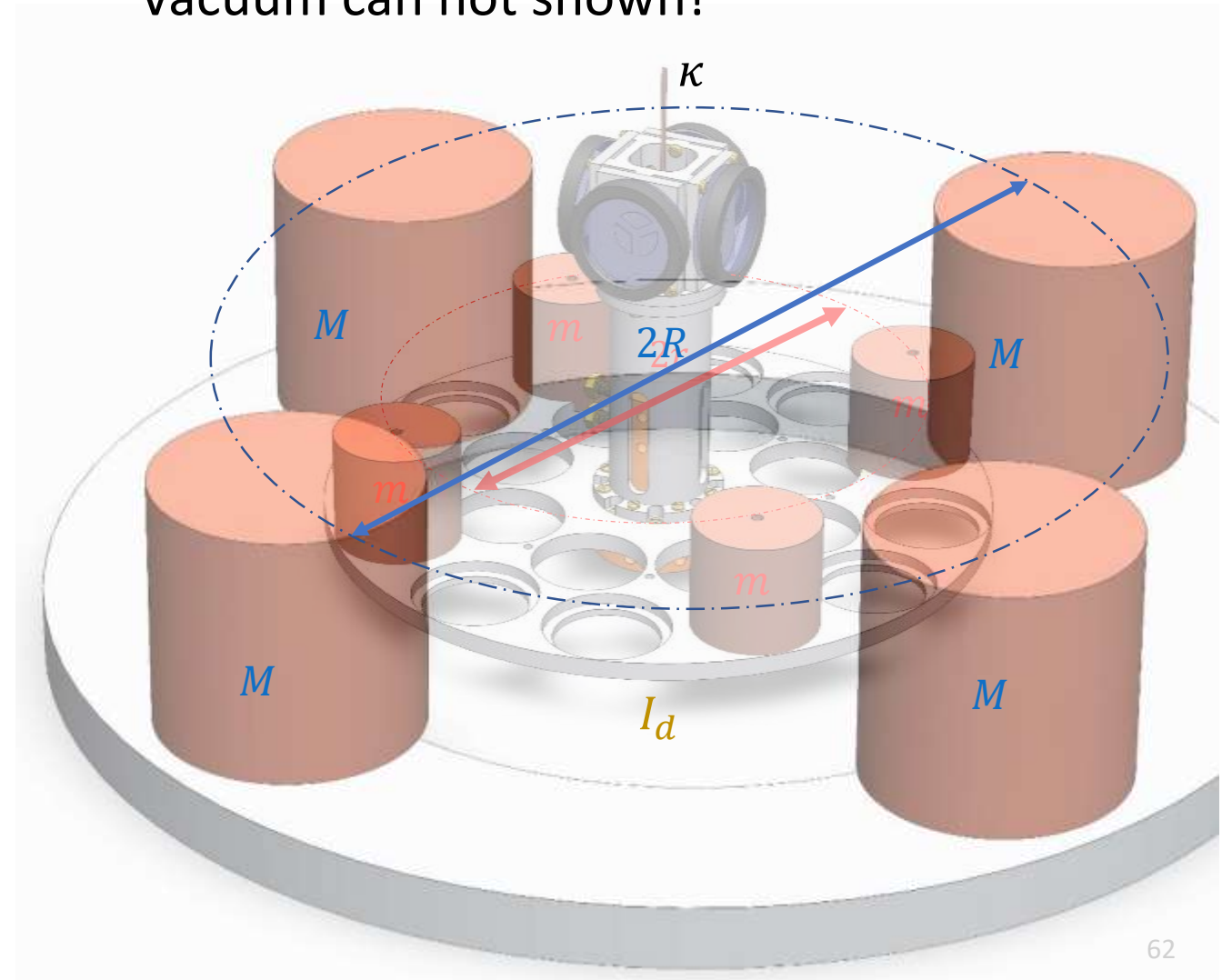


The BIPM instruments



m	1.2	kg
r	120	mm
κ	204	$\mu\text{N m}$
I_d	745	g dm^2
$I_d / (4m r^2)$	0.11	
T_o	121	s

Vacuum can not shown!



Remarkable Experiment

1. One instrument, two measurements: Cavendish and electrostatic servo

➤ Different sensitivities to different parameters:

$$G_{cav} \propto \phi$$

$$G_{cav} \propto m^{0.1}$$

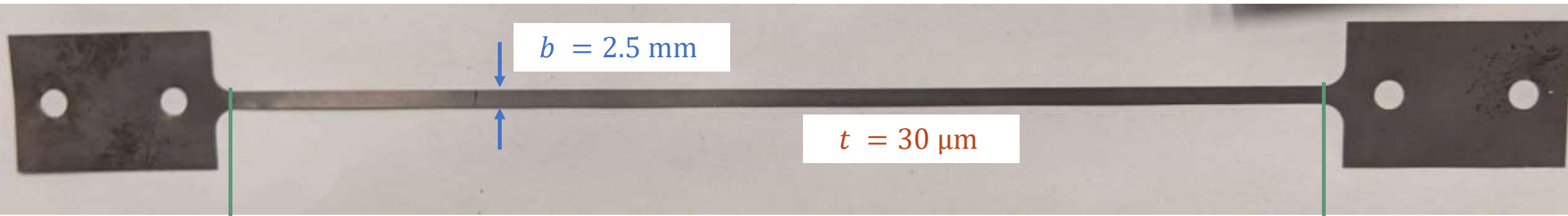
$$G_{cav} \propto r^{2.2}$$

$$G_{servo} \propto \phi^{-1}$$

$$G_{servo} \propto m$$

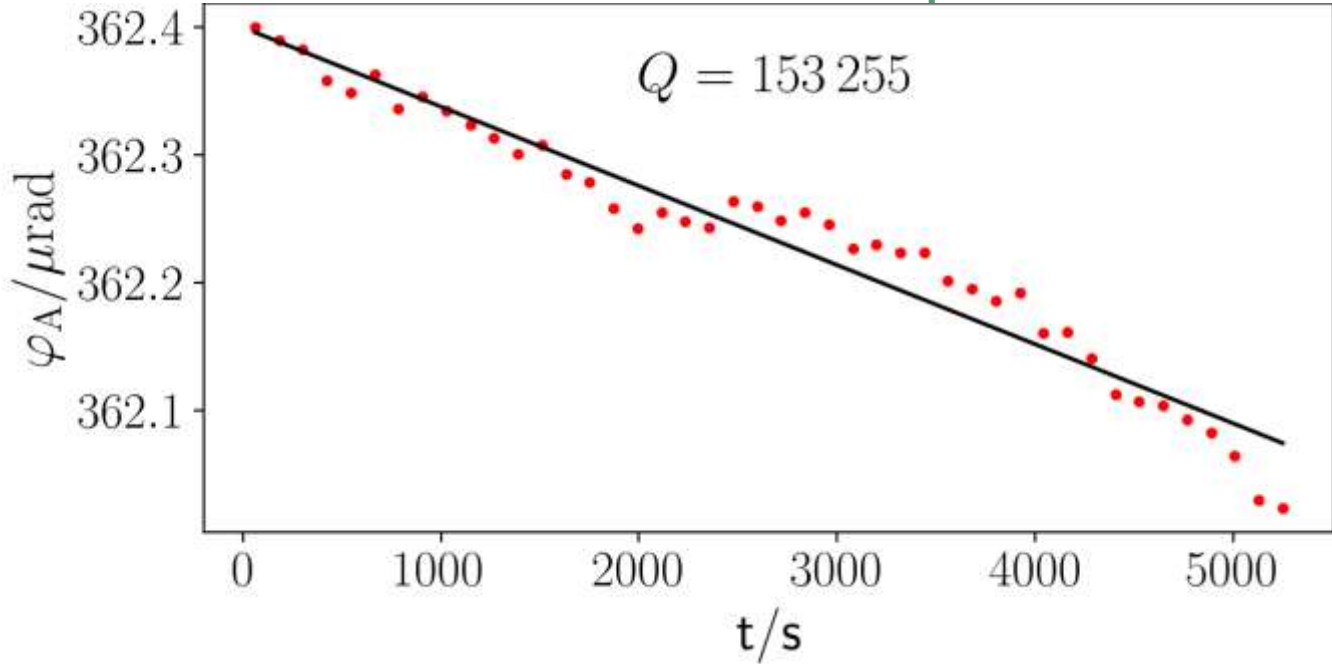
$$G_{servo} \propto r^4$$

The torsion strip



$L = 160 \text{ mm}$

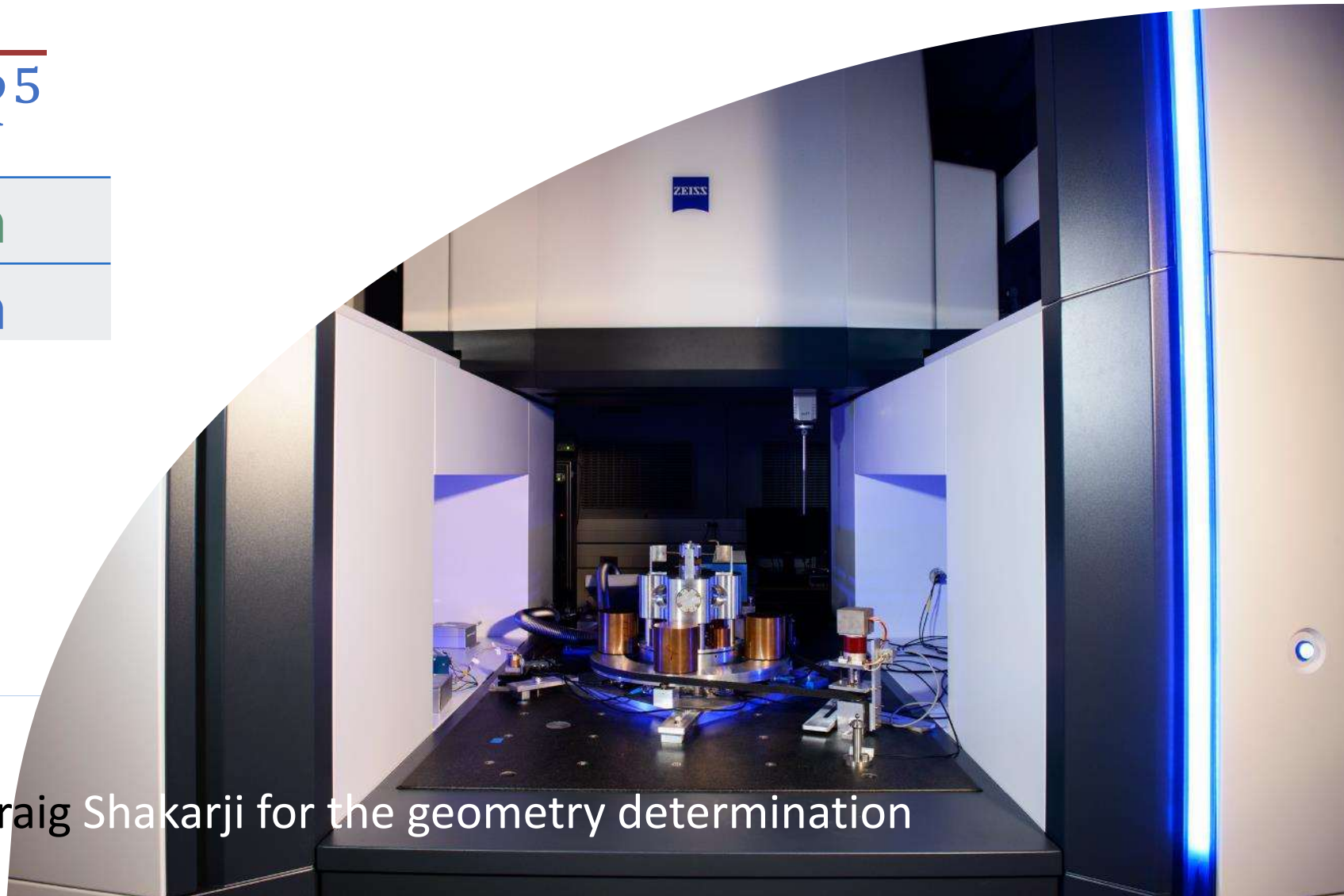
$$\kappa = \underbrace{\frac{bt^3 F}{3L}}_{\substack{\text{elastic} \\ 4\%}} + \underbrace{\frac{(4m + m_{disk})g b^2}{12L}}_{\substack{\text{conservative} \\ 96\%}}$$



Hexadecapole gravitational attraction

$$\Delta N \approx G 70 Mm \frac{r^4}{R^5}$$

r	120	mm
R	240	mm



Thank you, Vincent Lee and Craig Shakarji for the geometry determination

Blind measurement

$$M_{adj} = (1 + R)M$$

$$-10^{-3} \leq R \leq 10^{-3}$$

<i>Test</i>	σ_m/m	5×10^{-8}
<i>Copper</i>	σ_M/M	2×10^{-7}
<i>Sapphire</i>	σ_M/M	4×10^{-7}

Thank you, Patrick Abbott for the mass determination.



Cavendish mode

$$G_{Cav} = \Delta\varphi \frac{16\pi^2}{T_o^2} \frac{R^5}{70Mr^2} \left(1 + \frac{I_d}{4mr^2} \right)$$

To be measured:

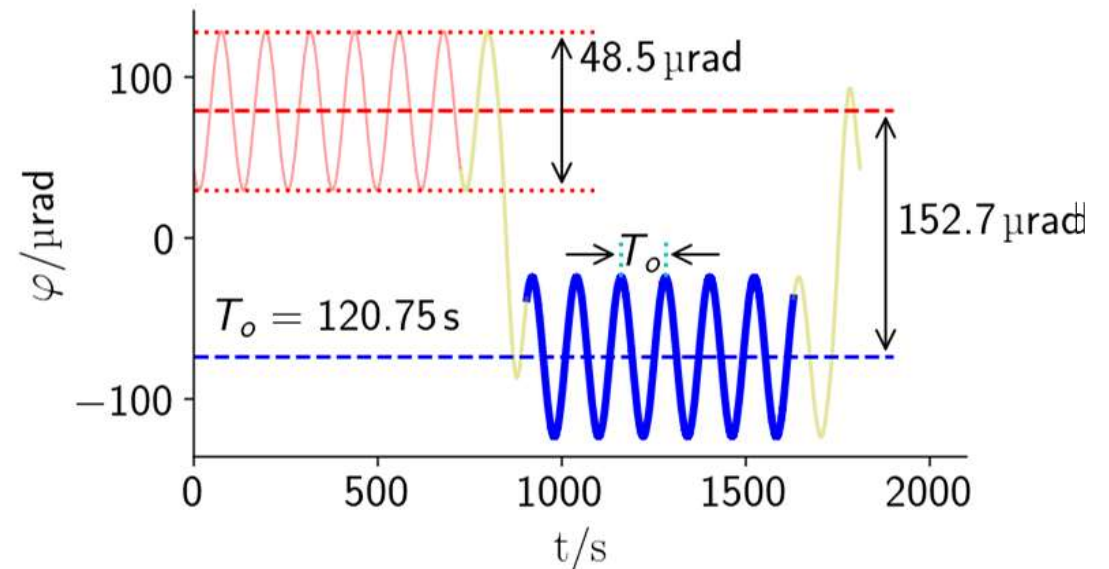
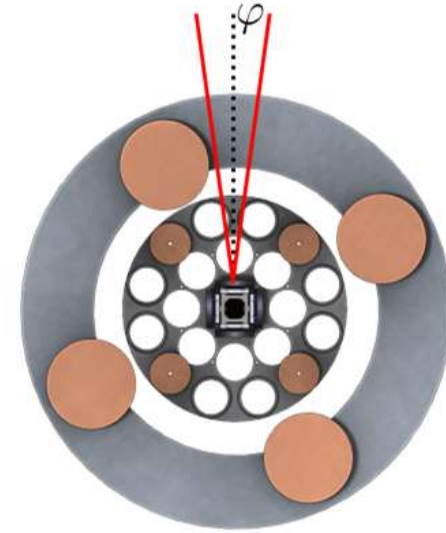
$T_o, \Delta\varphi$

Geometry and masses:

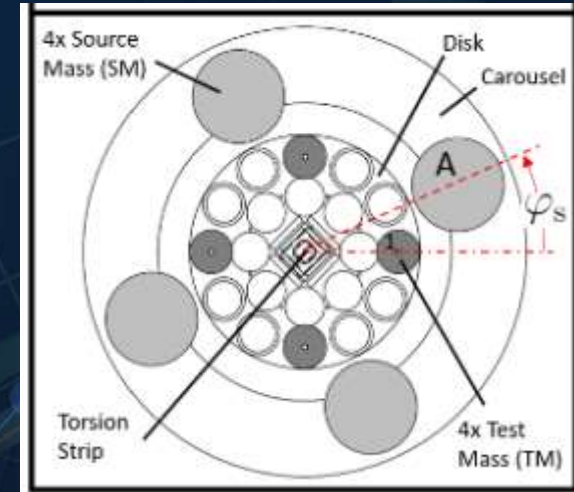
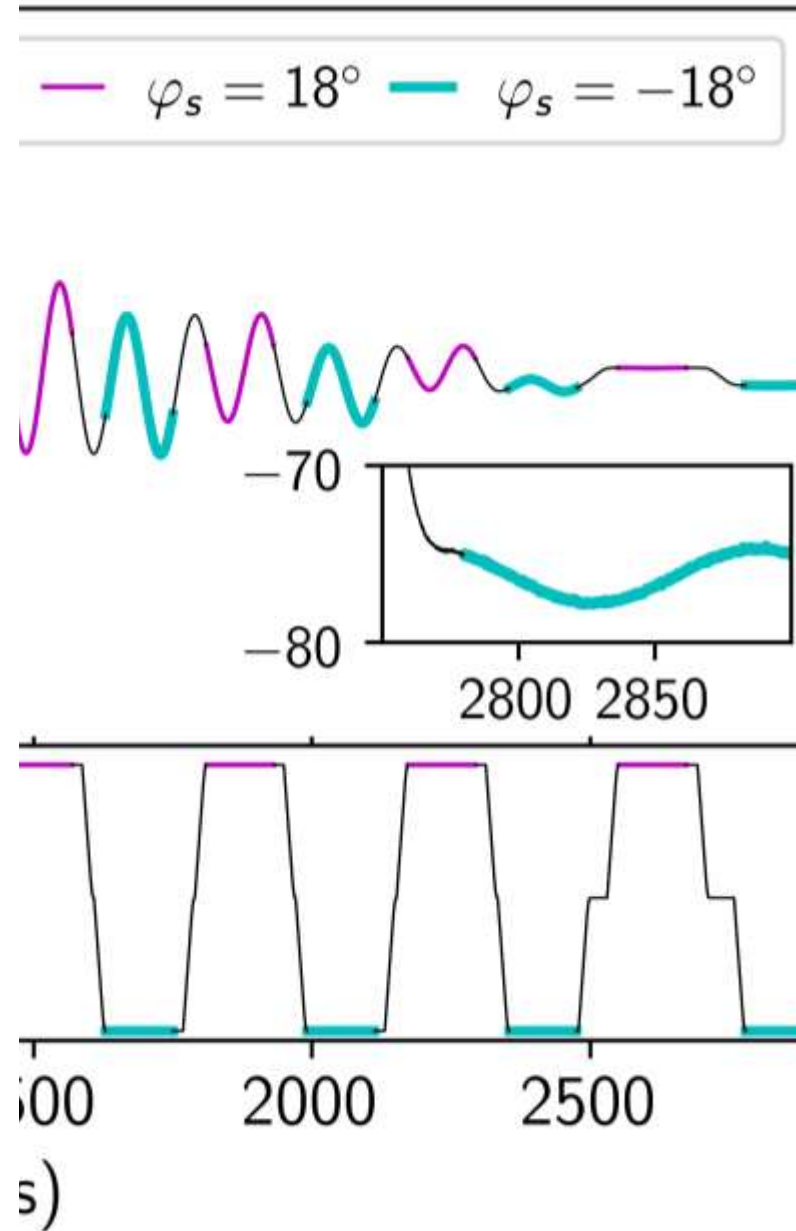
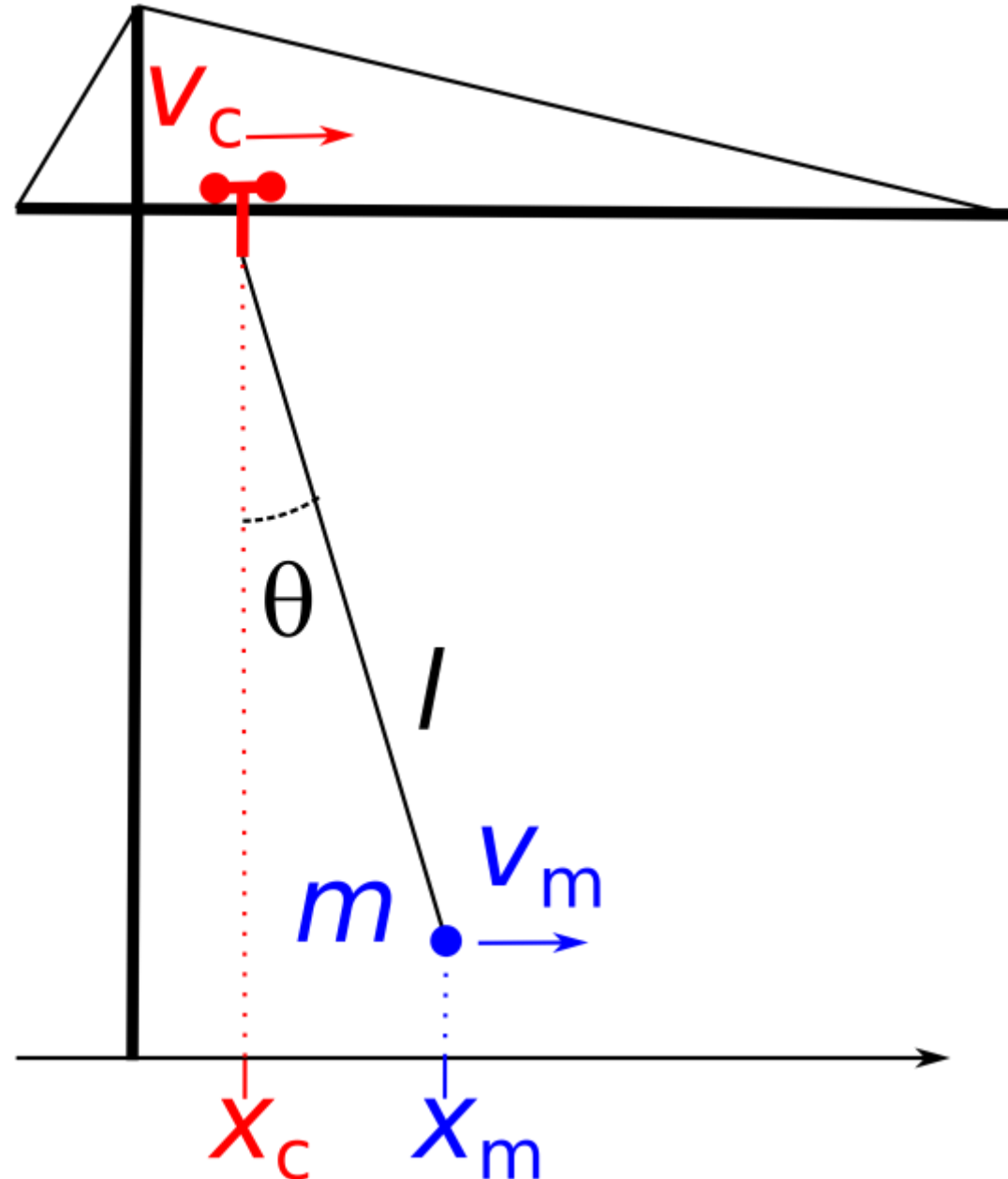
m, M, r, R

To be calculated:

I_d

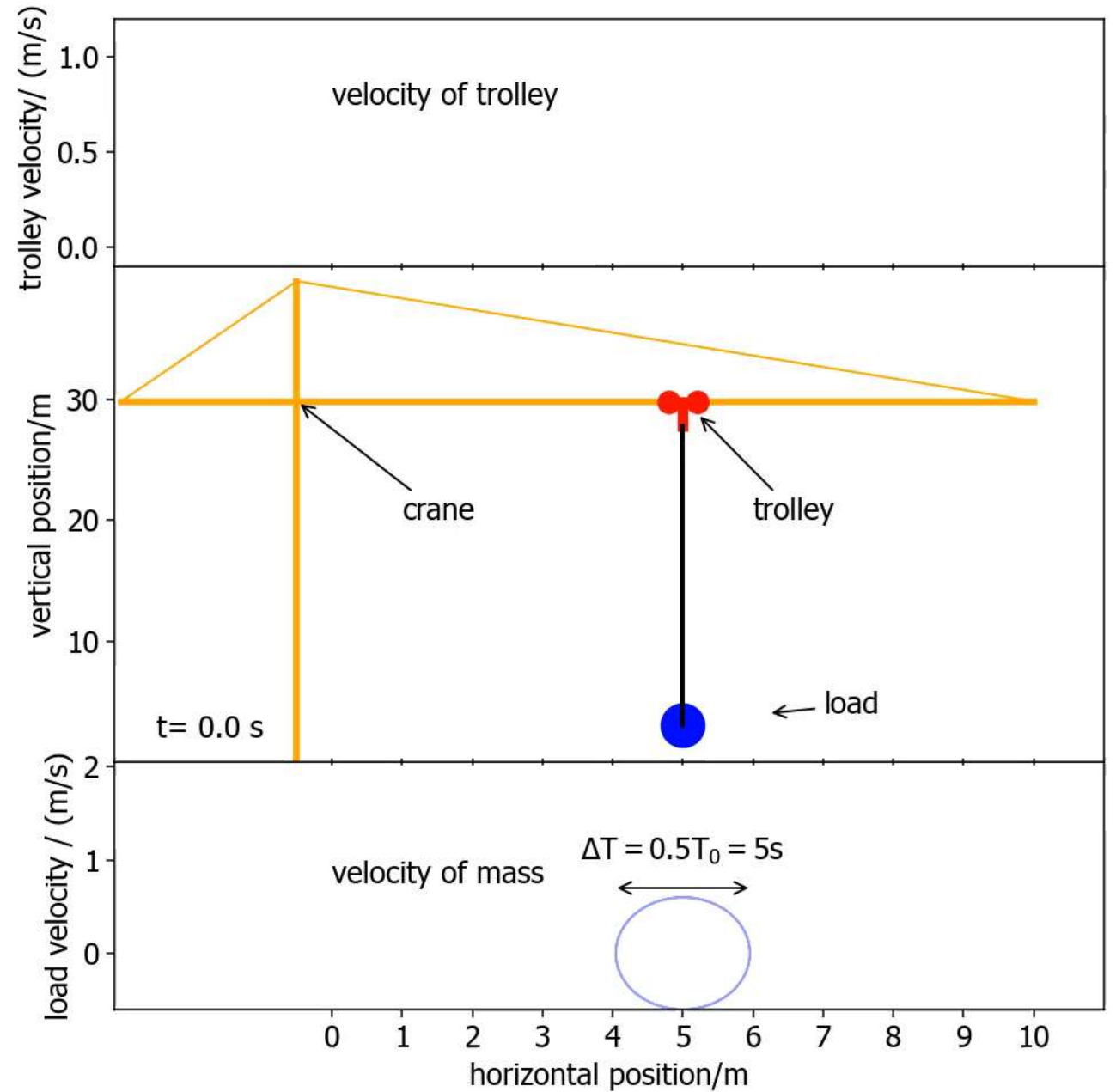


Our contribution to crane safety



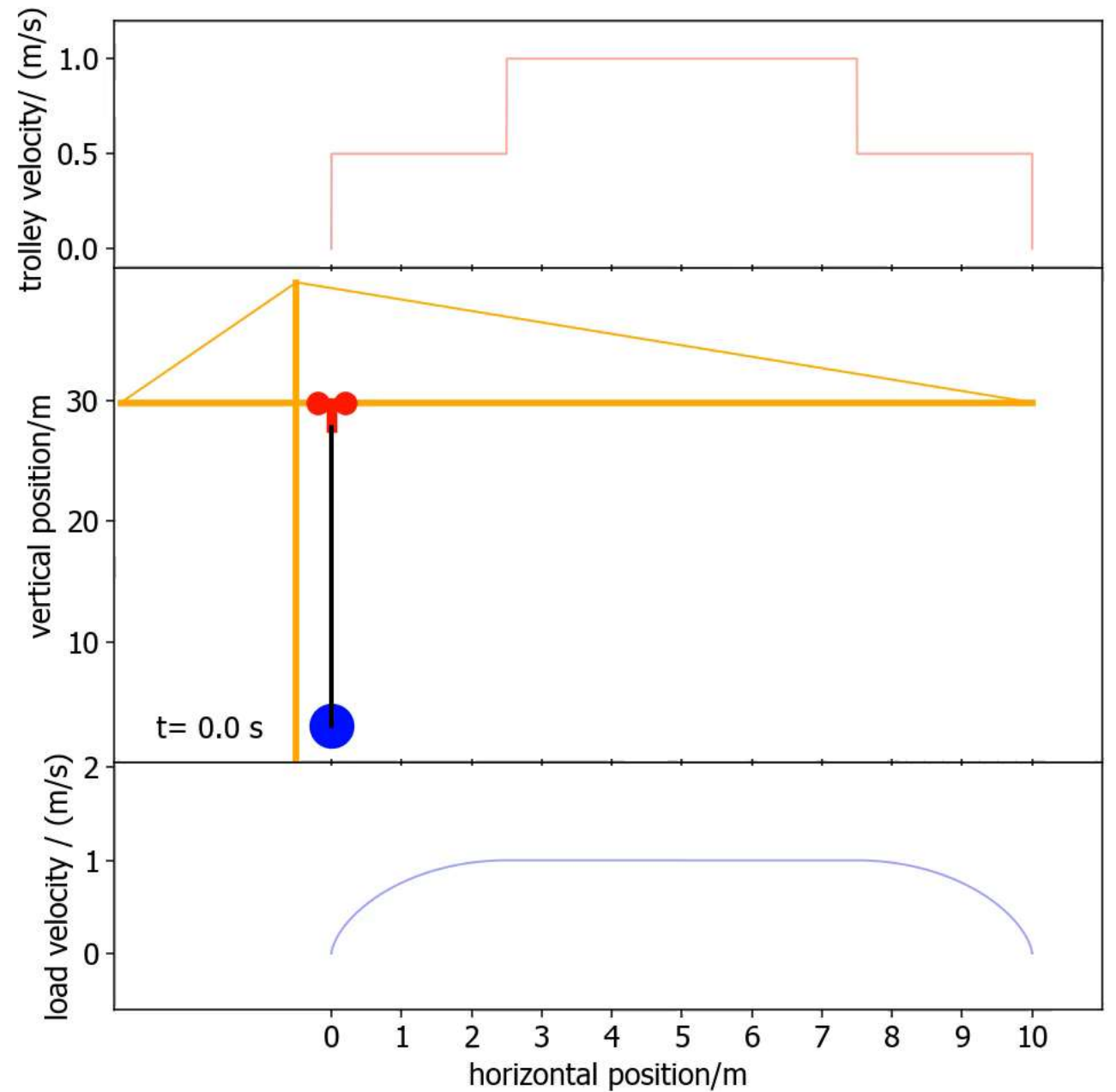
“The Crane Operator's Trick and other Shenanigans with a Pendulum”,
Am. J. Physics.

Crane analysis 1



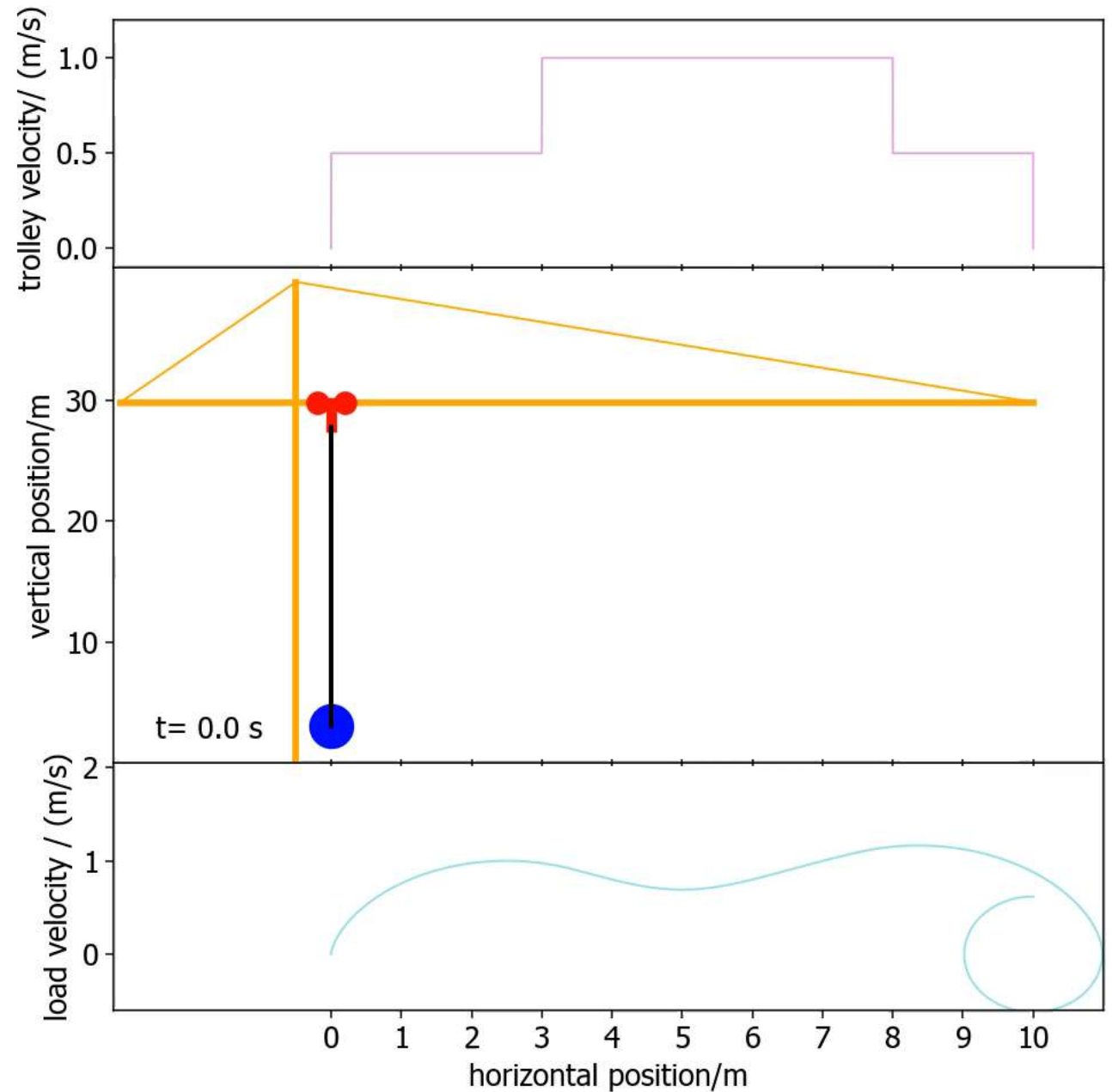
Crane analysis 2

HIRED



Crane analysis 3

FIRED

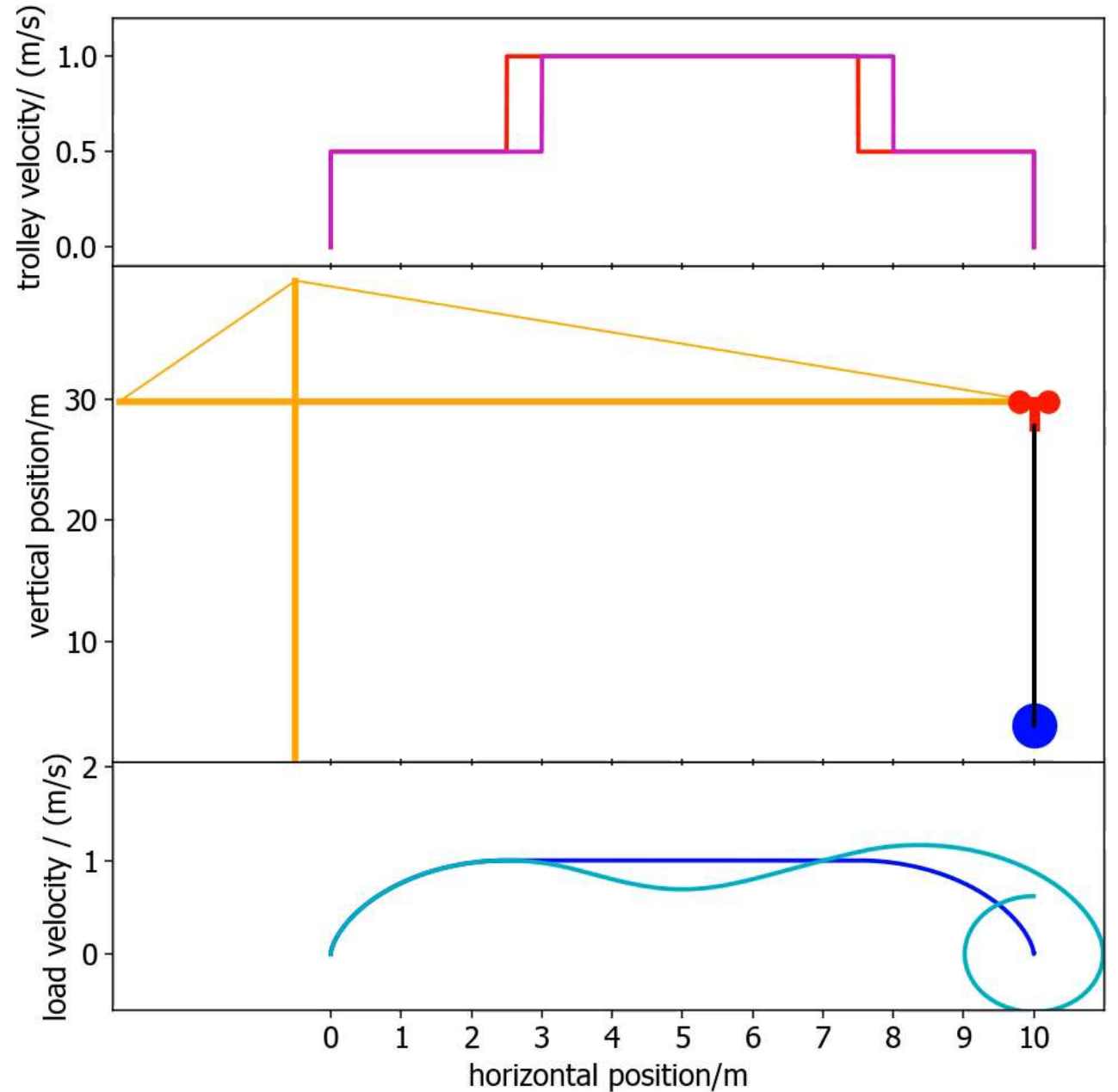


Crane analysis 4

HIRED

VS.

FIRED



Electrostatic servo

$$N_{el} = -\frac{k_{12}}{2} \langle V_{12}^2 \rangle - \frac{k_{13}}{2} \langle V_{13}^2 \rangle - \frac{k_{23}}{2} \langle V_{23}^2 \rangle$$

$$G_{servo} = \frac{R^5}{70Mmr^4} (\Delta N_{el})$$

To be measured:

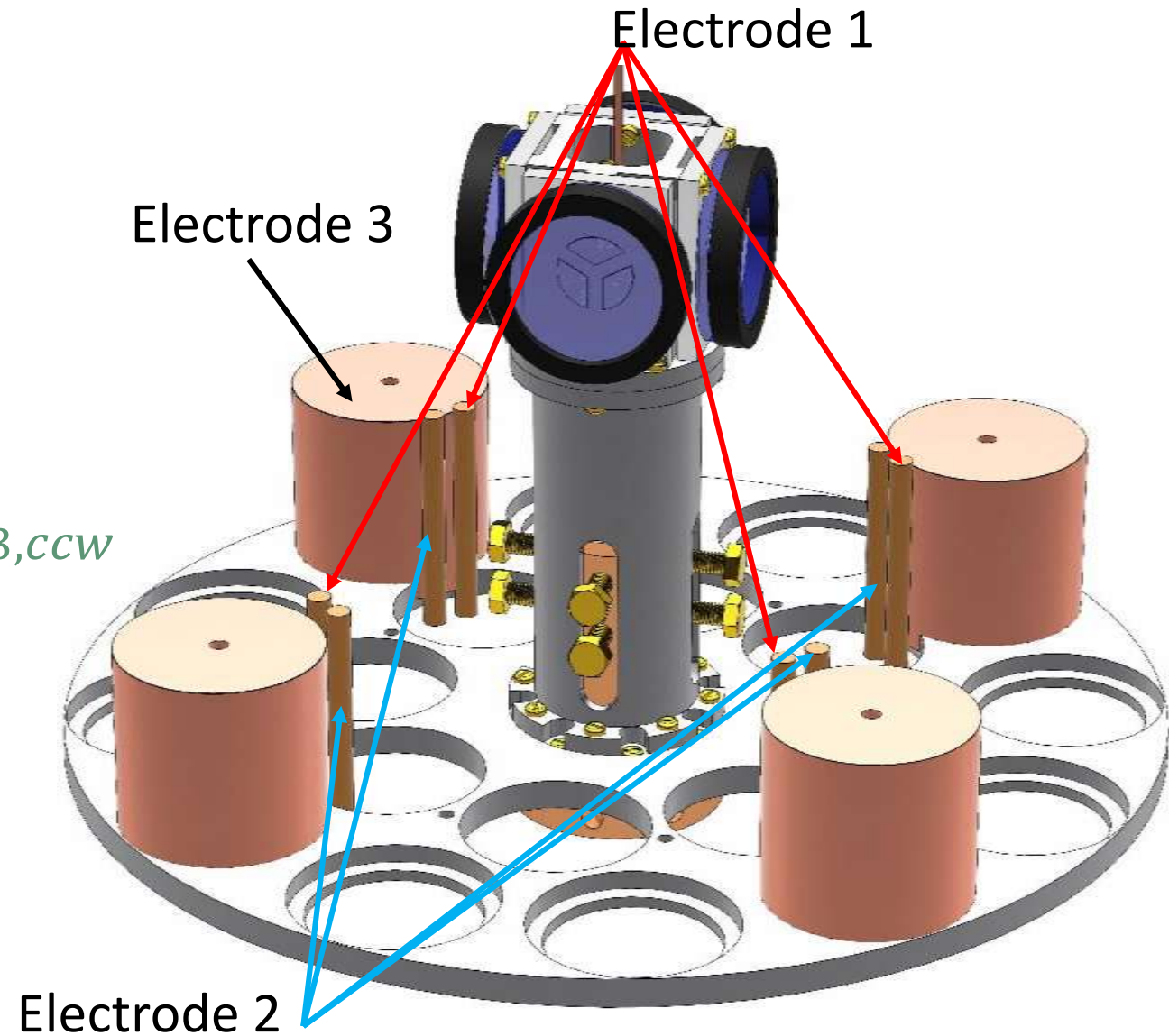
$k_{12}, k_{13}, k_{23}, V_{13,cw}, V_{23,cw}, V_{13,ccw}, V_{23,ccw}$

Geometry and masses:

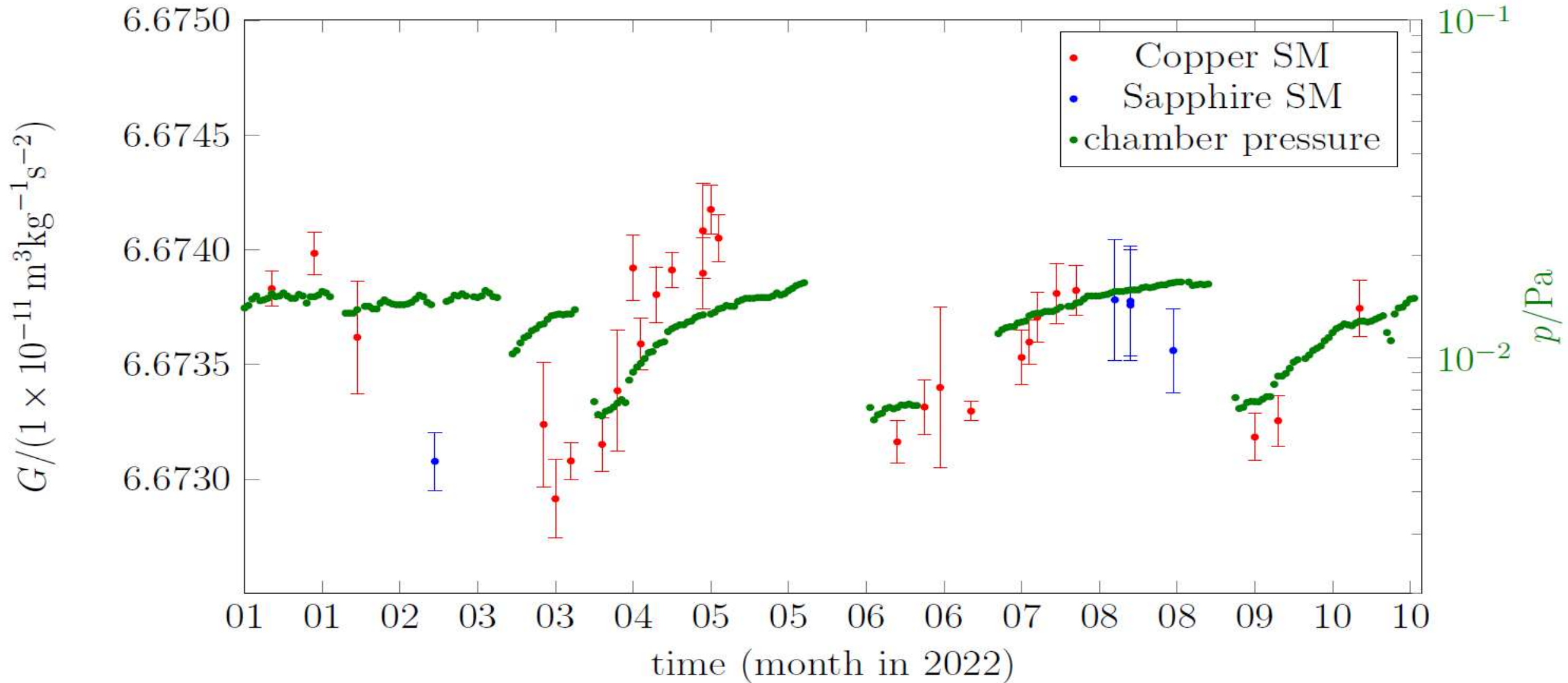
m, M, r, R

Note:

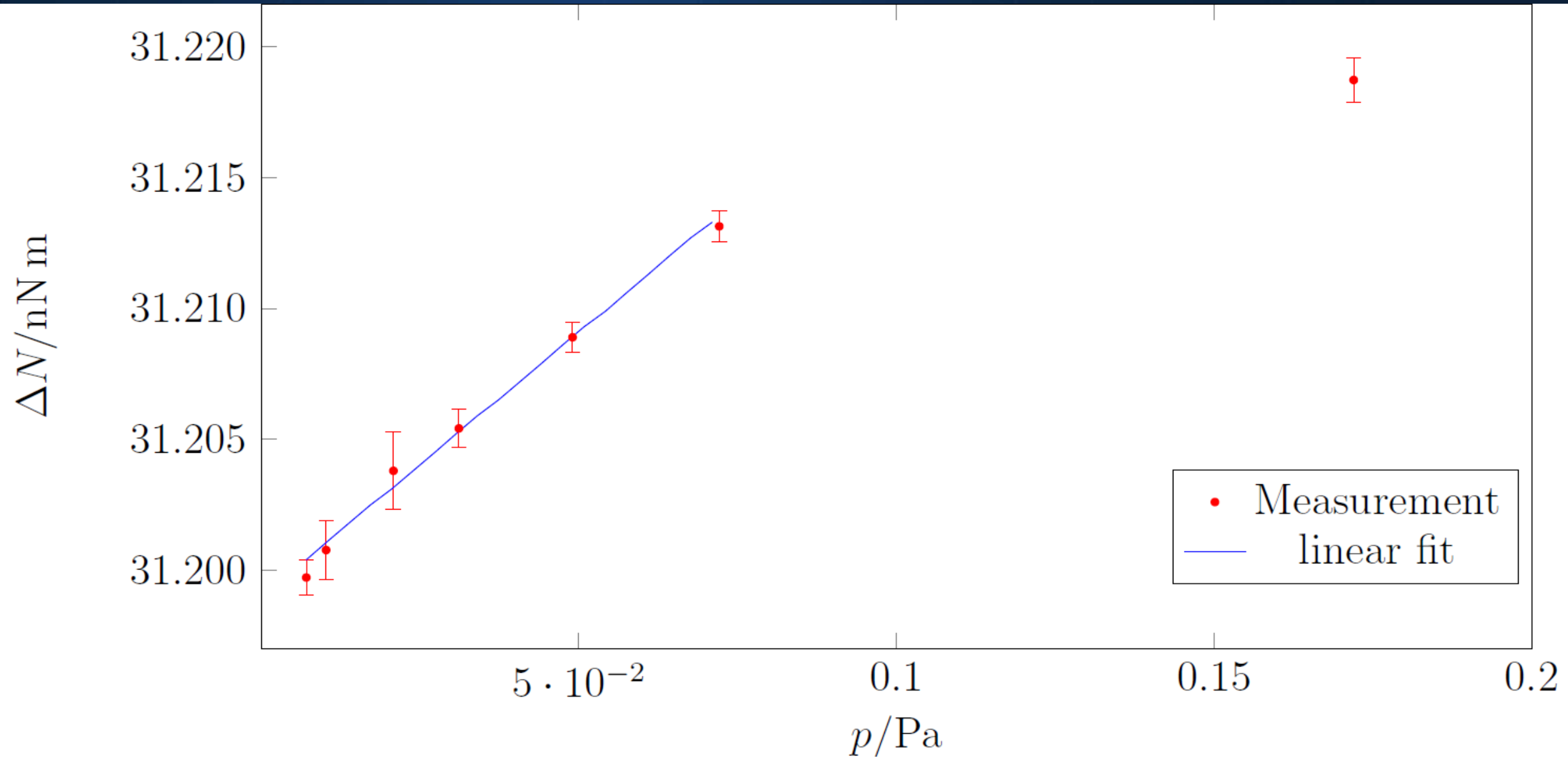
$$k_{ij} = \frac{\partial C_{ij}}{\partial \varphi}, V_{12} = V_{13} - V_{23}$$



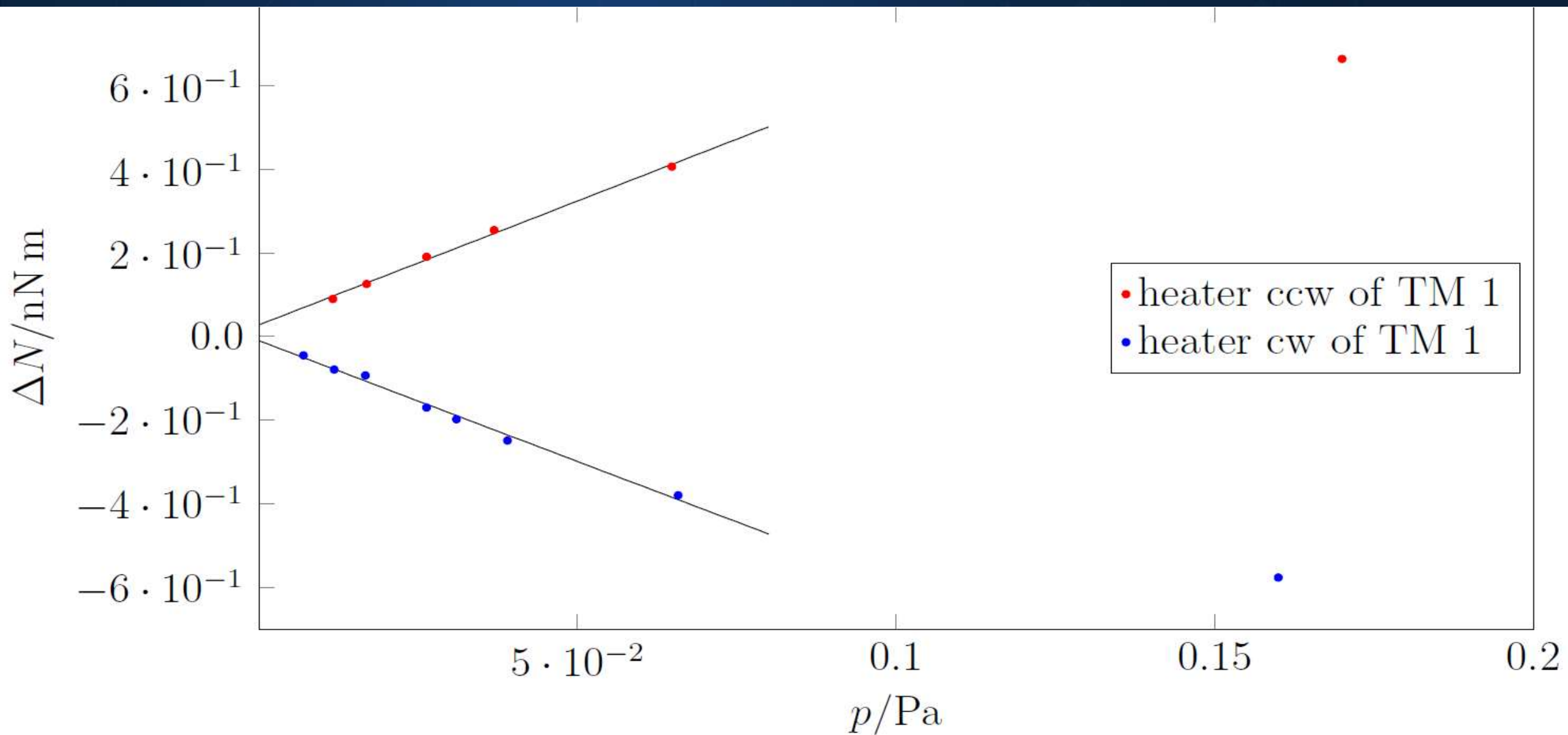
I wish, I could give you a new value for G



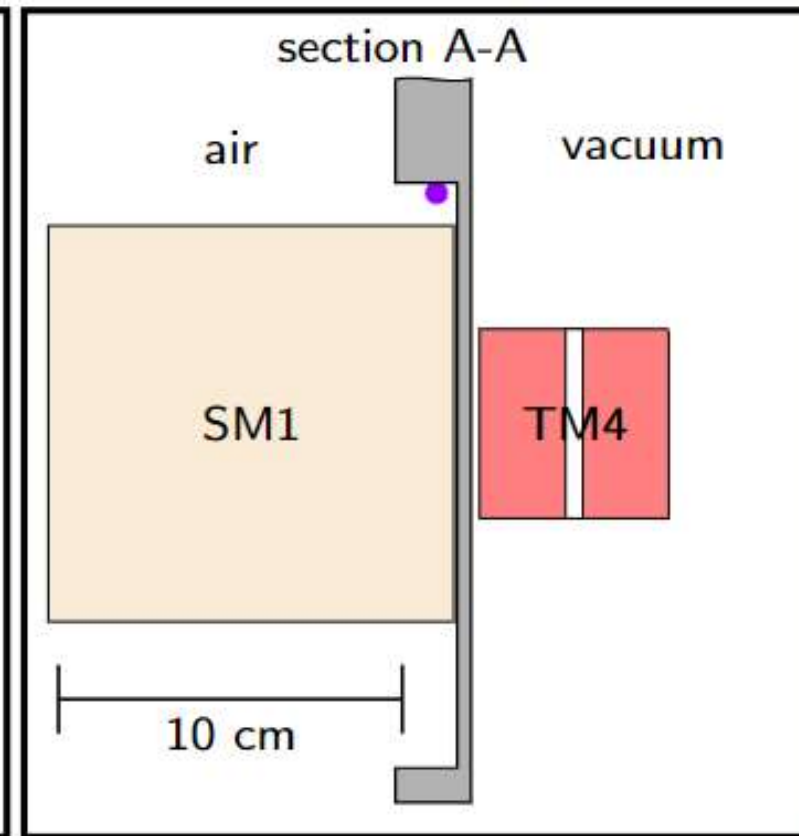
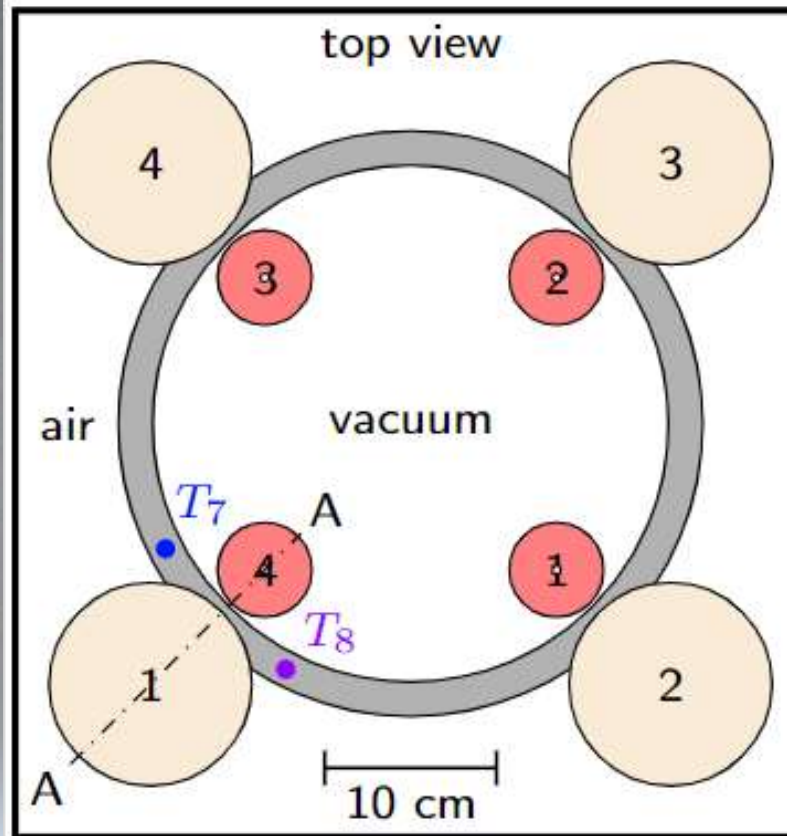
A more systematic investigation



Torque generated by heat



Crooke's radiometer



$$\Delta N_{spur} = p \Delta T$$

- How it all hangs together, new SI.
- How to measure forces.
- Persist!

Postdoc opportunity for an Electrical Engineer/Physicist at the National Institute of Standards and Technology

- You're passionate about mixed-signal design, digital techniques, data acquisition, and analysis.
- You're curious about fundamental metrology, units, and measurements.
- Opportunity to work with a world-class team on digital techniques to compare unlike impedances (C to R) with very small relative uncertainties (10^{-7} or better).
- Work will be conducted at NIST's Gaithersburg campus in Maryland – with at most one remote day per week.
- Overview: <https://www.nist.gov/programs-projects/farad-and-impedance-metrology>
- Patent pending technology:



Reach out to one or both us:

Yicheng Wang

yicheng.wang@nist.gov

Stephan Schlamminger

Stephan.Schlamminger@nist.gov

Thank you for your attention.



NIST

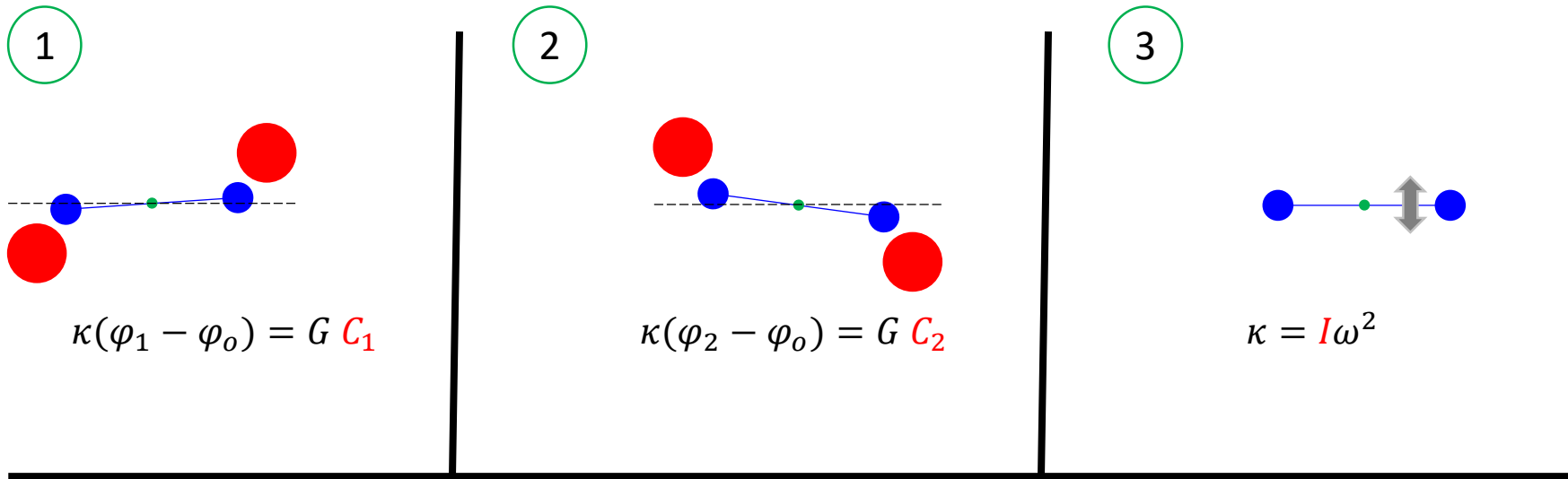
National Institute of
Standards and Technology
U.S. Department of Commerce



Remarkable features

- Two methods, one instrument: Cavendish and electrostatic servo.
- Torsion strip, not a round fiber.
- Hexadecapole mass configuration.

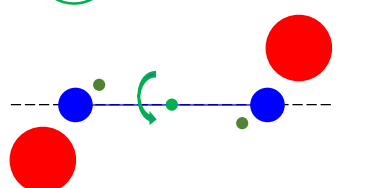
Cavendish method



$$G = \frac{(\varphi_1 - \varphi_2)\omega^2}{C_1 - C_2} I$$

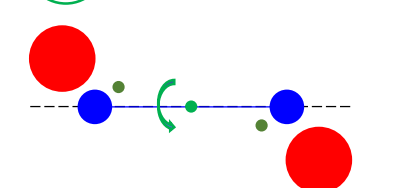
Servo method

1



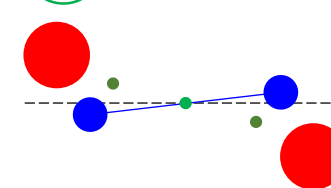
$$\kappa\varphi + \frac{1}{2} \frac{dC}{d\varphi} V_1^2 = G C_1$$

2



$$\kappa\varphi + \frac{1}{2} \frac{dC}{d\varphi} V_2^2 = G C_2$$

3

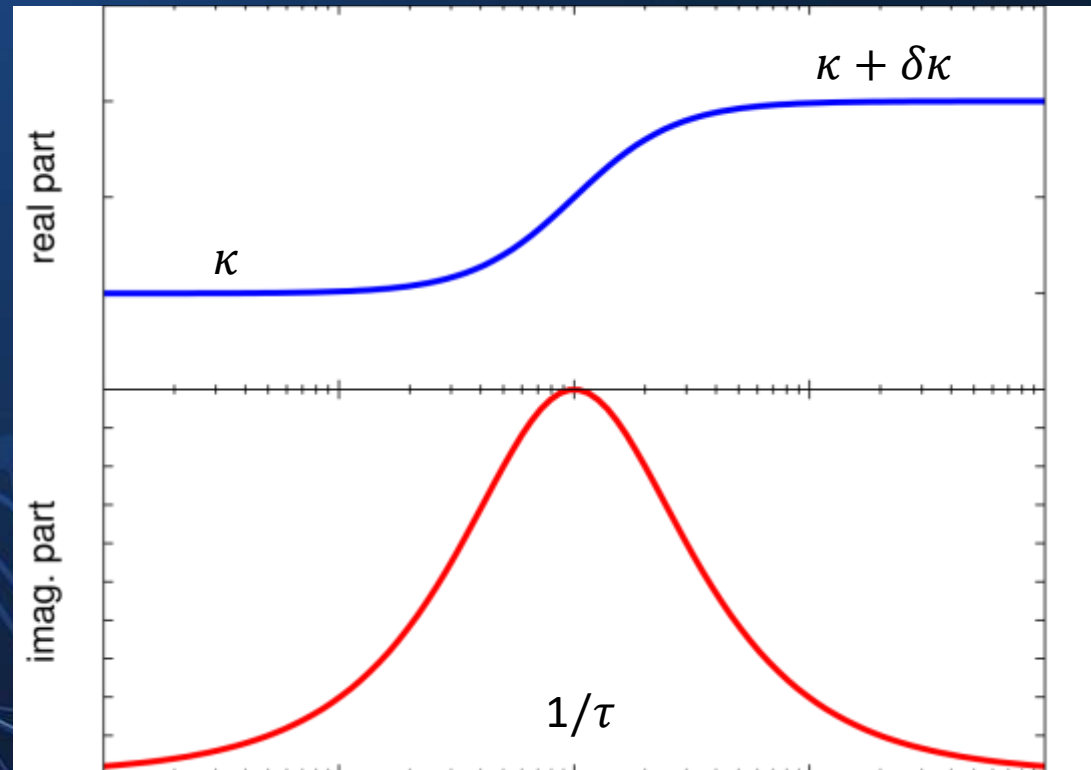
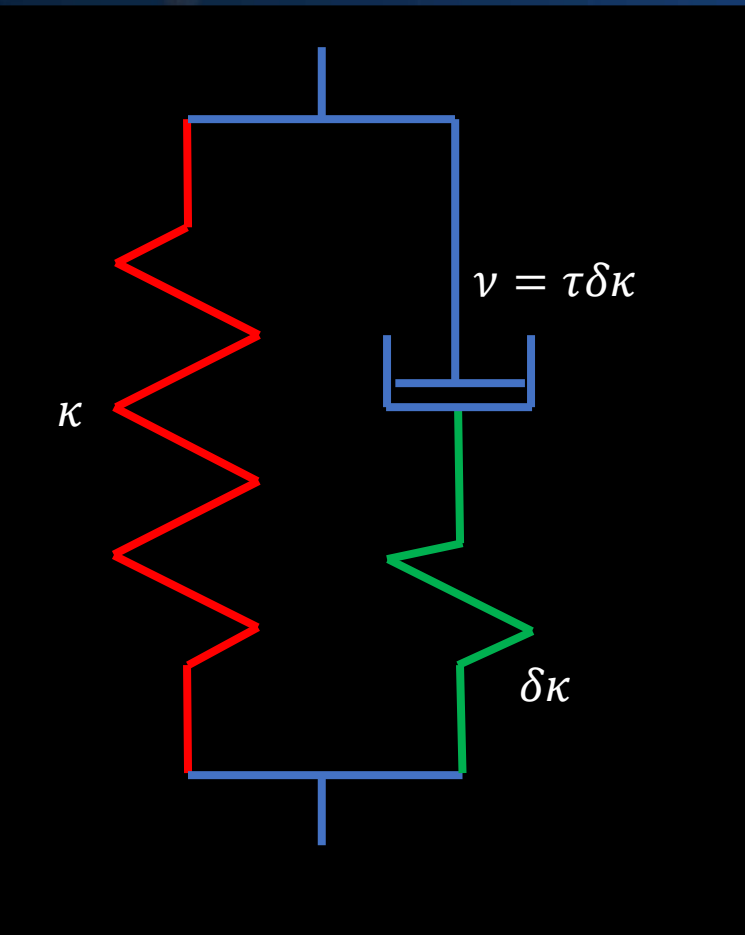


$$C(\varphi) \Rightarrow \frac{dC}{d\varphi}$$

$$G = \frac{1}{2} \frac{dC}{d\varphi} \frac{(V_1^2 - V_2^2)}{C_1 - C_2}$$

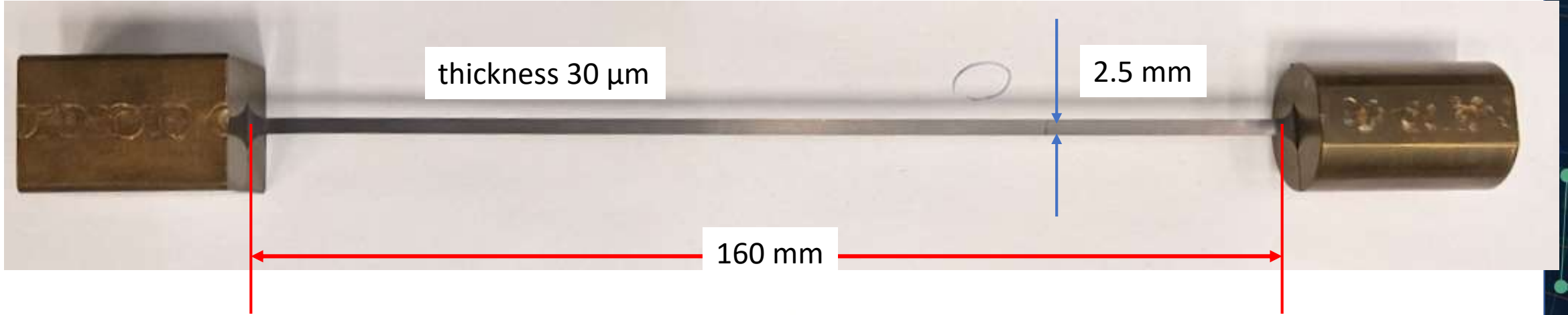
Mark I: DC servo
Mark 2: AC servo

Why is the Cavendish method trouble?



$$\frac{\sigma_G}{G} = \frac{1}{2Q}$$

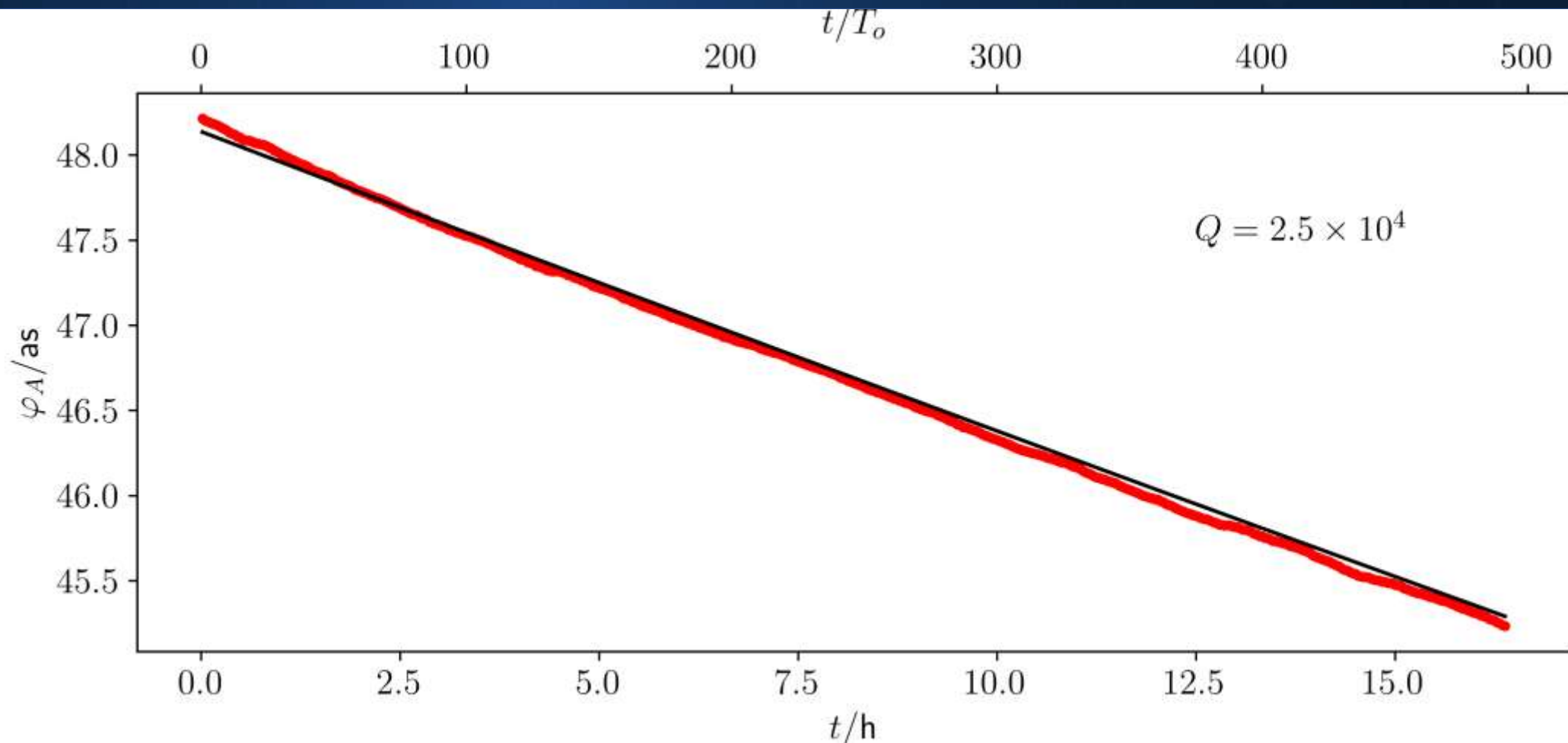
Torsion strip



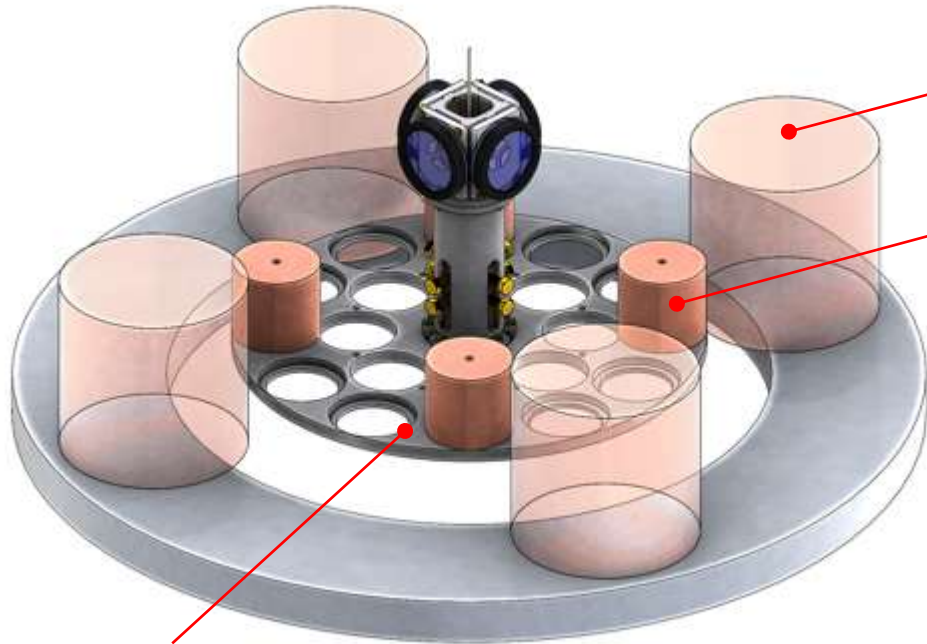
$$k = \underbrace{\frac{bt^3 F}{3L}}_{\text{elastic 4\%}} + \underbrace{\frac{M_p g b^2}{12L}}_{\text{conservative 96\%}} = 0.204 \text{ mNm}$$

can carry 9.9 kg

High Q



hexadecapole mass arrangement



disk, 1.78 kg

source mass $M = 11.2$ kg, located at $R = 214$ mm

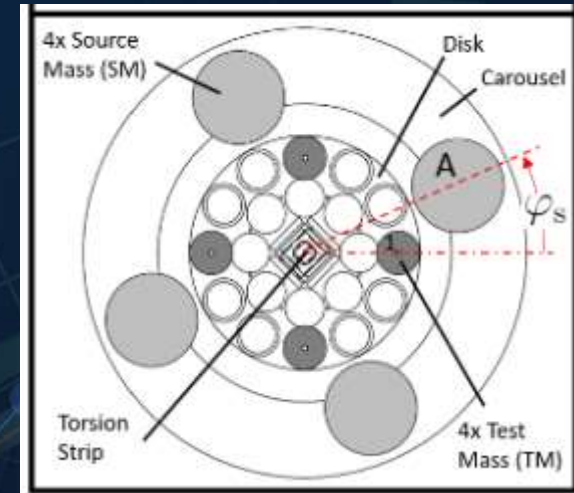
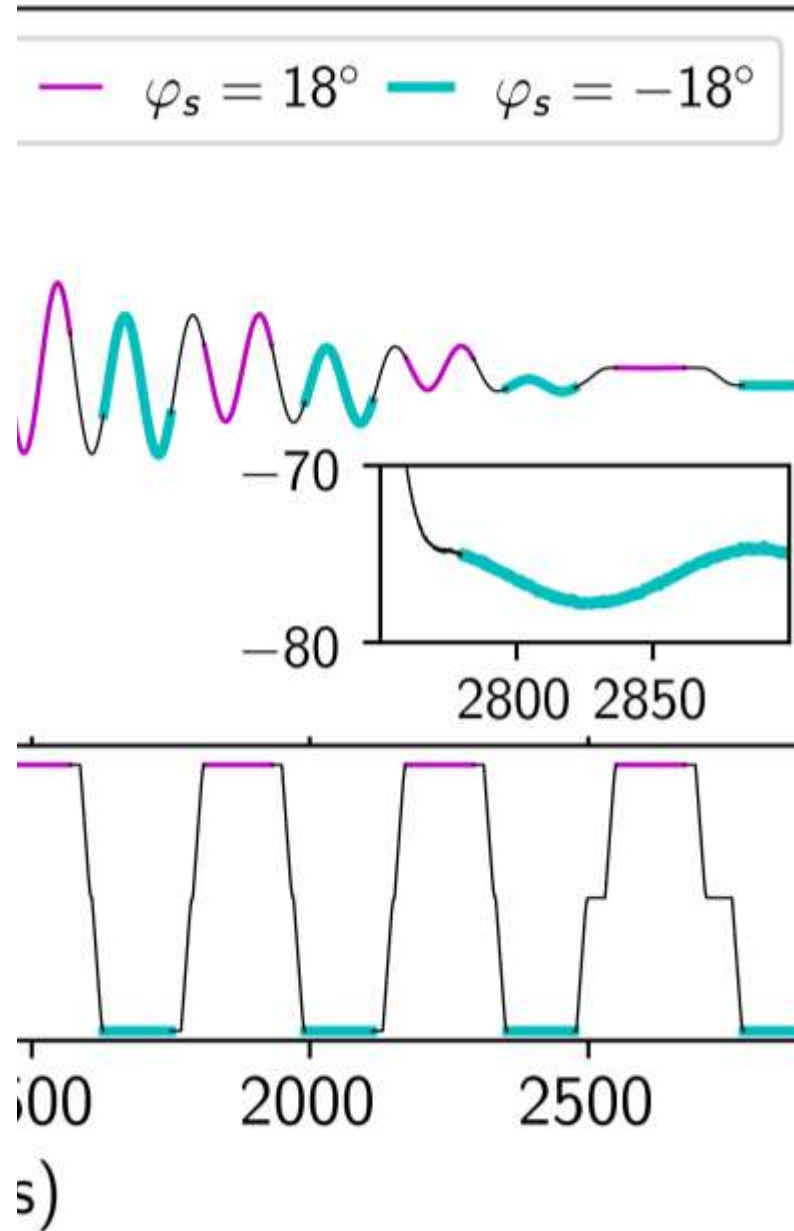
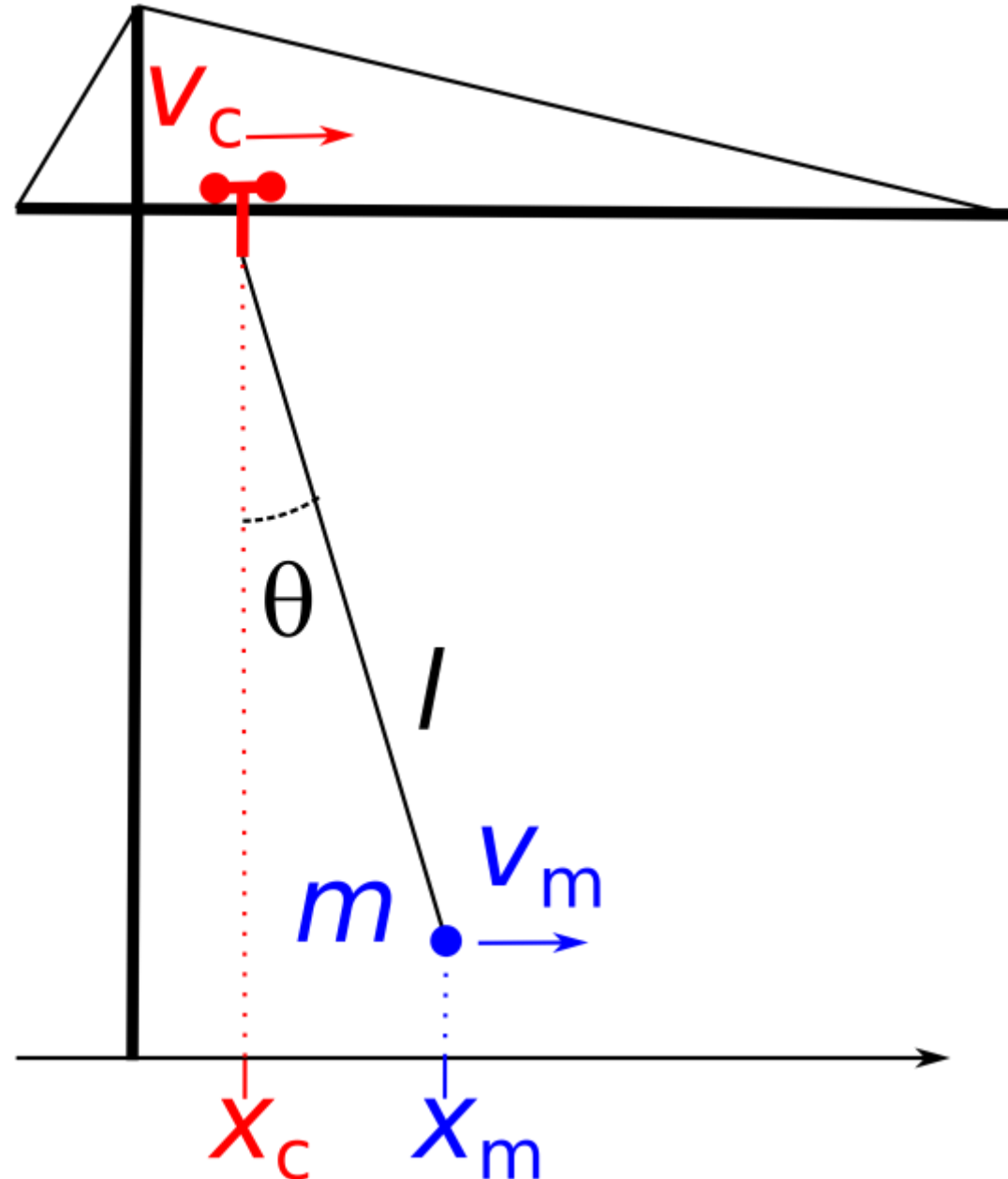
test mass $m = 1.15$ kg, located at $r = 120$ mm

$$N_{grav} = G 35 M m \frac{r^4}{R^5} \sin 4\theta = N_0 \sin 4\theta$$

$$N_0 \approx 13.9 \text{ nNm}$$

$$\frac{\sigma_N}{N} = \sqrt{2} 10^{-5} \Rightarrow \sigma_R = 0.43 \text{ } \mu\text{m} \text{ and } \sigma_r = 0.3 \text{ } \mu\text{m}$$

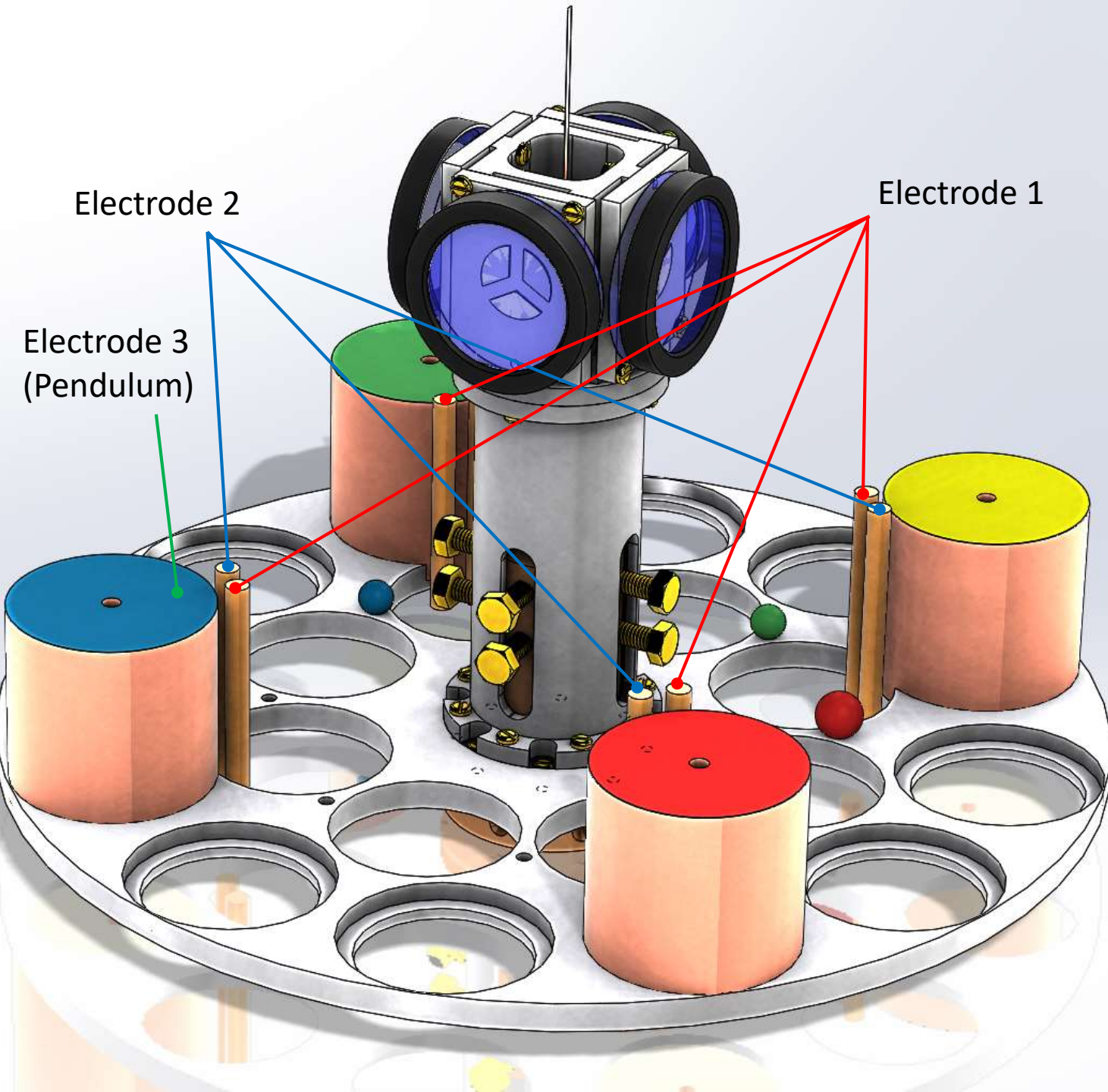
Our contribution to crane safety



“The Crane Operator's Trick and other Shenanigans with a Pendulum”,
Am. J. Physics.

Current status of the experiment

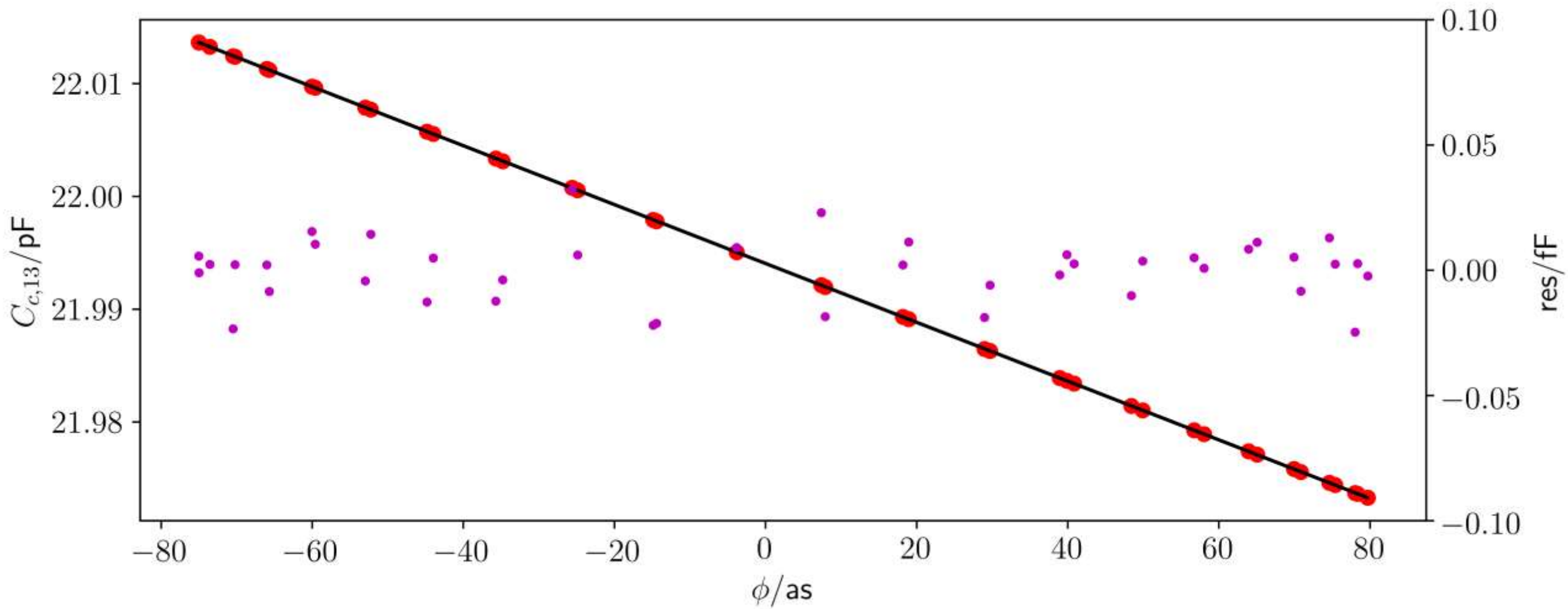
- We found a spurious torque that is proportional to .
- Electrostatic data since September 2021, work in progress.
- Uncertainty analysis in progress.
- Mass integration in progress.

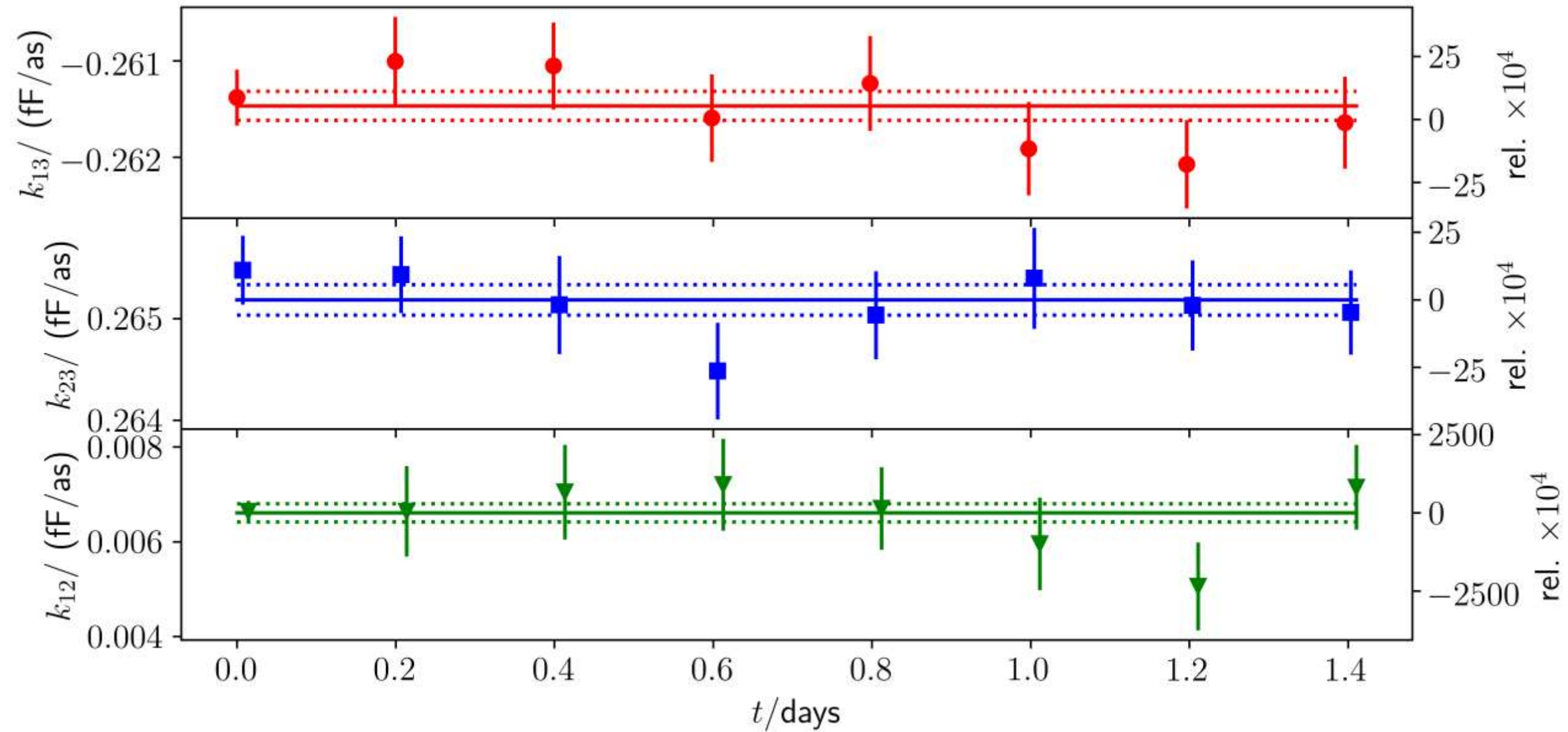


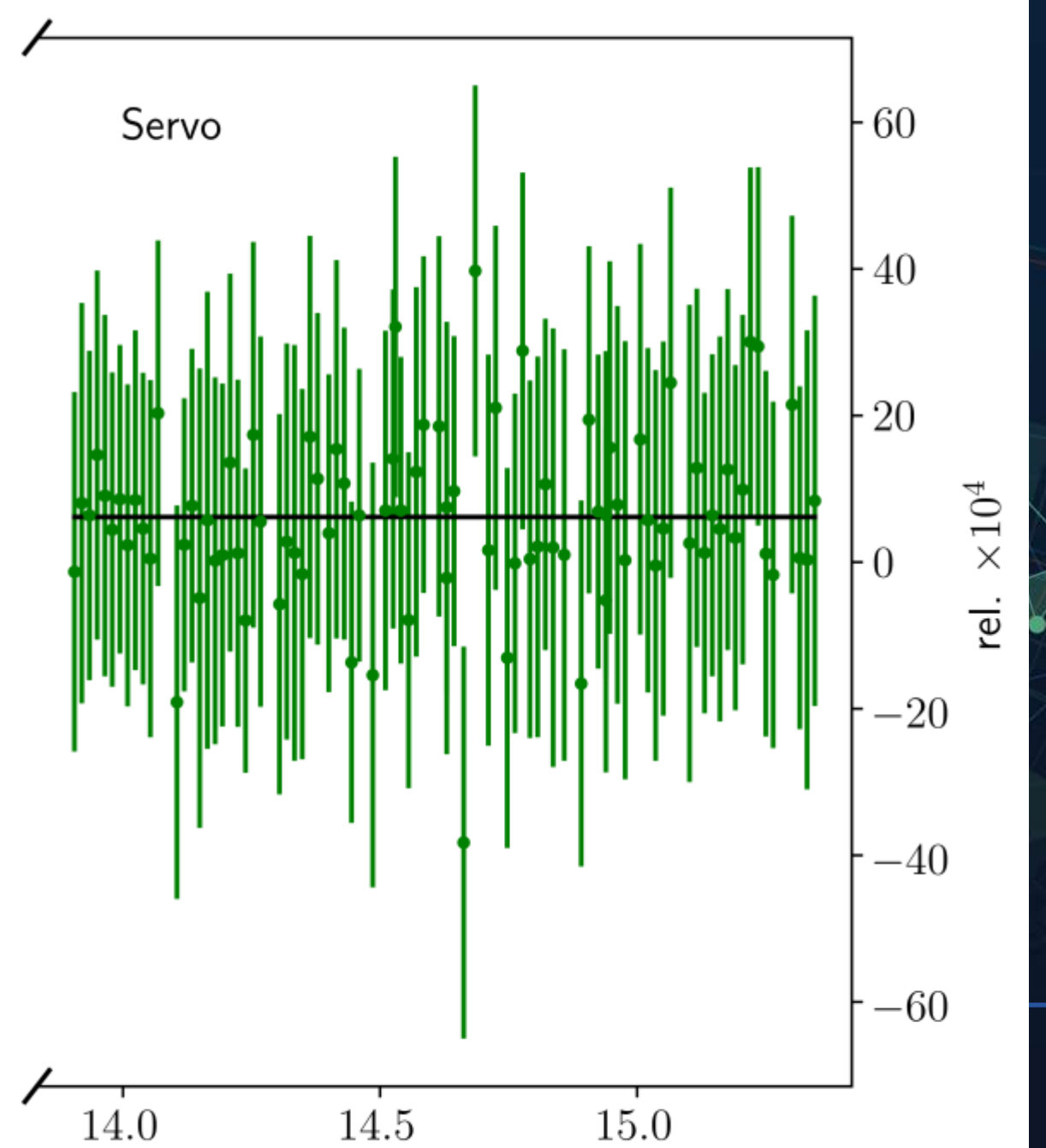
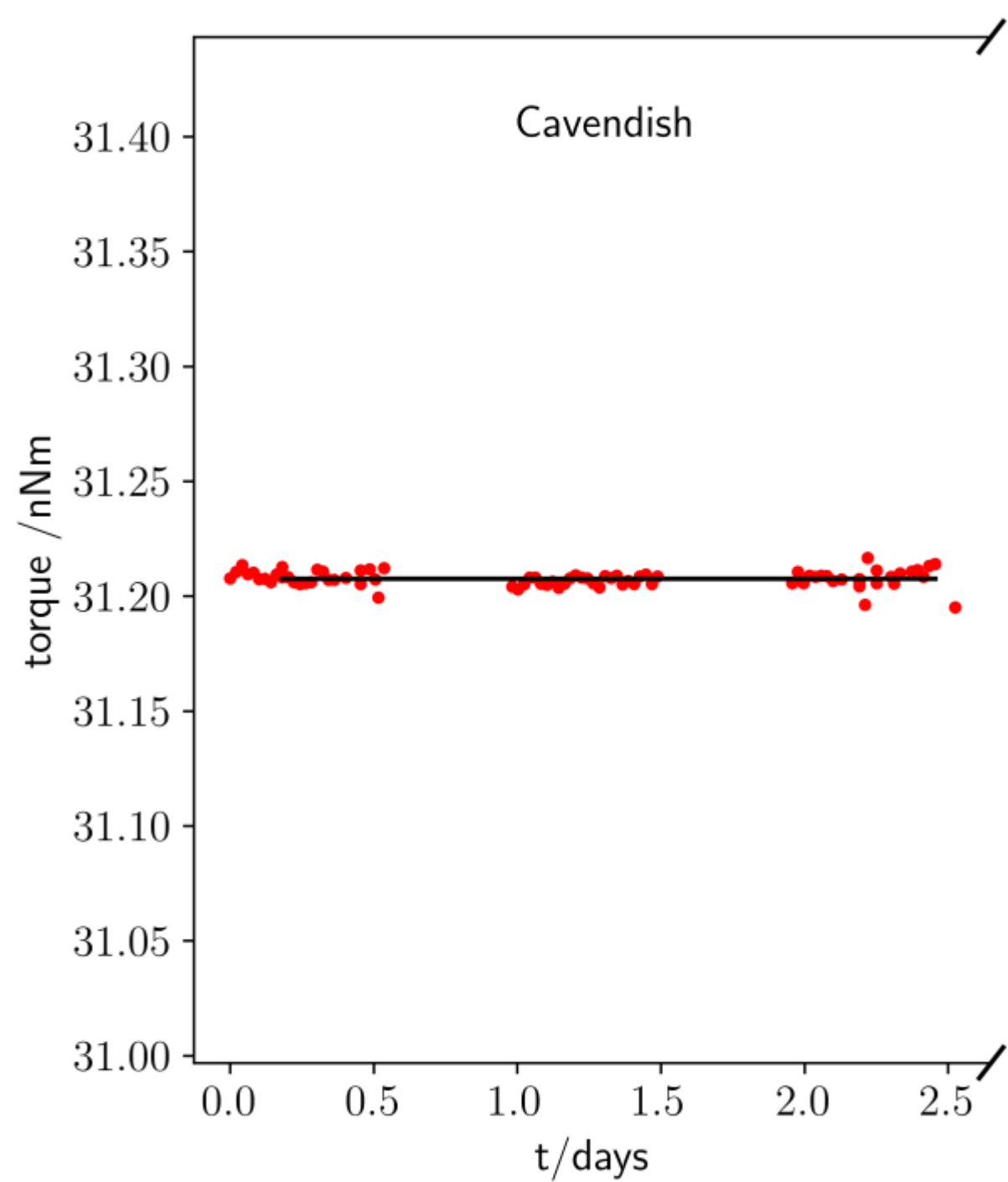
$$W = \frac{1}{2} \left(C_{c,12} (V_1 - V_2)^2 + C_{c,13} (V_1 - V_3)^2 + C_{c,23} (V_2 - V_3)^2 \right)$$

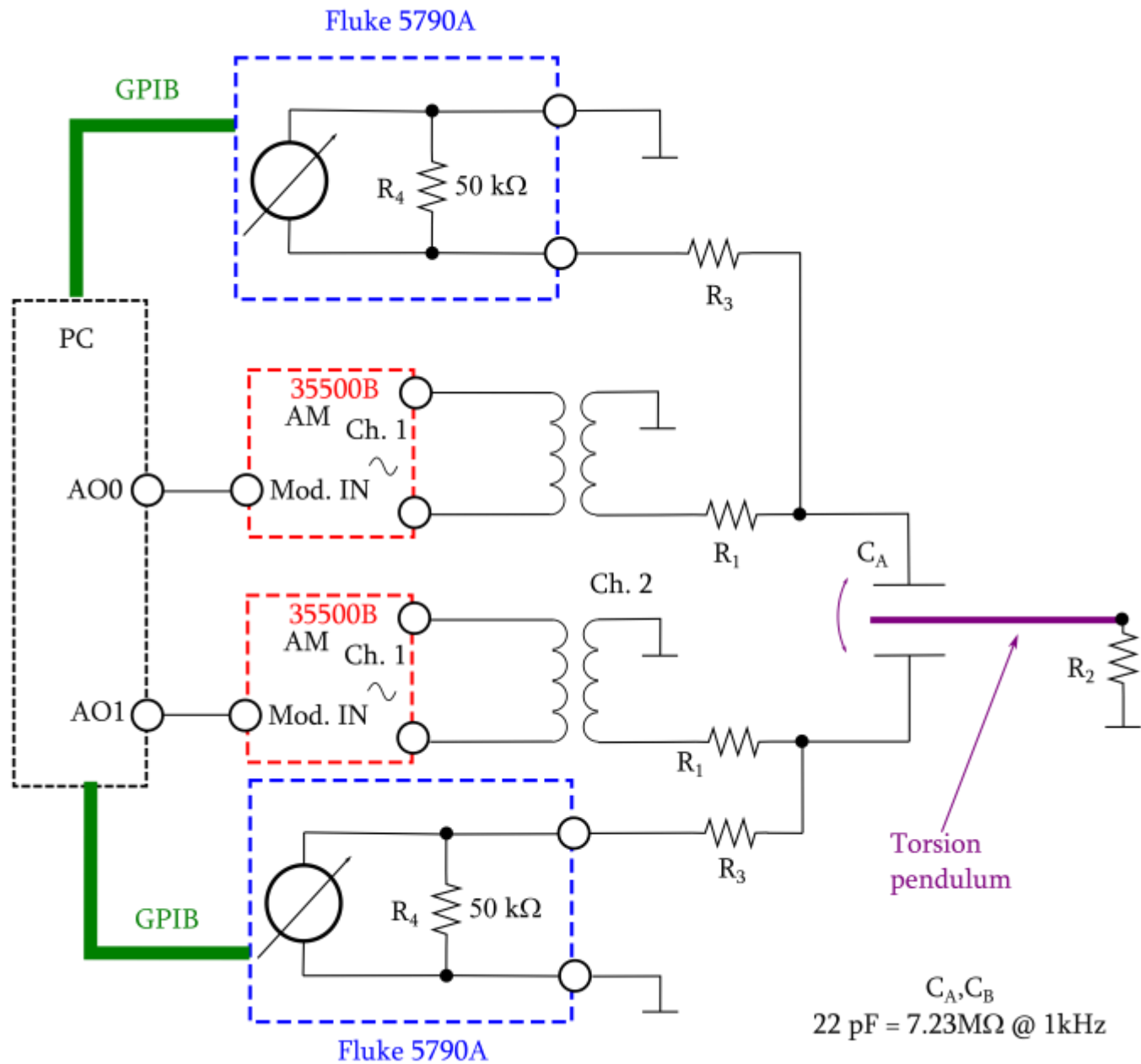
$$N = -\frac{dW}{d\theta} = -\frac{1}{2} \left(\frac{dC_{c,12}}{d\theta} (V_1 - V_2)^2 + \frac{dC_{c,13}}{d\theta} (V_1 - V_3)^2 + \frac{dC_{c,23}}{d\theta} (V_2 - V_3)^2 \right)$$

$$k_{ij} = \frac{dC_{c,ij}}{d\theta}$$









- as of now Cavendish and Servo method do not agree.
 - Electrostatic method produces data that is noisier.
 - stay tuned.
 - Blind measurement.
- public unblinding in 2022.

big G status

- We are making good progress with data taking.
- The noise in the servo mode is still high.
- A bias exists in the Servo mode. It has to be understood.
- We hope to have all data by the end of the year.
- The experiment is blind. We will unblind publicly in 2022, possibly @ APS April.

Summary

- Small forces and torques can be measured precisely using electromagnetic or electrostatic compensation.
- Relative uncertainties of 10^{-8} are possible.
- Applications include mass and torque calibration.
- The measurement of G and g depends on these techniques.

Thank you for your attention.

