Measuring gravity with satellites

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1. Precision gravity measurements before the space age

The Solar System: a lab for testing Newton’s theory of gravity

- Make predictions for the motion of planets and satellites using Newton’s theory of gravity (1687)
- Use the best available technology (+ your ingenuity in dealing with systematic and random errors) to make *measurements (observations)* as precise and accurate as possible
- If confirmed, use theory to determine unknown parameters.
- If not confirmed: Is something missing (e.g. “Neptune”, $J_{2\odot}$)? Or, does the theory need to be amended/changed?

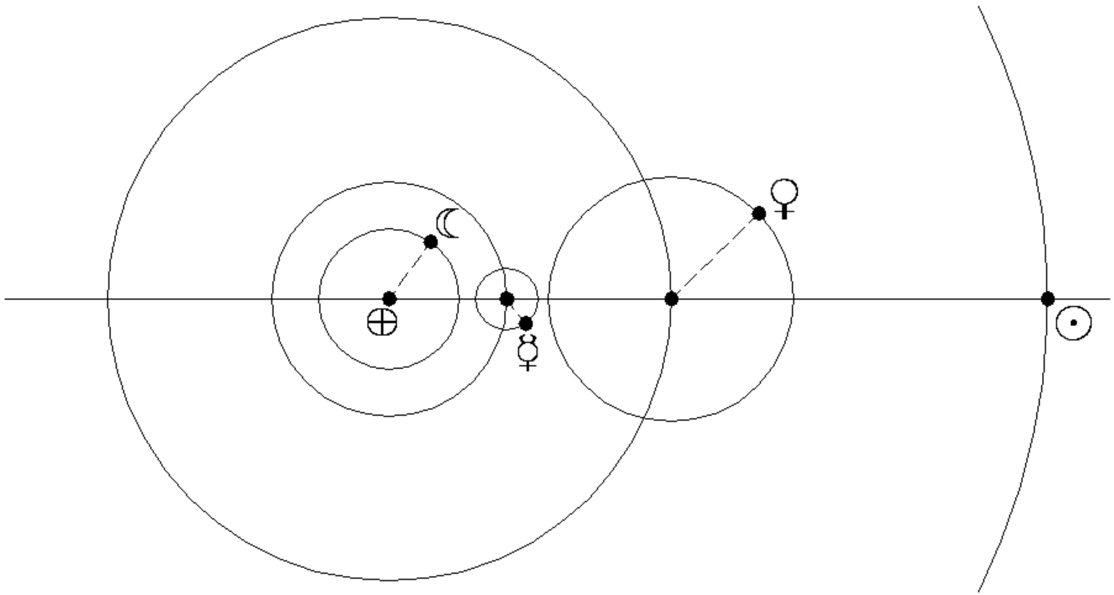
This is Celestial Mechanics and how it became the paradigm of exact science

15 centuries before Newton: Tolomeo

Claudio Tolomeo, “The Almagest” (The greatest, ~ 150 AD) by combining circular orbits, all run at uniform angular velocity, builds a model of all known non-fixed stars (the “errant” stars, 7 including the Sun) centered on a fixed Earth which can predict their angular positions to $2°$!

- The model/theory has a prediction power
- The accuracy of measurements is ensured by periodicity (as usual...)

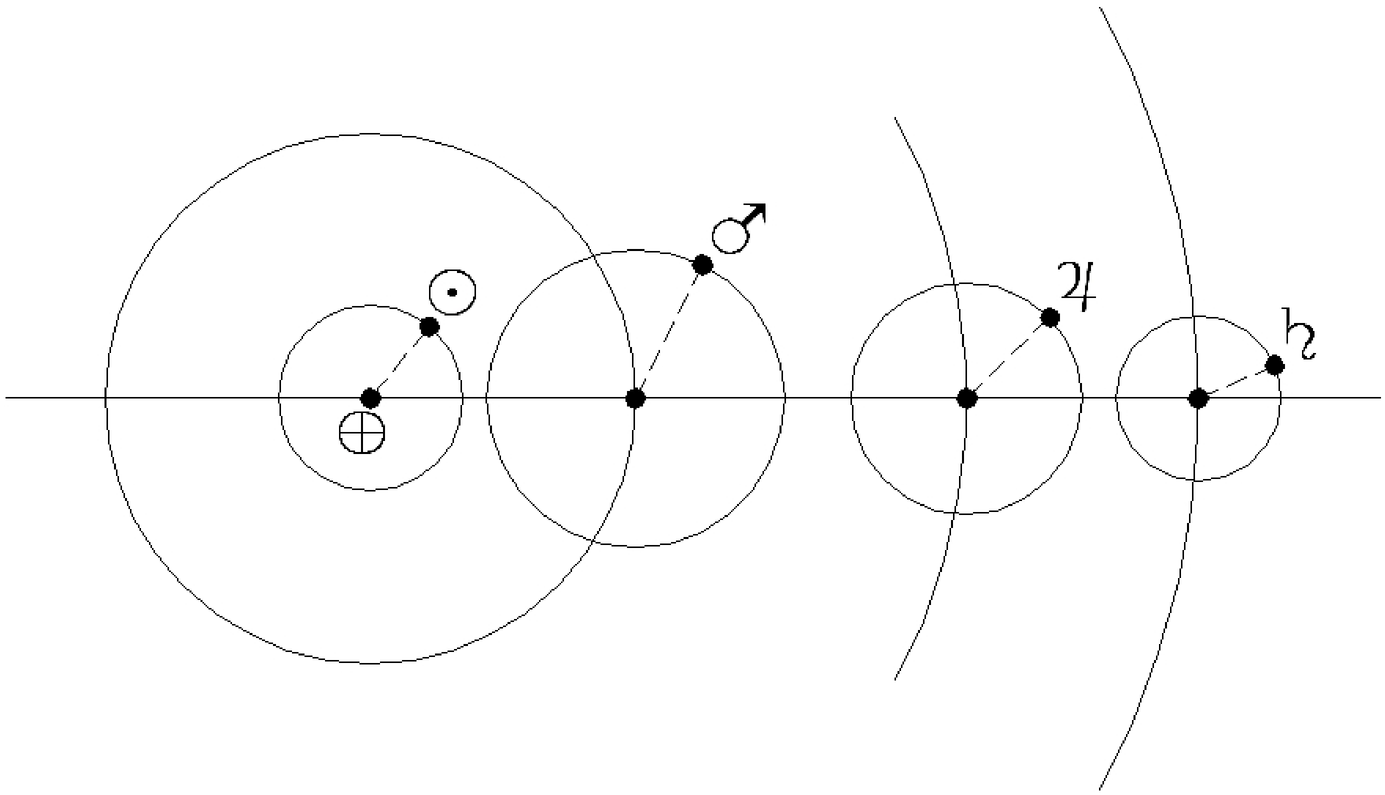
Tolomeo: Earth, Moon, Mercury, Venus, Sun



With appropriate $\omega_{1\odot}$, $\omega_{2\odot}$, $r_{1\odot}$, $r_{2\odot}$ (epicycle and deferent for the Sun) he can also account for the different time interval between subsequent equinoxes (due in fact to $e_{\oplus} \simeq 0.016$).

The problem is Mercury's large eccentricity $e_{☿} \simeq 0.21$ ....

Tolomeo: Earth, Sun, Mars Jupiter, Saturn



Cutting edge technology & a clever idea

Ulug Beg Observatory, Samarkand 1429-1449

Biggest quadrant (sextant?): the instrument was anchored to very deep rock to reduce seismic vibration noise

In 20 years provided the most precise observations since Hipparcos (129 BC) and Tolomeo (150 AD)

Science needs technology & money Uraniborg observatory

- Tycho Brahe obtains from Frederick II of Denmark construction and funding of Uraniborg Obesrvatory (Uraniborg=Castle of Urania, Castle of the Sky - Uranus was discovered 200 yrs later) on the (Swedish) island of Ven, ~ 1576
- With the best and most precise instruments of its time provides systematic observations to 2 arcmin accuracy. They clearly show the limits of both Tolomeo and Copernicus (De revolutionibus orbium coelestium, 1543) models
- 1588: Fredrick II dies
- 1599: Tycho moves to Prague (no more money...) and all Uraniborg data pass on to his student Kepler

Jump to Newton's Principia, 1687

G. B. Shaw honouring Einstein, London 1930

"Napoleon, and other great men of his type, they were makers of empire... but there is an order of men who get beyond that, they are not makers of empire but they are makers of Universe, their hands aren't stained with the blood of any human being on Earth ... Ptolemy made a Universe which lasted 14 hundred years ... Newton also made a Universe which lasted 3 hundred years ... Einstein has made a Universe and I can't tell you how long that will last."

Newton's theory of gravity & Celestial Mechanics

- Only the 2-body problem (with perfect spherical symmetry of the bodies) is integrable. For $N > 2$ and/or non spherical bodies, the problem is no longer integrable!
- Yet, just because we are still here and the Solar System has not destroyed itself, it is made of 2-body (or restricted 3-body) subsystems (hierarchical structure) + *"small" perturbations* (mind the Moon however...)

$$\Downarrow$$

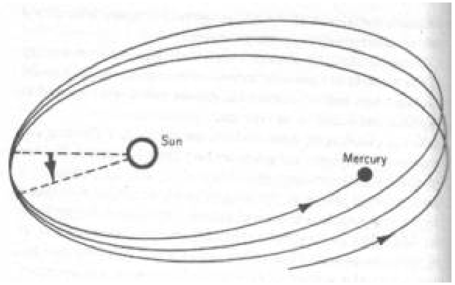
this is the contribution of the great celestial mechanicians of the 18th and 19th century: perturbation theory & "numerical" integration!

Neptune, 1845
The triumph of Newton's gravity and Celestial Mechanics

Neptune was found where Adams and Le Verrier had (independently!) predicted in order to explain the anomalies observed in the theoretical motion of Uranus as compared to observations.

J. Galle, in Berlin, first discovered Neptune thanks to a very good star catalogue that his observatory had just completed: *once he was told where to look*, he had a very good "map" of fixed stars where the "intruder" could be identified

Mercury the bad guy (as usual...)



The perihelion of Mercury ($e_{☿} \simeq 0.21$ ..very large..) *would not change in time within the 2-body Sun-Mercury problem (and spherically symmetric Sun), but precesses because of perturbations from other planets*

Predicetd by newtonian celestial mechanics :

- $278''$ /century due to Venus
- $153''$ /century due to Jupiter
- $90''$ /century due to Earth
- $10''$ /century due to other planets

$531''$ /century .... $\sim 43''$ /century missing!!!

Planet Vulcan? *Ring of mass particles?* $\dfrac{1}{r^{2.000000154}}$?

GR contribution to Mercury's perihelion precession

$$\dot{\omega}_{GR} = \frac{6\pi G M_\odot}{Pa(1-e^2)c^2}$$

It depends upon the measured values of AU and speed of light:

$$A_{1976} = 1.4959787 \times 10^{11}\ \mathrm{m}$$
$$c_{1976} = 299,792,458\ \mathrm{ms}^{-1}$$

$$\dot{\omega}_{GR_{☿}} = 42.98\left(\frac{A}{A_{1976}}\right)^2\left(\frac{c}{c_{1976}}\right)^{-2}\ ''/100\,\mathrm{yr}$$

Nobili and will, Nature 1986

Einstein calculated the contribution from GR to the perihelion advance of Mercury in 1915 and found that it adds $\sim 43''/century$, almost exactly the value that celestial mechanicians had been unable to reconcile with observations for so long.

"For a few days I was beside myself with joyous excitement"

Einstein's discovery relied on the very precise prediction & measurement of the much larger classical effect!

... but the Sun is not spherically symmetric

$J_{2\odot} = \frac{C-A}{MR^2_{e\odot}} \neq 0$ rotation implies flattening

$$\dot{\omega}_{J_{2\odot}\mercury} = -\frac{3}{4} n J_{2\odot} \frac{R^2_{e\odot}}{a^2} \frac{1-5\cos i}{(1-e^2)}$$

Can $\dot{\omega}_{J_{2\odot}\mercury}$ be neglected compared to $\dot{\omega}_{GR\mercury}$?

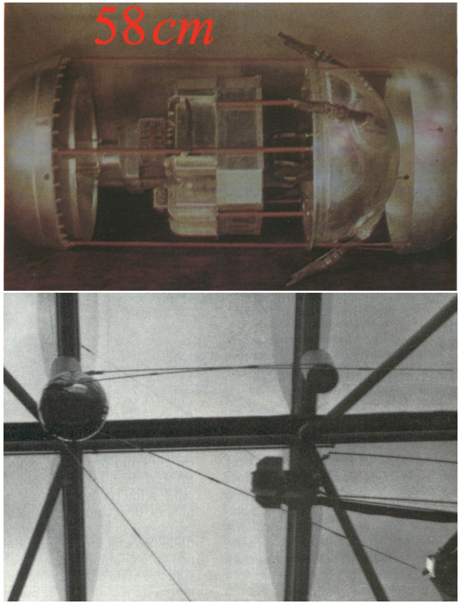
Measuring $J_{2\odot}$ very hard...
Dicke tried because Brans-Dicke theory alternative to GR predicted a smaller contribution to Mercury's perihelion advance

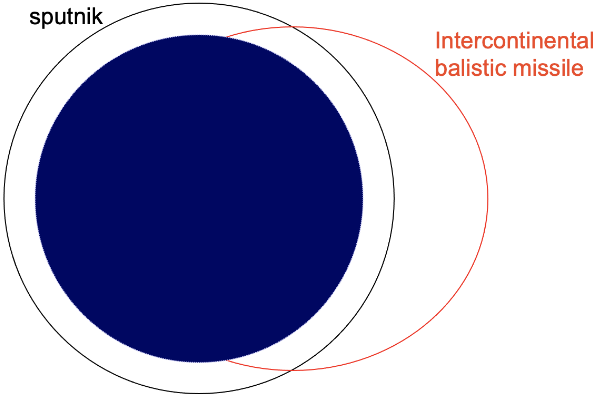
Only at the end of the 1980s helioseismology with the SOHO space mission gave $J_{2\odot}$ small enough to make $\dot{\omega}_{J_{2\odot}\mercury}$ dominate over $\dot{\omega}_{GR\mercury}$

Solar oscillations depend on $J_{2\odot}$, and their measurement was possible only in space

2. The birth of the space age and the original sin of space science

Sputnik: first artificial satellite to orbit the Earth and the original sin of space science





Small, low altitude, s/c
$58\,\mathrm{cm}$ radius $\sim 80\,\mathrm{kg}$ mass

Why a "national trauma" in the US?

$$E = -\frac{GM_\oplus}{2a}$$

The energy (per unit mass) of any object orbiting the Earth depends only on its semimajor axis $\Downarrow$

The soviet rocket that launched Sputnik could as well launch a nuclear warhead to Washington!

Sputnik
A national trauma & an opportunity for space science

"For those of us who remember the national trauma following the successful launch of the first Soviet satellite on October 4, 1957 there is little doubt that the military uses of space have provided the most powerful incentives of our subsequent efforts"

James Van Allen, January 1986

President Eisenhower and the foundation of NASA

- October 4, 1957: Sputnik launched
- 1958: *Science Advisory Committee* (SAC) appointed by Eisenhower
  SAC advises to set up NASA
- 1958: NASA founded
- SAC publishes *"The introduction to outer space"*, which divides scientific objectives in 4 categories: Early, Later, Still later, And much later still.
  Physics is 1st in the Early category with the words:

"Physicists are anxious to run one crucial and fairly simple gravity experiment as soon as possible. This experiment will test an important prediction made by Einstein's general theory of relativity, namely that a clock will run faster as the gravitational field around it is reduced." (GP-A)

↑

Gravity Probe-A: launch an atomic clock to high altitude and compare it with an identical clock on ground

3. GP-A and the controversy on gravitational redshift vs UFF/WEP

GP-A
The first fight over a space mission in fundamental physics

Does a measurement of the gravitational redshift really test GR?

- L. Schiff (AJP, Jan 1960)
  Gravitational redshift can be derived solely from WEP/UFF (Weak Equivalence Principle/Universality of Free Fall) and Special Relativity (both very well tested) <u>and is not a test of GR</u>

  "Terrestrial or satellite experiments that would go beyond supplying corroborative evidence for the equivalence principle and special relativity would be very difficult to perform, and would, for example require a frequency standard with an accuracy somewhat better than 1 *part in* 10^{18}*"*

  $\uparrow$

  (depends on the level of confirmation of WEP/UFF: at that time the best experiments by Eötvös had reached 10^{-8} – 10^{-9})

Does a measurement of the gravitational redshift really test GR? (II)

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- Questioned by Dicke (AJP, same issue, following page)
  Experimental evidence of UFF/WEP for ordinary matter does not necessarily apply to clocks

- Schiff's note added in proof (AJP, end of his paper, page before Dicke's one)
  "The Eötvös experiments show with considerable accuracy that the gravitational and inertial mass of normal matter are equal. This means that the ground state Hamiltonian for this matter appears equally in the inertial mass and in the interaction of this mass with a gravitational field. It would be quite remarkable if this could occur without the entire Hamiltonian being involved in the same way, in which case a clock composed of atoms whose motions are determined by this Hamiltonian would have its rate affected in the expected manner by a gravitational field".

- **1964**: Dicke changed his mind (*The Theoretical Significance of Experimental Relativity, Les Houches Lectures*):
  "The red shift can be obtained from the null result of the Eötvös experiment, mass energy equivalence, and the conservation of energy in a static gravitational field and static coordinate system."...

  .. and made the first WEP test after Eötvös reaching 10^{-11} in the field of the Sun, pushed by Braginsky to 10^{-12} in 1972 (**move signal from DC to diurnal frequency!**)

Roll, Krotkov & Dicke, Annals of Physics, 1964

Braginsky & Panov, Sov. Phys. JEPT, 1972

Does a measurement of the gravitational redshift really test GR? (III)

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- PRD 1973: Shiff's note of 1960 was formalized by Thorne et al. and became known as "Schiff's conjecture" despite a strong argument he had with Schiff and Thorne at the 1970 Caltech-JPL Conference on experimental tests of gravitational theories. Schiff was could never go back to the issue in writing because he died 2 months later

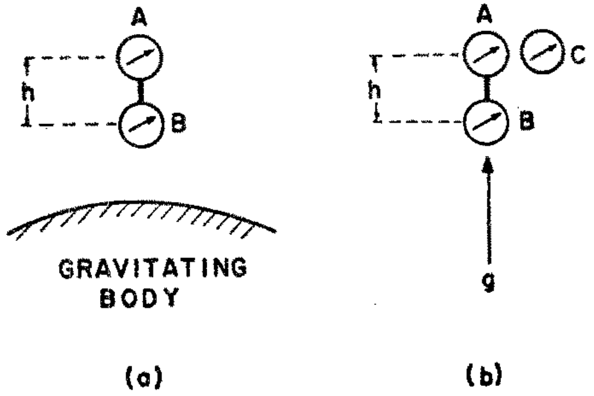
- PRD 1975: Nordtvedt, 1 one year before GP-A is launched

  WEP violation might affect clocks more strongly than ordinary masses, depending on amount of energy rearranged in generating the frequency standard. For H maser clock (as in GP-A) he estimated a stronger violation possibly by $\simeq 10^4$.

  In the meantime WEP/UFF had been confirmed by Dicke & *Braginsky experiments to* 10^{-11} – 10^{-12}

  $\downarrow$

  GP-A should now measure gravitational redshift to better than 10^{-7} – 10^{-8} ...no way...



(a) Two identically-constructed clocks, A and B, are at rest in a gravitational field

(b) The gravitating body is replaced by an upward acceleration g of clocks A and B in empty space; a stationary clock C is used to compare their frequencies

If WEP/UFF hold for all bodies, including clocks, (a) and (b) are "locally" equivalent ($h \ll R_\oplus$, tides are negligible)

Schiff chooses (b): clock C ticks with period T; clock A passes by clock C with velocity v_A and period T_A; clock B passes by clock C with velocity v_B and period T_B

Time dilation of Special Relativity ensures:

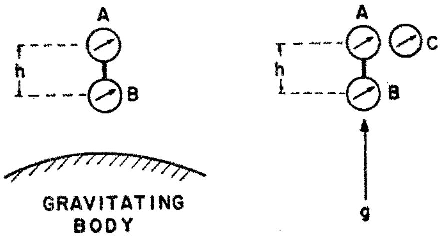
$$T_A = \frac{T}{\sqrt{1-\frac{v_A^2}{c^2}}}\,,\ T_B = \frac{T}{\sqrt{1-\frac{v_B^2}{c^2}}}$$

$$(v_B^2 = v_A^2 + 2gh)$$

To first order in gh/c^2:

$$T_B \simeq T_A\left(1+\frac{gh}{c^2}\right)$$

Schiff's derivation of the gravitational redshift (II)



(a) (b)

If h is not negligible (non uniform gravitational field) Schiff reasons in (a) and replaces gh with the gravitational potential difference between clock A and clock B at distance r_A, r_B from the center of mass of the gravitating body:

$$\Delta\Phi = -\frac{GM}{r_A} + \frac{GM}{r_B} > 0$$

Requires to perform a series of inter-comparisons between a number of identical clocks arranged in such a way that the gravitational field is nearly uniform from one to the next. Then (to first order):

$$\frac{\Delta\nu}{\nu} \simeq \frac{\Delta\Phi}{c^2}$$

Clock B at "lower altitude" is red-shifted w.r.t. clock A at "higher altitude"

Requires only WEP and Special Relativity; it does not require conservation of energy nor mass-energy equivalence

Measurements of the gravitational redshift as (poor) tests of WEP/UFF

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...but if WEP is violated, even if clocks are identical, they are attracted by the source body with a gravitational acceleration different then in case of no violation:

$$M_g = M_i(1+\eta_e) \qquad m_g = m_i(1+\eta_c)$$

$$g' = g(1+\eta+\mathcal{O}(\eta^2)) \quad (g = \frac{GM_i}{r^2} \quad \eta = \eta_e + \eta_c)$$

$$\Delta\Phi' = \Delta\Phi(1+\eta+\mathcal{O}(\eta^2))$$

Since clocks are equally attracted, Schiff's argument still holds, and the correct expression for gravitational redshift is

$$\left(\frac{\Delta\nu}{\nu}\right)_\eta = \frac{\Delta\Phi}{c^2}(1+\eta+\frac{7}{2}\frac{\Delta\Phi}{c^2}) + \mathcal{O}(\eta^2\frac{\Delta\Phi}{c^2}) + \mathcal{O}(\eta\frac{\Delta\Phi^2}{c^4}) + \mathcal{O}(\frac{\Delta\Phi^3}{c^6})$$

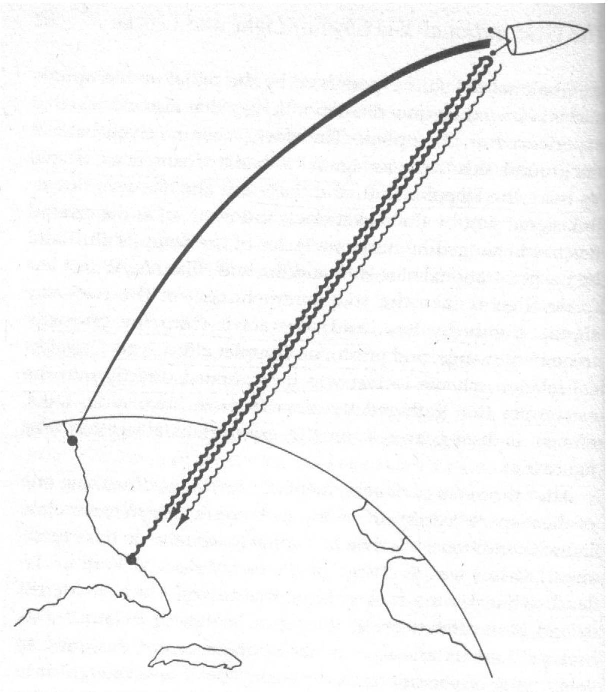
With $\Delta\Phi/c^2 \simeq 4.3\cdot 10^{-10}$ in GP-A, and $\eta \sim 10^{-8}$– 10^{-9} at the time of Schiff's paper, the requirement was a prohibitive frequency standard with an accuracy better than 10^{-18}!

...GP-A would only provide corroborative evidence for the equivalence principle. With current very much improved η the gap between the two kind of measurement has even widened...

Nobili et al. AJP, 2013

1976: The GP-A space experiment

Sub-orbital flight
Only about 2-hr duration



A clever way to cancel 1st order Doppler effect!!

As a hydrogen maser clock is sent aloft on a SCOUT D rocket, a signal from an identical maser clock is sent toward it. When the signal is received by the rocket it is sent back (*two-way signal not affected by redshift & Doppler: ground clock is emitter & receiver*), and a signal directly from the rocket clock is sent along with it (*one-way signal affected by redshift & Doppler: ground clock fixed, rocket clock at different position and moving*).

$\Downarrow$

take the frequency change of 2-way signal, divide by 2 and subtract it from 1-way frequency change

GP-A measurement of gravitational redshift

$$\left(\frac{\Delta\nu}{\nu}\right)_{GP-A} = [1 + (2.5 \pm 70)\cdot 10^{-6}]\cdot\left(\frac{\varphi_s - \varphi_e}{c^2} - \frac{|\vec{v}_s - \vec{v}_e|^2}{c^2} - \frac{\vec{r}_{se}\cdot\vec{a}_e}{c^2}\right)$$

Why tests of UFF/WEP are more accurate than measurements of gravitational redshift by so many orders of magnitude?

UFF/EP tests can be performed as *null experiments*

$\eta = \frac{\Delta a}{a}$ is derived from the differential acceleration Δa of the test bodies freely falling with average acceleration a.
If the experiment measures directly the differential acceleration, this gives η directly: no experiment signal, no violation (to the level of noise); the smaller the signal (or the noise), the better the test.

No precise prediction must be made to which the measured signal should be compared in order to obtain the physical quantity of interest

A measurement of gravitational redshift is an *absolute measurement*.

The measured frequency shift had to be compared with the sum of the 3 terms at the right hand side, whose values depend on various physical quantities, some of which to be measured during the experiment itself.

It is only by comparing the theoretical prediction and the measured shift that the authors could establish the ratio $[1 + (2.5 \pm 70)\cdot 10^{-6}]$.

No wonder it took 4 years to publish the results of an experiment that lasted only about 2 hours (Vessot et al. PRL, 1980)!

Müller, Peters & Chu ground measurement of gravitational redshift

Peters, Chung & Chu (Nature 1999) measured g (falling Cs atoms, atom interferometry):

$$\frac{\Delta g_{Cs}}{g} = 3 \cdot 10^{-9}$$

and compared it with g measured by FG-5 absolute gravimeter nearby (falling CCR glass, laser interferometry) testing UFF/WEP to

$$\eta_{Cs-glass} = (7 \pm 7) \cdot 10^{-9}$$

<u>With the same data</u> Müller, Peters & Chu (Nature 2010) measured the gravitational redshift (controversial..)

The frequency affected by gravitational redshift is the Compton frequency $\omega_C = m\hbar/c^2$ (m the rest mass of Cs atom). Gravitational redshift is recovered from the atom interferometry signal – which contains the local gravitational acceleration g – with g measured by the absolute gravimeter nearby (they need it in order to recover the frequency shift from the measured phase shift). The measured frequency shift differs from the predicted gravitational redshift by $\beta = (7 \pm 7) \cdot 10^{-9}$.

Since the mass-energy content is the full mass-energy of freely falling Cs atoms (as in 1999!) $\Rightarrow$ no Nordtvedt amplification factor $\Rightarrow$ it is a test of UFF/WEP

and they are getting exactly the same result $\eta_{Cs-glass} = (7 \pm 7) \cdot 10^{-9}$ as reported in 1999 (with the same data).

Do clocks obey UFF/WEP? *A ground based experiment*

In the gravitational field of the Sun the frequency rate of ground based clocks will be affected by gravitational redshift showing an annual variation. Clocks of different internal structure will show different annual variations if they violate UFF/WEP.

Ashby et al (PRL, 2007): Over a timespan of 7 years compared the frequencies of four H masers at NIST (USA) with one Cs fountain clock in the same lab, and also with three more Cs fountain clocks in Europe (in Germany, France and Italy).

The annual variation of the gravitational potential of the Sun is found to produce the same frequency shift on all pairs of clocks to

$$1.4 \cdot 10^{-6}$$

despite their different structure and also different location on Earth.

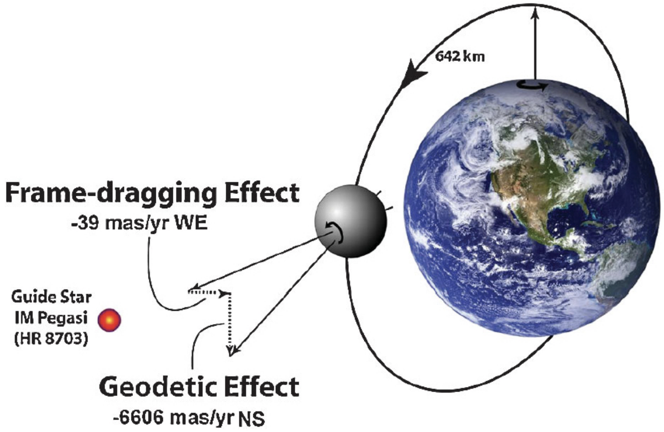
In a space experiment (e.g. STE-QUEST proposed mission) additional terms (to order $1/c^2$ and $1/c^3$) must be taken into account due to the motion of the clocks. Should a discrepancy be found, interpretation would be hard and highly disputable.

The theoretical bases of GP-B and Schiff's case for GP-B against GP-A

- *1915 Einstein* "Explanation of the Perihelion Motion of Mercury from the General Theory of Relativity" & "The Field Equations of Gravitation"
- *1917 De Sitter* "Planetary motion and the motion of the moon according to Einstein's theory" *(De Sitter precession, unrelated to rotation of the central body)*
- *1918 Lense & Thirring* "The influence of the *self rotation of central bodies* on the movements of the planets and the Moon according to Einstein's theory of gravitation"
- *1960-61 Schiff* In 3 papers Schiff solves the problem of the motion of a gyroscope according to Einstein's theory with a rotating central body and makes the case for GP-B as opposed to GP-A:
  measurements of the gravitational redshift (GP-A) do not test GR and would only corroborate UFF/WEP tests, while GP-B would measure the direct effects of GR on the precession of a spinning body

GP-B launched in 2004, 43 years later!

GP-B made simple

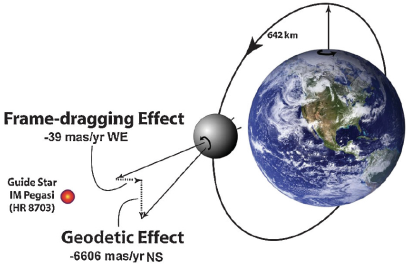


Geodetic (De Sitter) and Frame Dragging (Lense-Thirring) precessions (averaged over a polar orbit):

$$\vec{\Omega} = \frac{3}{2}\frac{(GM_\oplus)^{3/2}}{c^2 r^{5/2}}(\hat{r} \times \hat{v}) + \frac{1}{2}\frac{I_\oplus \omega_\oplus}{c^2 r^3}\,\hat{\omega}_\oplus$$

... but electrostatically suspended cryogenic rotors (to kill thermal distorsions & exploit supercunductivity for high precision readout) need a dewar of superfluid He to keep them at almost absolute zero temperature, drag-free control with very low noise thrusters, a telescope. In the end, more than $3000\,\mathrm{kg}$!

GP-B. The results and the unexpected



$$\Omega_{DSmeas} = -6601.8 \pm 18.3\,\mathrm{mas/yr}$$
$$\Omega_{DS-GR} = -6606.1\,\mathrm{mas/yr}$$

confirmed to 0.3%

(competitive with LLR)

$$\Omega_{LTmeas} = -37.2 \pm 7.2\,\mathrm{mas/yr}$$
$$\Omega_{LT-GR} = -39.2\,\mathrm{mas/yr}$$

confirmed to 19%

(not competitive with LAGEOS & LARES)

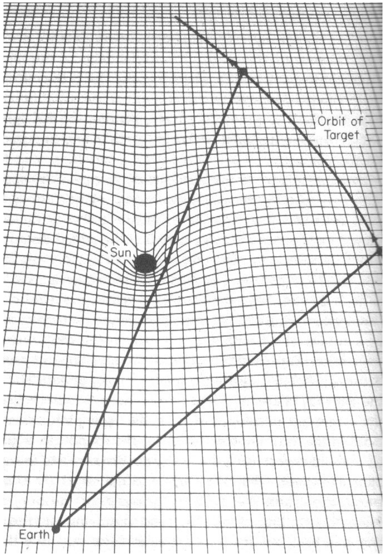
Everitt et al, PRL 2011

The gyro rotors and the gyro housing electrodes turned out to have surface patch potentials much higher than expected. The misalignment torque was explained with both patch potentials as high as $100\,\mathrm{mV}$ *(Note that the gap was only* $33\,\mu\mathrm{m}$*...)*

Buchman & Turneaure, RSI 2011

4. Fundamental Physics experiments while exploring the Solar System

How was it like in the early years
Measurement of the Shapiro time dealy



Mass tells spacetime how to curve, curved spacetime tells particles how to move… gravitational interaction is no longer there…

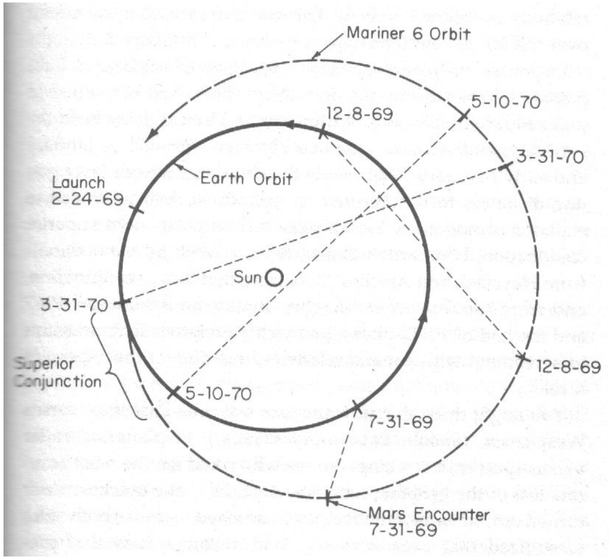
A radar signal to a planet (better to a s/c...) and back that passes near the Sun is delayed

Space-time is curved because of the mass of the Sun and this makes the signal travel a longer distance, giving rise to the

"Shapiro time delay"

computed by I. Shapiro in the early 1960s

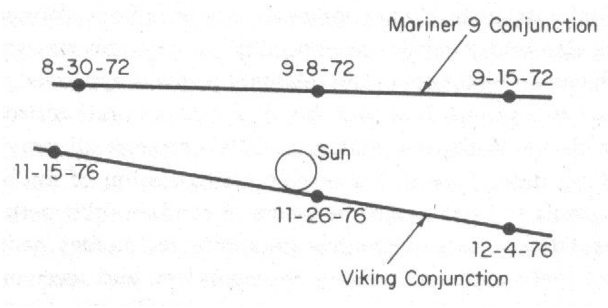
Shapiro time delay with Mariner 6



Measurements made at superior conjunctions of Mariner 6 s/c on March 31 and May 10, 1970
(Advantage of a transponder onboard the s/c...)

Experiment approved by NASA on December 8, 1969!!!

Shapiro time delay with Mariner 9 and Viking landers on Mars



Advantage of s/c: has a transponder .... but is affected by non-gravitational perturbations

Advantage of planet: non-gravitational perturbations are negligible
($\propto A/M \propto 1/R$ small for big planets, large for small s/c)
... but does not have a transponder

To get both advantages: anchor the s/c to the planet using an orbiter or, better, a lander

Measuring γ with CASSINI mission to Saturn

- CASSINI launched in 1997
- The deflection and delay of photons passing close to a mass is expressed by the γ parameter: $\gamma = 1$ in GR
- The longer the round trip travel time of the photons, the larger the effect
- Novel radio configuration to get rid of huge solar plasma noise: high frequency Ka-band i(n addition to X-band) and multi frequency link
- Measurement at solar conjunction in June 6-7, 2002:

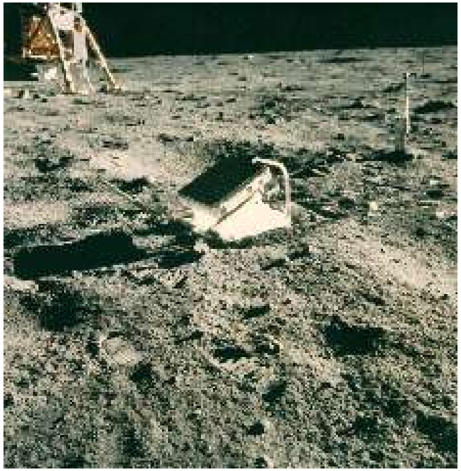
$$\gamma - 1 \simeq 10^{-5}$$

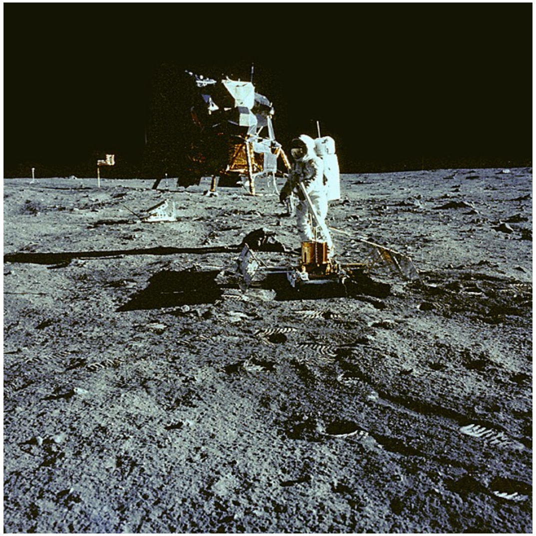
Bertotti et al. Nature, 2003

The race to the Moon and Lunar Laser Ranging

Q. <u>*NASA* (∼ 1968)</u>*:* “Is there a small, simple, possibly passive, instrument that you would like us to bring to the Moon to do some physics with it?”

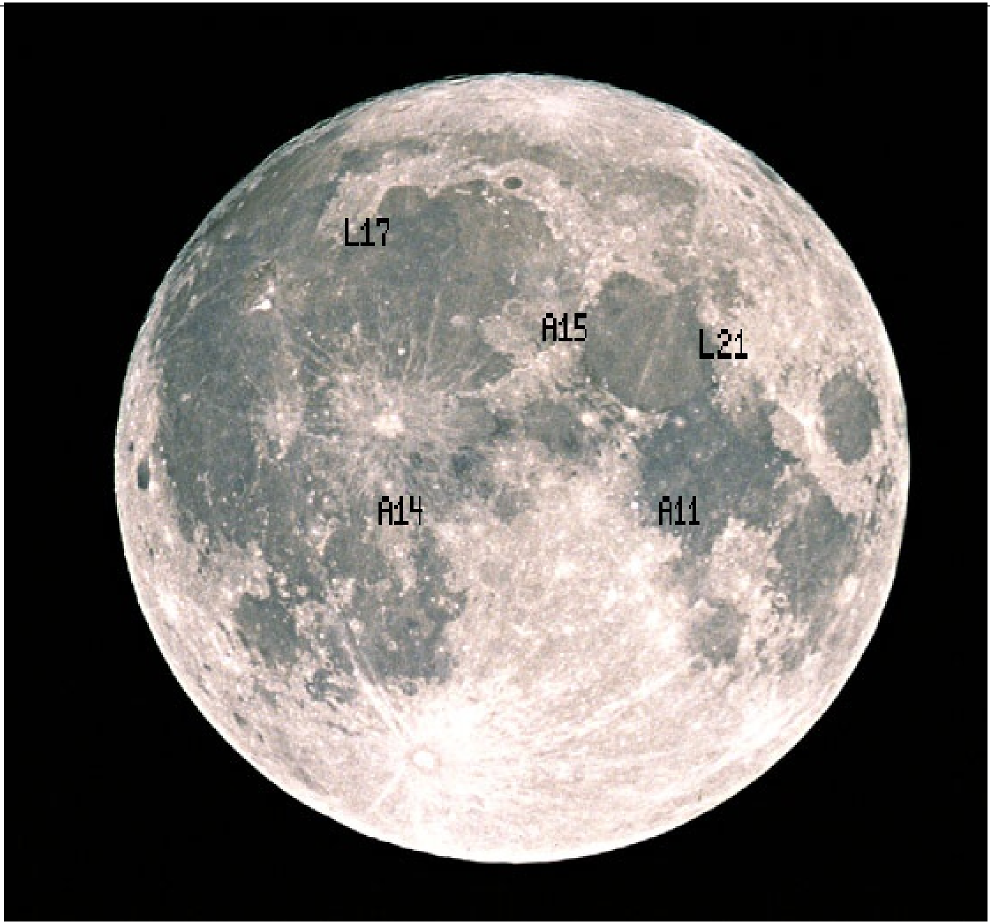
A. <u>*K. Nordtvedt*</u>*:* “A small array of laser retroreflectors”





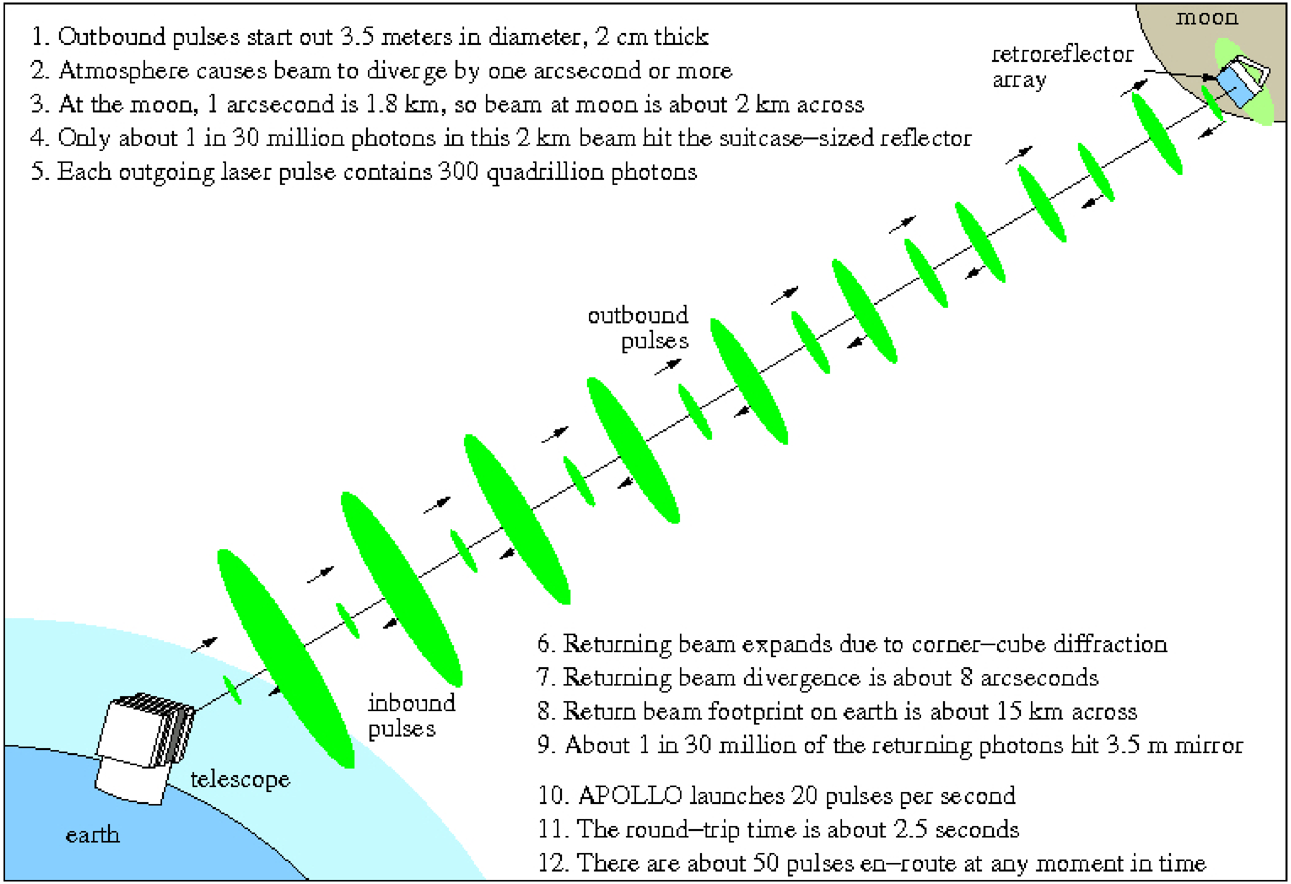
Apollo 11 retroreflector array & Astronaut Edwin Aldrin setting up the seismometer on the lunar surface (Tranquillity Sea) with Lunar Module and corner cube laser reflector in the background. (Apollo 11, July 1969). Two more corner cube reflectors were placed on the Moon during the Apollo 14 (at the Fra Mauro site) and Apollo 15 (in Hadlye/Appennine region) missions in 1971 (and two more from Soviet Lunakhod rovers)

Laser targets on the visible face of the Moon



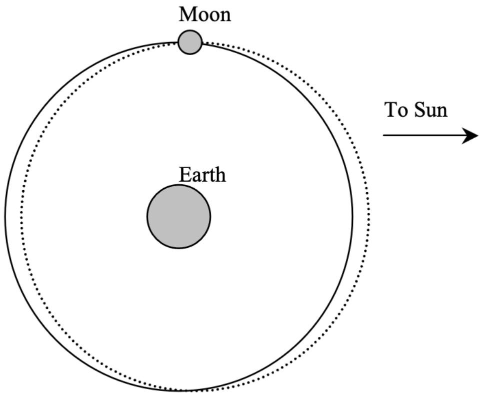
L1, on Soviet Lunakhod1 rover (1970), was unavailable till 2010 when it was laser ranged by Apache Point LLR station and has been working fine since then

Laser firing to the Moon



Hard job! APOLLO (Apache Point Observatory Lunar Laser-ranging Operation), in New Mexico, is the best LLR station

Equivalence Principle Tests with LLR



Polarization of the Moon's orbit towards the Sun due to solar perturbations on the Earth-Moon system (*classical parallactic inequality computed by Laplace*)
The polarization amplitude would slightly increase *if the Equivalence Principle were violated because of Earth-Moon different composition and/or different gravitational binding energy*

- Laser ranging to the Moon approaching mm level + improved lunar model (including librations, tides, tidal dissipation...) –both very hard...
  $\longrightarrow$ UFF test to:

$(-0.81 \pm 1.3) \times 10^{-13}$ *Williams et al IJMP D, 2012*
$(-0.3 \pm 0.66) \times 10^{-13}$ *Hoffmann & Müller CQG, 2018*
$(-0.38 \pm 0.71) \times 10^{-13}$ *Viswanathan et al. MNRAS, 2018*

i.e. between about $\pm(0.7...1.3) \times 10^{-13}$

- LLR tests for composition dependence (Fe-Ni rich Earth core vs silicate rich Moon) & difference in self gravitation binding energy, which reduces the mass of the Earth by 4.6×10^{-10} and that of the Moon by only 2×10^{-11} (does gravity obey UFF?) $\longrightarrow$ ambiguity!!
- Rotating torsion balance WEP test of Earth-like vs Moon-like test masses (self gravitation negligible...)

$(+1 \pm 1.4(stat) \pm 0.2(syst)) \times 10^{-13}$ *Adelberger CQG, 2001*

They must keep pace for ambiguity to be resolved!!!

more later on LLR...

Tests of General Relativity with BepiColombo mission to Mercury

- The ongoing BepiColombo mission to Mercury is designed fro extremely accurate tracking
- It carries the accelerometer ISA (Italian Spring Accelerometer, INAF, Rome) whose measurement of the inertial accelerations resulting from non gravitational force on the outer surface of the spacecraft is crucial to recover the purely gravitational motion of the s/c
- While determining its orbit around Mercury it is possible to indirectly observe the motion its center of mass with an accuracy several orders of magnitude better than it is possible by radar ranging to the surface of the planet near the Sun
- Altogether it makes possible a modern version of the classical tests of GR, based on Mercury's perihelion advance and the relativistic propagation of light near

Milani et al. PRD, 2002

5. Fundamental Physics with "test mass" laser tracked satellites

LAGEOS & LARES
Laser tracked test masses in orbit

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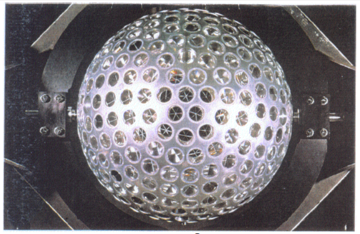
Parameter	Unit	Symbol	LAGEOS	LAGEOS II	LARES
Semi-major axis	km	a	12 270.00	12 162.08	7 820.31
Eccentricity	-	e	0.0044	0.0138	0.0012
Inclination	deg.	i	109.84	52.66	69.49
Radius	cm	R	30.0	30.0	18.2
Mass	kg	M	406.9	405.4	383.8
Area/Mass	m^2/kg	A/M	6.94×10^{-4}	6.97×10^{-4}	2.69×10^{-4}

LAGEOS (NASA, 1976)



LAGEOS II (ASI/NASA, 1992)

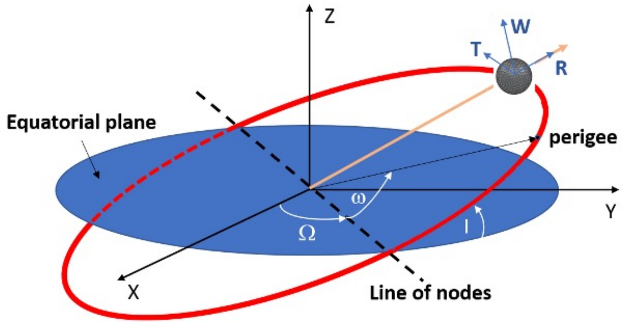
LARES (ASI, 2012)

- LAGEOS originally devoted to space geodesy (to improve high order harmonics of the geopotential whose inaccuracies would impair recovery of the global field) and Earth rotation
- *Precise orbit determination (laser ranging to mm level& very good physical model), ever increasing measurement time series $\rightarrow$ great contributions to fundamental physics (hence LAGEOS II and LARES)*

..follow the very good work by David Lucchesi & colleaugues

Laser ranging from all over the world

GR effects
Orbit of test mass instead of gyroscope



$$\dot{\Omega}_{LT} = \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}}$$

$$\dot{\omega}_{Schw} = \frac{3(GM_\oplus)^{3/2}}{c^2 a^{5/2}(1-e^2)}$$

$$\dot{M}_{Schw} = -\sqrt{1-e^2}\frac{3(GM_\oplus)^{3/2}}{c^2 a^{5/2}(1-e^2)}$$

Rate (mas/yr)	LAGEOS	LAGEOS II	LARES
$\dot{\omega}_{Schw}$	+ 3270.78	+ 3352.58	+ 10,110.15
$\dot{\omega}_{LT}$	+ 31.23	– 57.33	– 124.53
$\dot{\omega}_{J2}^{dir}$	– 3.26	+ 2.85	– 23.38
$\dot{\omega}_{J2}^{indir}$	– 0.36	+ 0.16	– 2.65
Total	+ 3306.38	+ 3298.26	+ 9959.59
$\dot{M}_{Schw}$	– 3278.75	– 3352.26	– 10,110.14
$\dot{M}_{J_2 rel}$	– 0.92	+ 0.15	– 6.71
Total	– 3278.75	– 3352.11	– 10,116.85
$\dot{\Omega}_{LT}$	+ 30.67	+ 31.51	+ 118.47
$\dot{\Omega}_{dS}$	+ 17.64	+ 17.64	+ 17.64
$\dot{\Omega}_{J2}^{dir}$	+ 1.95	– 3.63	– 15.31
$\dot{\Omega}_{J2}^{indir}$	+ 0.08	– 0.15	– 0.64
Total	+ 50.34	+ 45.37	+ 120.16

Lense-Thirring precession measurement with LAGEOS I, LAGEOS II and LARES

Università di Pisa

$$\begin{cases} \dot{\Omega}_2^{L1}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{L1}\delta\bar{C}_{4,0} + \dot{\Omega}_{LT}^{L1}\mu + \cdots = \delta\dot{\Omega}_{res}^{L1} \\ \dot{\Omega}_2^{L2}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{L2}\delta\bar{C}_{4,0} + \dot{\Omega}_{LT}^{L2}\mu + \cdots = \delta\dot{\Omega}_{res}^{L2} \\ \dot{\Omega}_2^{LR}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{LR}\delta\bar{C}_{4,0} + \dot{\Omega}_{LT}^{LR}\mu + \cdots = \delta\dot{\Omega}_{res}^{LR} \end{cases}$$

$$(\mu, \delta\bar{C}_{2,0}, \delta\bar{C}_{4,0})$$

$$\dot{\Omega}_{GR}^{comb} = 50.17\ mas/yr$$

$$\mu = \frac{\dot{\Omega}^{comb}}{\dot{\Omega}_{GR}^{comb}} = \begin{cases} 1 & \text{• In General Relativity} \\ 0 & \text{• In Newtonian physics} \end{cases}$$

$$\dot{\Omega}^{comb} = \delta\dot{\Omega}_{res}^{L1} + k_1\delta\dot{\Omega}_{res}^{L2} + k_2\delta\dot{\Omega}_{res}^{LR}$$

$$k_1 \cong 0.345$$
$$k_2 \cong 0.073$$

Lucchesi et al. arXiv:1910.01941, 2019 & Universe, 2020

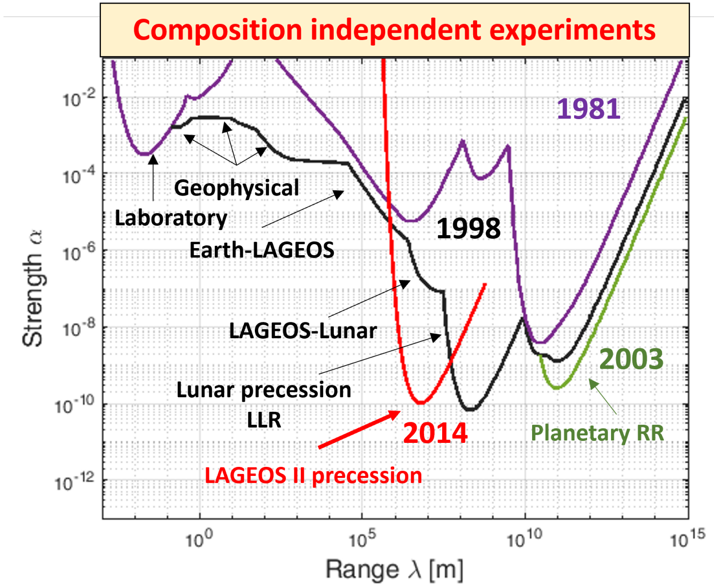
$\mu_{meas} = 1.0015 \pm 7.4 \times 10^{-3} \pm \mathbf{0.016}$

in agreement with *Ciufolini et al. EPJ C, 2019* $\mu_{meas} = 0.9910 \pm \mathbf{0.02}$

both a factor 10 better than GP-B *Everitt et al. PRL, 2011*

***Note**: measurement of De Sitter precession needs a gyroscope in orbit & LAGEOS as gyroscope cannot equal GP-B high quality gyroscopes*

New constraint on long-range Yukawa interaction with LAGEOS II



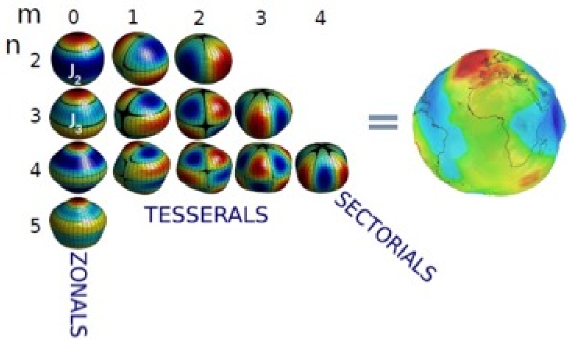
$$|\alpha| \simeq 1 \cdot 10^{-10}$$

$$\lambda \simeq 6081\,\mathrm{km} \simeq 1\mathrm{R}_{\oplus}$$

Lucchesi & Peron PRD, 2014

Earth gravitational perturbations

$$V(r,\varphi,\lambda) = -\frac{GM_\oplus}{r}\left[1 + \left(\frac{R_\oplus}{r}\right)^\ell P_{\ell m}(\sin\varphi)(\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{lm}\sin m\lambda)\right]$$



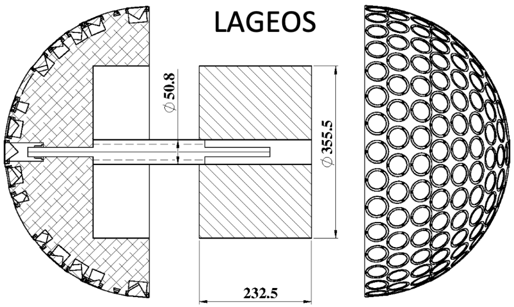
Even zonal harmonics produce (fully classical) secular effects directly competing with the GR effects to be measured & much bigger

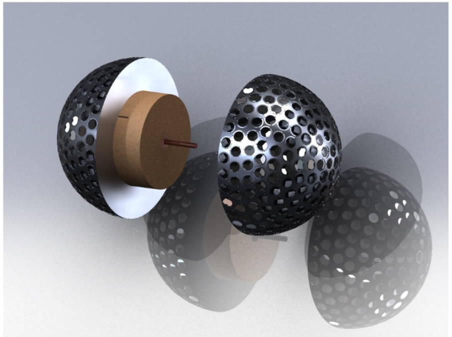
$\Downarrow$

Low order even zonal harmonics must be modelled over long timescales $\Rightarrow$ *dynamical geopotential model needed (provided by GRACE & GRACE Follow On space geodesy missions)*

Lucchesi IJMP D, 2005
Lucchesi & Peron PRD, 2014
Lucchesi et al. Universe, 2020

Non gravitational perturbations





LAGEOS like satellites are designed to have very low area-to-mass ratio: dense & compact + spherical symmetry

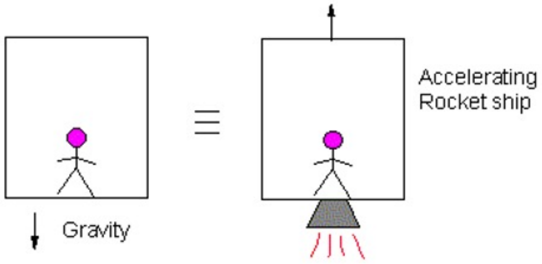
Yet, at high accuracy levels non gravitational perturbations by thermal thrust force from Earth & Sun. must be modelled which in turn require a good spin model

Neutral drag model also needed

Visco & Lucchesi Adv, Space Res, 2016
& PRD, 2018
Pardini et al. Acta Astr., 2017

6. Fundamental Physics with dedicated missions. The case for WEP and MICROSCOPE results

From Newton's UFF/WEP to EEP



An observer inside Einstein elevator close to the Earth cannot tell –before hitting the ground!– whether he is falling with the local gravitational acceleration (no gravity gradient) or else he is moving with an acceleration g in empty space, far away from all masses

Einstein, 1907: Hypothesis of complete physical equivalence between a gravitational field and an accelerated frame

... obviously assumes UFF (later referred to as WEP)

Dicke, 1964: Einstein Equivalence Principle (EEP), referred to as strong equivalence principle by Dicke in his Les Huches Lectures)

"The strong equivalence principle might be defined as the assumption that in a freely falling, nonrotating, laboratory the local laws of physics take on some standard form, including a standard numerical content, independent of the position of the laboratory in space and time. It is of course implicit in this statement that the effects of gradients in the gravitational field strength are negligibly small, i.e., tidal interaction effects are negligible"

New geometry needed

- If all accelerated frames are equivalent, then Euclidean geometry cannot hold in all of them. Gravity is locally replaced by a uniformly accelerated system, hence globally there cannot be just one such system

- A new geometry is needed:

  "Describing the physical laws without reference to geometry is similar to describing our thought without words. We need words in order to express ourselves"

  Luckily, it was available thanks to great mathematicians...

  "...It took me eight more years until I finally obtained the complete solution"

  Einstein, 1916: The foundation of the General Theory of relativity

Experimental evidence: Einstein & Eötvös

GR is based on the assumption that UFF/WEP holds

1916: "The foundation of the General Theory of relativity"
§ 2 The need for an extension of the postulate of relativity.
Einstein writes:

"This view is made possible for us by the teaching of experience as to the existence of a field of force, namely the gravitational field, which possesses the remarkable property of imparting the same acceleration to all bodies.1)

Footnote 1)***Eötvös has proved experimentally that the gravitational field has this property in great accuracy."***

Diese Auffassung wird dadurch ermöglicht, daß uns die Erfahrung die Existenz eines Kraftfeldes (nämlich des Gravitationsfeldes) gelehrt hat, welches die merkwürdige Eigenschaft hat, allen Körpern dieselbe Beschleunigung zu erteilen.1)

1) Daß das Gravitationsfeld diese Eigenschaft mit großer Genauigkeit besitzt, hat Eötvös experimentell bewiesen.

UFF/WEP experiments: the key facts

UFF/WEP experiments are unique in that:

- they test the foundation of GR (its universal coupling to all forms of matter-energy) not its predictions $\rightarrow$ much higher probing power
- they test **composition-dependence** (new expected long-range interactions are composition dependent...)

They can reach astonishing high precision because:

- they **are <u>not</u> absolute measurements** $\rightarrow$ the measured signal does not need to be compared with a theoretical prediction (often depending on poorly known parameters) to yield the physical quantity of interest (like measurements of G, gravitational redshift, Lense-Thirring precession, PPN parameter γ ...)
- if the sensor is differential like the violation signal, they are **null experiments**, known as the most precise experiments in physics
  $\eta = \Delta \mathrm{a}/\mathrm{a}$ no violation $\rightarrow$ no effect

In the footsteps of LLR, why not using SLR to LAGEOS & LARES?

In the field of the Earth:

The theoretical background is the 2-body problem *<u>with WEP violation</u>*

$$M_{g\oplus} = M_{i\oplus}(1+\eta_\oplus) \quad , \quad m_{g-sat} = m_{i-sat}(1+\eta_{sat})$$

$$\ddot{\vec{r}} = \frac{GM_{i\oplus}(1+\eta)}{r^3}\vec{r} \qquad \eta = \eta_\oplus + \eta_{sat} \qquad \eta_\oplus \eta_{sat} \ \text{ negligible}$$

The solution shows that WEP violation ($\eta \neq 0$) rescales the orbit size

$$a_{EP} = a_{cl}(1+\frac{1}{3}\eta) \quad \rightarrow \quad \eta = 3\frac{\Delta a_{EP}}{a_{cl}}$$

... hence it is limited to η_{min} by the semimajor axis measurement error Δa_{meas}

$$\eta_{min} = 3\frac{\Delta a_{meas}}{a_{cl}}$$

With numbers:

SLR range at about mm level rms
But semimajor axis determined at cm level (hard to model gravitational and non gravitational perturbations)

$$\eta_{min} \simeq 3\frac{10^{-2}\,\mathrm{m}}{1.23\times 10^{7}\,\mathrm{m}} \simeq 2.4\times 10^{-9}$$

Check via 3rd Kepler's law

$$\frac{\Delta(GM_\oplus)}{GM_\oplus} = 3\frac{\Delta a_{meas}}{a_{cl}}$$

with $GM_\oplus$ given by IERS (2010):

$$\frac{\Delta(GM_\oplus)}{GM_\oplus} \simeq \frac{8\cdot 10^{5}\,\mathrm{m^3s^{-2}}}{3.986004418\cdot 10^{14}\,\mathrm{m^3s^{-2}}} \simeq 2\times 10^{-9}$$

Not competitive for EP by orders of magnitude .... but can test Yukawa over $1R_\oplus$ range

Nobili et al GRG, 2008

What makes LLR so much better?

The Earth-Moon system falls **in the field of the Sun**, hence the semimajor axis of the orbit to be rescaled in case of violation ($\eta \neq 0$) is $1\,\mathrm{AU}$ (not $\sim 2R_{\oplus}$ as for LAGEOS). Assuming that the semimajor axis of the orbit of the Moon is determined to $3\,\mathrm{mm}$, the minimum limit from known physics is:

$$\eta_{min-LLR} \simeq 3\frac{\Delta a_{meas}}{d_{\oplus\odot}} \simeq 3\frac{3\cdot 10^{-3}\,\mathrm{m}}{1.5\cdot 10^{-13}\,\mathrm{m}} \simeq 0.6\times 10^{13}$$

Agreement with current results. Nothing magic!
... It is only the very large distance from the source body that makes the difference... not the fact that the Moon is much less affected from non gravitational perturbations due to its low area-to-mass ratio as very often stated (by the way (even solar radiation pressure is going to matter at mm level...)

PS: A LAGEOS-Earth system in the field of the Sun (like the Earth-Moon system of LLR) is better than LAGEOS in the field of the Earth, but not competitive with LLR because too close to the Earth, hence less affected by the Sun

Nobili et al GRG, 2008

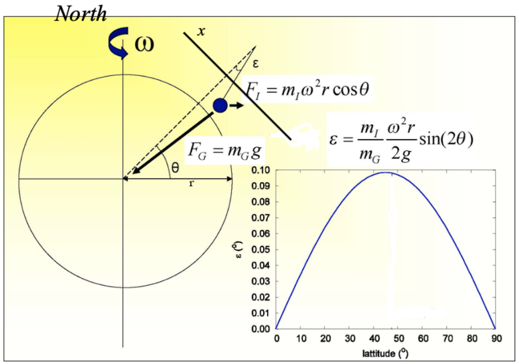
... yet LLR is unlikely to be the winner in testing the equivalence principle

Laser ranging relies on the **absolute measurement** –from the surface of the Earth– of the semimajor axis of the orbiting bodies, which contains a lot of information
(LAGEOS means LAser GEOdynamic Satellite since that is what is was designed for, and LLR has yielded a lot more about the motion of the Moon than just EP tests...)

However, UFF/WEP test srequire to measure only the **relative (differential)** accelerations between two test masses of different cvomposition (and the **differential** displacements they give rise to), which can be measured *in situ*, not from very far away

Measuring the Earth-Moon distance to mm accuracy is extremely hard, while mm level displacements are huge for accelerometers, both on ground and in space!

Learn from ground WEP labs: basics first



$F_I = m_I \omega^2 r \cos\theta \sin\theta$
$F_G = m_G g \sin\epsilon$

Do plumb lines of different composition deviate by the same amount?

Eötvös: better couple them on a torsion balance..

For suspended test masses the driving signal from Earth is:

$$a_{dr} = \omega^2 r \cos\theta \sin\theta \lesssim 0.016\,\mathrm{ms}^{-2}$$

a factor 600 smaller than g in mass dropping Galileo-type tests!
Yet torsion balances have done much better (due to initial condition errors in mass dropping)

Note: Violation signal from Earth is DC. In the field of the Sun is at diurnal frequency without rotating the balance (Dicke's experiment 1964, motivated by Schiff's view; improved by Braginsky 1971). Then came the rotating torsion balances of Eöt-Wash (motivated by Fischbach's claim in 1986 of possible fifth-force ...)

A WEP lab in low Earth orbit
How to get the real big gain

Strength of driving signal for WEP experiments on ground and in Low Earth Orbit (in $m\,s^{-2}$)

	Earth's field		Sun's field	
	Ground	LEO	Ground	LEO
mass dropping (Galileo – like tests)	9.8	$\simeq 8$	—	—
suspended masses (regardless of the suspension type : mechanic, electrostatic, superconducting coils...)	$\simeq 0.016$	$\simeq 8$	$\simeq 0.0057$	$\simeq 0.0057$

factor 1.2 loss (mass dropping: Ground → LEO, Earth's field)

factor 600 loss (Earth's field, Ground: mass dropping → suspended masses)

factor 2.8 loss (suspended masses, Ground: Earth's field → Sun's field)

factor 500 gain! (suspended masses, Earth's field: Ground → LEO)

Tests with suspended masses, which have been the best on ground, get a 500 times bigger driving signal inside a lab in low Earth orbit

Initial conditions/release errors in orbit

Blaser CQG, 2001; Nobili et al. GRG, 2008; Nobili PRA, 2016

A WEP lab in low earth orbit
There is a lot more to gain

- "Absence" of weight $\rightarrow$ *very weak suspensions needed (whatever the kind of suspensions) $\rightarrow$ high sensitivity to very small forces*

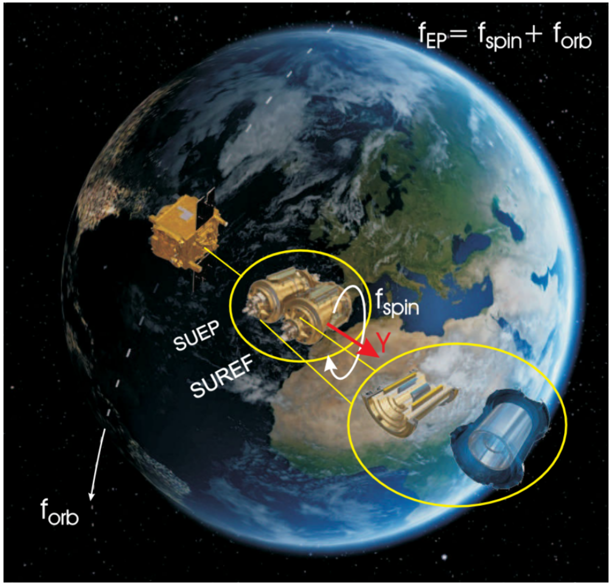
- Test masses as "concentric" cylinders (general agreement) $\rightarrow$ *reduced classical gravity gradient (tidal) effects*
  ... the more concentric the better...

- Space is (almost) empty $\rightarrow$ *no terrain* $\rightarrow$ *no terrain tilts, no microseismicity* (sonic vibration noise at high frequencies, which are not those of interest in WEP tests)

- Rotation, the faster the better, as we have learned from rotating torsion balances
  In space the whole s/c rotates $\rightarrow$ *no stator needed* $\rightarrow$ *no stator/rotor noise* $\rightarrow$ *rotation can be totally passive (angular momentum conservation around symmetry axis)*

... more later with "take away lessons from Microscope"

A WEP lab in LEO
What to fight



Microscope in orbit & rotating.
Rotation up-converts frequency of violation signal as well as along-trcak drag

Largest effect from air drag on outer surface of s/c (next solar radiation)

$a_{ng} \propto \frac{A}{M}$ (cannot be as small as for LAGEOS)

$a_i = -a_{ng}$ "same" inertial ("ideally" common mode !) acceleration on all TMs suspended inside s/c

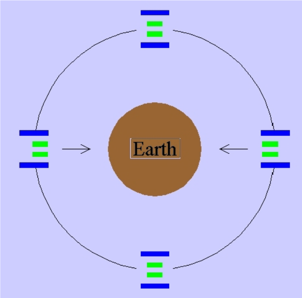
It is huge compared to the signal!! $\Downarrow$

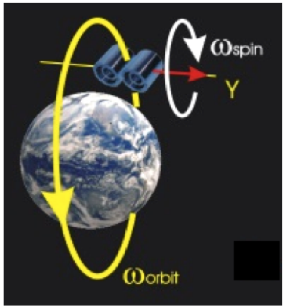
Share the burden:

- make s/c drag free
- design the TMs to reject common mode effects (exploit that the signal is differential!)

... more later with "take away lessons from Microscope"

MICROSCOPE's main features





- 2 Sensor Units, each with 2 coaxial cylinders (SUEP Pt-Ti; SUREF Pt-Pt)
- each cylinder is suspended electrostatically and coupled to its cage ($600\ \mu m$ gap) only via a tiny gold wire (for electric grounding and polarization voltage)
- each cylinder's axis is the sensitive axis (in the orbit plane) $\rightarrow$ to up-convert the signal to frequency higher than orbital frequency rotation axis must be $\perp$ to the orbit plane and to the cylinders' sensitive/symmetry axis (not nice.. symmetries do matter in physics..)
- a set of electrodes is used to apply voltages to: i) measure the displacements (by capacitance sensing) and ii) apply forces as required to keep the test mass "motionless" at the "capacitive" zero of the cage (closed control loop)
- each test cylinder is part of an independent accelerometer (scale factor matching needed in orbit)
- one Sensor Unit is used at a time in drag free control while the other collects science data

MICROSCOPE is a success

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- Launched 25th April 2016
- Goal: test WEP to $\eta \simeq 10^{-15}$ in the field of the Earth
- Limited by differential acceleration random noise attributed to internal damping (losses) in the gold wires because of its typical $1/\sqrt{\nu}$ dependence of spectral density

  SUEP: 19 measurement sessions properly combined to give final WEP result of the experiment

$$\eta = (-1.5 \pm 2.3) \times 10^{-15}$$

  about two orders of magnitude improvement over best ground results by the Eöt-Wash group with rotating torsion balances

Small, low cost space mission (by space mission standards..) $300\,\mathrm{kg}$ *total mass*
Small, well controlled experiment

Touboul et al. PRL, 2022

A set of ancillary papers by the Science Team on CQG, 2022

... *yet*

Space experiments are expensive and unlikely to be replicated,
while the essence of science is repeatibility

↓

results must be taken with caution and cannot be above criticism based on
experience in similar experiments & driven by sound physics arguments

Nobili & Anselmi arXiv:2302.06400v2, 2023

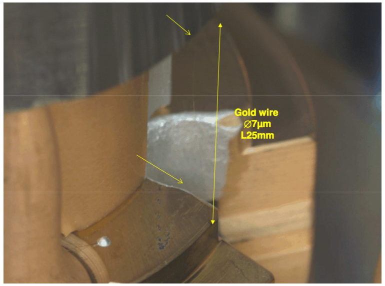
New version in preparation

Missed heritage from GOCE mission

GOCE: single cubic mass accelerometer, 2 ultra sensitive axes along the directions perpendicular to the gold wire, 1 low sensitive axis parallel to the wire, because $k_{||} \gg k_{\perp}$

GOCE report:
$k_{||} \simeq 10^{-3}\,\mathrm{N/m}$
$k_{\perp} \simeq 10^{-5}\,\mathrm{N/m}$ *GO-TN-AI-0084, 2002*

The high stiffness, low sensitive axis was used for ground testing (with very small $32\mu\mathrm{m}$ gap)



In Microscope the gold wire is parallel to the sensitive/symmetry axis (inevitable for concentric cylinders) $\rightarrow$
$k_{||}$ is the stiffness that matters, and $\simeq 10^{-3}\,\mathrm{N/m}$ is its value (very similar wire)

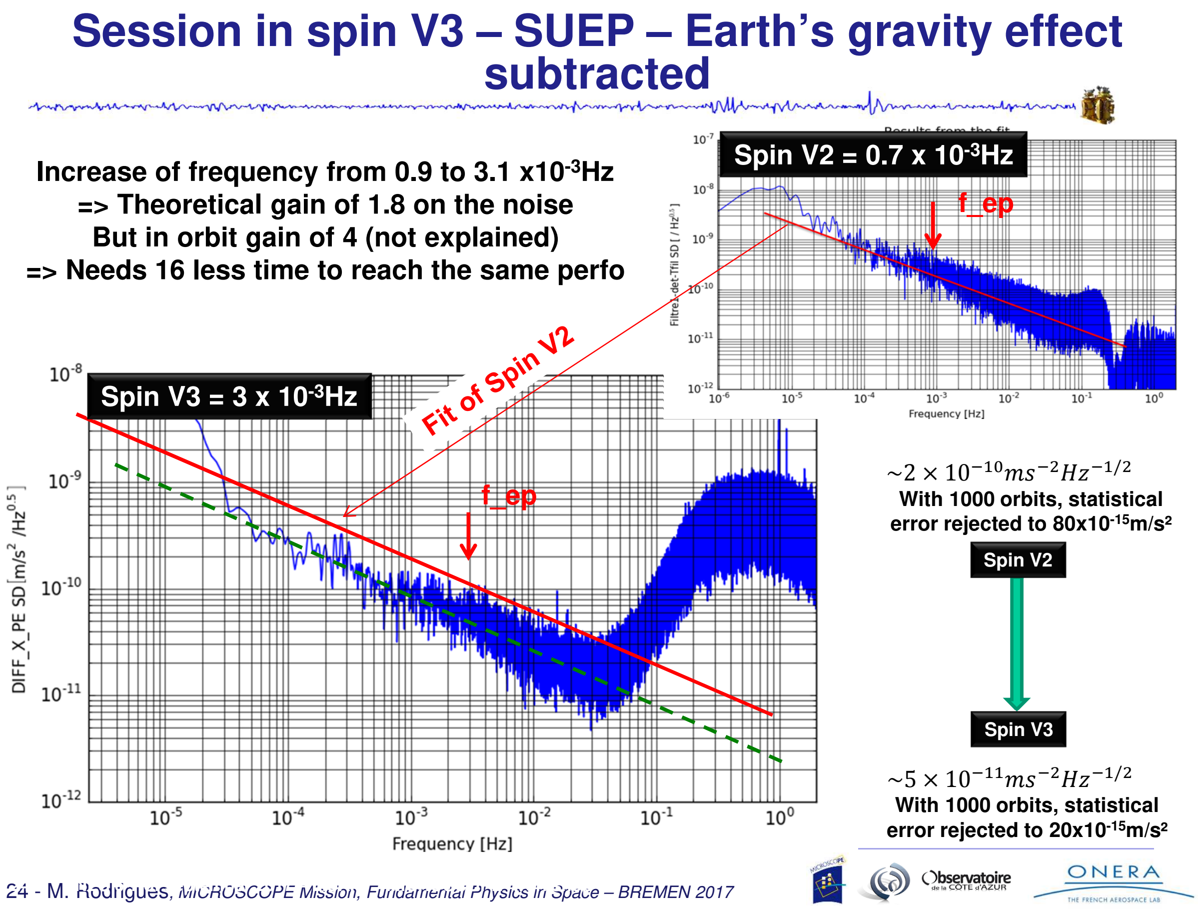
Ground tests of the stiffness in view of the Microscope mission measured $k_{\perp}$ confirming the very low value of $10^{-5}\,\mathrm{N/m}$, which has become the "expected" value for the space experiment

Willemenìont & Touboul, RSI 2000

Bergé et al, CQG 2022

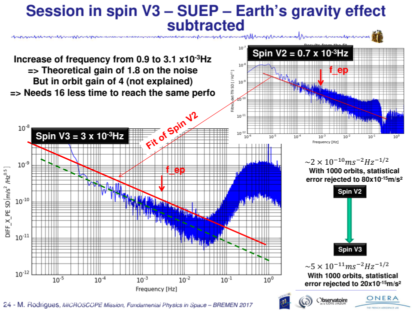
Once measured in orbit for all 4 cylinders the value was as measured in GOCE for $k_{||}$, i.e. 100 times higher than "expected" $k_{\perp}$ $\rightarrow$ 10 times larger internal damping noise & issue with capacitive zero...

Anomalous acceleration noise jump reported …



Slide 24 from Rodrigues' talk, Bremen 2017

.. not mentioned ever since



This jump was reported as unexplained but has not been mentioned in later publications on Microscope results: PRL 2017, CQG 2019, PRL 2022, CQG issue 2022

A table of the spectral density of acceleration noise at signal frequency for all measurement sessions, which might show if similar jumps occurred, has not been published

Inertial mode (zero spin) was originally planned. No way. Noise far too high

Initial spin mode at V2 was already at the higher end of the planned spin rates. Still noise too high

V3 spin mode was at much higher spin rate than planned (there was no choice but to increase spin rate!), and crucial to reduce noise (still at least a factor 2 higher than expected)

Eventually, faster spin was ok and saved the mission (even though spin is not around symmetry axis → *shows the enormous advantage unique to the experiment in space of rotating the entire lab!)*

More anomalous jumps found in sequential sessions

Spectral density of acceleration noise at ν_{EP} for each session reconstructed from published δ (WEP test value of the session) and session duration t_{int}

Noise is random:

$$\delta \cdot g_{drive} = S^{1/2}(\nu_{EP})/\sqrt{t_{int}}$$

Noise is due to losses in gold wires (k, Q):

$$S^{1/2}(\nu_{EP}) = f_{TkQ}\frac{1}{\sqrt{2\pi\nu_{EP}}} \quad ms^{-2}/\sqrt{Hz}$$

$$f_{TkQ} = \sqrt{4k_BT\left(\frac{k_1}{m_1^2}+\frac{k_2}{m_2^2}\right)\frac{1}{Q}} \quad ms^{-2}$$

$$Q \propto S(\nu_{EP})$$

TABLE I. SUEP sensor. Session number in V2/V3 spin mode; session δ and percentage of glitches G from Tables 6,7 and 4,5 of [7]; calculated values of $S^{1/2}_{\Delta a_{th}}(\nu_{EP})$, Q, k/Q. ↓ indicates a jump between sequential sessions; * an M vs A discrepancy.

	δ 10^{-15}		$S^{1/2}_{\Delta a_{th}}(\nu_{EP})$ $10^{-11} ms^{-2}/\sqrt{Hz}$		Q		k/Q $10^{-3} N/m$		G %
Session	M	A	M	A	M	A	M	A	
↓210 (V3)	−30.1	−29.2	13	13	0.75	0.79	1.2	1.1	18
212 (V3)	10.4	9.5	4.9	4.5	5.2	6.3	0.17	0.15	17
*218 (V3)	3.6	6.7	2.4	4.5	22	6.3	0.042	0.14	15
234 (V3)	5.6	5.9	3.3	3.4	12	11	0.078	0.086	18
↓236 (V3)	2.7	2.6	1.8	1.7	39	42	0.024	0.022	21
238 (V3)	6.1	5.8	4.1	3.9	7.6	8.4	0.12	0.11	24
252 (V3)	−14.7	−14.9	9.2	9.3	1.5	1.4	0.62	0.64	26
↓254 (V3)	−14.2	−14.1	9.5	9.4	1.4	1.4	0.65	0.64	27
256 (V3)	−4.7	−5.3	3.1	3.5	13	10	0.071	0.091	28
326-1 (V3)	−10.1	−16.3	4.8	7.7	5.5	2.1	0.16	0.43	12
326-2 (V3)	−11.1	−10.4	3.9	3.7	8.1	9.2	0.11	0.099	7
358 (V3)	15.4	15.8	9.0	9.2	1.6	1.5	0.59	0.62	14
402 (V2)	27.3	28.4	7.1	7.3	8.5	7.9	0.11	0.12	35
*404 (V3)	6.3	4.7	4.2	3.1	7.1	13	0.13	0.071	23
406 (V3)	6.0	5.9	1.6	1.6	47	49	0.019	0.019	23
*438 (V2)	−12.5	−23.4	4.3	8.1	23	6.5	0.040	0.14	21
*442 (V2)	−10.7	−1.5	4.1	0.58	25	1273	0.037	0.00072	21
*748 (V2)	−17.5	−23.4	5.2	7.0	15	8.7	0.059	0.11	25
750 (V3)	66.6	66.9	11	12	0.97	0.95	0.94	0.97	19

N&A arXiv:2302.06400v2, 2023

The jumps in Q required to explain jumps of acceleration noise spectral density in sequential sessions (no calibration in between) are very hard to explain!

Another message from GOCE

- Is noise from internal damping from the gold wires the only one with $1/\sqrt{\nu}$ dependence?

- GOCE has found that fluctuating potential due to patch charges has $1/\sqrt{\nu}$ dependence below $\simeq 10^{-2}\,\mathrm{Hz}$
- too small to matter ... at first glance ..

GO-TN-AI-0084, 2002

- spurious force due to patch fluctuating potential

$$\propto \delta V_{pflp} \cdot V_{applied}$$

$V_{applied} \simeq \mu\mathrm{Volt}$ during fine control of test cylinder in closed loop $\rightarrow$ spurious force too small

- but ... how is the "capacitive zero" maintained?
Forces to fight are: i) tidal force from Earth & ii) restoring force of spring/wire ("capacitive zero" very unlikely to coincide with "mechanical zero" , $0_{cap} \neq 0_{k_{||}}$)

restoring force from $k_{||}$ dominates with $k_{||} \simeq 10^{-3}\,\mathrm{N/m}$
(if $k_{||}$ smaller, next comes electrostatic stiffness ...)
$\Downarrow$ $V_{applied}$ needed are of several Volts, no longer of the order of $\mu\mathrm{Volt}$ $\rightarrow$ patch fluctuating potentials can be relevant !!

$\Downarrow$

There is indeed another source of $1/\sqrt{\nu}$ noise ... moreover: it can fluctuate... !!

When a new measurement session is initialized (no new calibration) the patches of charges rearrange themselves in an unpredictable way and the level of patch potential at ν_{EP} may change, thus changing the level of the total measured acceleration noise (jumps ... both up or down...) ***because of a "fluctuating zero" a real nightmare in high precision measurements!!!!***

Glitches acceleration spikes Heritage from GOCE missed

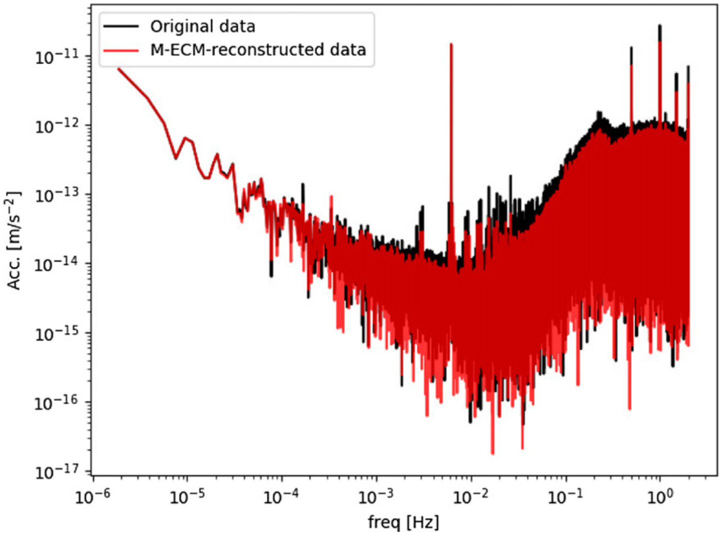
Microscope spacecraft wrapped with MLI

- Multi Layer Insulation wrapping is crucial for thermal stability
- Thermal input stresses from Earth (infrared radiation) and Sun hit the s/c as the satellite moves from facing the hot Earth/Sun to facing the cold deep space. $\Rightarrow$ they occur at the synodic frequencies of the s/c relative to Earth/Sun (& their harmonics)

$\nu_{syn_\oplus} = \nu_{orb} + \nu_{spin} = \nu_{EP}$ $\qquad$ $\nu_{syn\odot} \simeq \nu_{spin}$

- Stress energy accumulates on MLI spots at these frequencies $\Rightarrow$ cracks on MLI spots, spacecraft-quakes $\Rightarrow$ huge acceleration spikes on all 4 test cylinders ...
- Release events are erratic but tend to occur at these frequencies ... we have a problem!
- GOCE (launched 2009) solved the problem by a rigid MLI: no glitches observed ... could have been solved for Microscope too!

Data gaps and artificial reconstructed data



SUEP session #380. Figure taken from Bergé et al. CQG, 2022
FFT of differential accelerations before removing spikes (black) and after filling the gaps with reconstructed data (red)
Gaps amount to 46% of session data $\rightarrow$ session split into two much shorter ones in final analysis

- Glitches spikes at synodic frequencies & harmonics visible.
- Each line is double, two lines very close by because synodic frequencies are very close
- Line at $\nu_{syn\oplus} - \nu_{syn\odot} = \nu_{orb} \simeq 1.t \cdot 10^{-4}$ Hz also visible
- Glitches are removed (46% of data in session #380) & resulting gaps are replaced by reconstructed artificial data
- FFT after glitches removal and gaps replacement with artificial data (black): lines are reduced but not eliminated... because gaps occur at the synodic frequencies and harmonics, hence retain memory of the glitches

Artificial data in precision experiments ... scary

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SUEP measurement sessions: up to 35% of artificial data. Each session yields a WEP test to $\delta_i \pm \sigma_i$ from which the final published result is obtained :

$$\delta_{fin} = \frac{\frac{\delta_i}{\sigma_i^2}}{\sum_i \frac{1}{\sigma_i^2}}$$

$$\sigma_{fin}^2 = \frac{1}{\sum_i \frac{1}{\sigma_i^2}}$$

thus, artificial data contribute to σ_i, hence to δ_{fin} and σ_{fin} $\Rightarrow$ Paradox: the more artificial data, the better the final test of WEP!

... luckily Eöt-Wash group as shown that there is a way out. Note that they had only 7% missing data (because of ion pump). They did as follows:

WEP violation signal has known frequency and phase, only sign & amplitude are unknown (holds for torsion balances on ground as well as for Microscope in space)

$\Downarrow$

eliminate outliers, leave gaps empty, fit a violation signal **to real measured data only** and determine amplitude and sign of possible violation, if any

Schlamminger et al. PRL, 2008

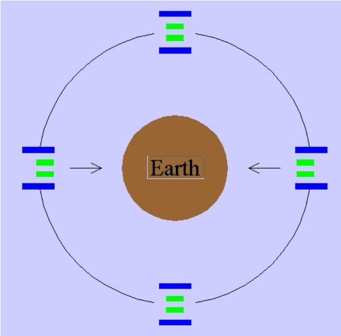
Should such an analysis test be made by Microscope scientists, the final result might be somewhat worse than they have obtained using artificial data, but it would be much more robust and much less questionable!!

Take aways lessons from Microscope space test of UFF/WEP

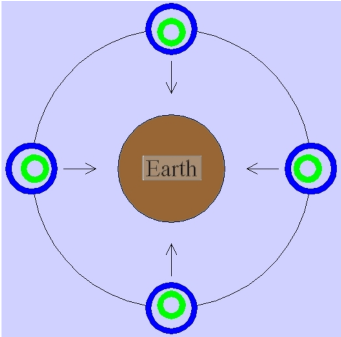
- Microscope has been successful despite several serious problems ... look carefully at previous relevant missions ...
- It shows that WEP test in low Earth orbit has huge potential for improvement (ground Eöt-Wash have still measured the smallest differential acceleration, but Microscope has 500 times bigger driving signal on its side..)
- Huge driving signal, weak coupling, high sensitivity, low platform noise
- s/c isolated, whole s/c rotates, very low rotation noise (no stator)
- Mechanical suspensions (gold wires) are a problem in Microscope.. if designed properly they can be the solution: i) couple test bodies to reject common mode effects; ii) ensure one single zero (equilibrium position) dictated by physics, not arbitrary, not fluctuating...
- Capacitance measurement of displacements is ok (certainly not a limitation), yet it is time to use laser interferometry ... especially if you don’t need electrostatic forces to hold the masses since suspensions do that ..

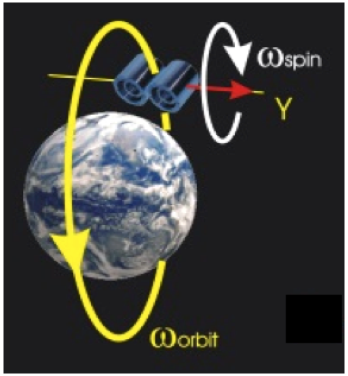
How it could look like....

Microscope: $\simeq 10^{-15}$



Galileo Galilei–GG: 10^{-17} ***target***





GG: passive spin around symmetry axis at 1 Hz (conservation of angular momentum!), self-centering ensured by physics & tested in lab

Pegna et al. PRL 2011; Nobili et al. PRD 2014

A balance for testing WEP in space

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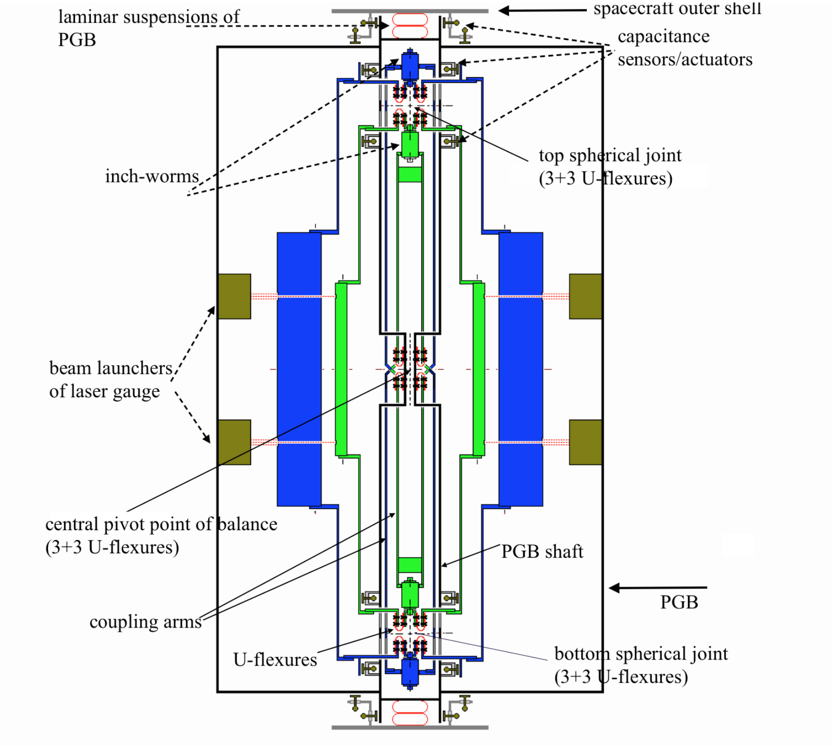

- ***Mechanical suspensions are the solution, not the problem.*** *They can be predicted analytically, and simulated numerically; k and Q can be measured in the lab for the expected conditions of operation.. there is nothing misterious about using them in space (in fact, weightlessness is an advantage!)*
- *Most precise gravitational experiments use mechanical suspensions (Rotating Torsion Balances, GW ground detectors..)*
- *You don't need to fly a flexure to know how it will work in orbit! Micorscope knew, but forgot …*

GG balance animation