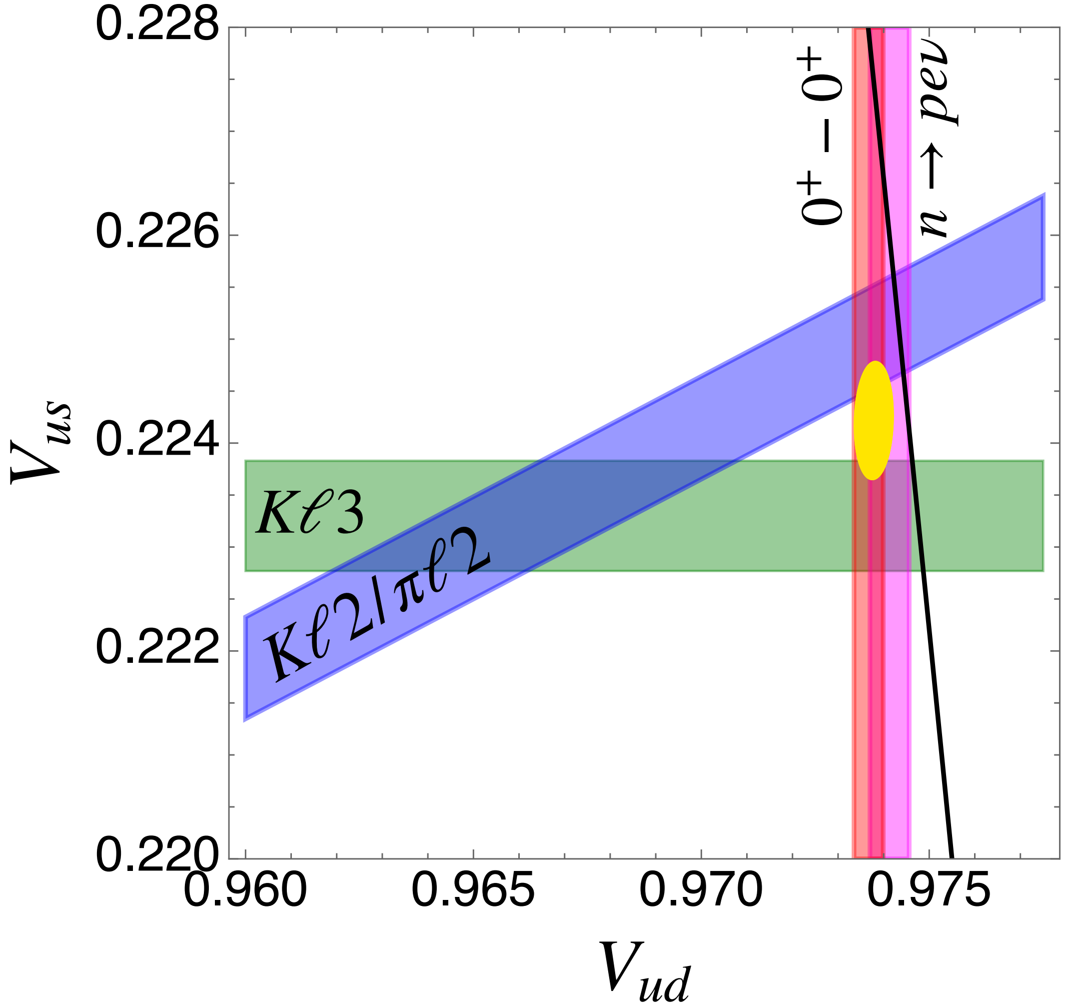
Cabibbo Unitarity: Status and Outlook



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Neutron beta decay review: MG, Seng, Universe **2023**, 9(9), 422, arXiv:**2307.01145**
Nuclear beta decay review: MG, Seng, Ann.Rev.Part.Nucl.Sci. 74 (2024) 23-47, arXiv:**2311.00044**

Present and Future Perspectives in Hadron Physics
Laboratori Nazionali di Frascati & INFN, 17-21 June, 2024

Outline

Status of Cabibbo Angle Anomaly

RC to β-decays: overall setup, scale separation

Nucleon γW-box: Dispersion Theory, lattice QCD and EFT

Nuclear corrections: ab-initio & EFT

BSM in β-decays

Summary & Outlook

Quark Mixing & CKM Unitarity

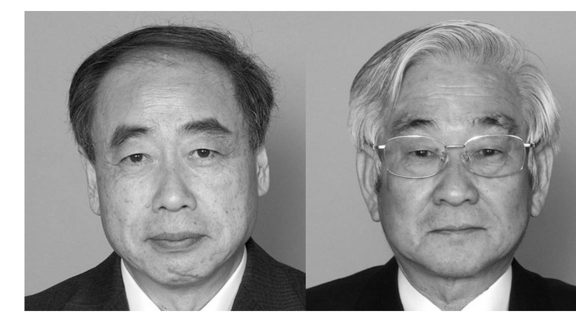
Rate of weak decays of d,s quarks lower than that of μ

Cabibbo: mass and flavor eigenstates connected by Cabibbo angle θ_C
Strength of weak interaction is redistributed

$$G_V^{\Delta S=0} = \cos\theta_C G_\mu$$
$$G_V^{\Delta S=1} = \sin\theta_C G_\mu$$

Kobayashi & Maskawa: 3 flavors, CP-violation

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



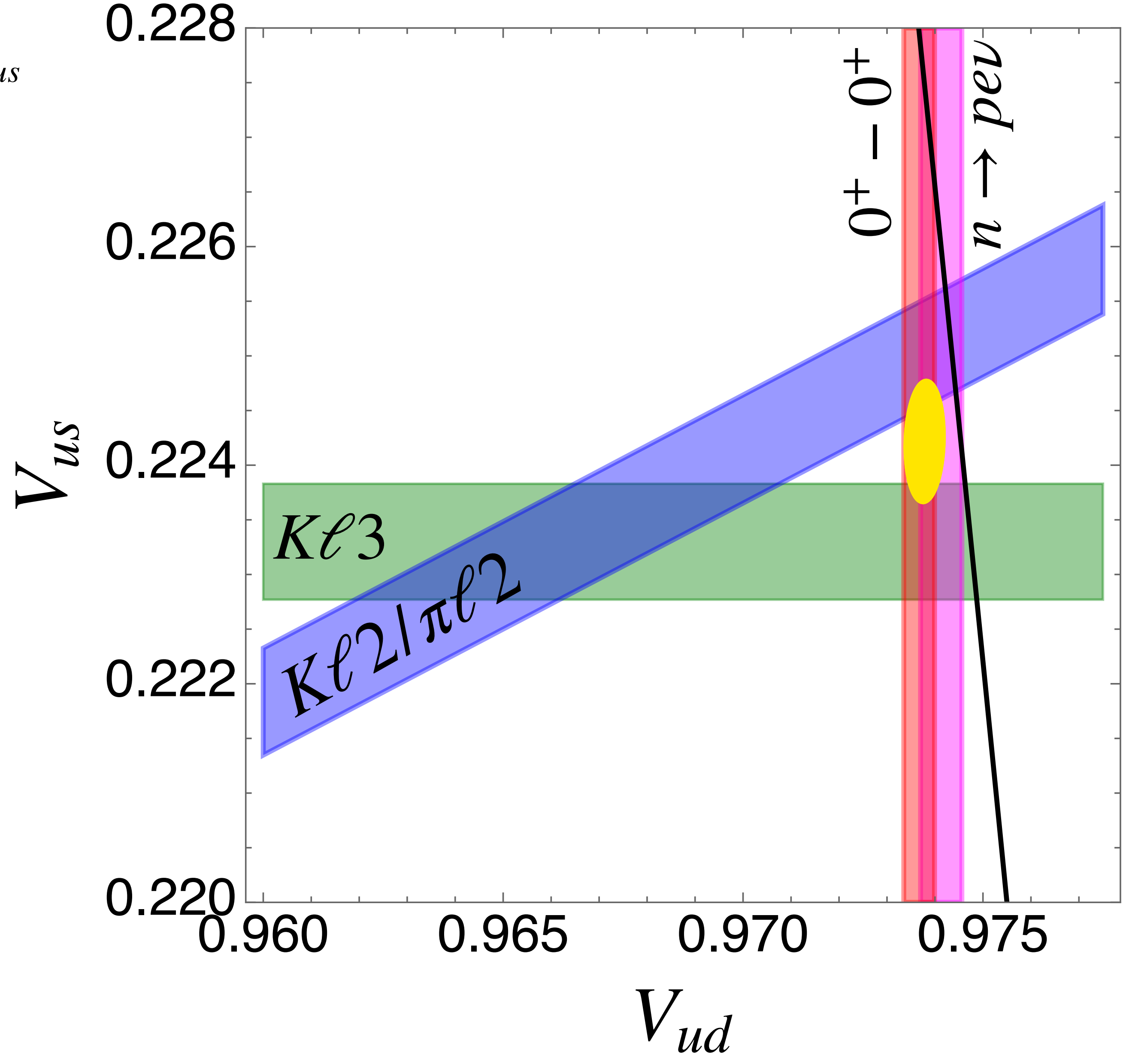
CKM unitarity - measure of completeness of the SM: $VV^\dagger = \mathbf{1}$

Top-row unitarity constraint $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

CKM unitarity is among our best precision tools to test the SM!

Status of Cabibbo unitarity

$$\underset{\sim 0.95}{|V_{ud}|^2} + \underset{\sim 0.05}{|V_{us}|^2} + \underset{\sim 10^{-5}}{\cancel{|V_{ub}|^2}} = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$



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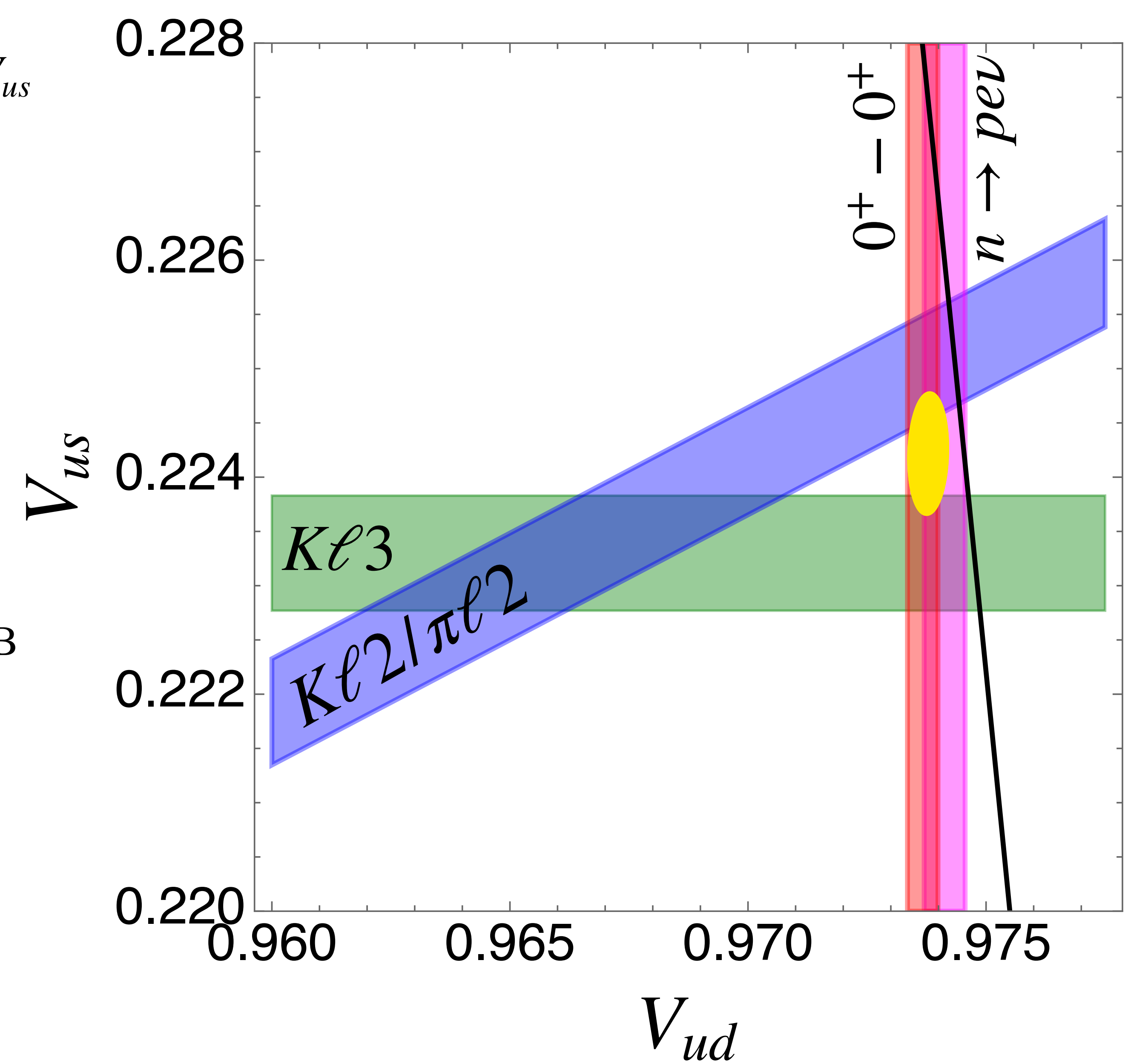
$$V_{ud}^{0^+-0^+} = 0.9737\,(1)_{exp,\,nucl}(3)_{NS}(1)_{RC}[3]_{total}$$

$$V_{ud}^{\text{free n}} = 0.9743\,(3)_{\tau_n}\,(8)_{g_A}\,(1)_{RC}[9]_{total}$$

$$K\ell 2: \quad V_{us}/V_{ud} = 0.23108(23)_{\text{exp}}(42)_{\text{lat}}(16)_{\text{IB}}$$

$$K_{\ell 3}: \quad V_{us} = 0.22330(35)_{\text{exp}}(39)_{\text{lat}}(8)_{\text{IB}}$$



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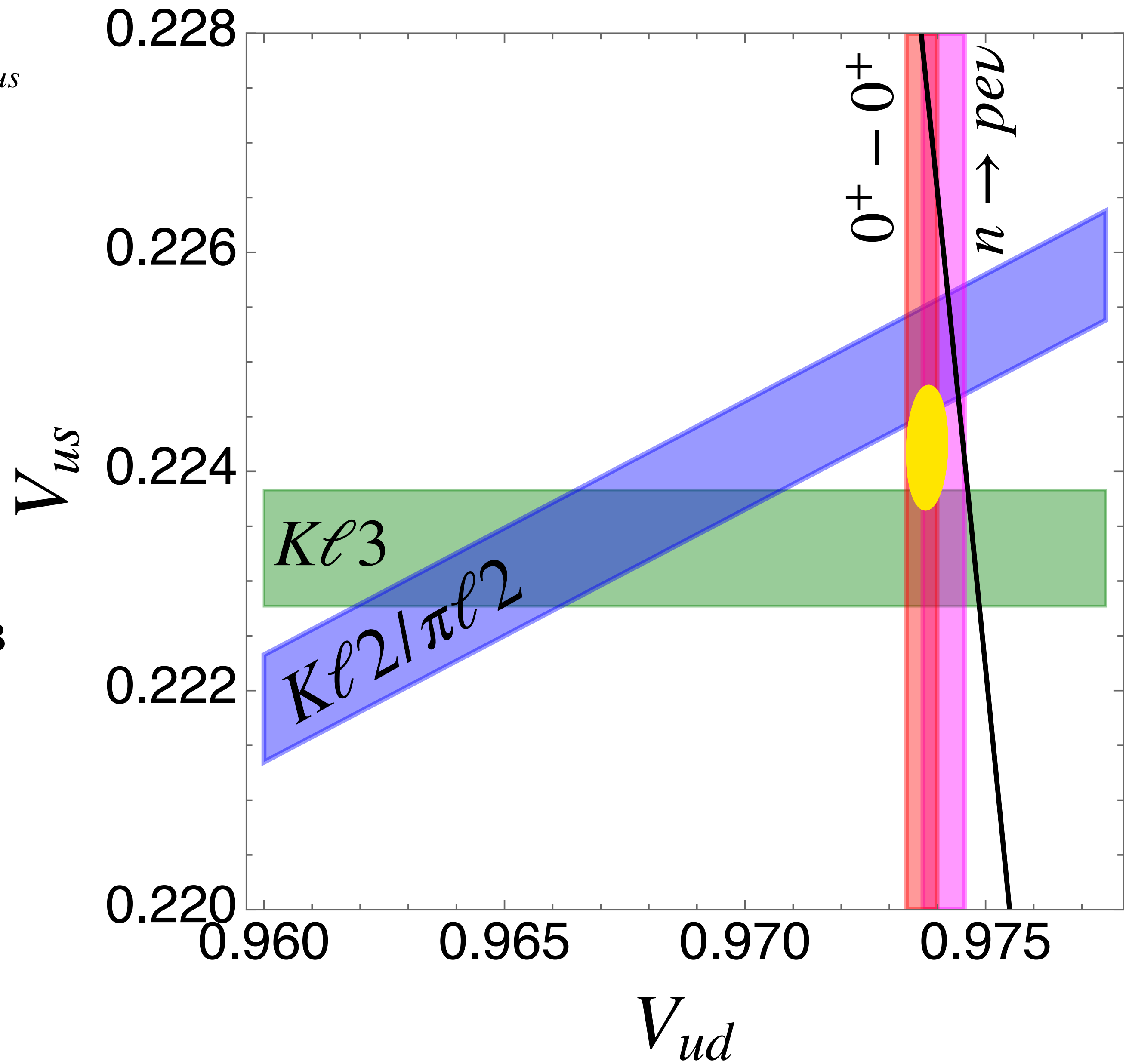

Inconsistencies between measurements of V_{ud} and V_{us} and SM predictions

Main reason for Cabibbo-angle anomaly: shift in V_{ud}

Status of V_{ud}

V_{ud} from neutron decay

Neutron decay: 2 measurements needed

$$V_{ud}{}^2 = \frac{5024.7\,\mathrm{s}}{\tau_n(1 + 3g_A{}^2)(1+\Delta_R^V)}$$

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RC Δ_R^V: bottleneck since 40 years

Pre-2018: $\Delta_R^V = 0.02361(38)$ *Marciano, Sirlin PRL 2006*

Post-2018: $\Delta_R^V = 0.02479(21)$ *MG, Seng Universe 2023*

Since 2018: DR+data+pQCD+EFT+LQCD

Δ_R^V uncertainty: factor 2 reduction

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Experiment: factor 3-5 uncertainties improvement; discrepancies in τ_n and g_A

3.4σ

$g_A = -1.27641(56)$ **PERKEO-III** *B. Märkisch et al, Phys.Rev.Lett. 122 (2019) 24, 242501*

$g_A = -1.2677(28)$ **aSPECT** *M. Beck et al, Phys. Rev. C101 (2020) 5, 055506; 2308.16170*

4σ

$\tau_n = 877.75(28)^{+16}_{-12}$ **UCNτ** *F. M. Gonzalez et al. Phys. Rev. Lett. 127 (2021) 162501*

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PDG average

$$V_{ud}^{\mathrm{free\,n}} = 0.9743\,(3)_{\tau_n}\,(8)_{g_A}\,(1)_{RC}[9]_{total}$$

Single best measurements only

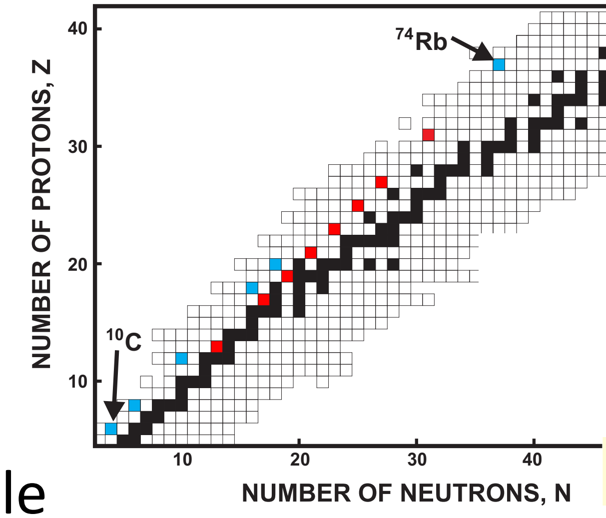
$$V_{ud}^{\mathrm{free\,n}} = 0.9740\,(2)_{\tau_n}\,(3)_{g_A}\,(1)_{RC}[4]_{total}$$

Future exp coming! RC under control

V_{ud} from superallowed decays

Advantages:

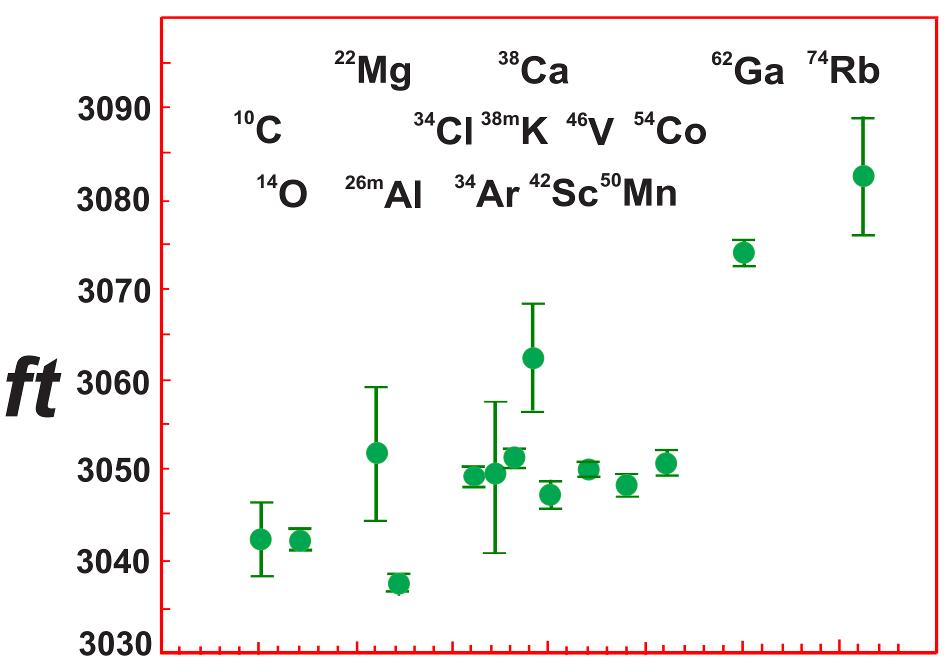
1. Only conserved vector current
2. 15 measured to better than 0.2%
3. 5 measured better than τ_n
4. Internal consistency as a check
5. SU(2) good —> corrections ~small
6. We know a lot about nuclei
7. Only scalar (or vector) BSM accessible



${}^{10}_{6}C \to {}^{10}_{5}B$	${}^{26m}_{13}Al \to {}^{26}_{12}Mg$
${}^{14}_{8}O \to {}^{14}_{7}N$	${}^{34}_{17}Cl \to {}^{34}_{16}S$
${}^{18}_{10}Ne \to {}^{18}_{9}F$	${}^{38m}_{19}K \to {}^{38}_{18}Ar$
${}^{22}_{12}Mg \to {}^{22}_{11}Na$	${}^{42}_{21}Sc \to {}^{42}_{20}Ca$
${}^{26}_{14}Si \to {}^{26}_{13}Al$	${}^{46}_{23}V \to {}^{46}_{22}Ti$
${}^{30}_{16}S \to {}^{30}_{15}P$	${}^{50}_{25}Mn \to {}^{50}_{24}Cr$
${}^{34}_{18}Ar \to {}^{34}_{17}Cl$	${}^{54}_{27}Co \to {}^{54}_{26}Fe$
${}^{38}_{20}Ca \to {}^{38}_{19}K$	${}^{62}_{31}Ga \to {}^{62}_{30}Zn$
${}^{42}_{22}Ti \to {}^{42}_{21}Sc$	${}^{66}_{33}As \to {}^{66}_{32}Ge$
${}^{46}_{24}Cr \to {}^{46}_{23}V$	${}^{70}_{35}Br \to {}^{70}_{34}Se$
${}^{50}_{26}Fe \to {}^{50}_{25}Mn$	${}^{74}_{37}Rb \to {}^{74}_{36}Kr$
${}^{54}_{28}Ni \to {}^{54}_{27}Co$	

Exp.: **f** - phase space (Q value)

t - partial half-life ($t_{1/2}$, branching ratio)



ft values: same within ~2% but not exactly!

Reason: SU(2) slightly broken

a. RC (e.m. interaction does not conserve isospin)

b. Nuclear WF are not SU(2) symmetric

(proton and neutron distribution not the same)

V_{ud} extraction: Universal RC and Universal Ft

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1+\mathrm{RC}+\mathrm{ISB}) = \mathscr{F}\mathrm{t}(1+\Delta_R^V) = \mathrm{ft}(1+\delta_R')(1-\delta_C+\delta_{NS})(1+\Delta_R^V)$$

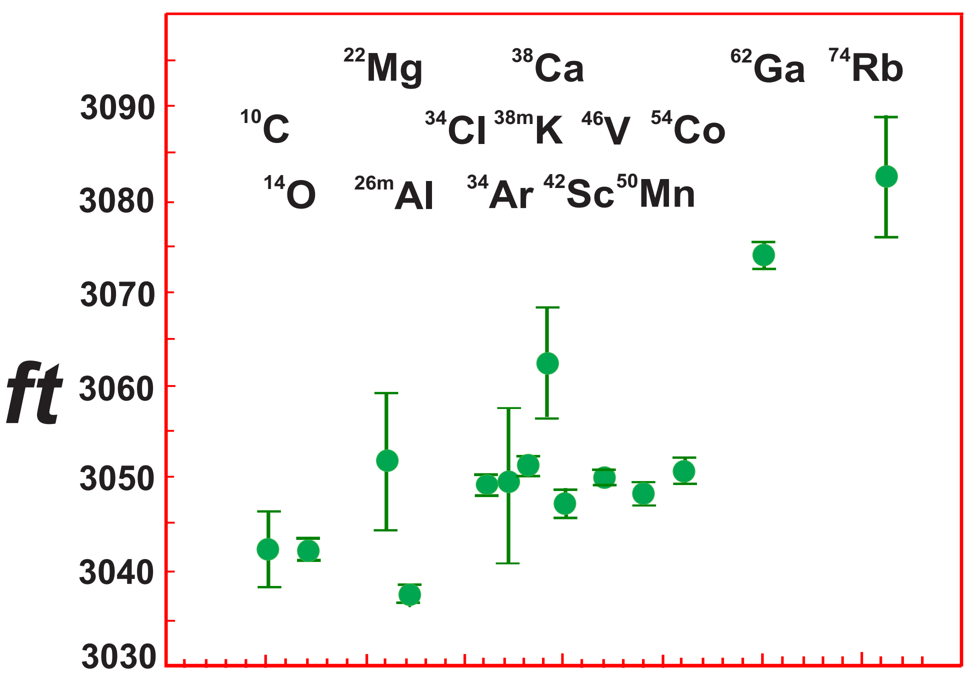
~ Measured

QED

Isospin-breaking

Nuclear structure

Universal inner



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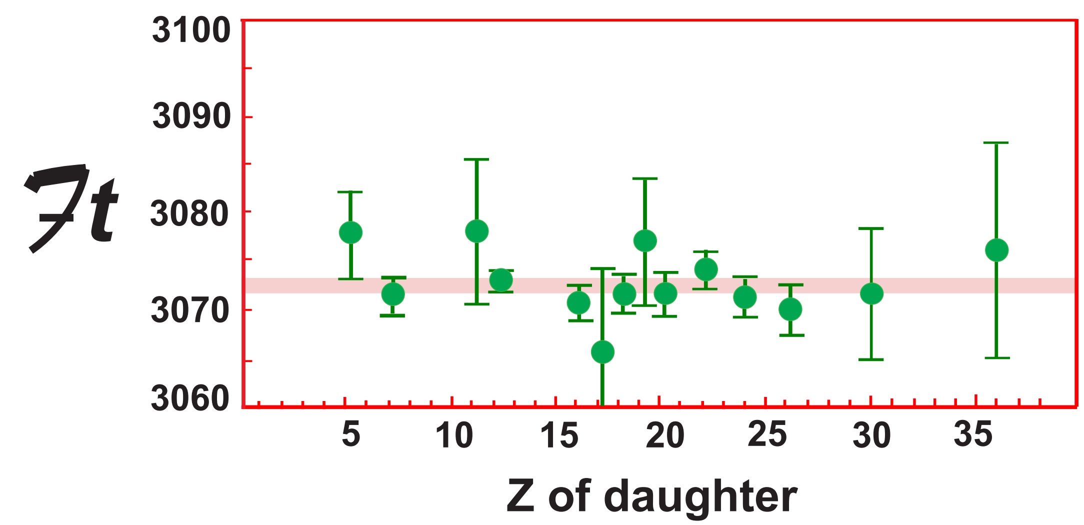


Average of 14 decays Hardy, Towner 1972 - 2020

$$V_{ud}{}^2 = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

Pre-2018: $\overline{\mathscr{F}t} = 3072.1 \pm 0.7\, s$

PDG 2022: $\overline{\mathscr{F}t} = 3072 \pm 2\, s$

$$V_{ud}^{0^+-0^+} = 0.9737\,(1)_{exp,\,nucl}(3)_{NS}(1)_{RC}[3]_{total}$$

V_{ud} from semileptonic pion decay

Pion decay $\pi^+ \to \pi^0 e^+ \nu_e$: theoretically cleanest, experimentally tough

$$V_{ud}{}^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi\ell 3}}{0.3988(23)\,\mathrm{s}^{-1}} \qquad V_{ud}^{\pi\ell 3} = 0.9739\,(27)_{exp}\,(1)_{RC}$$

RC to semileptonic pion decay δ uncertainty: factor 3 reduction

ChPT: $\delta = -0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}}$ *Cirigliano et al, 2003; Passera et al, 2011*

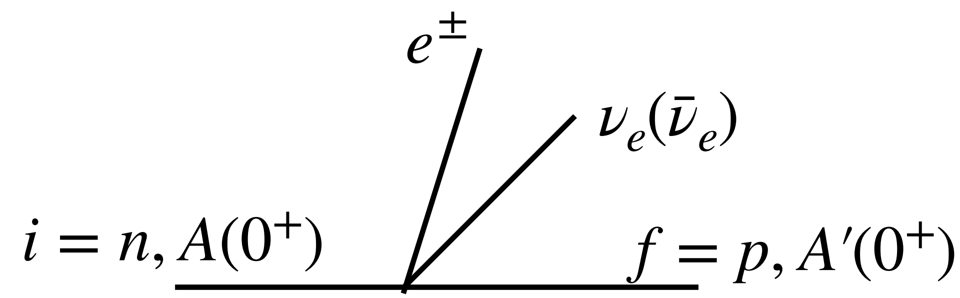
DR + LQCD + ChPT: $\delta = 0.0332(1)_{\gamma W}(3)_{\mathrm{HO}}$ *Feng et al, 2020; Yoo et al, 2023*

Future exp: 1 o.o.m. (PIONEER @ PSI)

RC to beta decay

RC to beta decay: overall setup

Tree-level amplitude



$\sim V_{ud}$

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal for V_{ud} extraction 1×10^{-4}

Electron carries away energy E < Q-value of a decay

E-dep RC: $\frac{\alpha}{2\pi}\left(\frac{E}{\Lambda}, \ln\frac{E}{\Lambda}, \ldots\right)$

Energy scales Λ

Λ

Weak boson scale
$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale
$\Lambda_{\mathrm{had}} = 300\,\mathrm{MeV}$

Universal

Nuclear scale
$\Lambda_{\mathrm{nuc}} = 10 - 30\,\mathrm{MeV}$

Nuclear structure dependent
(QCD)

Decay Q-value (endpoint energy)
$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Nucleus-specific

Electron mass
$m_e \approx 0.5\,\mathrm{MeV}$

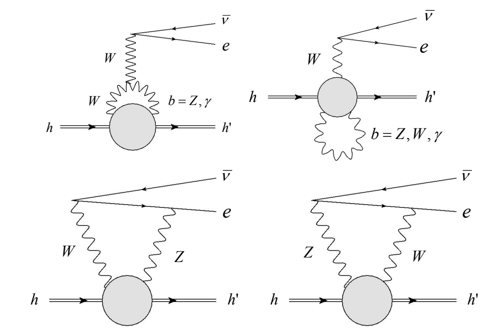
Nuclear structure independent
(QED)

RC to beta decay: separating scales

Generically: only IR and UV extremes feature large logarithms!
Works by Sirlin (1930-2022) and collaborators: all large logs under control

IR: Fermi function (Dirac-Coulomb problem) + Sirlin function (soft Bremsstrahlung)

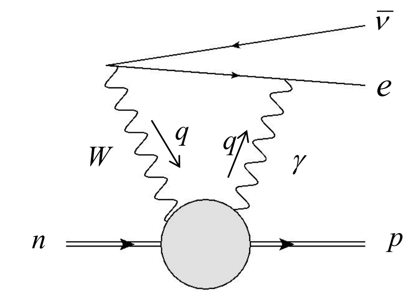
UV: large EW logs + pQCD corrections



Inner RC:
energy- and model-independent

W,Z - loops
UV structure of SM

γW**-box: sensitive to all scales**



New method for computing EW boxes: dispersion theory
Combine exp. data with pQCD, lattice, EFT, ab-initio nuclear

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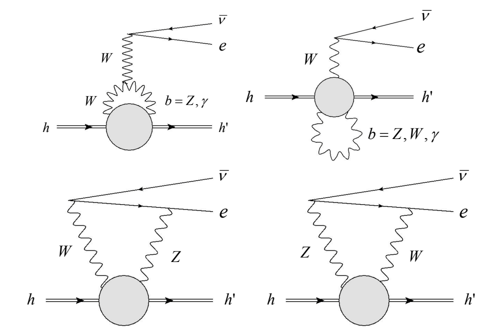

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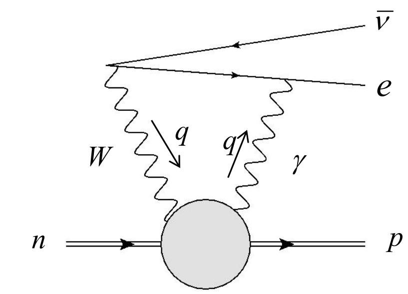

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UV-sensitive γW-box on free neutron Δ_R^V: Sirlin, Marciano, Czarnecki 1967 - 2006

$$g_V^2 = V_{ud}{}^{2}\left[1 + \frac{\alpha}{2\pi}\left\{3\ln\frac{M_Z}{M_p} + \ln\frac{M_Z}{M_W} + \tilde{a}_g\right\} + \delta_{\rm QED}^{HO} + 2\,\Box_{\gamma W}\right]$$

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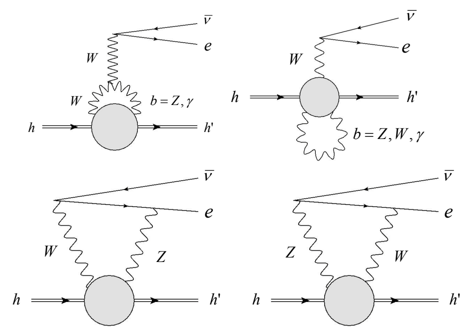

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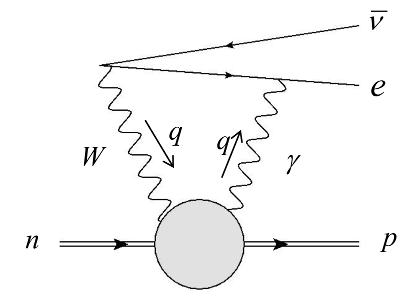

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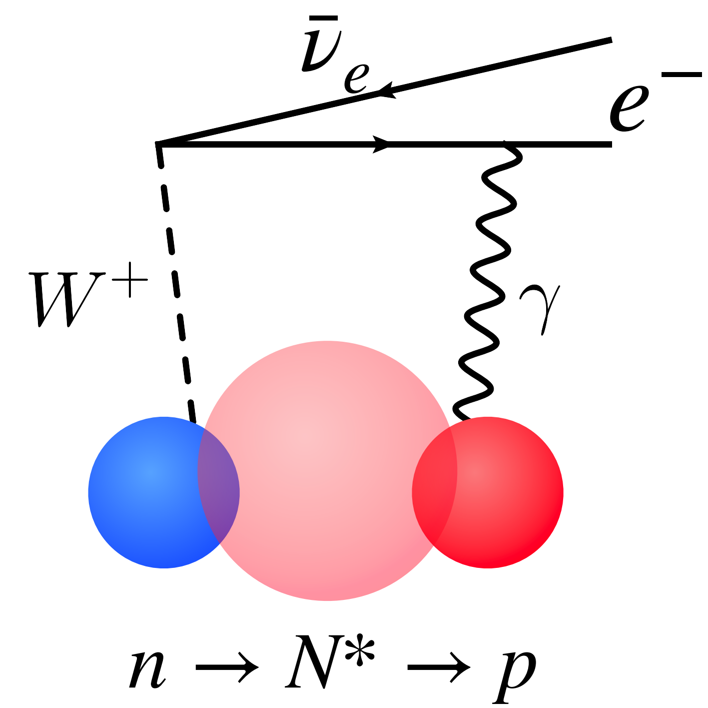
Nuclear structure: $\delta_{\mathrm{NS}} = 2(\square_{\gamma W}^{\mathrm{Nucl}} - \square_{\gamma W}^{\mathrm{free\,n}})$

All non-enhanced terms $\sim \alpha/2\pi \sim 10^{-3}$ — only need to ~10%

Δ_R^V from dispersion relations, LQCD, EFT

Universal RC from dispersion relations

UV large log — model independent (Parton model + pQCD)
Sensitivity to nonperturbative QCD: inclusive hadron spectrum



Interference γW structure functions

$$\mathrm{Im}T^{\mu\nu}_{\gamma W} = \ldots + \frac{i\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2(pq)} F_3^{\gamma W}(x, Q^2)$$

After some algebra

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) = \frac{\alpha}{\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+2\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^2}\frac{F_{3,-}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^2)$$

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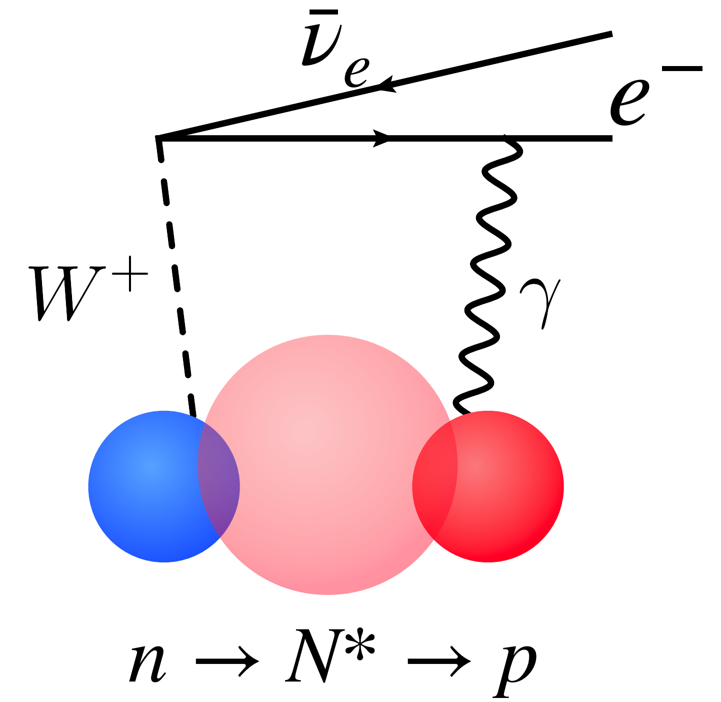

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2-fold integral: depending on Q^2 different physical pictures dominate

Explicit energy dependence quantifiable (earlier was neglected)

Input into dispersion integral - $\nu/\bar{\nu}$ data

Mixed CC-NC γW SF (no data) <—> Purely CC WW SF (inclusive neutrino data)

Isospin symmetry: vector-isoscalar current related to vector-isovector current

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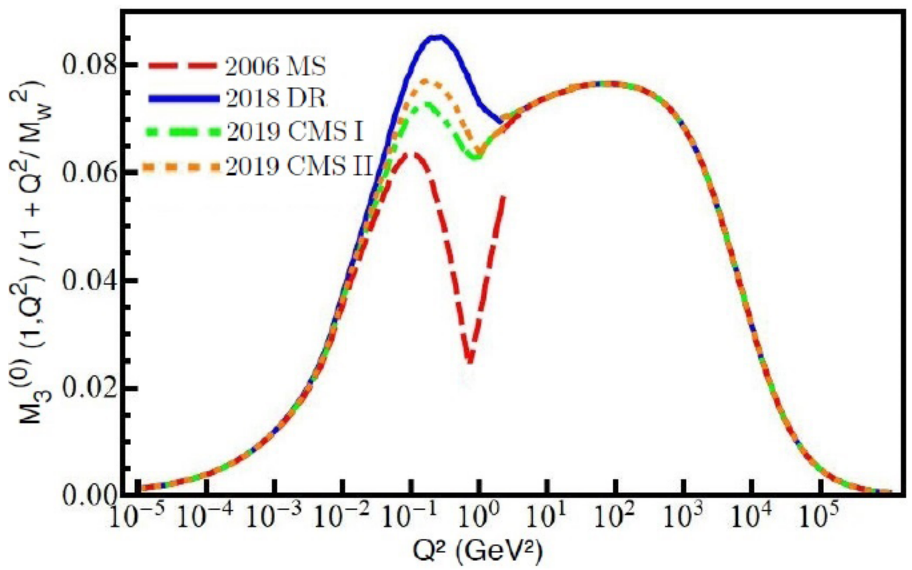
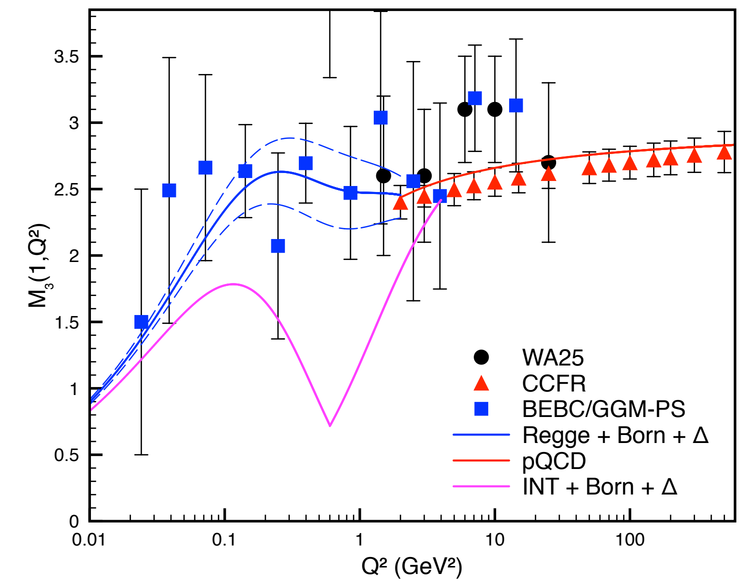
Free neutron γW box



Neutrino scattering data



Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $V_{ud} = 0.97420(10)_{Ft}(18)_{RC}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22)$ —> $V_{ud} = 0.97370(10)_{Ft}(10)_{RC}$

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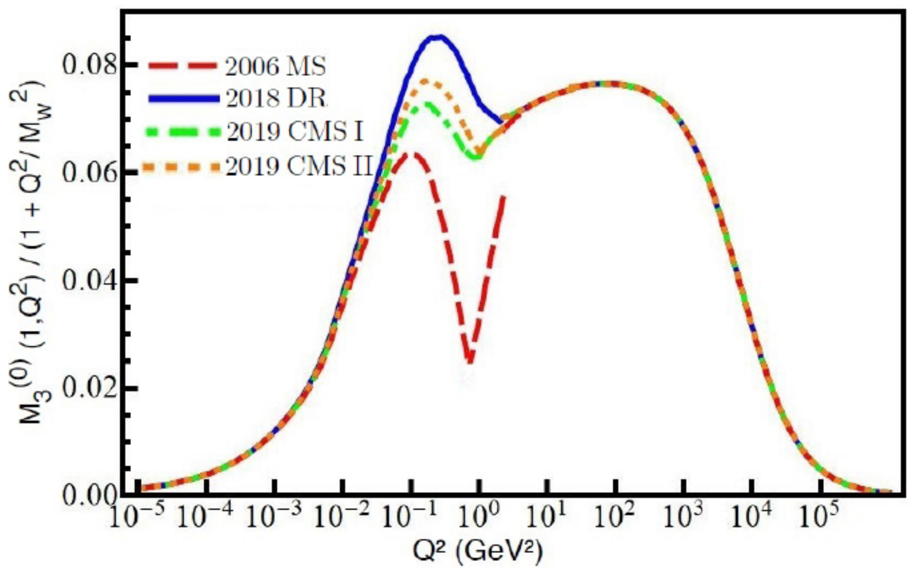

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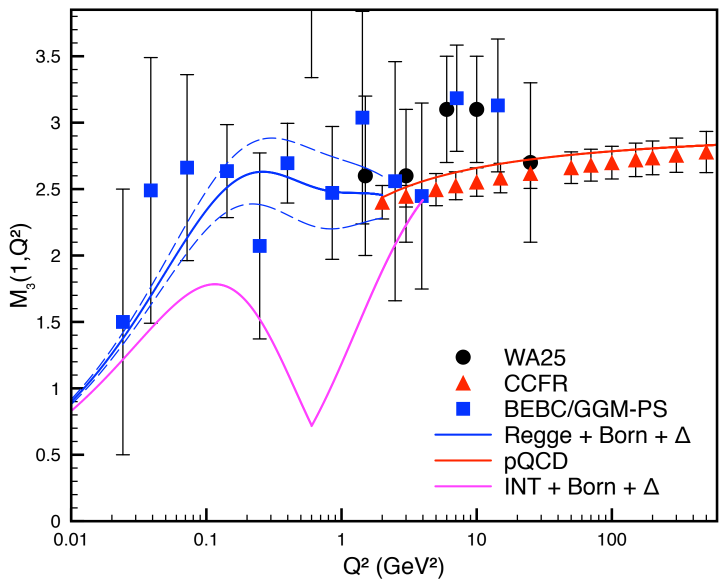


Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $V_{ud} = 0.97420(10)_{Ft}(18)_{RC}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22)$ —> $V_{ud} = 0.97370(10)_{Ft}(10)_{RC}$

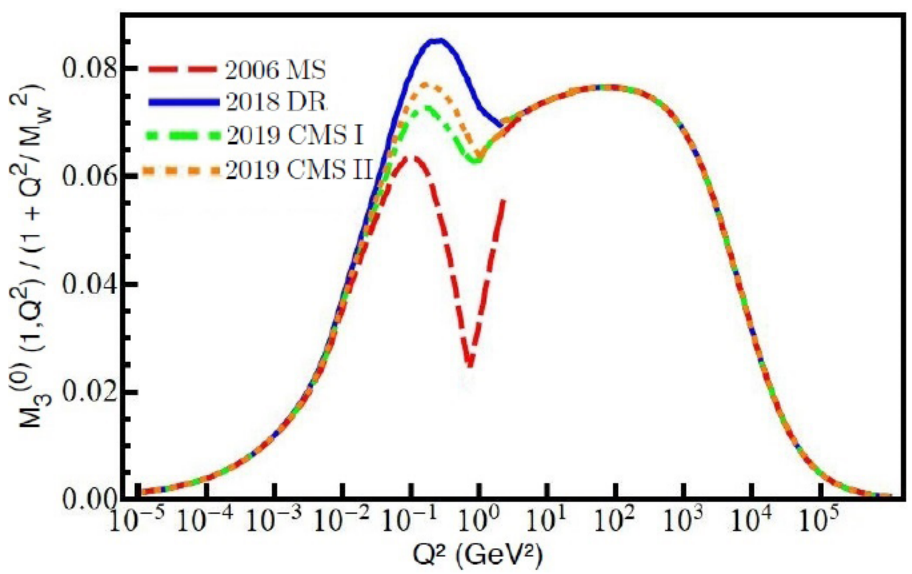
Shift upwards by 3σ + reduction of uncertainty by factor 2

Input into dispersion integral - $\nu/\bar{\nu}$ data

Mixed CC-NC γW SF (no data) <—> Purely CC WW SF (inclusive neutrino data)

Isospin symmetry: vector-isoscalar current related to vector-isovector current

Free neutron γW box



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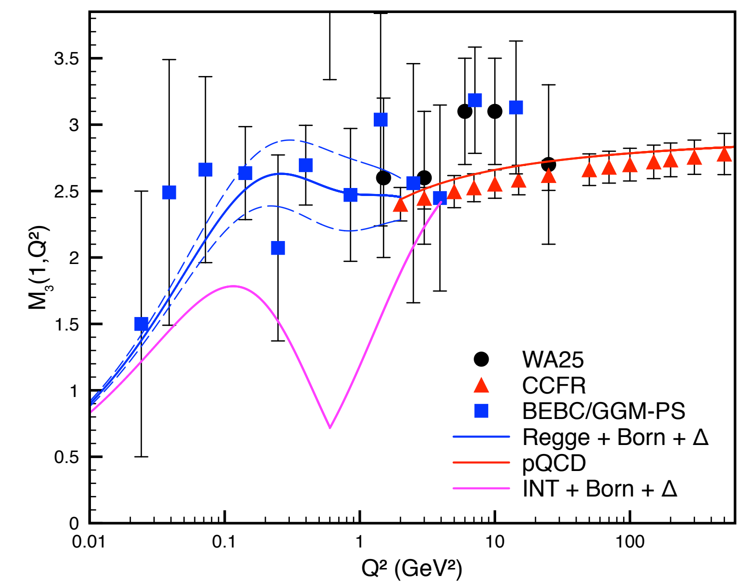

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Confirmed by lattice QCD:

LQCD on pion + pheno: $\Delta_R^V = 0.02477(24)_{\mathrm{LQCD}^\pi+\mathrm{pheno}}$ **Seng, MG, Feng, Jin, 2003.11264** **Yoo et all, 2305.03198**

LQCD on neutron: $\Delta_R^V = 0.02439(19)_{\mathrm{LQCD}^n}$ **Ma, Feng, MG et al 2308.16755**

EFT: scale separation for free n

Cirigliano et al, 2306.03138

Effective Field Theory: explicit separation of scales

- SM —> LEFT (no H,t,Z,W) —> ChPT —> NR QED
- Formal consistency built in, RGE, transparent error estimation (naturalness)
- Precision limited by matching (LEC) and HO — relies on inputs (e.g. γW-box from DR)
- To improve: need to go to higher order — new LECs, still tractable?
- At present: order $O(\alpha, \alpha\alpha_s, \alpha^2)$ — realistic to go beyond?

$$\frac{\mathrm{d}\Gamma_n}{\mathrm{d}E_e} = \frac{G_F^2\,|V_{ud}|^2}{(2\pi)^5}(1+3\lambda^2)\; p_e E_e (E_0 - E_e)^2 \,[g_V(\mu_\chi)]^2\, F_{NR}(\beta)\Big(1+\delta_{\mathrm{RC}}(E_e,\mu_\chi)\Big)\Big(1+\delta_{\mathrm{recoil}}(E_e)\Big)$$

$\lambda = g_A/g_V$
Extract from Experiment

vector coupling

$\pi^2, 1/\beta$
Enhanced

$\mathcal{O}(\alpha)$
[no logs]

$\mathcal{O}(m_e/m_N)$

Total RC: $1 + \Delta_{\mathrm{TOT}} = 1.07761(27)\,\%$

Total RC from DR: $1 + \Delta_{\mathrm{TOT}} = 1.07735(27)\,\%$

Good agreement within errors!

Dispersion Formulation of δ_{NS}

δ_{NS} from dispersion relations

Same formulas for free neutron and nuclei;

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) \;=\; \frac{\alpha}{\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\rm thr}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+2\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^2}\frac{F_{3,-}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^2)$$

$$\Box^{b,\mathrm{o}}_{\gamma W}(E_e) \;=\; \frac{2\alpha E_e}{3\pi}\int_0^\infty dQ^2 \int_{\nu_{\rm thr}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+3\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^3}\frac{F_{3,+}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^3)$$

NS correction reflects extraction of the free box $\quad \delta_{\rm NS} = 2[\,\Box_{\gamma W}^{\rm VA,\,nucl} - \Box_{\gamma W}^{\rm VA,\,free\,n}\,]$

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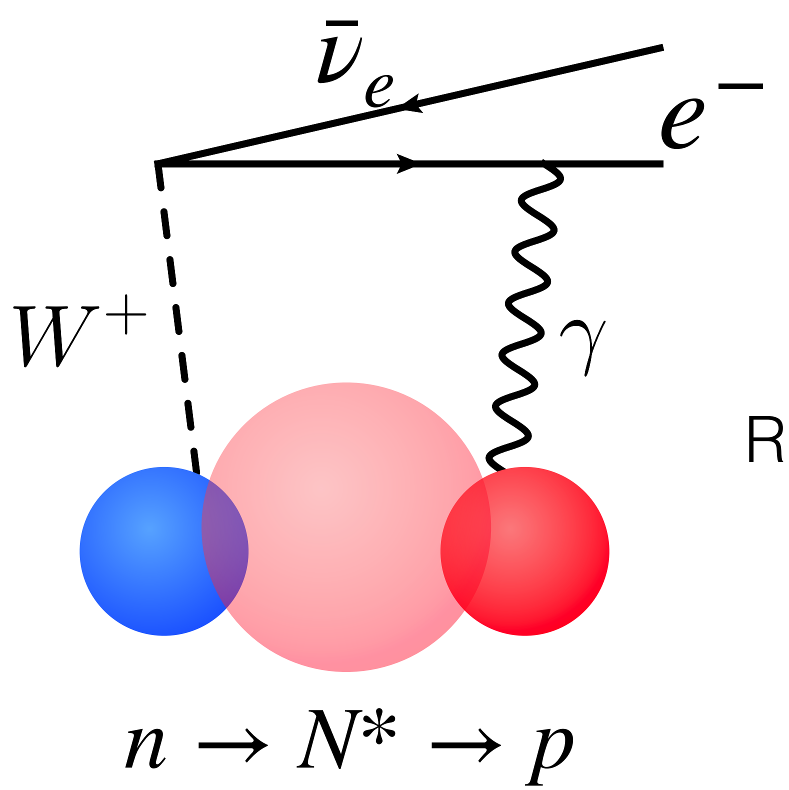
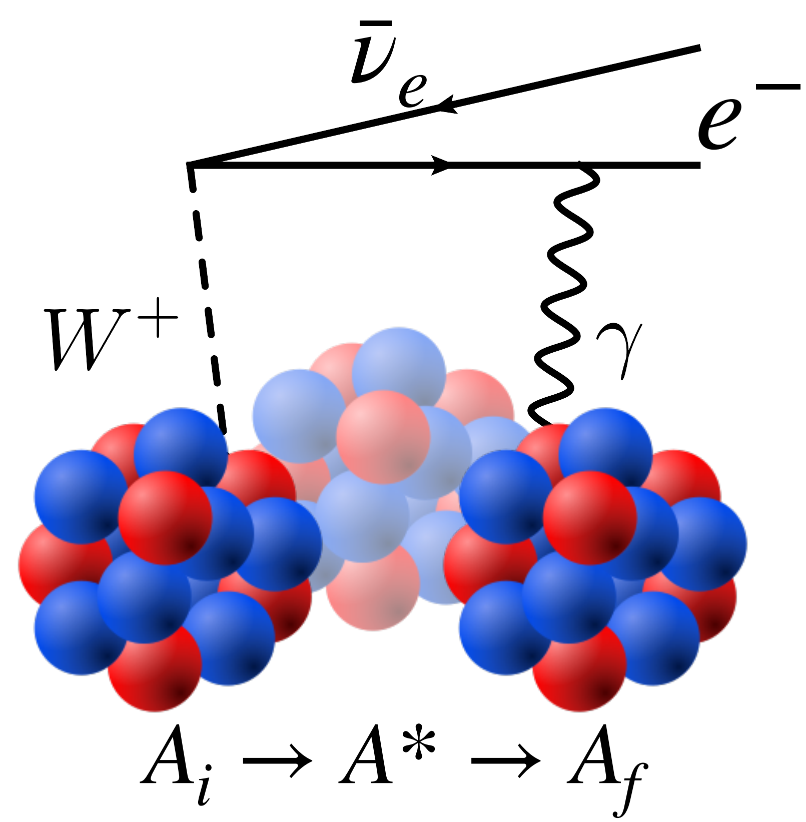


Differences due to:

Richer excitation spectrum in nuclei

Different quantum numbers
(spin, isospin)



δ_{NS} in ab-initio nuclear theory

M. Gennari, M. Drissi, MG, P. Navratil, C.-Y. Seng, arXiv: **2405.19281**

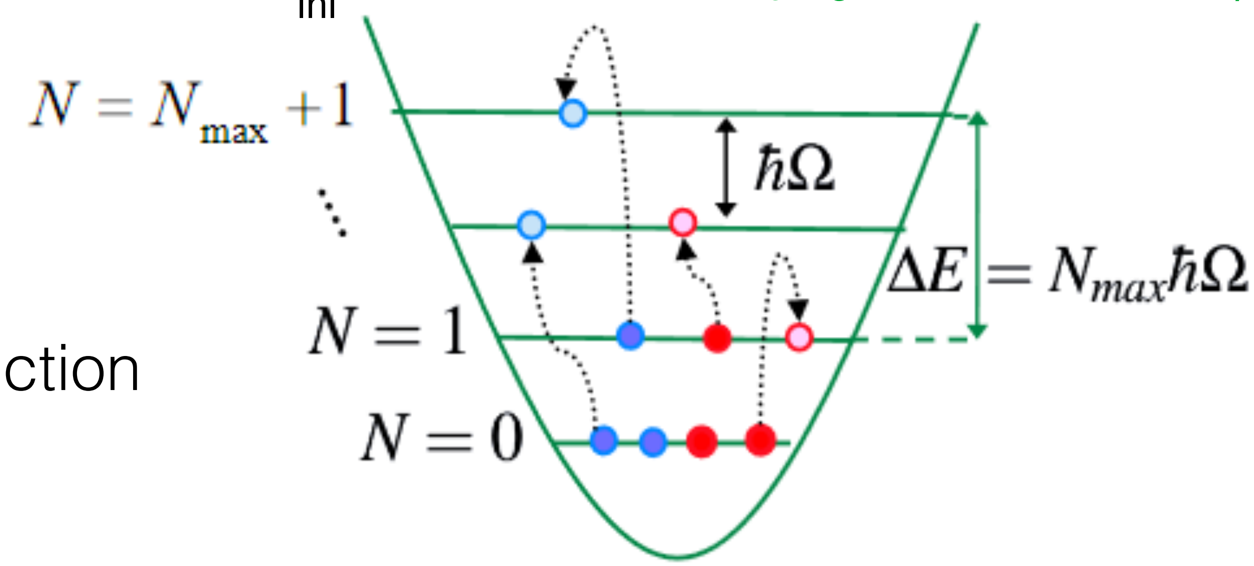
Low-momentum part of the loop: account for nucleon d.o.f. only

First case study: ${}^{10}C \rightarrow {}^{10}B$ in No-Core Shell Model (NCSM)

Many-body problem in HO basis with separation Ω and up to $N = N_{max} + N_{Pauli}$

- Nuclear interactions from Chiral EFT:
  - NN-N^{4}LO+3N$_{lnl}$
  - NN-N^{4}LO+3N*$_{lnl}$

Entem, Machleidt and Nosyk, 2017 PRC;
Gysbers et al., 2019 Nature;
Kravvaris, Navrátil, Quaglioni, Hebborn and Hupin, 2023 PLB



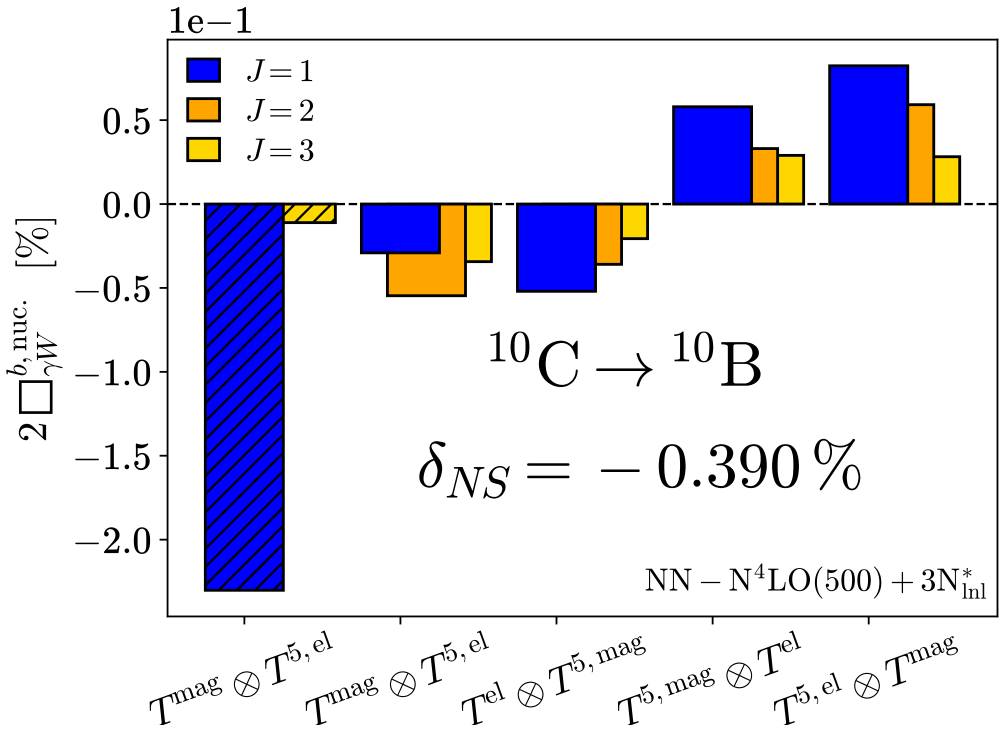
Evaluate the m.e. of nuclear Green's function

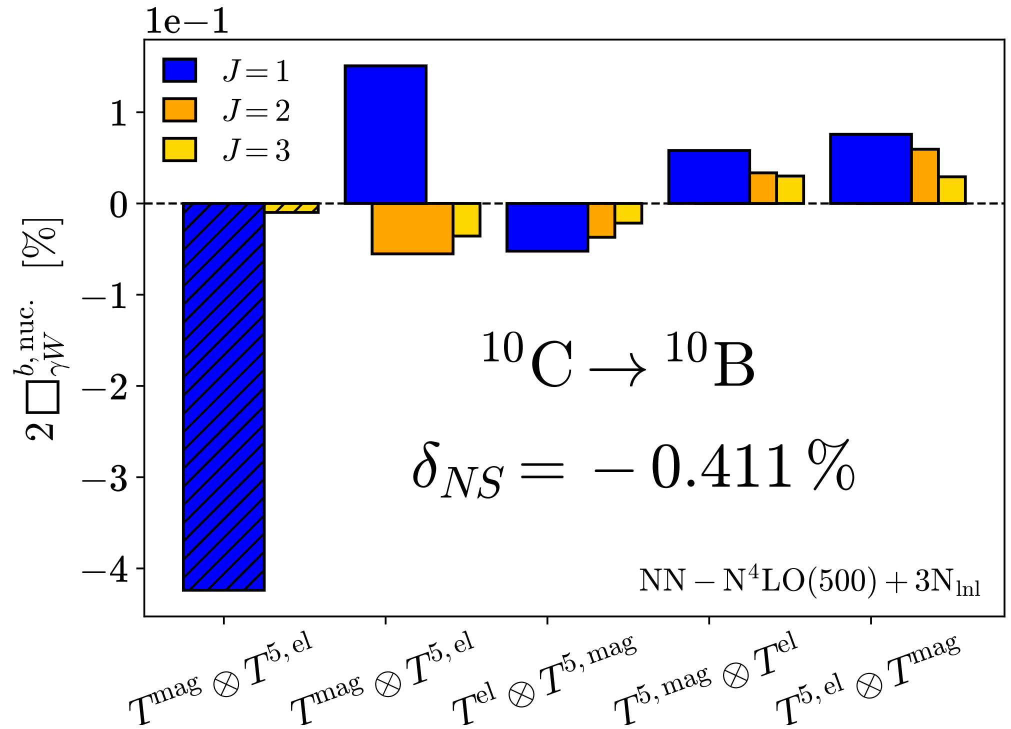
$$G(z) \equiv \frac{1}{z - H_0}$$

Difficulty: Inverting a large matrix!

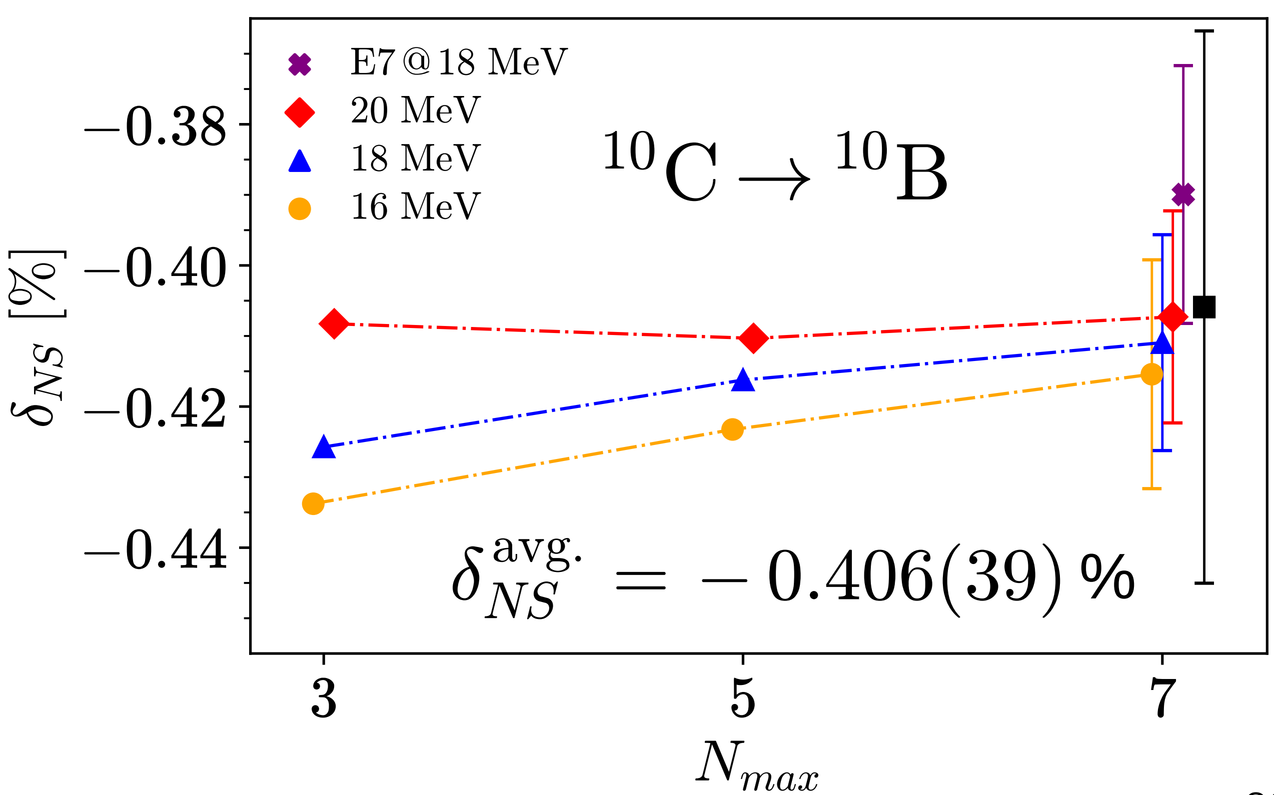
Lanczos continuous fraction method

Ab-initio δ_{NS}: numerical results





Check **Ω-independence** and **convergence w.r.t. N_{max}**



Final result for $^{10}C \to {}^{10}B$:

$$\delta_{\text{NS}} = -0.406(39)\,\%$$

arXiv: **2405.19281**

Compare to Hardy-Towner (old-fashion SM)

$$\delta_{\text{NS}} = -0.347(35)\,\% \qquad (2014)$$

$$\delta_{\text{NS}} = -0.400(50)\,\% \qquad (2020)$$

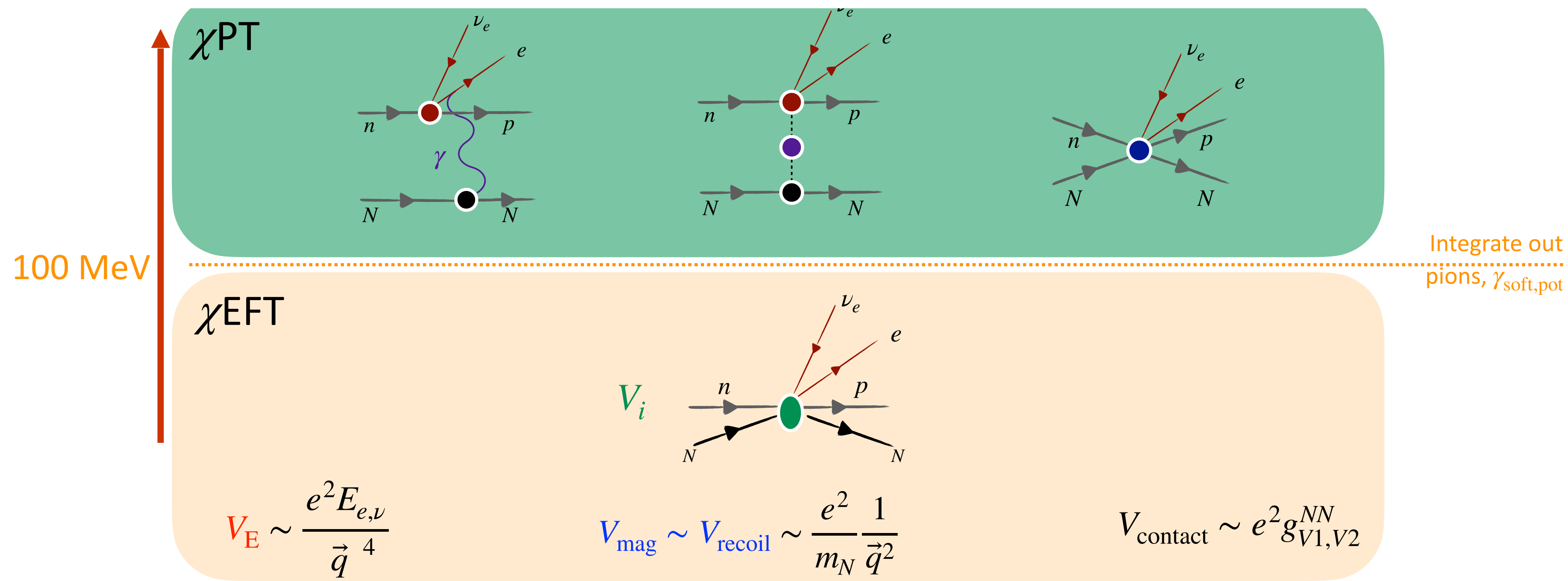
Ab-initio δ_{NS} in EFT: $^{14}O \to {}^{14}N$ with Variational Monte Carlo

V. Cirigliano et al, arXiv: **2405.18469**

$$\delta_{NS}^{(0)} = -\,(1.76 + 0.11 \pm 0.88) \cdot 10^{-3}$$

Uncertainty:
assuming unknown
counter term to be of
"natural size"

$$g^{NN}_{V1,V2} = 1/(4 m_N F_\pi^2)$$



Compare to Hardy-Towner 2020: $\delta_{NS,B} = -\,1.96(50) \cdot 10^{-3}$

Promising avenue: all logs under control and are consistent
Downside: EFT non-renormalizable —> unknown counter terms external to the theory
Need extra input (dispersion theory; explicit modeling; fit to data)

Interpretation of Cabibbo Angle Anomaly

CAA summary - 3 anomalies!

3 observables: $|V_{us}|^{K\ell 3}$, $|V_{us}/V_{ud}|^{K\mu 2}$, V_{ud}
2 quantities to determine: V_{us}, V_{ud}

▶ **3 ways to test unitarity**

$$\Delta^{(1)}_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}^{K_{\ell 3}}|^2 - 1$$ **= –0.00176(56) –3.1σ**

$$\Delta^{(2)}_{\mathrm{CKM}} = |V_{ud}|^2 \left[1 + \left(\left|\frac{V_{us}}{V_{ud}}\right|^{K_{\mu 2}}\right)^2\right] - 1$$ **= –0.00098(58) –1.7σ**

$K_{\mu 2}$ result shows better agreement with unitarity than $K_{\ell 3}$ result when $|V_{ud}|$ obtained from beta decays:

$$\Delta V_{us}(K_{\ell 3} - K_{\mu 2}) = V_{us}^{K_{\ell 3}} - V_{ud}\left(\frac{V_{us}}{V_{ud}}\right)^{K_{\mu 2}}$$ **= –0.0174(73) –2.4σ**

$\Delta^{(3)}_{\mathrm{CKM}}$ uses no information from β decays:

$$\Delta^{(3)}_{\mathrm{CKM}} = |V_{us}^{K_{\ell 3}}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K_{\mu 2}}}\right)^2 + 1\right] - 1$$ **= –0.0164(63) –2.6σ**

CAA in presence of RH currents

Cirigliano et al.
PLB 838 (2023)

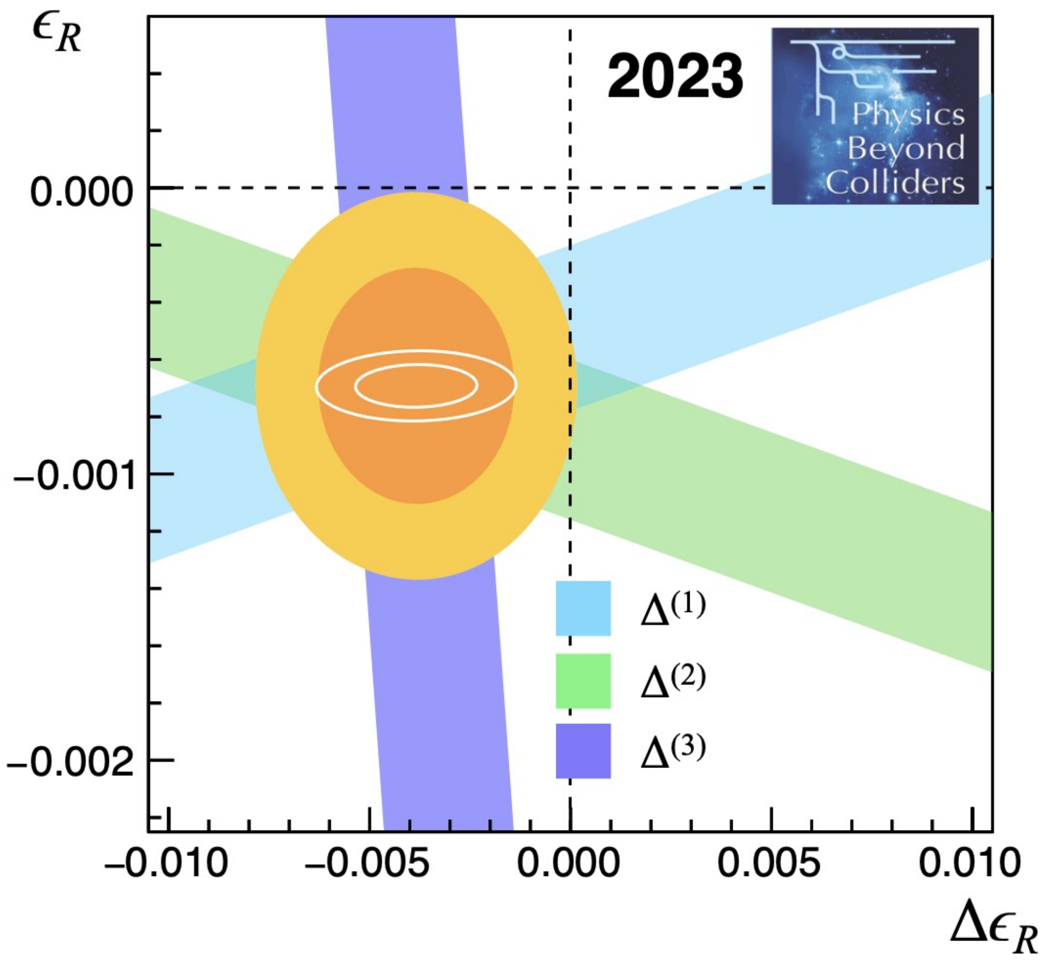
- In SM, W couples only to LH chiral fermion states
- New physics with couplings to RH currents could explain both unitarity deficit and $K_{\ell 3}$-$K_{\mu 2}$ difference
- Define ϵ_R = admixture of RH currents in non-strange sector
  $\epsilon_R + \Delta\epsilon_R$ = admixture of RH currents in strange sector

$$\Delta^{(1)}_{\mathrm{CKM}} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(2)}_{\mathrm{CKM}} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(3)}_{\mathrm{CKM}} = 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2)$$

$$r \equiv \left(\frac{1+\Delta^{(2)}_{\mathrm{CKM}}}{1+\Delta^{(3)}_{\mathrm{CKM}}}\right)^{1/2} = \frac{\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$

From current fit:

$\epsilon_R = -0.69(27) \times 10^{-3}$ (2.5σ)
$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$ (2.4σ)
$\epsilon_R = \Delta\epsilon_R = 0$ excluded at 3.1σ



Summary and Outlook

Summary & Outlook

Cabibbo unitarity deficit at 2-3σ observed
Great improvement in theory in past 5 years (STRONG-2020)

Nuclear uncertainties under scrutiny: δ_{NS} in ab-initio and EFT
δ_C & δ_{NS} for 15 decays from ^{10}C to ^{74}Rb
Community effort required! (STRONG-2030?)

Future neutron experiments:
UCNτ, τSPECT ($\delta\tau_n$: $0.4 \rightarrow 0.1s$); PERC, NAB ($\delta g_A : 4 \rightarrow 1 \times 10^{-4}$)

Kaon decays: NA62 (R.I.P. HIKE)

Cabibbo anomaly interpretable in terms of BSM

Superallowed decays: bounds on scalar BSM from dataset consistency (nuclear theory involved)