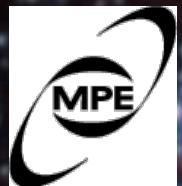


# GRAVITY

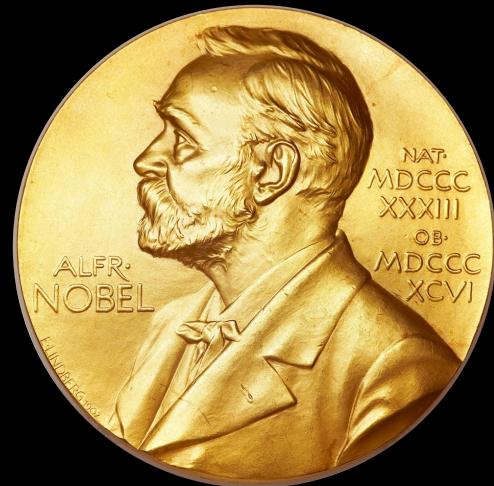
## Stars orbiting the Galactic Black Hole

Stefan Gillessen  
on behalf of

The GRAVITY  
collaboration



The mass measurement of Sgr A\*,  
the massive black hole candidate in the Galactic Center,  
was honored with the Nobel prize



# Outline

- Part I        The mass of Sgr A\*
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  - Interferometry
  - Testing General Relativity in the Galactic Center
  - SgrA\* flares
  - (A funny gas cloud)

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$$R_S = \frac{2\,G\,M}{c^2}$$

# How to find a black hole

1. Find an object that has an event horizon
2. Show that a certain amount of mass is confined to a volume smaller than  $R_S$
3. Show that an object is unstable against its self-gravity



Note: Density is ill-defined for black holes

$$\rho \sim \frac{M}{\frac{4\pi}{3}R_S^3} \sim \frac{1}{M^2}$$

# Black holes are astronomical objects

- 1kg:  $R_S = 10^{-27}$  m;  
(proton:  $10^{-15}$  m)  $\rho = 10^{77}$  g/cm<sup>3</sup>
- Earth:  $R_S = 1$  cm;  $\rho = 10^{27}$  g/cm<sup>3</sup>
- Sun:  $R_S = 3$  km;  $\rho = 10^{16}$  g/cm<sup>3</sup>
- Star cluster:  
( $10^5$  stars)  $R_S = 3 \times 10^8$  m;  
(distance to the moon)  $\rho = 10^6$  g/cm<sup>3</sup>
- Galaxy  
( $10^{11}$  stars)  $R_S = 3 \times 10^{14}$  m;  
(2000 AU)  $\rho = 10^{-6}$  g/cm<sup>3</sup>  
(thinner than air)

# Types of black holes

- Particle physics scale
  - LHC, cosmic radiation
  - Should be bright & unstable due to their Hawking radiation
- Planetary mass – primordial black holes
  - primordial
- Stellar mass black holes ( $1 - 10 M_{\text{sun}}$ )
  - End products of massive stars
- Intermediate mass black holes ( $10^3 - 10^5 M_{\text{sun}}$ )
  - Primordial, or first generation of stars
- Supermassive black holes ( $10^6 - 10^9 M_{\text{sun}}$ )
  - in the center of galaxies
  - Quasars, Active Galactic Nuclei

???

??

✓

?

✓

# A short history of black holes

1783 John Michell

Basic idea

1915 Albert Einstein:  
General relativity

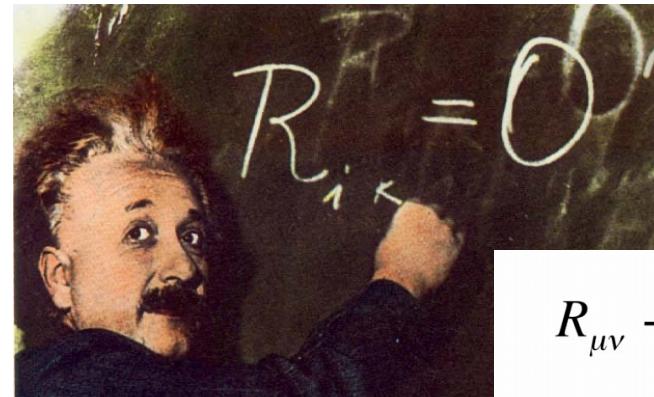
1916 Karl Schwarzschild  
Black hole solution

1964 John Wheeler  
“Black hole”

1969 Quasars speculated

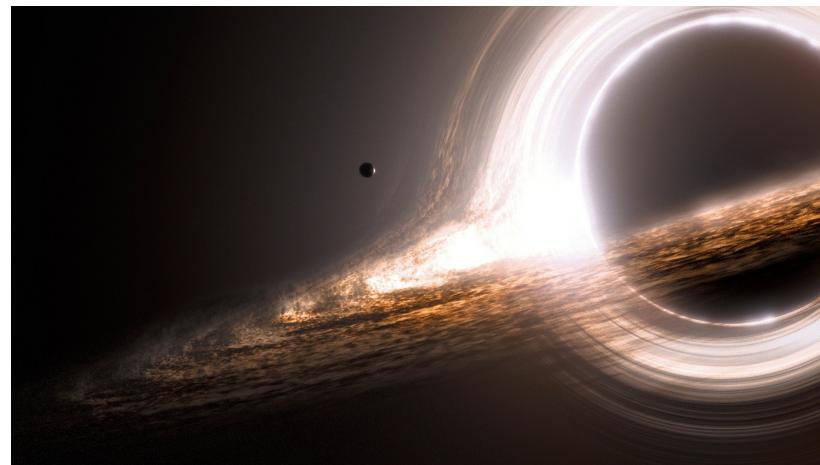
1971 Cyg X-1's mass

2002 Galactic Center



$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$ds^2 = c^2 \left(1 - \frac{2GM}{c^2r}\right) dt^2 - \left(1 - \frac{2GM}{c^2r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$



PHILOSOPHICAL  
TRANSACTIONS:  
OF THE  
ROYAL SOCIETY  
OF  
LONDON.

VOL. LXXIV. For the Year 1784.

PARTS I. & 2



LONDON,

SOLD BY LOCKYER DAVIS, AND PETER ELMSLY,  
PRINTERS TO THE ROYAL SOCIETY.

MDCCLXXXIV.

VII. *On the Means of discovering the Distance, Magnitud:, &c. of the Fixed Stars, in consequence of the Diminution of the Velocity of their Light, in case such a Diminution should be found to take place in any of them, and such other Data should be procured from Observations, as would be farther necessary for that Purpose.* By the Rev. John Michell, B. D. F. R. S. In a Letter to Henry Cavendish, Esq. F. R. S. and A. S.

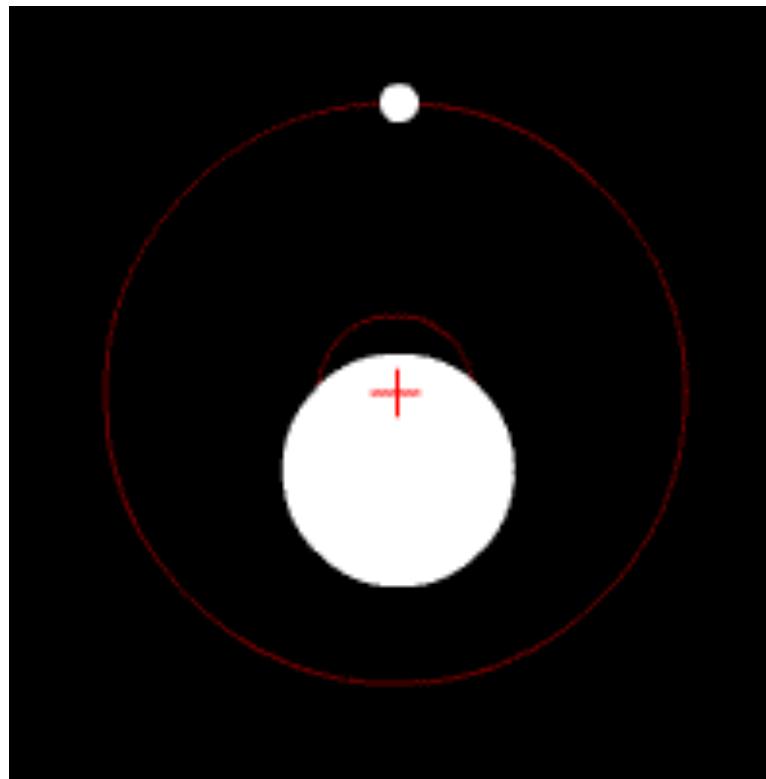
P. 35.

16. Hence, according to article 10, if the semi-diameter of a sphære of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiarum, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.

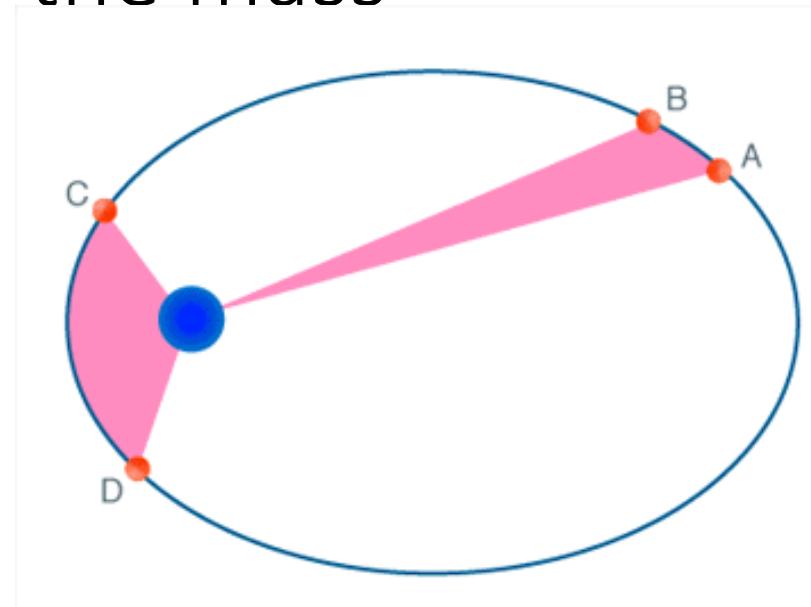
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Unseen mass can shake a visible object

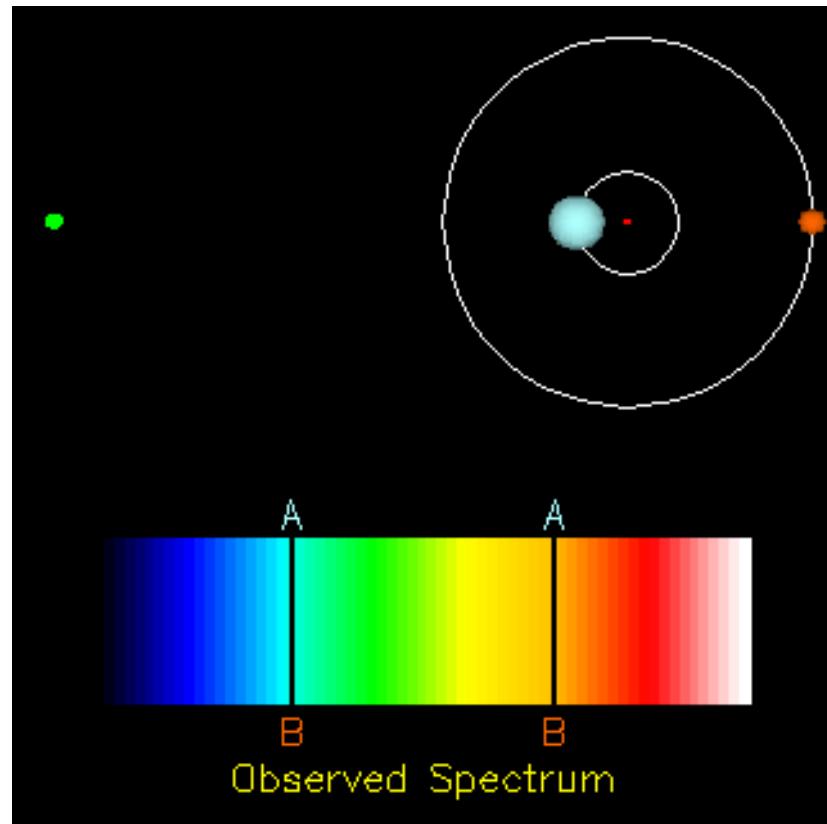


# Kepler ellipses yield the mass

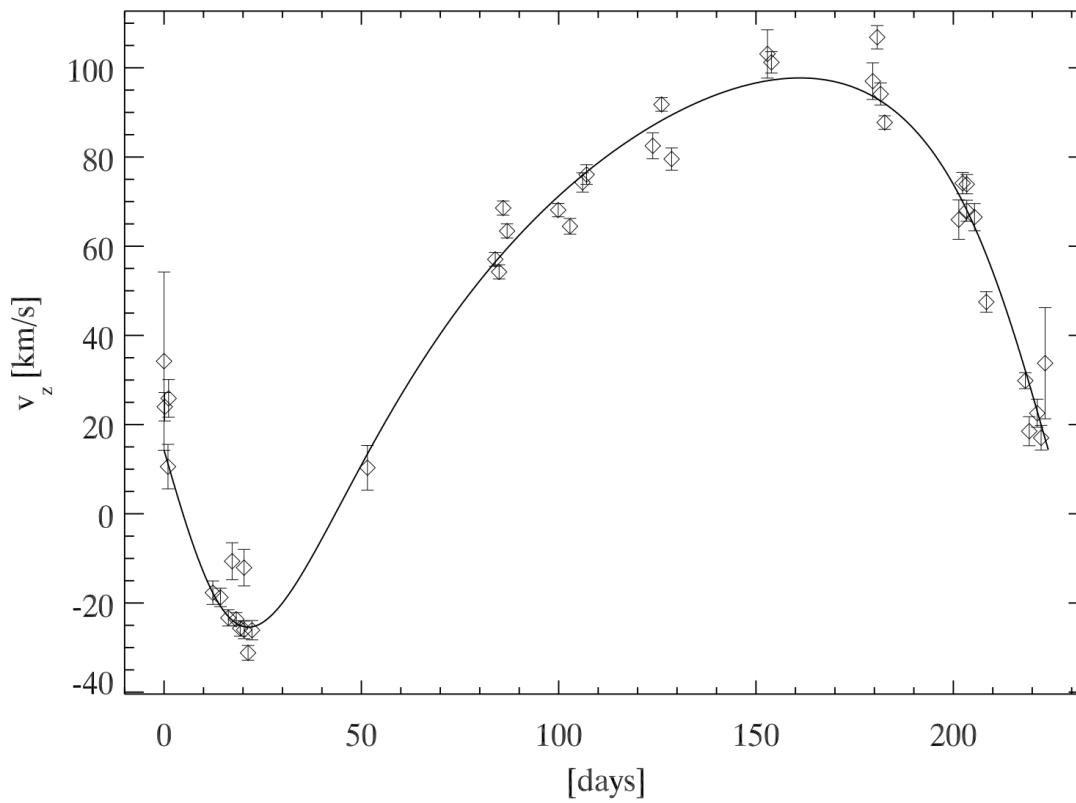


$$M = \frac{4\pi^2}{G} \frac{a^3}{T^2}$$

Most successful route:  
Radial velocity measurements using the Doppler effect

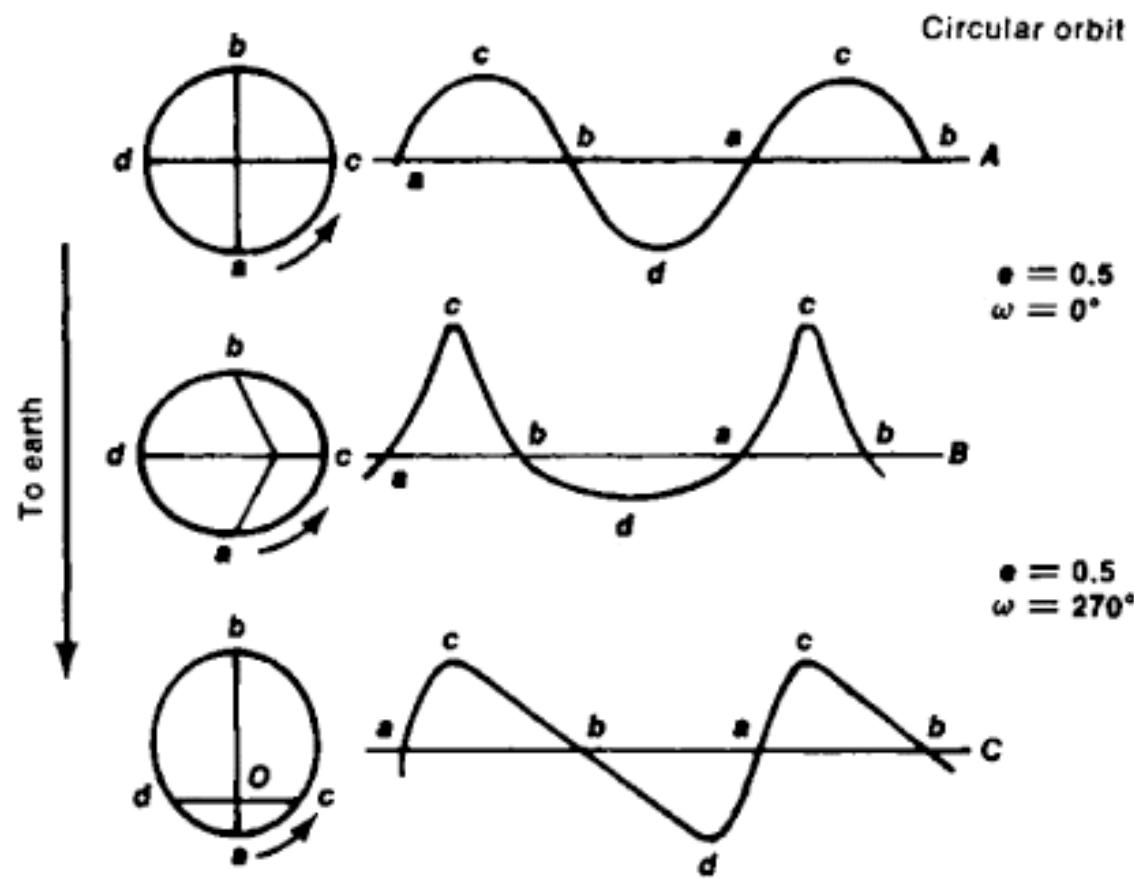


# A typical example



Pfuhl et al. 2014

# Shape and orientation of the orbit matter



# Determine the orbital elements

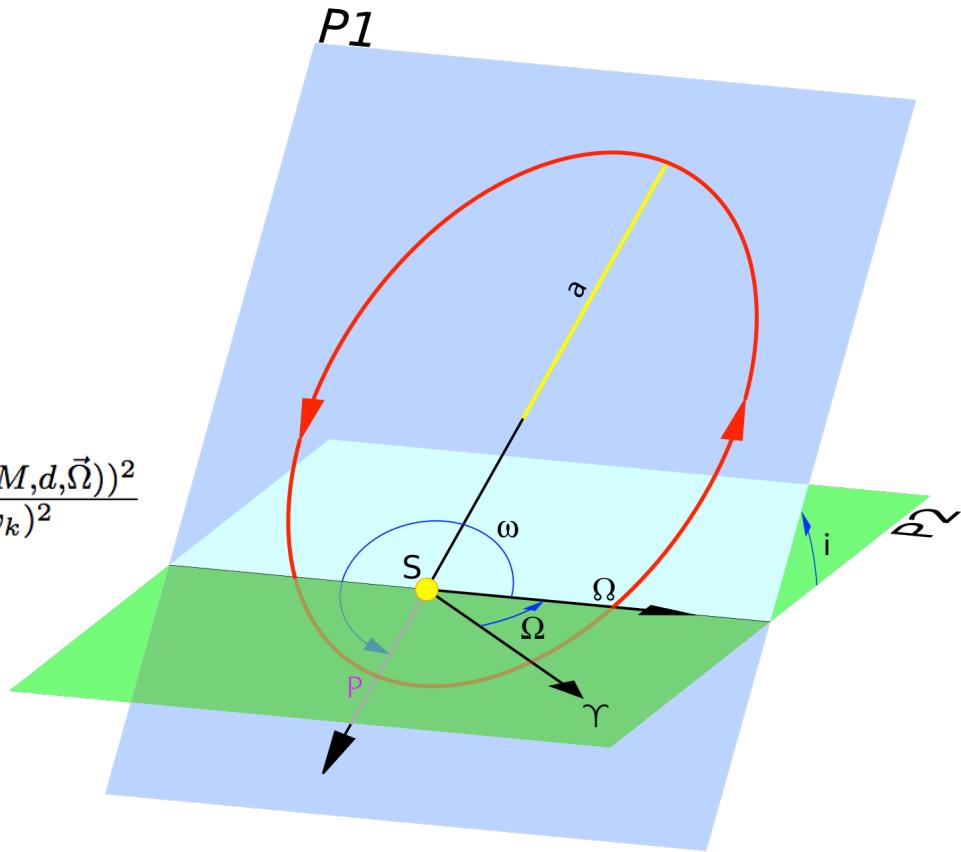
$$\{v_k \pm \Delta v_k\}$$

$$\vec{\Omega} = \{a, e, i, \Omega, \omega, t_0\}$$

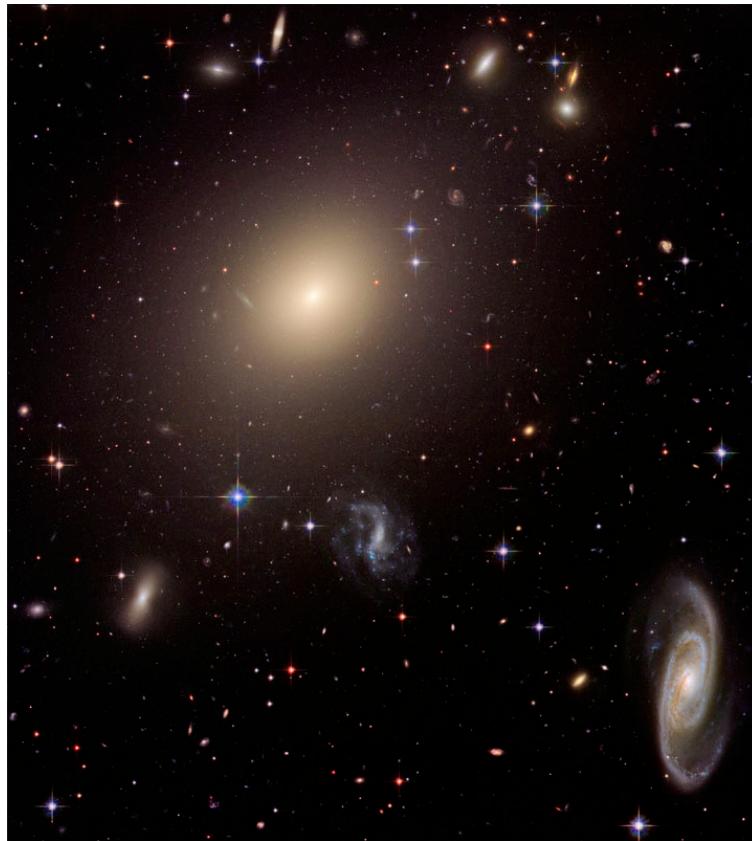
$$v(t) = v(t; M, d, \vec{\Omega})$$

$$\chi^2(M, d, \vec{\Omega}) = \sum_{k=1}^{\max} \frac{(v_k - v(t; M, d, \vec{\Omega}))^2}{(\Delta v_k)^2}$$

$$\min |_{(M, d, \vec{\Omega})} \chi^2(M, d, \vec{\Omega})$$



# More difficult: Unresolved stellar system



You can measure:

- Projected light profile  
→ proxy for  $n(R)$
- Radial velocity profile  
→  $v(R), \sigma(R)$

# Solution: Jeans modeling

Distribution function:

$$f = f(\vec{x}, \vec{v}, t)$$

“How many stars are at any given time  
at a certain position with a certain velocity”

Number density of particles:

$$n(\vec{x}, t) = \int f(\vec{x}, \vec{v}, t) d^3v$$

Mean velocity:

$$\langle v \rangle = \int v f d^3v$$

# Solution: Jeans modeling

Full differential:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial t}$$

Newton's law:

$$\frac{\partial v}{\partial t} = a = -\frac{\partial \Phi}{\partial x}$$

yields:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \frac{\partial \Phi}{\partial x} \frac{\partial f}{\partial v}$$

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \nabla \Phi \frac{\partial f}{\partial \vec{v}}$$

# Solution: Jeans modeling

Collisionless, incompressible system:

“CBE”       $0 = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \nabla \Phi \frac{\partial f}{\partial \vec{v}}$

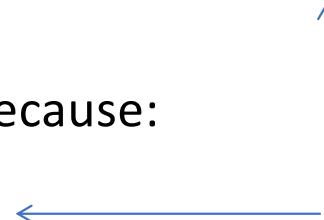
$\int \text{CBE } d^3v$  yields continuity equation:       $0 = \frac{\partial n}{\partial t} + \frac{\partial(n \cdot \langle \vec{v} \rangle)}{\partial \vec{x}}$

$\int v \text{ CBE } d^3v$  yields Jeans equation

$$-\nabla \Phi = \frac{\partial \langle \vec{v} \rangle}{\partial t} + \langle \vec{v}_i \rangle \frac{\partial \langle \vec{v} \rangle}{\partial x_i} + \frac{1}{n} \nabla(n \sigma^2)$$

Velocity dispersion appears here, because:

$$\langle v^2 \rangle = \langle v \rangle^2 + \sigma^2$$



# Solution: Jeans modeling

Simplest case: steady state, spherical symmetric, non-rotating system:

The potential is Newtonian       $\Phi(r) = \frac{G M(r)}{r}$

Only radial part of Jeans equations survives, and many terms are 0:

$$-\frac{d\Phi}{dr} = \frac{1}{n} \frac{d(n\sigma^2)}{dr}$$

Integrating:

$$n(r)\sigma^2(r) = \int_r^\infty n(r') \frac{d\Phi(r')}{dr'} dr'$$

# Solution: Jeans modeling

Slight complication: We measure **projected** values only

$$r^2 = R^2 + z^2$$

$$\Sigma(R) = \int_{-\infty}^{\infty} n(R, z) dz$$

change of variables  $z \rightarrow r$

$$\Sigma(R) = \int_R^{\infty} \frac{r}{\sqrt{r^2 - R^2}} n(r) dr$$

Similar for velocity dispersion

$$\Sigma(R)\sigma_P^2(R) = \int_R^{\infty} \frac{r}{\sqrt{r^2 - R^2}} n(r)\sigma^2(r) dr$$

Some maths ...

$$\sigma_P^2(R) = G \frac{\int_R^{\infty} (r^2 - R^2)^{1/2} r^{-2} M(r)n(r)dr}{\int_R^{\infty} (r^2 - R^2)^{-1/2} r n(r)dr}$$

## Now we can “Jeans model”

1. Define your model

$$M(r) = M_{\text{central}} + 4\pi \int_0^r r^2 \rho(r) dr$$

$$\rho(r) = \rho_0 r^\gamma$$

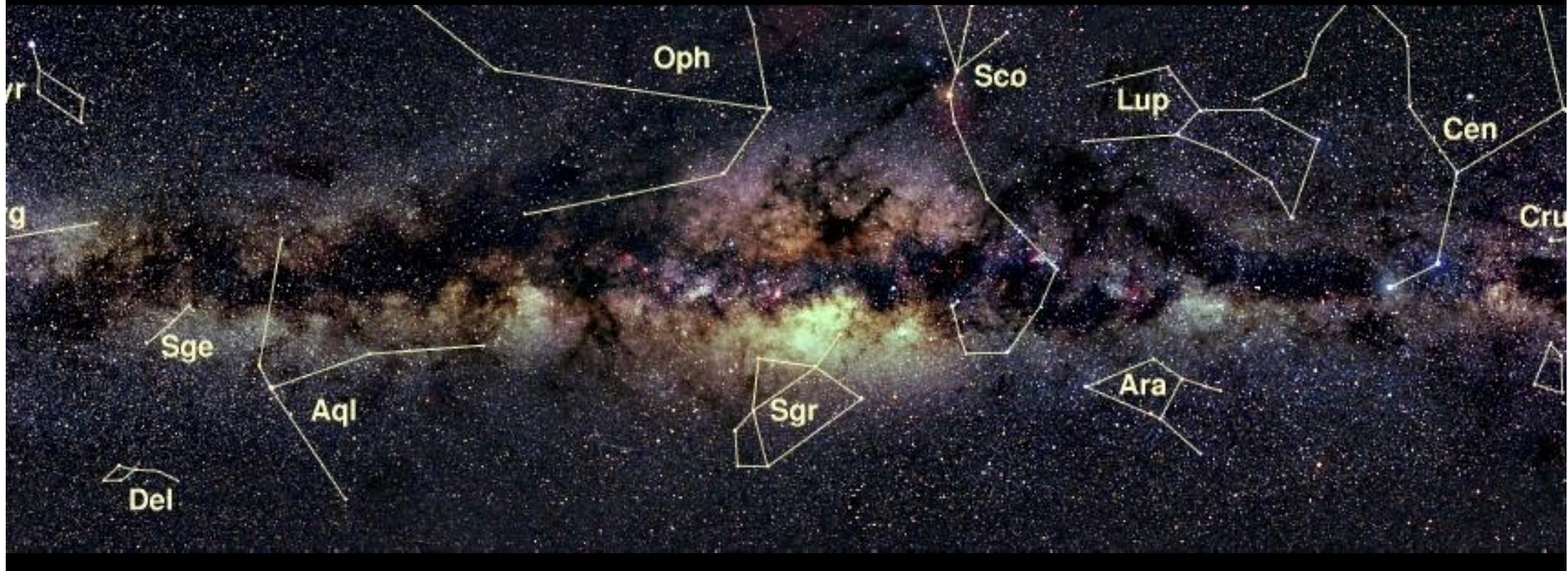
$$n(r) \propto r^\Gamma$$

1. Determine  $n(r)$  from  $\Sigma(R)$ ,  
i.e. fit  $\Gamma$  to reproduce measured density profile
2. Remaining free 3 parameters:  $M_{\text{central}}, \rho_0, \gamma$ .
3. For each set of these 3, one can evaluate  $\sigma_P^2(R)$
4. Minimize  $\chi^2$  to measured values of  $\sigma_P(R_k)$
5. You have determined  $M(r)$  !

# Outline

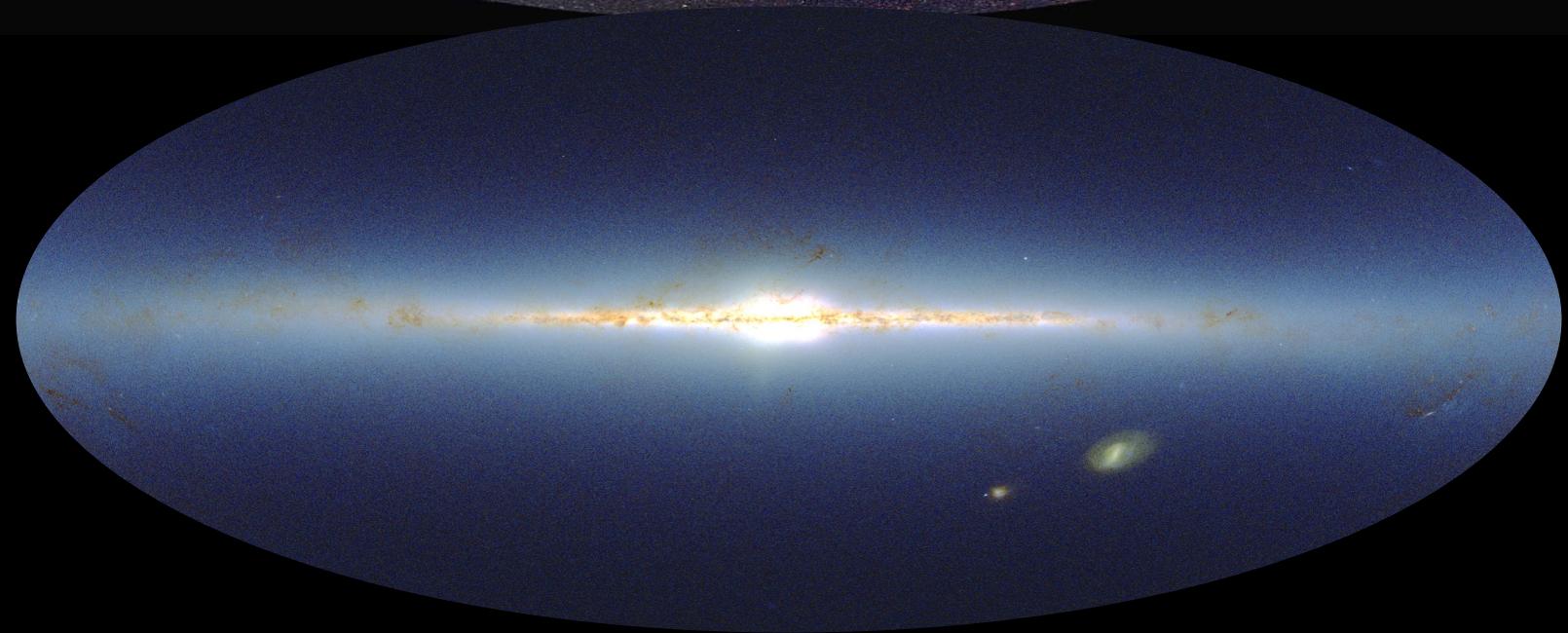
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# The Galactic Center is highly obscured

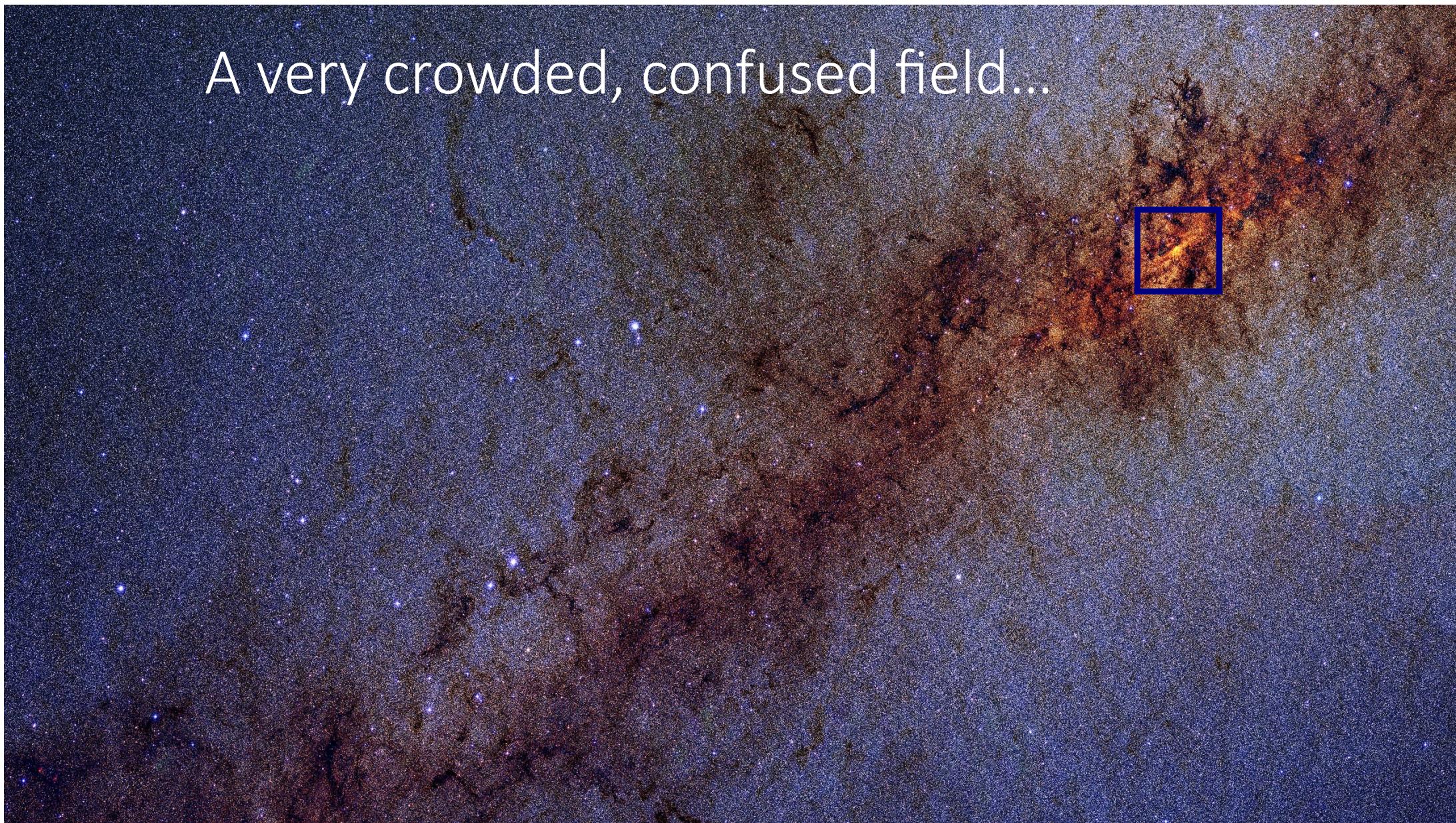


Using infrared light allows one to look through the dust

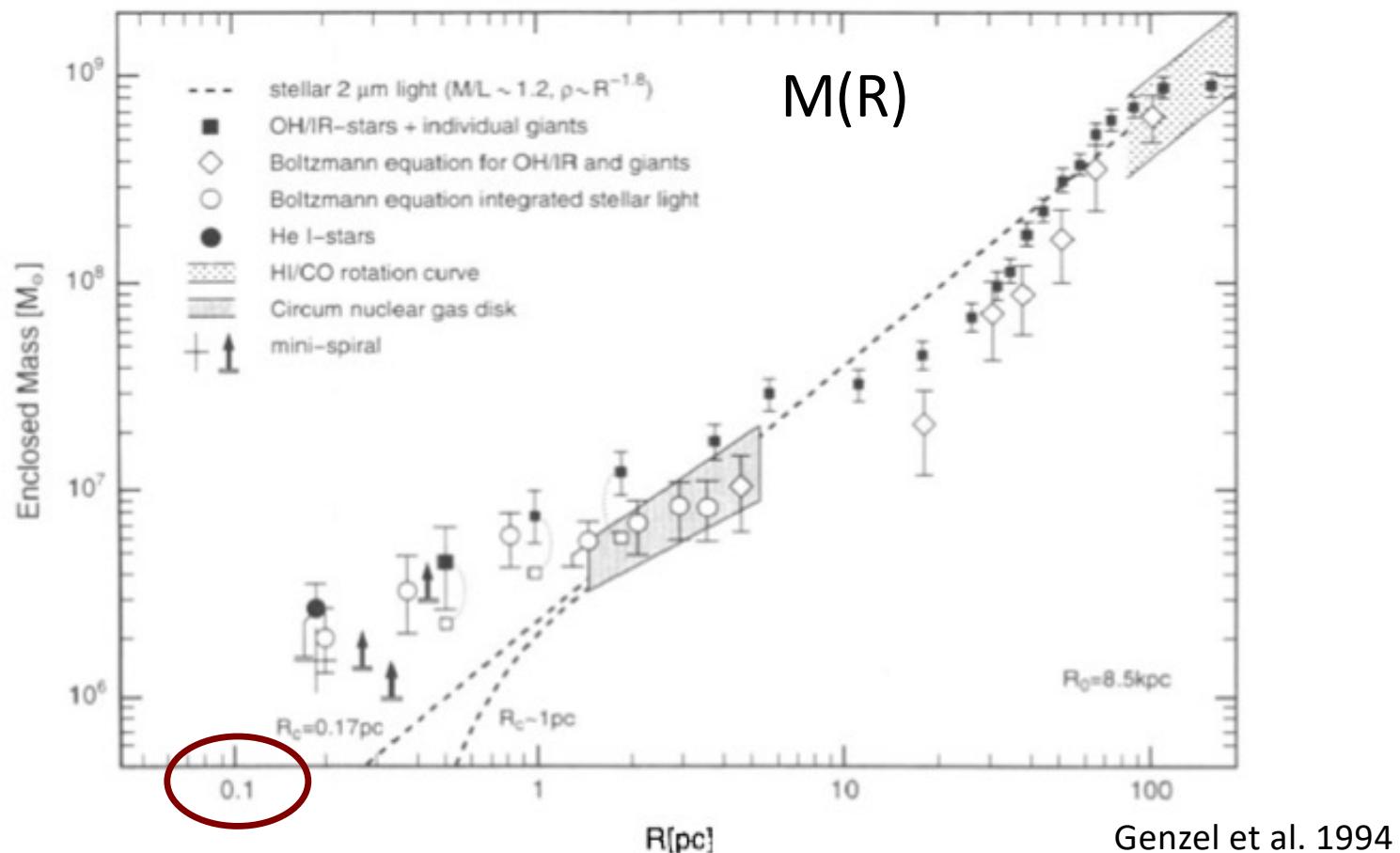




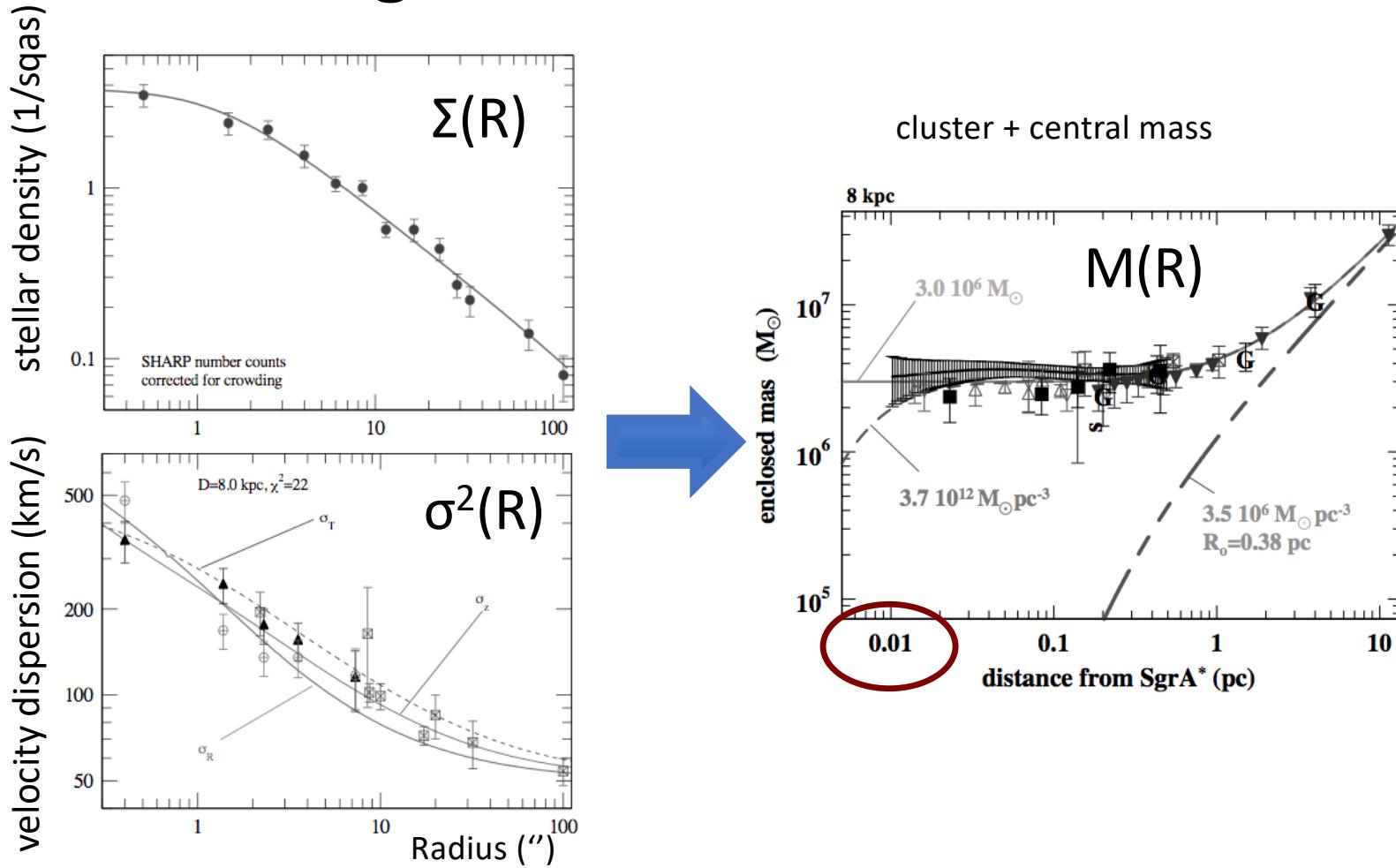
A very crowded, confused field...



# Jeans modeling in the Galactic Center – 1990's



# Jeans modeling in the Galactic Center – 2000

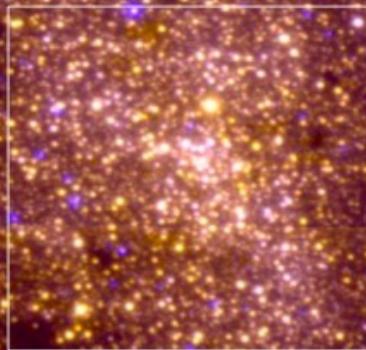


Genzel et al. 2000

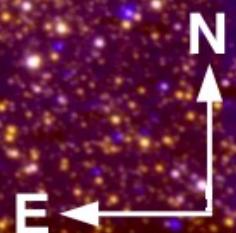
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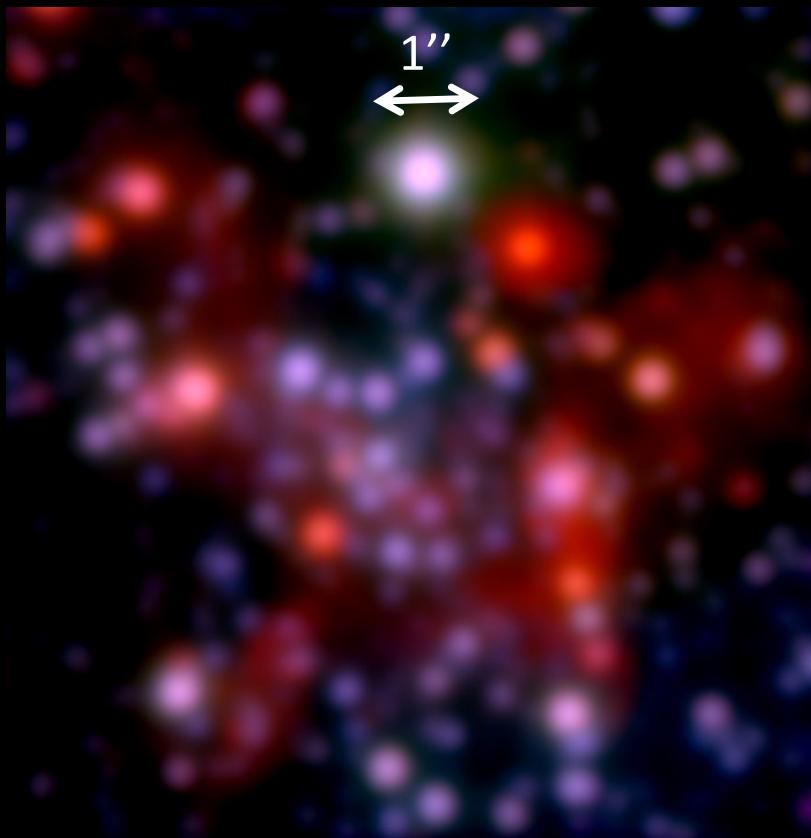
The Galactic Center contains an extremely dense star cluster



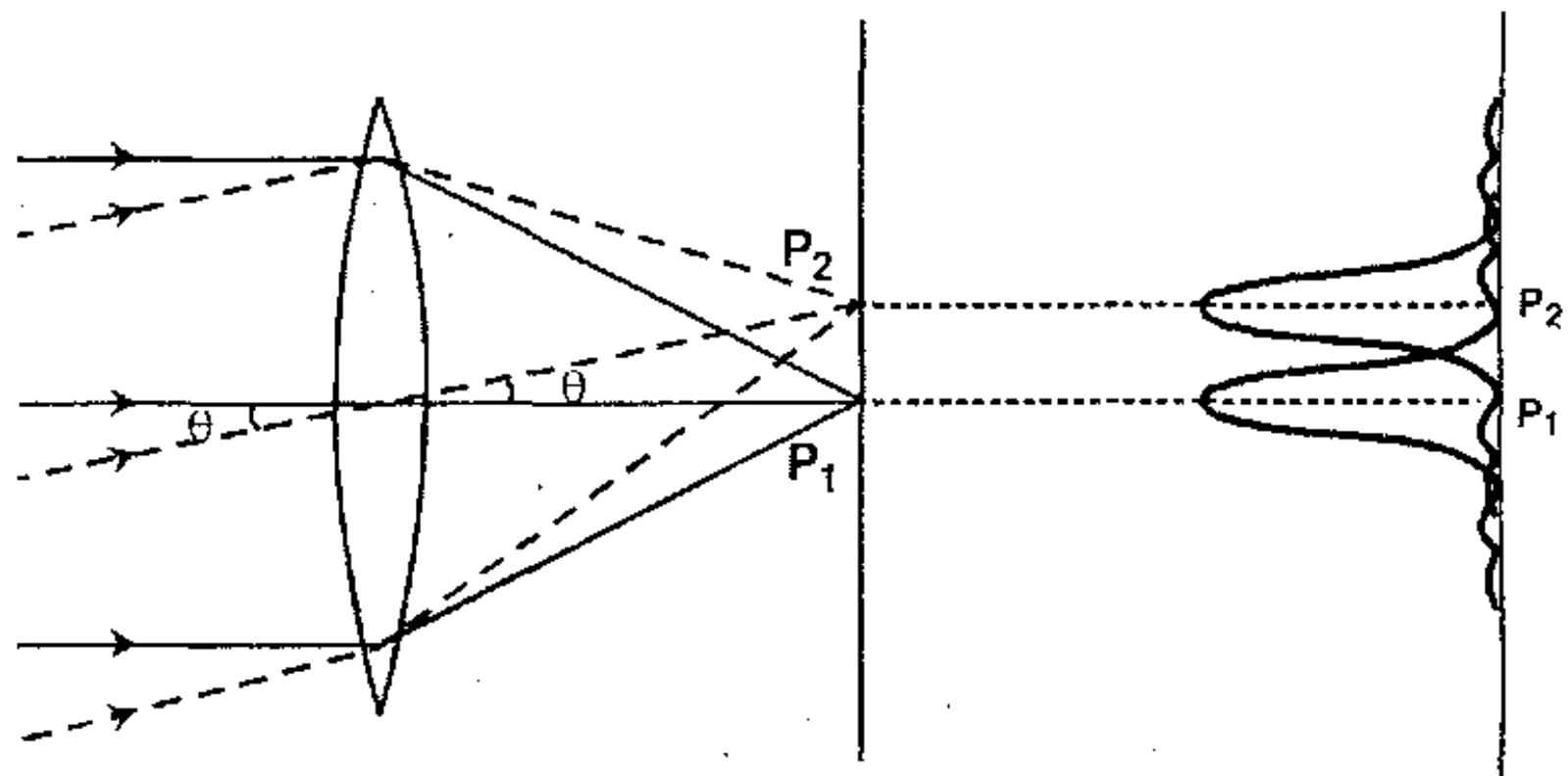
30'' = 4 lightyears



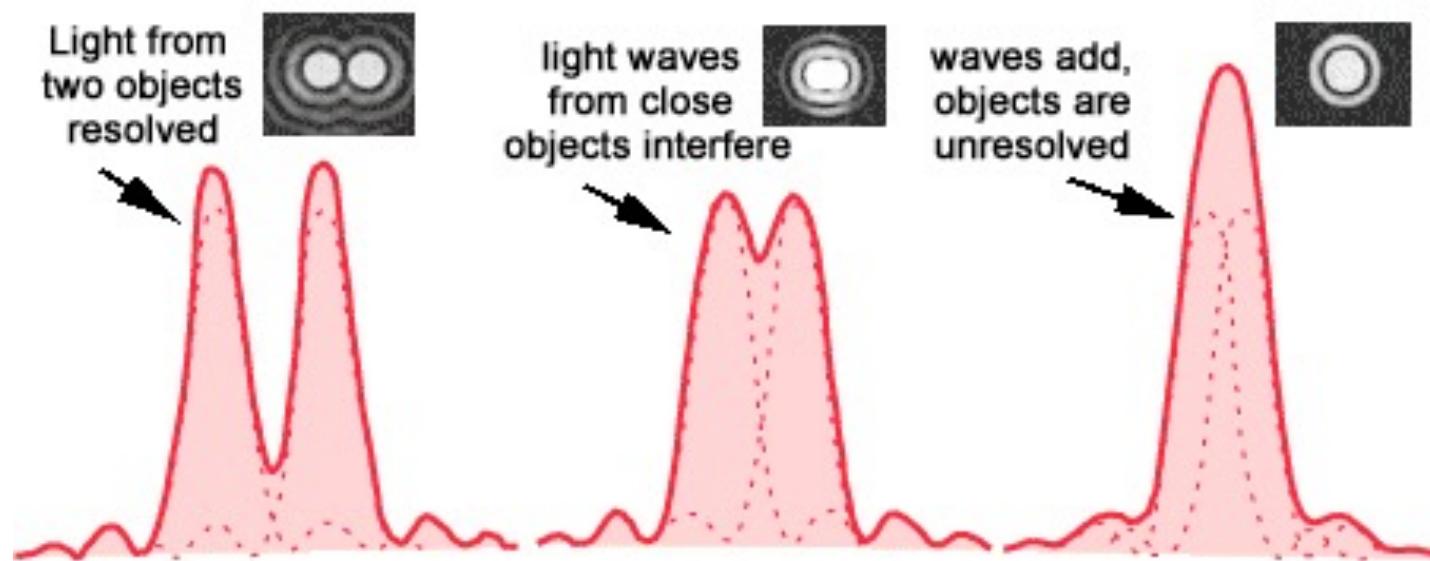
The central 20'': Not very sharp



Why are the point-like stars not images as points?



Objects are resolved when they are further apart than  $1.22 \times \lambda / D$



Rayleigh criterion:  $\theta > 1.22 \times \lambda / D$

$\lambda = 2\mu\text{m}$ ;  $D = 8\text{m}$  should be:  $\theta = 0.05''$   
but we observe  $\theta = 1''$  ?!

The "Seeing" smears out the images



# That is severely limiting

Light wavefronts are aberrated

Wavefront error  
for telescope diameter D:

$$\sigma^2 = 1.030 \left( \frac{D}{r_0} \right)^{5/3}$$

complete decorrelation:  $\sigma^2 = 1 \text{ rad} \rightarrow D \approx r_0$

Turbulence limits the resolution of a telescope to an effective size of  $r_0$ .

$R \sim \lambda/r_0$  instead of  $R \sim \lambda/D$ .

$$\lambda = 2.2 \mu\text{m}, \theta = 1''$$



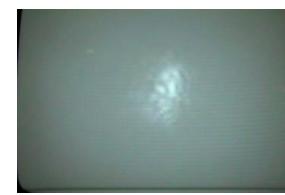
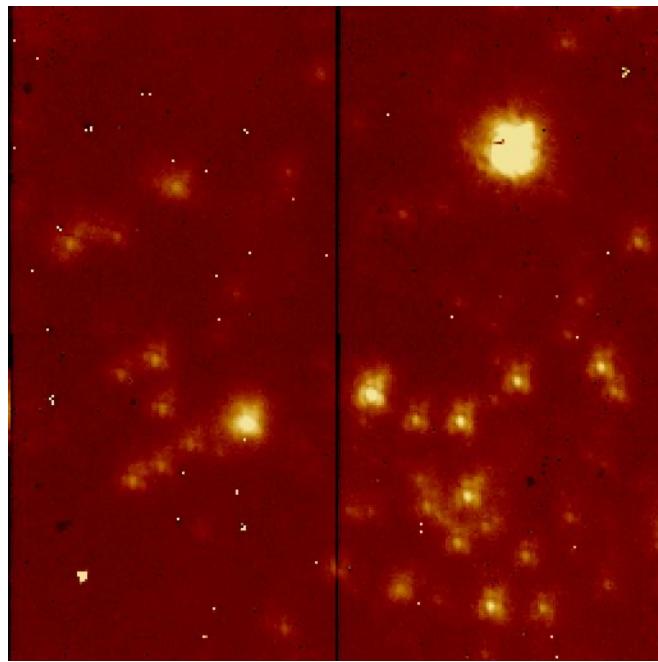
$$r_0 = 40 \text{ cm}$$

La Silla, Chile

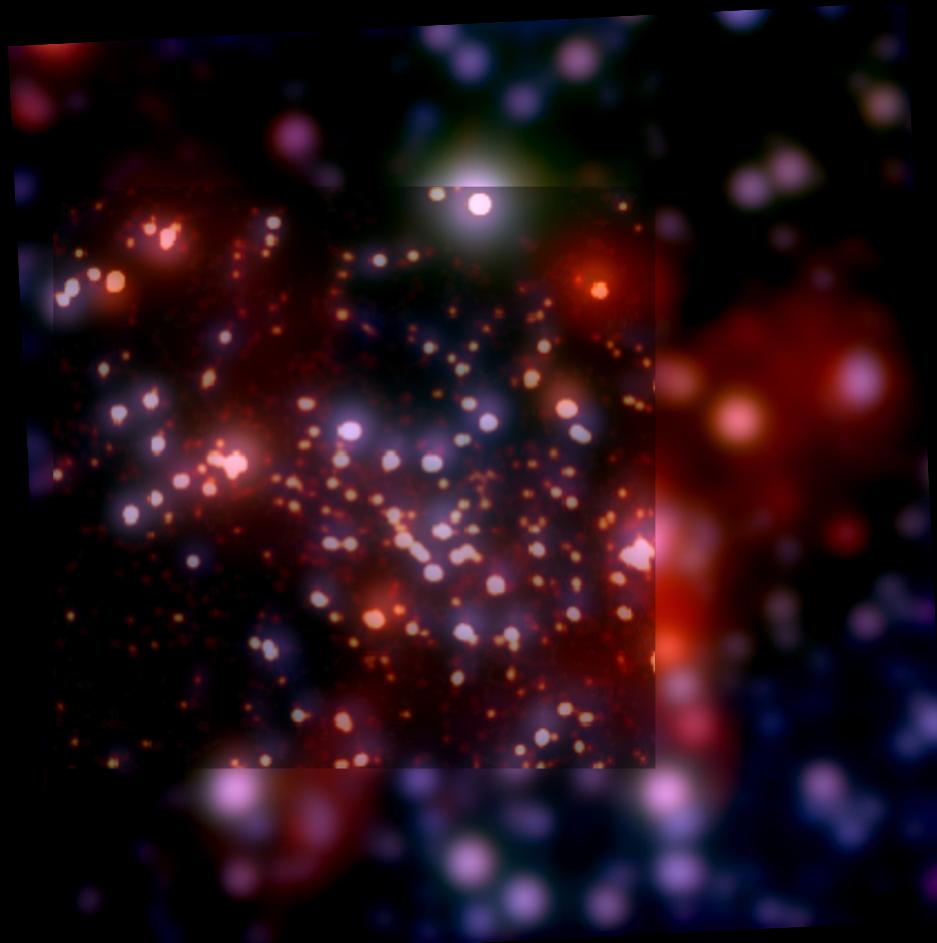


© Serge Brunier

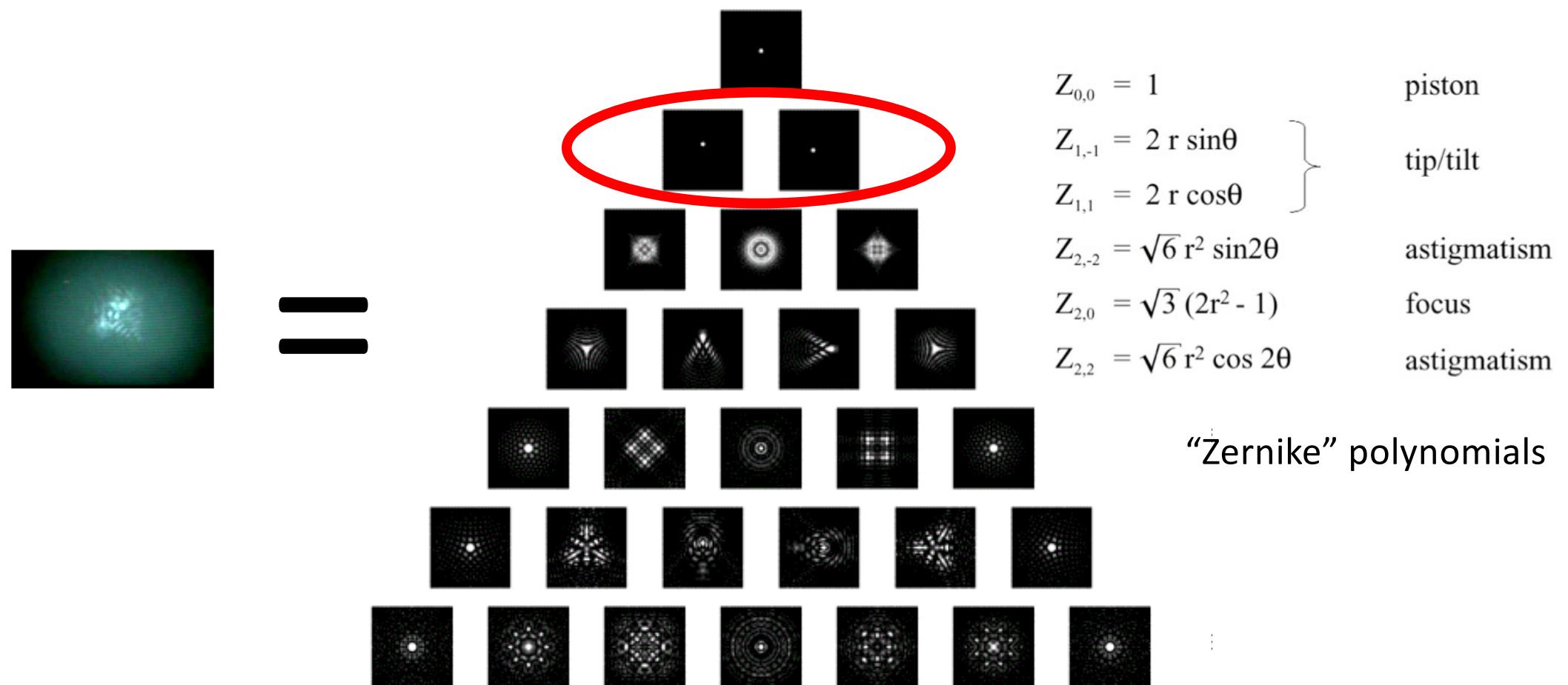
# 1992 - 2001: Speckle-Imaging

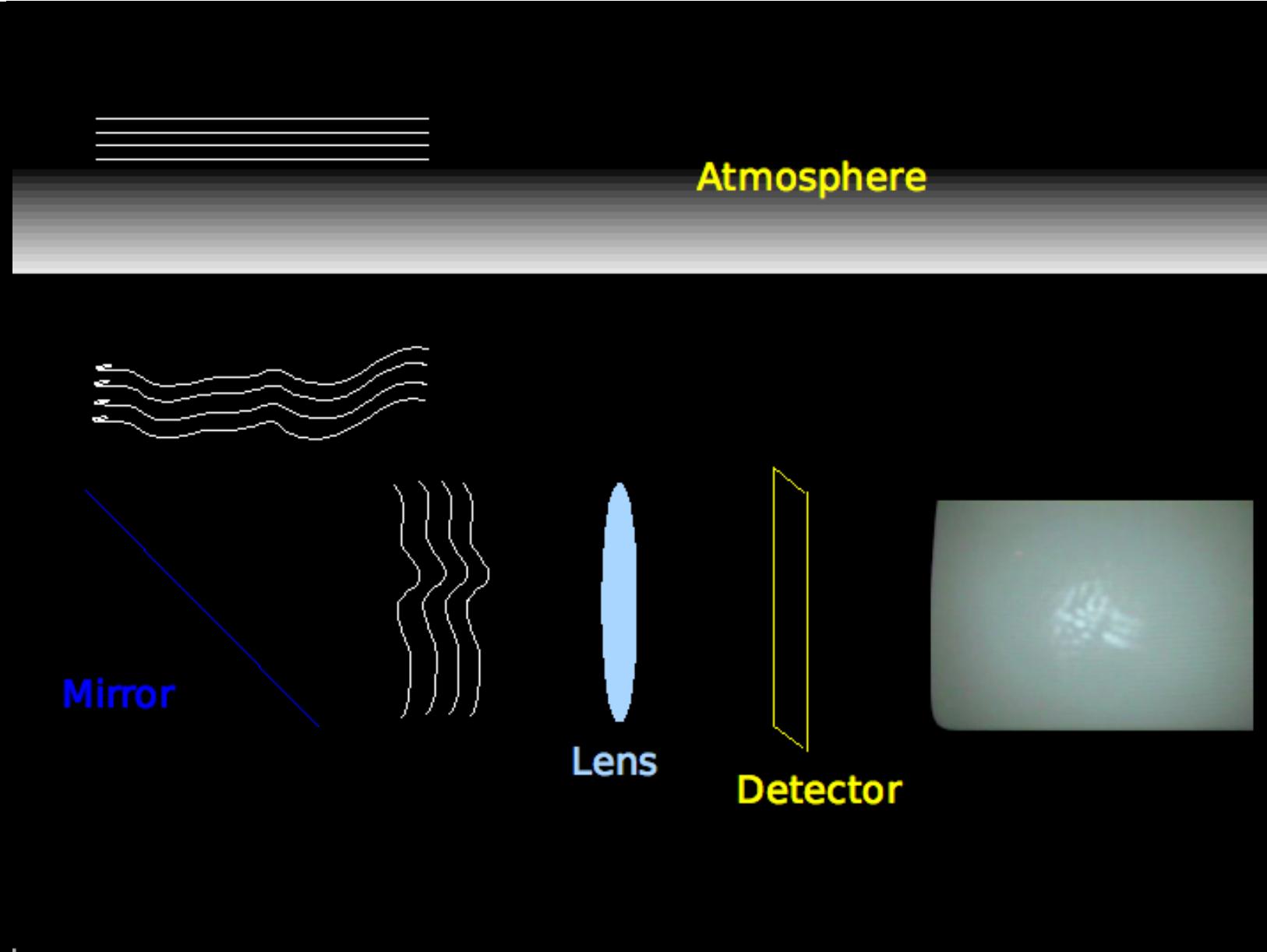


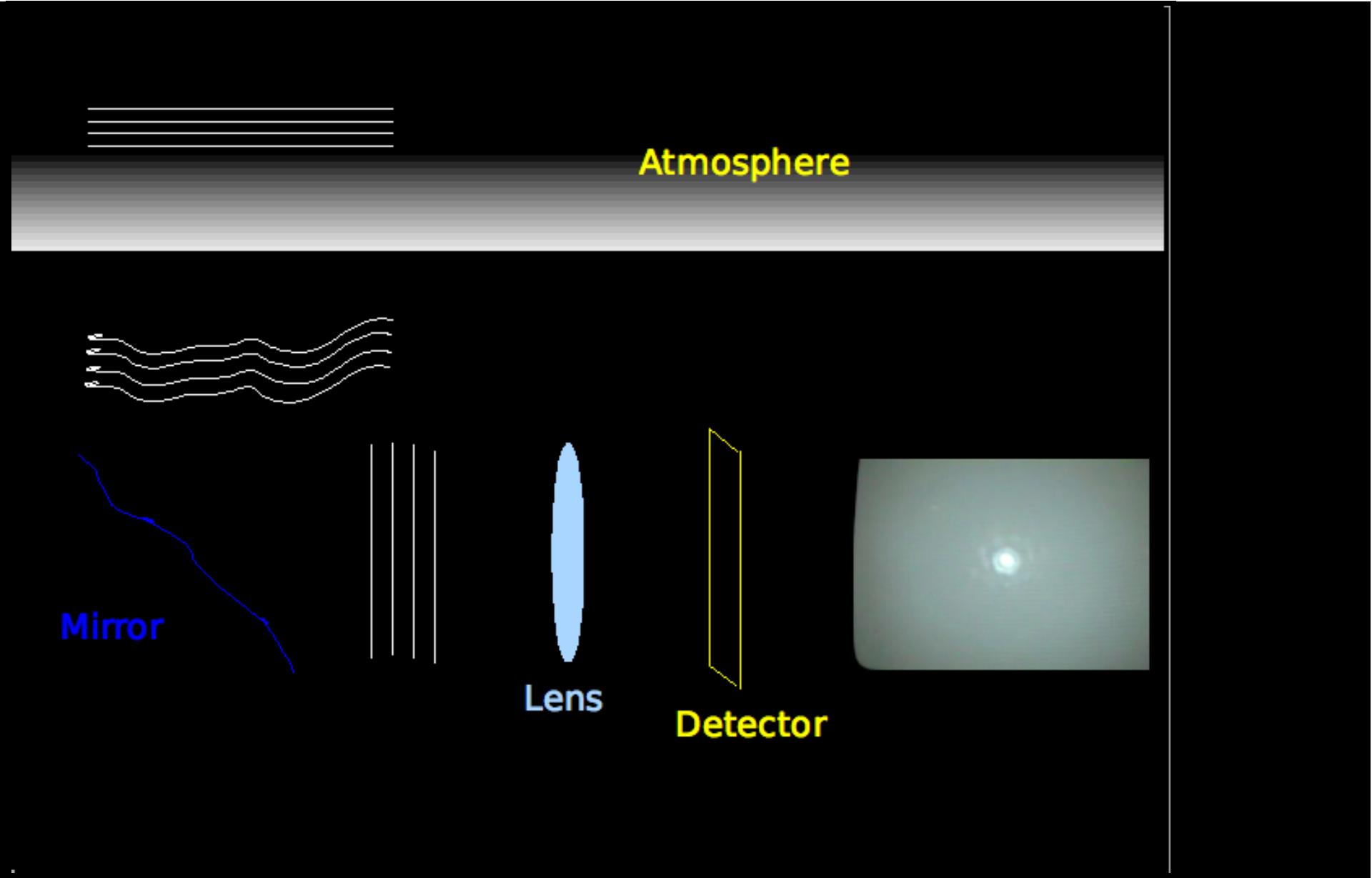
From seeing-limited to diffraction-limited



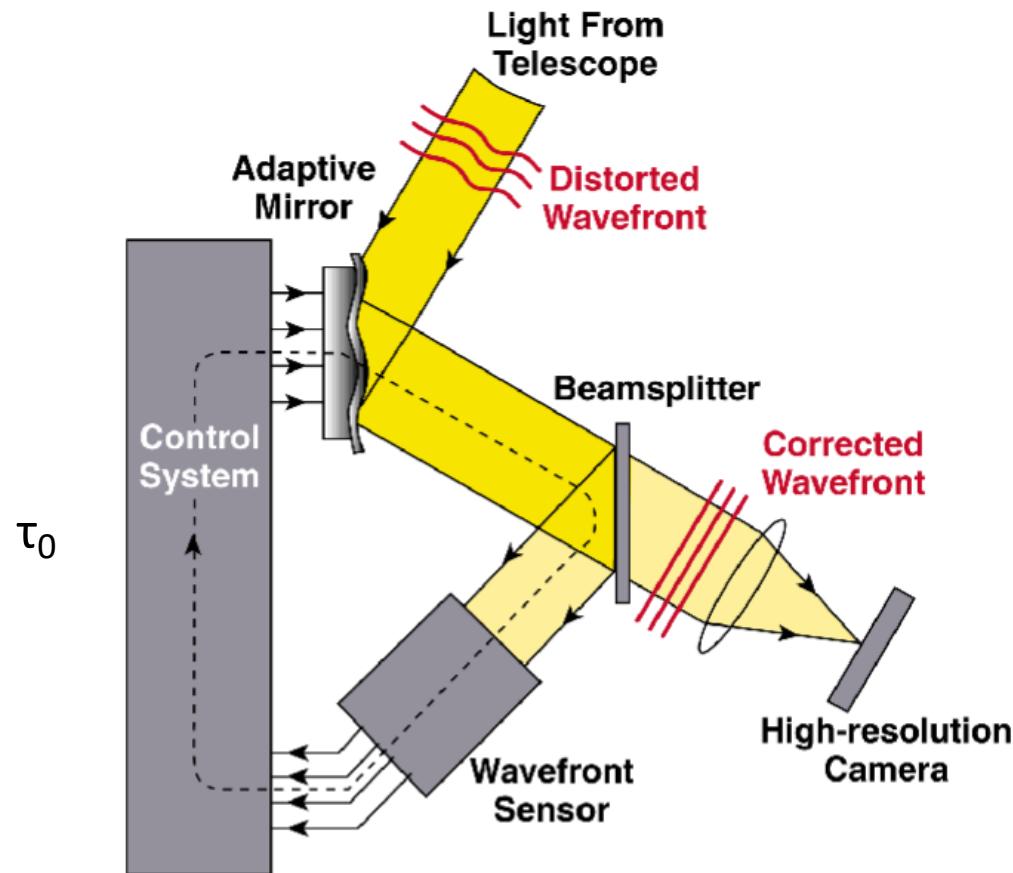
“Simple shift and add” only corrects two terms of wavefront aberrations



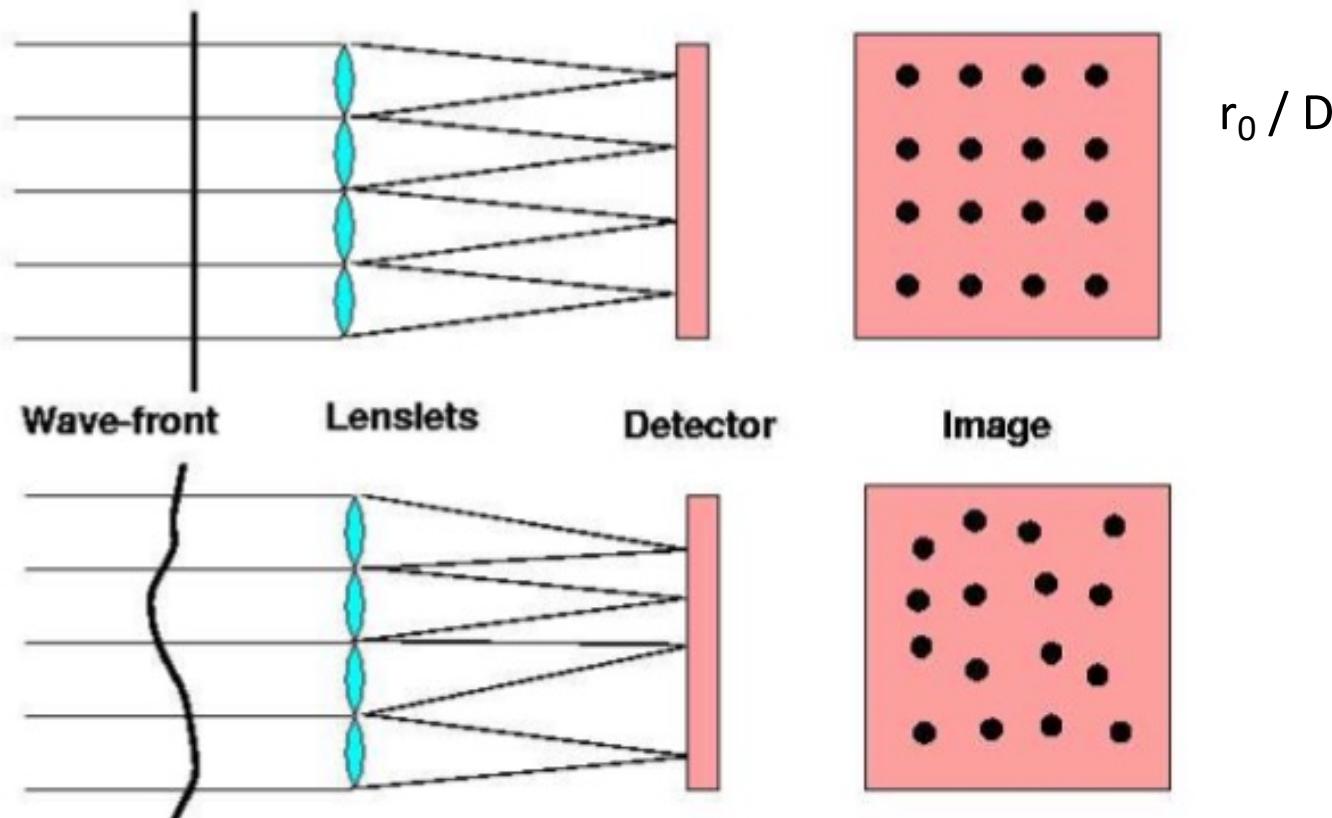




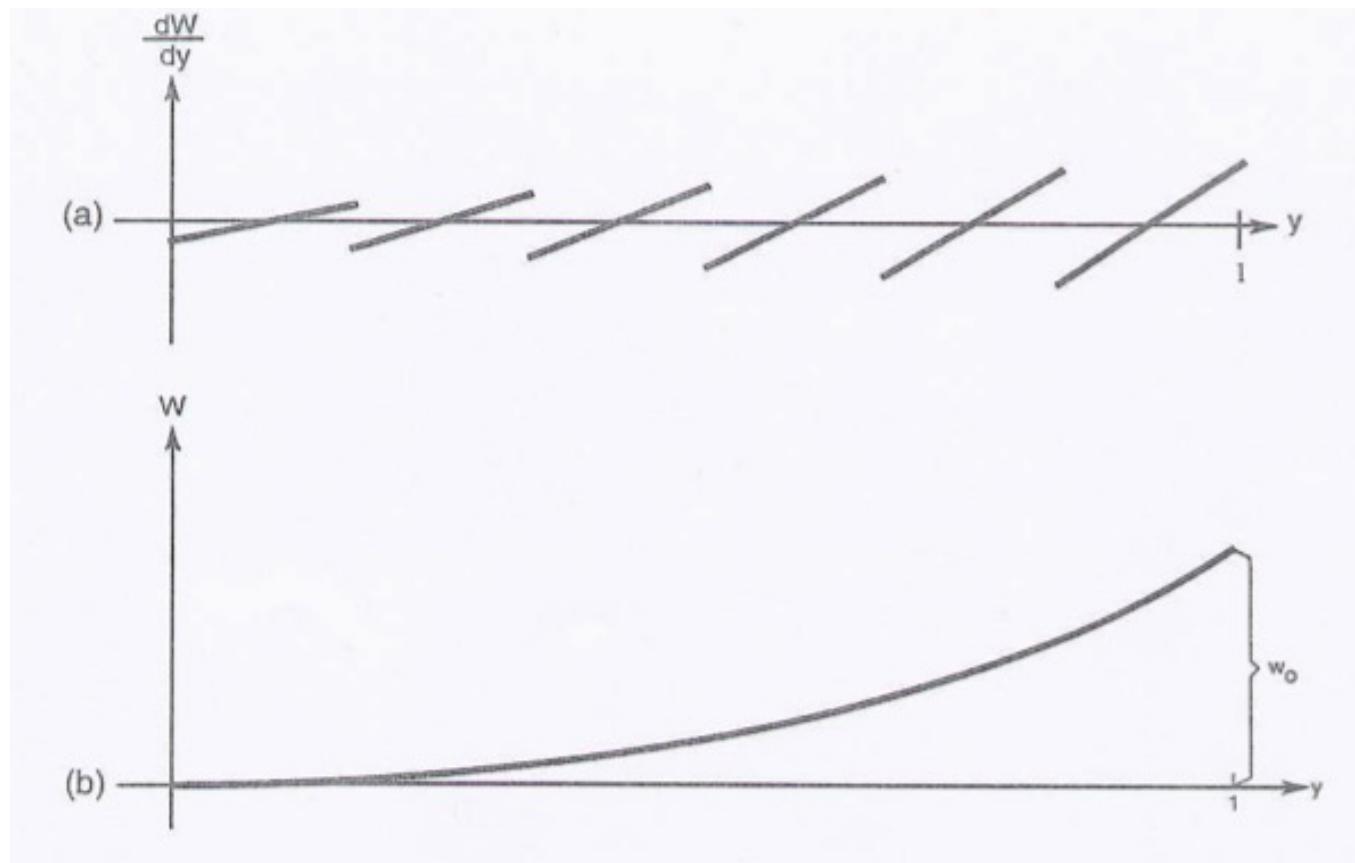
# We need... An Adaptive Optics system



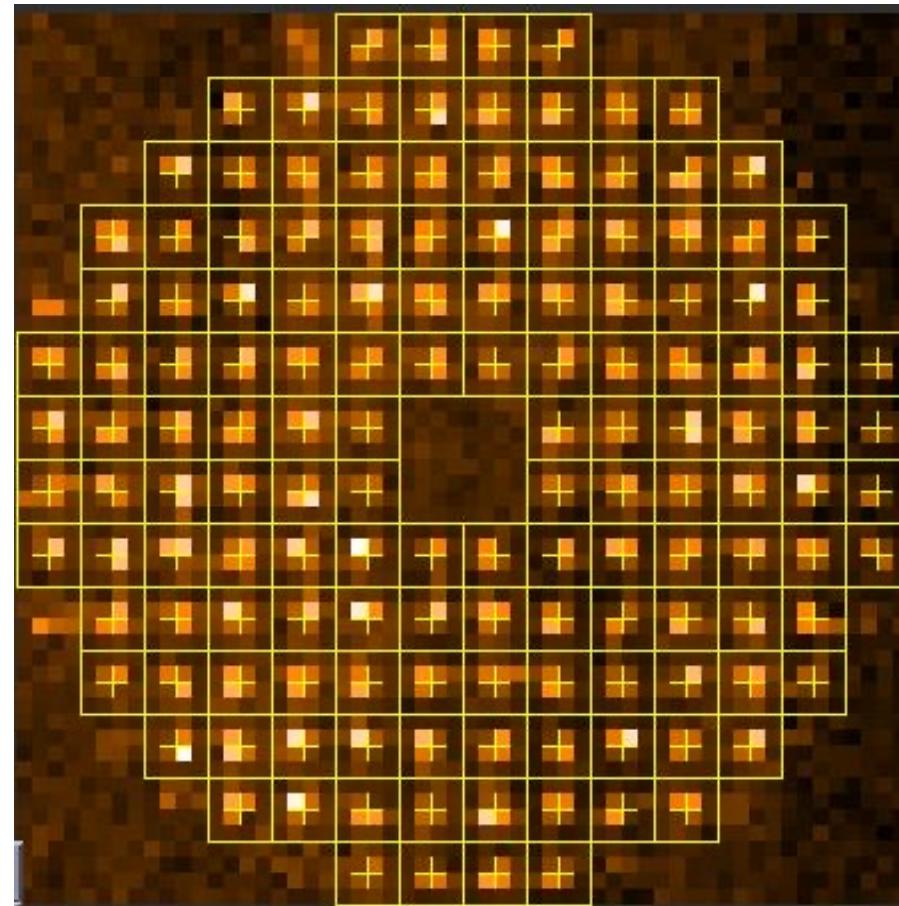
# Shack-Hartmann-WFS: samples **slope** of wave front



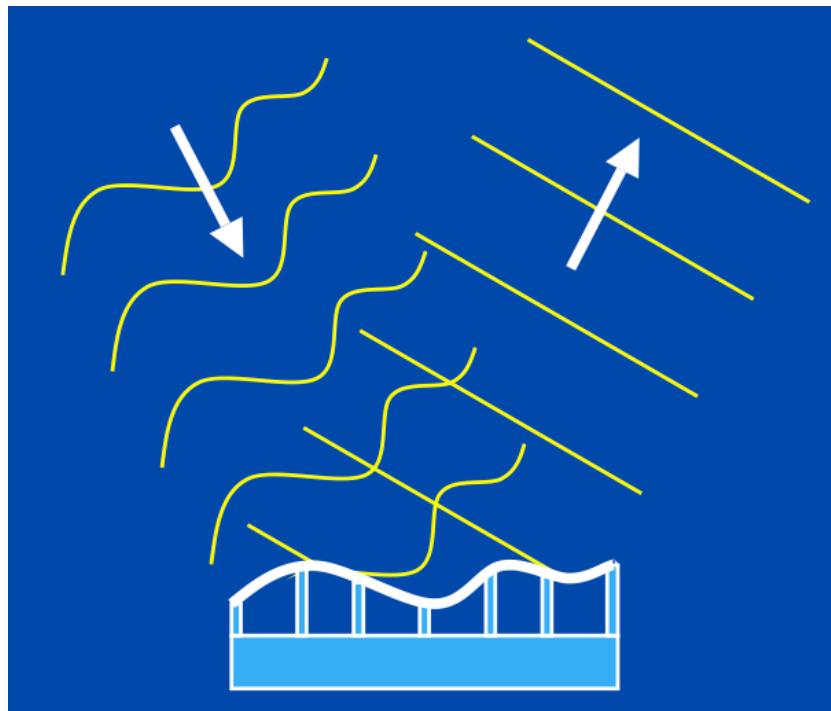
# From slopes to the wavefront



# A real world detector: NACO-WFS



# Deformable Mirrors



# Paranal, Chile



# NACO & SINFONI: For almost 20 years the Adaptive optics instruments at the VLT

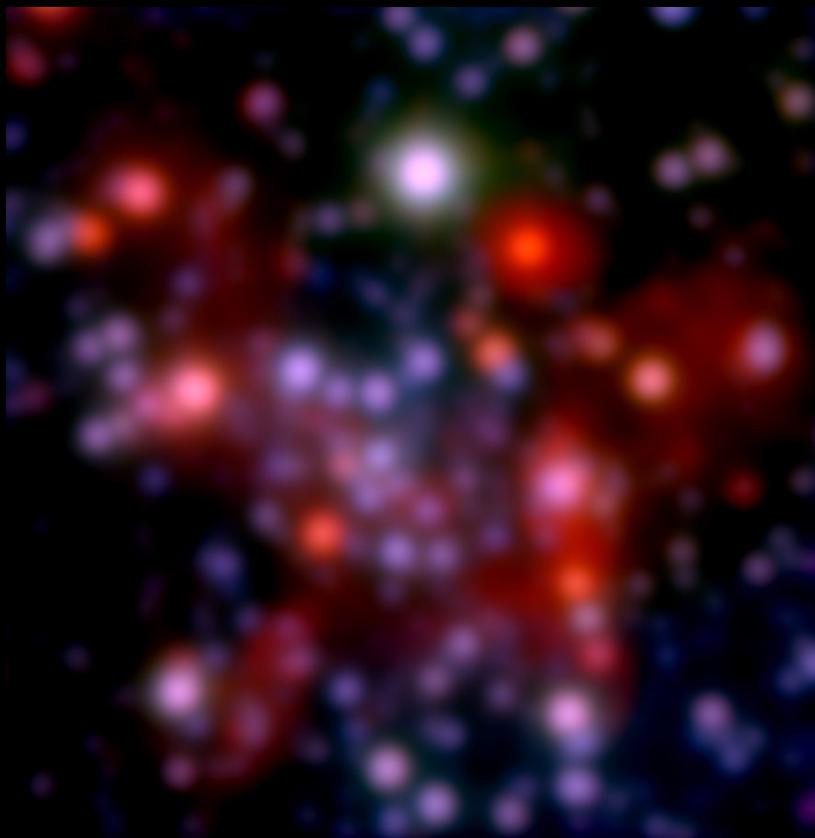


NACO (2002 – 2019):  
Astrometry with  $300 \mu\text{as}$

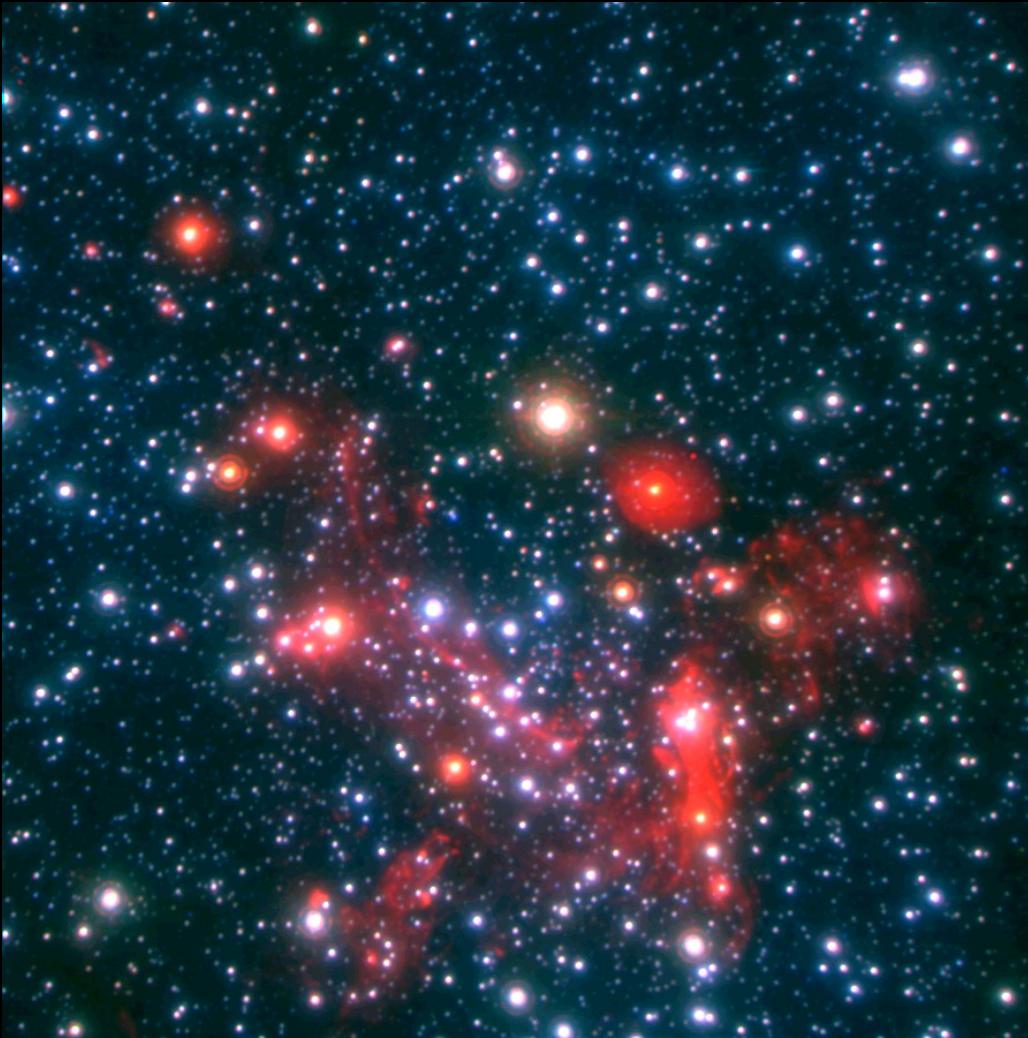


SINFONI (2003 – 2019):  
Spectroscopy with  $7 \text{ km/s}$

Really a big step forward: AO



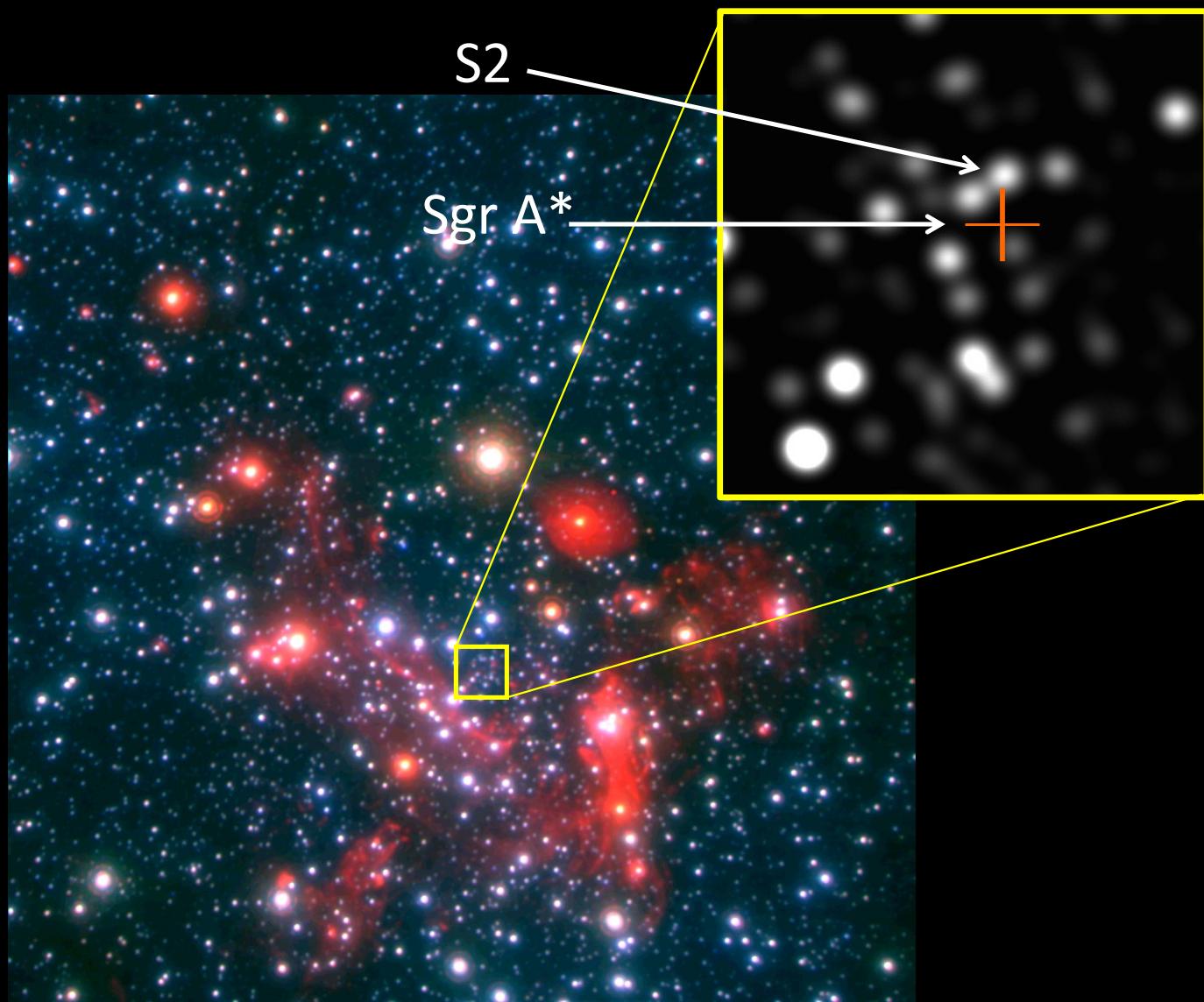
Strehl ratio  
40%



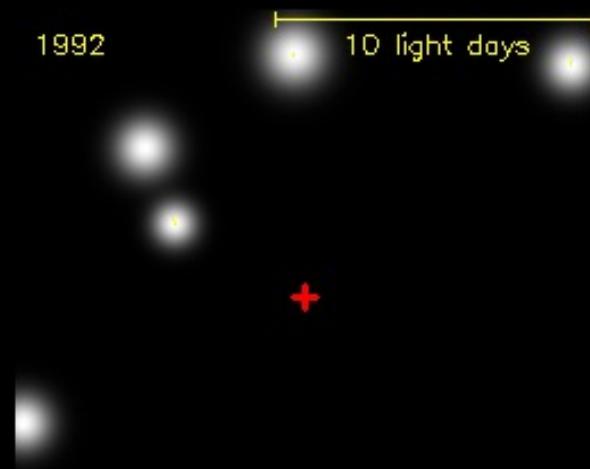
NACO,  
HKL color composite

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# The stars move on Keplerian ellipses



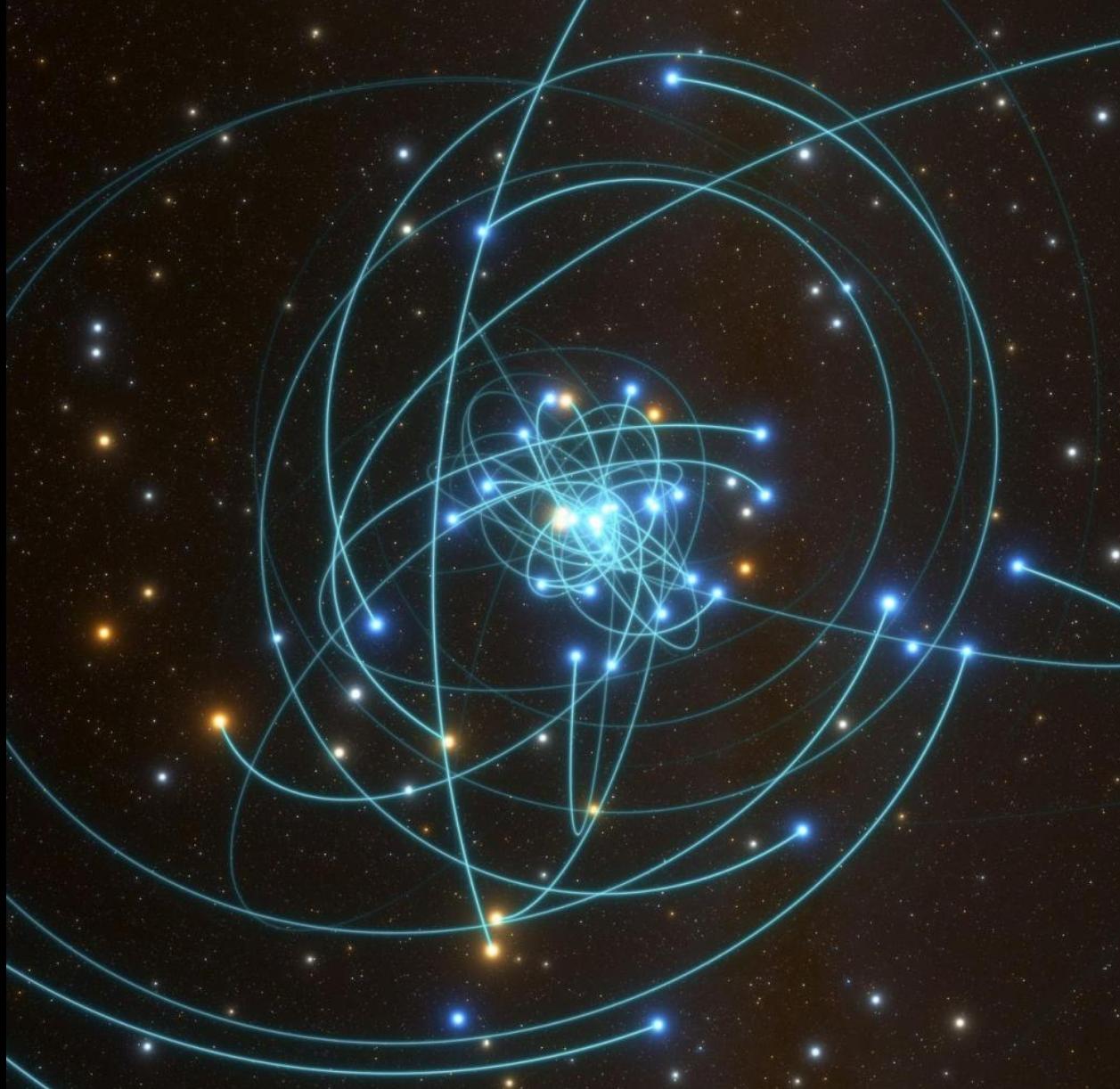
Resulting mass:

$$M = \frac{4\pi^2}{G} \frac{a^3}{P^2}$$

$$M = \frac{4\pi^2}{6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}} \frac{(125'' \times 8.3 \text{ kpc})^3}{(16 \text{ yr})^2} =$$

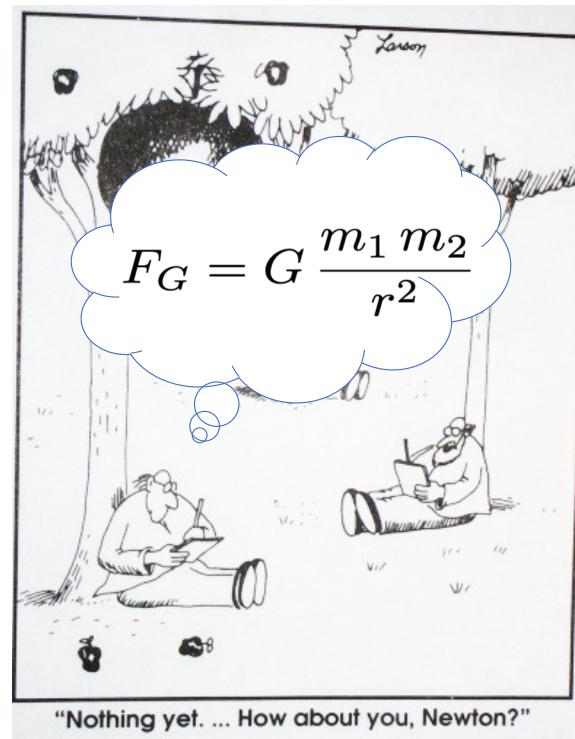
$$4.3 \times 10^6 M_\odot$$

WOW!

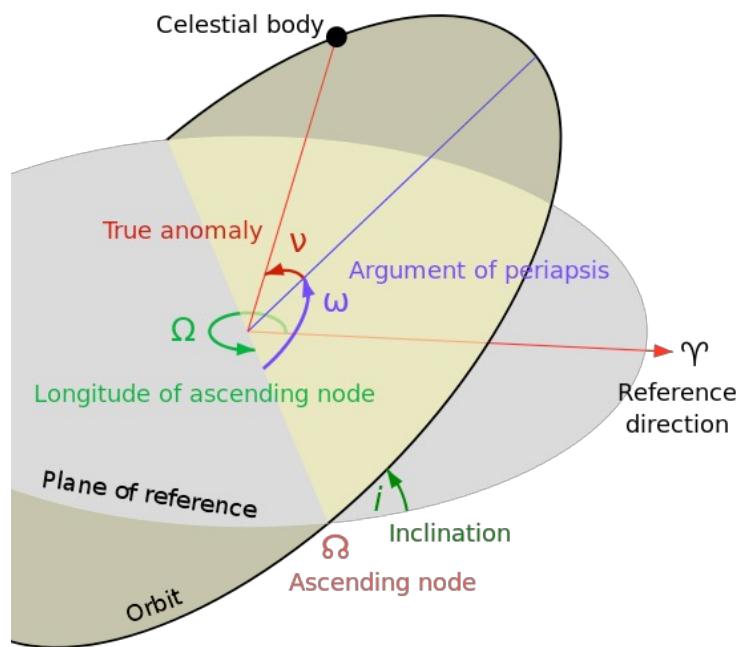


Currently known:  
around 50 orbits

Newton would have understood this  
but he had no chance to see it



# How to measure stellar orbits



- 6 orbital elements:  
( $a, e, i, \omega, \Omega, t$ )
- Need 6 dynamical quantities
- more if information on potential needs to be inferred
- Imaging data:  
 $x, y, v_x, v_y$
- Spectroscopy:  
 $v_z$
- At least one more number needed:  
an acceleration
  - $a_{2D}$  from imaging
  - $a_z = dv_z / dt$  from spectroscopy

# Determine the orbital elements and black hole parameters

$$\{x_j \pm \Delta x_j, y_j \pm \Delta y_j, v_k \pm \Delta v_k\}$$

$$\vec{\Omega} = \{a, e, i, \Omega, \omega, t_0\}$$

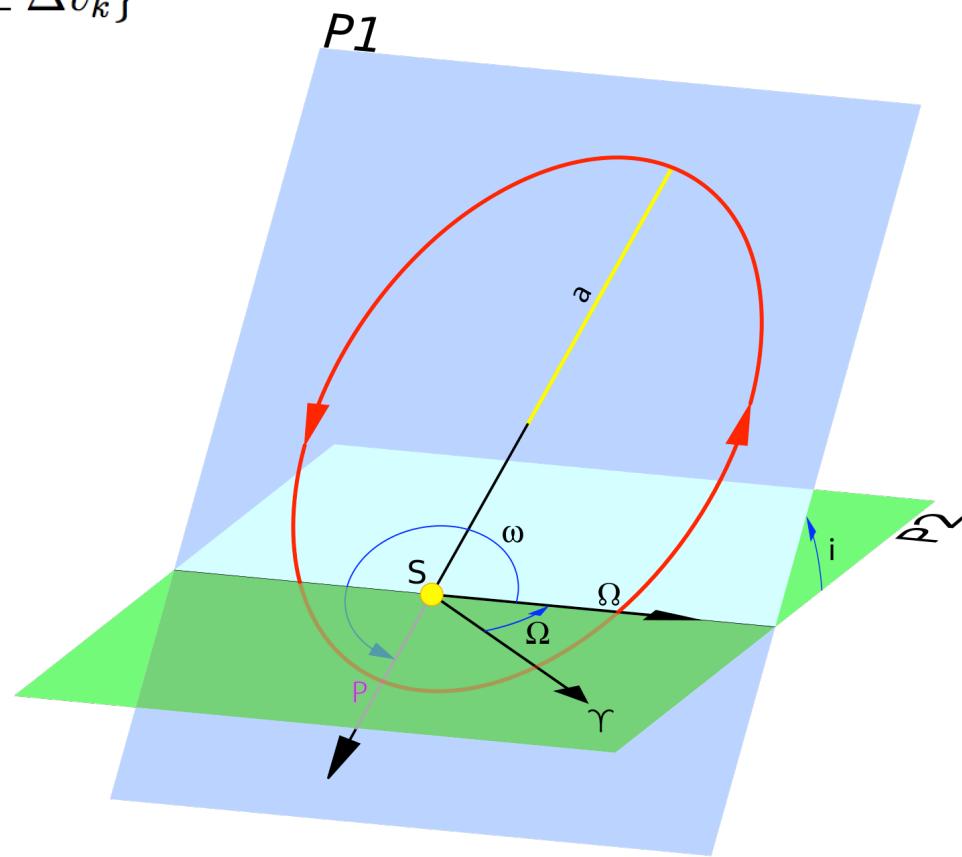
$$x(t) = x(t; M, d, \vec{\Omega})$$

$$y(t) = y(t; M, d, \vec{\Omega})$$

$$v(t) = v(t; M, d, \vec{\Omega})$$

$$\begin{aligned} \chi^2(M, d, \vec{\Omega}) = & \\ \sum_{j=1}^{\max} \frac{(x_j - x(t; M, d, \vec{\Omega}))^2}{(\Delta x_j)^2} + & \\ \sum_{j=1}^{\max} \frac{(y_j - y(t; M, d, \vec{\Omega}))^2}{(\Delta y_j)^2} + & \\ \sum_{k=1}^{\max} \frac{(v_k - v(t; M, d, \vec{\Omega}))^2}{(\Delta v_k)^2} & \end{aligned}$$

$$\min |_{(M, d, \vec{\Omega})} \chi^2(M, d, \vec{\Omega})$$



# Outline

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# Why we are sure



# Sgr A\* is a black hole

Eckart & Genzel  
1996, Ghez+ 1998,  
2003, 2005,  
Schödel+ 2002,  
Eisenhauer+ 2005,  
Gillessen+ 2009,  
2017, Böhle+  
2016, GRAVITY  
Coll 2018, 2022

Reid & Brunthaler  
2004, 2020

	mass	position	faintness	size
Eckart & Genzel 1996, Ghez+ 1998, 2003, 2005, Schödel+ 2002, Eisenhauer+ 2005, Gillessen+ 2009, 2017, Böhle+ 2016, GRAVITY Coll 2018, 2022	IR <b>stellar orbits</b> $4.3 \times 10^6 M_\odot$ $\pm 0.3\%$  VLBA : <b>radio Sgr A*</b> $> 1 \times 10^6 M_\odot$	IR + VLA  <b>mass and radio Sgr A*</b> coincide  Reid+ 2007 Plewa+ 2015	radio + IR  <b>Sgr A*</b> cannot have a surface  Broderick & Narayan 2006 Broderick+ 2009	IR + X-ray timescales $\approx 0.1$ AU  VLBI / EHT 0.4 AU  GRAVITY flares 0.5 AU
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# Why we are sure

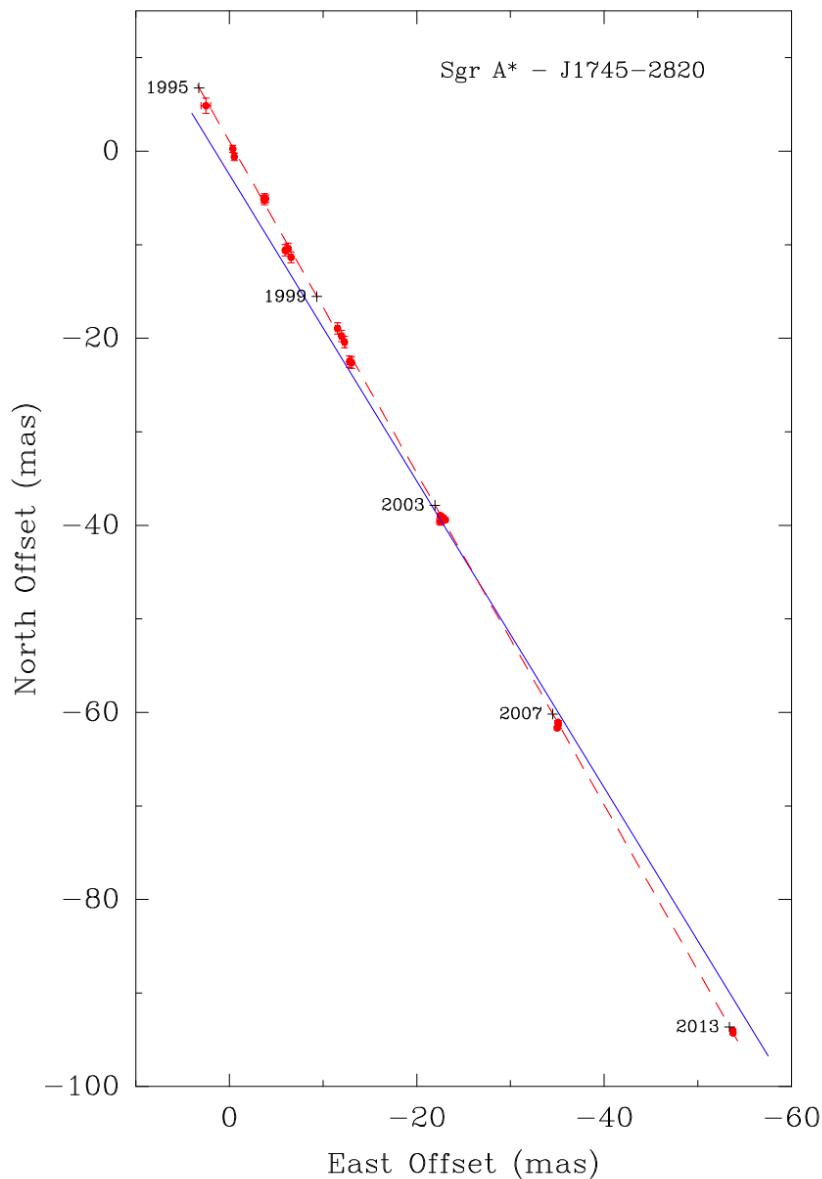


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Reid & Brunthaler 2004, 2020				Baganoff+ 2001, Genzel+ 2003, Dodds-Eden+ 2009  Doeleman+ 2008, Akiyama+ 2022,  GRAVITY Coll. 2018b



## Sgr A\* must be very heavy

- perfectly linear motion
  - reflex motion of Sun ( $\sim 200$  km/s)
- intrinsic motion
  - gal. I :  $-7.2 \pm 8.5$  km/s
  - gal. b:  $-0.4 \pm 0.9$  km/s
- Sgr A\* is much heavier than surrounding stars
  - mass  $> 10^6 M_{\odot}$

Reid 2007, 2020

# Why we are sure



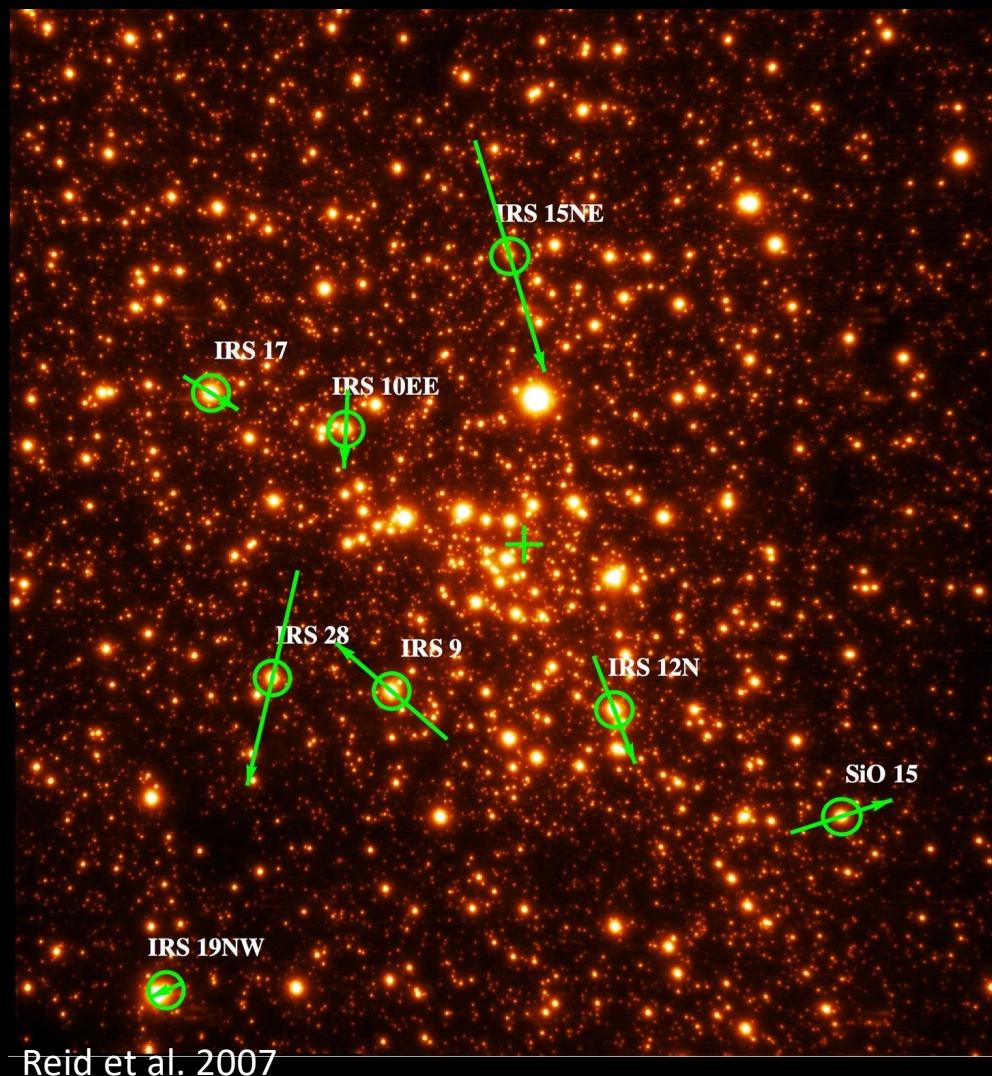
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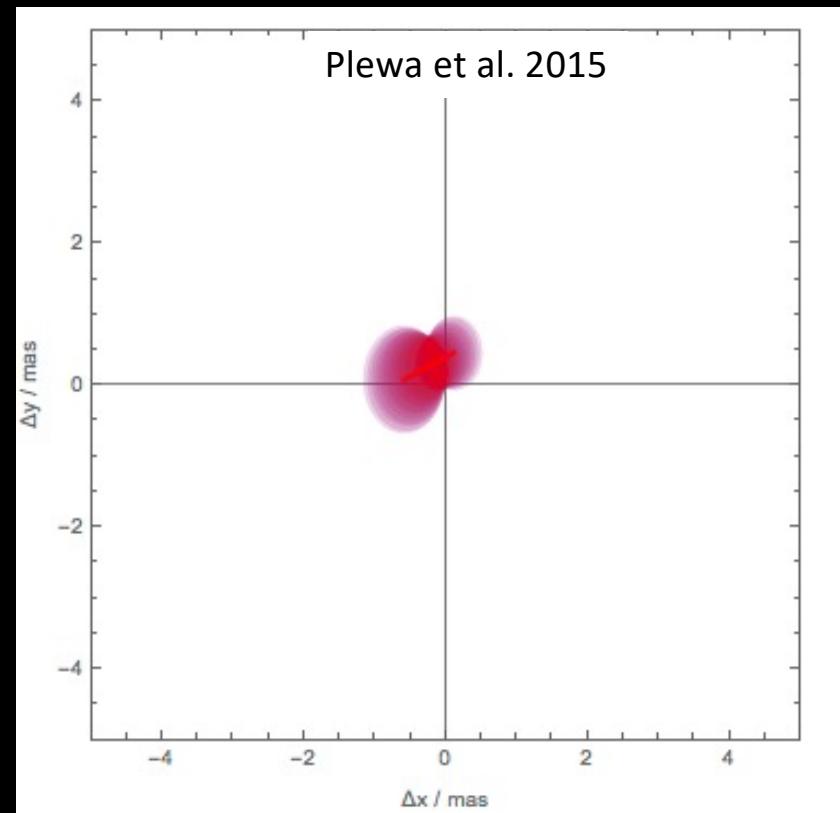
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# The positions of mass & radio-Sgr A\* agree to within 1 mas



- Sgr A\*
  - radio source
- SiO maser stars
  - IR sources
  - radio sources



# Why we are sure



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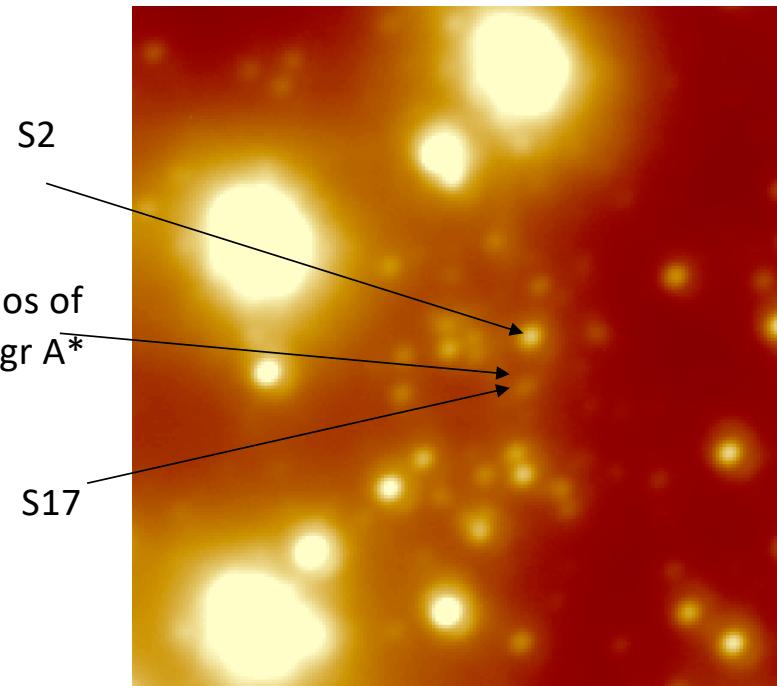
Sgr A\* should be bright - but is not

Limit: Eddington luminosity  
*radiation pressure = gravitation*

$$L_{Edd} = \frac{4\pi G m_p c}{\sigma_T} M$$

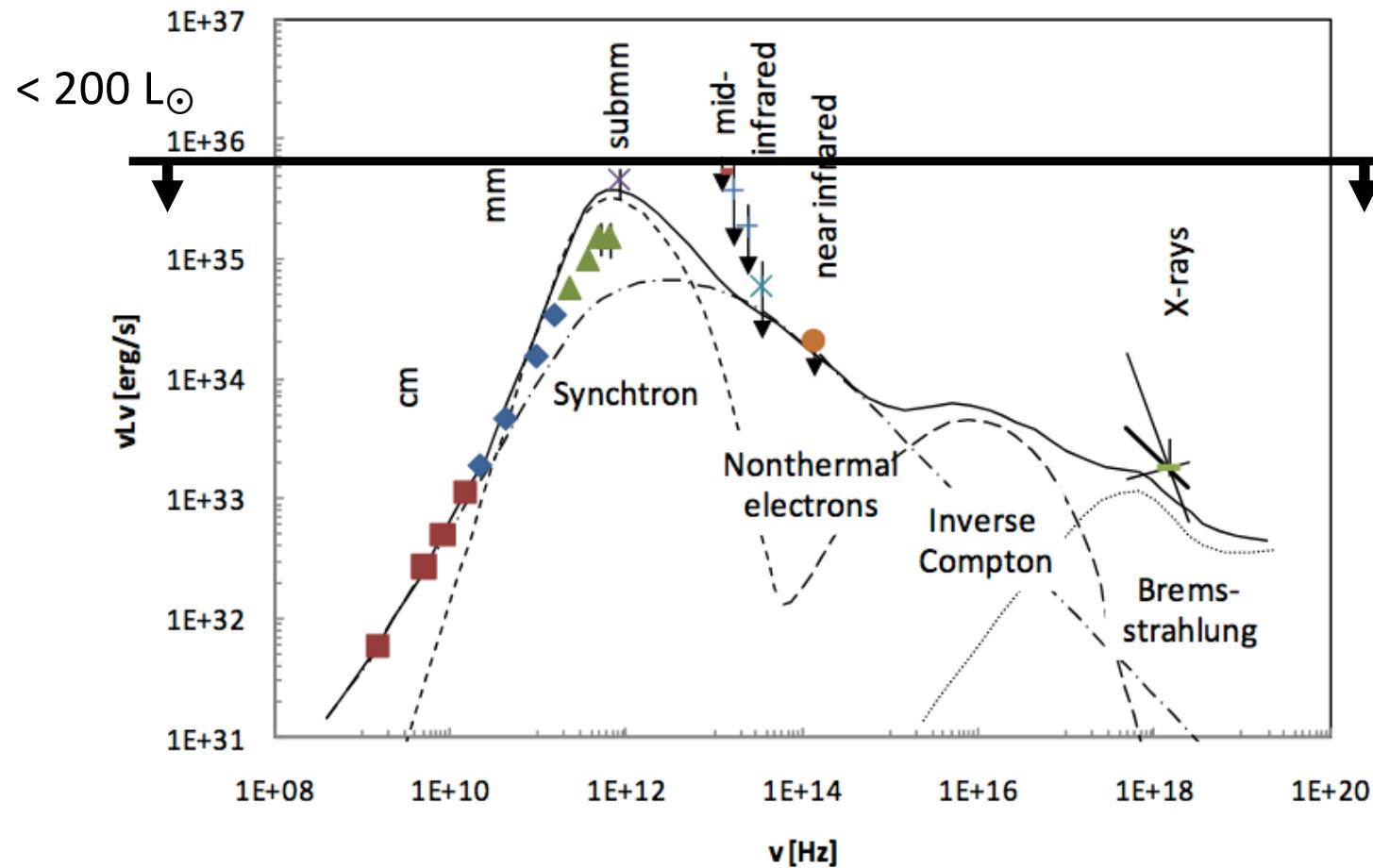
$$L = \eta \times 5 \times 10^{44} \text{ erg/s}$$

$$= \eta \times 10^{11} L_\odot$$



Sgr A\* is dim at all wavelengths:  
 $\eta \sim 10^{-8}$

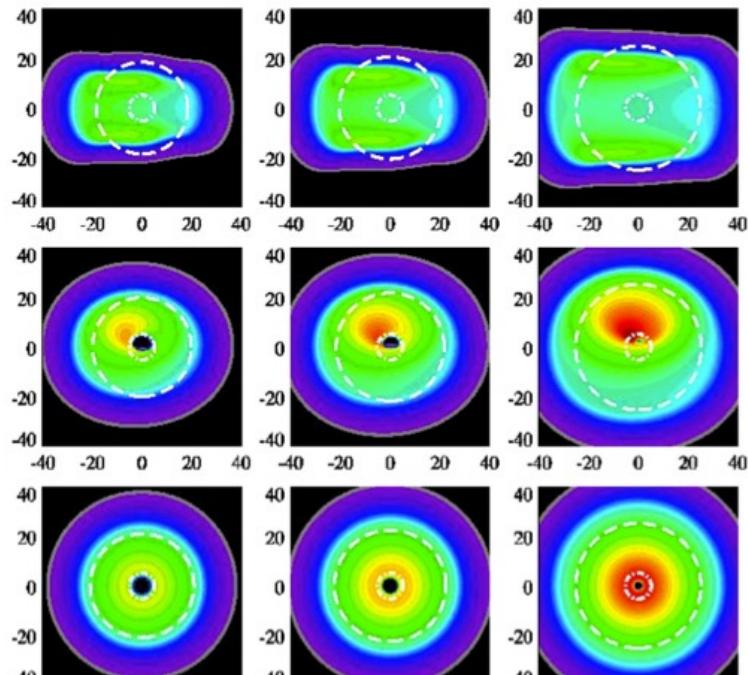
Genzel et al. 2010



# Radiatively Inefficient Accretion Flow

low  $L/L_{\text{Edd}}$  is a combination of:

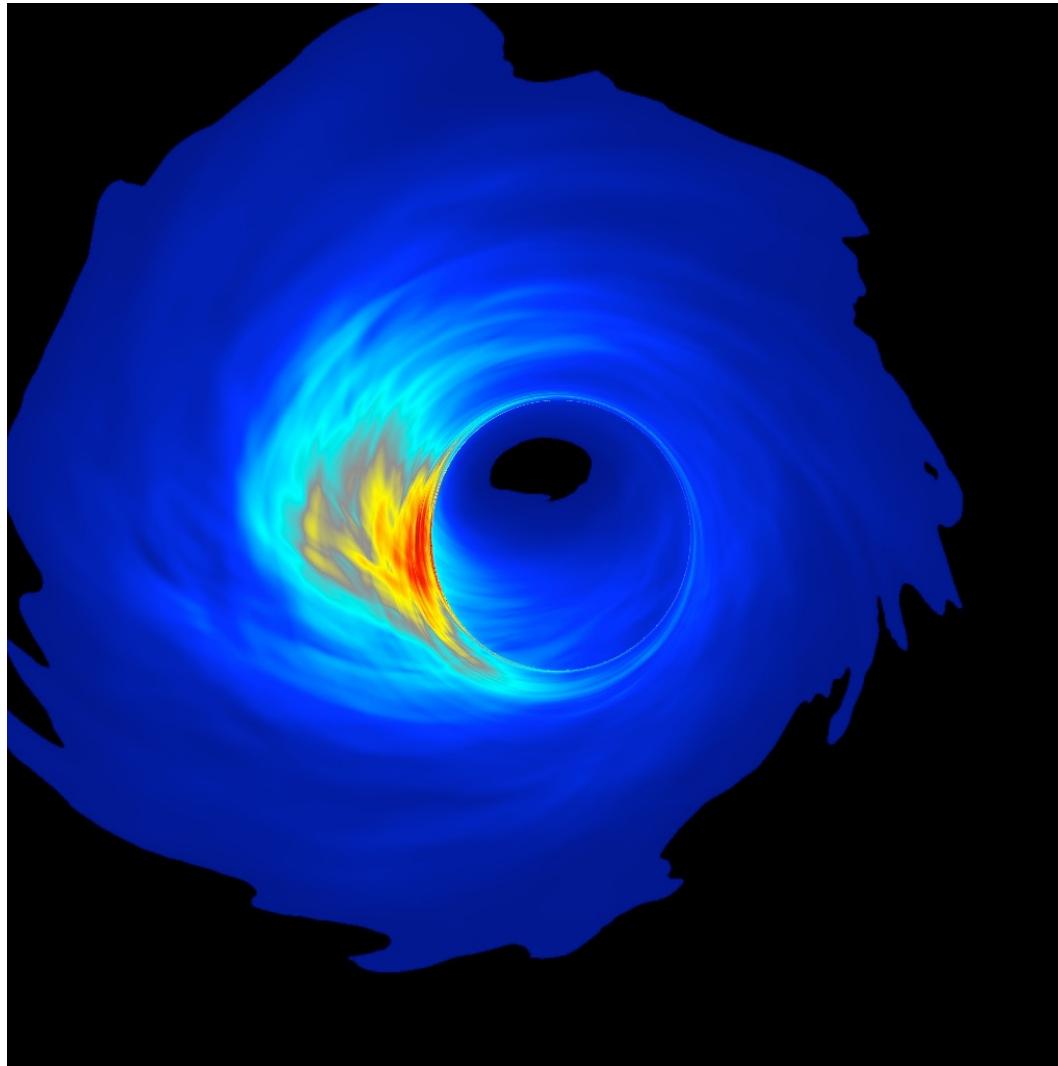
- low accretion rate at Bondi radius
- low efficiency angular momentum transport
- low efficiency energy transfer protons to electrons
- most of the gas arriving at a few  $R_s$  ejected back out



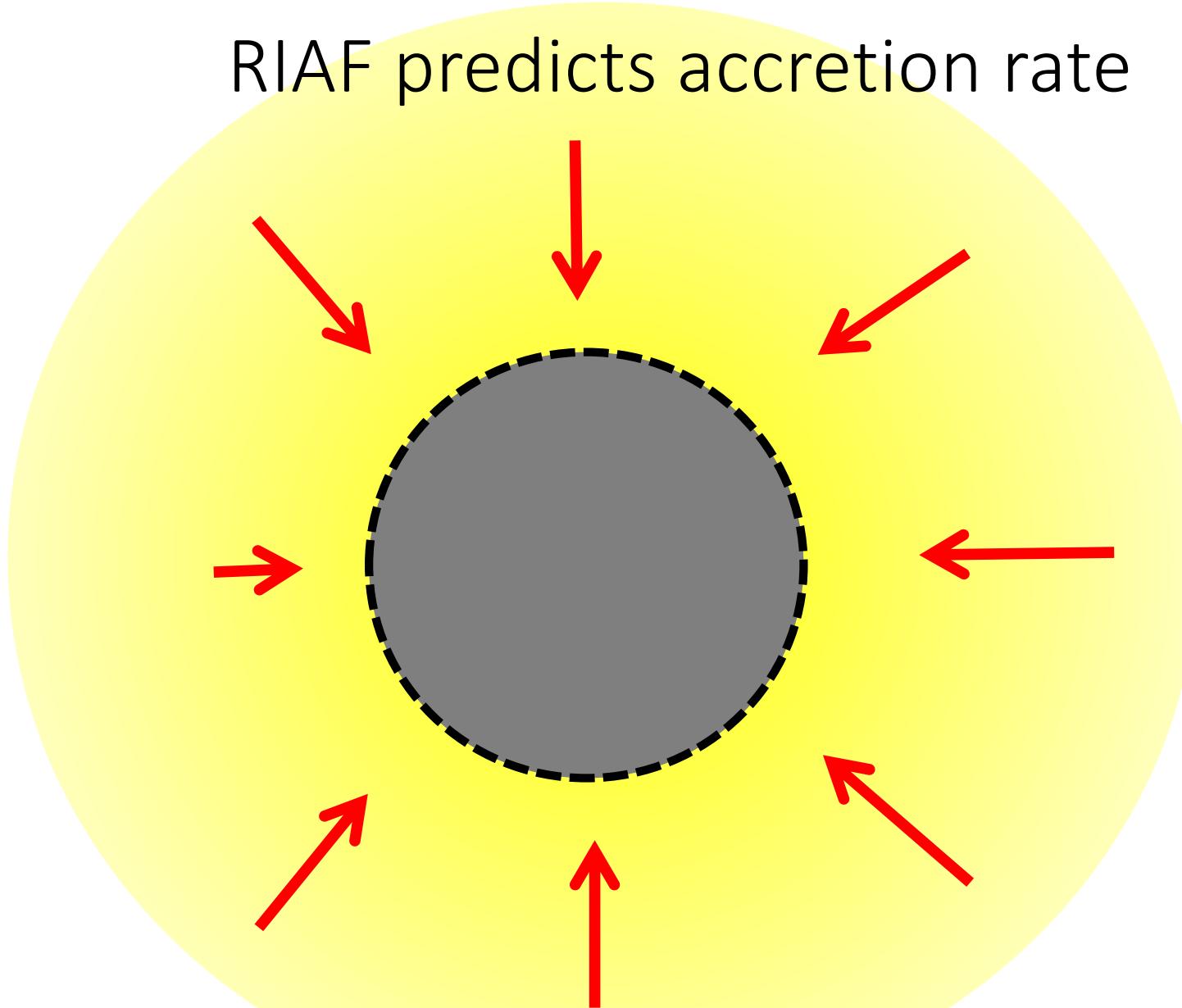
Yuan et al. 2009

Begelman, Bower, Blandford, Cuadra, De Villers, Falcke, Hawley, Krolik, Liu, Melia, Markoff, Marrone, Narayan, Quataert, Rees, Revnitshev, Stone, Wang, Yuan 1995-2013

# MHD simulation with GR ray-tracing



# RIAF predicts accretion rate



accretion flow

radio emission:  
RIAF

accretion rate:  
 $\sim 5 \times 10^{-8} M_{\odot}/\text{yr}$

99% of that  
reaches  
“surface”

Assume, material is crashing onto surface

$$P = \sigma A T^4$$

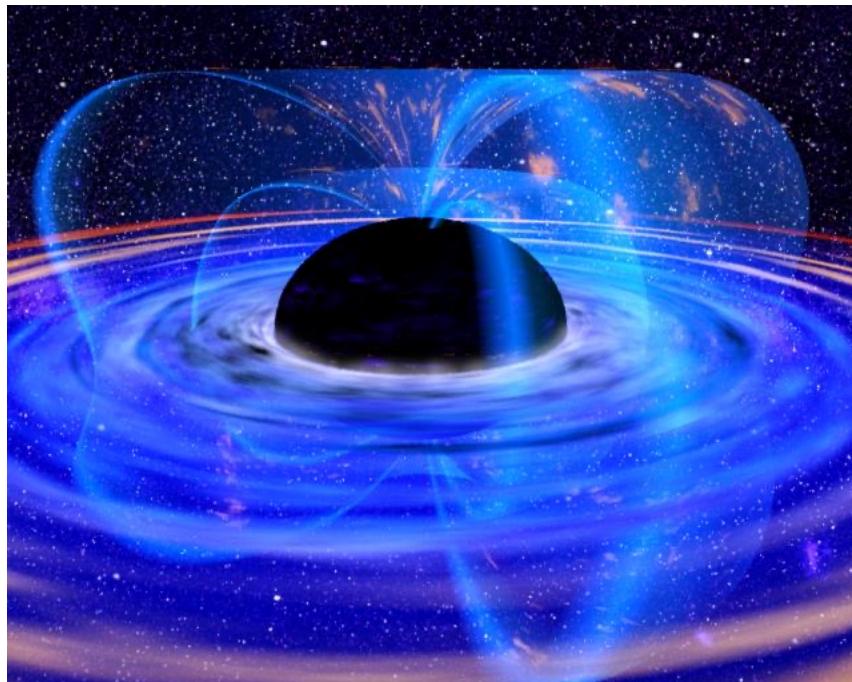
The diagram illustrates the components of the Stefan-Boltzmann law. At the top is the equation  $P = \sigma A T^4$ . Below it, arrows point to each term:  $P = \dot{M}c^2$  points to "Accretion Rate"; "constant" points to "measured size < 4 Rs"; "area" points to a green oval labeled "temperature"; and  $T^4$  points to a red oval labeled "20000 K".

$P = \dot{M}c^2$  constant area temperature

Accretion Rate measured size < 4 Rs 20000 K

# Sgr A\* has an event horizon

- Hot surface: 20000 K
  - infrared light
- Size: 1/10 Earth orbit
  - like a normal star
  - extremely luminous
- But normal stars can be seen
- Sgr A\* cannot have a surface



# Why we are sure



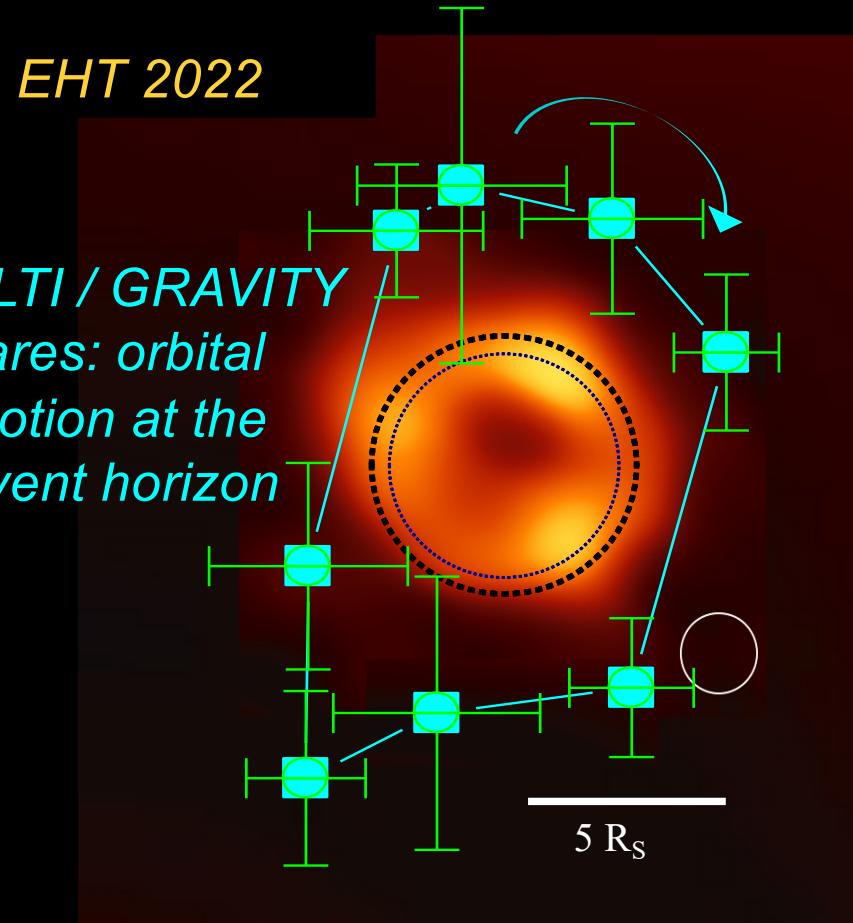
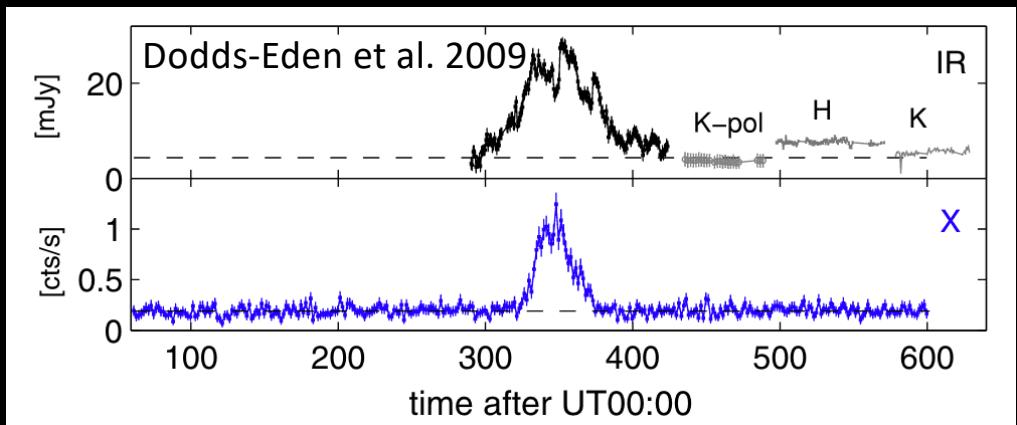
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# Sgr A\* is very compact



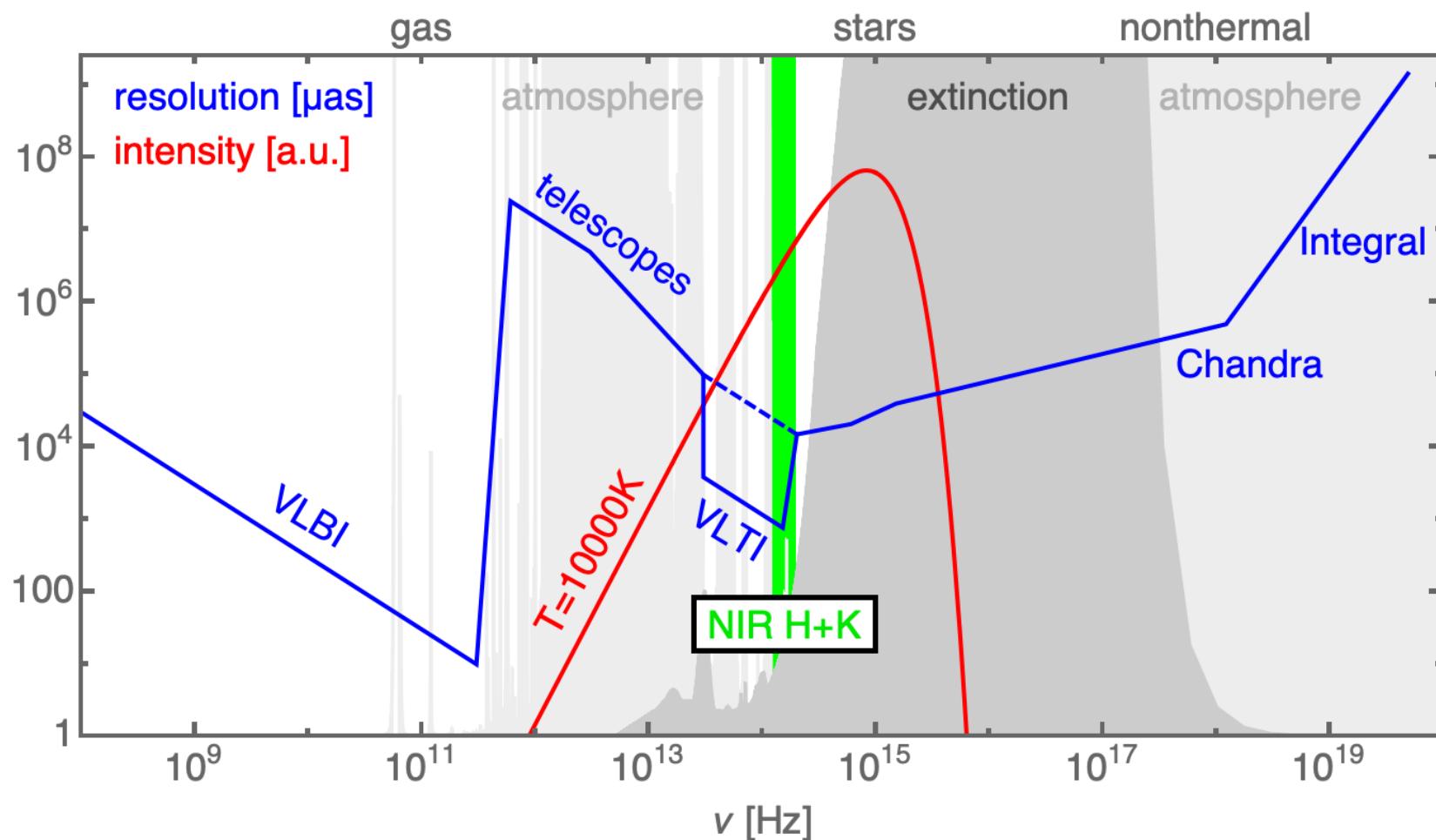
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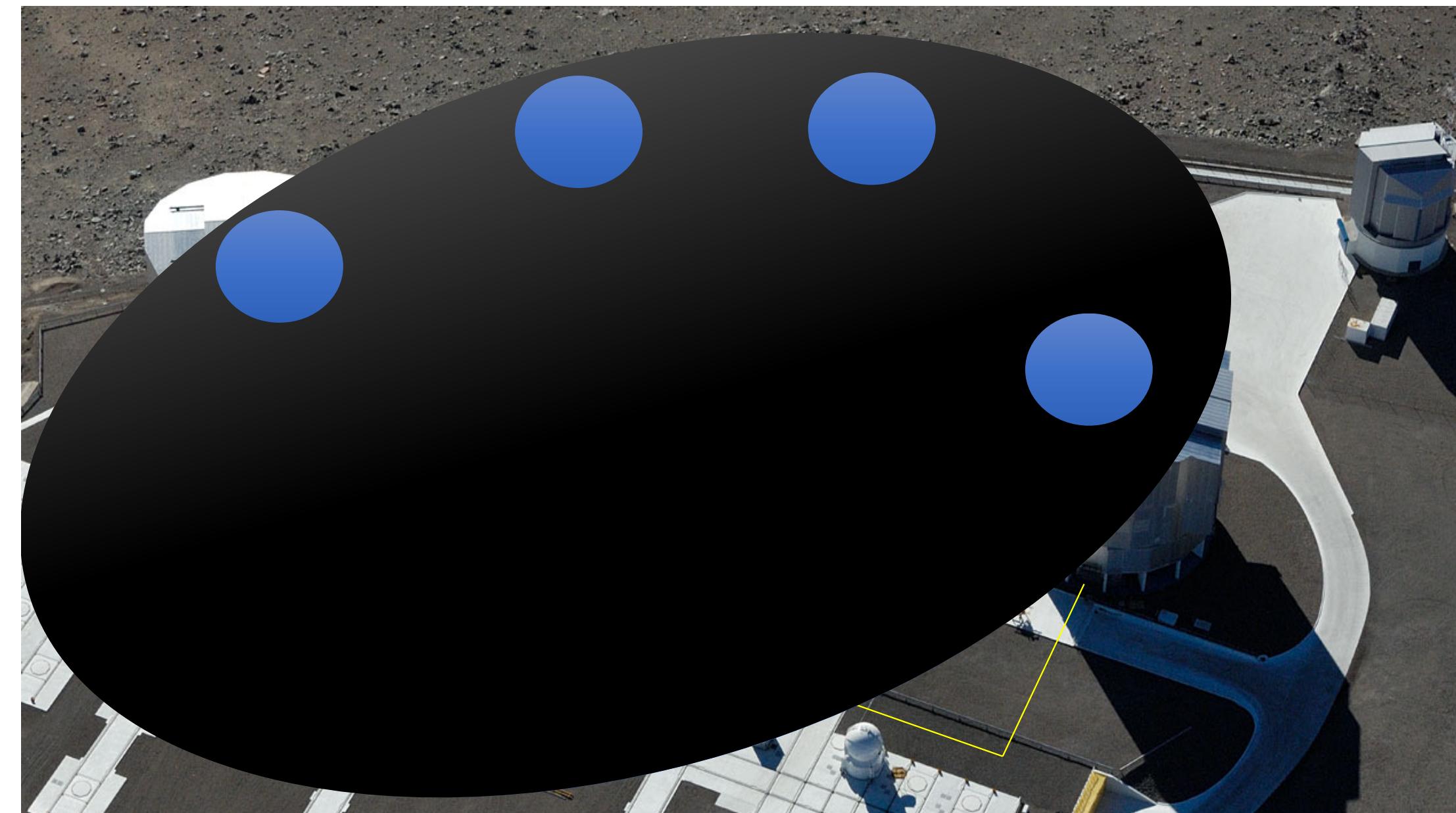
# How to zoom in further ?



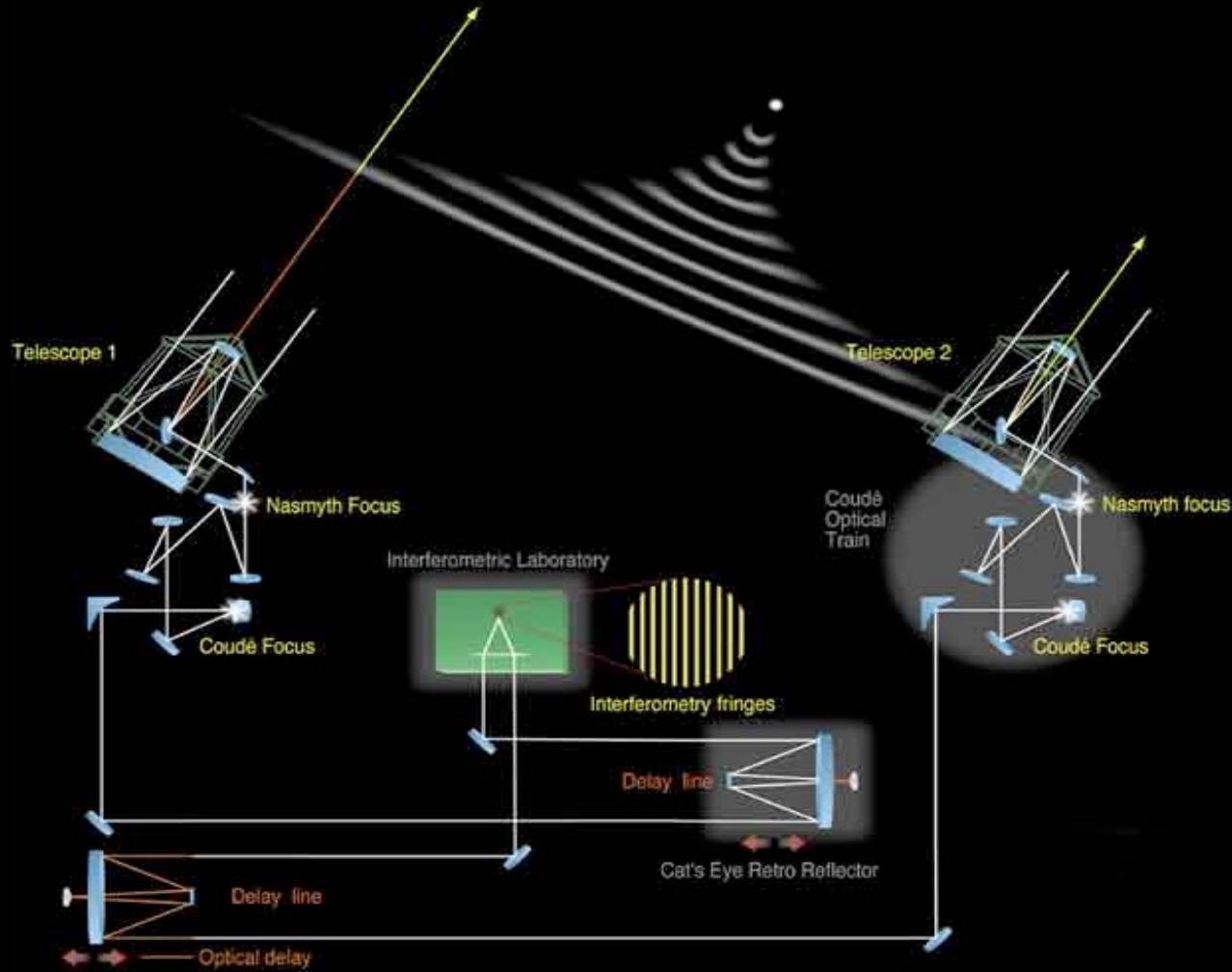
# The sweet spot in wavelength: The near-infrared







# But: Optical interferometry is hard



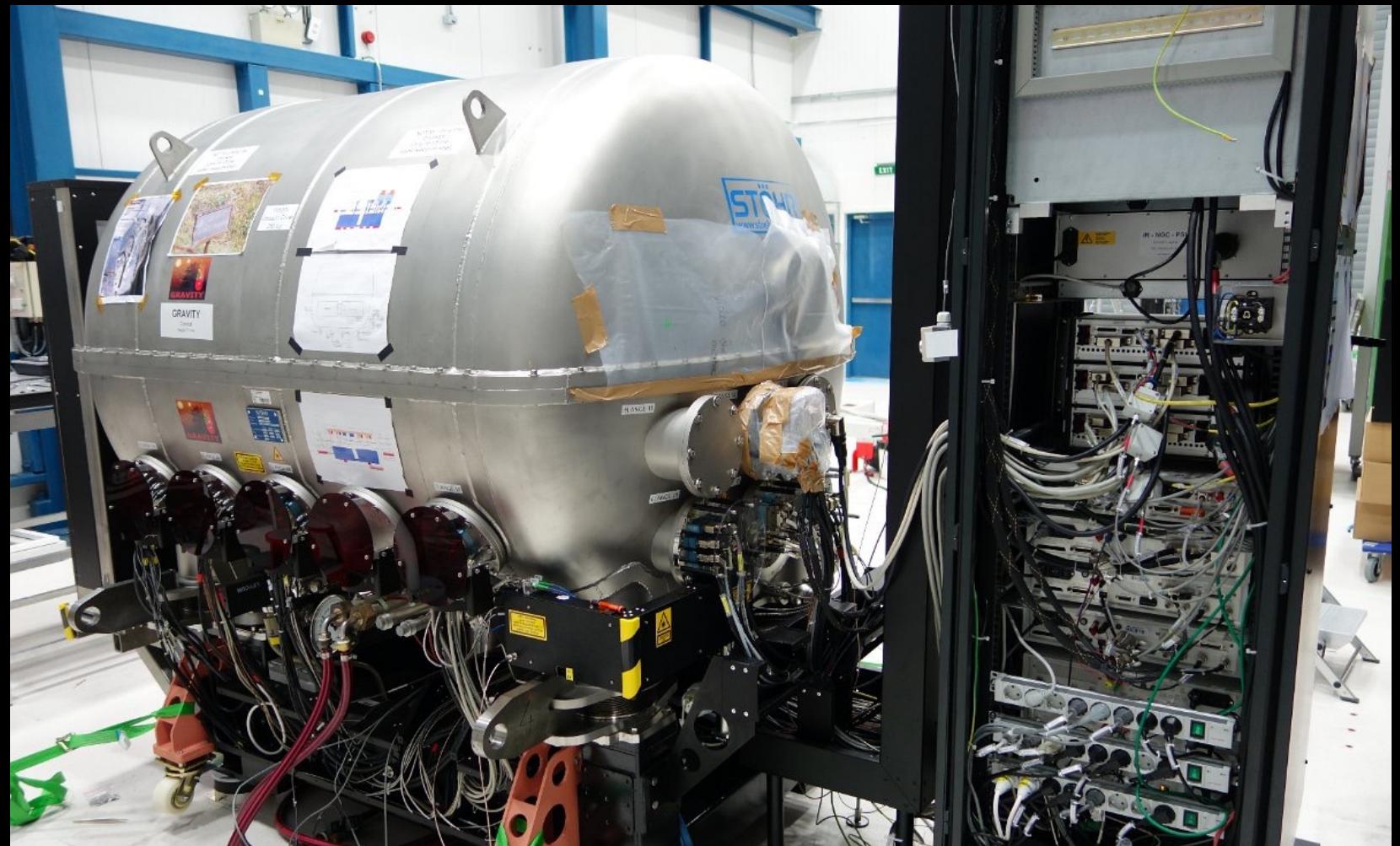
Online  
cophasing to  
nano-meters



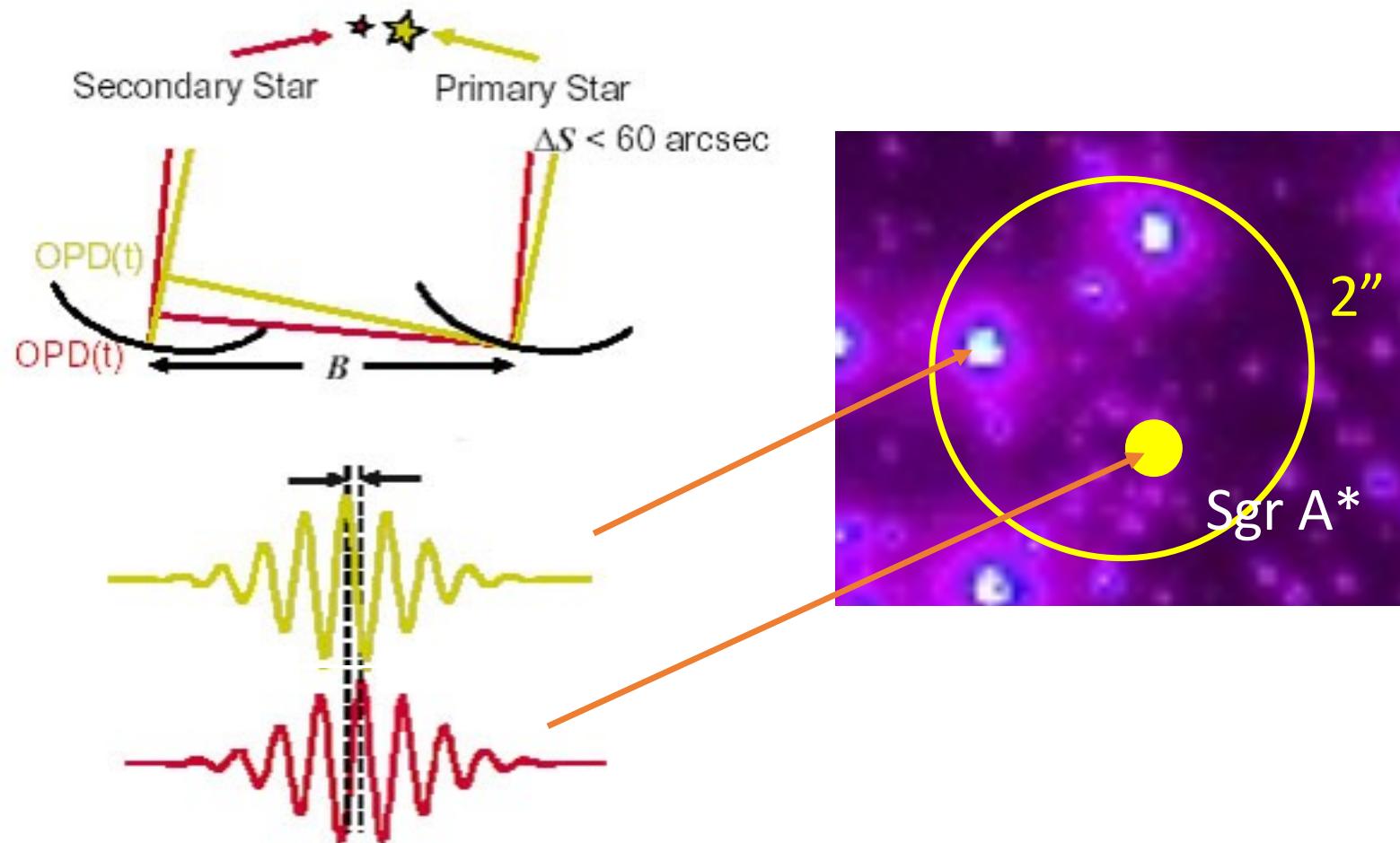
Partenariat Haute résolution Angulaire Sol-Espace



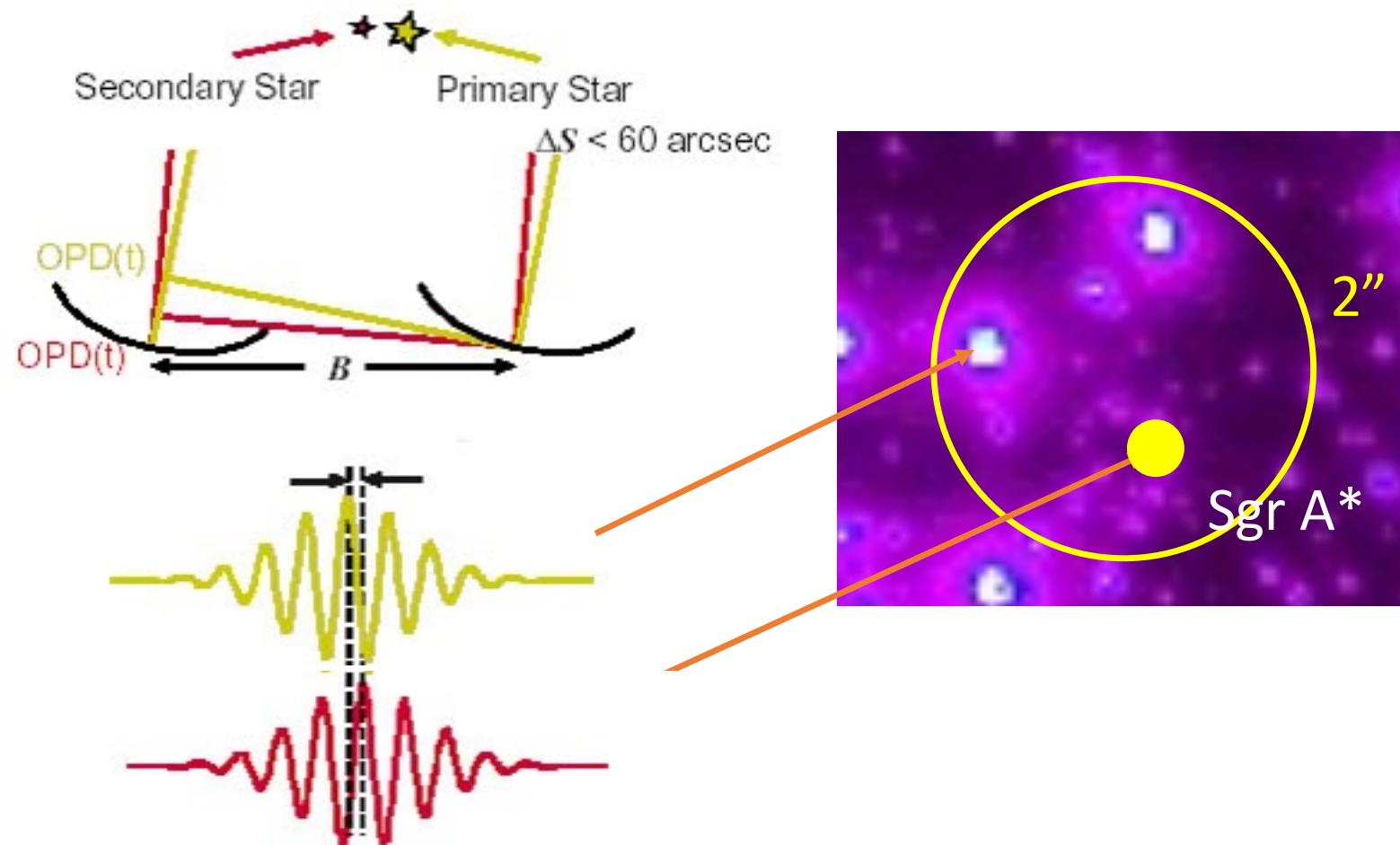
# Dual feed, 4-telescope, adaptive optics-assisted, fringe-tracking beam combiner instrument



# Phase referenced imaging



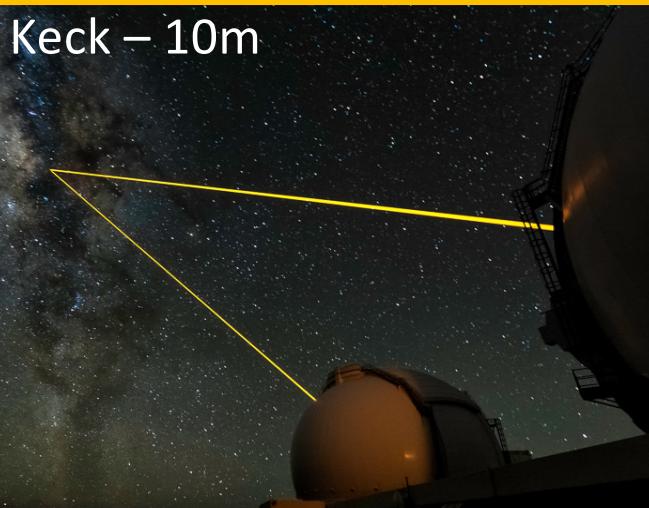
# Dual beam astrometry



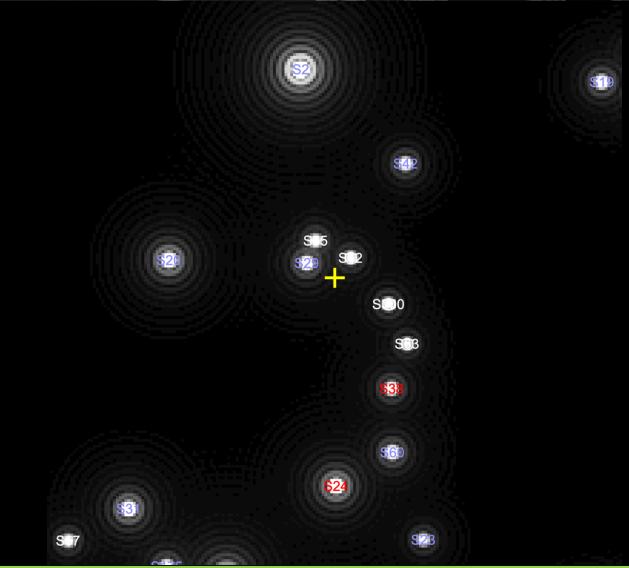
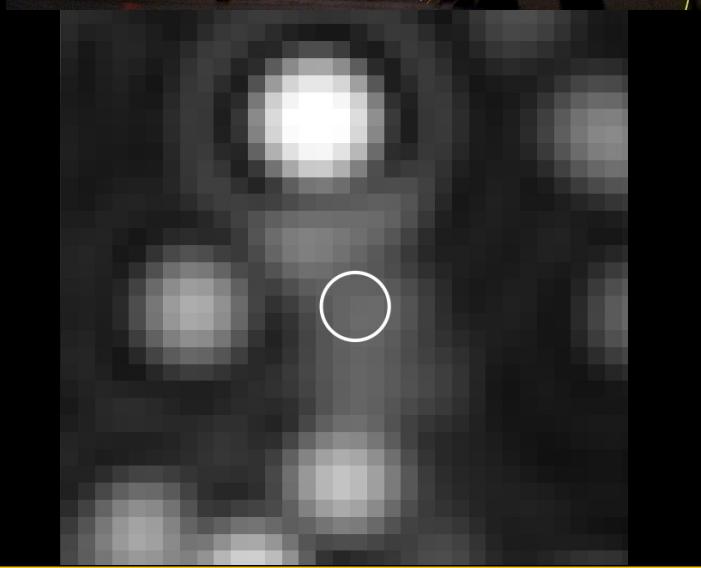
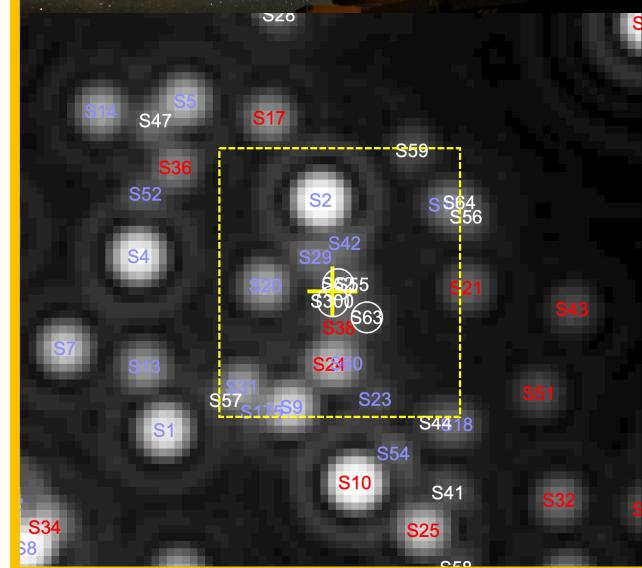


# Laser guide star adaptive optic is 15x less sharp

Keck – 10m

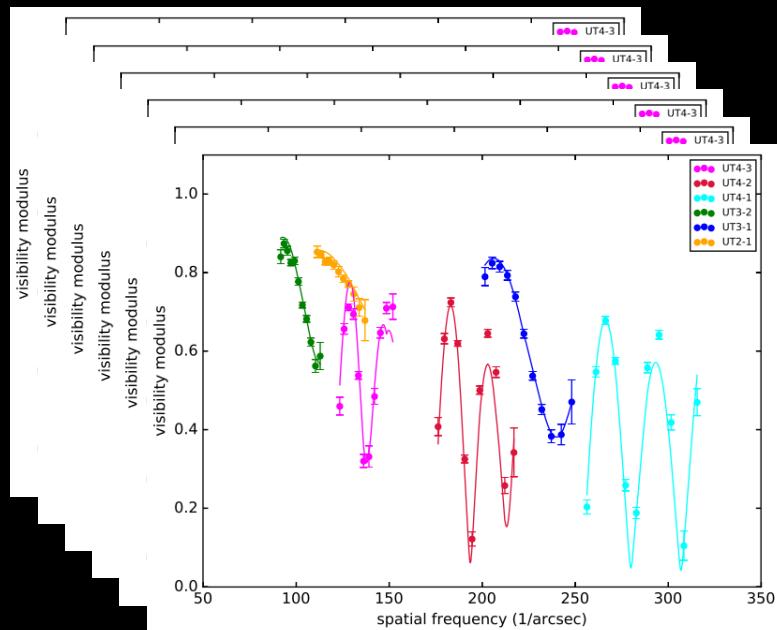


VLT – 8m



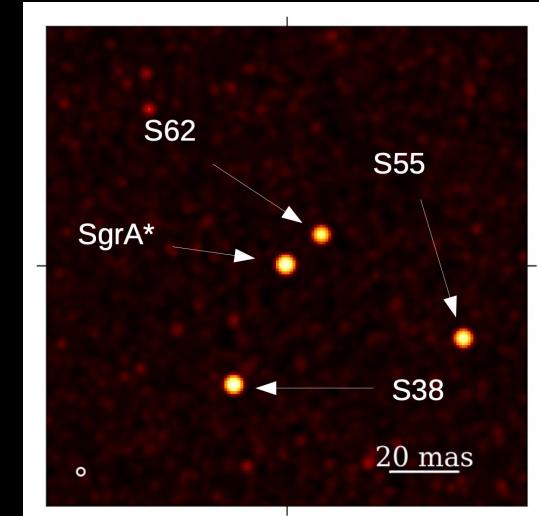
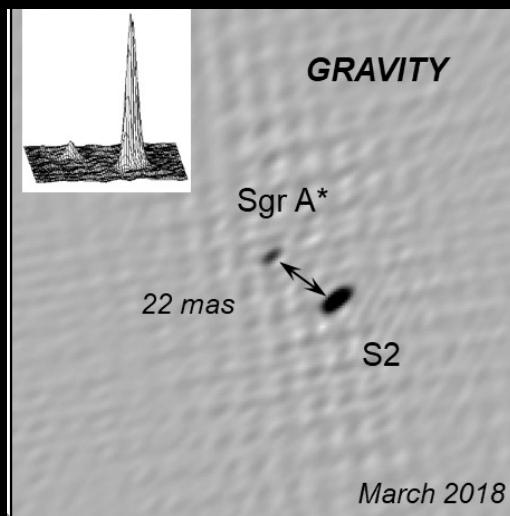
# GRAVITY interferometry of Sgr A\* and S2

## Fringe contrast and phase

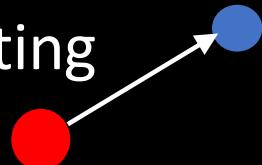


## Images

GRAVITY coll. 2017, 2018a



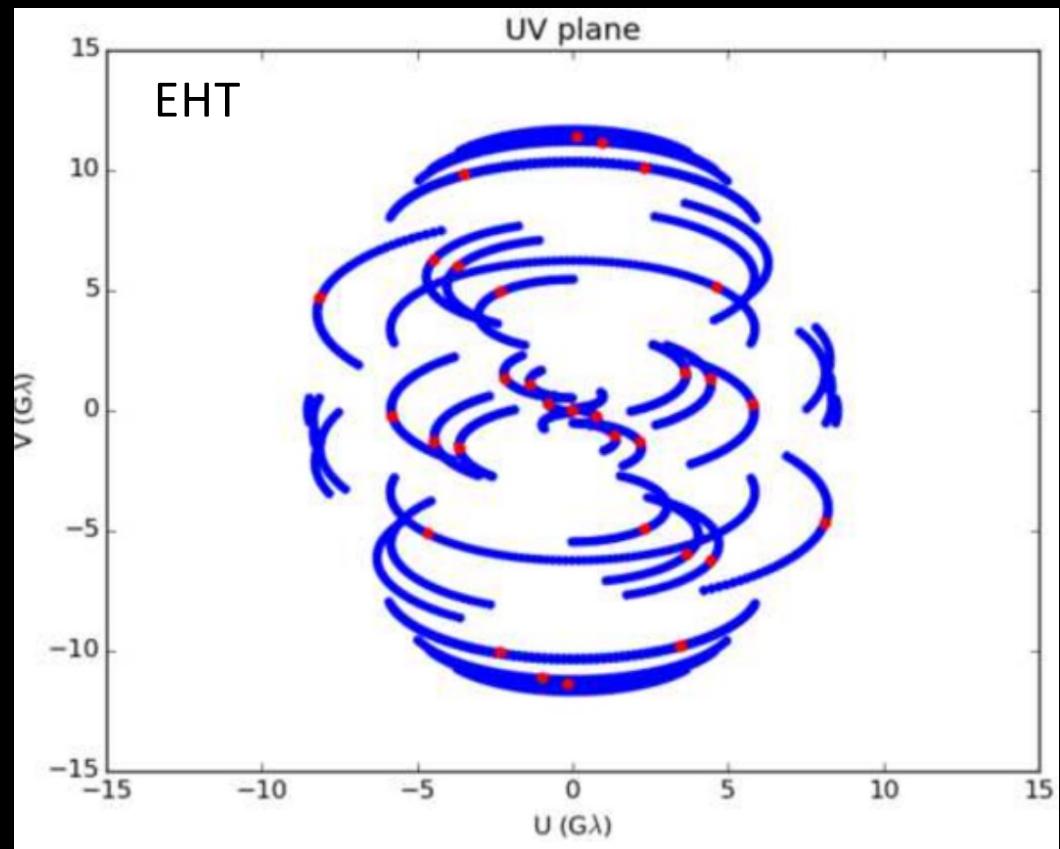
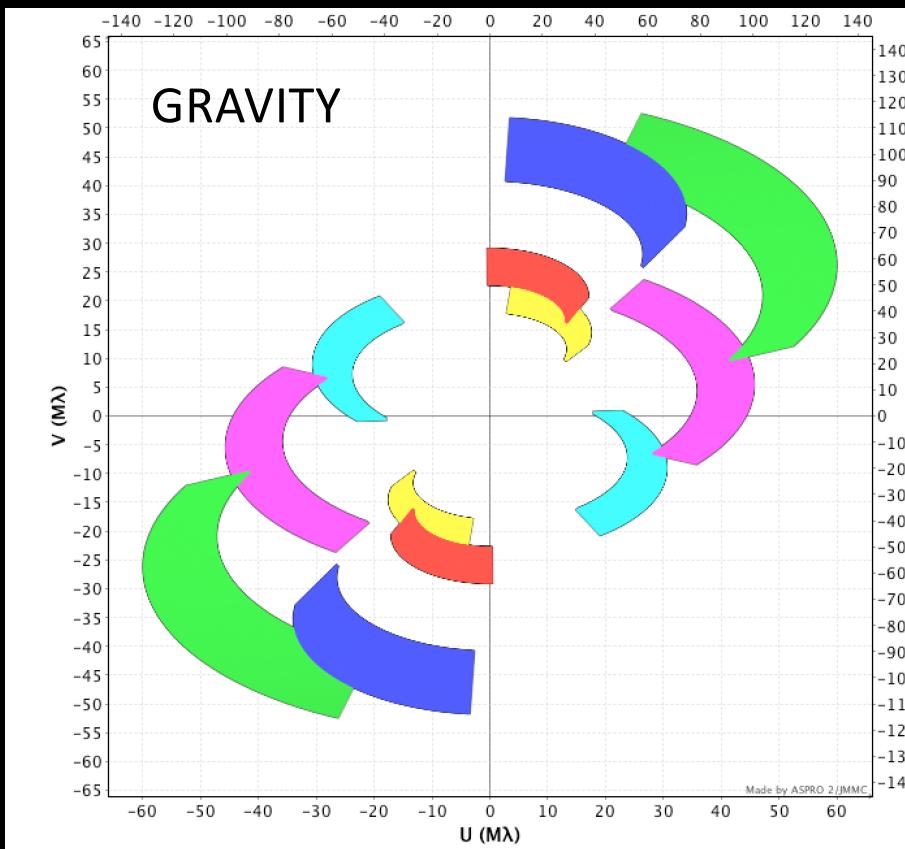
Model fitting

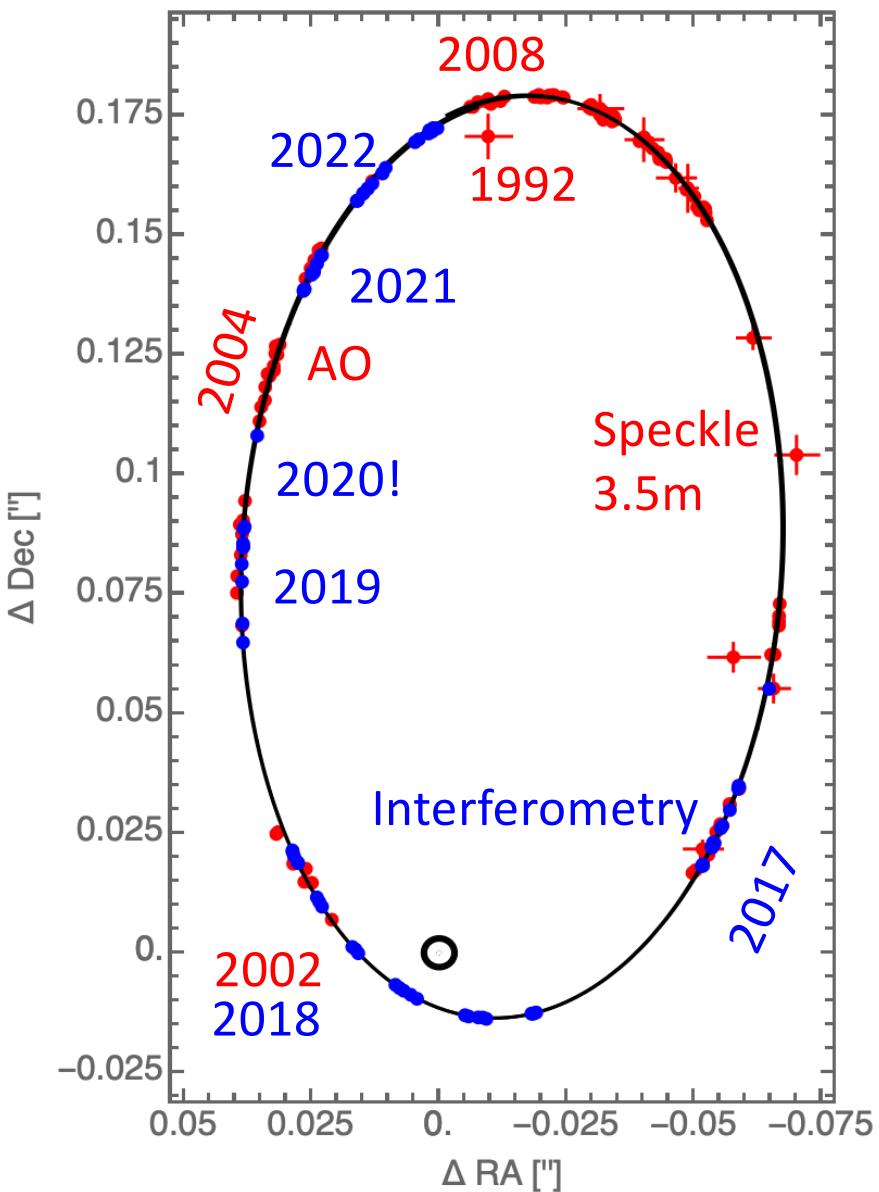


Source & instrument model:

- Separation vector
- Brightness-ratio
- source colors
- Bandwidth smearing
- Injection per telescope

UV coverage good even with 6 baselines  
- due to the 20% width of the K-band

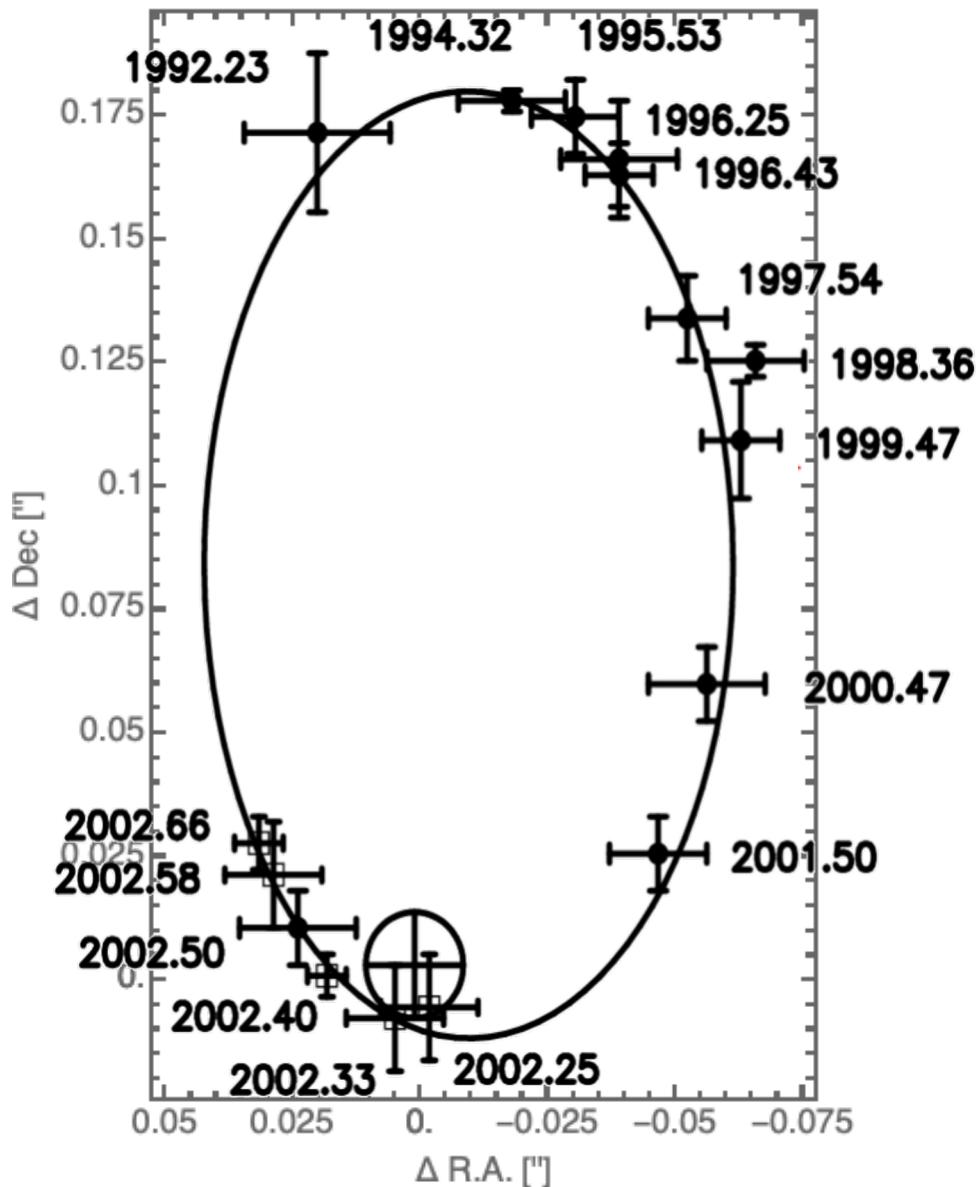




## S2: the showcase star Adaptive Optics & Interferometry data

(Gillessen et al. 2017, Boehle et al. 2016,  
GRAVITY collaboration 2018a, 2019a,  
2020, 2021, 2022)

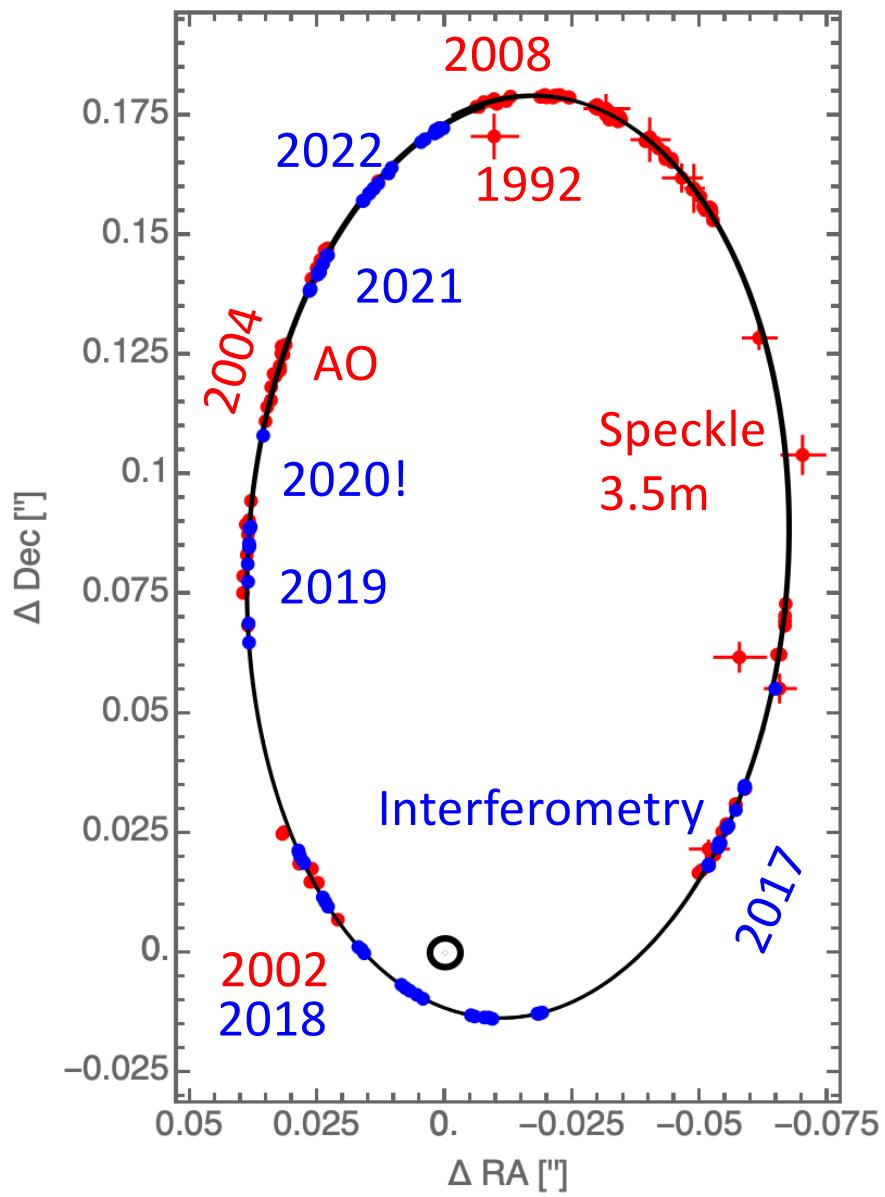
- period: 16 years
- semi major axis: 125 mas
- eccentricity 0.88
- angular momentum and energy have errors of 0.2%
- pericenter: 19 May 2018



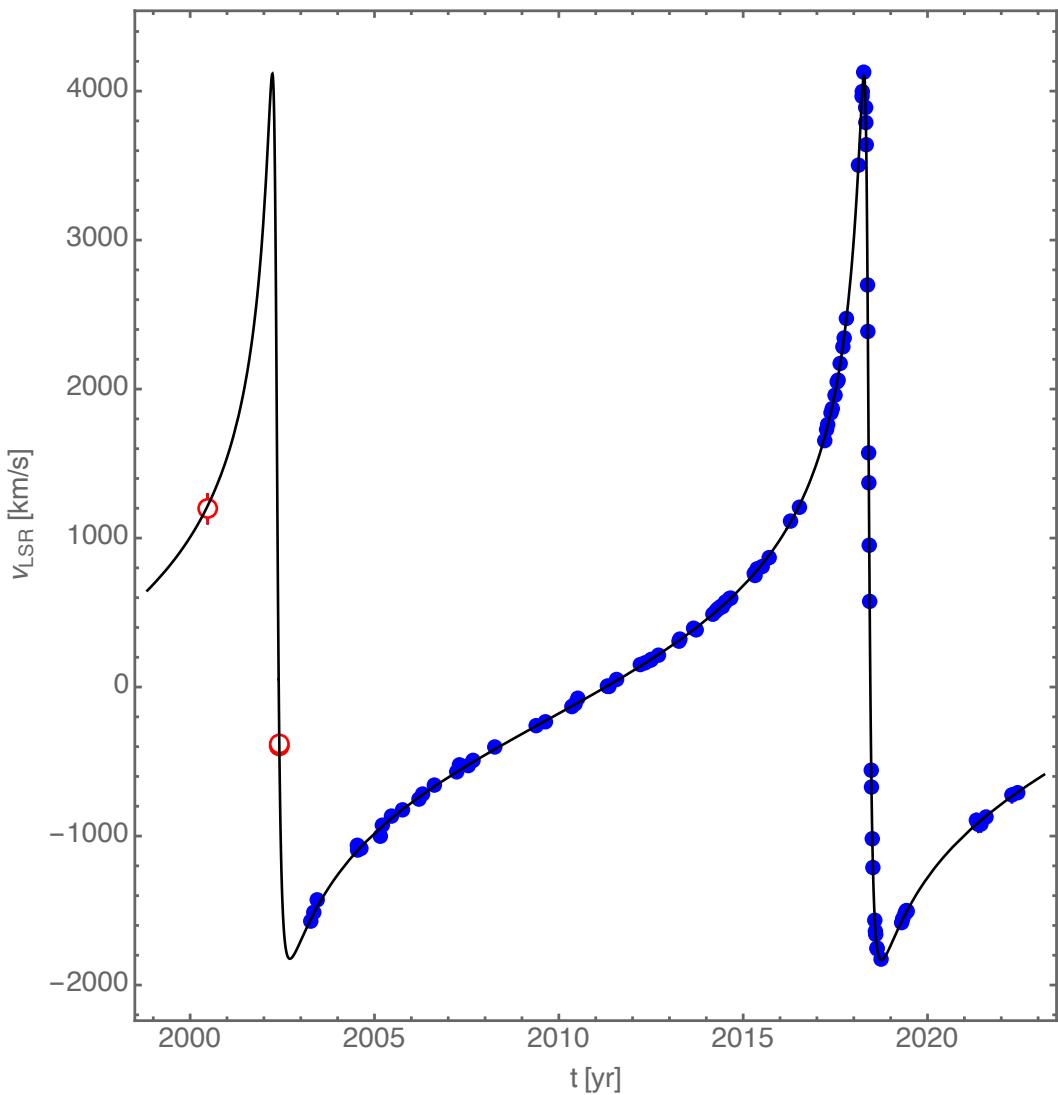
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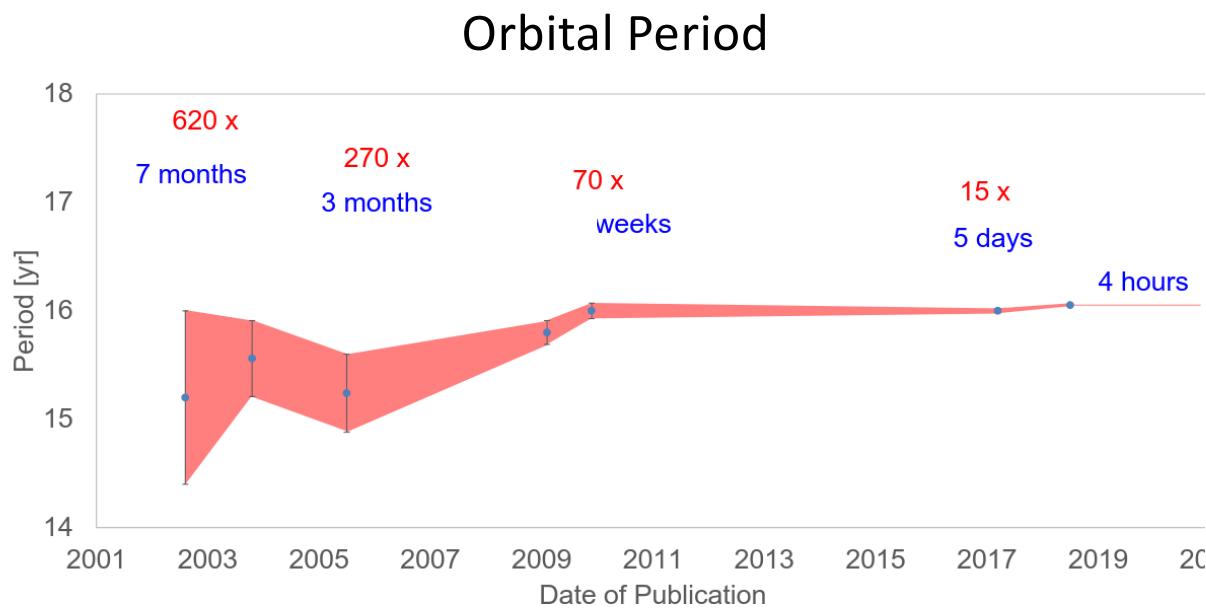
The S2 orbit is very fruitful



The S2 orbit is very fruitful

- geometric distance
- relativistic redshift
- test of equivalence principle
- relativistic precession
- limits on extended mass

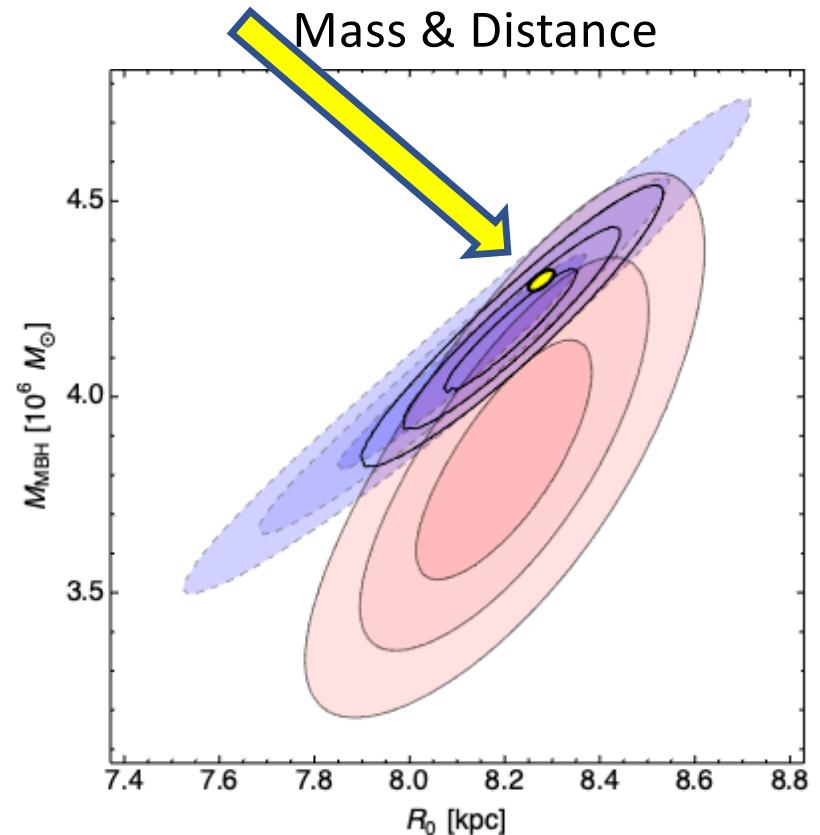
# Improvements by orders of magnitudes



Images: Positions, proper motions, angular velocity in mas/yr

Spectra: Radial velocity in km/s

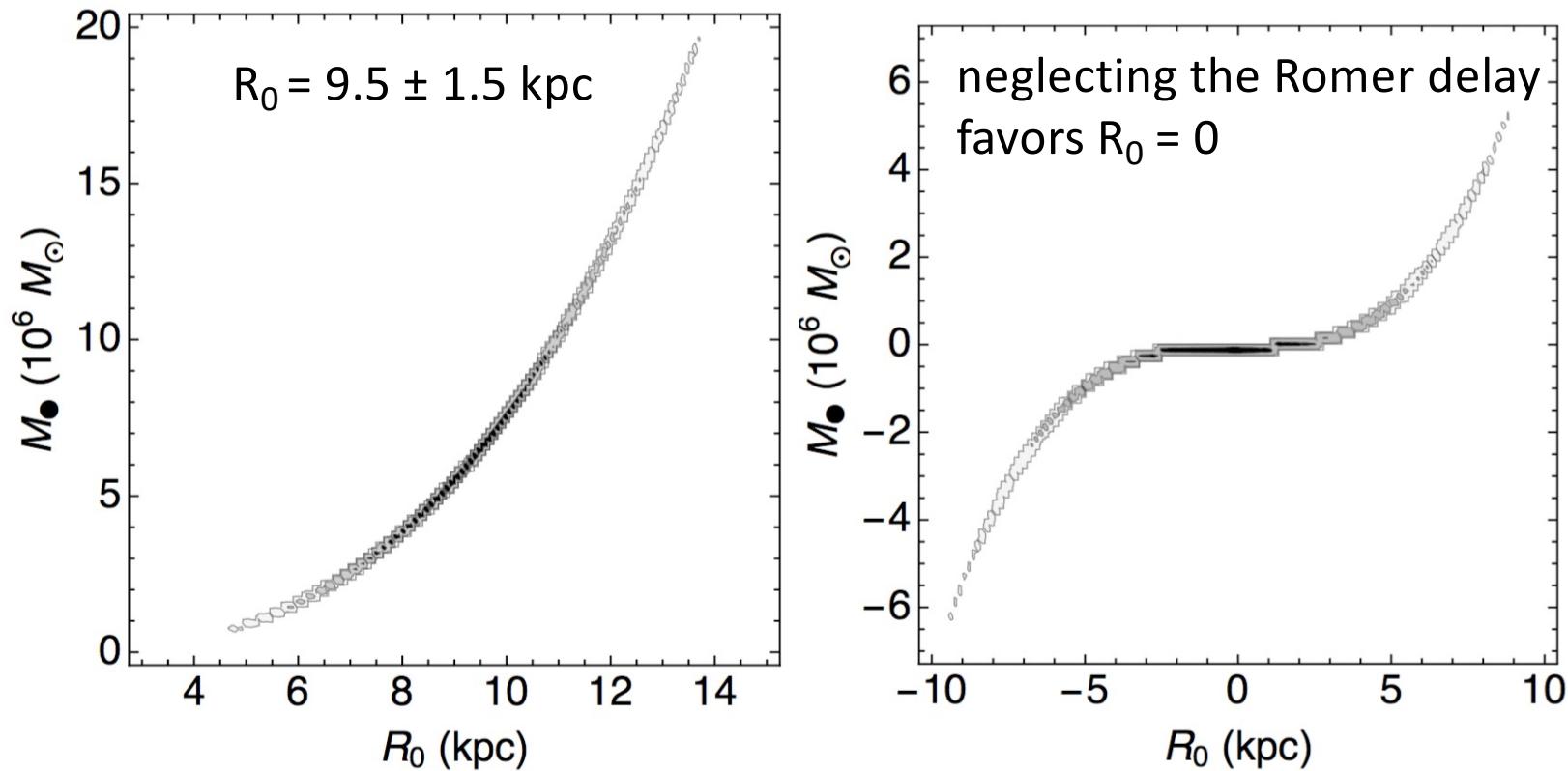
Conversion of angles to absolute length: **The distance**



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  - SgrA\* flares
  - (A funny gas cloud)

We can measure  $R_0$  without radial velocities  
via the Romer delay in the astrometry

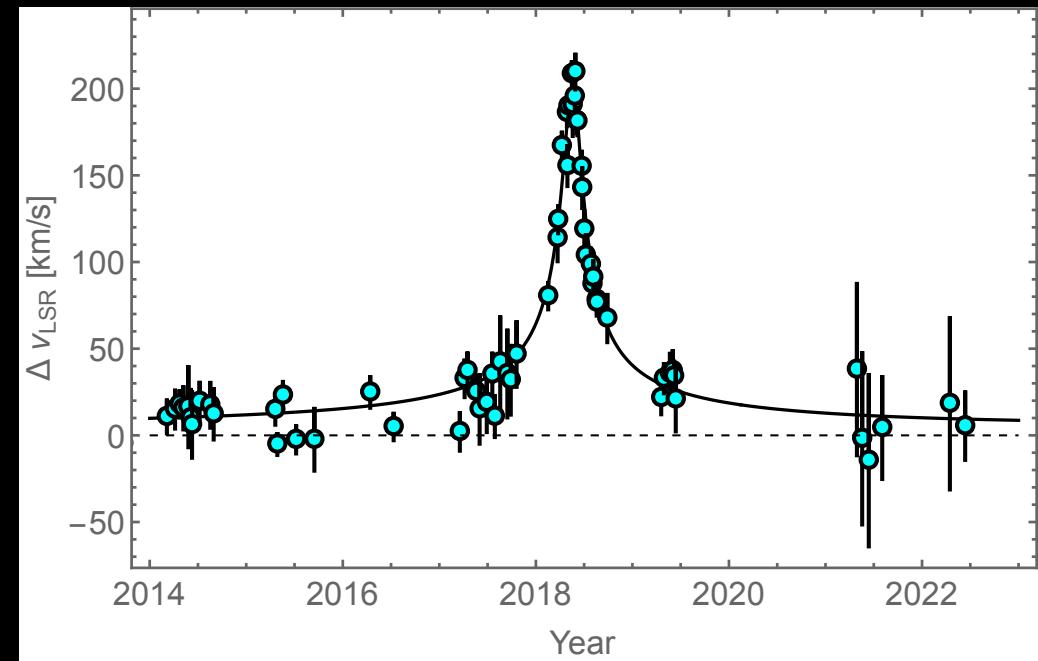
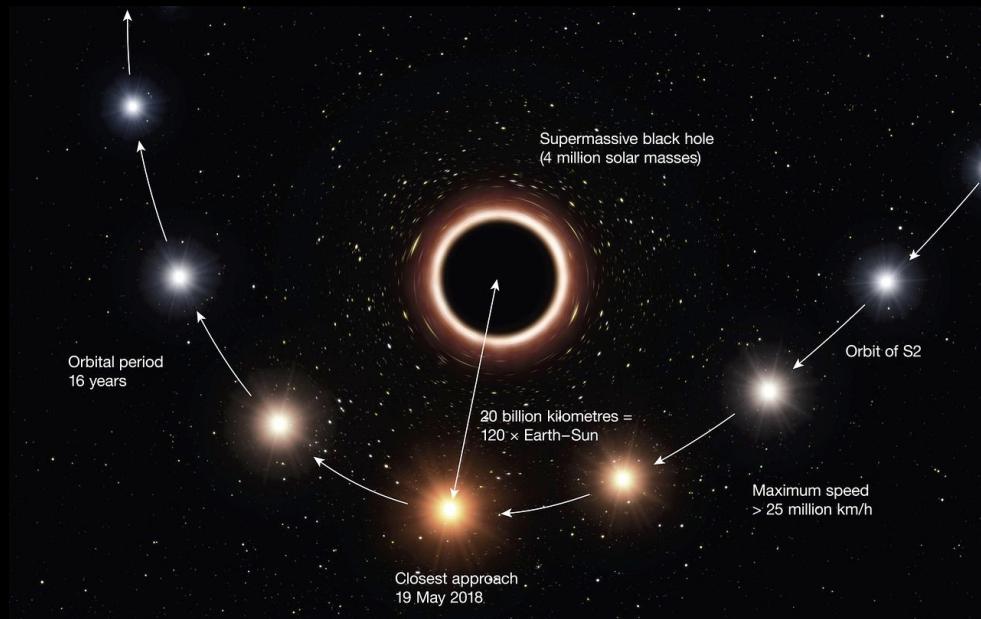


Anglada-Escudé & Torra 2006, GRAVITY collaboration 2019a

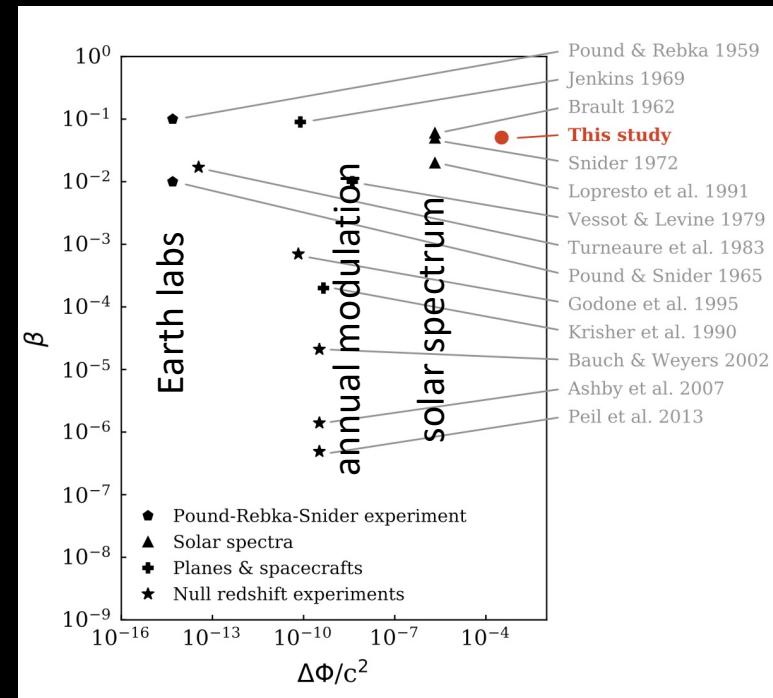
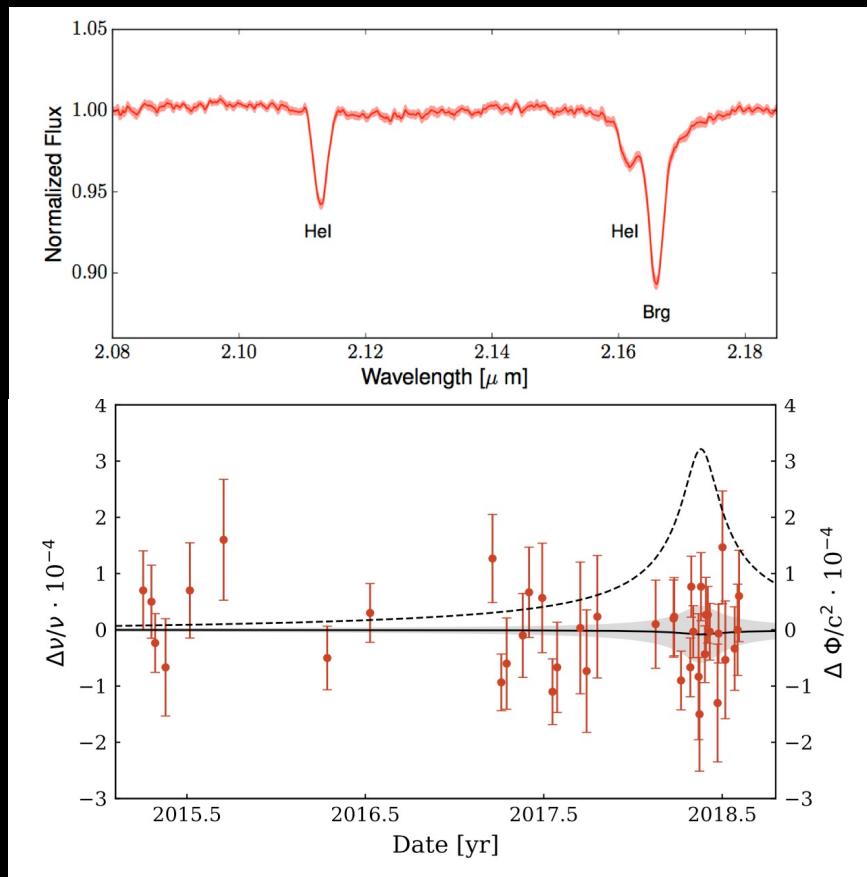
At closest approach of 120 AU, S2 reached 8000 km/s

- **transverse Doppler effect**
- **gravitational redshift from Sgr A\***

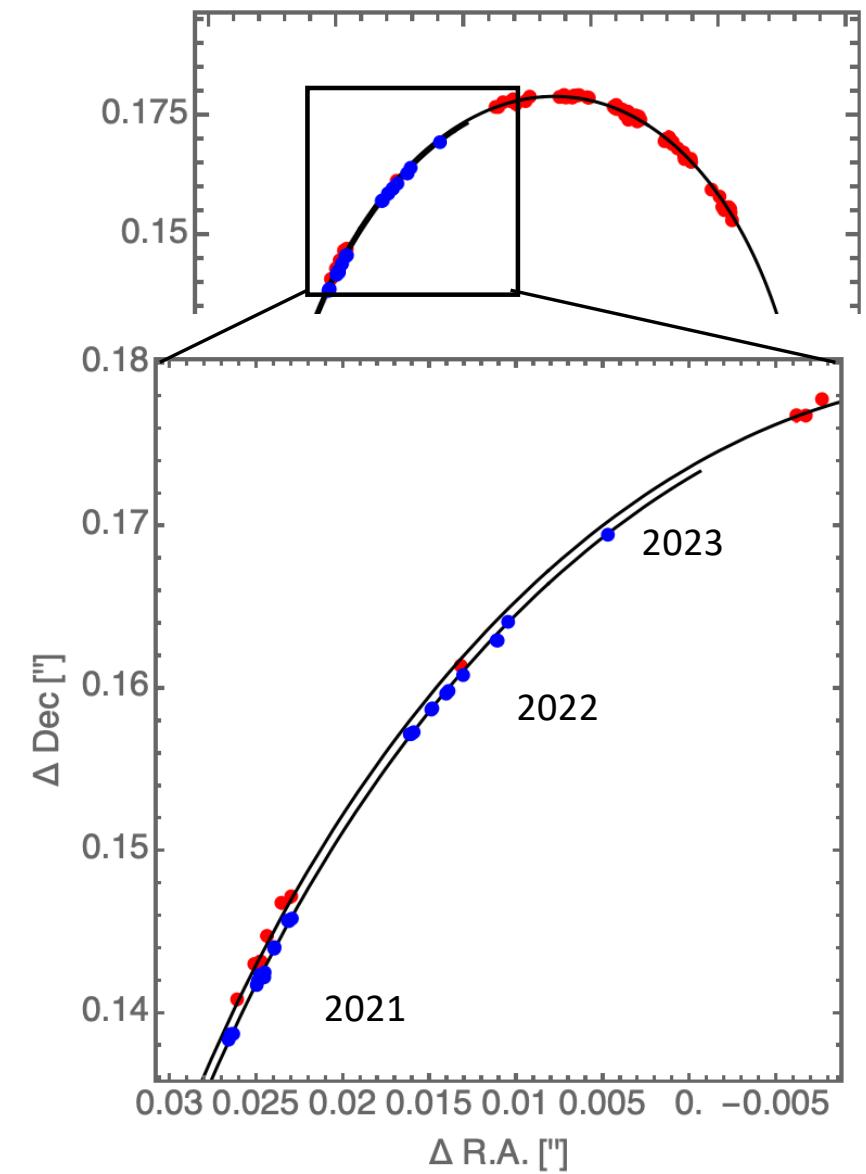
GRAVITY Coll. 2018a



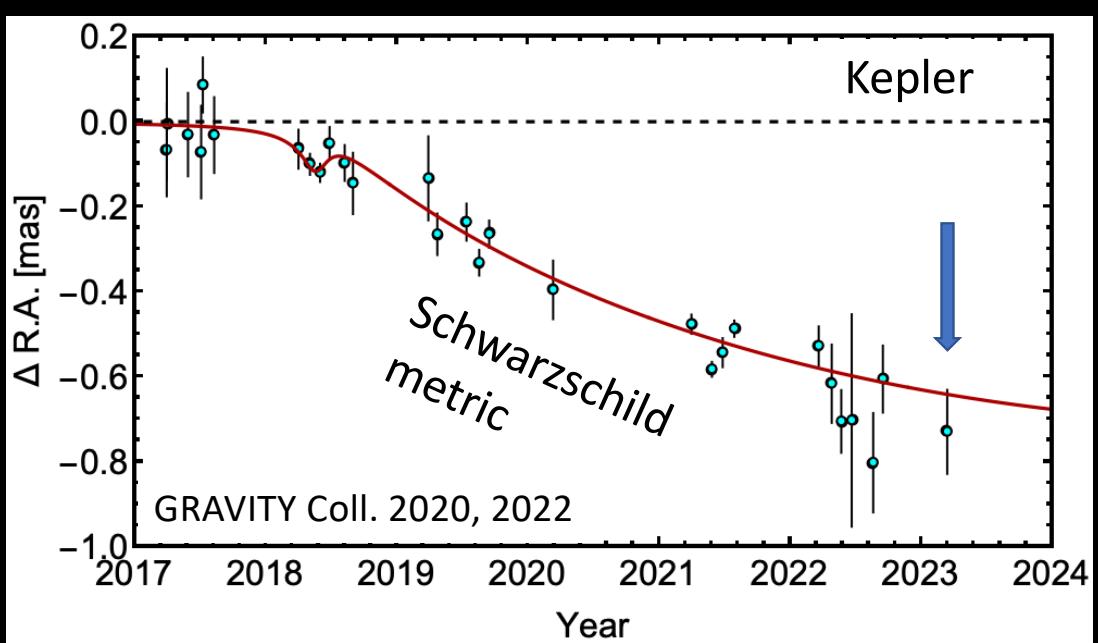
# Comparing the redshift for two atomic lines tests the equivalence principle



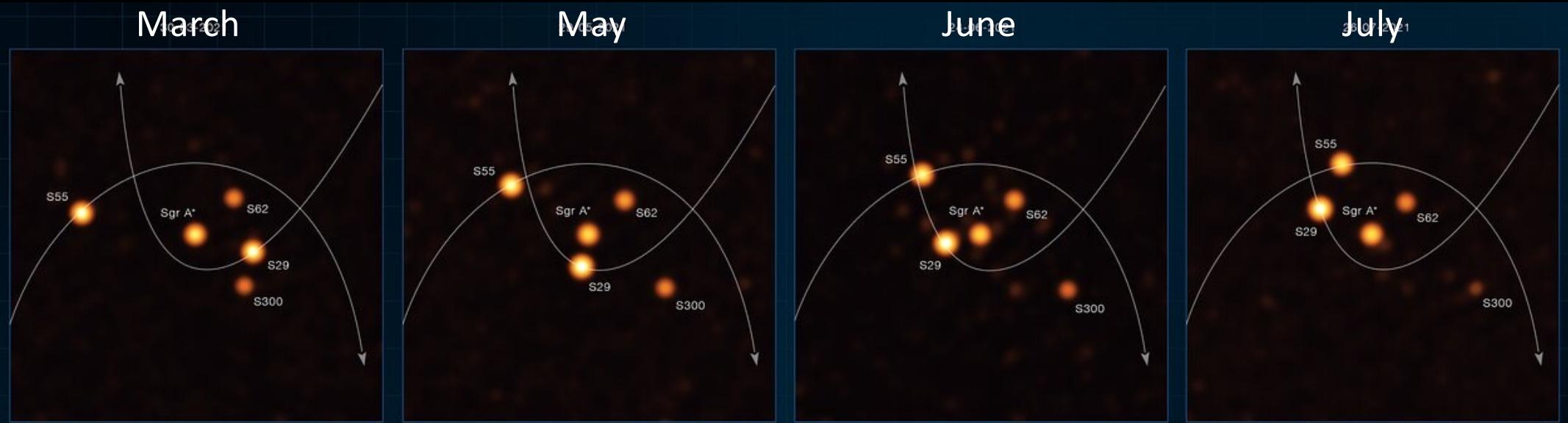
GRAVITY Coll. 2019



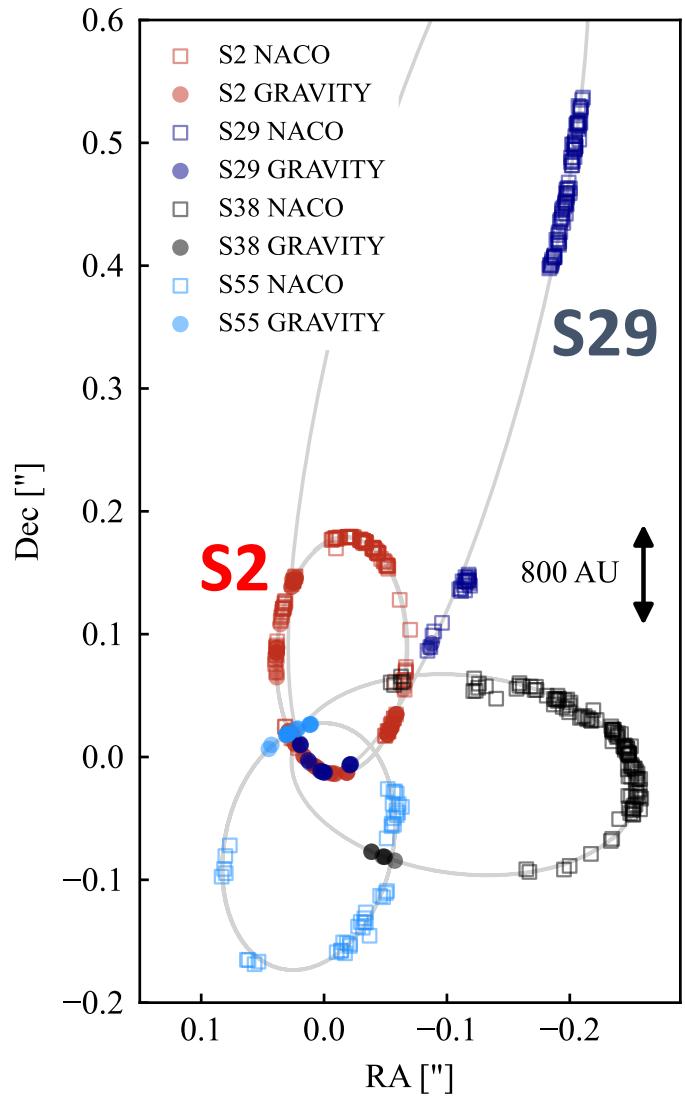
The Schwarzschild precession  
is detected at  $>10\sigma$



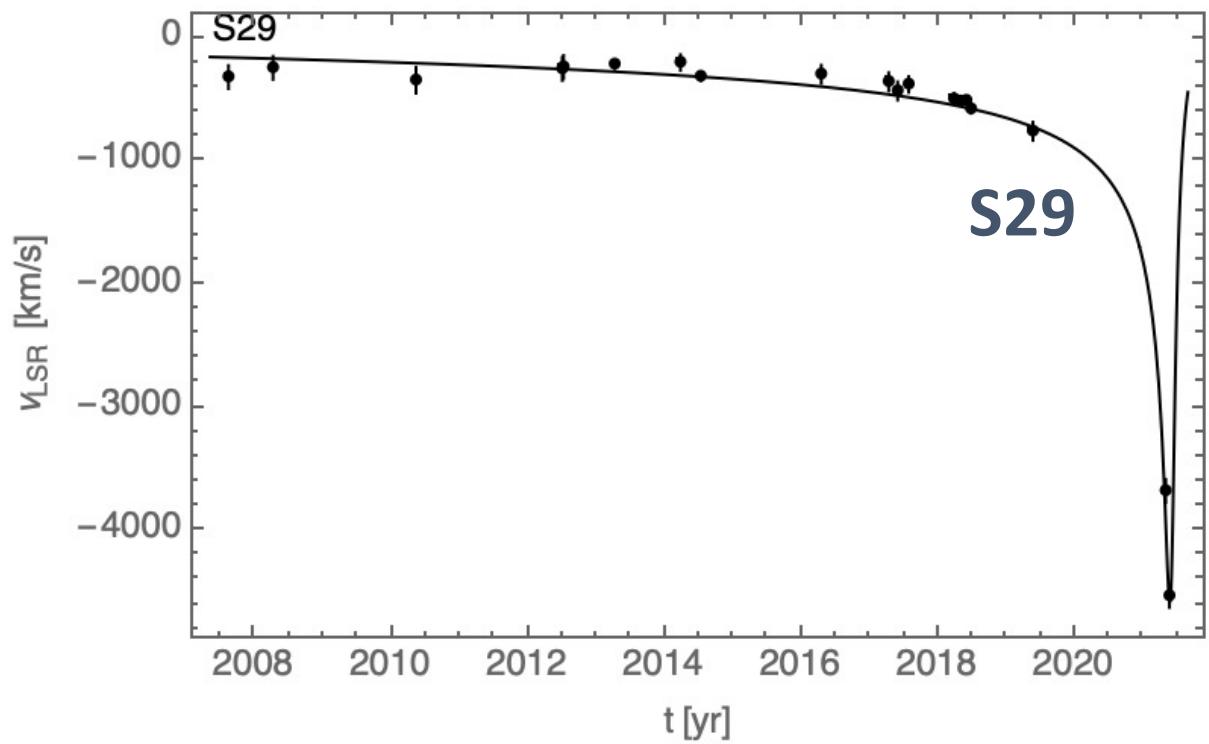
# New since 2021: GRAVITY astrometry of more stars (S29, S38, S42, S55, ...)



GRAVITY collab. 2022a,b

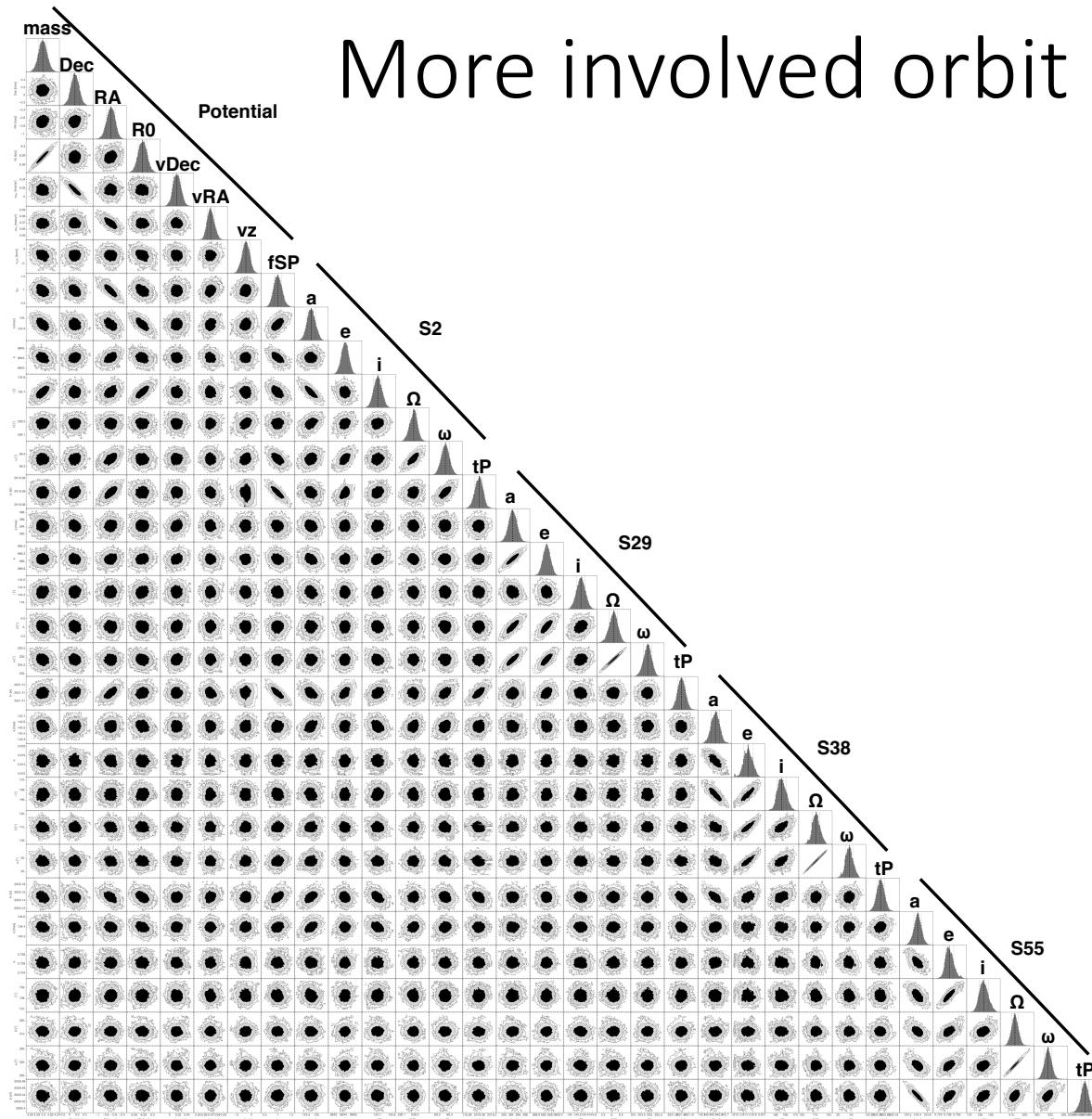


We had previous coverage  
for these stars



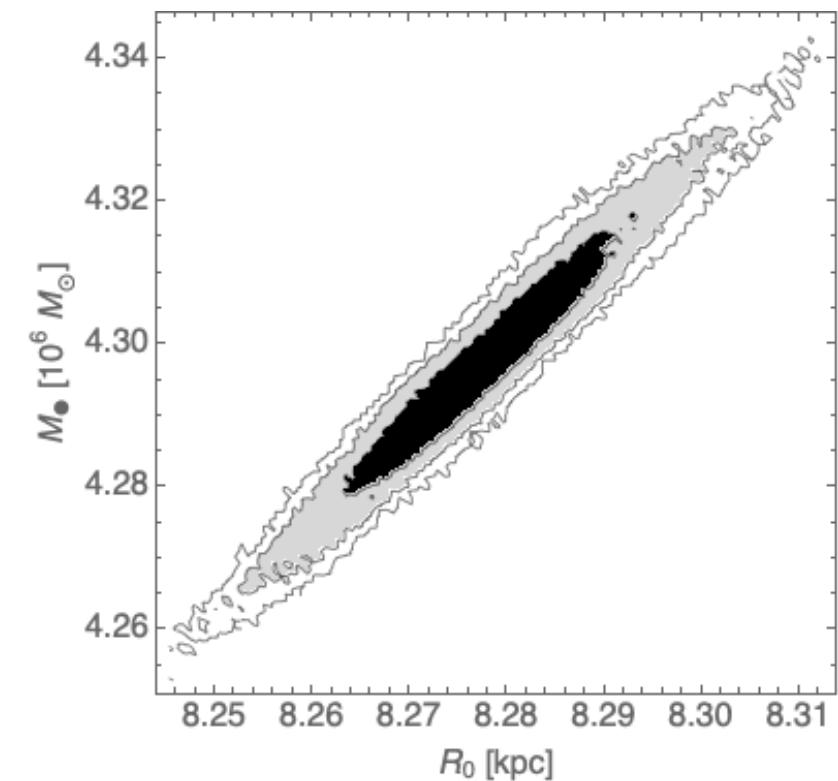
GRAVITY collab. 2022a

# More involved orbit fit...



$$R_0 = 8.277 \pm 0.009_{\text{stat}} \pm 0.035_{\text{sys}} \text{ kpc}$$

$$M = 4.297 \pm 0.012_{\text{stat}} \pm 0.040_{\text{sys}} \times 10^6 M_\odot$$

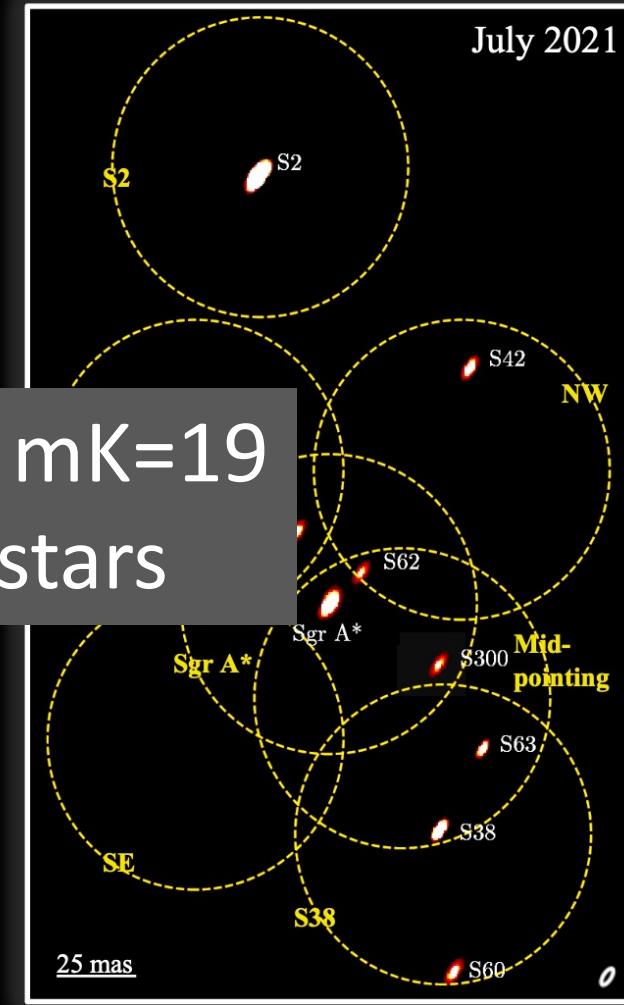


GRAVITY collab. 2022a

# Imaging at milli-arcsec resolution

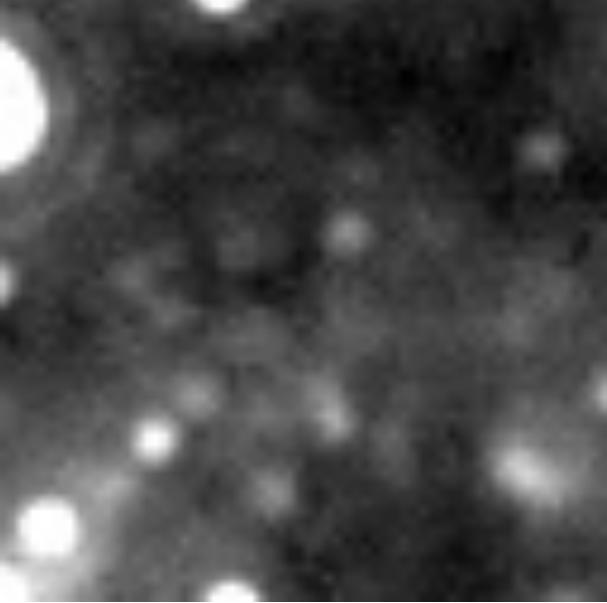


- Tracking stars down to  $mK=19$
- Discovering new, faint stars

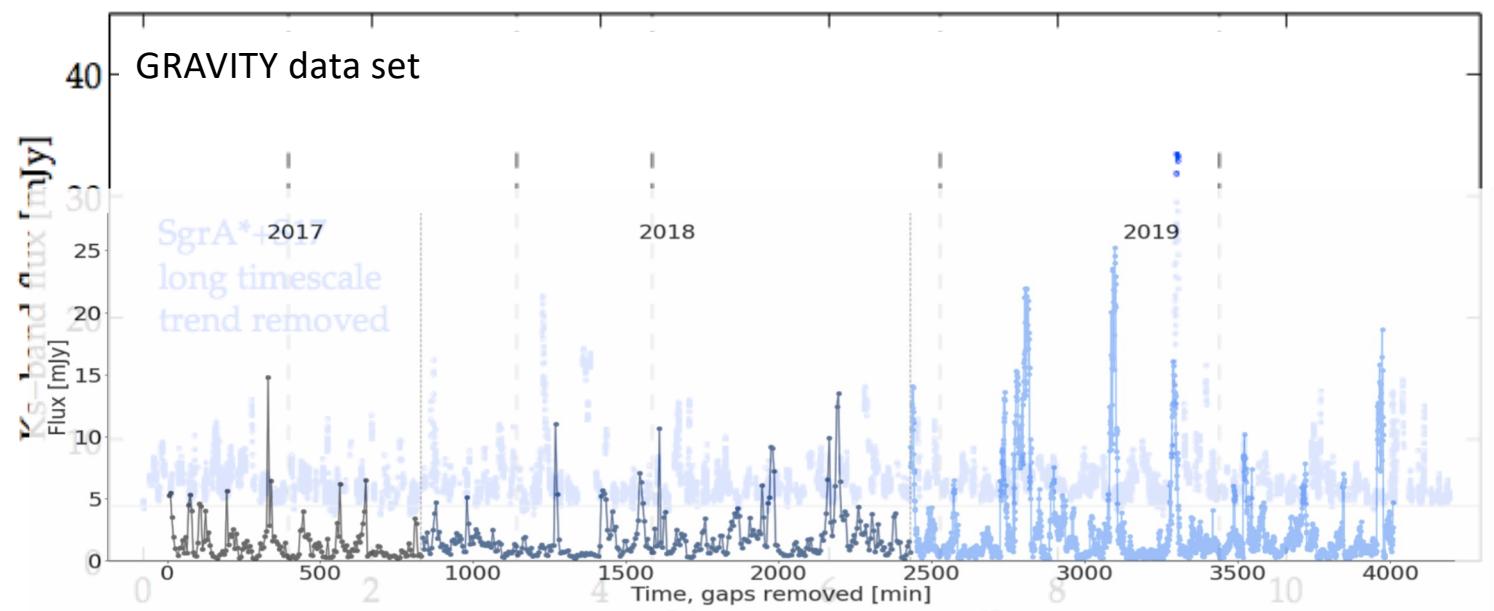


# Outline

- Part I        The mass of Sgr A\*
  - Astronomical black holes
  - Measuring mass
  - Infrared observations
  - Adaptive Optics
  - Stellar Orbits
- Part II      Errors, Fitting and all that
- Part III     Testing the black hole paradigm
  - The black hole nature of Sgr A\*
  - Interferometry
  - Testing General Relativity in the Galactic Center
  - **SgrA\* flares**
  - (A funny gas cloud)

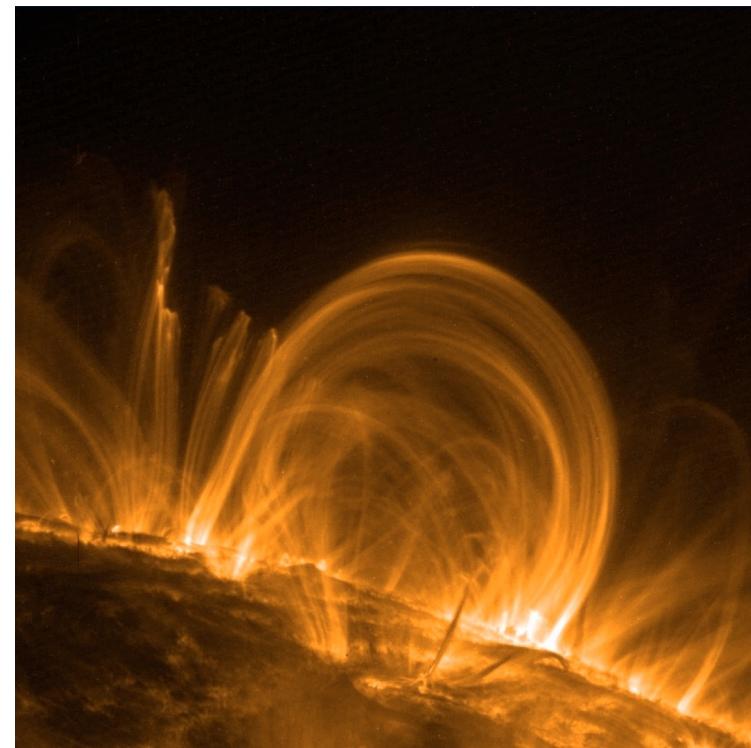
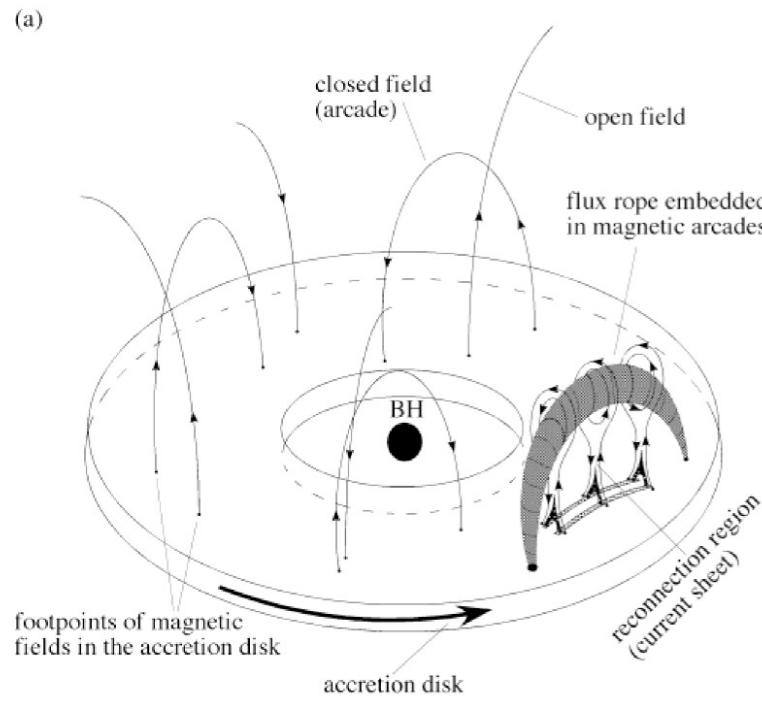


# Sgr A\* is flaring in the infrared



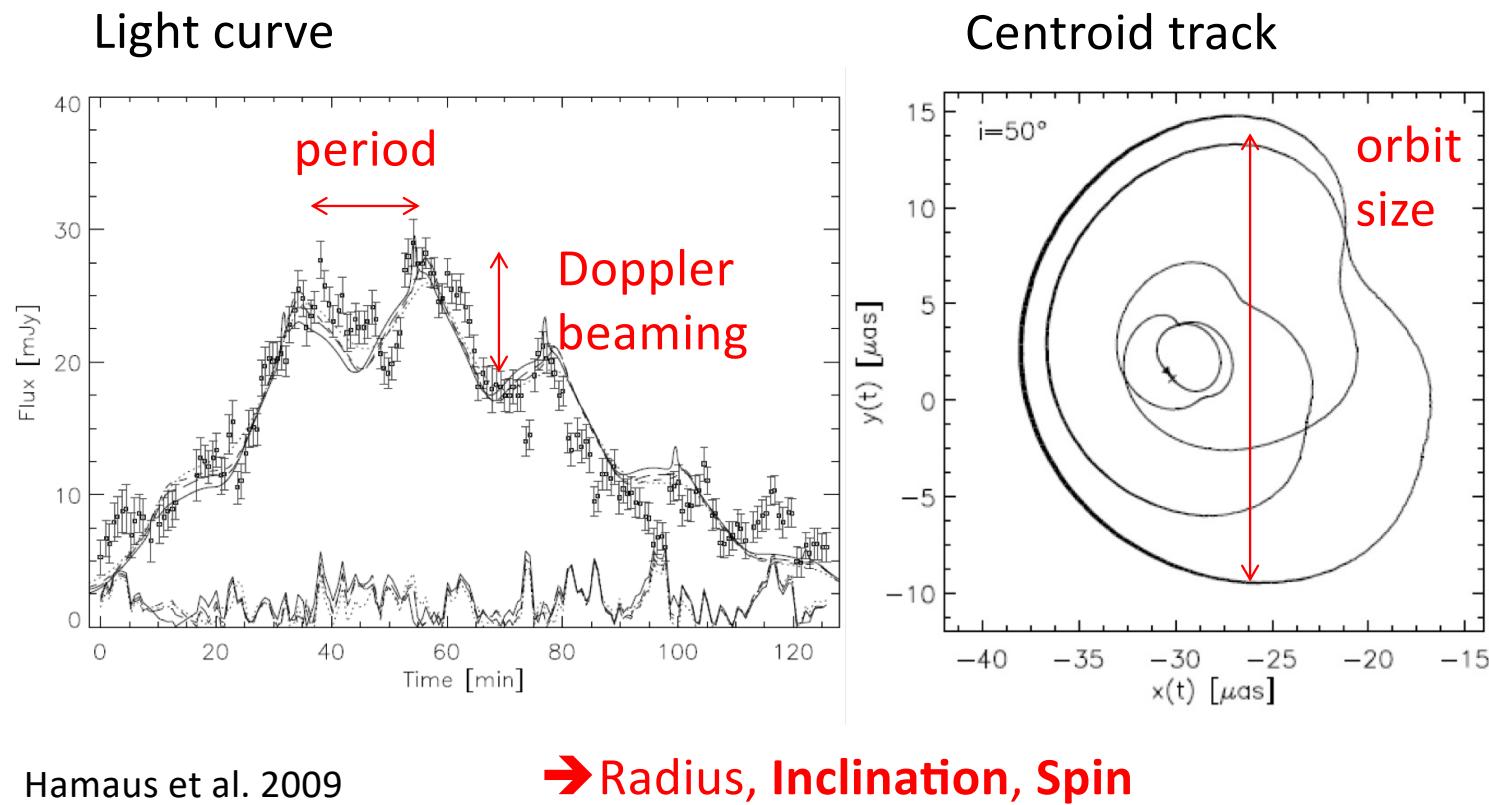
Genzel et al. 2003, Dodds-Eden et al. 2011, Witzel et al. 2012, 2018, GRAVITY coll. 2020b

# Speculation: An analog to solar flares

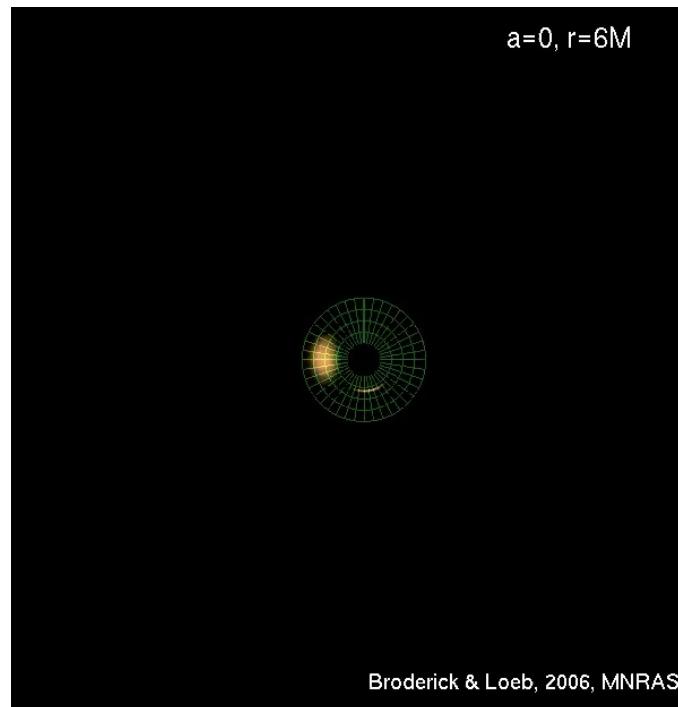


Yuan et al. 2009

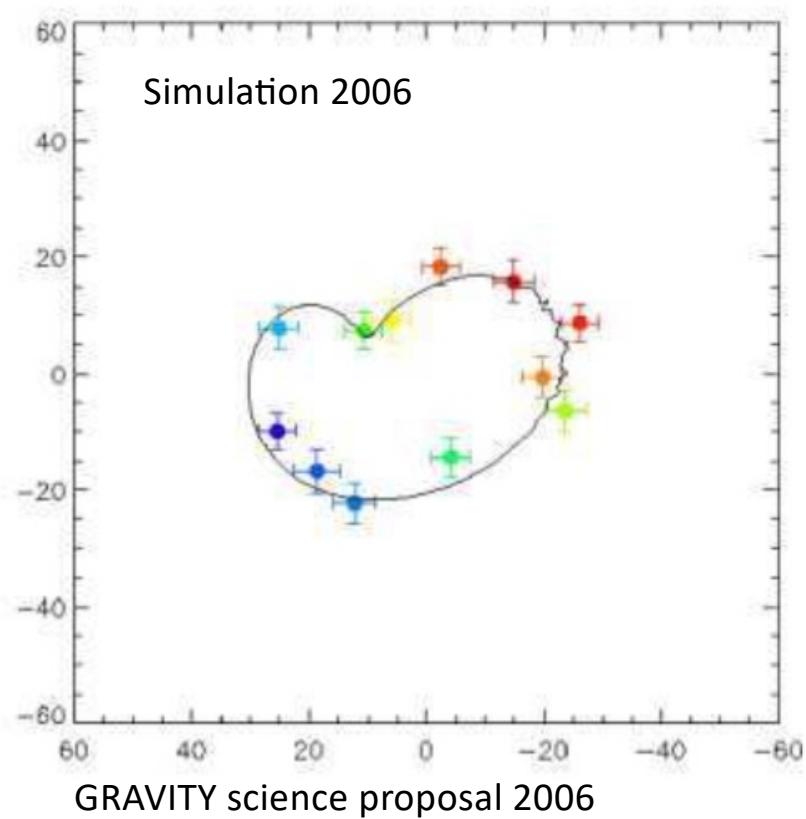
# What can one learn from flare orbits?



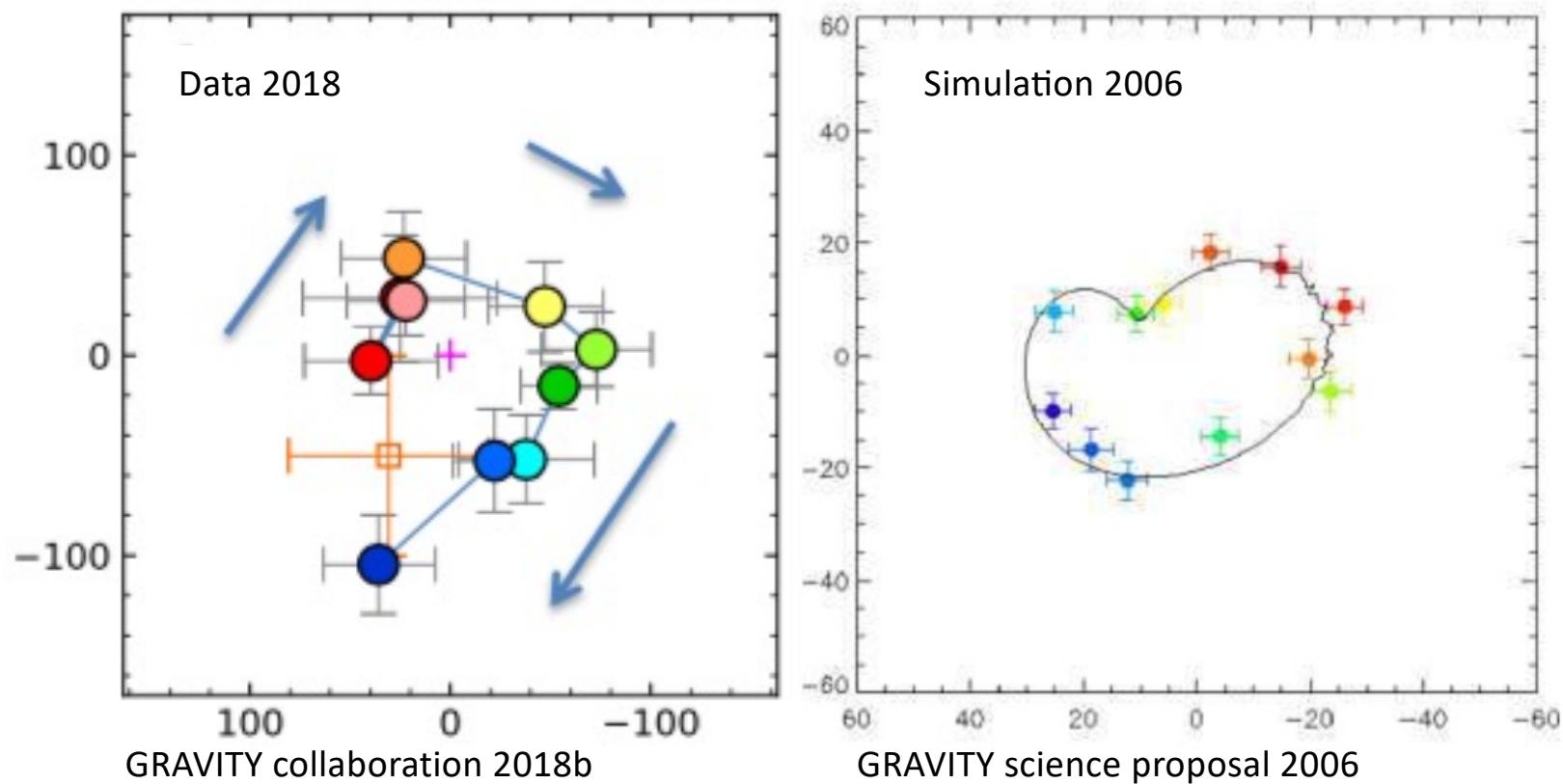
The flares should orbit at a few  $10\mu\text{as}$  radius -  
in reach for GRAVITY astrometry



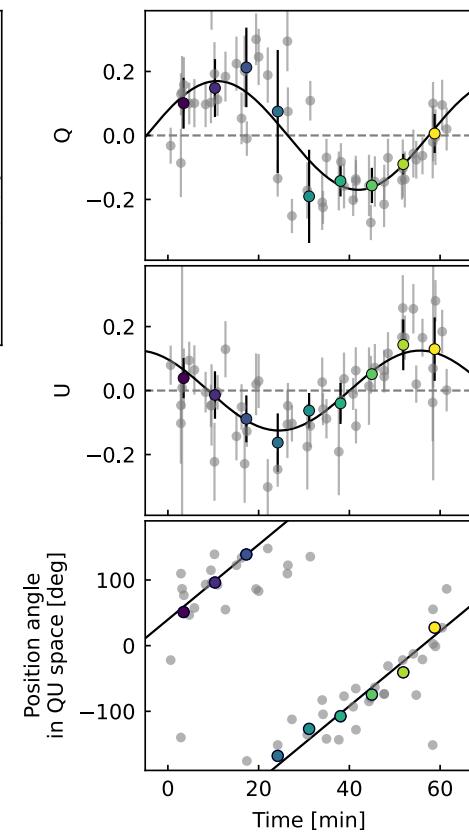
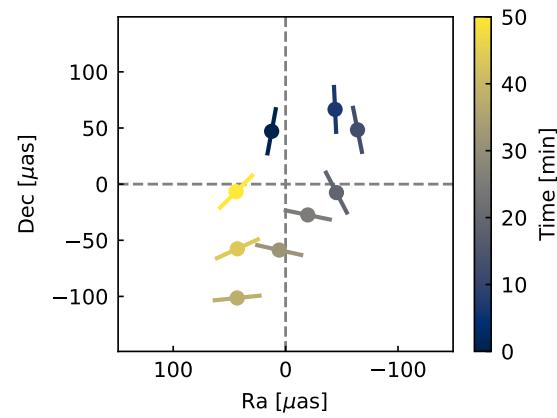
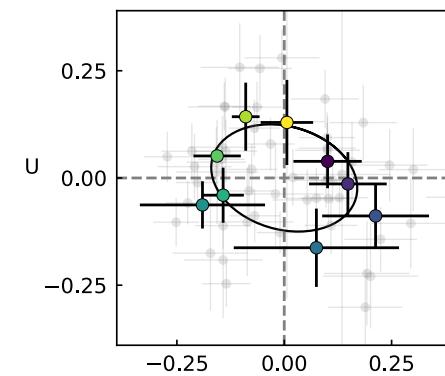
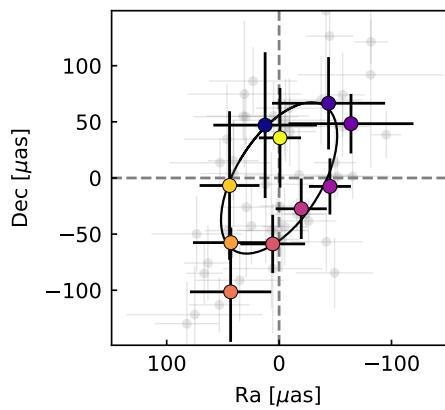
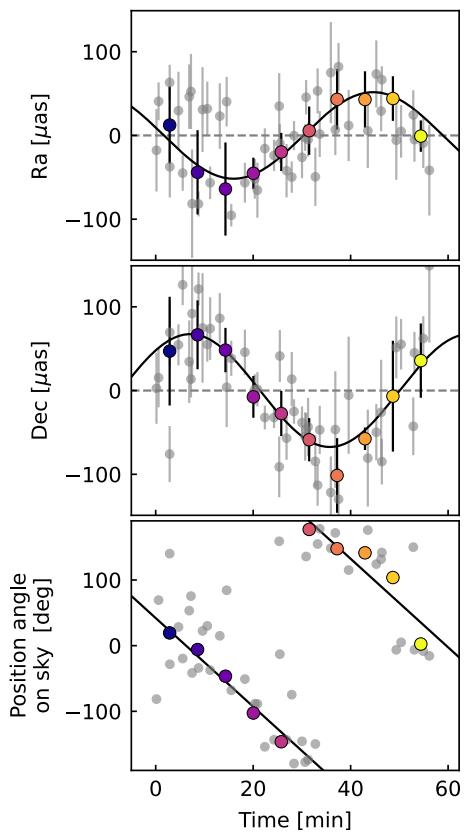
Broderick & Loeb 2005, 2006



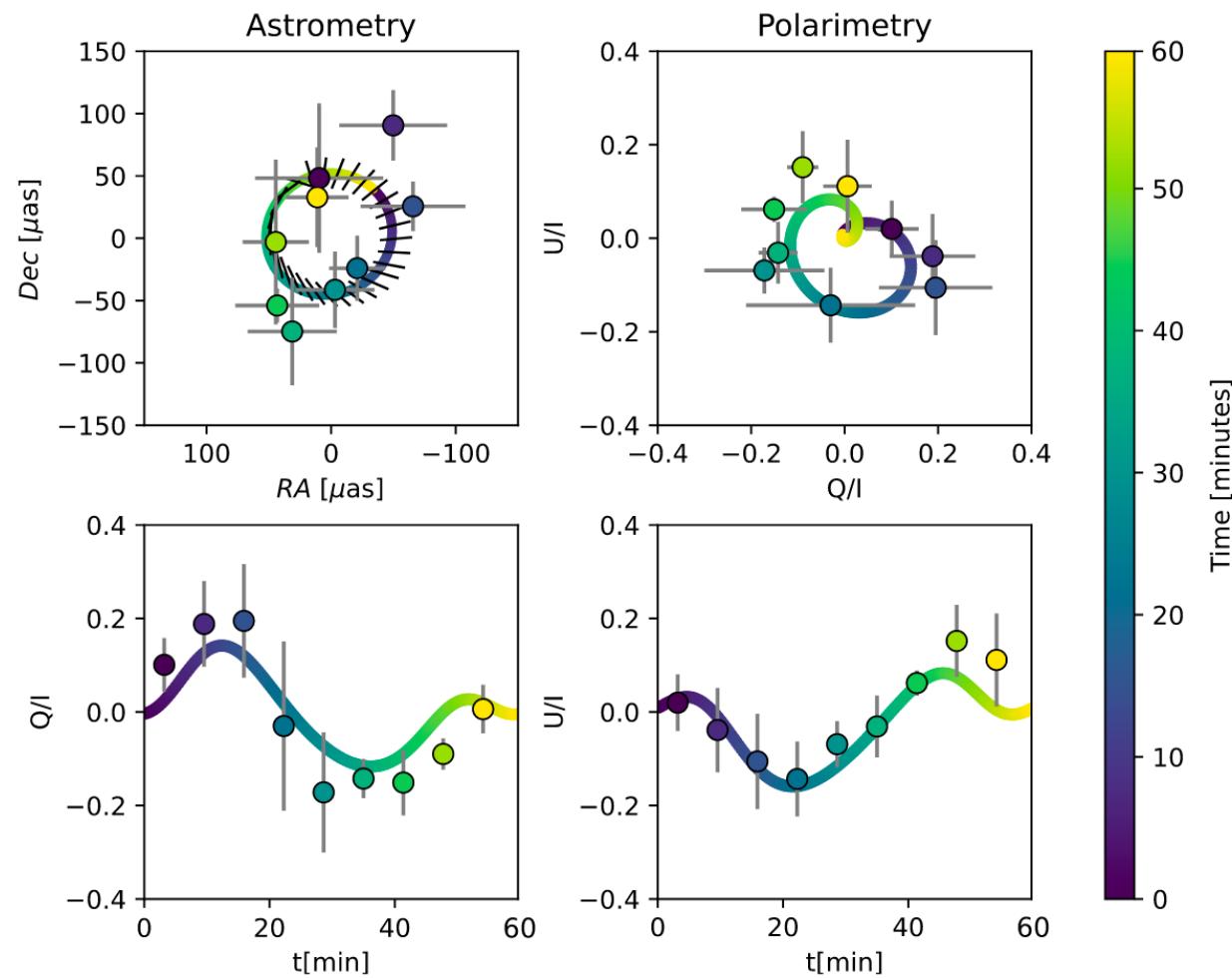
# GRAVITY observations 2018: Orbiting hot spots (4x)



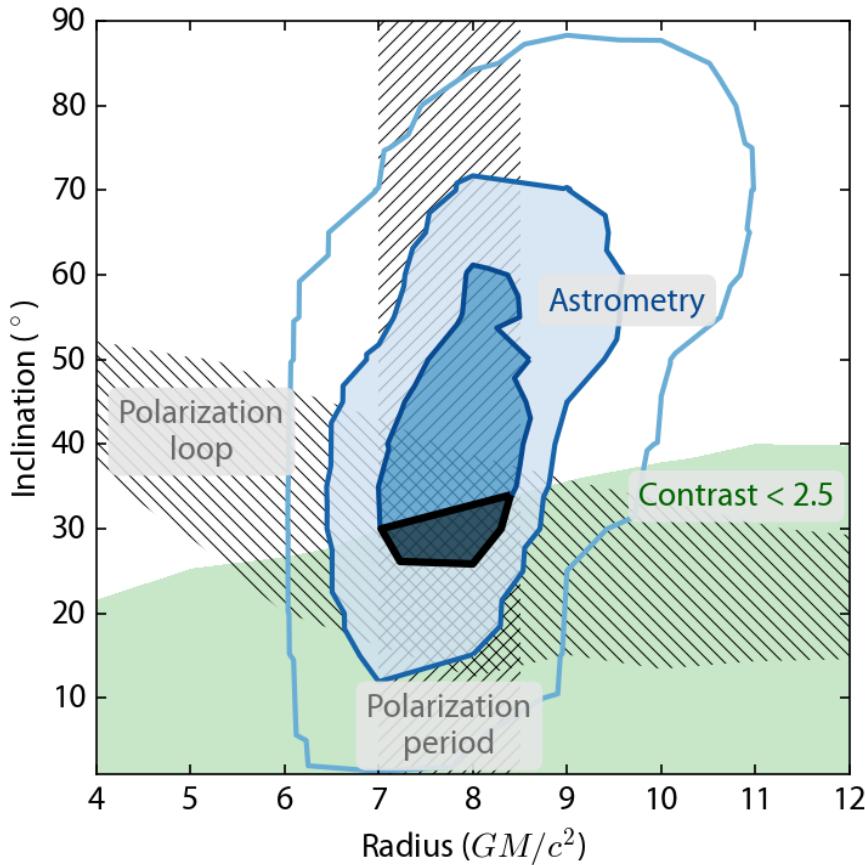
Status 2023:    4 flares with astrometric loop  
                      6 flares with polarimetric loop



# An orbiting, polarized hot spot can describe the data

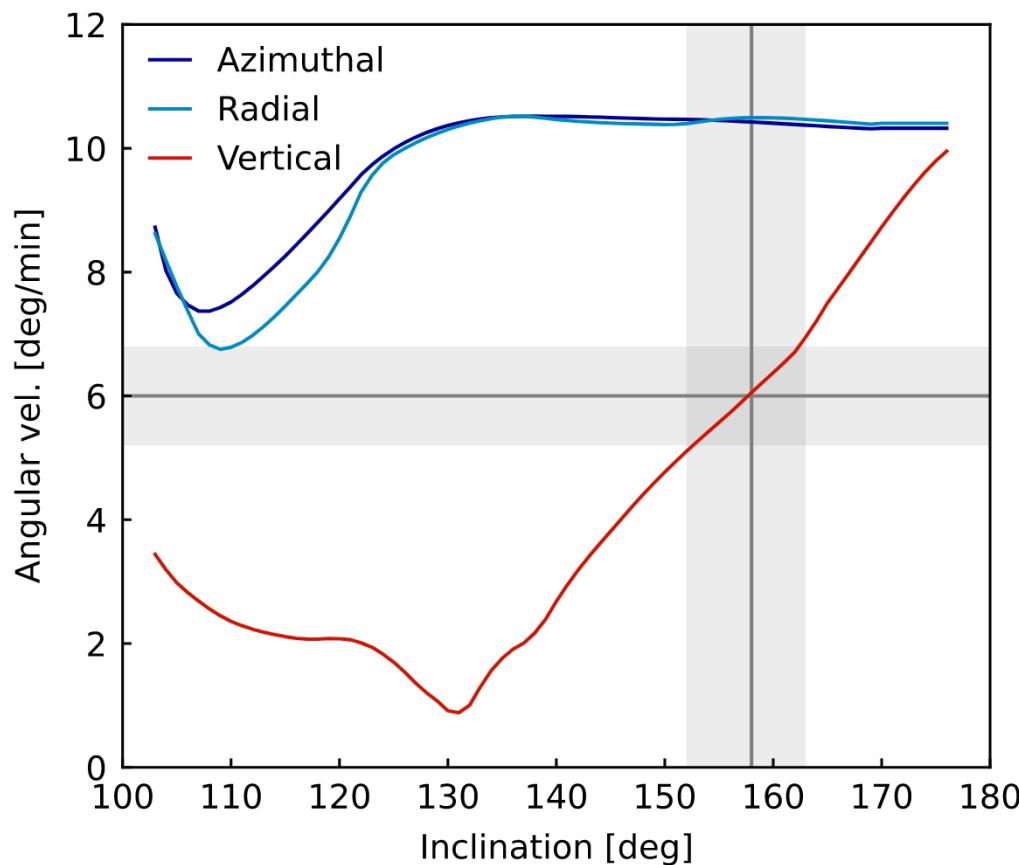


# A face-on view and a vertical magnetic field for SgrA\* is favored

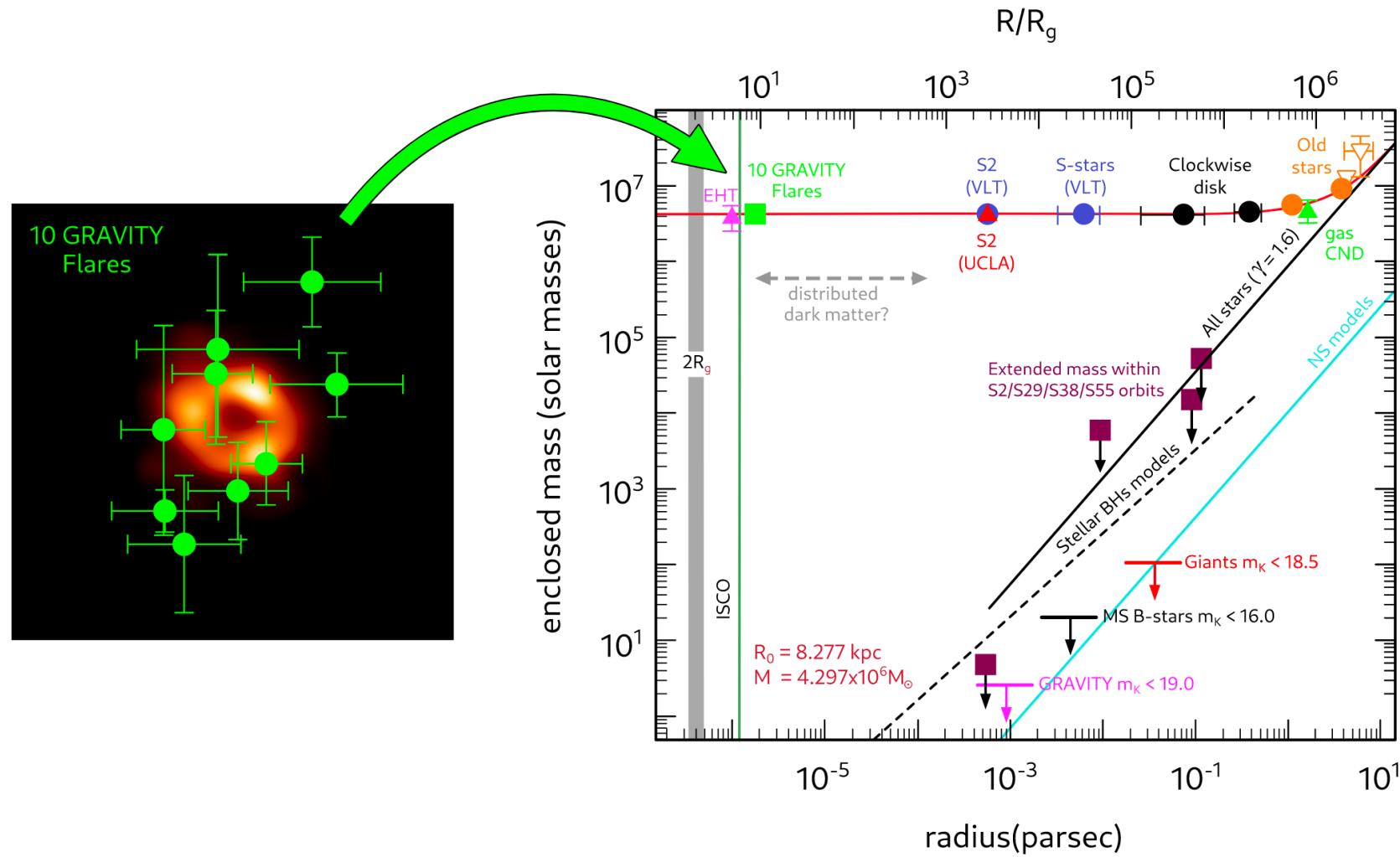


GRAVITY collaboration 2018b

# Key for the inclination: Angular velocity of polarization vector rotation

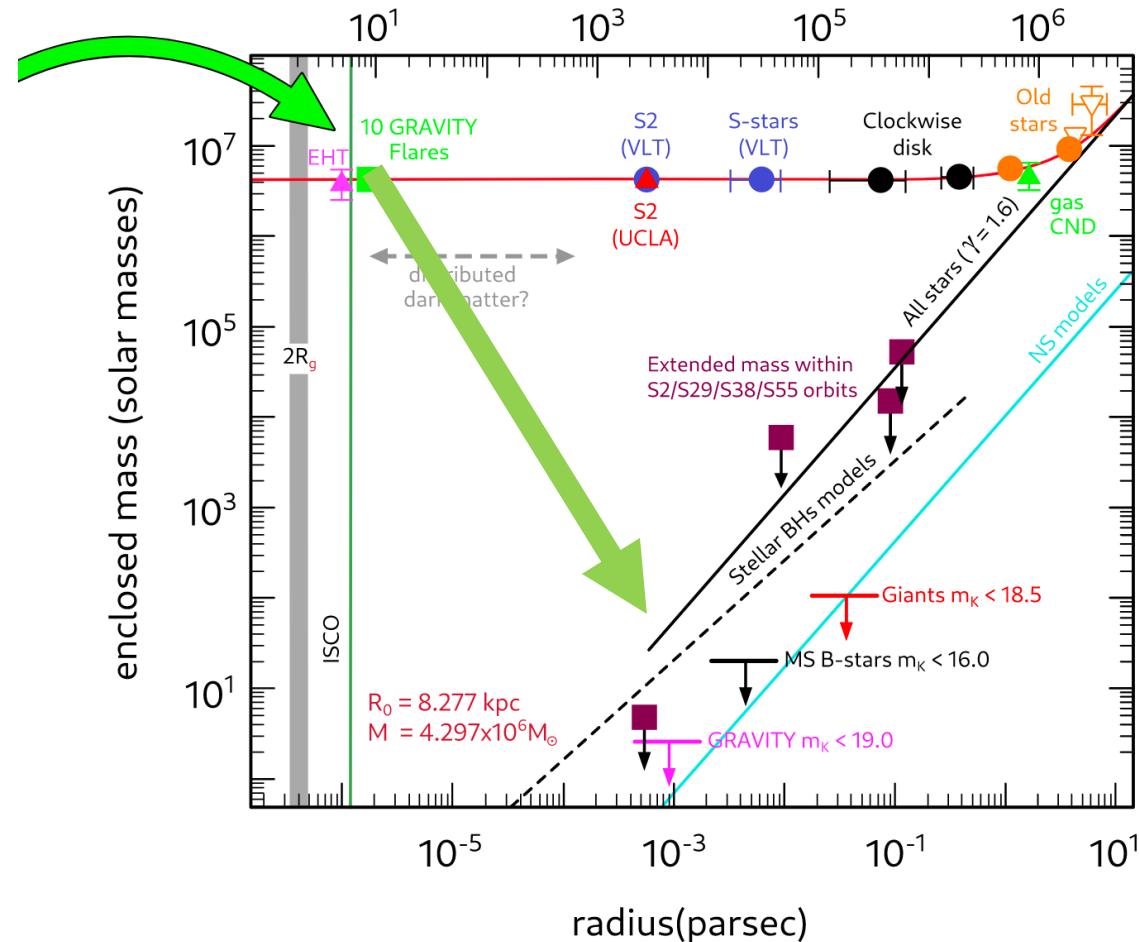


Period known, radius known  mass known

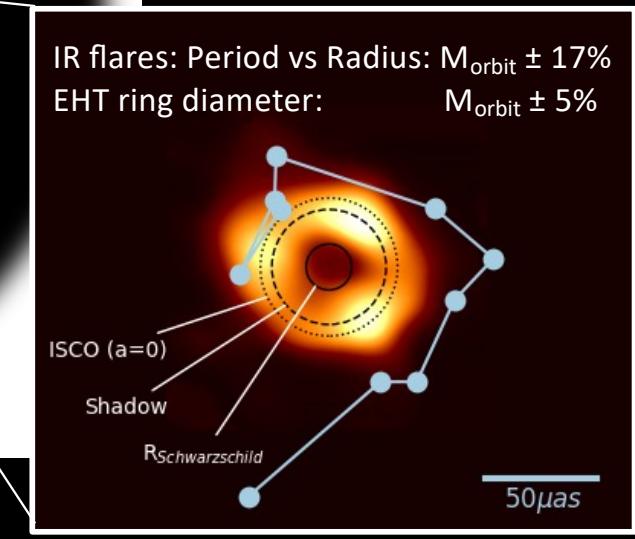
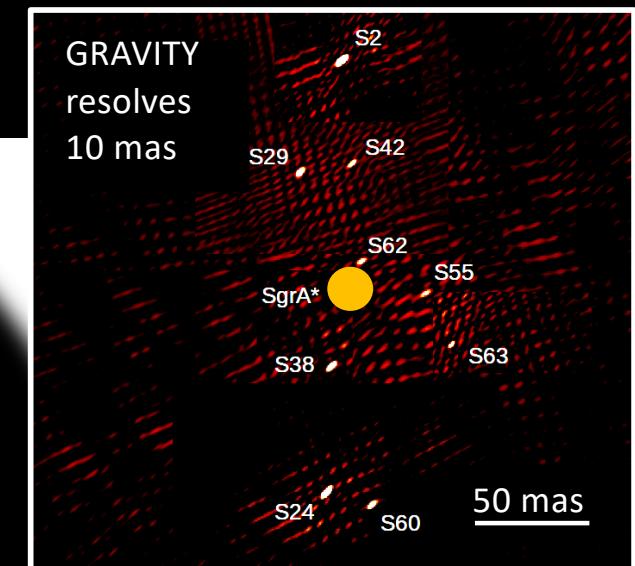
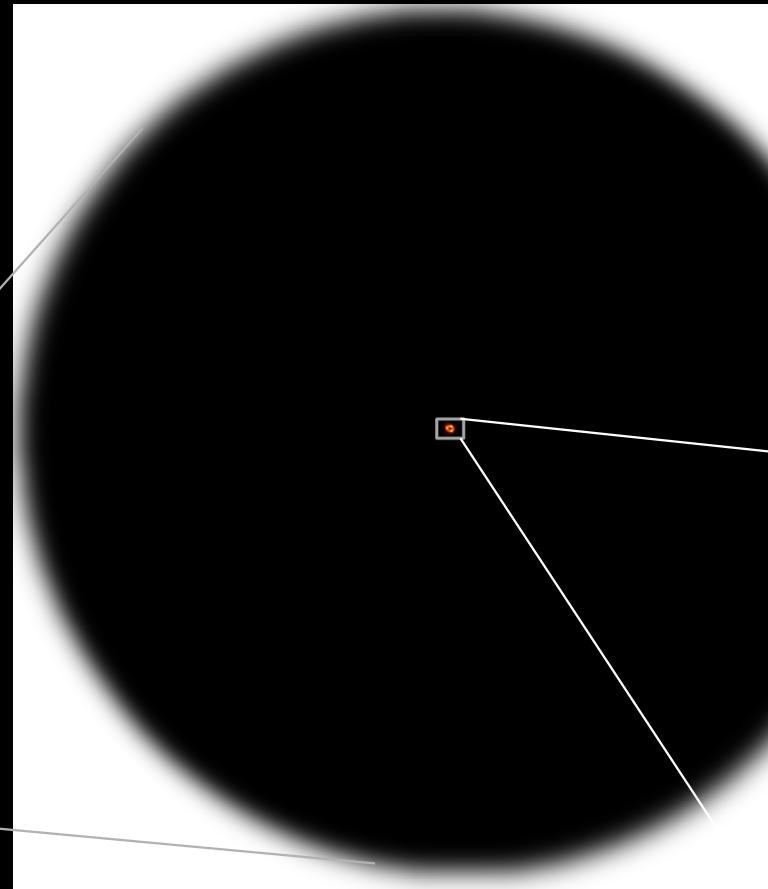
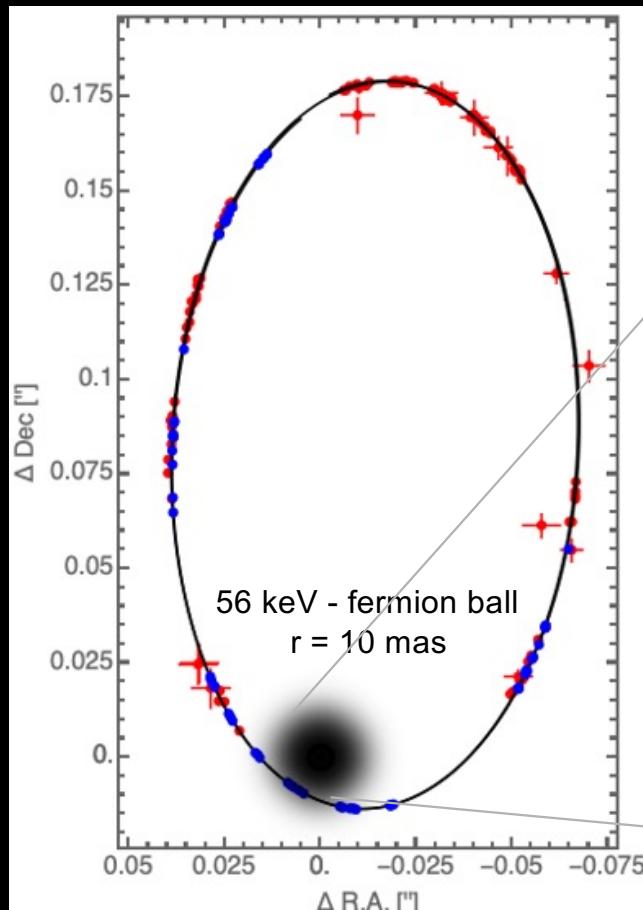


99.9% of the mass of Sgr A\* is  
inside of S29's apo =  $100 \text{ AU}$   
 $R/R_g$

At most  
 $\sim 0.1\%$   $\sim 4000 M_\odot$   
can be in an extended  
configuration



# Example of a model completely ruled out: Fermion ball



Argüelles et al. 2019

# GRAVITY- starting to set limits

Gravitational redshift  $f_{RS}$



Equivalence principle  $1 + \Delta\beta$



Schwarzschild precession  $f_{SP}$



$P_{flare}/P_{Kepler}$



White hole metric  $\alpha_{WH}/R_G$



Black hole spin  $X_{spin}$

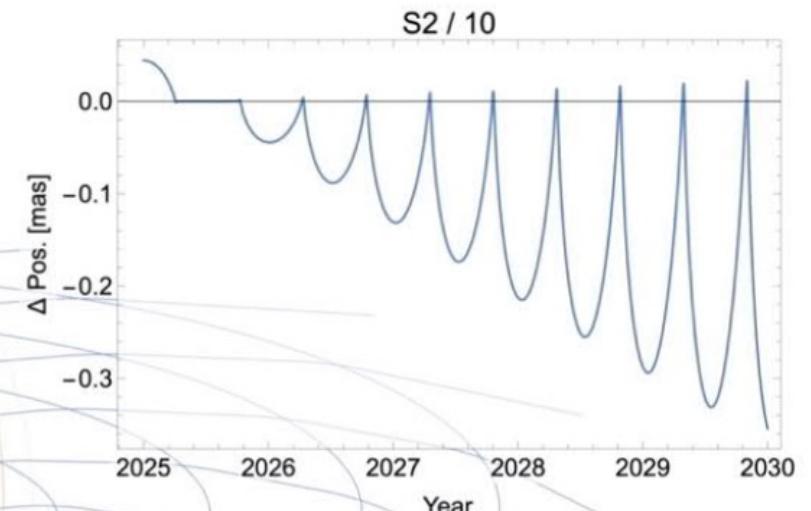
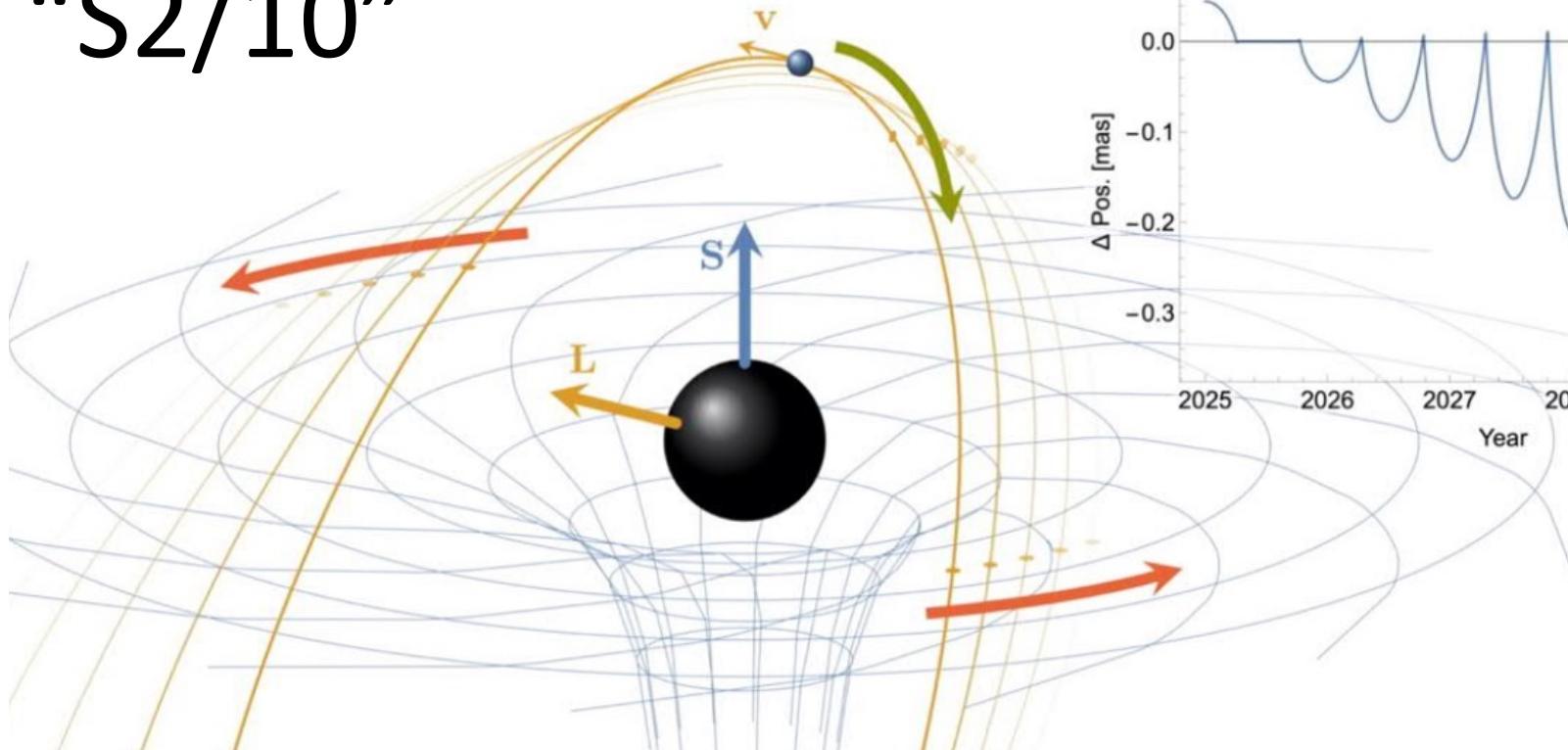


2PN term  $f_{2PN}$

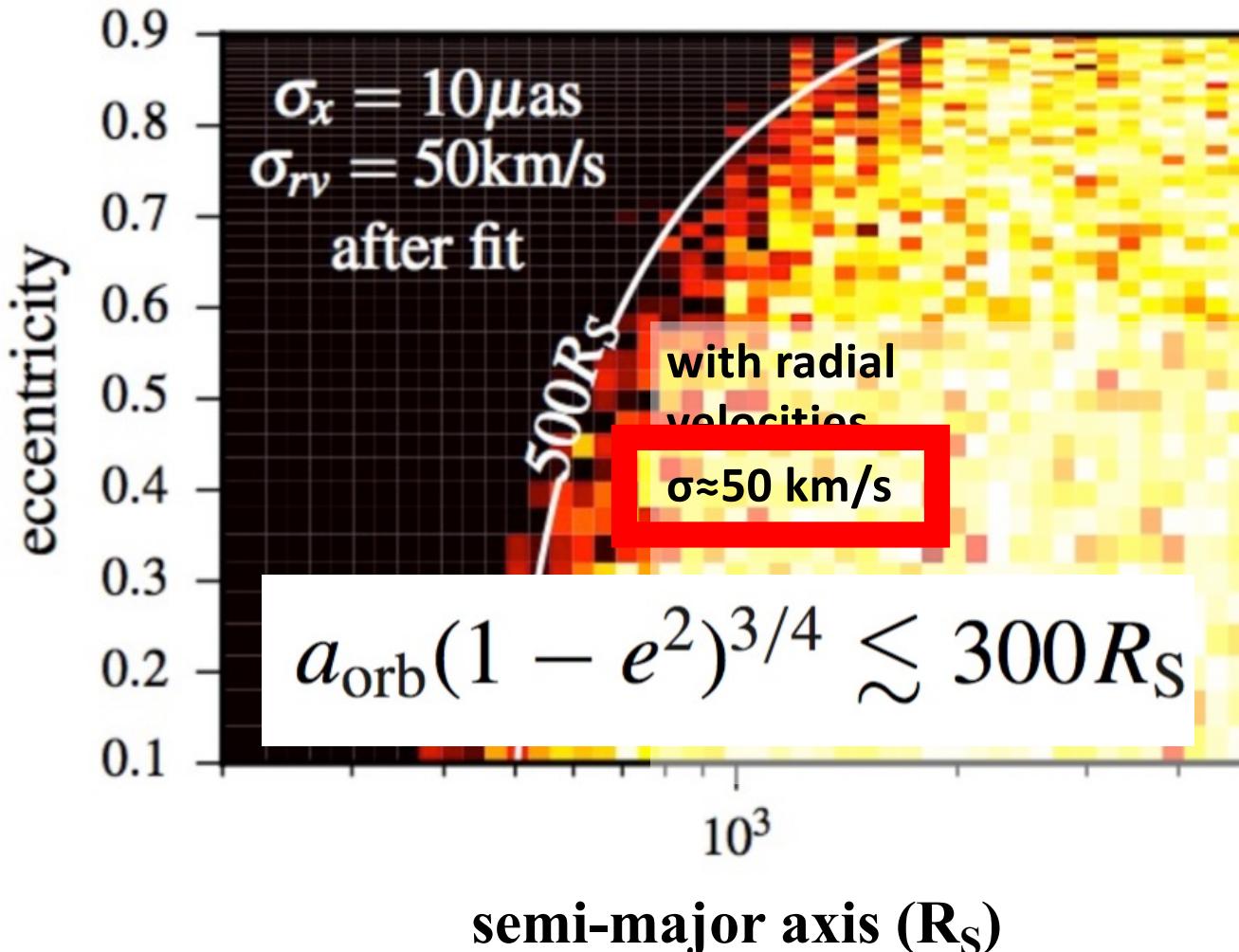


# How far can we get in the Galactic Center?

“S2/10”



# What star is needed to detect the spin?



- Spin effects:  $r^{-3}$
- stars further in
- astrometry harder,  
spectroscopy easier
- very conservative  
radial velocity error
  - we don't need to  
limit us to one star

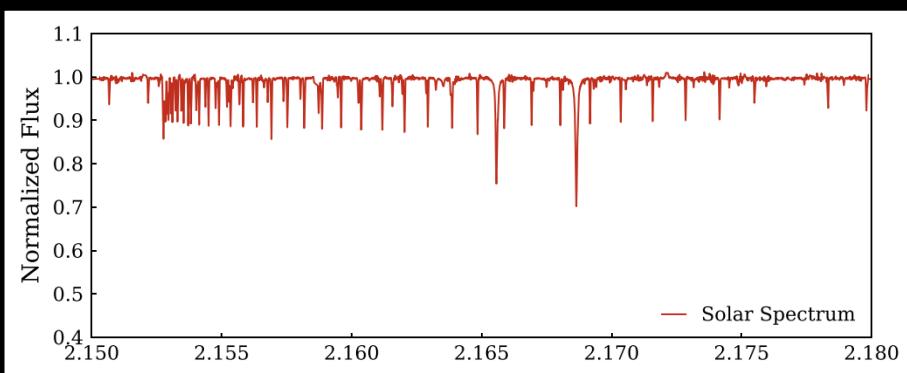
Waisberg et al. 2018

# Medium-term goal: The spin of Sgr A\*

## ELT Spectroscopy

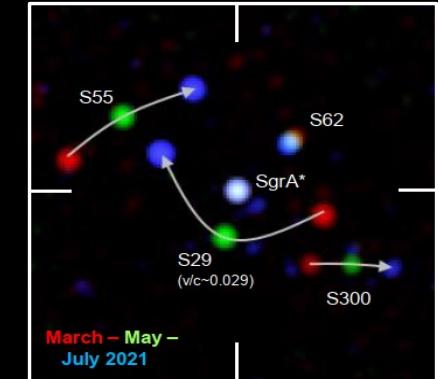


## Deep Interferometric Imaging

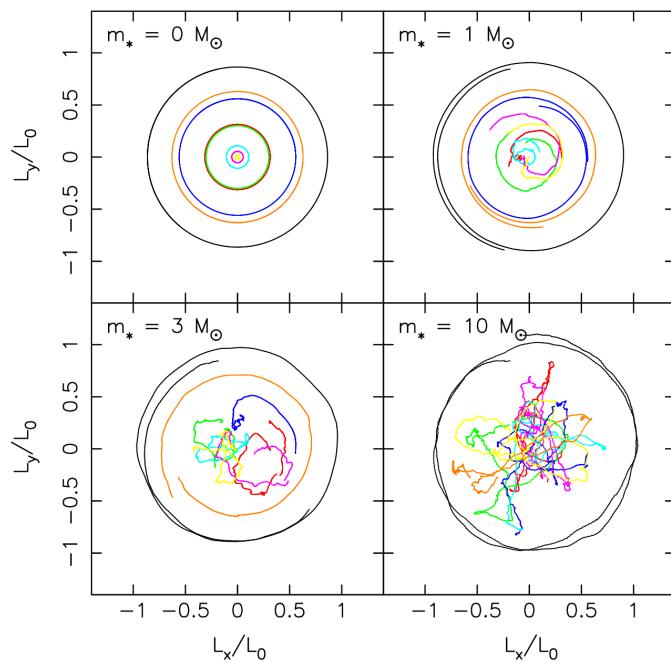


$\Delta v < 1 \text{ km/s}$

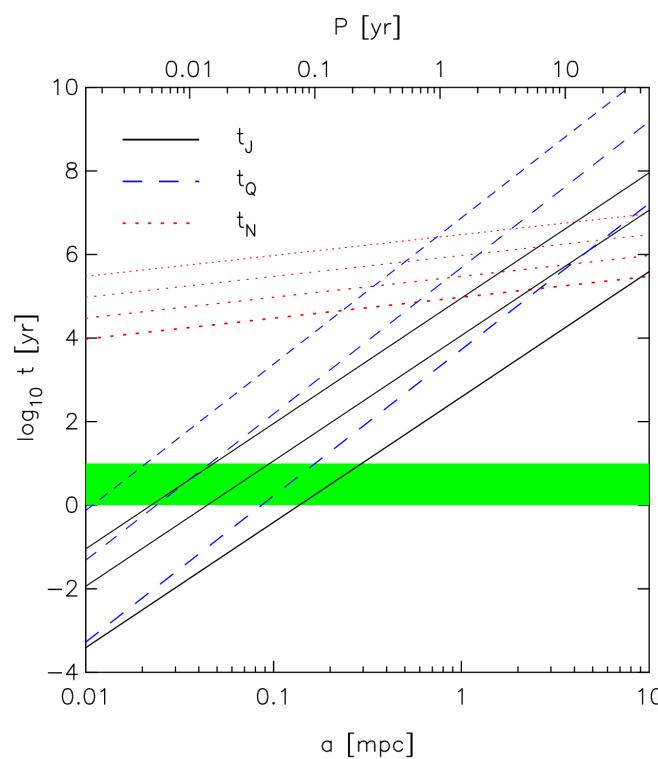
$mK > 21$



# Theoretical arguments were too pessimistic

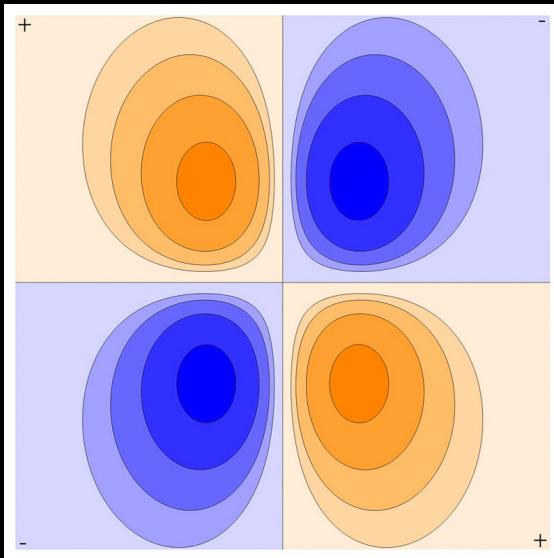


Merritt+ 2010



- arguments based on time-averaged changes
- but observers see “abrupt” changes
- “if a change happens at pericenter, it was the MBH”

# Longer term goal: No-hair theorem



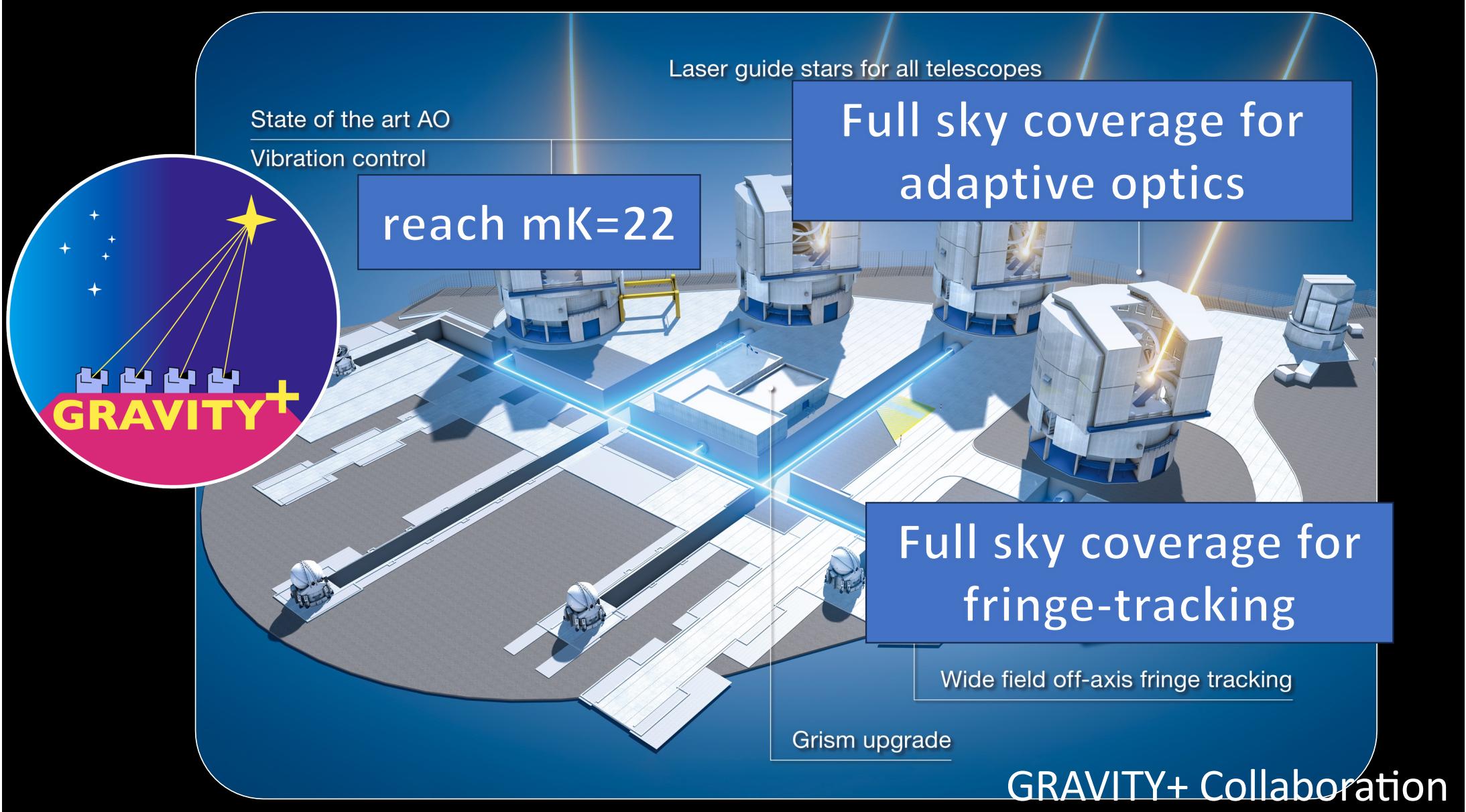
Will 208

$$\begin{aligned}
 \boldsymbol{a} = & -\frac{M\boldsymbol{x}}{r^3} + \frac{M\boldsymbol{x}}{r^3} \left( 4\frac{M}{r} - v^2 \right) + 4\frac{M\dot{r}}{r^2} \boldsymbol{v} \\
 & - \frac{J}{r^3} [2\boldsymbol{v} \times \hat{\boldsymbol{J}} - 3\boldsymbol{m} \times \hat{\boldsymbol{J}}] \\
 & + \frac{3Q_2}{2r^4} [5\boldsymbol{n}(\boldsymbol{n} \cdot \hat{\boldsymbol{J}})^2 - 2(\boldsymbol{n} \cdot \hat{\boldsymbol{J}})\hat{\boldsymbol{J}} - \boldsymbol{n}],
 \end{aligned}$$

In general relativity:  
 $Q_2 = -J^2/M$

Quadrupole moment:  
observational quantity

Rad err [km/s]	Schwarz- schild	Lense- Thirring	Lensing & Delay	Quad- rupole
10	0.6%	3%	-	-
1	0.4%	0.7%	35%	-
0.1	0.02%	0.04%	4%	11%

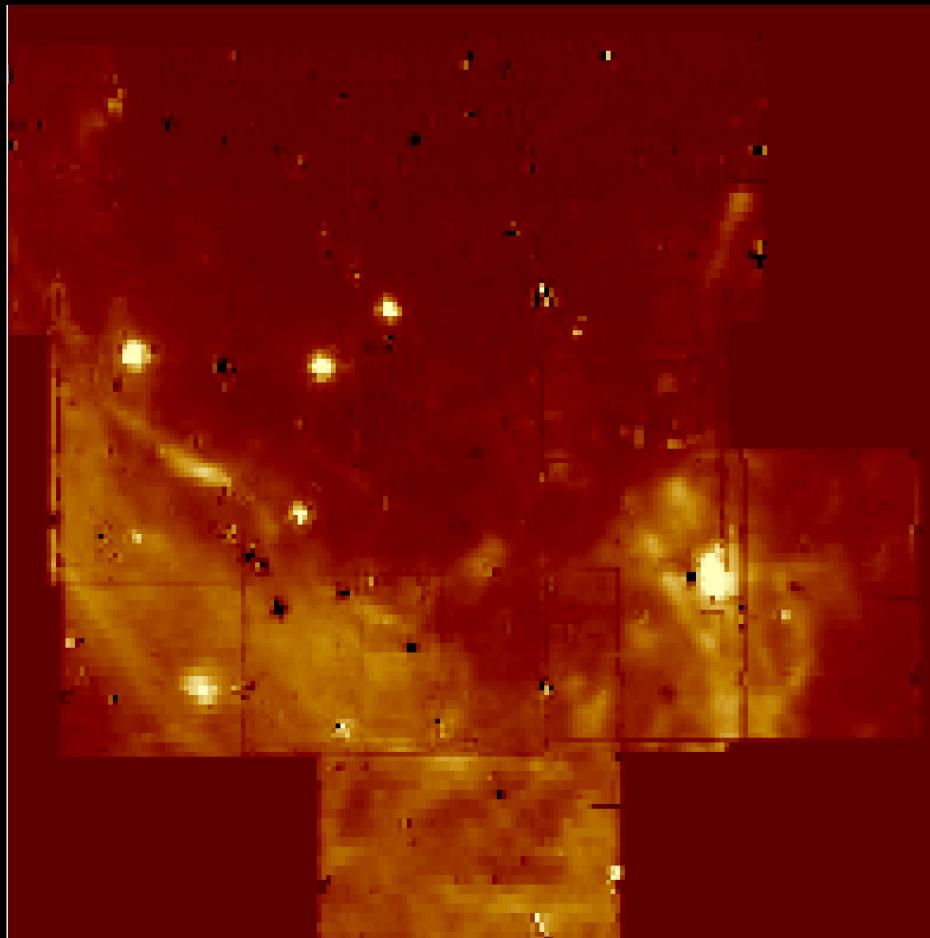
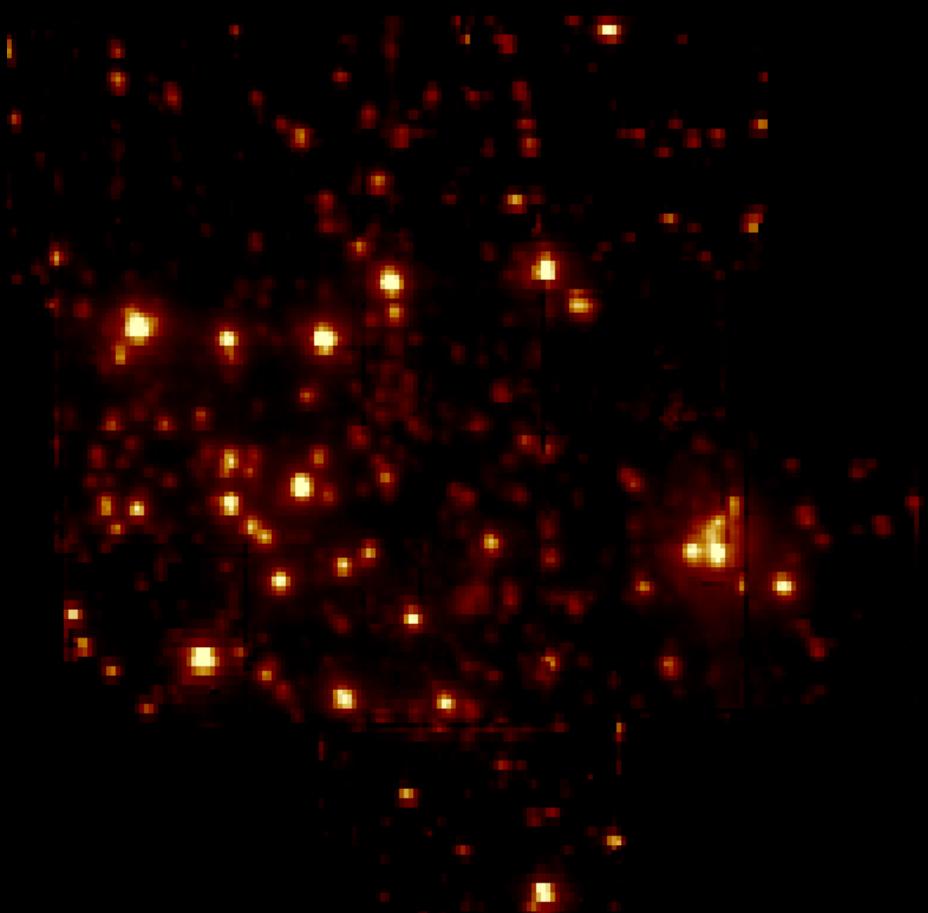


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  - SgrA\* flares
  - A funny gas cloud

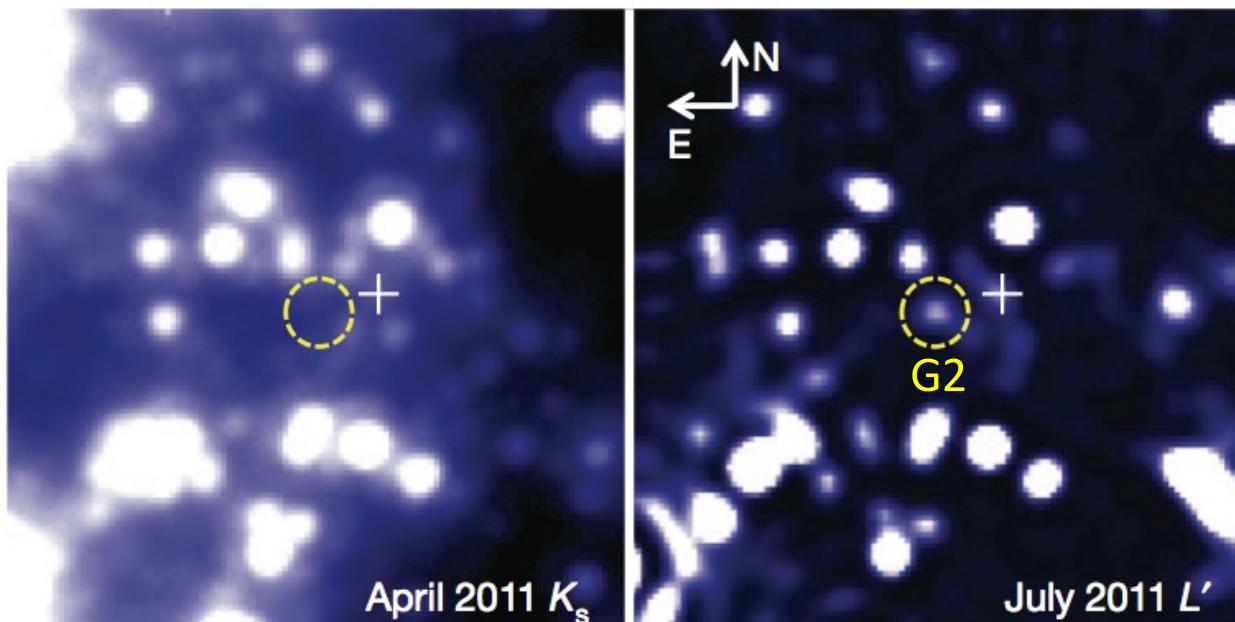
# The Galactic Center is gas rich

ERIS, 100mas, 2.166 $\mu$ m, R=11200



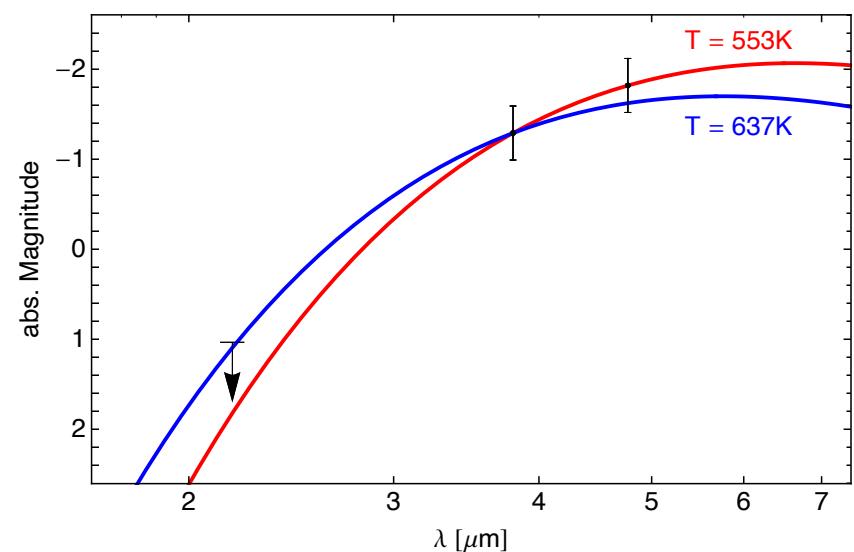
# A cool object: G2 - $T = 600$ K

Gillessen et al. 2012



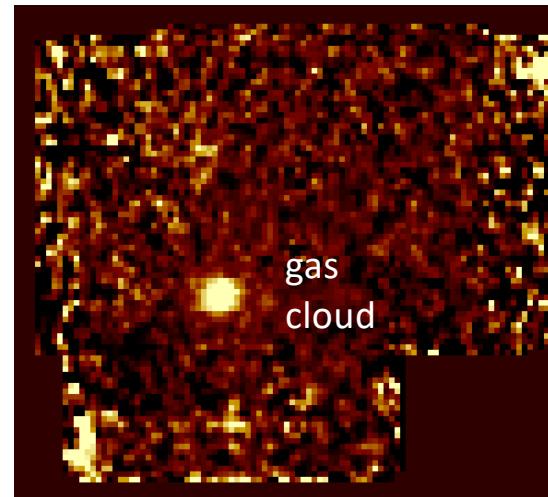
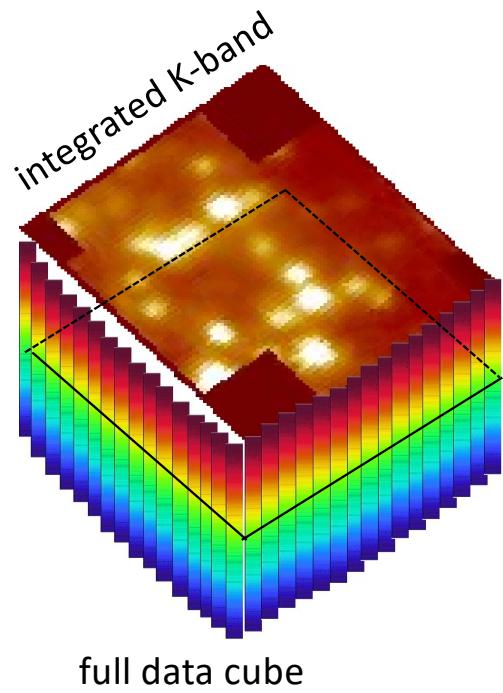
2.15  $\mu\text{m}$

3.8  $\mu\text{m}$



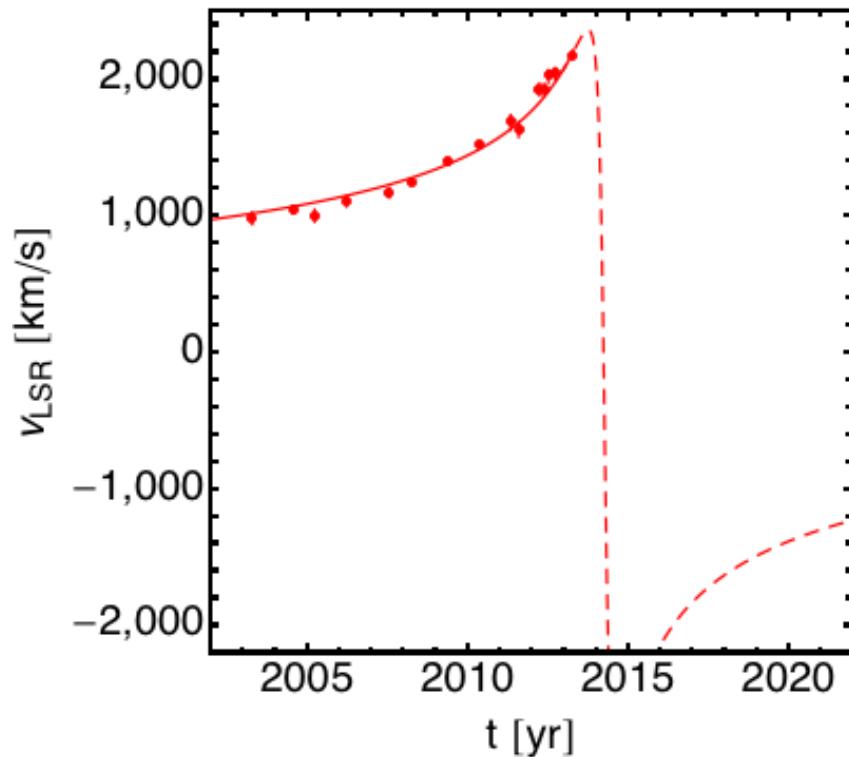
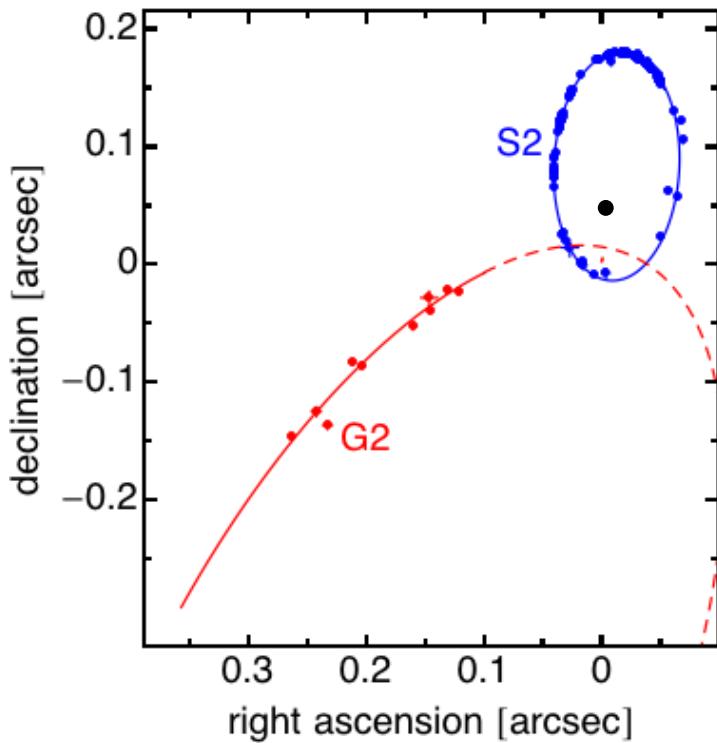
# G2 is a dusty, ionized gas cloud

Integral-field spectroscopy: SINFONI



channel map at  
 $\text{Br}-\gamma + 1300 \text{ km/s}$   
( $2.175 \mu\text{m}$ )

# G2 has an interesting orbit



- pericenter 2014 @  $2000 R_s$
- highly eccentric

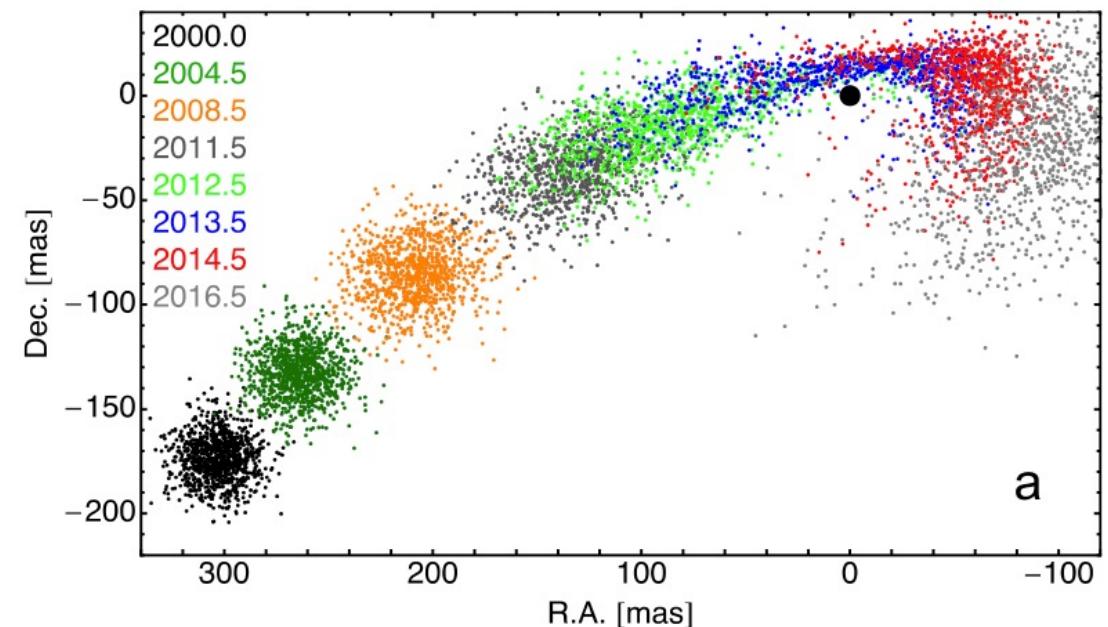
Phifer et al. 2013  
Gillessen et al. 2013

# For the gas dynamics the source model does not matter

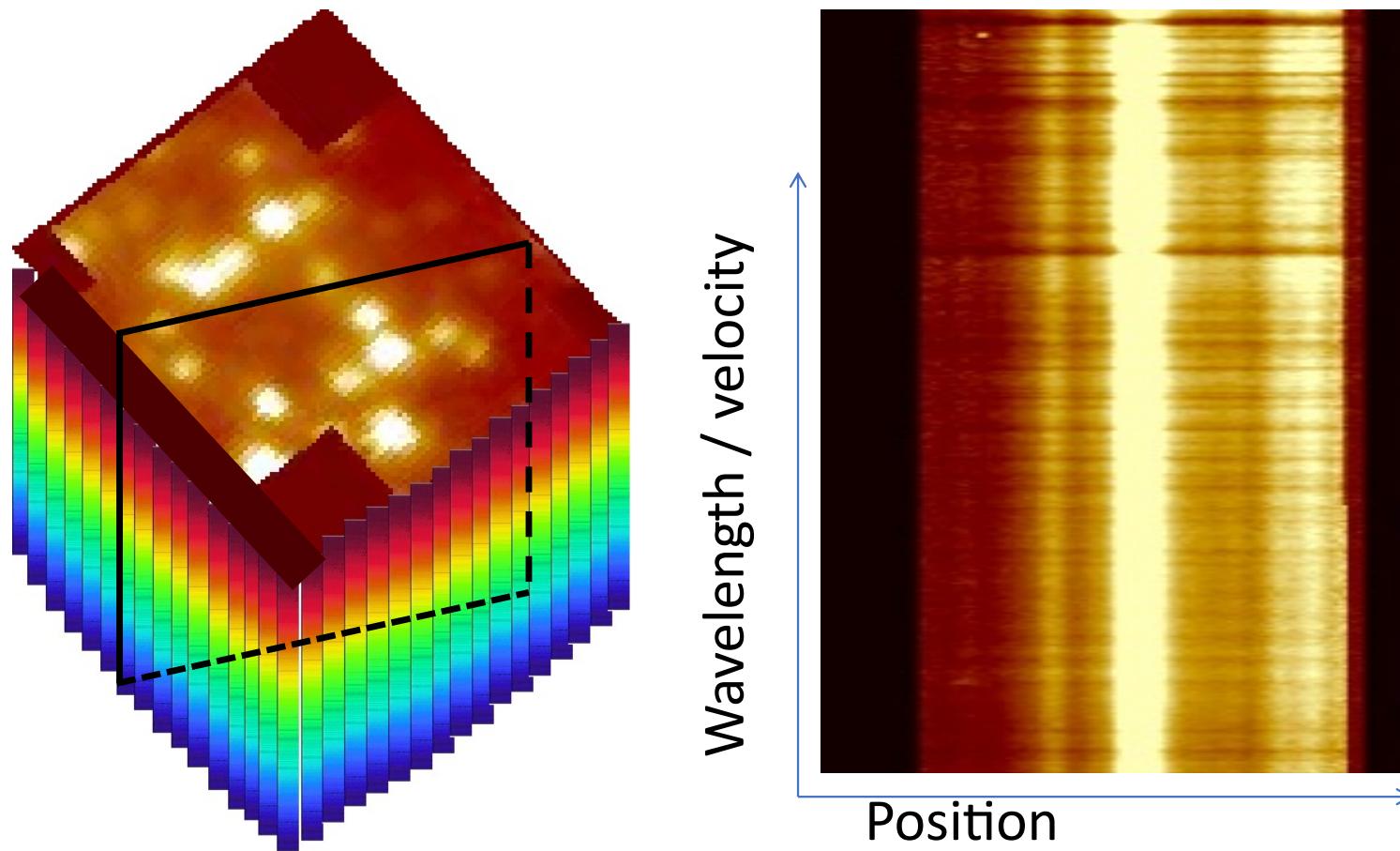
**Gravitationally binding G2's  
gas in the tidal field of Sgr A\*:  
 $M \gg 100M_{\odot}$**

**The gas that we see  
cannot be bound to any  
central object**

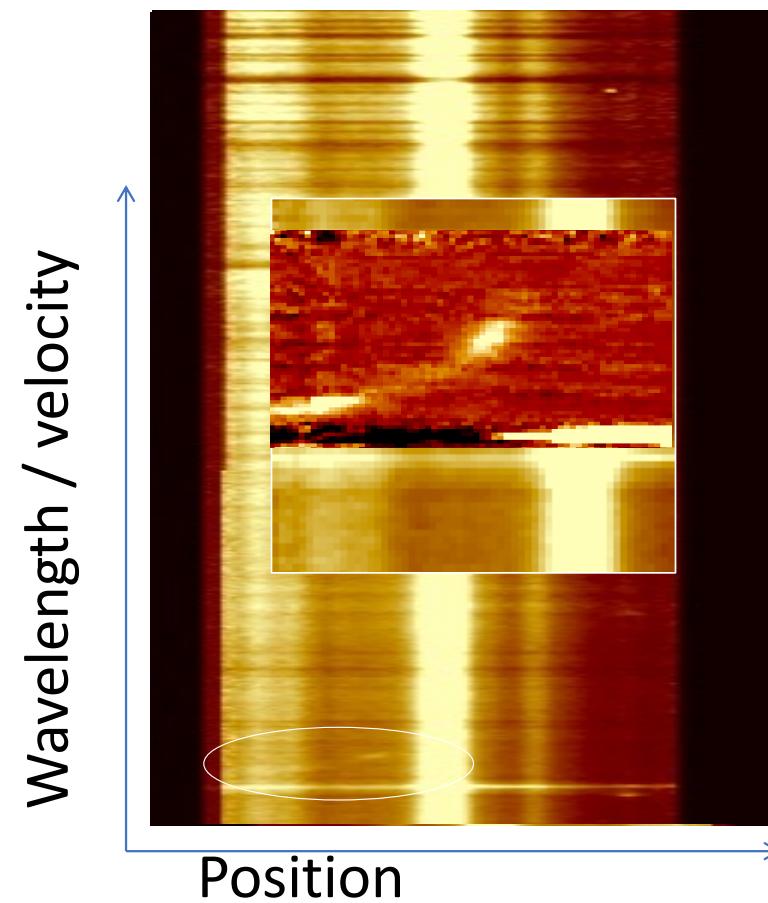
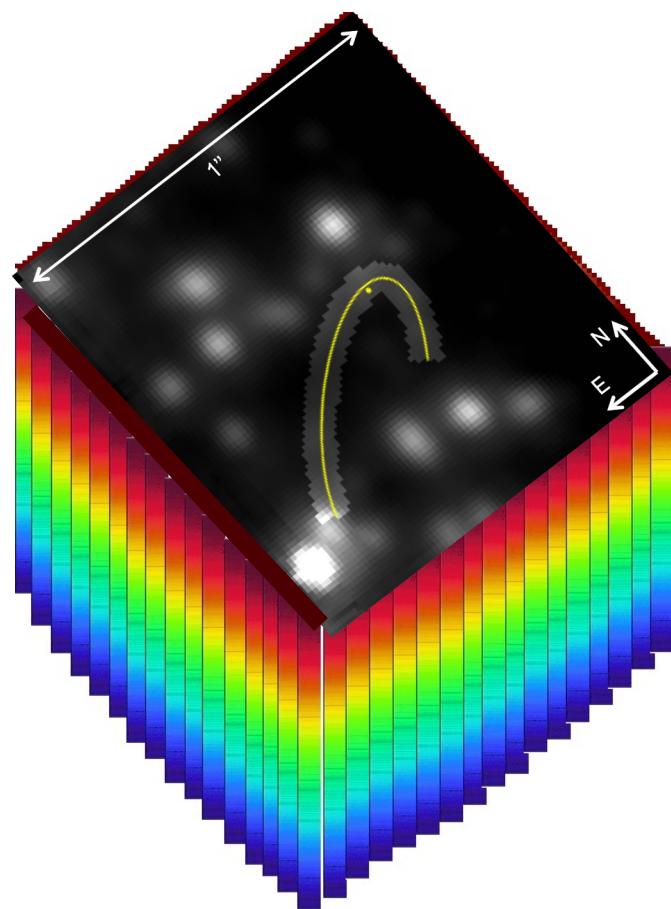
**We should see G2  
tidally evolve**



# Position – Velocity - Diagram

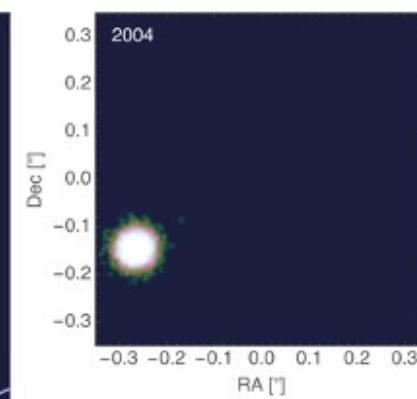
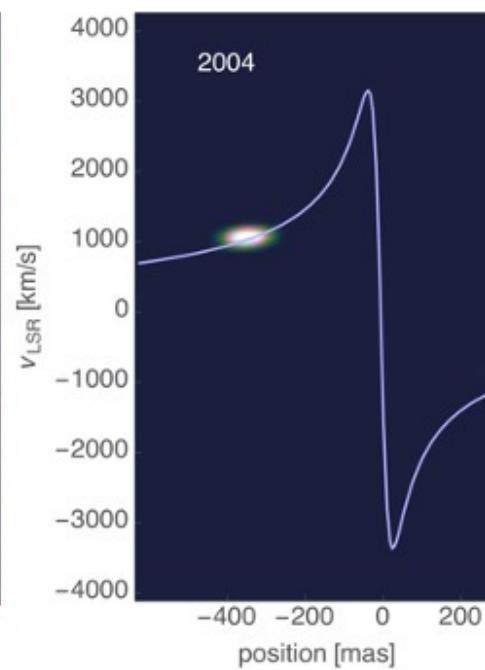
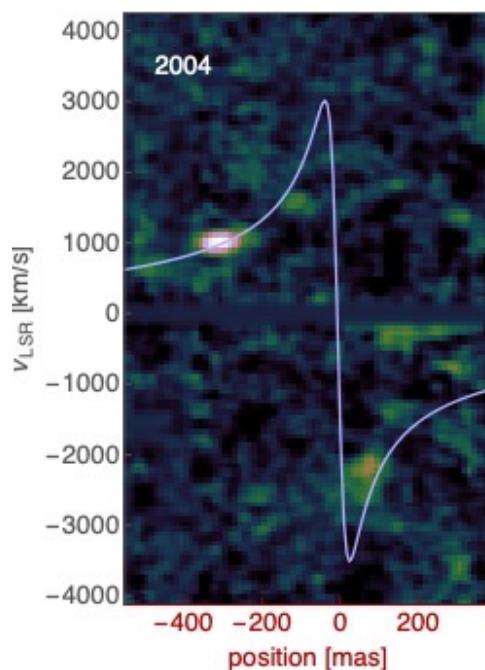


# Position – Velocity - Diagram



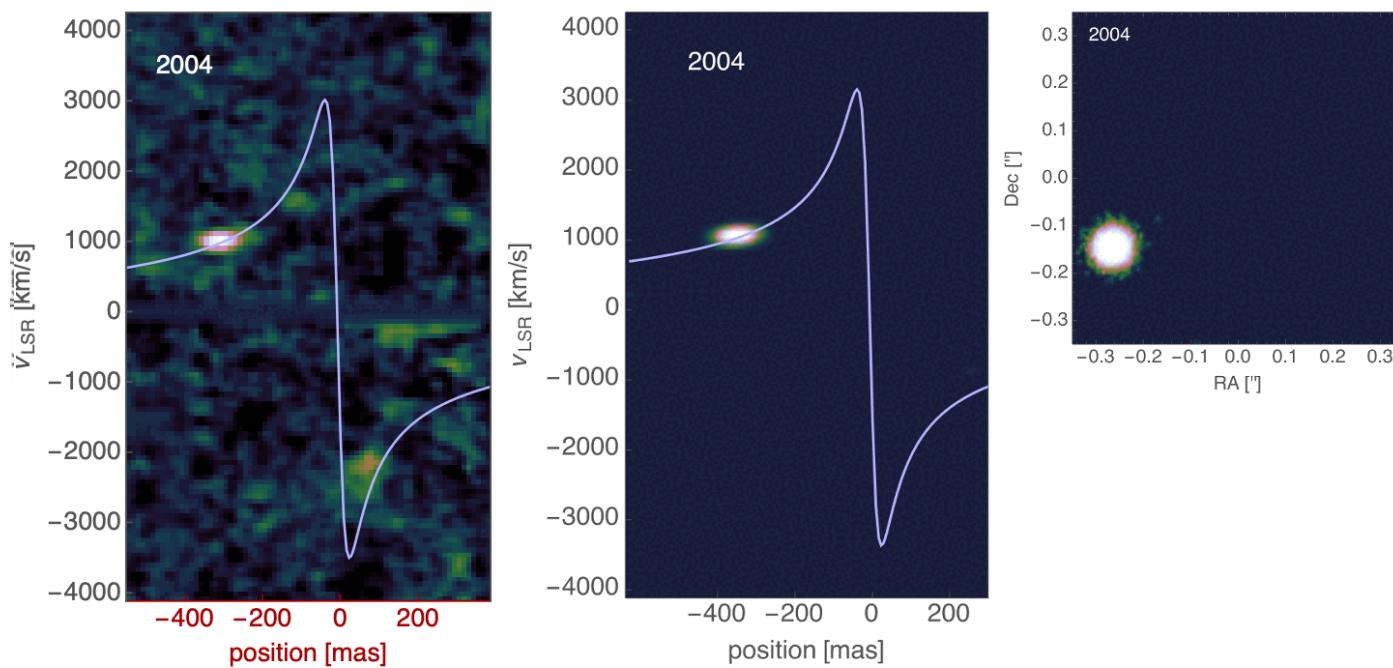
# Too beautiful to watch: The tidal evolution of G2

2004



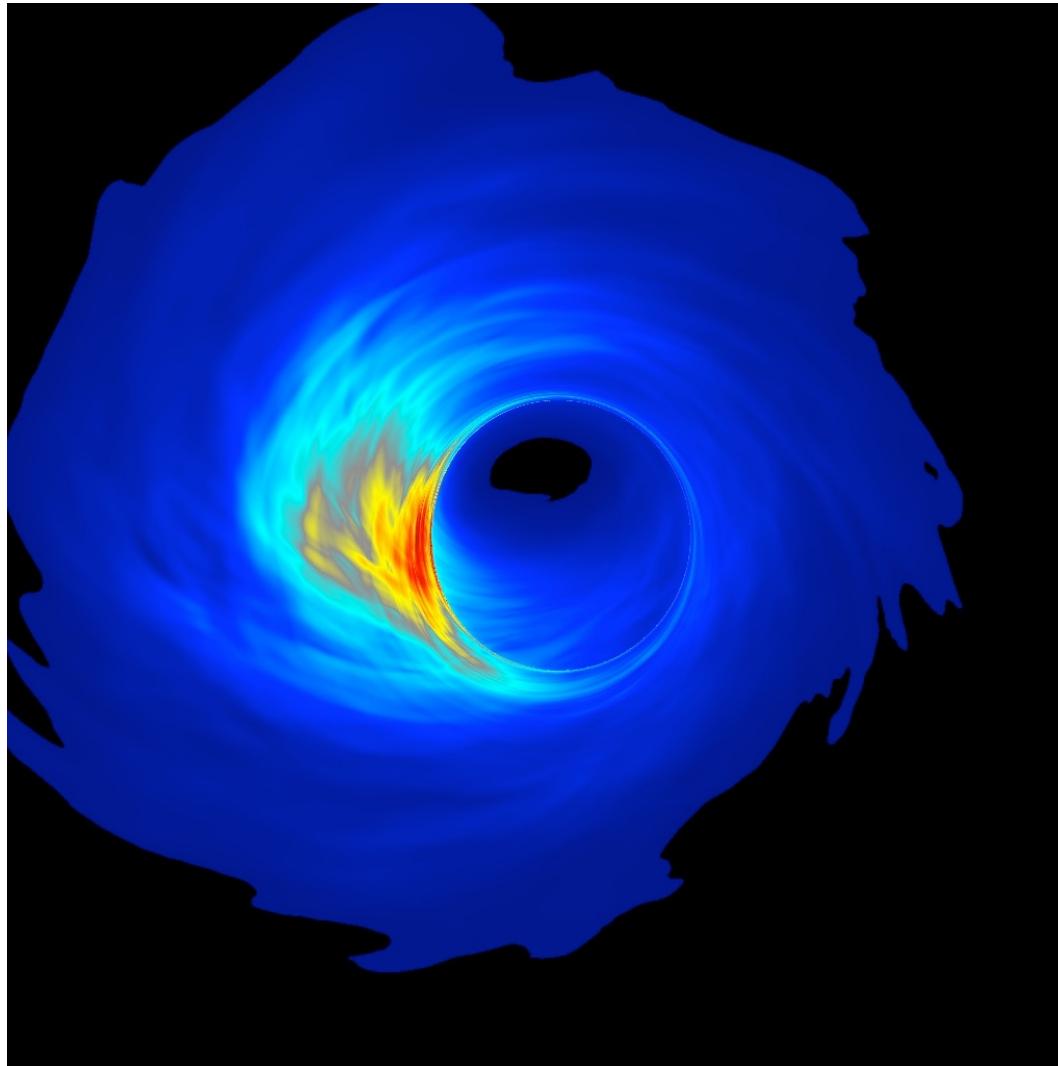
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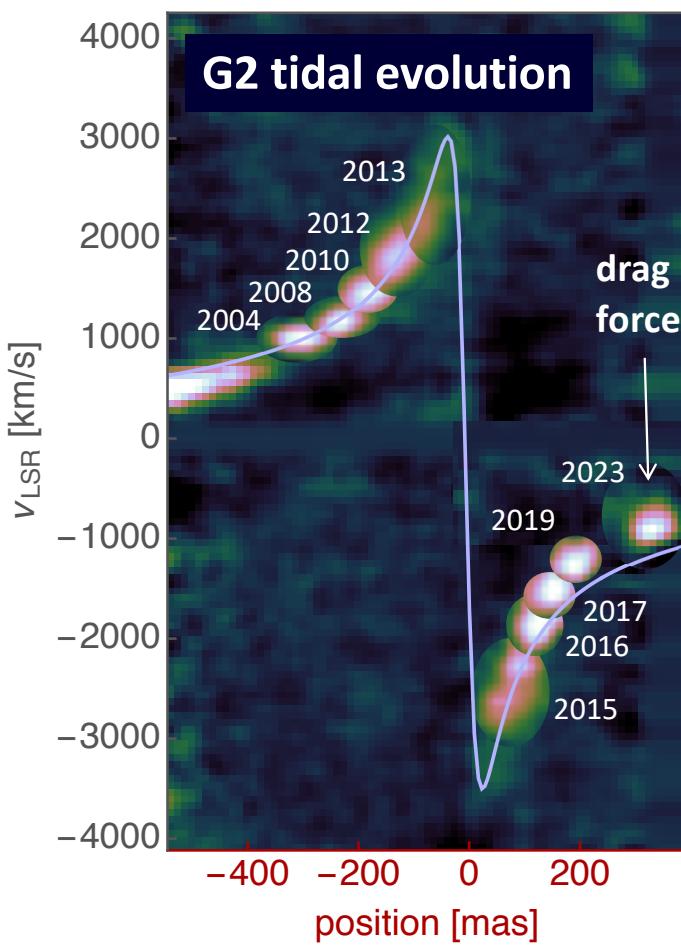




# Missing: Interaction with Accretion Flow



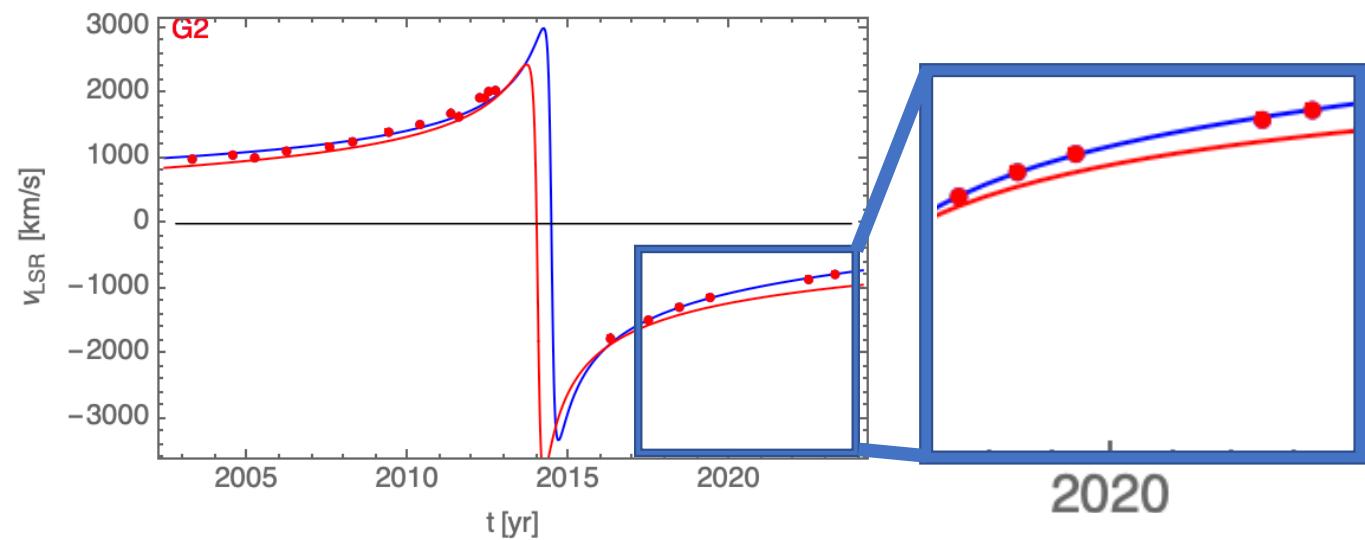
# The G2 data since 2017 show a deceleration



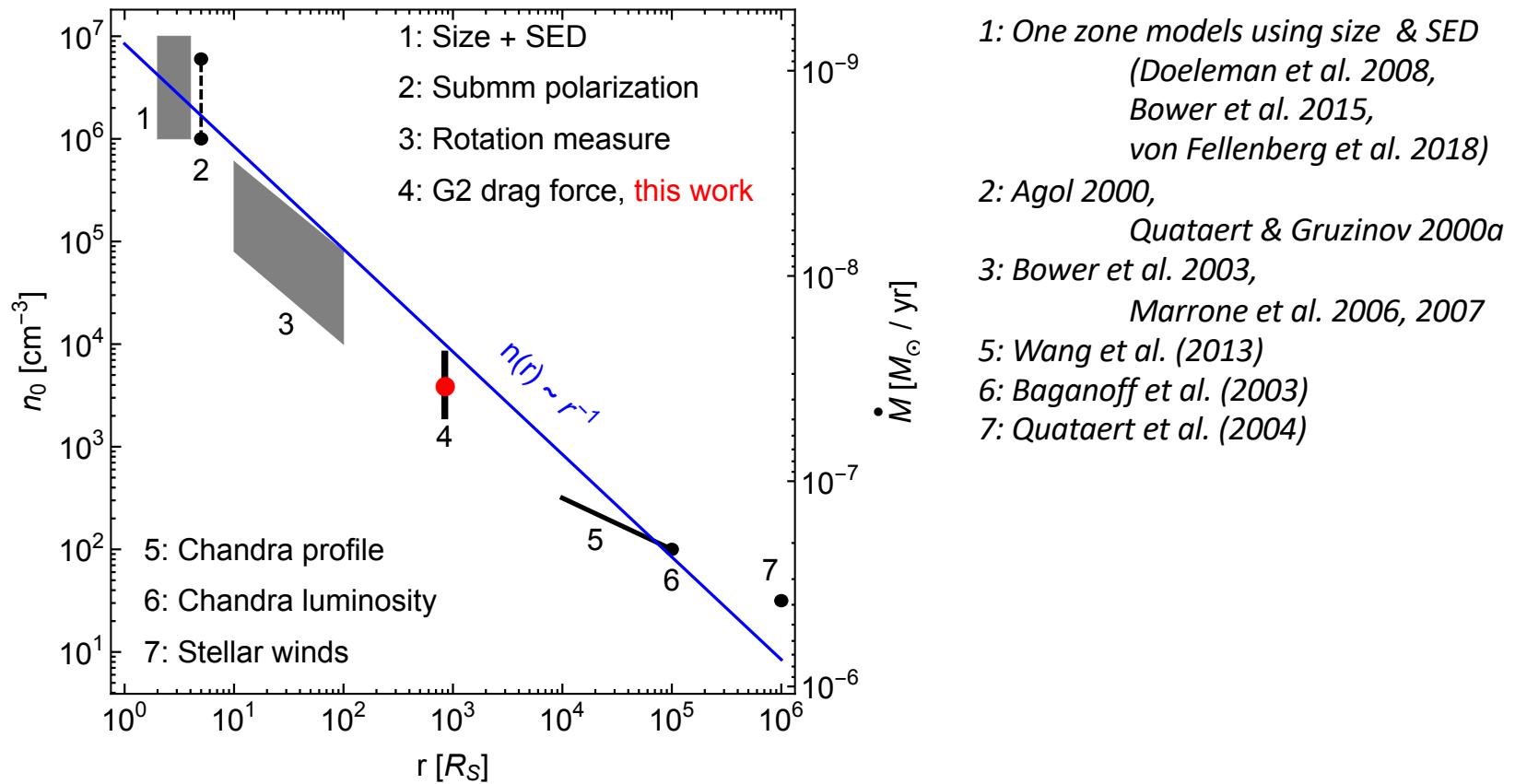
$$F_{\text{drag}} \propto s^2 \rho(r) v^2$$

↓

$$\rho(r) \propto r^{-1}$$



# A very valuable constraint of the accretion flow density



# Outline

- Part I        The mass of Sgr A\*
  - Astronomical black holes ..... stellar and massive
  - Measuring mass ..... orbits, Jeans-modeling
  - Infrared observations ..... see through the dust
  - Adaptive Optics ..... see sharp – diffraction limit
  - Stellar Orbits ..... measure the 4.3 million  $M_{\text{sun}}$
- Part II      Errors, Fitting and all that
- Part III     Testing the black hole paradigm
  - The black hole nature of Sgr A\* ..... mass, size, position, faintness
  - Interferometry ..... breaks the diffraction limit
  - Testing G.R. in the Gal. Center ..... redshift, precession, ... spin?
  - SgrA\* flares ..... accretion flow geometry
  - A funny gas cloud ..... accretion flow density



Thanks to the  
**GRAVITY team**

