## Earth-based Gravitational-Wave Experiments Part II: Sources, data analysis, science

## Frédérique Marion



SIGRAV International School 2024 - Measuring Gravity

## PANORAMA OF GW SOURCES AND DATA ANALYSIS TECHNIQUES

## **GW** emission

### Quadrupole formula

$$h_{ij} = \frac{G}{c^4} \frac{2}{r} \frac{d^2 Q_{ij}}{dt^2} \qquad Q_{ij} = \int \rho \ x_i \, x_j \ d^3 x$$

Expected strength of signal



## **Sources detectable from Earth**

#### Merging black holes, neutron stars



Burst sources

**Transient signals** 



### Two broad classes

- > Transient signals
- Persistent signals
- Search strategies
  - > Waveform known
  - > Waveform unknown



#### Spinning neutron stars



#### Stochastic backgrounds



## **Compact binary coalescences**



- 91 detection candidates in O1-O2-O3 data
  - Many binary black holes
    - Most with ~ equal masses
    - Discovery signal GW150914 turned out to be quite typical
  - Binary neutron stars: GW170817, GW190425
  - > Neutron star-black hole: GW200105, GW200115



## The (inspiral) signal in a nutshell



# Matched filtering

$$S = (s|T) = 4 \int_0^\infty \frac{\tilde{s}(f)\tilde{T}^*(f)}{S_n(f)} df$$

- If we know what we're
   looking for, and we know
   the properties of detector
   noise
- Correlation of data with
   expected signal, weighted
   by sensitivity curve

$$E[S] = \alpha$$
 if  $\tilde{s} = \alpha \tilde{T} + \tilde{n}$   
and T is properly normalized



# Matched filtering (cont.)

□ As a function of the (unknown) arrival time

$$S(t_c) = 4 \int_0^\infty \frac{\tilde{s}(f)\tilde{T}^*(f)}{S_n(f)} e^{-i2\pi f t_c} df$$

Maximize over unknown phase

$$S(t_c) = 4 \left| \int_0^\infty \frac{\tilde{s}(f)(\tilde{T}_{0^\circ}(f) - i\tilde{T}_{90^\circ}(f))^*}{S_n(f)} e^{-i2\pi f t_c} df \right|$$

 $\Box$  Record *trigger* at  $t_c$  if  $S(t_c)$  exceeds some threshold

# Matched filtering is "optimal"



In Gaussian, stationary noise with known PSD...

- **D** Noise SNR distribution:  $\chi^2$  with 2 degrees of freedom
- Signal SNR distribution: non-central  $\chi^2$  distribution
  - ~ Gaussian distribution if signal strong enough

□ Matched filter optimizes SNR  $SNR = \frac{1}{\sqrt{E}}$ 

 $\frac{E[S]}{\sqrt{E[(S-E[S])^2]}}$ 





 Selecting triggers by setting threshold on SNR  $\rho > \rho^*$  guarantees lowest false alarm probability for given detection probability

But...



Noise spectrum



- Detector noise spectrum has complex structure
  - Broadband noise
  - Narrow features
  - Large dynamic range
- Noise spectrum is not stationary
- Estimated by averaging consecutive FFTs
  - Over time large enough to get smooth estimate, short enough to follow medium-term variations



# Waveforms

## Approximate analytical solutions

- Perturbative approaches
  - Post-Newtonian expansion
  - Effective-one-body approach
  - Final black hole ringdown
- > Accurate for inspiral and ringdown, loses accuracy close to merger

## Hybrid models

- Combining results from analytical and numerical approaches
- > Provide full inspiral-merger-ringdown waveforms

## Numerical solutions

- Solving Einstein's equations directly with numerical evolution methods
- Computationally expensive
  - Cannot be used to model many orbits
- Can model merger

## **Parameters**

 $e^D_{\epsilon}$ 

### □ In general, compact binary is described by up to 19 parameters

- Intrinsic parameters drive system dynamics
  - Masses (2)
  - Spins (6)
  - Deformability for neutron stars (2)
  - Eccentricity (2)
- Extrinsic parameters impact measured signal
  - Position : luminosity distance, right ascension, declination (3)
  - Orientation: inclination, polarization (2)
  - Time and phase at coalescence (2)
- Searching a reduced parameter space
  - Assume that there is no eccentricity
  - > Assume that there is no precession of the orbital plane
  - Assume that both bodies are black holes
  - Restrict to the dominant, quadrupolar mode of the signal
  - Orientation and location parameters now enter as overall scale, time or phase shifts, easily maximized over

 $\succ$  Scan a 4-dimensional space:  $m_1, m_2, S_{1z}, S_{2z}$ 



## Search parameter space



## Noise is not Gaussian

- Environmental or instrumental artefacts are common in the data
  - > Aka glitches
  - Responsible for long tails in SNR distributions
- Coping strategies
  - Use data quality tools to diagnose and flag issues where possible
  - Go beyond SNR by considering additional observables to distinguish between astrophysical signals and glitches
    - Combine SNR with outcome of signal consistency tests to rank triggers
  - Estimate background from data
    - Requiring coincidence between detectors both reduces the background and provides ways to estimate it



CQG 33 134001

arXiv:2101.1167

# **Background estimation & IFAR plots**

### With time slides



## Without time slides

- > Use all pairs of single-detector triggers
  - Account for probability that they could form a coincidence



### □ Cumulative number of triggers with IFAR ≥ x-axis value

- > Average background distribution follows n = T/IFAR
- Foreground candidate events appear as outliers

## **Burst sources**

### $\Box$ Generic GW Bursts with < ~1 – 10 s duration

- ➢ Some long-lived transient signals considered too, duration < 10<sup>4</sup> s
- Many poorly modeled transient sources
  - CBC post-merger signal
  - Core-collapse supernovae
  - Long Gamma-ray bursts
  - Neutron star instabilities
  - > Soft Gamma-ray repeater flares
  - ▶ ...
  - > ???
- □ Some well modeled (speculative) sources
  - > Cosmic strings

Cusps 
$$\tilde{h} \propto f^{-4/3}$$
 Kinks  $\tilde{h} \propto f^{-5/3}$ 



- Robust search paradigm
  - > Look for excess power in time-frequency space
    - Using Fourier or wavelet decomposition
  - > Require coherent signals in multiple detectors
    - Common features at ~same time, consistent with single sky location
      - Using direction-dependent antenna response

# **Core-Collapse Supernovae (i)**



- Process still poorly understood
- GW expected, mainly from protoneutron star oscillations
  - Oscillations excited by multi-dimensional hydrodynamic instabilities
    - Convection
    - Possibly large-scale non-radial oscillations of shock (SASI)
  - GW carry information about dynamics of central engine



- Efficiency of GW emission strongly parameter and model dependent
  - $E_{GW} \sim 10^{-11} 10^{-7} \ M_{\odot} \ c^2$

# **Core-Collapse Supernovae (ii)**



O3 sensitivity

$$h_{\rm rss} = \sqrt{\int (h_+(t)^2 + h_{\times}(t)^2) dt} \sim 10^{-22} / \sqrt{\rm Hz}$$

$$\succ~E_{GW}\,{\sim}10^{\text{--}10}~M_{\odot}\,c^2$$
 @ 10 kpc,  ${\sim}100~Hz$ 

- Could detect GW signal from Galactic supernova for some models
- Put constraints on extreme scenarios for supernova in the local group
- Next-gen detectors needed for robust and detailed observations

## **Multi-messenger searches**

### Triggered searches

- Search for GW signals in coincidence wit remarkable events
  - GRBs, Magnetar flares, Pulsar glitches, Supernovae, High energy neutrinos...
- > Are more sensitive than their all-sky counterparts

## □ The electromagnetic follow-up program

- > Agreements with partners allowed successful follow-up in O1/O2
  - Spectacular results for GW170817
- > Moved to open public alerts since O3 run

## **Sources detectable from Earth**

#### Merging black holes, neutron stars



Burst sources

**Transient signals** 



### Two broad classes

- > Transient signals
- Persistent signals
- Search strategies
  - > Waveform known
  - > Waveform unknown



#### Spinning neutron stars



#### Stochastic backgrounds



## **Continuous wave sources**

### GW signal from non axisymmetric rotating neutron star

- > O(10<sup>6</sup> 10<sup>7</sup>) neutron stars within 5 kpc
- > ~2000 known pulsars, ~ 10% in frequency band of ground-based detectors

$$h = 3.10^{-27} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{10 \text{ kpc}}{D}\right) \left(\frac{I}{10^{45} \text{ g cm}^2}\right) \left(\frac{f}{200 \text{ Hz}}\right)^2$$

- □ Amplitude of GW signal driven by ellipticity, many uncertainties
  - $\succ\,$  Maximum sustainable  $\epsilon$  depends on NS structure
  - $\succ$  Processes to produce/sustain  $\epsilon$ 
    - NS born with bumpy crust
    - Strong internal magnetic fields
    - Accretion  $\pm$  unstable r-mode oscillations
    - Free precession
- Emission frequency
  - $\succ$  Depends on emission mechanism  $f=2~f_{
    m rot}, f_{
    m rot}\dots$
- f a Amplitude very small, but integrating signal over time makes SNR grow  $\,\propto T^{1/2}$



21

 $\sim 10^{-12} < \epsilon < \sim 10^{-5}$ 

 $\epsilon < 10^{-7} - 10^{-5}$ 

# **CW** search challenges

### Computationally limited searches

- > Coherent analysis needs to account for Doppler modulation of signal due to Earth motion
- Need to scan an enormous parameter space
  - Sky location x Frequency x Frequency derivative(s) x Inclination x Polarization
- Coherent analysis is expensive
  - Cost  $\alpha$  (coherence time)<sup>6</sup> x (band upper frequency)<sup>3</sup>
- □ Pick your battles: choose your search mix well
  - Coherent / Semi-coherent, Targeted/Directed/All sky, Isolated neutron stars / In binaries (accretion!)

### Data quality

- > Chase wandering lines of instrumental or environmental origin
- Electromagnetic information
  - Pulsar ephemerides, glitches...

# Known pulsars: upper limits

Astrophys. J. 935, 1 (2022)

- Spin-down limit surpassed for 23 pulsars
- GW emission <0.009% of spindown luminosity for Crab pulsar
  - Mountains < 2 cm</p>
- □ J1745–0952: smallest upper limit on GW amplitude
   > h < 4.72 10<sup>-27</sup>
- J0711-6830: smallest upper limit on ellipticity
   ε < 5.26 10<sup>-9</sup>



# Stochastic gravitational wave backgrounds



## Stochastic gravitational-wave backgrounds expected from

- Cosmological sources
  - Inflation models, Cosmic strings, Phase transitions...
  - Production processes typically involve energies inaccessible to particle colliders
    - Discovery window
- > Astrophysical sources
  - Superposition of unresolved sources
    - Pulsars in Milky Way
    - BNS and BBH mergers in Universe

# Searching for stochastic backgrounds

- Search for isotropic background by cross-correlating data streams from detector pairs
  - > Optimal filter
  - > Assume power law spectrum for signal

$$\Omega_{\rm GW}(f) = \Omega_{\alpha} \left( \frac{f}{f_{\rm ref}} \right)$$

- α = 0 (cosmologically motivated)
- $\alpha$  = 3 (astrophysically motivated)
- $\alpha$  = 2/3 (dominated by CBC sources)
- > Optimal filter depends on detector pair overlap function γ(f)
  - Determined by network geometry

$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{\rm GW}}{df}$$

$$Y = \int \tilde{s_1}^*(f) \tilde{Q}(f) \tilde{s_2}(f) df$$

$$\tilde{Q}(f) \propto \frac{\gamma(f) \Omega_{\rm GW}(f)}{f^3 S_1(f) S_2(f)}$$

## **Background from compact binaries**

Detections have revealed population of BBH with relatively high mass

- > Boosts expected background from BBH
  - Dominated by inspiral phase



## FOCUS ON COMPACT BINARY COALESCENCES

## From signals to sources to science



- Detailed features of signal reveal source properties
  - > Used for astrophysics, cosmology, fundamental physics

- Characterizing sources, extracting science: mostly through Bayesian analyses of
  - Individual events
  - Collections of events

## **Parameter estimation via Bayesian inference**

 $\Box$  Assume data **d** are described by model *M* with parameters  $\vec{\theta}$ 

Use Bayes' theorem to infer posterior probability distribution for parameters  $\overrightarrow{\theta}$ , given data **d** 



# **Likelihood** $p(\boldsymbol{d}|\overrightarrow{\theta}, M)$

 $\begin{array}{c} \square \ p(d|\overrightarrow{\theta}) \text{ is probability of drawing residual } d - R[h] \text{ from noise distribution} \\ \hline \end{array} \\ \begin{array}{c} \square \ Once \text{ we have a signal model, the noise model defines the likelihood} \\ p(n) \propto e^{-\frac{1}{2}(n|n)} \quad (n|n) \equiv 4 \int_{0}^{\infty} \frac{\tilde{n}(f)\tilde{n}^{*}(f)}{S_{n}(f)} df \end{array} \\ \end{array}$ 

## Prior, posterior, evidence

 $p(\boldsymbol{d} \mid \boldsymbol{\theta})$  $\boldsymbol{d}, M$ 

Prior Potentially influential choices

### Posterior

Sampling algorithm provides set of (n-dim) parameter values that together give a fair representation of the posterior pdf

 n-dim posterior samples are end result of inference

Evidence = marginal likelihood  

$$p(d|M) = \int_{\Omega_{\overrightarrow{\theta}}} p(d|\overrightarrow{\theta}, M) p(\overrightarrow{\theta}|M) d\overrightarrow{\theta}$$
  
Important for model selection

Results often presented using 2-D corner plots

- > Marginalizing on other parameters, e.g.  $p(m_1, m_2 | d) = \int_{\vec{a}} p(\vec{\theta}_{other}, m_1, m_2 | d) d\vec{\theta}_{other}$
- Parameter correlations

# **Rapid parameter estimation**

Parameter estimation requires long computing times

- A few hours for short BBH signals
- Weeks for BNS signals
- Driven by evaluating likelihood (including computing waveform) at each step

### Low-latency localization of sources for electromagnetic follow-up

- Focus is on extrinsic parameters
  - Fix intrinsic parameters to values reported by search pipelines
- Information crucial for localization is encapsulated in matched-filter estimates of times, amplitudes, and phases on arrival at the detectors
- Compute posterior distribution of extrinsic parameters, provide (good!) approximate marginal posterior distribution of sky location within minutes



L12 (2017

ApJL 848



# System dynamics & intrinsic parameters

0.6

0.5

-0.02

0.02

- □ Inspiral phase evolution: post-Newtonian expansion in powers of  $(v/c)^2$ □ At leading order: driven by chirp mass  $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$ □ At higher orders > Mass ratio  $q = m_2/m_1$ > Effective spin  $\chi_{\text{eff}} = \frac{m_1 \chi_{1z} + m_2 \chi_{2z}}{m_1 + m_2}$ PRX 9, 011001 (2019) Correlations GW170817 > Between  $m_1$  and  $m_2$ > Between q and  $\chi_{eff}$ 0.7
- For high-mass systems, mergerringdown significant part of signal, driven by total mass



## **Extrinsic parameters**

From GW signal, difficult to distinguish distant, well-oriented source from nearby, ill-oriented source

Correlation between luminosity distance and inclination (and direction)



## More features: higher-order modes

$$h_{+} - ih_{\times} = \sum_{l \ge 2} \sum_{m=-l}^{l} {}_{-2}Y_{lm}(\iota, \phi_c)h_{lm}$$

Quadrupolar mode dominates

 $l = 2 m = \pm 2$ 

- Higher-order modes significant
  For binaries with asymmetric masses
  - For binaries seen edge-on





#### Asymmetric system GW190412

> 30+8 M<sub>0</sub>

 Presence of higher-order modes helps lifting degeneracy between distance and inclination



See movie

## **More features: precession**



# More features: matter effects

GW170817

- Relevant for BNS and NSBH binaries
- Point-particle approximation breaks down before end of inspiral
- Tidal field of companion induces massquadrupole moment and accelerates inspiral
  - > Induced quadrupole moment depends on unknown NS tidal deformability  $\Lambda$
  - Impact on waveform phase potentially observable above a few hundred Hz
- $\hfill\square$  Upper limits on  $\Lambda$  constrain NS compactness and radius
- **GW170817** 
  - Equations of state predicting less compact stars are disfavored
  - » NS radii ~12 km



# More features: post-merger signal

### Black hole ringdown

- Merger remnant likely BH in most cases
- > Reaches equilibrium by radiating GW quasinormal modes
  - Superposition of exponentially damped sinusoidal oscillations
  - Frequencies and damping times determined by mass and spin of remnant BH
  - Energy radiated via ringdown <  $\sim 1\% M_{BH}$

### BNS case

- Prompt BH formation
- Formation of a short-lived or long-lived NS
  - NS oscillations potentially excited and detectable

Black holes have no hair (?)

## More features: lensing?

- Like electromagnetic waves, gravitational waves can be gravitationally lensed
- □ Lenses rare in universe probed with current sensitivities → unlikely
  - > Expected for  $1:10^{3-4}$  events
- Various signatures

### Signal amplification



#### Multiple images Time delays: minutes, months, years



### Waveform distortion from microlensing



## **Multi-messenger counterparts**

□ Counterparts expected for BNS and some NSBH mergers

- > Electromagnetic emission
- Possibly neutrinos

□ The famous case of GW170817

- Coincident with a short Gamma-ray burst
- Extensive follow-up led to discovery of optical transient, then X-ray, radio
- > Optical transient linked to kilonova
  - Nucleosynthesis of heavy elements in ejecta



## **Standard sirens & Hubble constant**

- GW signal provides luminosity distance standard sirens
- Universe expansion rate: recession velocity / distance
- GW signal typically does not provide redshift
  - Full mass-redshift degeneracy for inspiral
- □ How do we get the redshift ?
  - From possible electromagnetic counterpart GW170817!
  - Statistically, from reliable galaxy catalog
  - > Statistically, from known features in NS / BH mass distribution
  - From tidal effects if NS equation of state is known
  - From post-merger signal if observed and NS EoS is known
- High statistics will provide precise measurements

Present

## Source population: merger rates

Detection

### 100 **Constant Binary Total Mass Constant Binary Mass Ratio** Secondary Mass (Solar Masses) 10 GW190412 GW17060 GW190814 NSBF GW170817 GW200105 162426 100 Primary Mass (Solar Masses)

**Observed sample** 

Merger rates

$$\mathcal{R}_{\rm BNS} = 13$$
 - 1900  ${
m Gpc}^{-3}{
m yr}^{-1}$ 

 $\mathcal{R}_{\text{NSBH}} = 7.4 - 320 \text{ Gpc}^{-3} \text{yr}^{-1}$ 

$$\mathcal{R}_{\rm BBH} = 16 - 130 \ {\rm Gpc}^{-3} {\rm yr}^{-1}$$

Intrinsically rarer but dominate observed sample – louder sources detectable at larger distances

BBH merger rate increases with redshift





# Source population: formation scenarios

- Understanding binary formation and evolution of progenitor stars
  - > Merger rates
  - Mass distribution
  - > Spin distribution
- Two main classes of formation channels for merging binaries
  - > Isolated binary evolution
  - > Dynamical formation





### GW190521: challenges and clues

- > 66 + 85 ⇒142 M<sub>☉</sub>
- > How were the initial BHs formed?
- Remnant is an intermediate-mass BH



## **Testing some GR cornerstones**



### GW propagation speed

- > GW170817 GRB 170817A: delay of  $1.74 \pm 0.05$  s over > 85 million years propagation
- > Assume γ emission delayed by [0,10]s

$$-3 \times 10^{-15} \le \frac{v_{\text{GW}} - v_{\text{EM}}}{v_{\text{EM}}} \le 7 \times 10^{-16}$$

Astrophys. J. Lett. 848, L13 (2017)

## Equivalence principle

EM radiation and GWs affected by background gravitational potentials in the same way ?

> Shapiro delay 
$$\delta t_{\rm S} = -\frac{1+\gamma}{c^3} \int_{\mathbf{r}_{\rm e}}^{\mathbf{r}_{\rm o}} U(\mathbf{r}(l)) dl$$
  
 $-2.6 \times 10^{-7} \le \gamma_{\rm GW} - \gamma_{\rm EM} \le 1.2 \times 10^{-6}$ 

Many alternative theories of gravity ruled out



## **Further tests of GR**

### □ GW polarization

> Are signals recorded in different detectors consistent with two tensor polarizations?

Dispersion

Any sign of waveform distortion due to different frequencies propagating at different speeds?

Source dynamics

- Consistency of inspiral waveform with GR prediction
- Consistency of inspiral and ringdown parts of signal
- Test of BH no-hair theorem with ringdown spectroscopy



## **CLOSING THE LOOP : SCIENCE & DETECTORS**

## More sensitive detectors for more science

### Sensitivity

- More statistics to characterize source populations
- > Higher signal-to-noise ratio, i.e. precision, for exceptional events
- > Potential for new discoveries

## Bandwidth

- Low-frequency sensitivity
  - High-mass BBH mergers
  - More accurate parameter estimation
- > Mid- and high-frequency sensitivity
  - Black hole spectroscopy
  - Post-merger signal



## Network size and robustness

- > Duty cycle
- > 3-detector observations
  - Improved sky localization

## Multi-messenger approach

- Low-latency alerts
  - Possibly early warning
- > And multi-wavelength
  - Some sources expected to be visible from space then from Earth

## Potential of next-gen detectors re. BNS & BBH



Stellar-mass BHs and NSs throughout cosmic time

- Map population of compact objects across time
- Remnants of first stars

# **Further reading**

## Two recent Scholarpedia articles

- Gravitational Waves: Ground-Based Interferometric Detectors
- Gravitational Waves: Science with Compact Binary Coalescences