THERMAL HADRON YIELDS FROM A COUPLED-CHANNEL ANALYSIS

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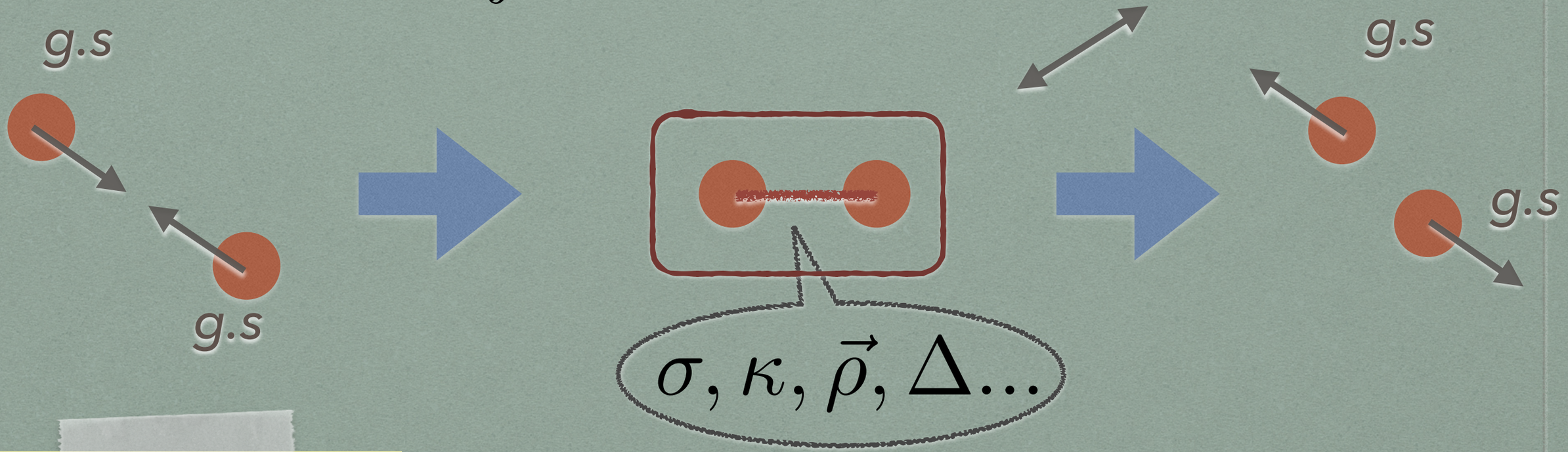
17-19.06.2024
PRESENT AND FUTURE PERSPECTIVES IN HADRON PHYSICS, FRASCATI

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE\, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \mathrm{tr}\, (\delta_E)\,.$$



+ repulsions

PWA
X
S-matrix thermo.

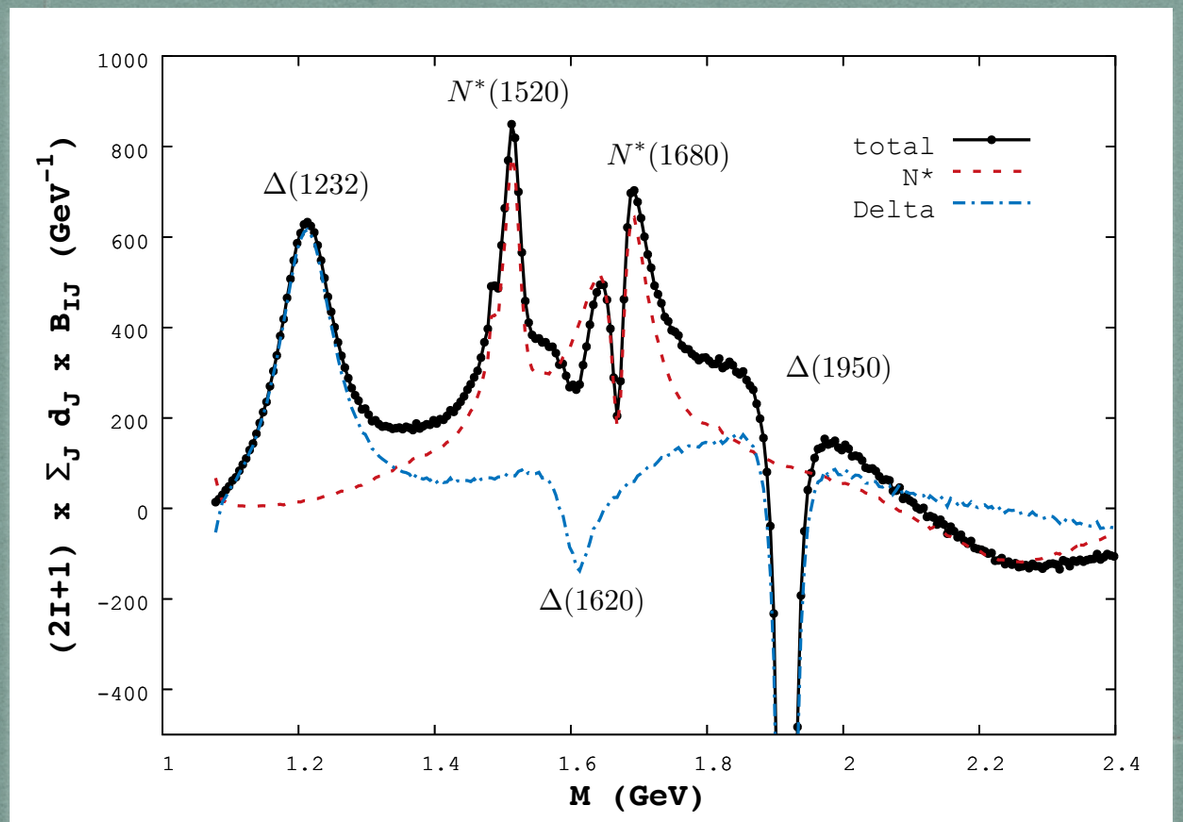
$$\delta \longrightarrow \mathcal{Q}(M) \equiv \frac{1}{2} \mathrm{Im}\, (\mathrm{tr} \ln\, \mathrm{S})$$

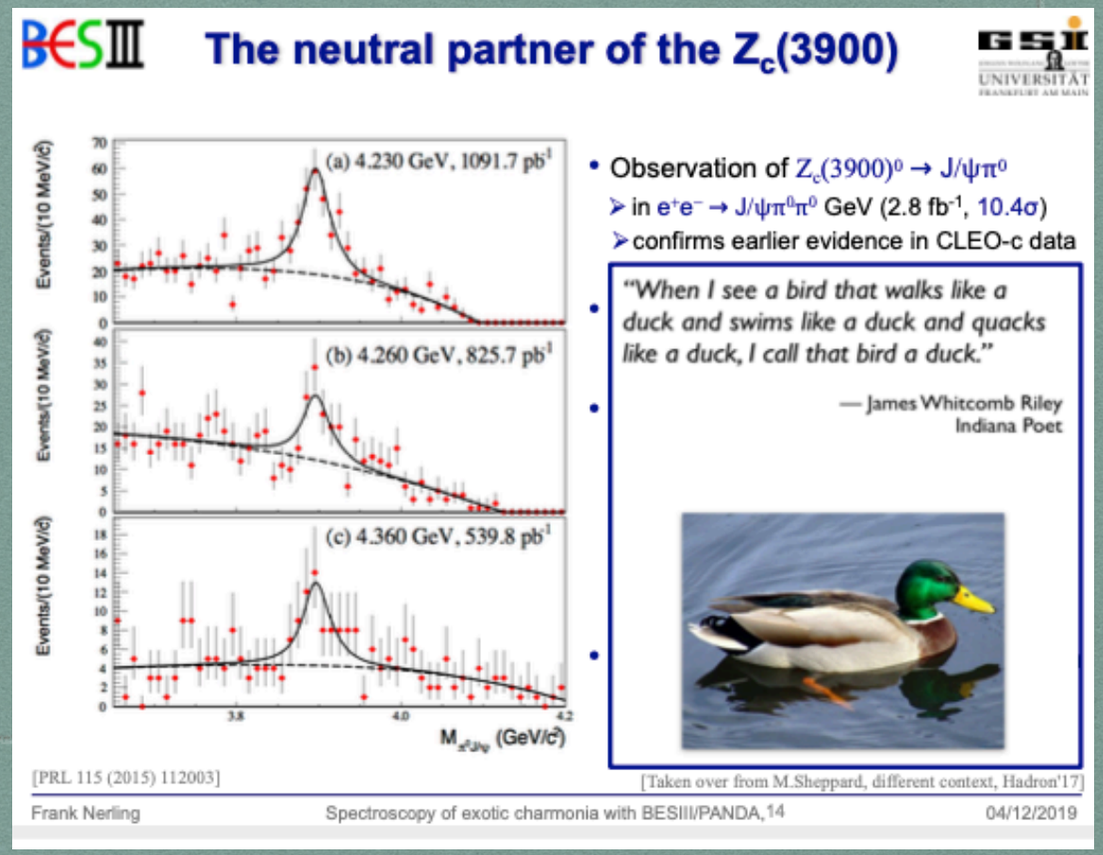
RESONANCES / EXCITATIONS VIA SCATTERING STATES

- broad /overlapping resonances
- molecular states
- threshold effects /cusps

non-resonant interactions: +/-

$$\pi N \to \Delta \to \pi N$$

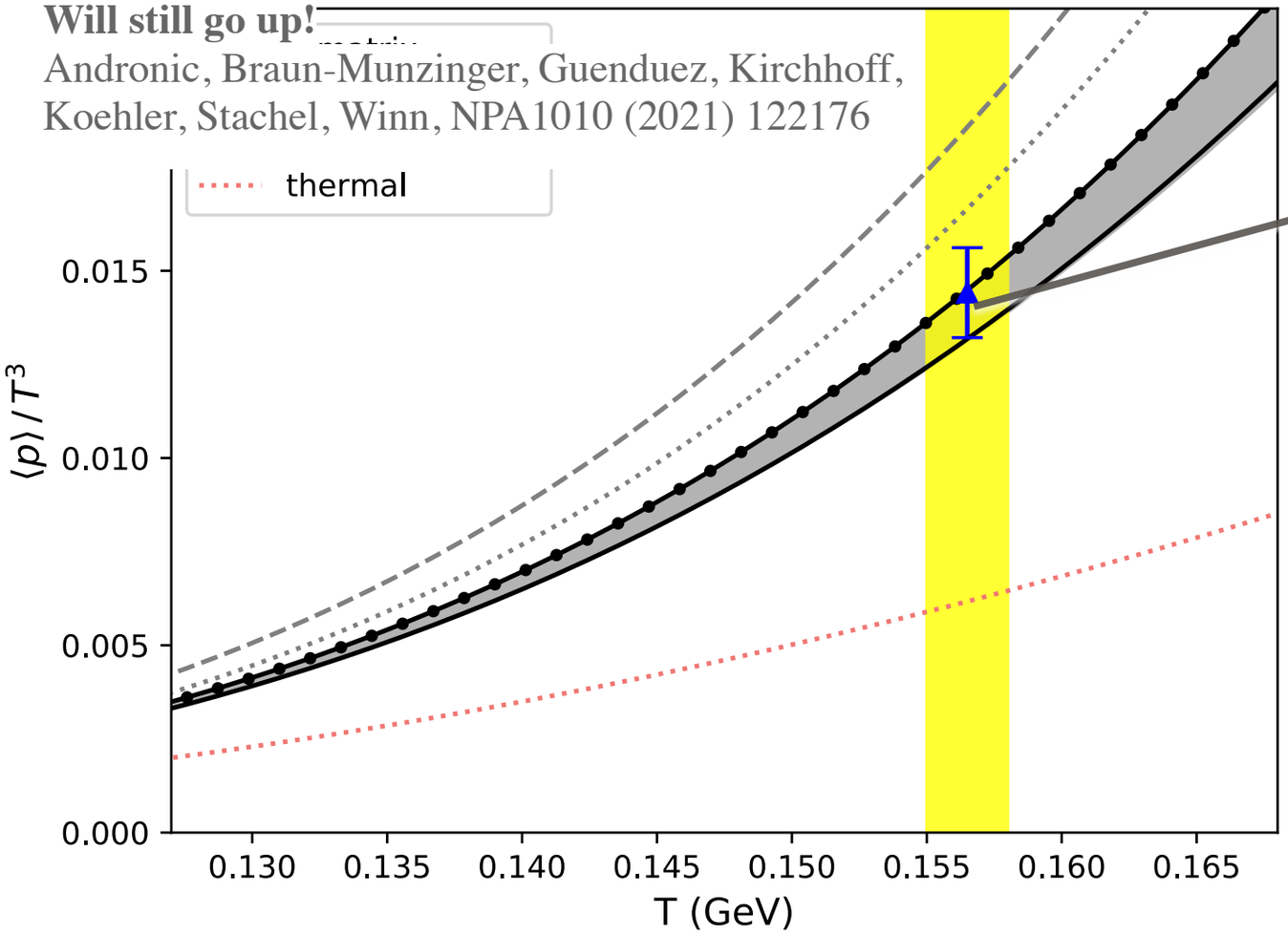




KEY RESULTS

Will still go up!

Andronic, Braun-Munzinger, Guenduez, Kirchhoff, Koehler, Stachel, Winn, NPA1010 (2021) 122176



ALICE proton yield

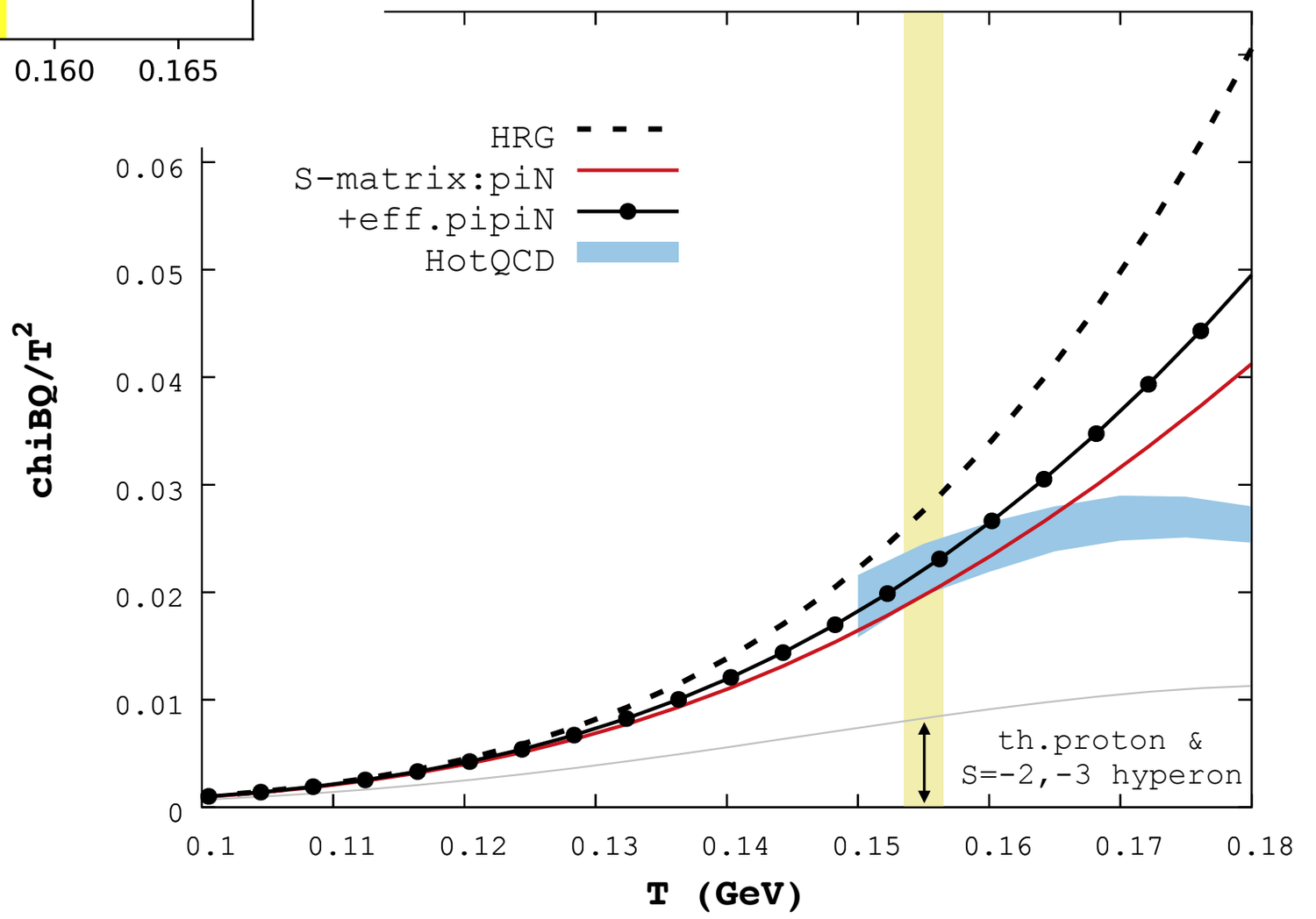
Pb-Pb @ 2.76 TeV

thermal model est.

LQCD result on chiBQ

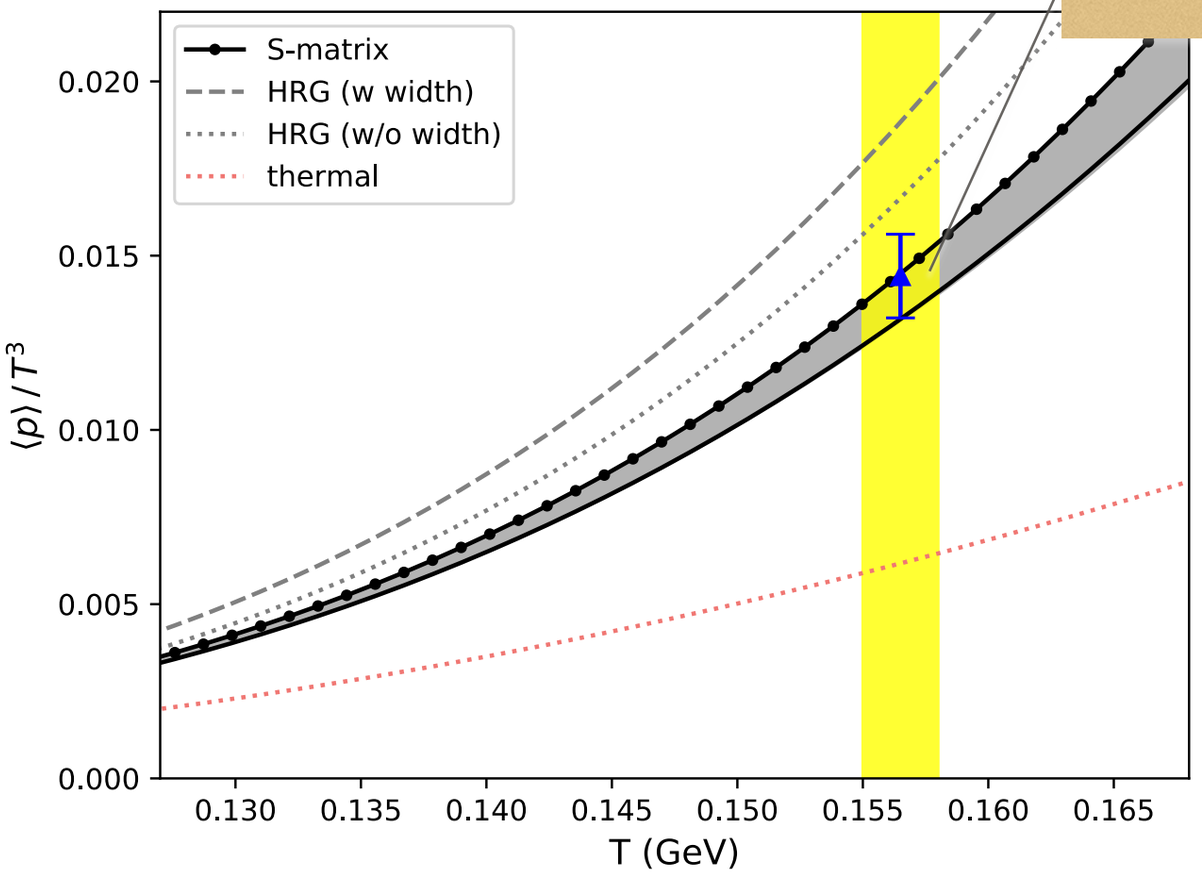
A. Bazavov, et al.,
Phys. Rev. D 86 (2012) 034509.

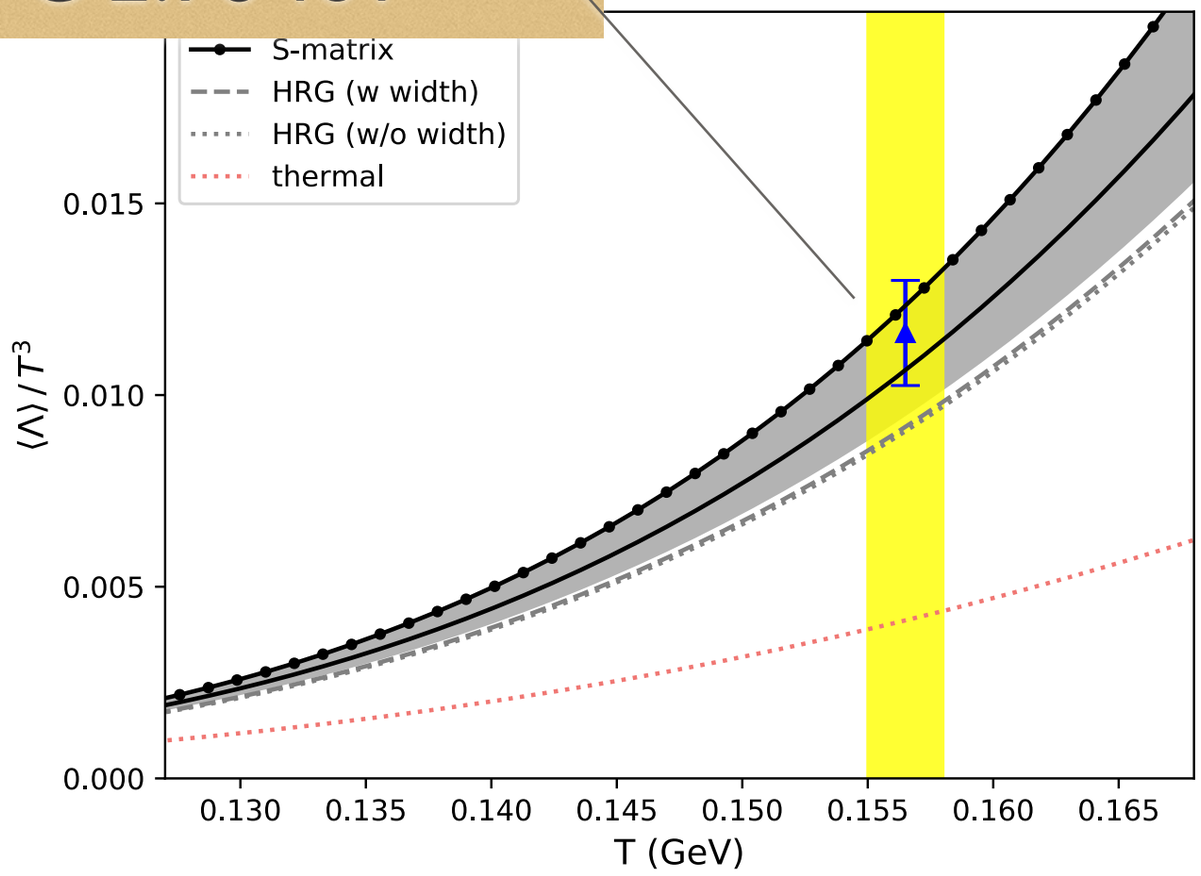
see also
Bellwied et al.
Phys. Rev. D 101, 034506 (2020)



S-matrix VS HRG

ALICE proton yield
@ 2.76 TeV





GW INS

Jefferson Lab
Thomas Jefferson National Accelerator Facility

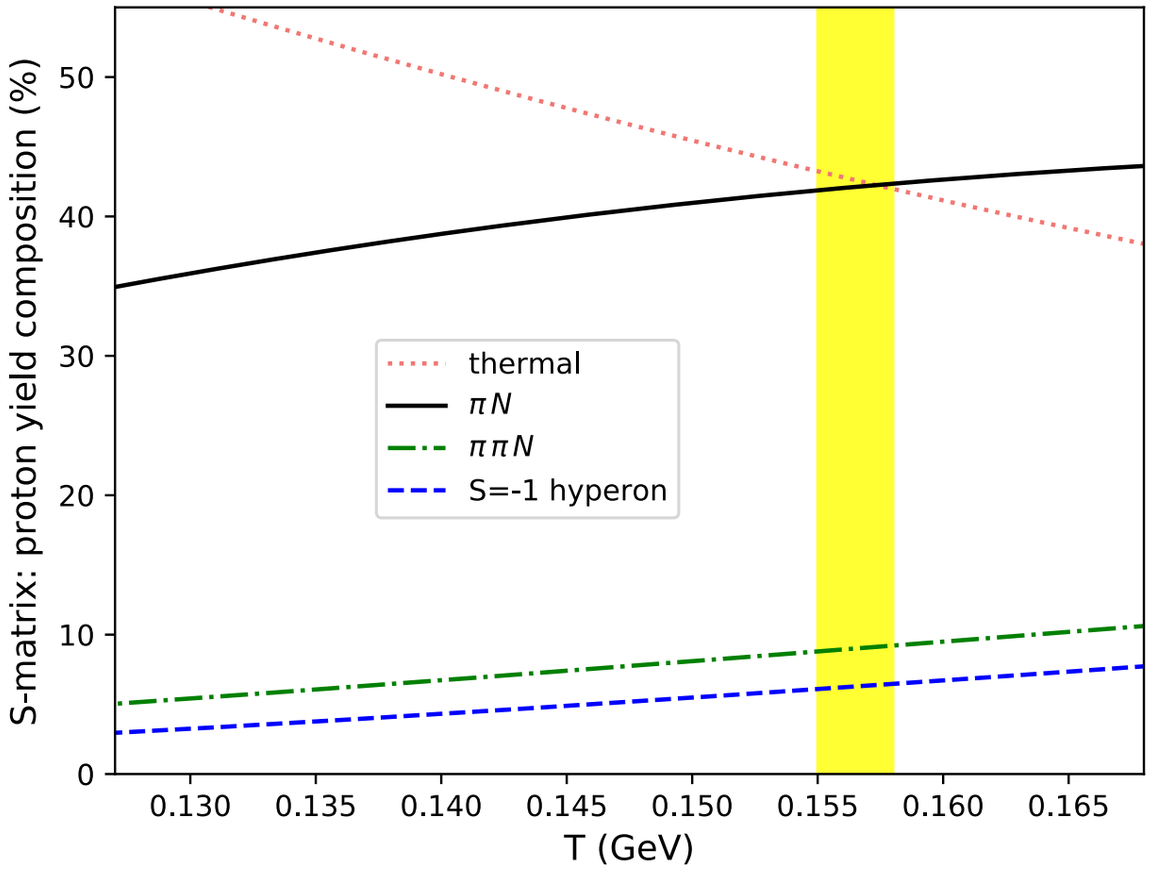
piN phase shifts
pipiN BGs
hyperons

Coupled-Channel model:

$\bar{k}N, \pi\Lambda, \pi\Sigma, \ldots$

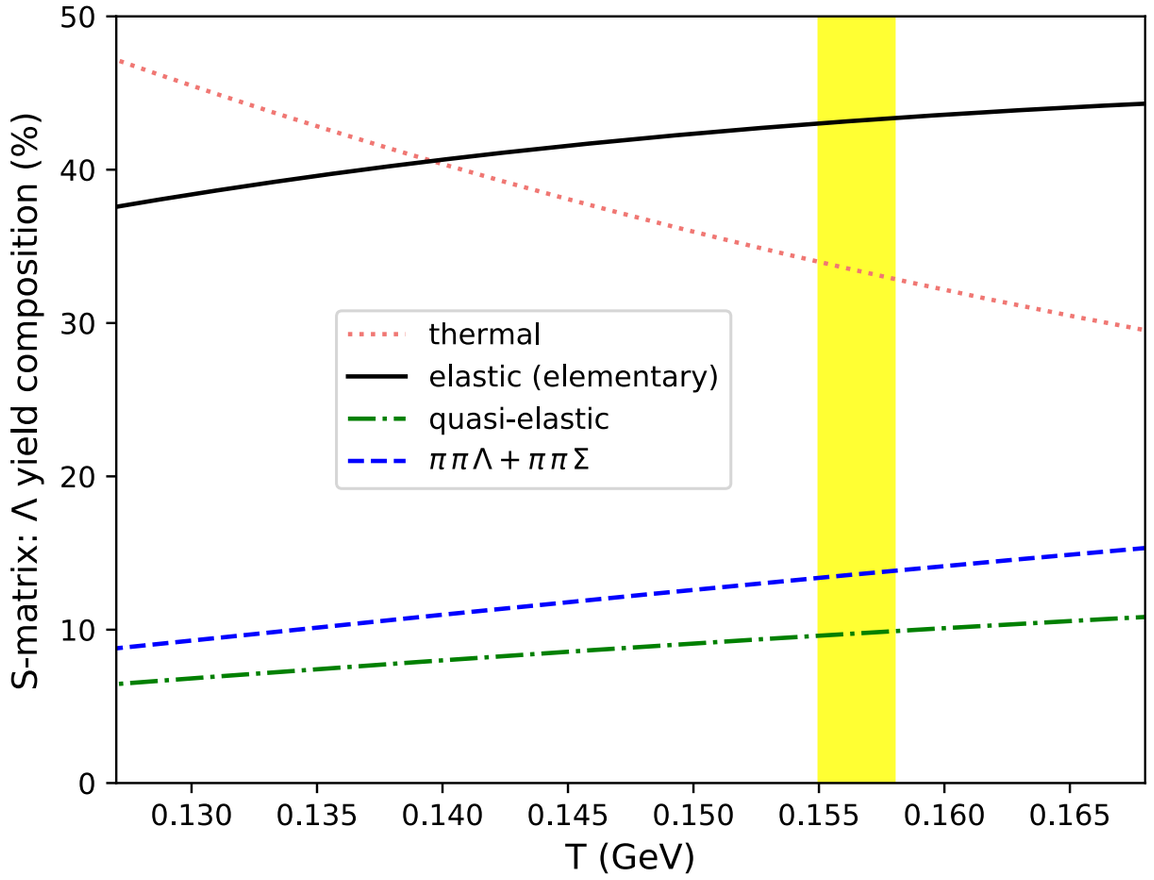
extra hyperon states
beyond PDG
unitarity BGs

consistent treatment of res and non-res. int.



SAID GWU

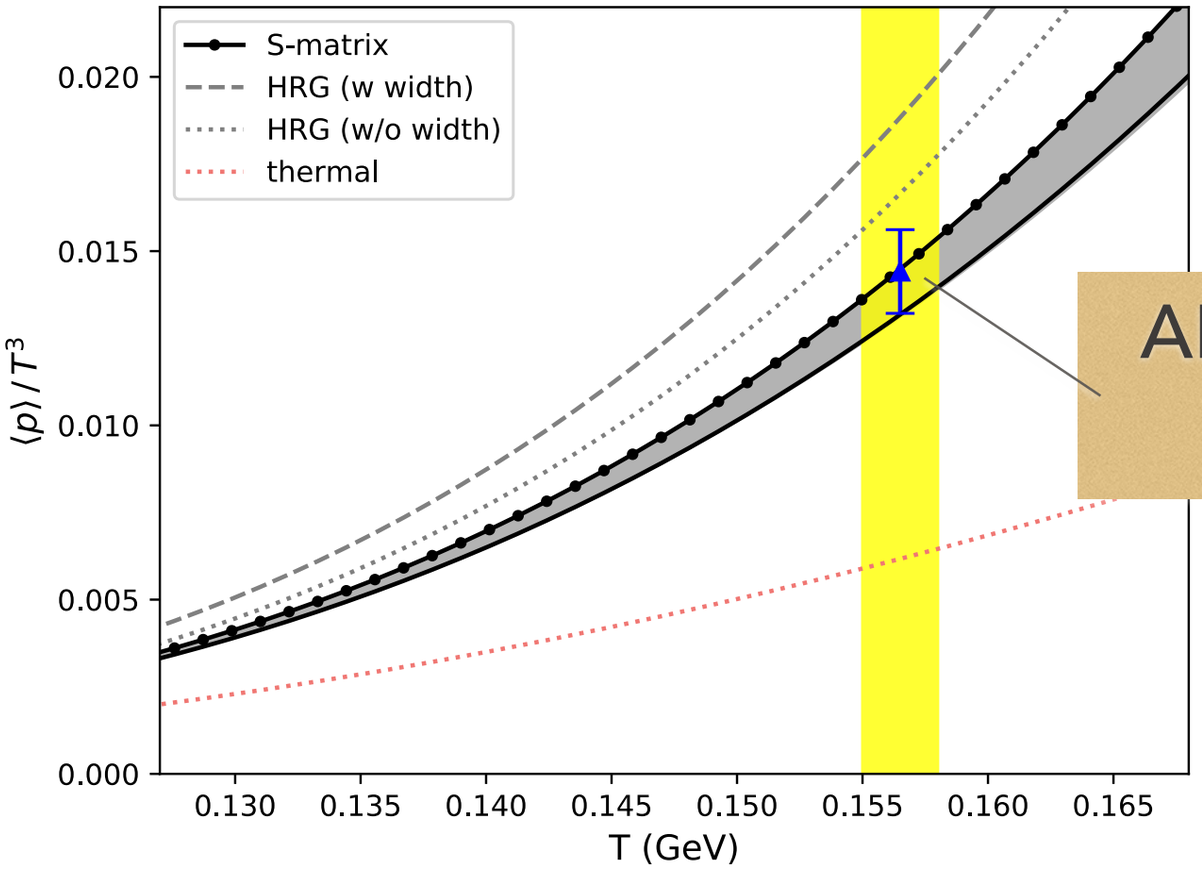
piN phase shifts
pipiN BGs
hyperons



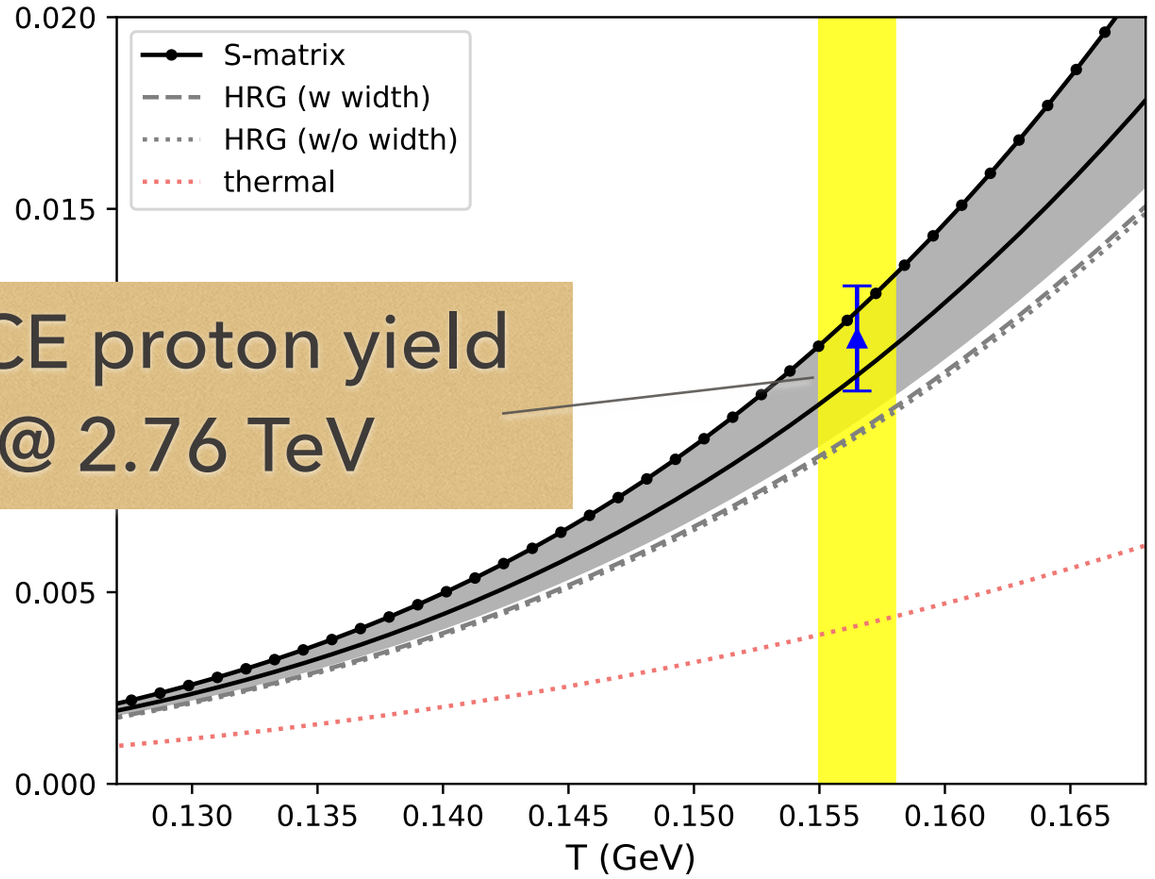
JPAC

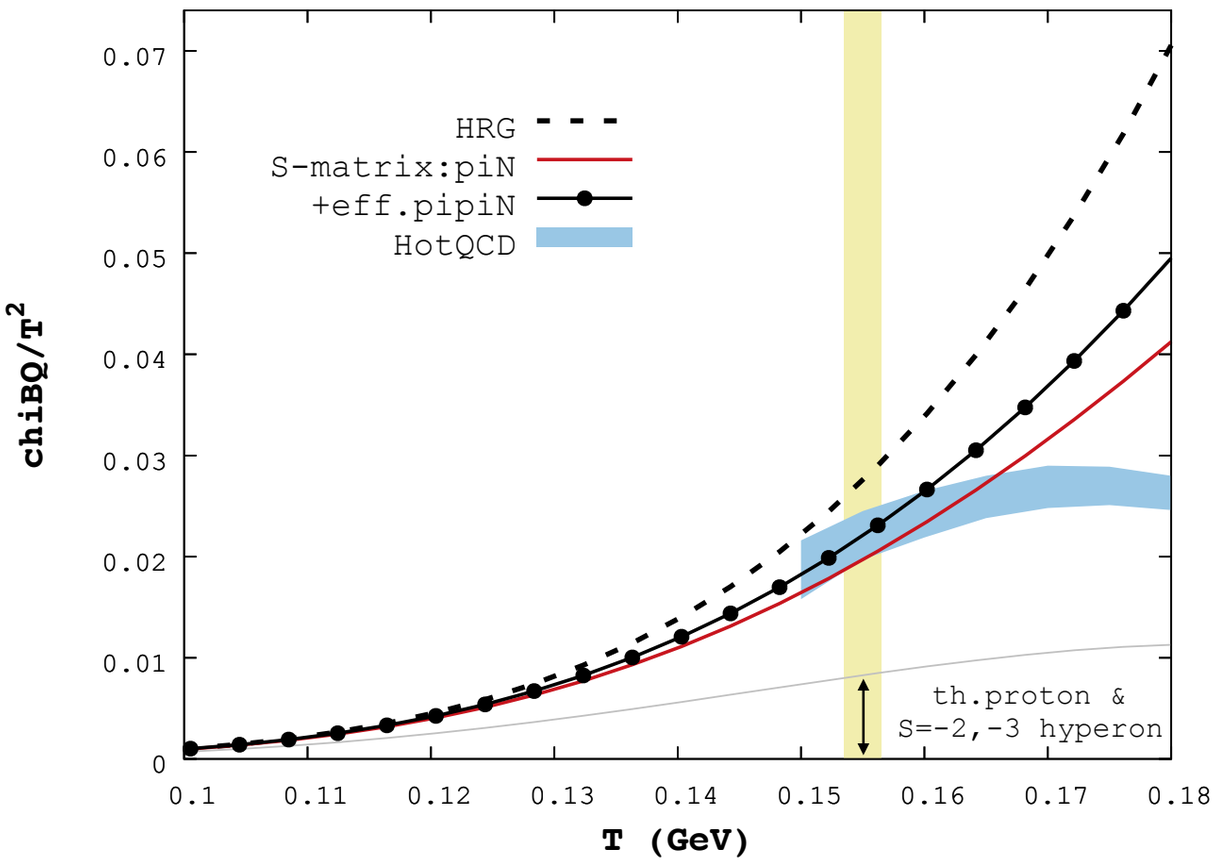
Coupled-Channel system:
$\bar{k}N, \pi\Lambda, \pi\Sigma, ...$
extra hyperon states
beyond PDG
unitarity BGs

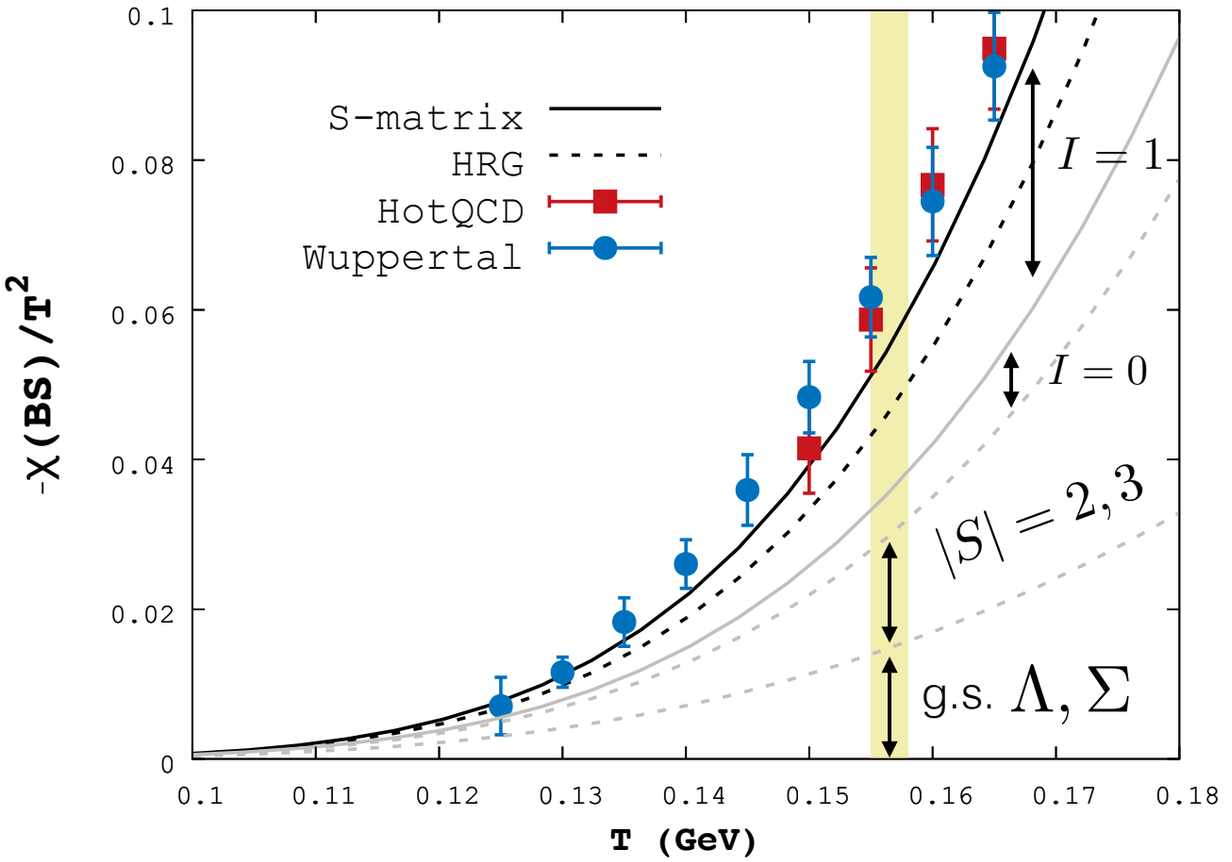
consistent treatment of res and non-res. int.

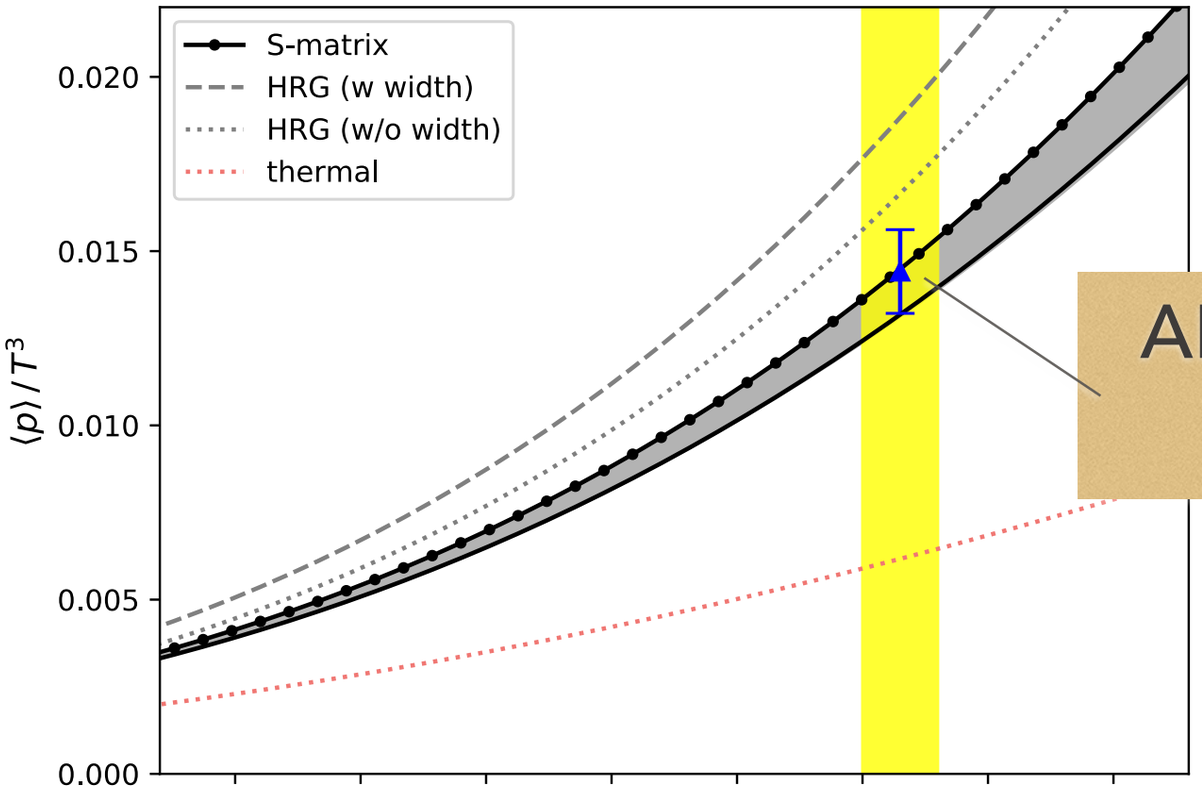


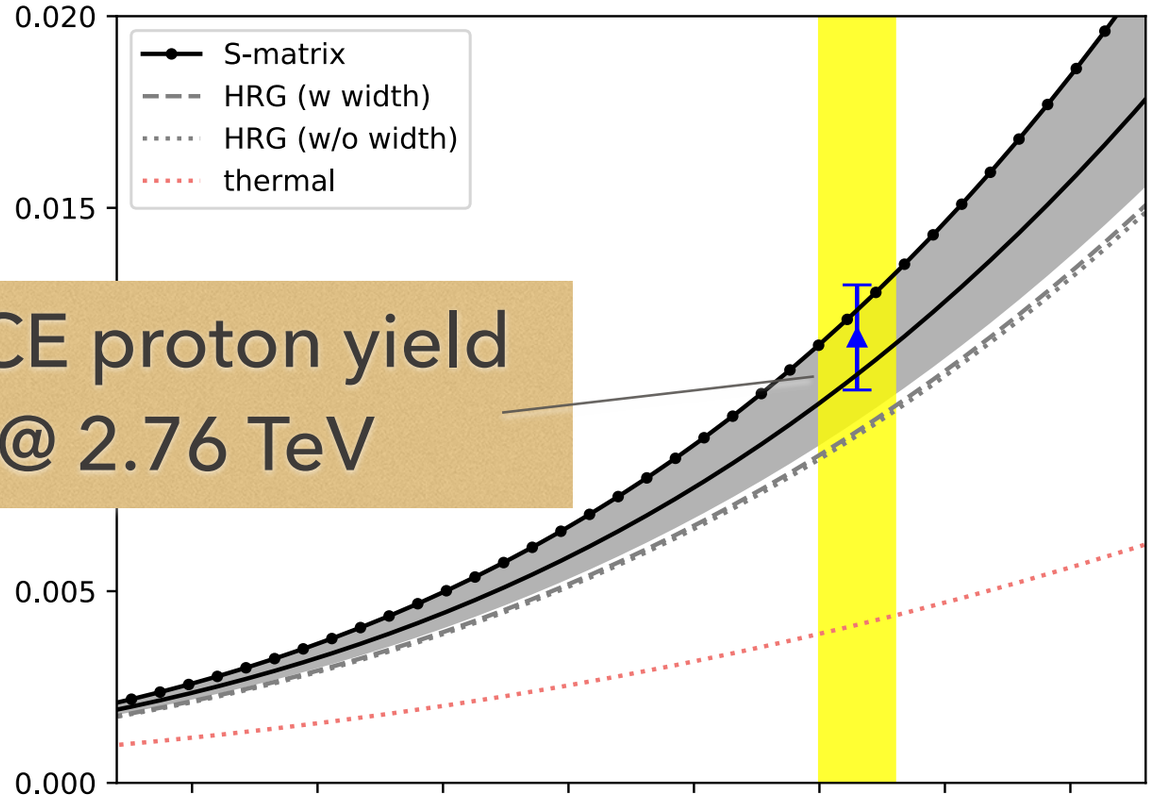
ALICE proton yield
@ 2.76 TeV





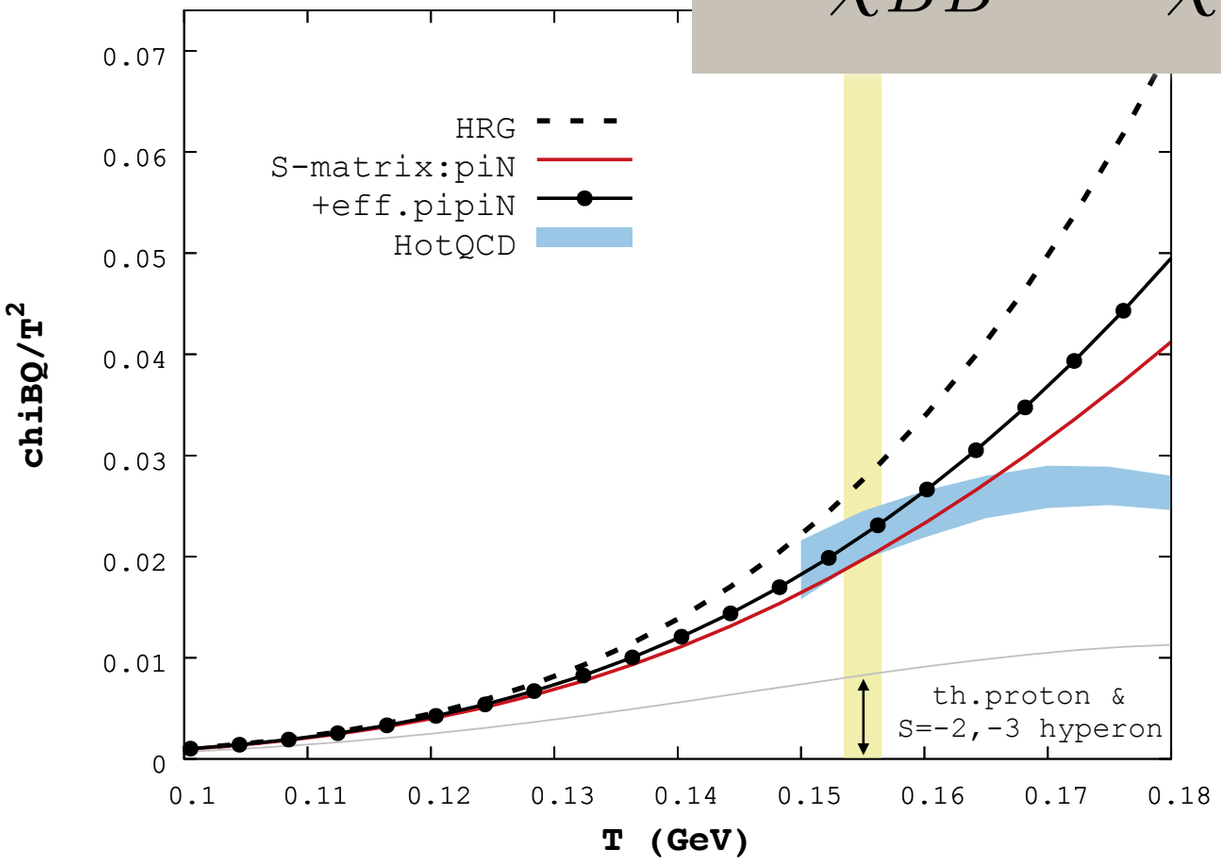


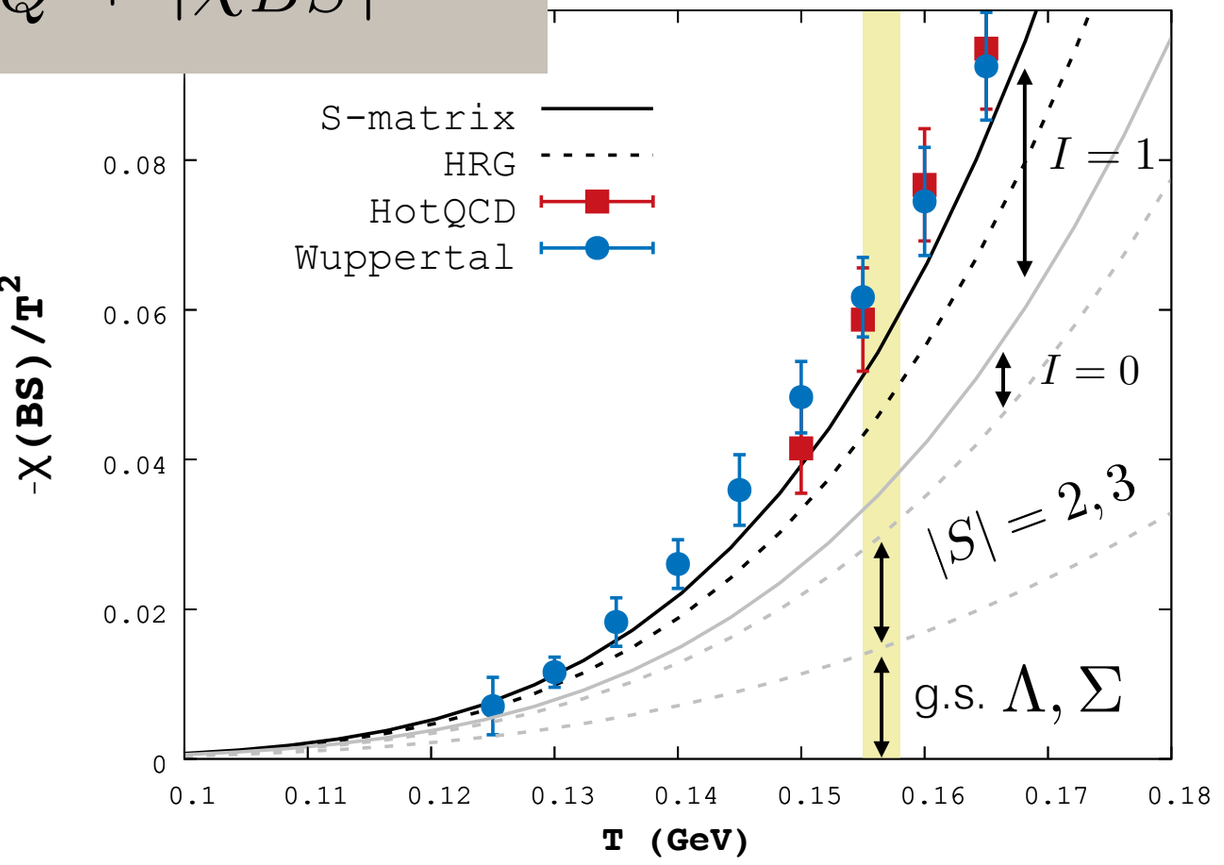




ALICE proton yield
@ 2.76 TeV

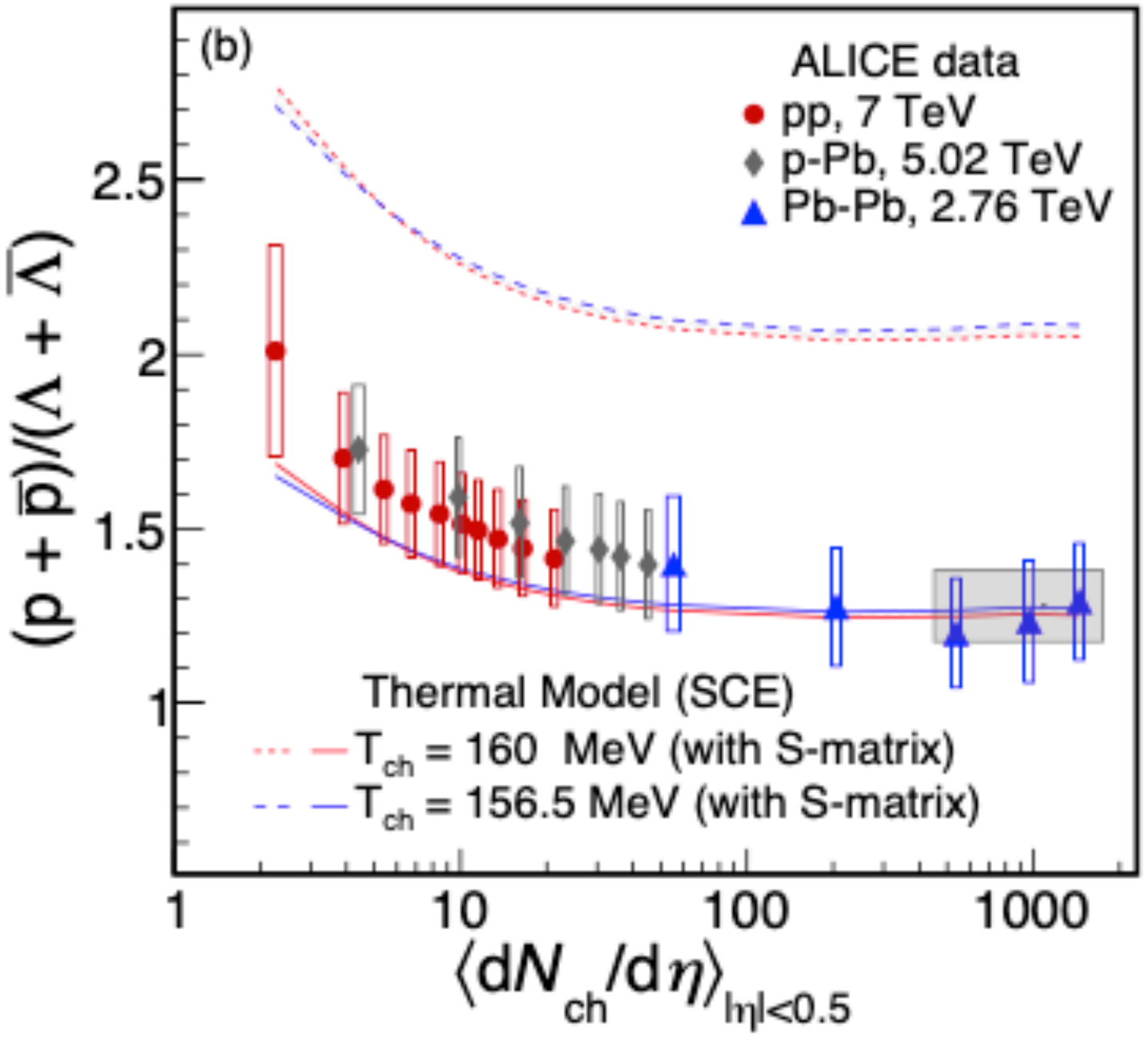
$$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$$





less protons

more lambdas



hrg

smat

Phys. Rev. C 103, 014904 (2021).

Phys. Lett. B 792, 304 (2019).

THEORETICAL ISSUES

muB = 0 @T = 155 MeV

- LHC conditions = pion rich: p = pbar; $\langle\pi\rangle/\langle p\rangle \approx 15$
  Need to Take Pions Seriously!
  NN is heavily (Boltzmann) suppressed compared to pi N

- How to include a resonance?
- Why it is NOT a Breit-Wigner?
- In-medium Effects from S-matrix

HOW TO RELATE PHASE SHIFTS TO THERMODYNAMICS?

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

single channel, elastic $\frac{1}{\pi} \frac{d}{dE} \delta$

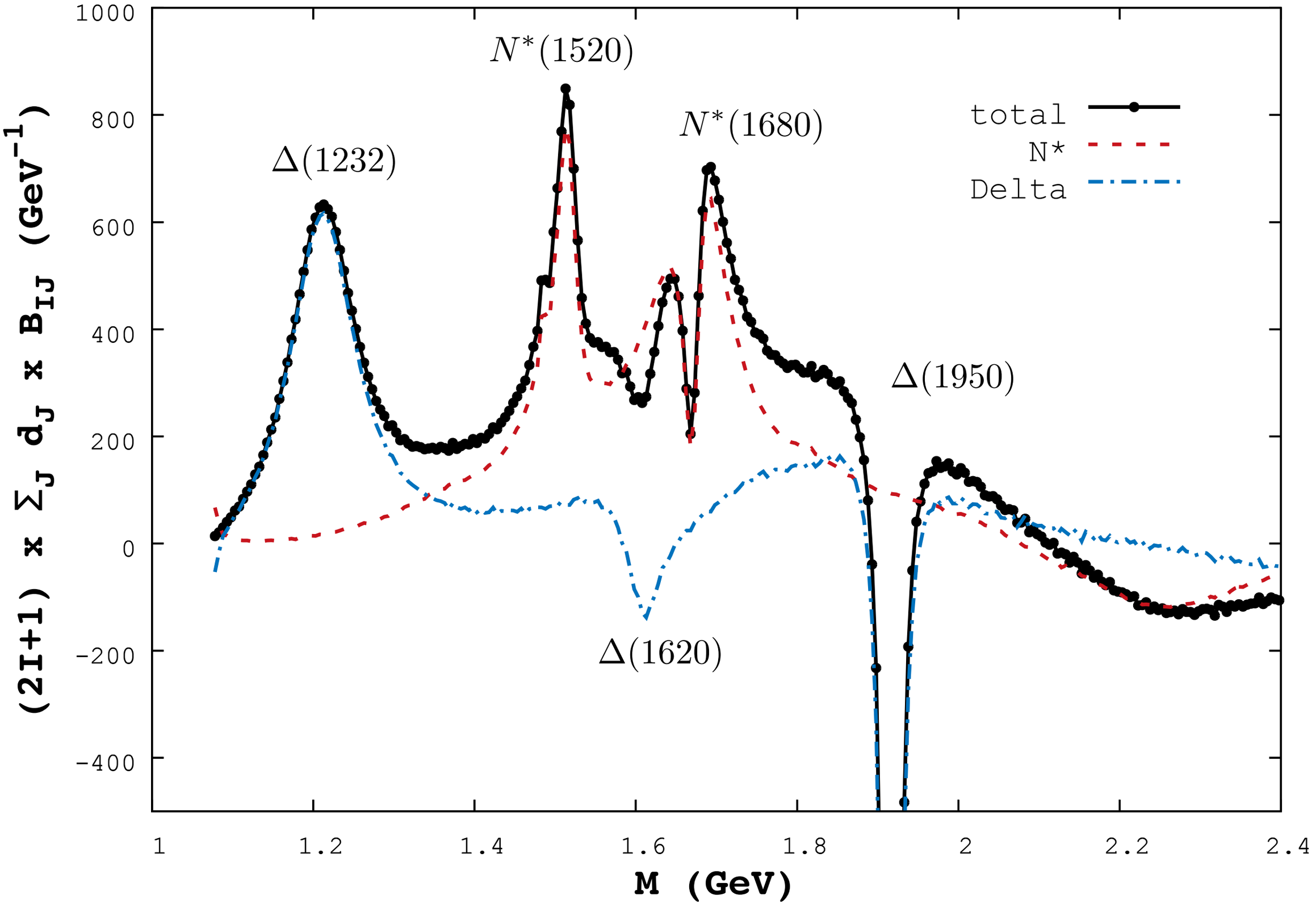
N-body &
Coupled-Channel problem

multi (coupled) channel $\frac{1}{\pi} \frac{d}{dE} \mathcal{Q}$

$$\mathcal{Q} = \frac{1}{2} \mathrm{Im Tr}\ \ln S$$
$$= \sum_{\text{channels}} \lambda_i$$

SINGLE CHANNEL -> MULTI-CHANNEL

- Instead of phase shifts -> on-shell S/T-matrix
  - N-channels, inelasticity, etc.
- Tricks of hadron physics
  - Non-resonant interactions
  - Quasi-elastic scattering (N > 2)
  - Unitarity BGs
  - Coupled-channel techniques


$N^*(1520)$
$N^*(1680)$
$\Delta(1232)$
$\Delta(1950)$
$\Delta(1620)$
total
N*
Delta
$(2I+1) \times \Sigma_J\ d_J \times B_{IJ}$ (GeV^{-1})
M (GeV)
1000
800
600
400
200
0
-200
-400
1
1.2
1.4
1.6
1.8
2
2.2
2.4

JPAC, PRD **93,** 034029 (2016)

PHASE SHIFT FROM PWA

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Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

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Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
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Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosphy. If your
parameters do not make sense, the output will not make sense either.
2 - You can use, share and modify this code under your own responsability.
3 - This code is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
4 - No PhD students or postdocs were severely damaged during the development
of this project.

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```

- 1 $\to \bar{K}N$,
- 2 $\to \pi\Sigma$,
- 3 $\to \pi\Lambda$,
- 4 $\to \eta\Lambda$,
- 5 $\to \eta\Sigma$,
- 6 $\to \bar{K}_1 N$,
- 7 $\to \left[\bar{K}_3 N\right]_-$,
- 8 $\to \left[\bar{K}_3 N\right]_+$,
- 9 $\to [\pi\Sigma^*]_-$,
- 10 $\to [\pi\Sigma^*]_+$,
- 11 $\to \left[\bar{K}\Delta\right]_-$,
- 12 $\to \left[\bar{K}\Delta\right]_+$,
- 13 $\to [\pi\Lambda(1520)]_-$,
- 14 $\to [\pi\Lambda(1520)]_+$,
- 15 $\to \pi\pi\Lambda$,
- 16 $\to \pi\pi\Sigma$.

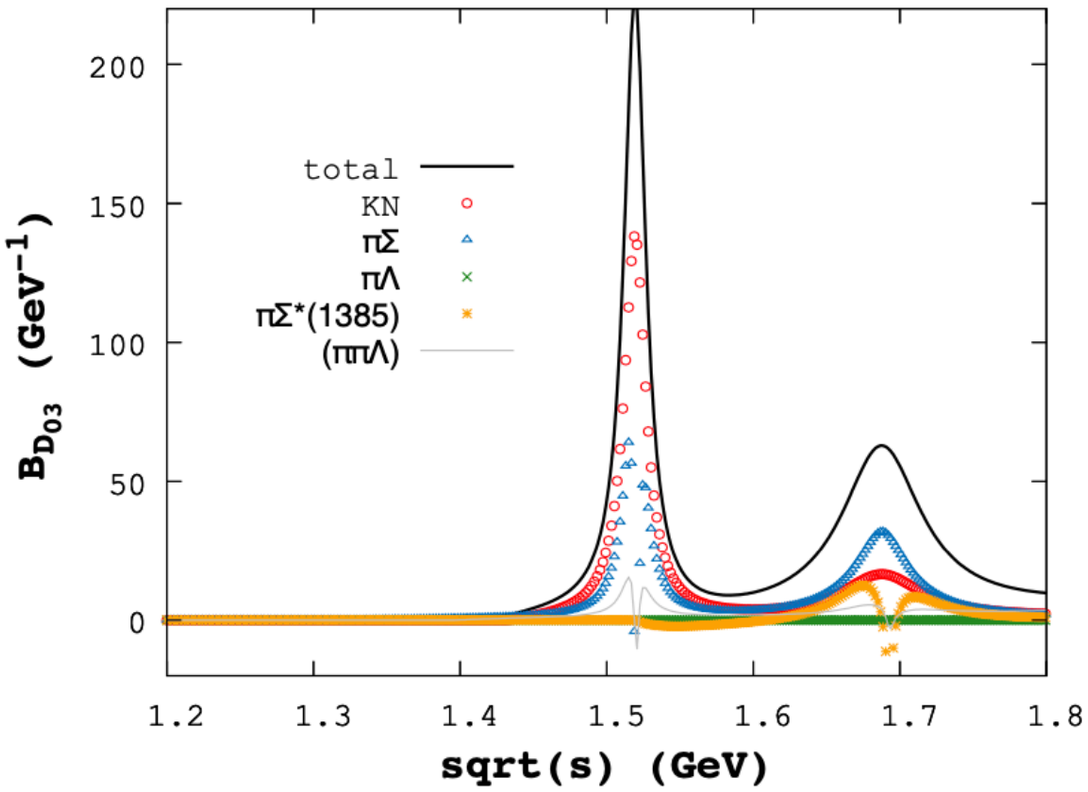
elastic scatterings (elementary)

Effective elementarity

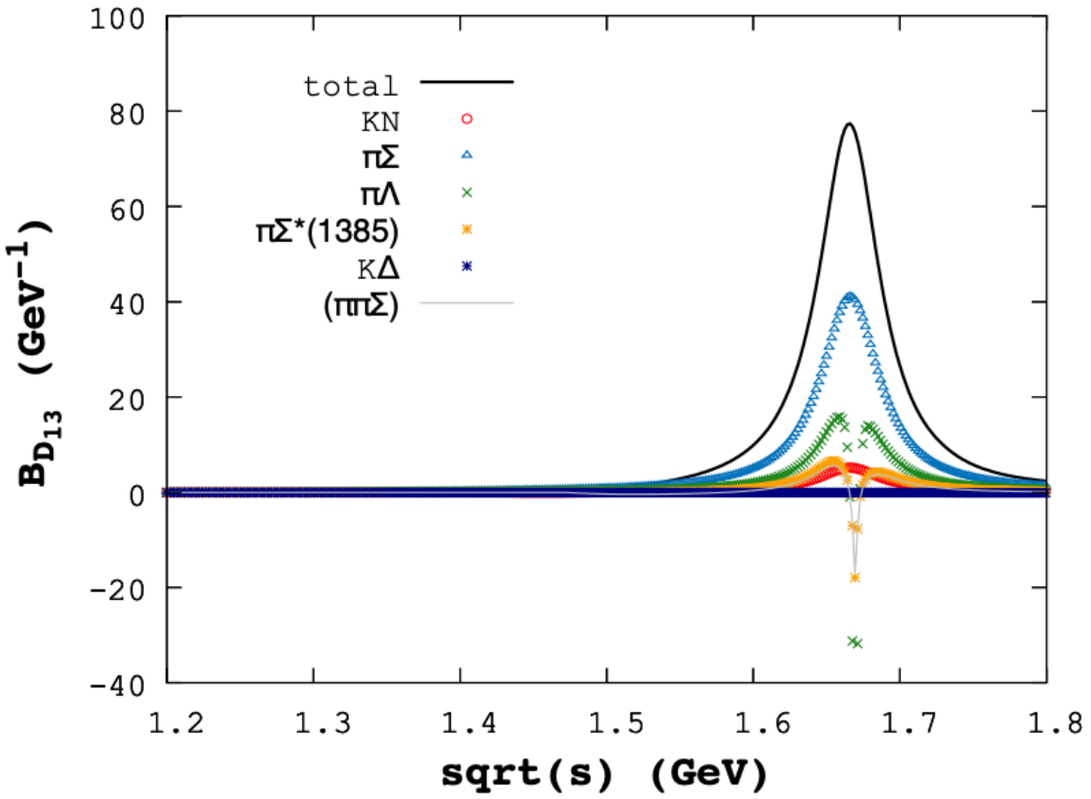
quasi elastic scatterings

unitarity background

1520, 1690



1670



Λ(1520) 3/2⁻ $I(J^P) = 0(\frac{3}{2}^-)$

Mass $m = 1519.5 \pm 1.0$ MeV [d]
Full width $\Gamma = 15.6 \pm 1.0$ MeV [d]
$p_{\text{beam}} = 0.39$ GeV/c $4\pi\lambda\!\!\!^{-2} = 82.8$ mb

Λ(1520) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	45 ± 1%	243
$\Sigma\pi$	42 ± 1%	268
$\Lambda\pi\pi$	10 ± 1%	259
$\Sigma\pi\pi$	0.9 ± 0.1%	169
$\Lambda\gamma$	0.85 ± 0.15%	350

Λ(1690) 3/2⁻ $I(J^P) = 0(\frac{3}{2}^-)$

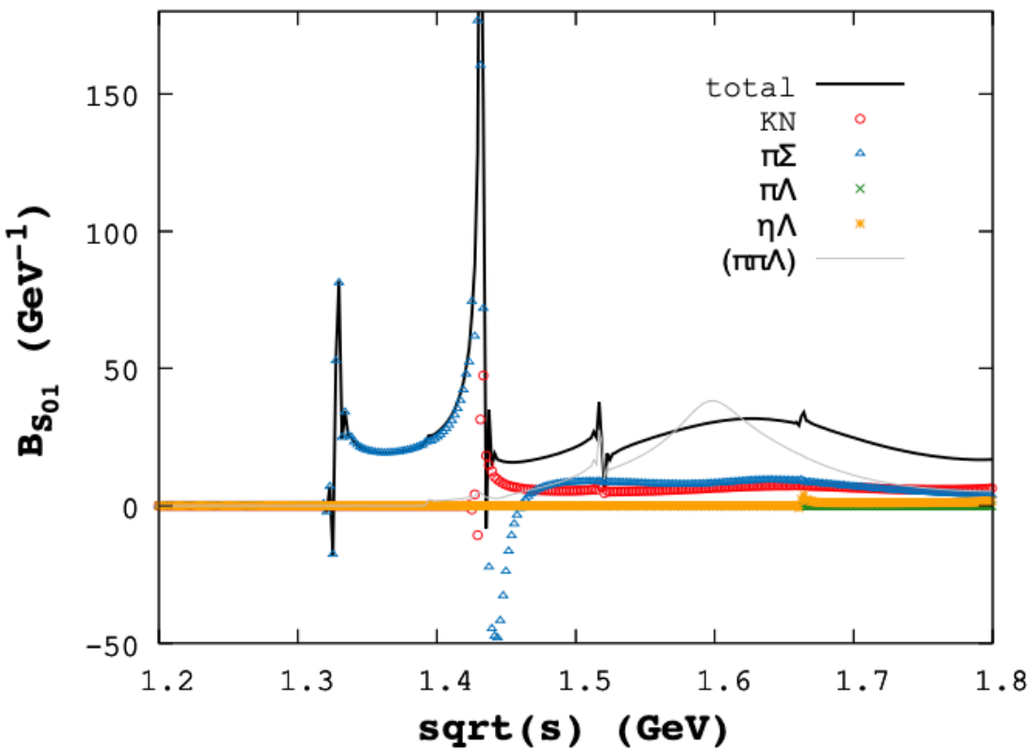
Mass $m = 1685$ to 1695 (≈ 1690) MeV
Full width $\Gamma = 50$ to 70 (≈ 60) MeV
$p_{\text{beam}} = 0.78$ GeV/c $4\pi\lambda\!\!\!^{-2} = 26.1$ mb

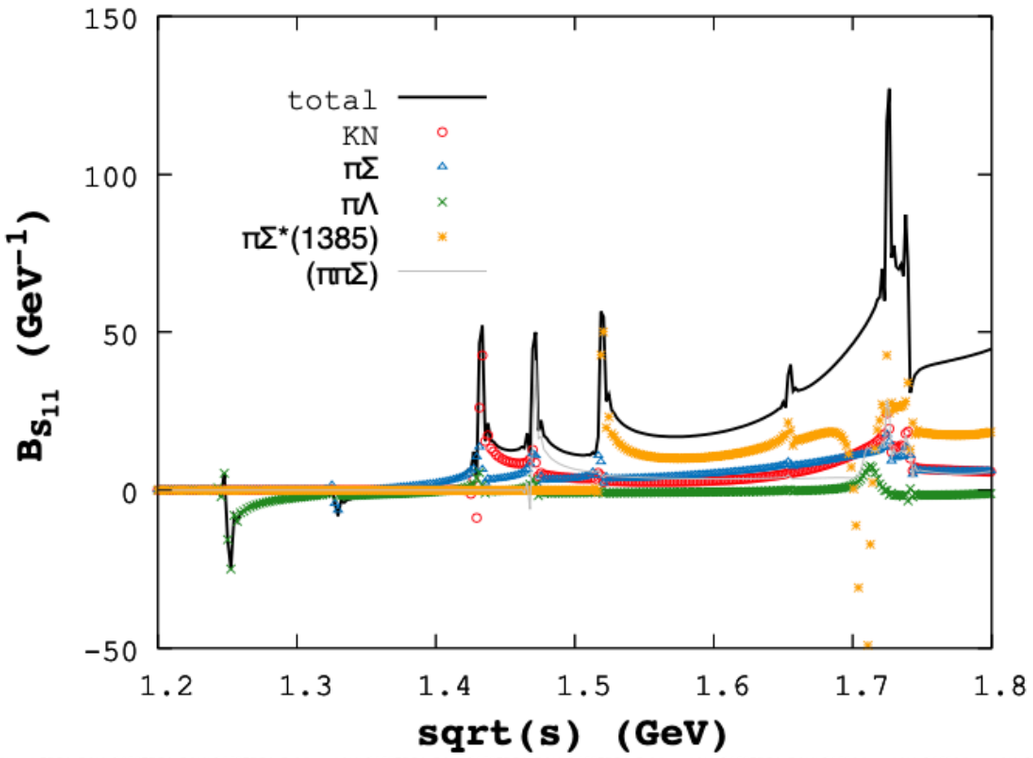
Λ(1690) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	20–30 %	433
$\Sigma\pi$	20–40 %	410
$\Lambda\pi\pi$	~ 25 %	419
$\Sigma\pi\pi$	~ 20 %	358

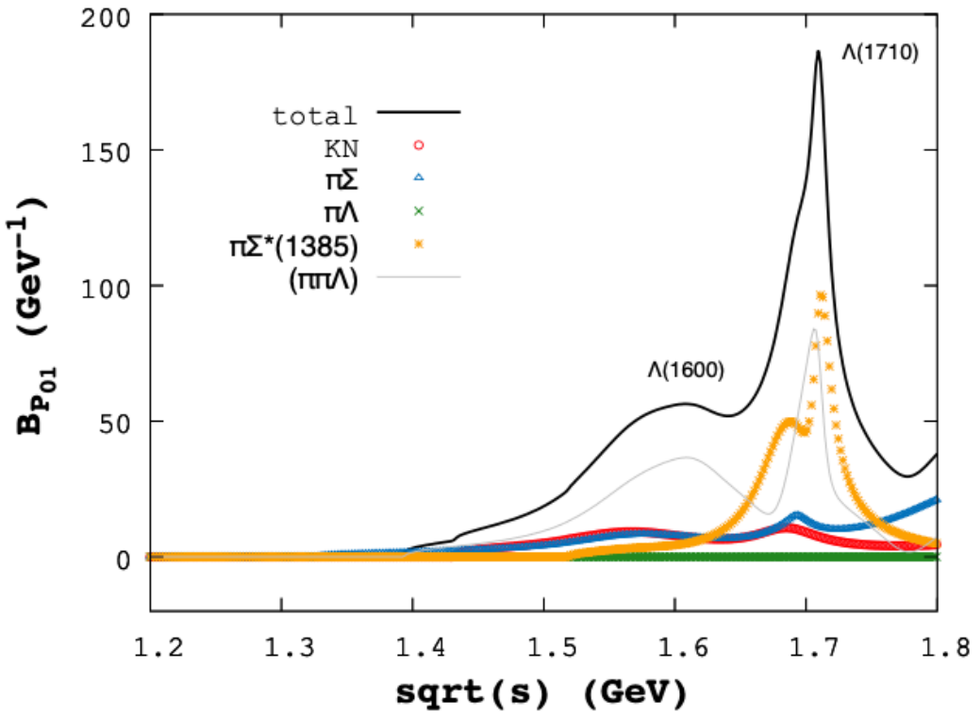
Σ(1670) 3/2⁻ $I(J^P) = 1(\frac{3}{2}^-)$

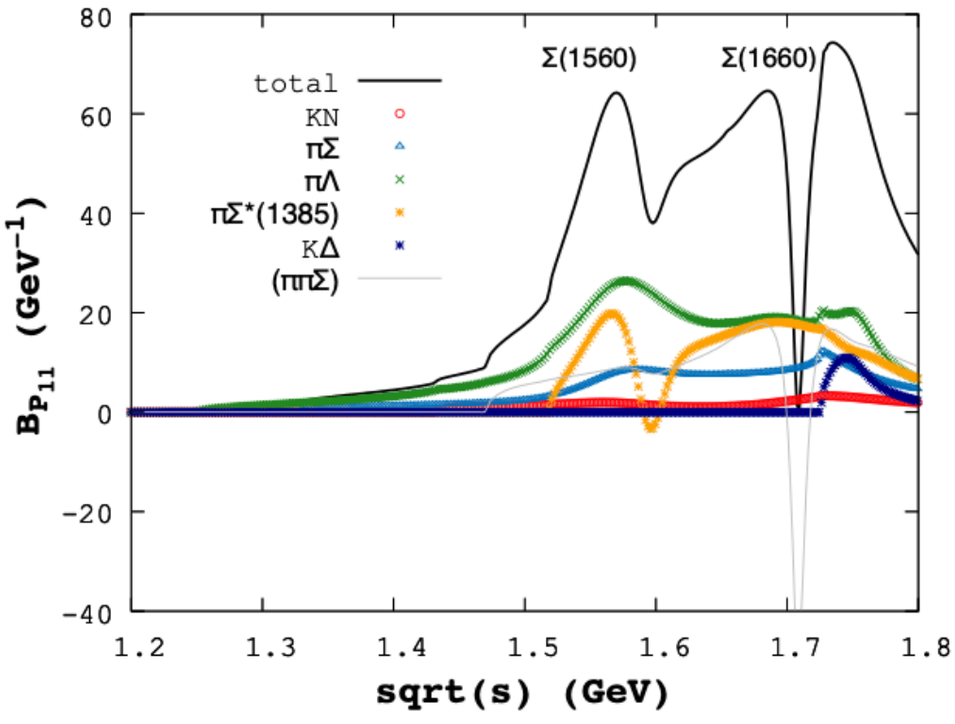
Mass $m = 1665$ to 1685 (≈ 1670) MeV
Full width $\Gamma = 40$ to 80 (≈ 60) MeV
$p_{\text{beam}} = 0.74$ GeV/c $4\pi\lambda\!\!\!^{-2} = 28.5$ mb

Σ(1670) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	7–13 %	414
$\Lambda\pi$	5–15 %	448
$\Sigma\pi$	30–60 %	394

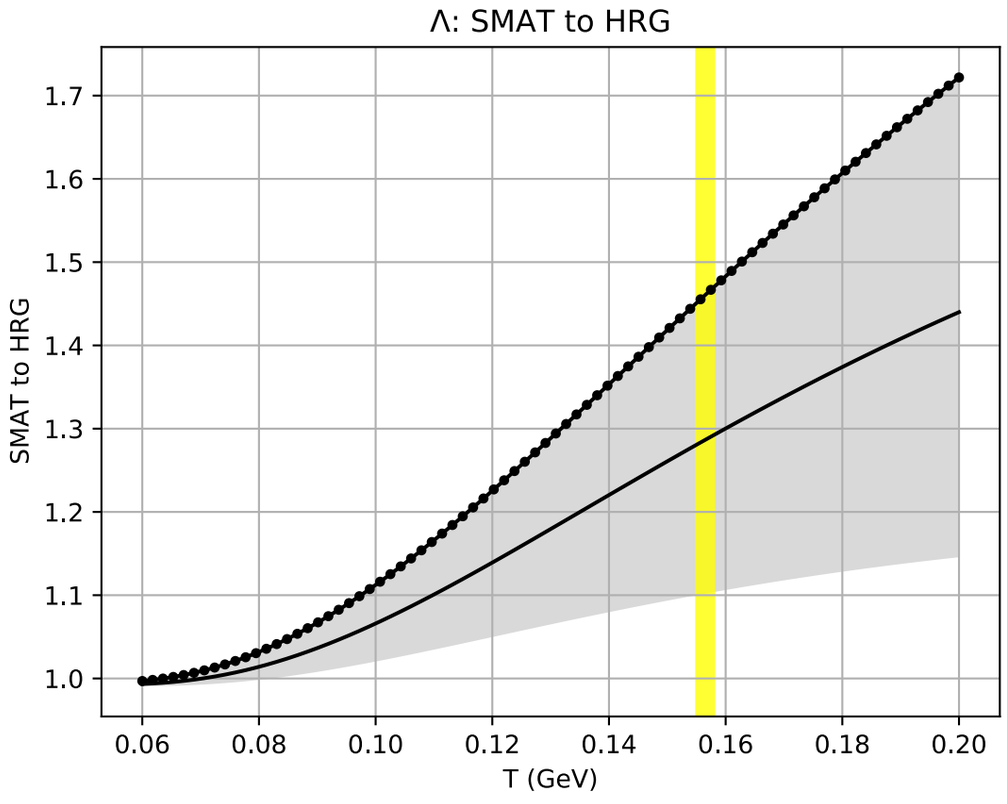


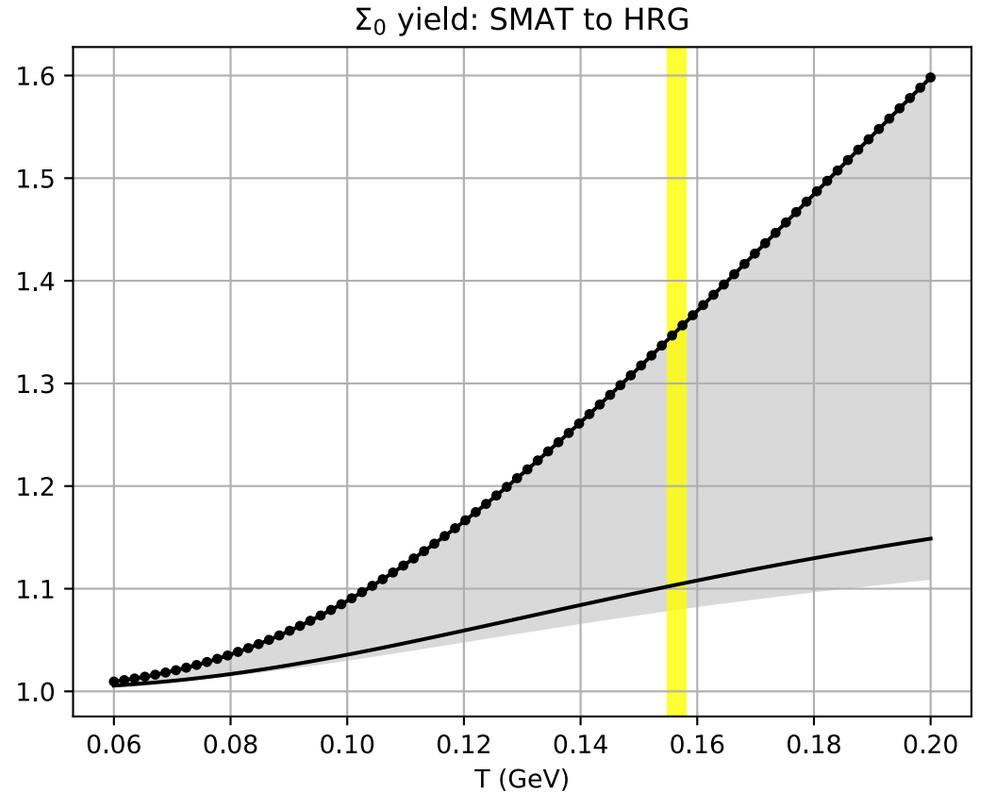


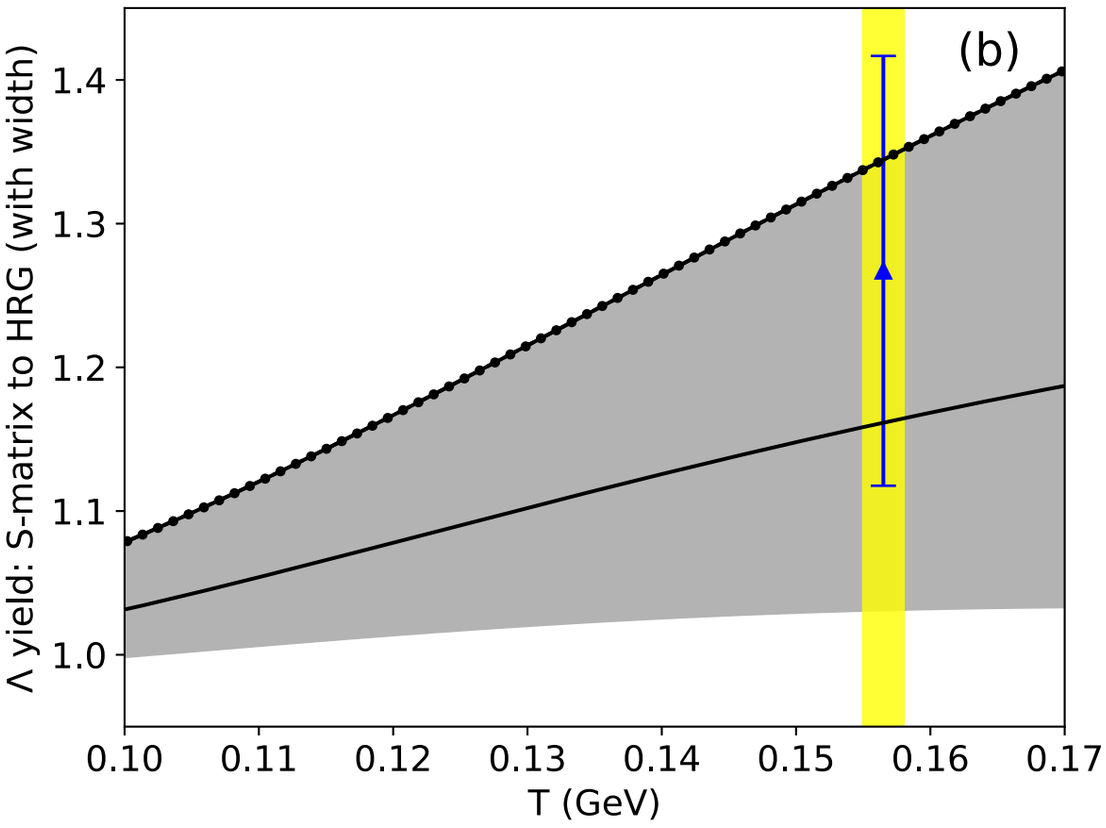


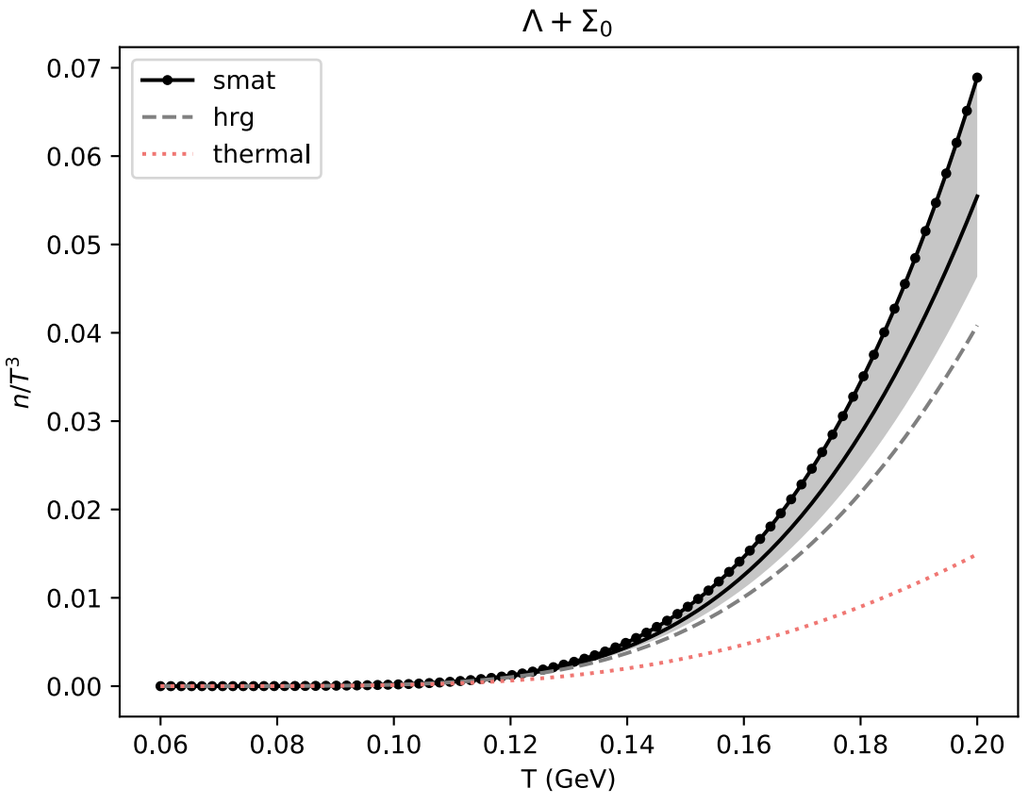


branching ratio?

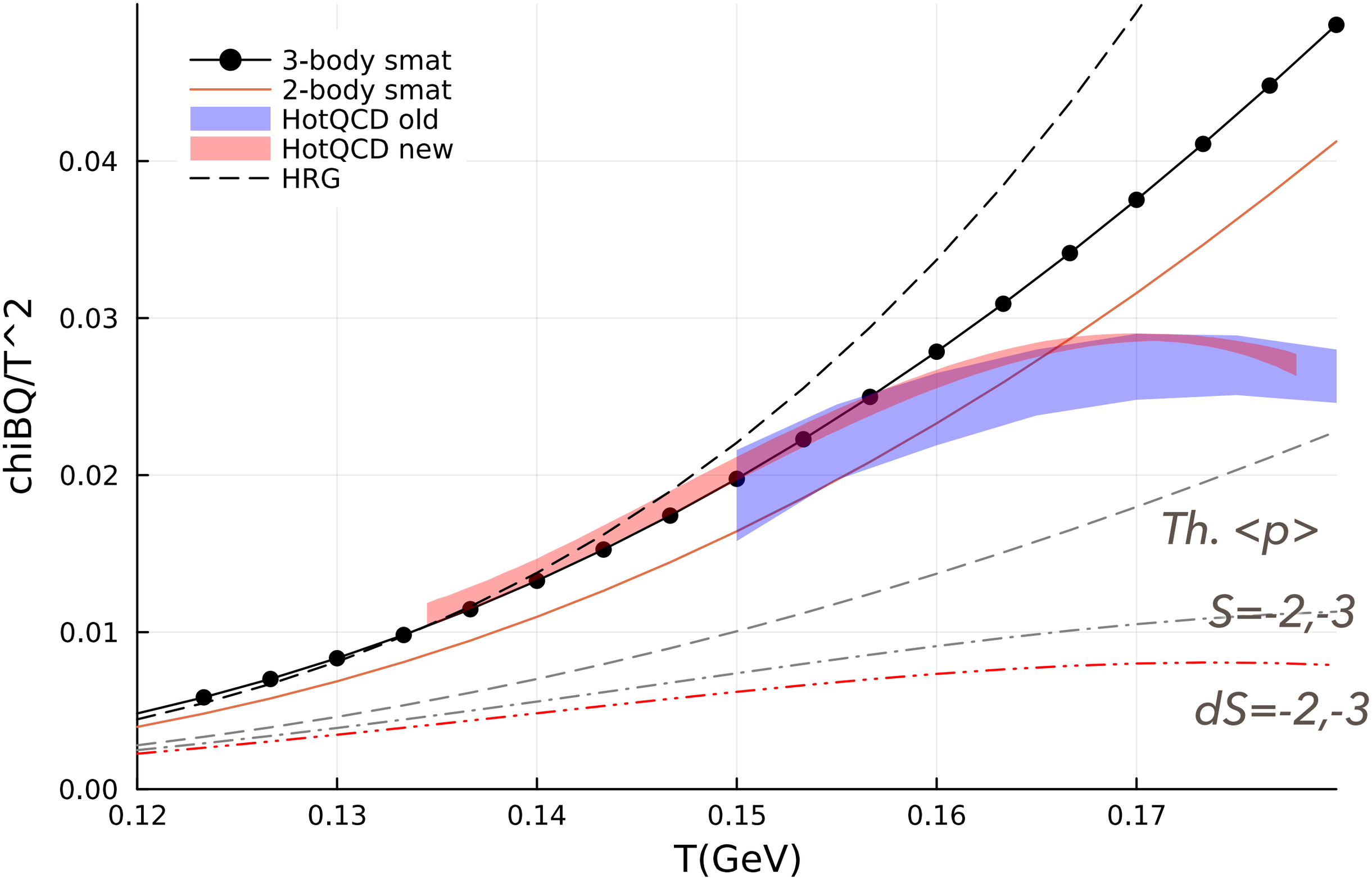

Λ: SMAT to HRG
SMAT to HRG
T (GeV)
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20


Σ0 yield: SMAT to HRG
T (GeV)
1.0
1.1
1.2
1.3
1.4
1.5
1.6
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20

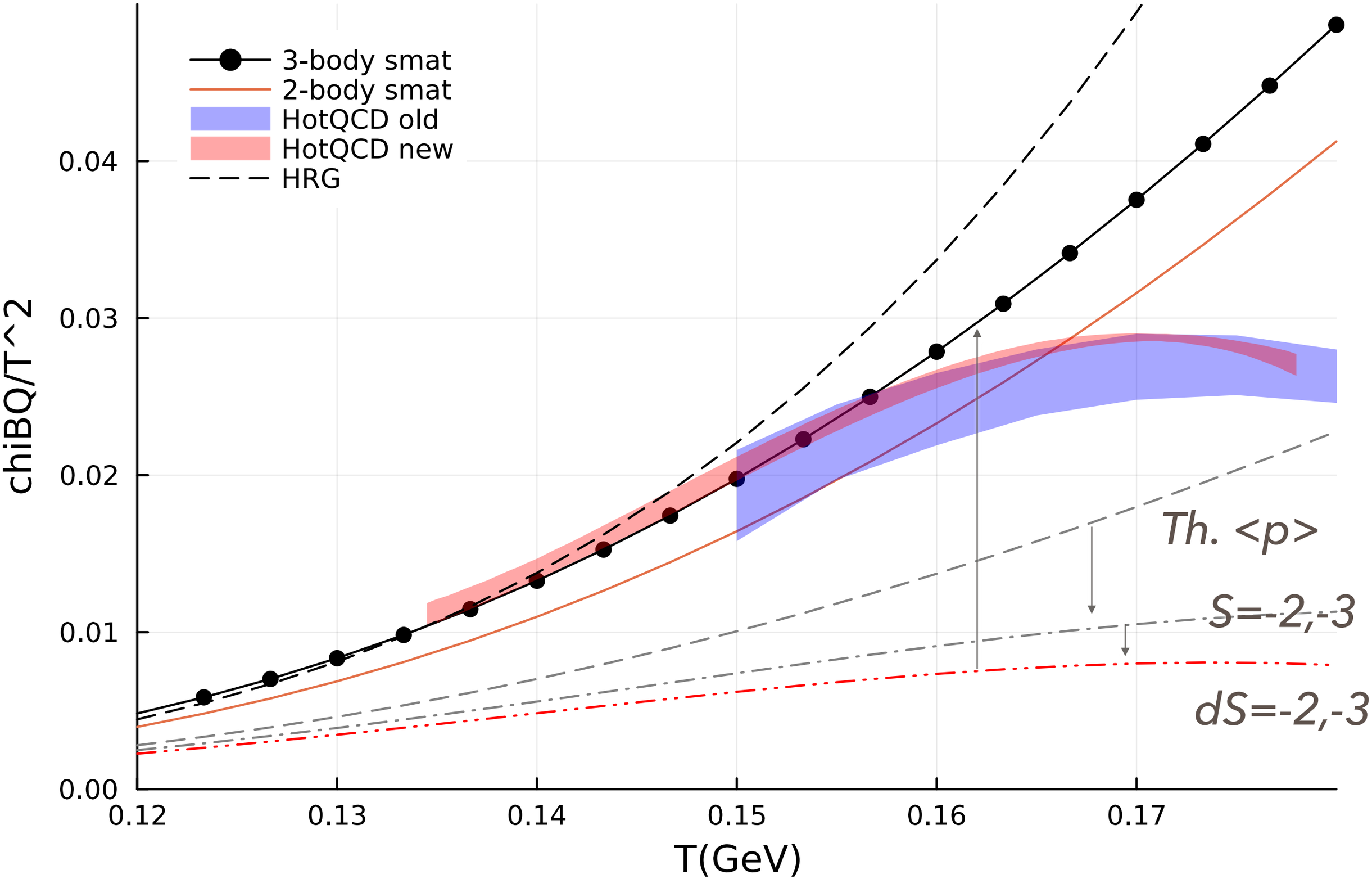

(b)
Λ yield: S-matrix to HRG (with width)
T (GeV)
1.0
1.1
1.2
1.3
1.4
0.10
0.11
0.12
0.13
0.14
0.15
0.16
0.17

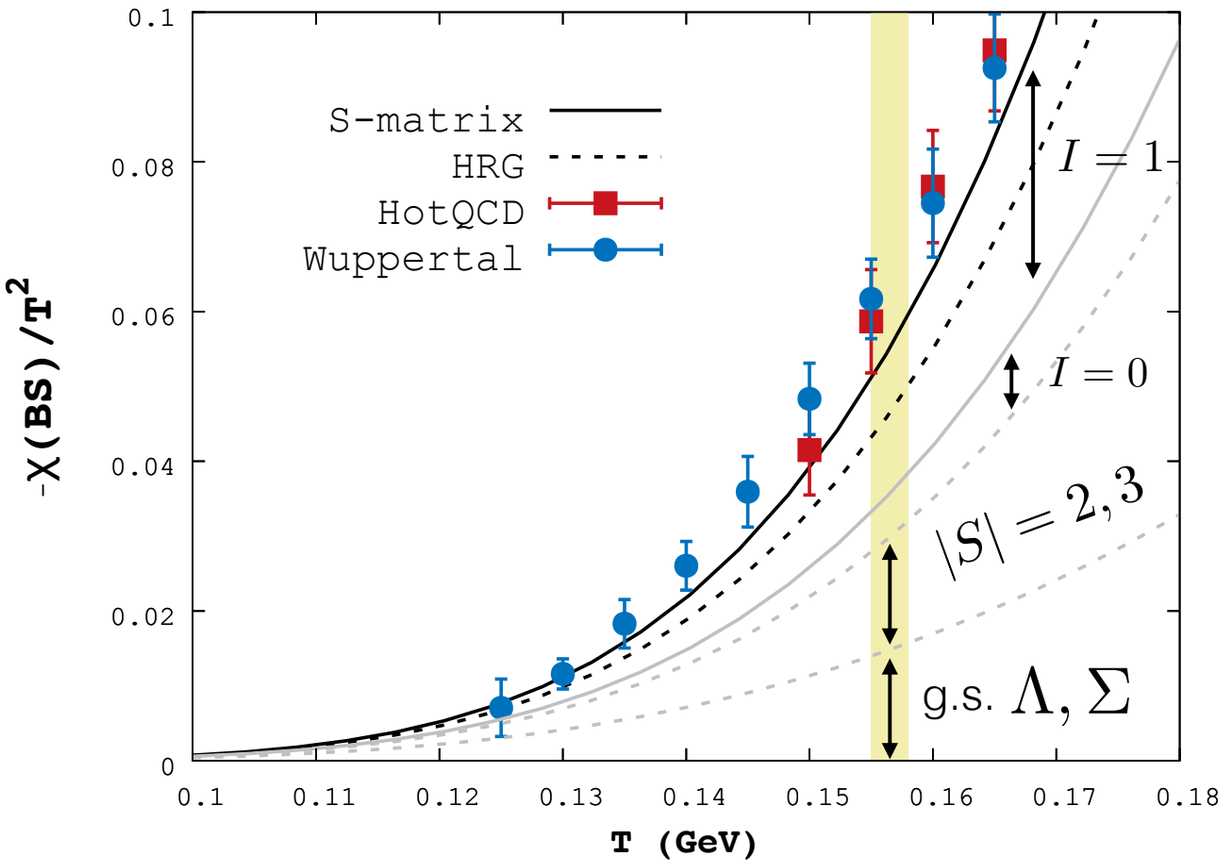

Λ + Σ0
smat
hrg
thermal
n/T3
T (GeV)
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20

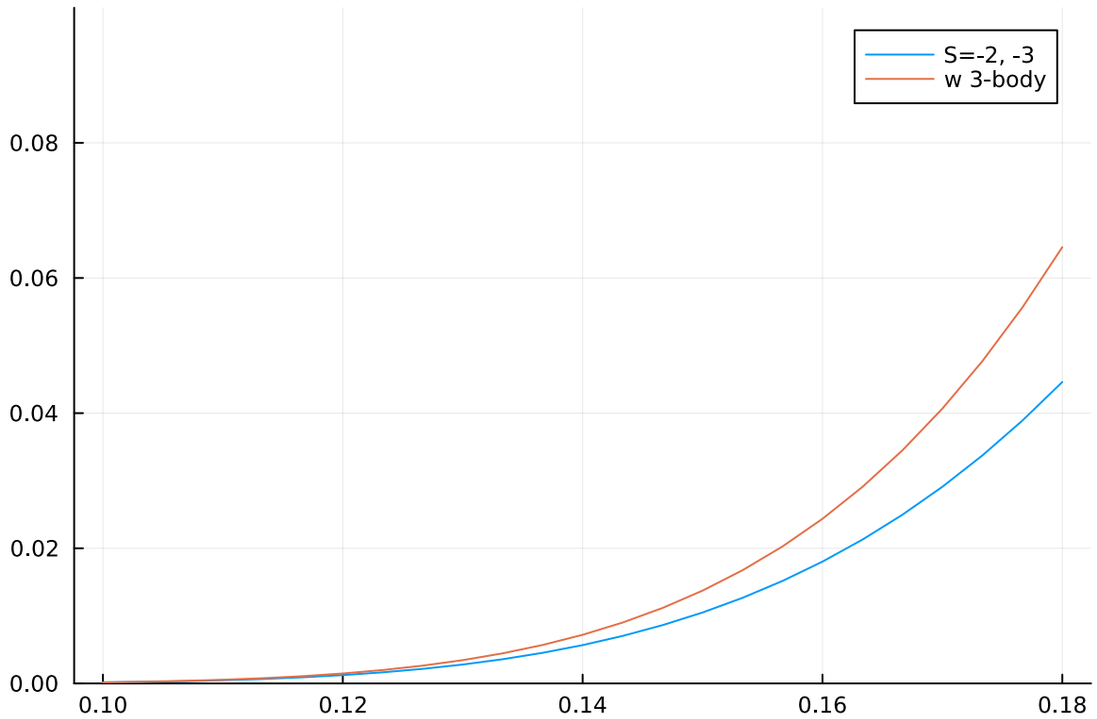
Prelim


3-body smat
2-body smat
HotQCD old
HotQCD new
HRG
Th. <p>
S=-2,-3
dS=-2,-3
chiBQ/T^2
0.00
0.01
0.02
0.03
0.04
0.12
0.13
0.14
0.15
0.16
0.17
T(GeV)

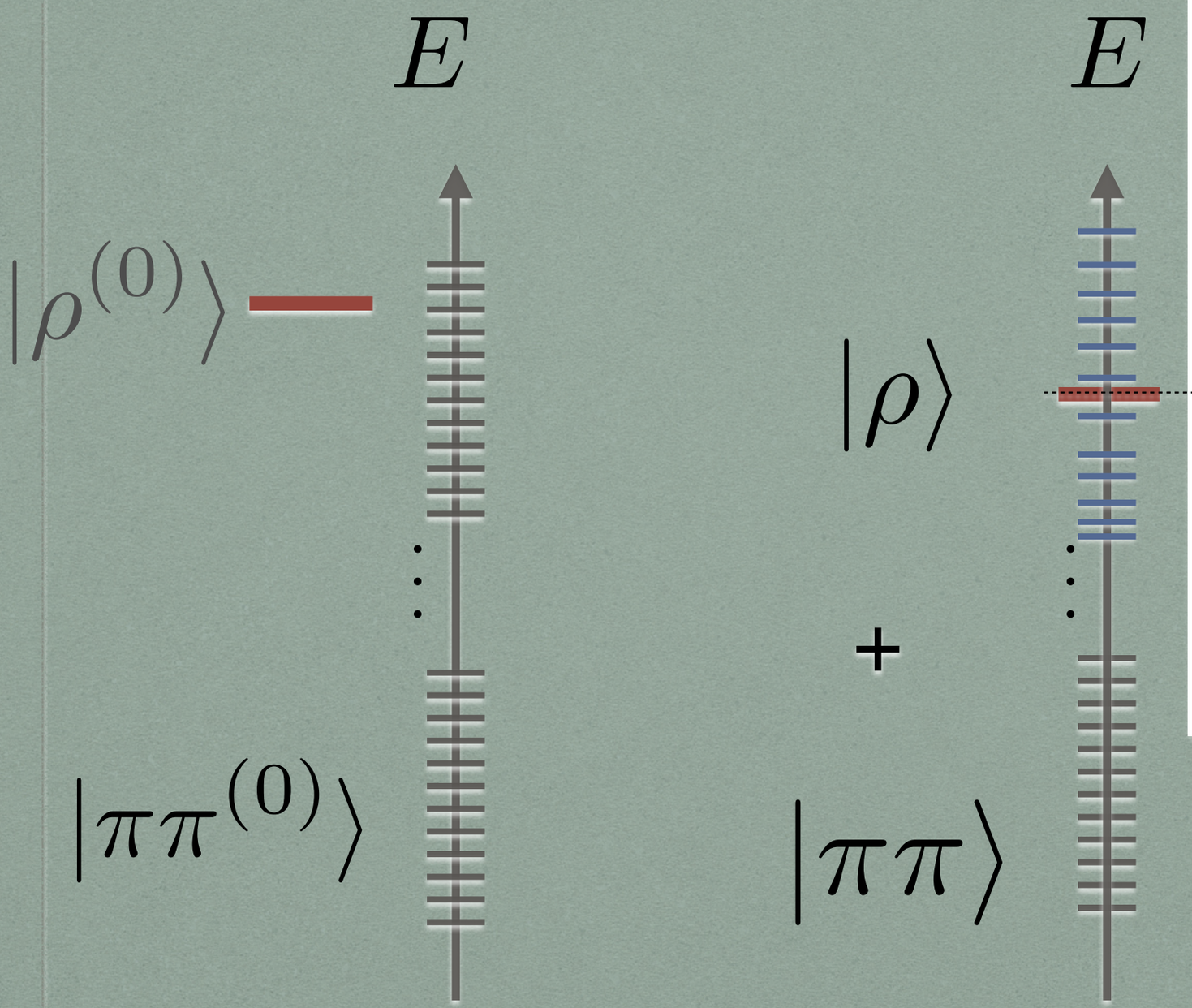
Prelim

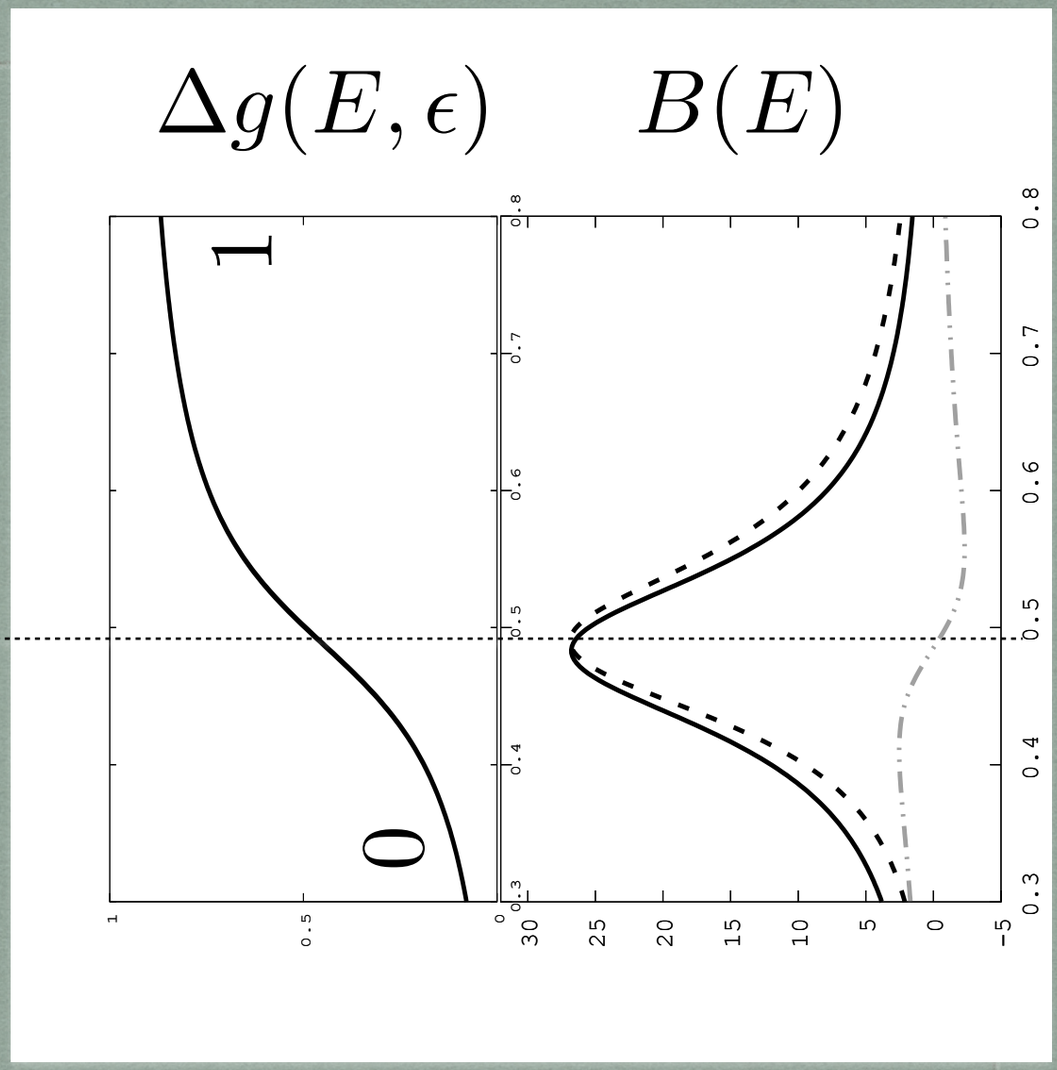

3-body smat
2-body smat
HotQCD old
HotQCD new
HRG
0.04
0.03
0.02
0.01
0.00
chiBQ/T^2
0.12
0.13
0.14
0.15
0.16
0.17
T(GeV)
Th. <p>
S=-2,-3
dS=-2,-3


S-matrix
HRG
HotQCD
Wuppertal
$I = 1$
$I = 0$
$|S| = 2,3$
g.s. Λ, Σ
-χ(BS) /T²
T (GeV)
0.1
0.08
0.06
0.04
0.02
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18


S=-2, -3
w 3-body
0.08
0.06
0.04
0.02
0.00
0.10
0.12
0.14
0.16
0.18

IN-MEDIUM EFFECTS





$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E,\epsilon)$$

$$= A_\rho + \Delta A_{\pi\pi}$$

$$\mathrm{Tr}\, e^{-\beta \mathcal{H}_0} \quad \textit{vs} \quad \mathrm{Tr}\, e^{-\beta \mathcal{H}}$$

PML, Francesco Giocosa EPJC **79**(2019)

PHYSICS OF B

$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

$$B = 2\frac{\partial}{\partial E}\delta$$

$$= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1}$$

$$= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial\Sigma_\rho}{\partial E}G_\rho]$$

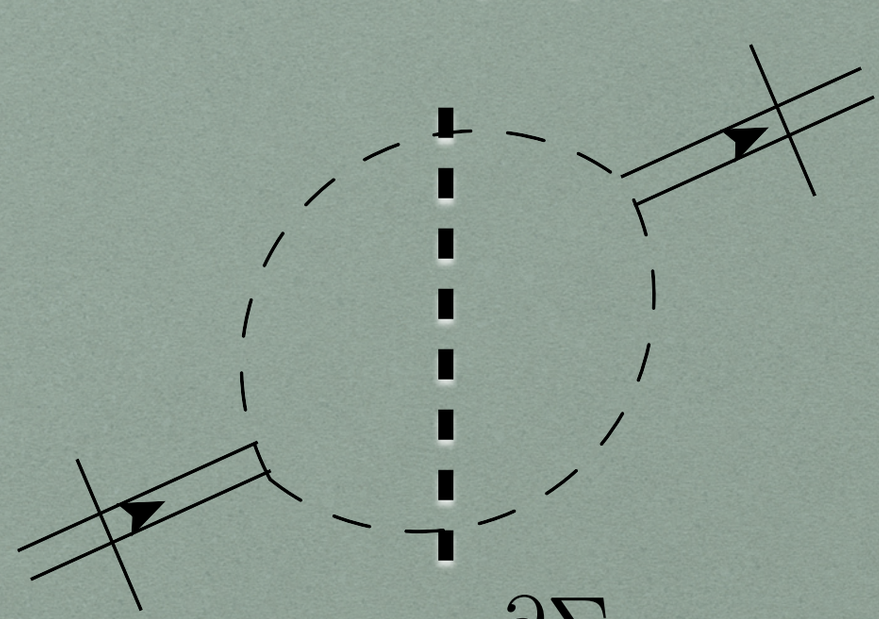
$$= A_\rho(E) + \Delta A_{\pi\pi}$$

$$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$$

pipi -> pipi

physical interpretation:

contribution from correlated pi pi pair



$$\frac{\partial\Sigma_\rho}{\partial E}$$

PHYSICS OF B

to rho or not to rho?
that's out of the question!

resonance's picture:

$$B(E) = A_\rho(E) + \Delta A_{\pi\pi}$$

rho

scattering picture:

$$B_1 = \frac{\partial}{\partial E}\,\mathrm{Tr}\,\hat{t}_{\mathrm{re}}$$

pipi -> pipi

$$B_2 = \frac{1}{2}\,\mathrm{Im}\,\mathrm{Tr}\,\hat{t}^\dagger \overleftrightarrow{\partial_E}\,\hat{t}$$

$\delta = -\mathrm{Im}\,\mathrm{T}$

$B = 2\,\frac{\partial}{\partial E}\delta$

$= -2\,\mathrm{Im}\frac{\partial}{\partial E}$

$= -2\,\mathrm{Im}[G$

$= A_\rho(E) + \Delta A_{\pi\pi}$

$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$ pipi -> pipi

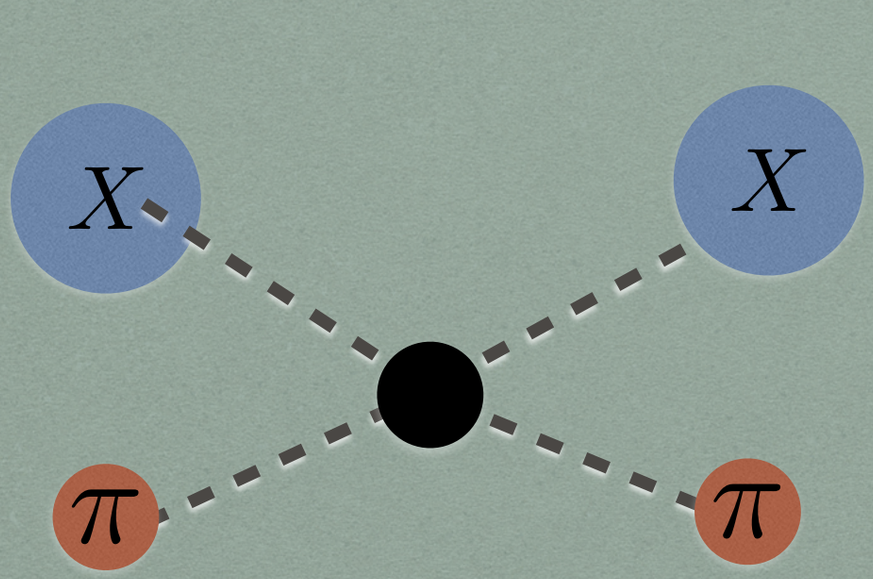
$\frac{\partial \Sigma_\rho}{\partial E}$

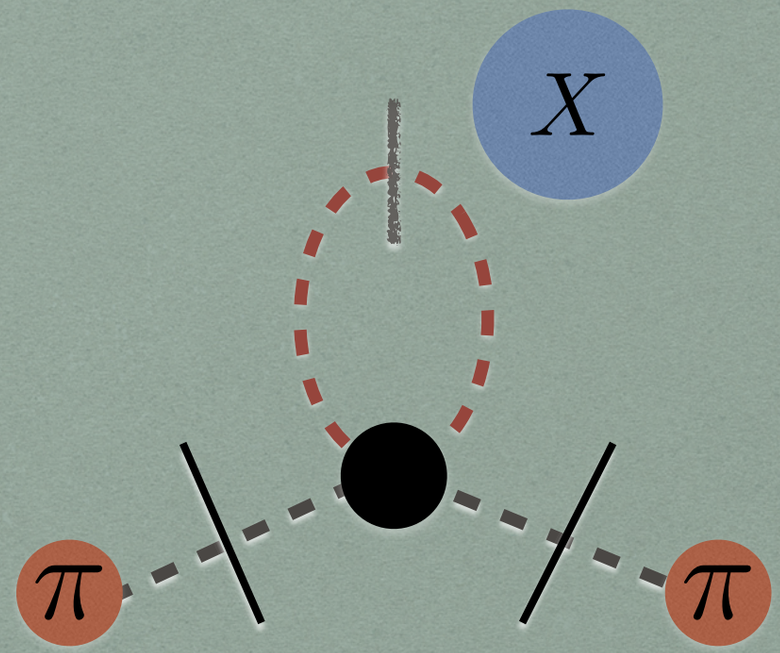
on:

m

air

IN-MEDIUM EFFECTS FROM S-MATRIX





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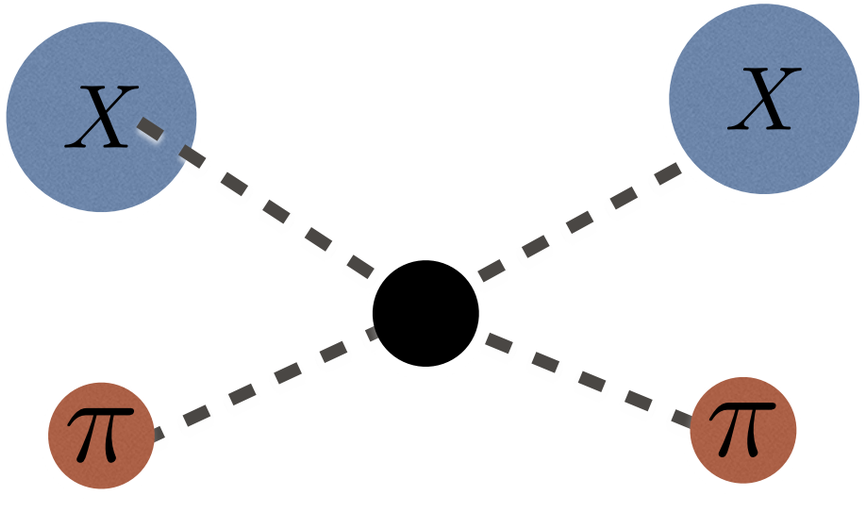


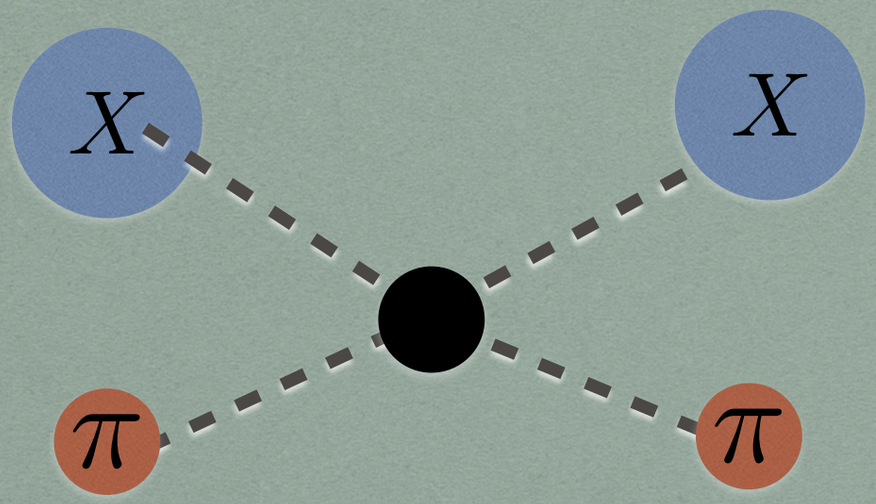
$$T_{\rm nr} \approx -\frac{4\pi f}{2m_{\rm red}}.$$

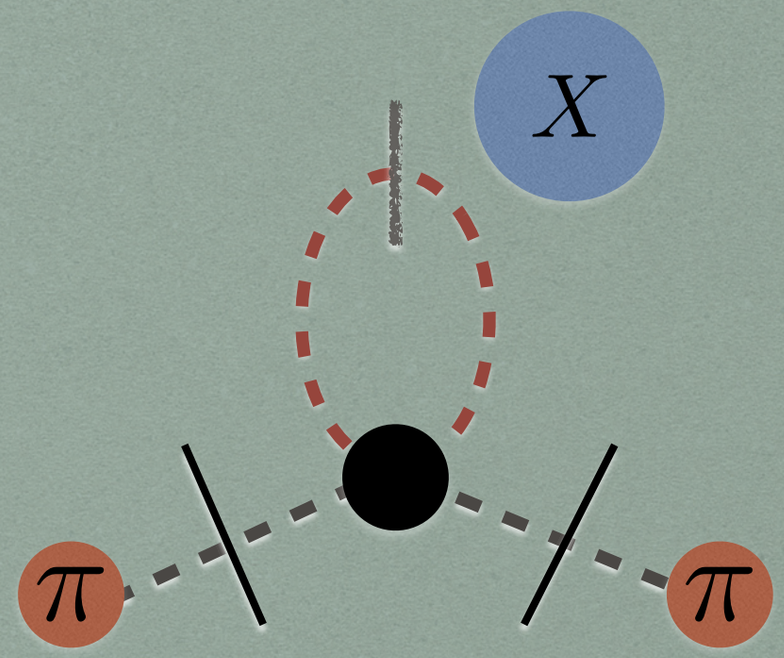
$$\begin{aligned}
\Delta P &\approx \int \frac{d^3P}{(2\pi)^3}\frac{dE'}{(2\pi)} e^{-\beta(m_{\rm tot}+\frac{P^2}{2m_{\rm tot}}+E')} 2\mathcal{Q}(E') \\
&= \int \frac{d^3P}{(2\pi)^3}\frac{d^3q}{(2\pi)^3} e^{-\beta(m_{\rm tot}+\frac{P^2}{2m_{\rm tot}}+\frac{q^2}{2m_{\rm red}})} (-T_{\rm nr}) \\
&\approx N^A_{\rm th} N^B_{\rm th} \times (-T_{\rm nr}).
\end{aligned}$$

$$\begin{aligned}
\Delta P &\approx T \int \frac{d^3p_A}{(2\pi)^3} e^{-\beta(m_A+\frac{p_A^2}{2m_A})} (-\beta \Delta m_A) \\
&= -\Delta m_A N^A_{\rm th} \\
&= N^A_{\rm th} N^B_{\rm th} \times \frac{4\pi f}{2m_{\rm red}}.
\end{aligned}$$

Change of pressure to due "Dressed mass"

IN-MEDIUM EFFECTS FROM S-MATRIX





$$\Delta P = N_{\rm th}^A \, N_{\rm th}^B \times \frac{4\pi f}{2m_{\rm red}}.$$

$$\Sigma_A(E_A) = \int \frac{d^3k_B}{(2\pi)^3} \frac{1}{2E_B} \, n_{\rm th}(E_B) \, T(AB \to AB).$$

$$\Delta m_A = \frac{1}{2E_A} \, {\rm Re}\, \Sigma_A(p)$$

$$\approx N_{\rm th}^B \times \frac{-4\pi f}{2m_{\rm red}}.$$

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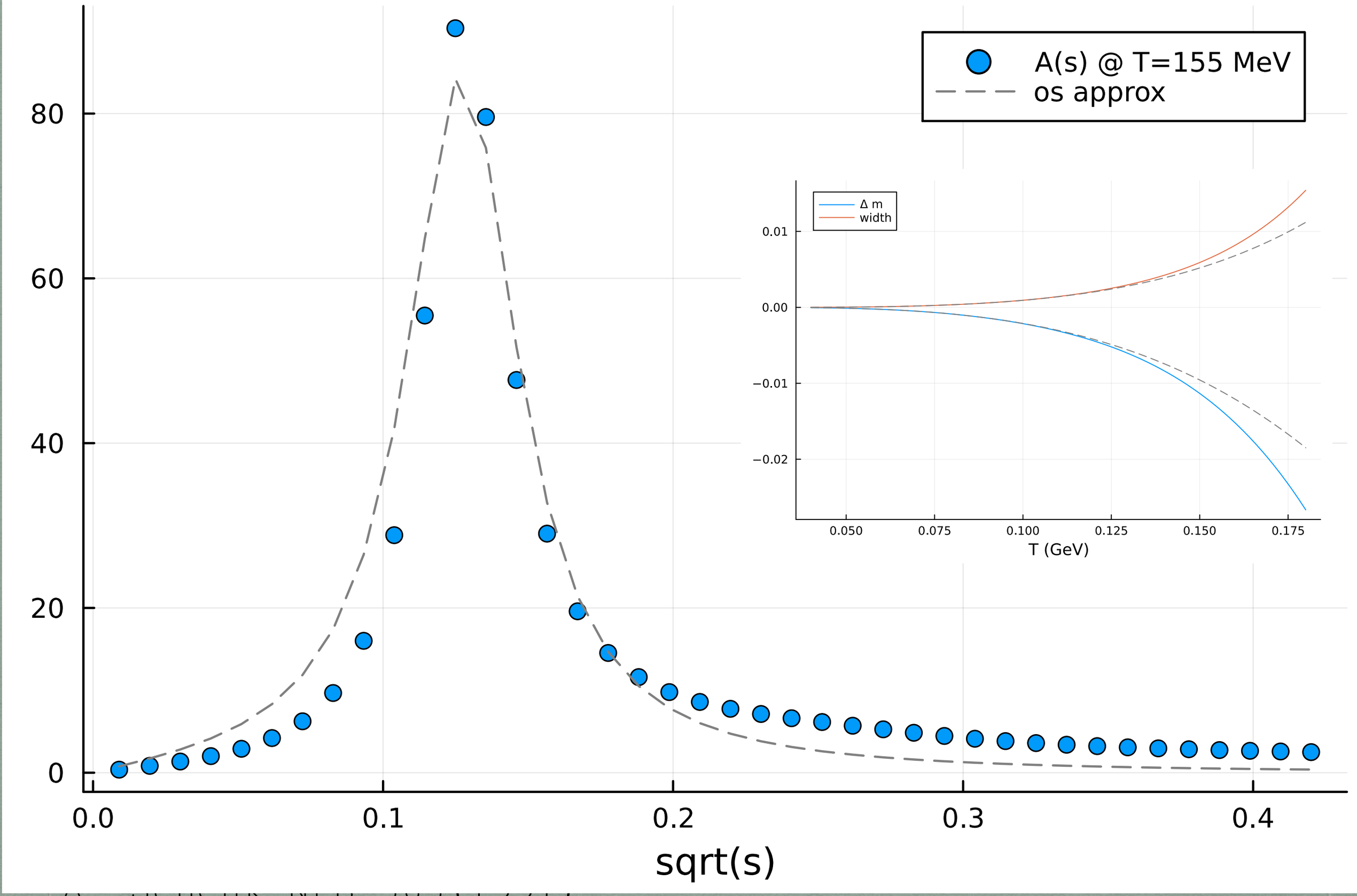

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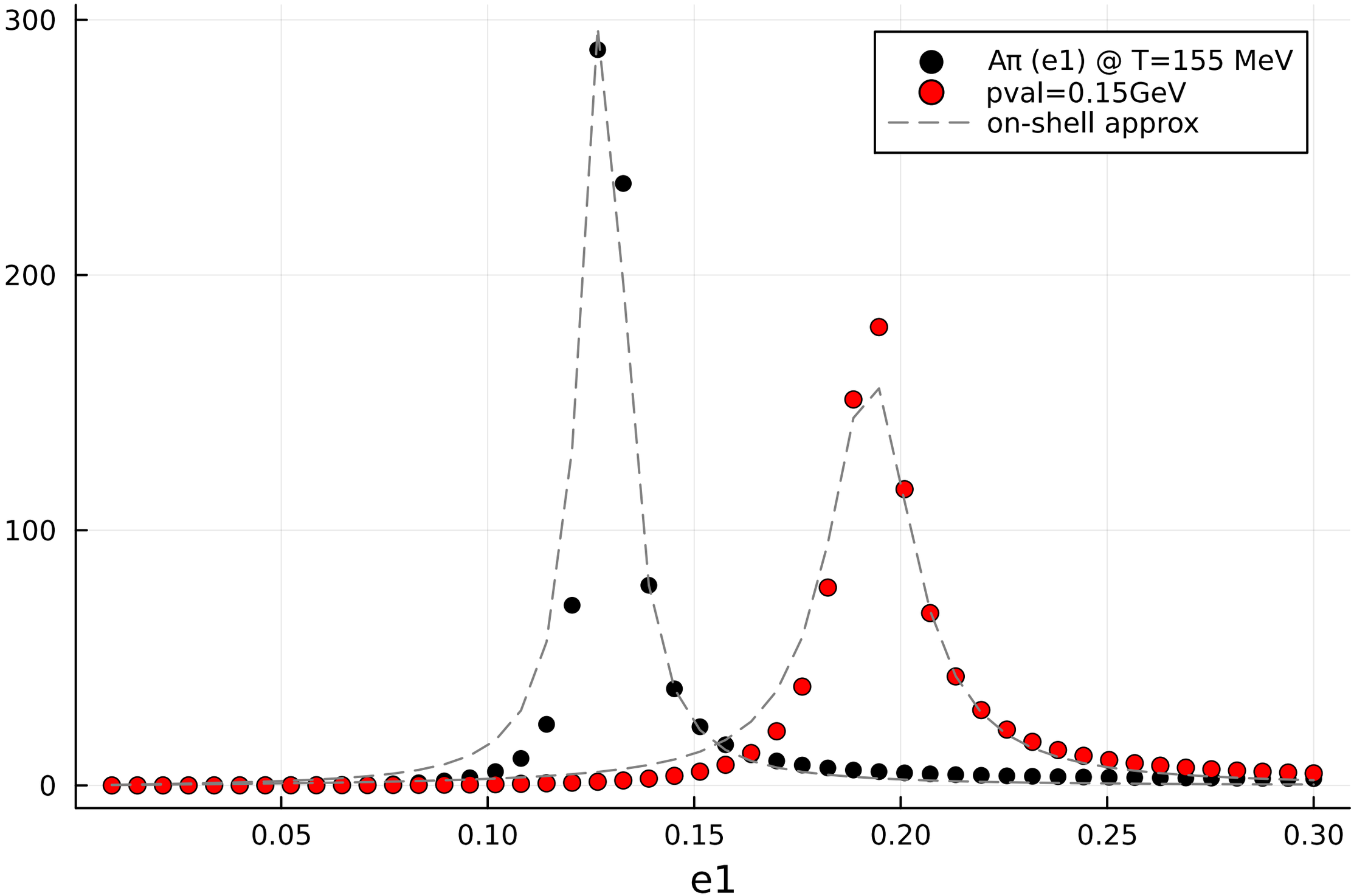
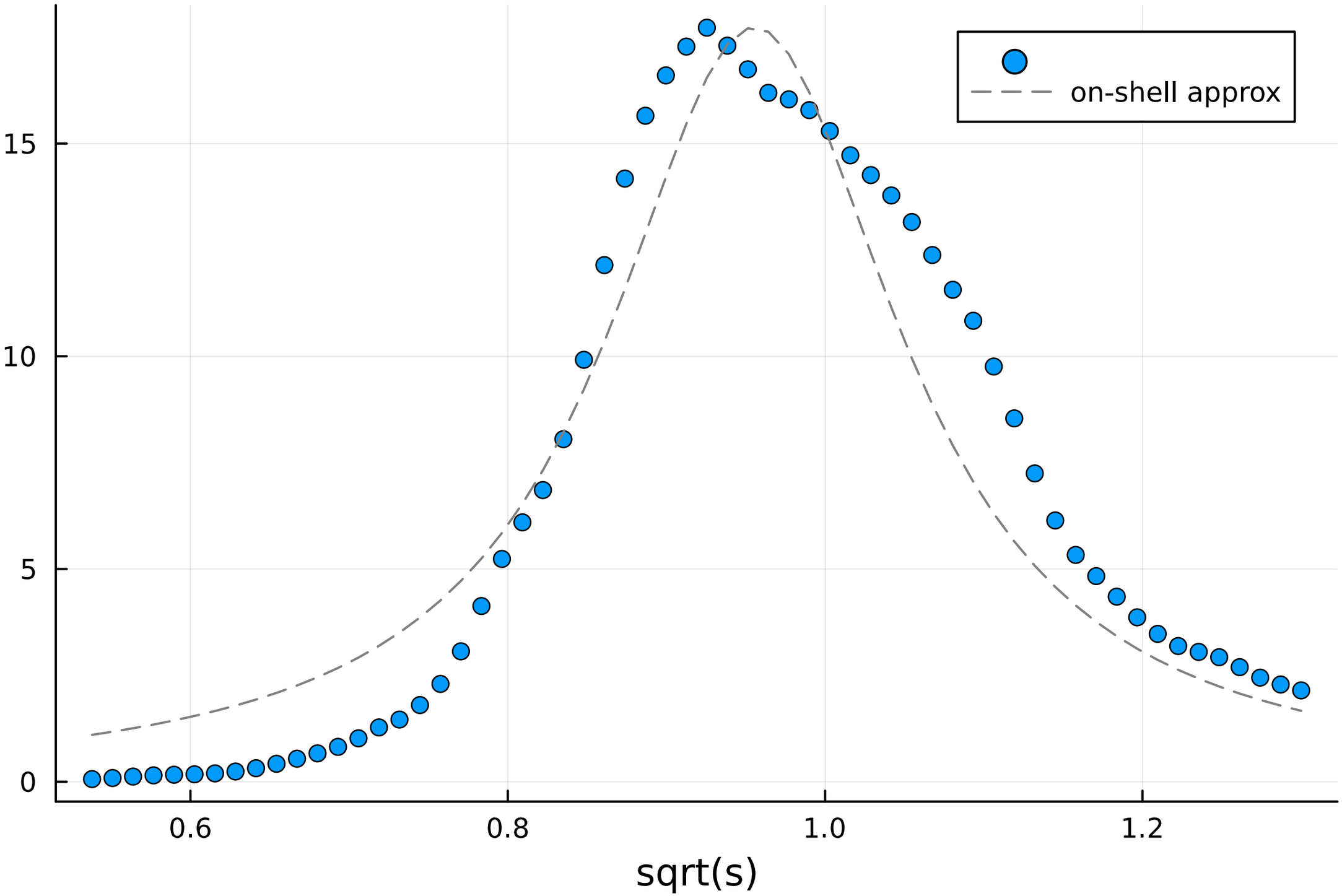
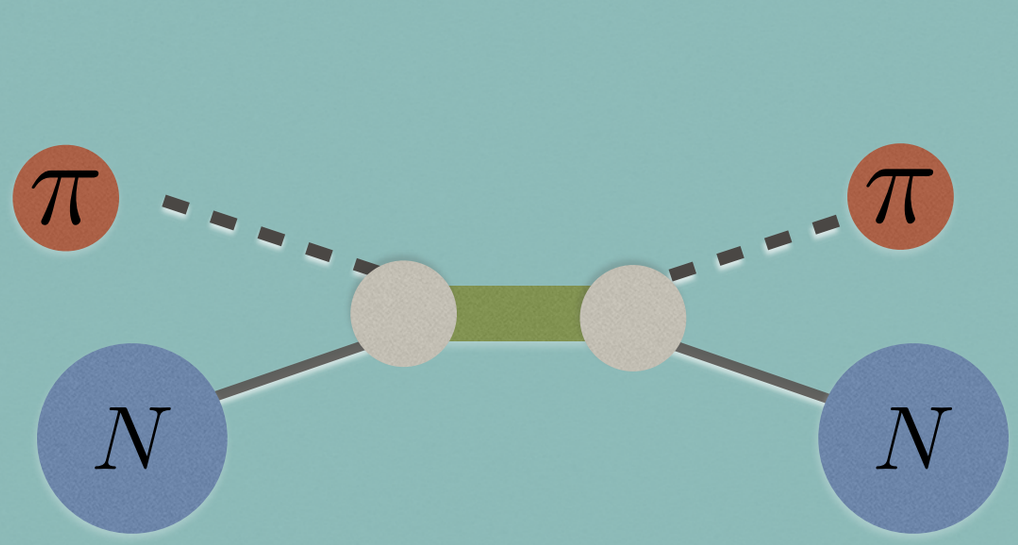
Pion spectral function

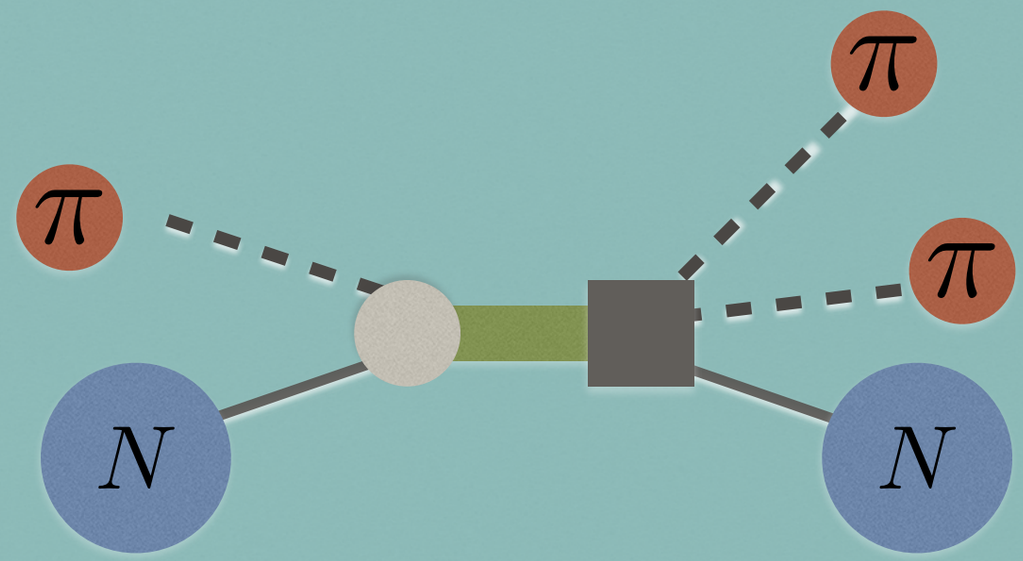


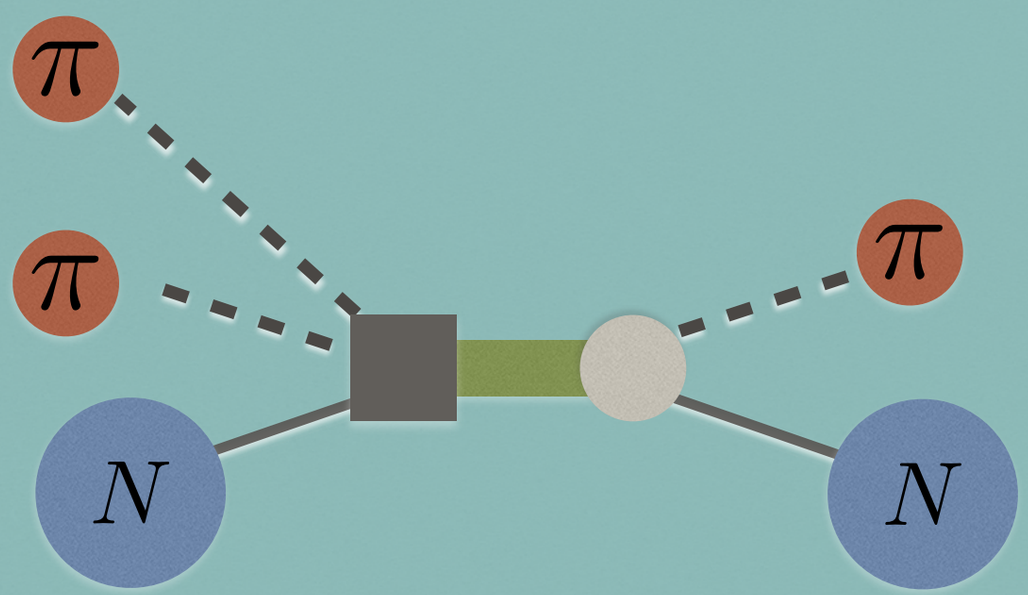
Proton spectral function

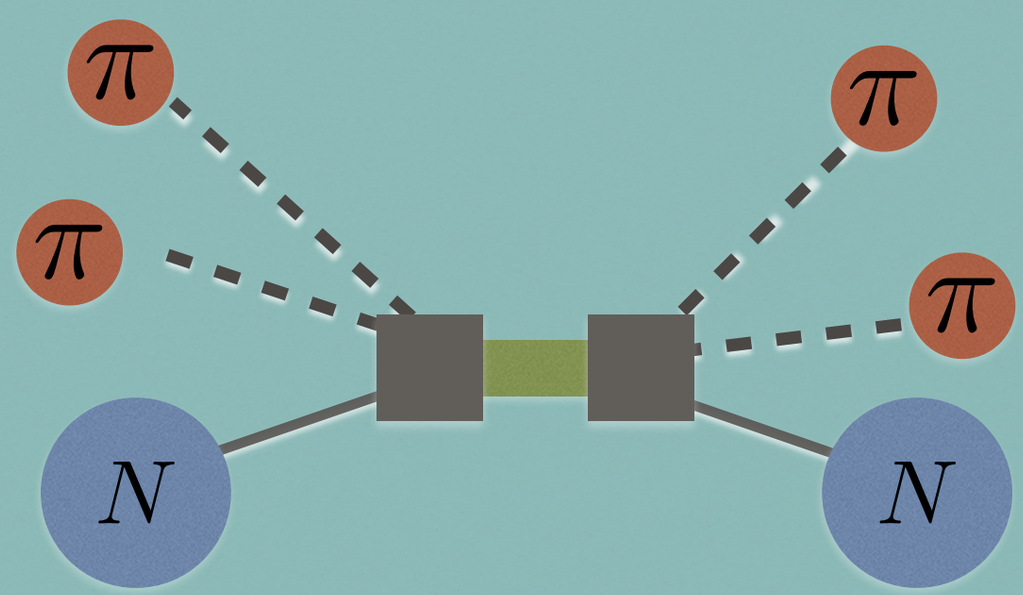


ISOBAR MODEL





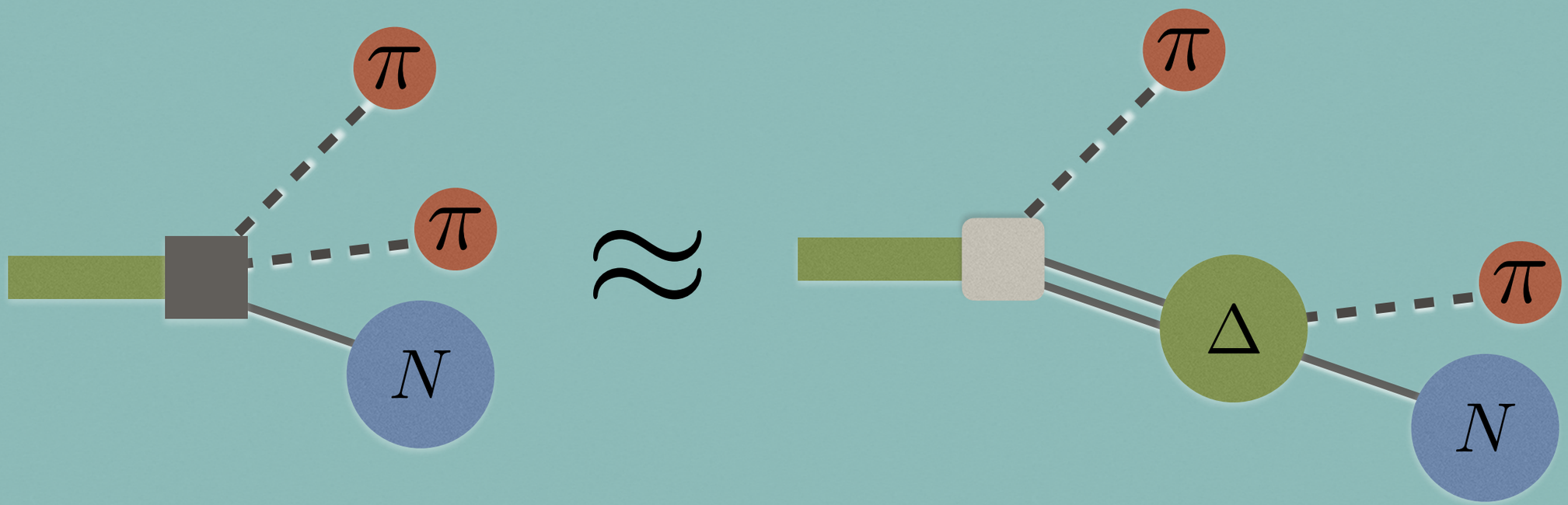




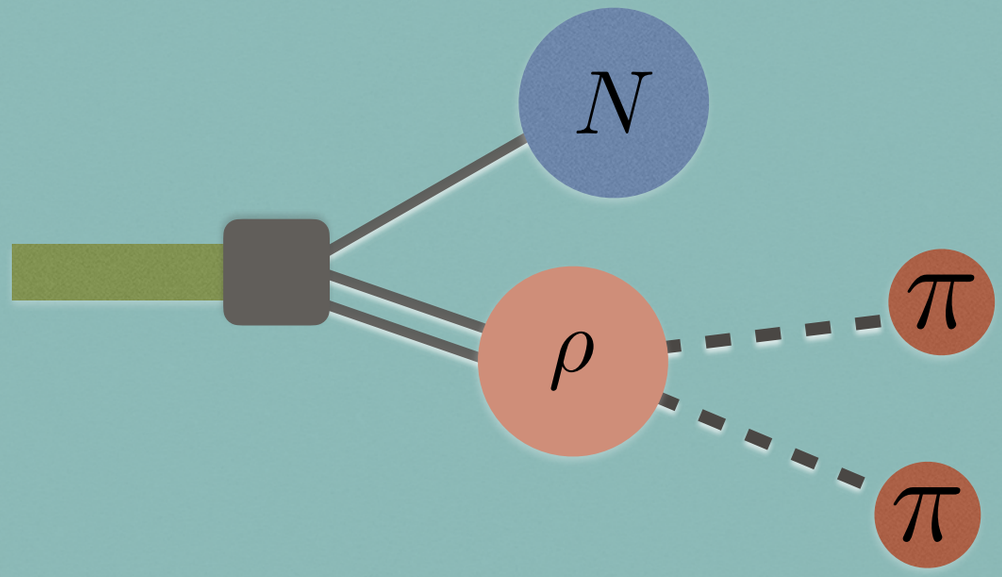
NEED THIS!

ISOBAR MODEL

sequential decay model



and / or



LET'S WORK IT OUT

- Scattering and Thermodynamics
  bulk
  thermal production
- B VS A
  resonance, R-sheets, virtual states
- In-medium & N>2-body prototype
- HIC phenols: invar. mass spec. & femto
- Virial expansion approach to dense(r) matter

THANK YOU