THERMAL HADRON YIELDS FROM A COUPLED-CHANNEL ANALYSIS

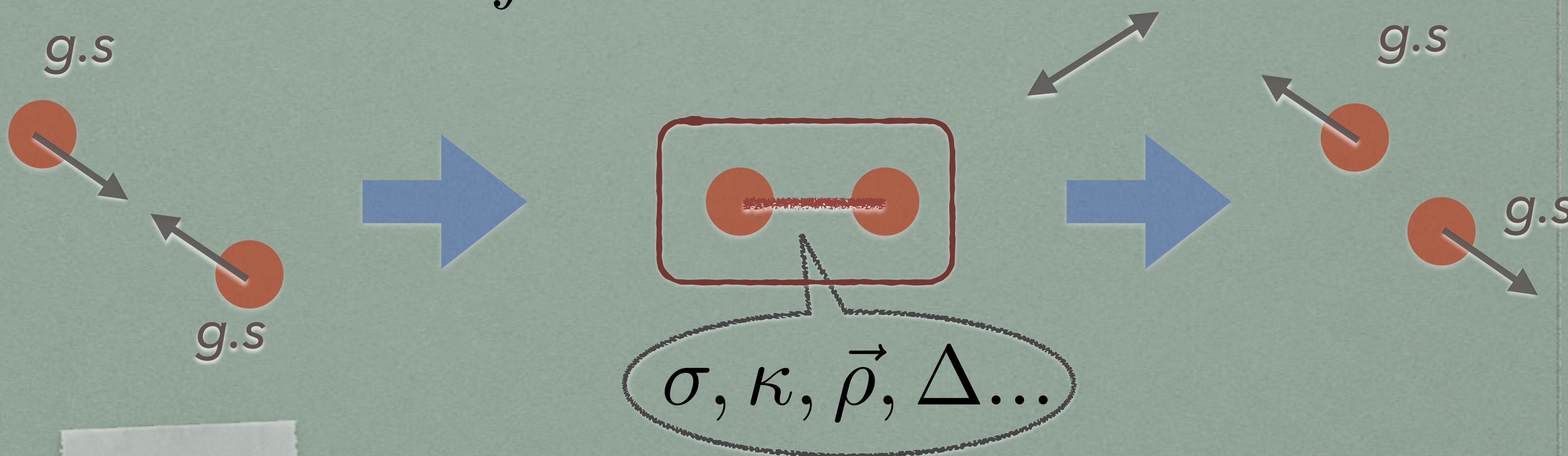
POK MAN LO (盧博文)

University of Wroclaw

17-19.06.2024
PRESENT AND FUTURE PERSPECTIVES IN HADRON PHYSICS, FRASCATI

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE \, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \mathrm{tr} \, (\delta_E) \, .$$



+ repulsions

$$\delta \longrightarrow \mathcal{Q}(M) \equiv \frac{1}{2} \, \mathrm{Im} \, (\mathrm{tr} \ln \mathrm{S})$$

PWA

X

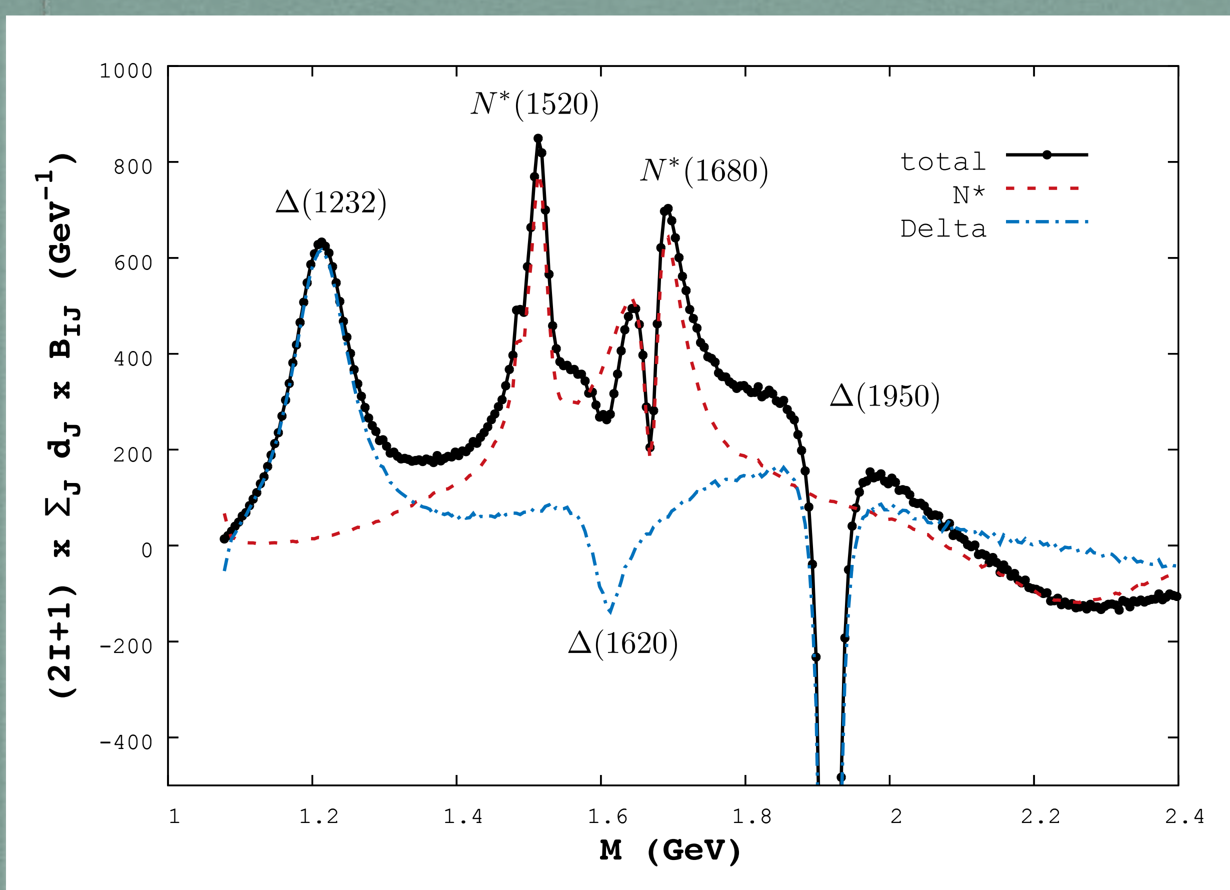
S-matrix thermo.

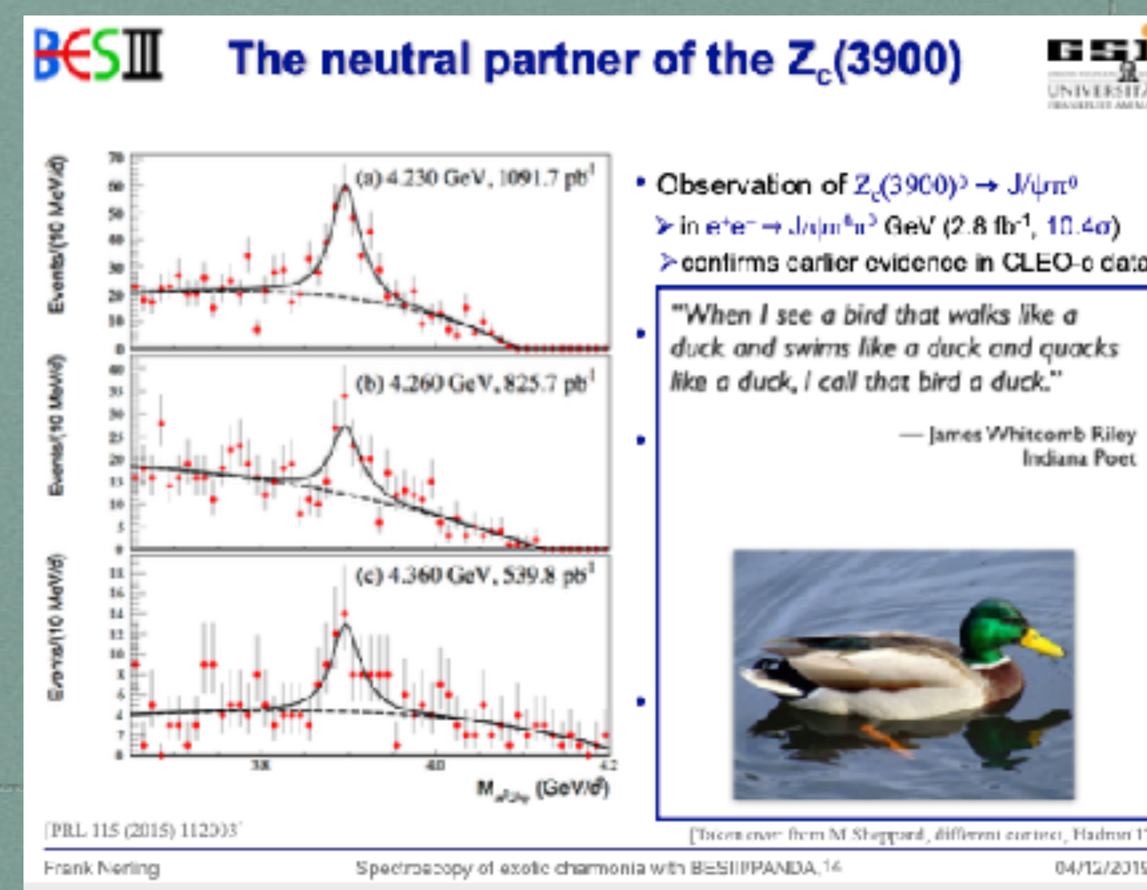
RESONANCES / EXCITATIONS VIA SCATTERING STATES

- broad /overlapping resonances
- molecular states
- threshold effects /cusps

non-resonant interactions: +/-

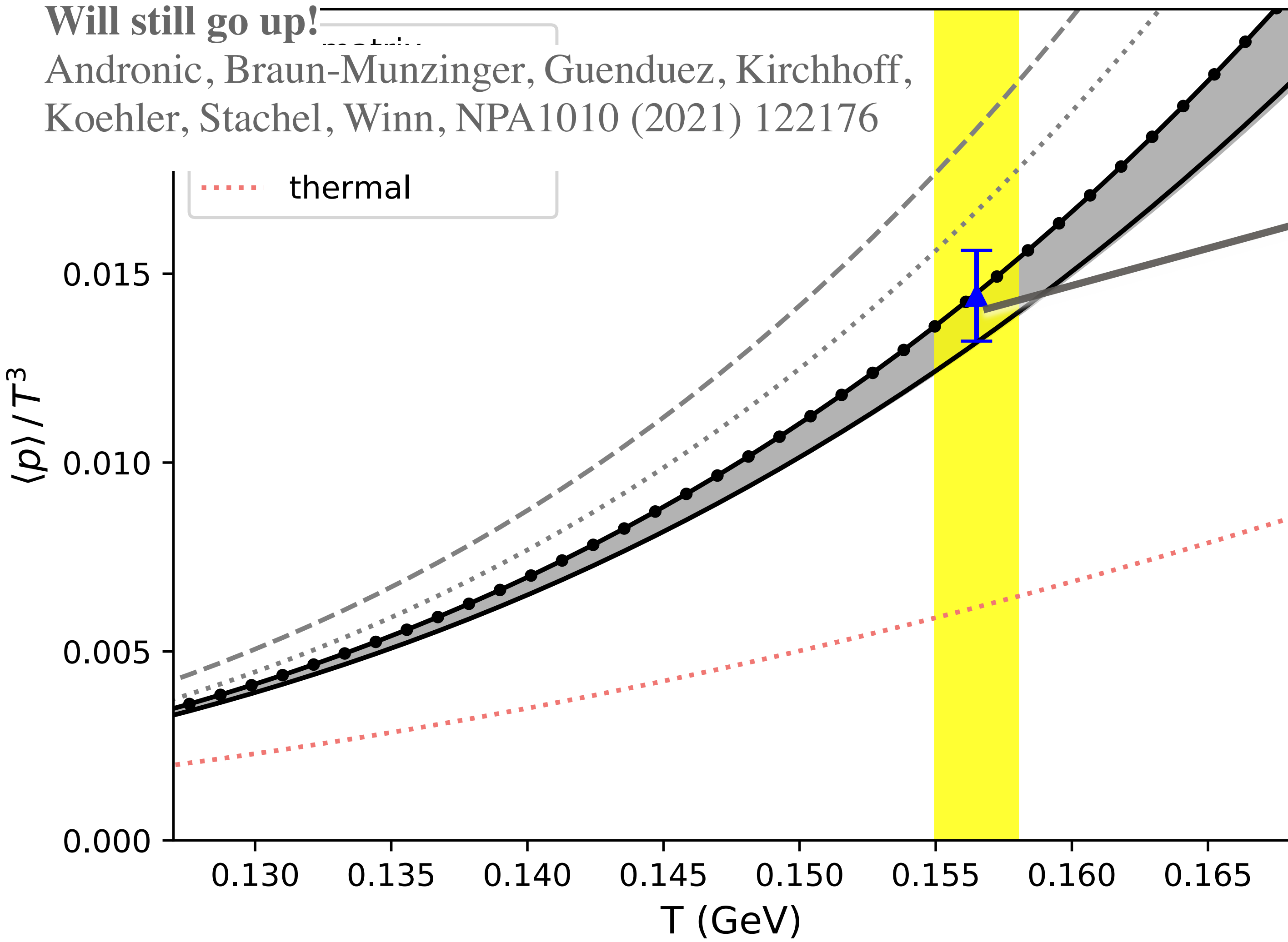
$$\pi N \to \Delta \to \pi N$$





KEY RESULTS

Will still go up!
Andronic, Braun-Munzinger, Guenduez, Kirchhoff, Koehler, Stachel, Winn, NPA1010 (2021) 122176

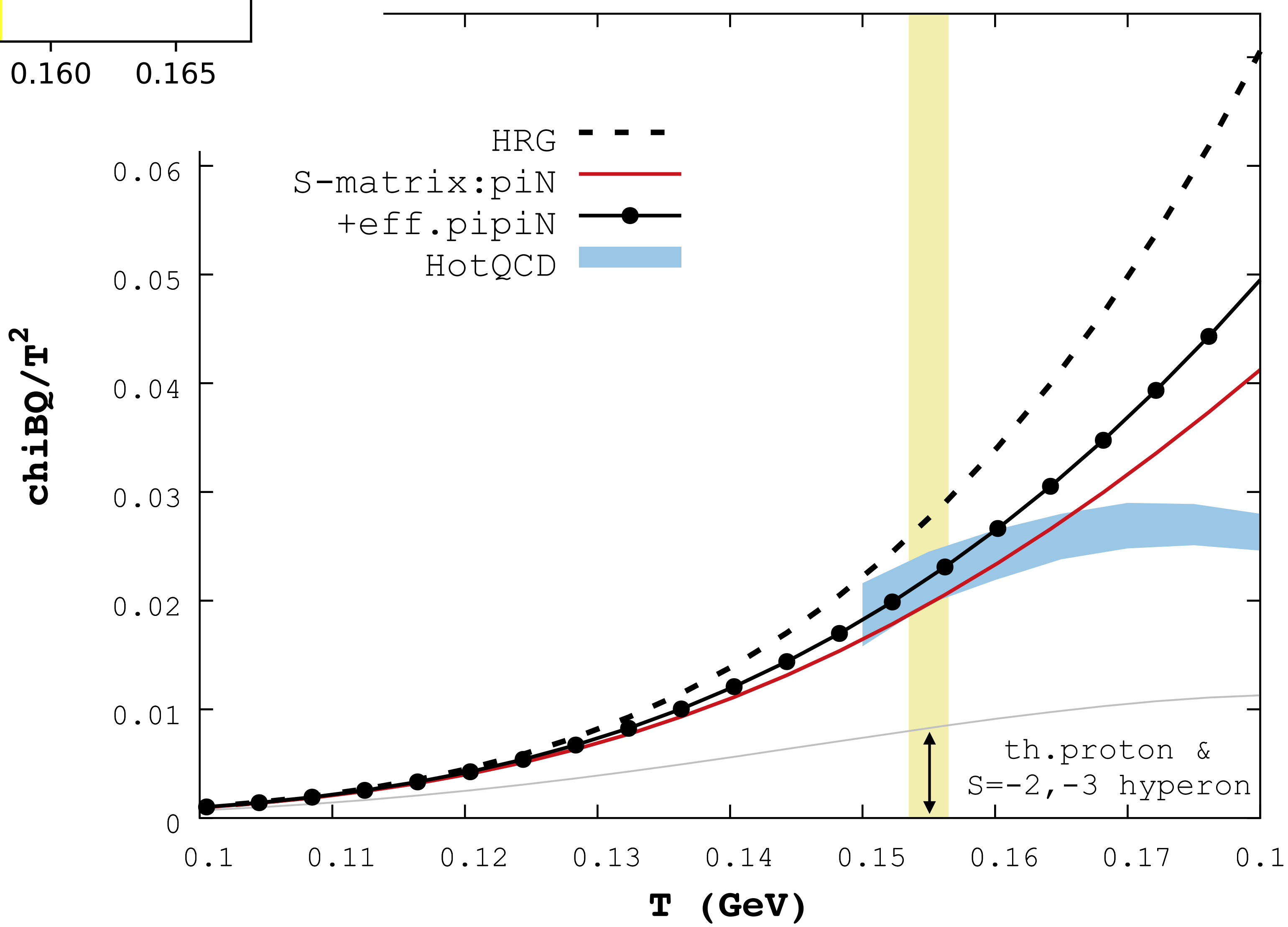


ALICE proton yield
Pb-Pb @ 2.76 TeV
thermal model est.

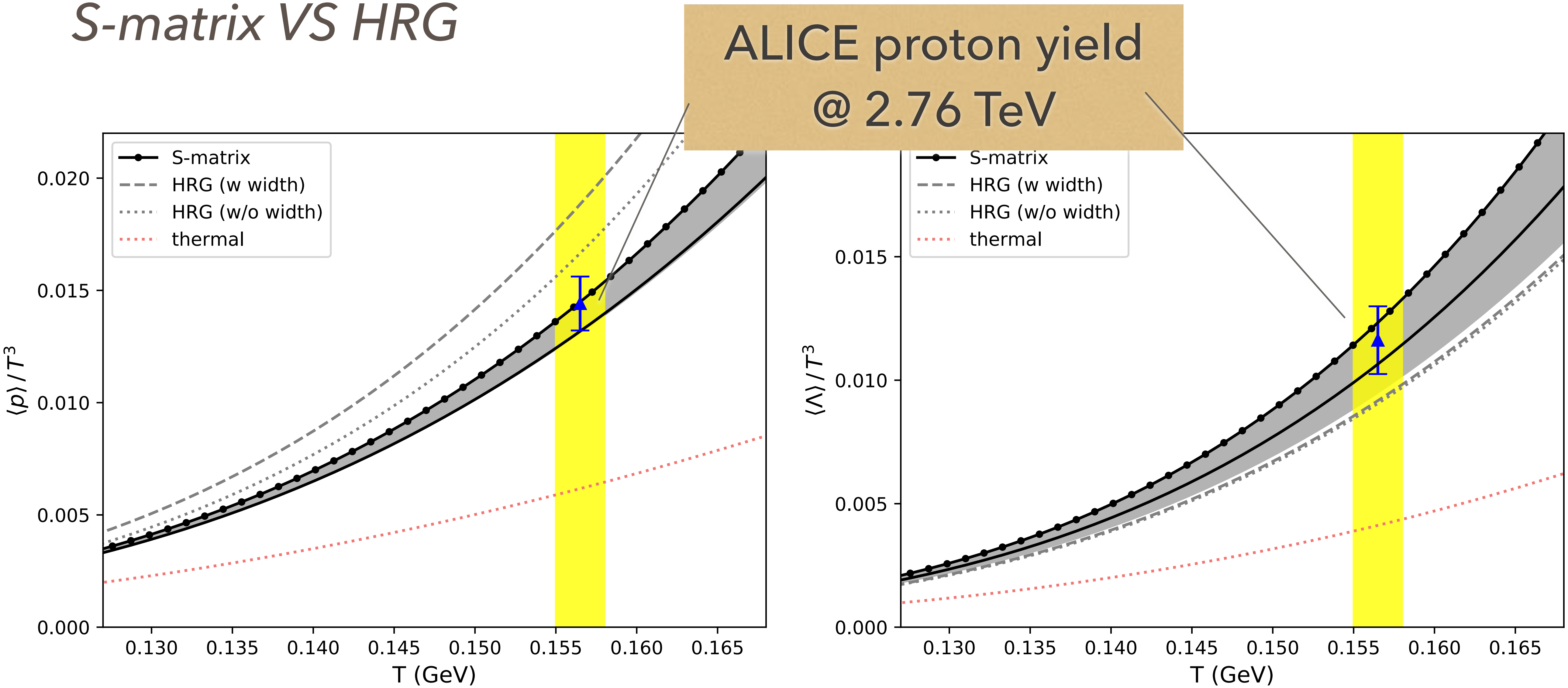
LQCD result on chiBQ

A. Bazavov, et al.,
Phys. Rev. D 86 (2012) 034509.

see also
Bellwied et al.
Phys. Rev. D 101, 034506 (2020)



S-matrix VS HRG



Jefferson Lab
Thomas Jefferson National Accelerator Facility

piN phase shifts
pipiN BGs
hyperons

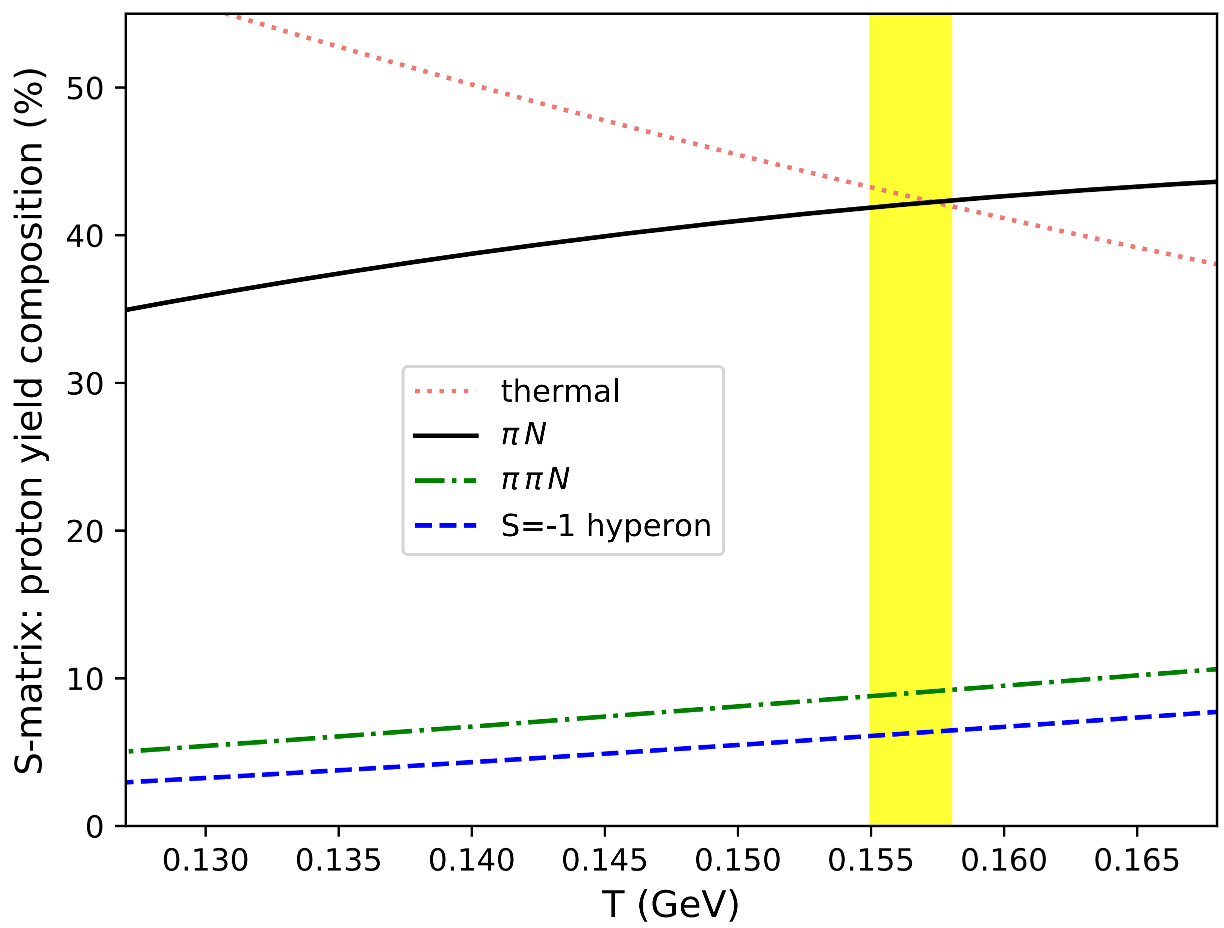
consistent treatment of res and non-res. int.

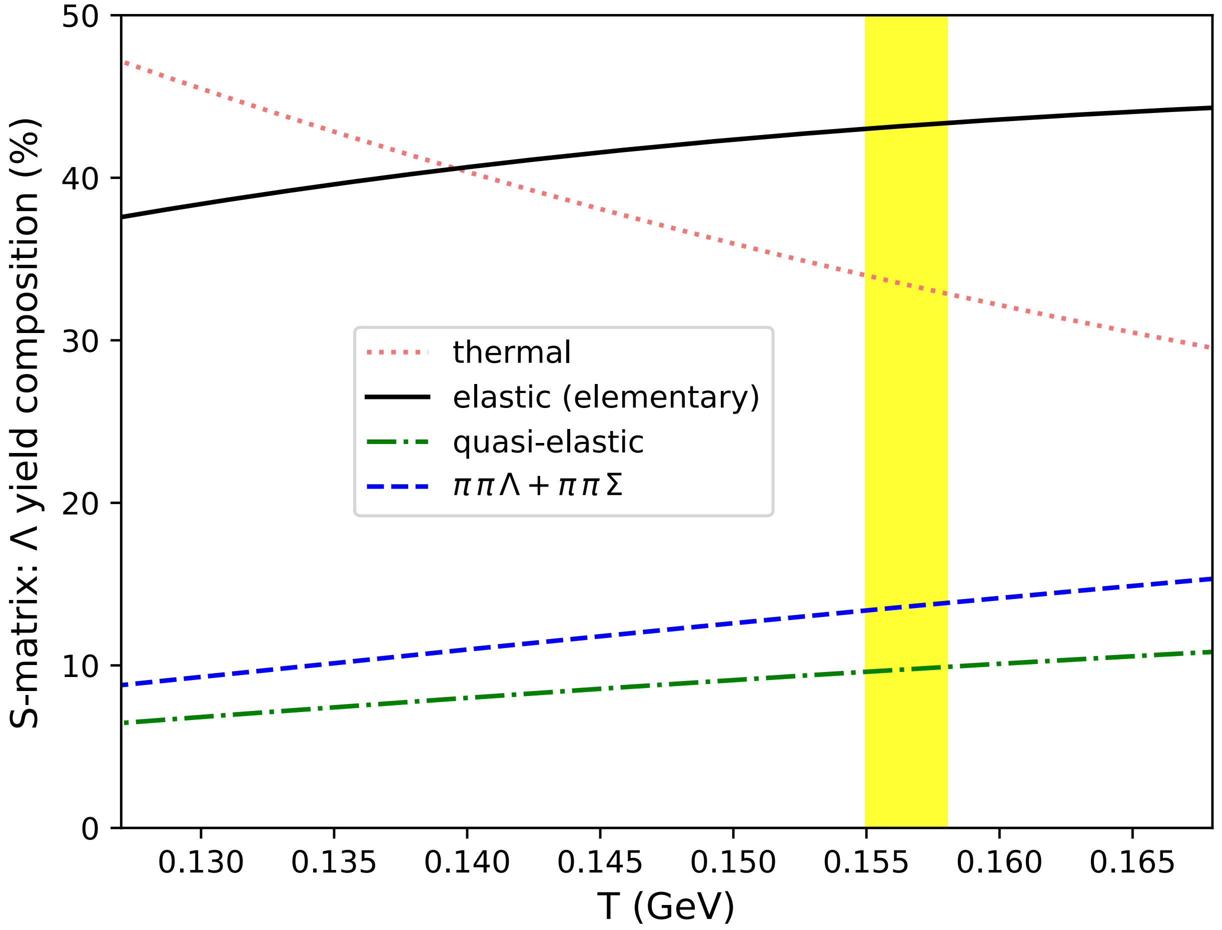
Coupled-Channel model:

$$\bar{k}N, \pi\Lambda, \pi\Sigma, \ldots$$

extra hyperon states beyond PDG

unitarity BGs





SAID GWU

JPAC

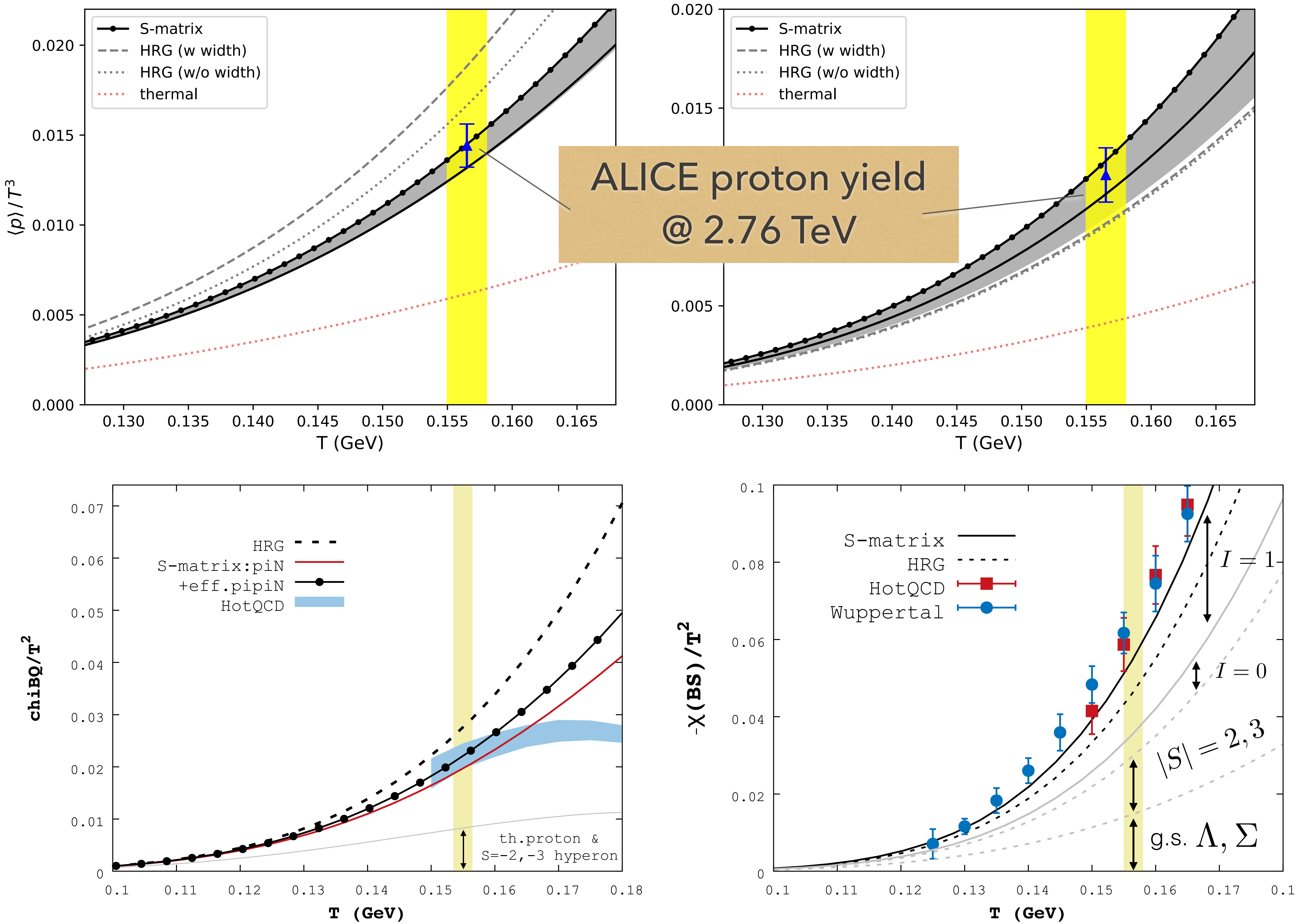
piN phase shifts
pipiN BGs
hyperons

Coupled-Channel system:

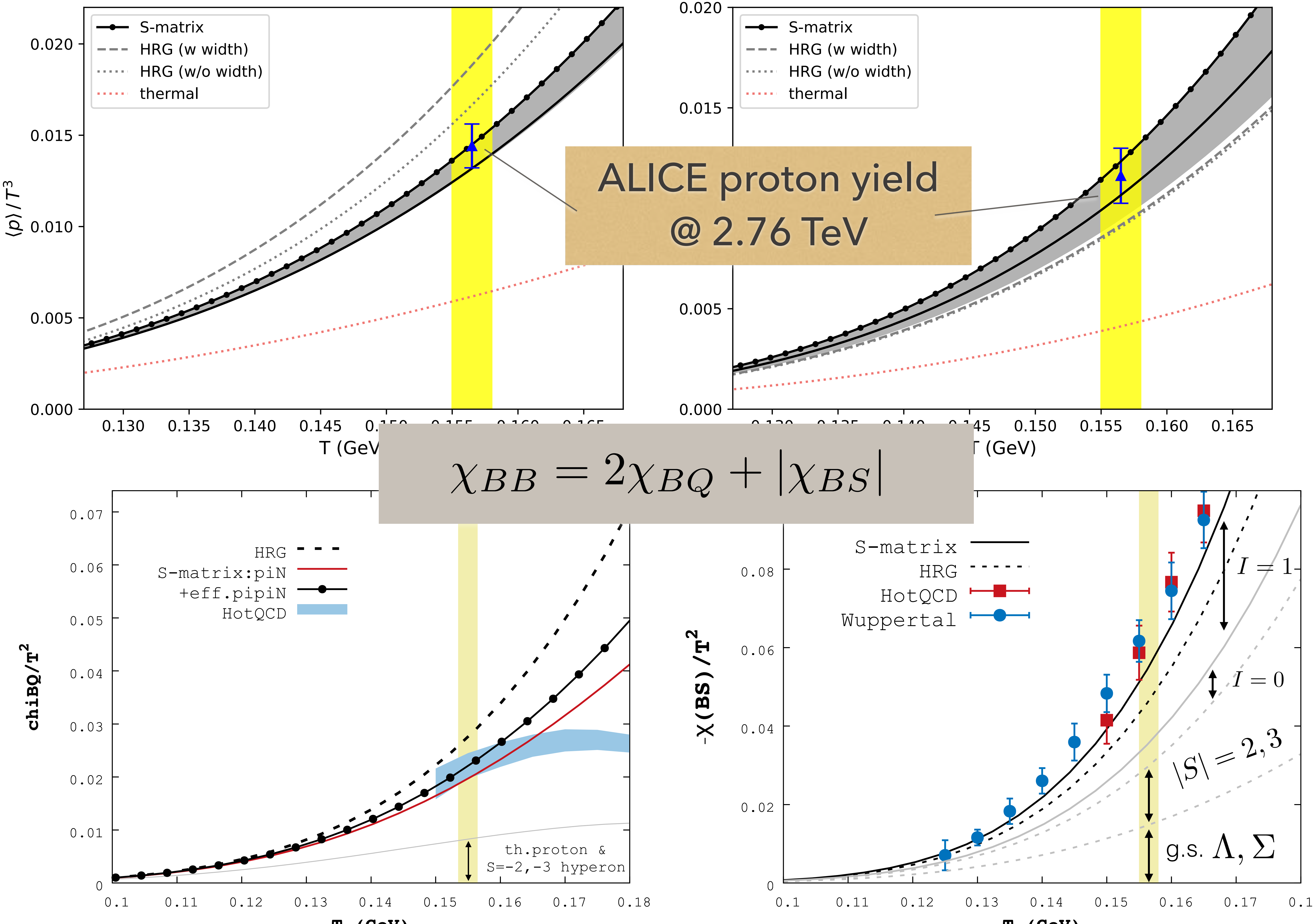
$\bar{k}N, \pi\Lambda, \pi\Sigma, ...$

extra hyperon states
beyond PDG
unitarity BGs

consistent treatment of res and non-res. int.

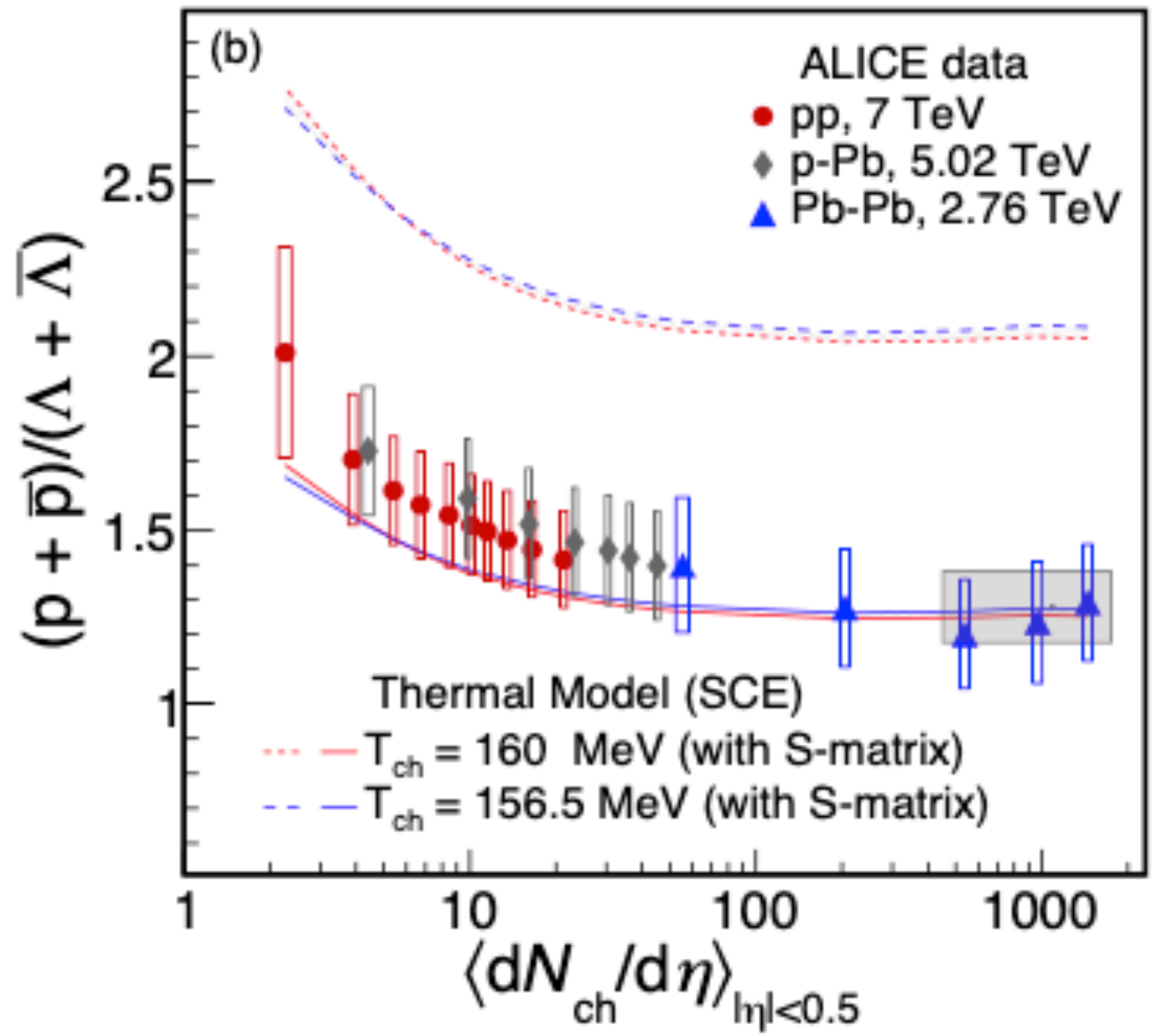


S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨p⟩ / T^3
T (GeV)
ALICE proton yield
@ 2.76 TeV
HRG
S-matrix:piN
+eff.pipiN
HotQCD
chiBQ/T^2
th.proton &
S=-2,-3 hyperon
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
-χ(BS)/T^2
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ
T (GeV)


S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨p⟩ / T^3
T (GeV)
ALICE proton yield
@ 2.76 TeV
$$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$$
HRG
S-matrix:piN
+eff.pipiN
HotQCD
chiBQ/T^2
th.proton &
S=-2,-3 hyperon
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
-χ(BS)/T^2
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ
T (GeV)

less protons

more lambdas



hrg

smat

Phys. Rev. C 103, 014904 (2021).

Phys. Lett. B 792, 304 (2019).

THEORETICAL ISSUES

muB = 0 @T = 155 MeV

- LHC conditions = pion rich: p = pbar; $\langle \pi \rangle / \langle p \rangle \approx 15$
  Need to Take Pions Seriously!
  NN is heavily (Boltzmann) suppressed compared to pi N
- How to include a resonance?
- Why it is NOT a Breit-Wigner?
- In-medium Effects from S-matrix

HOW TO RELATE PHASE SHIFTS TO THERMODYNAMICS?

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

single channel, elastic $\frac{1}{\pi} \frac{d}{dE} \delta$

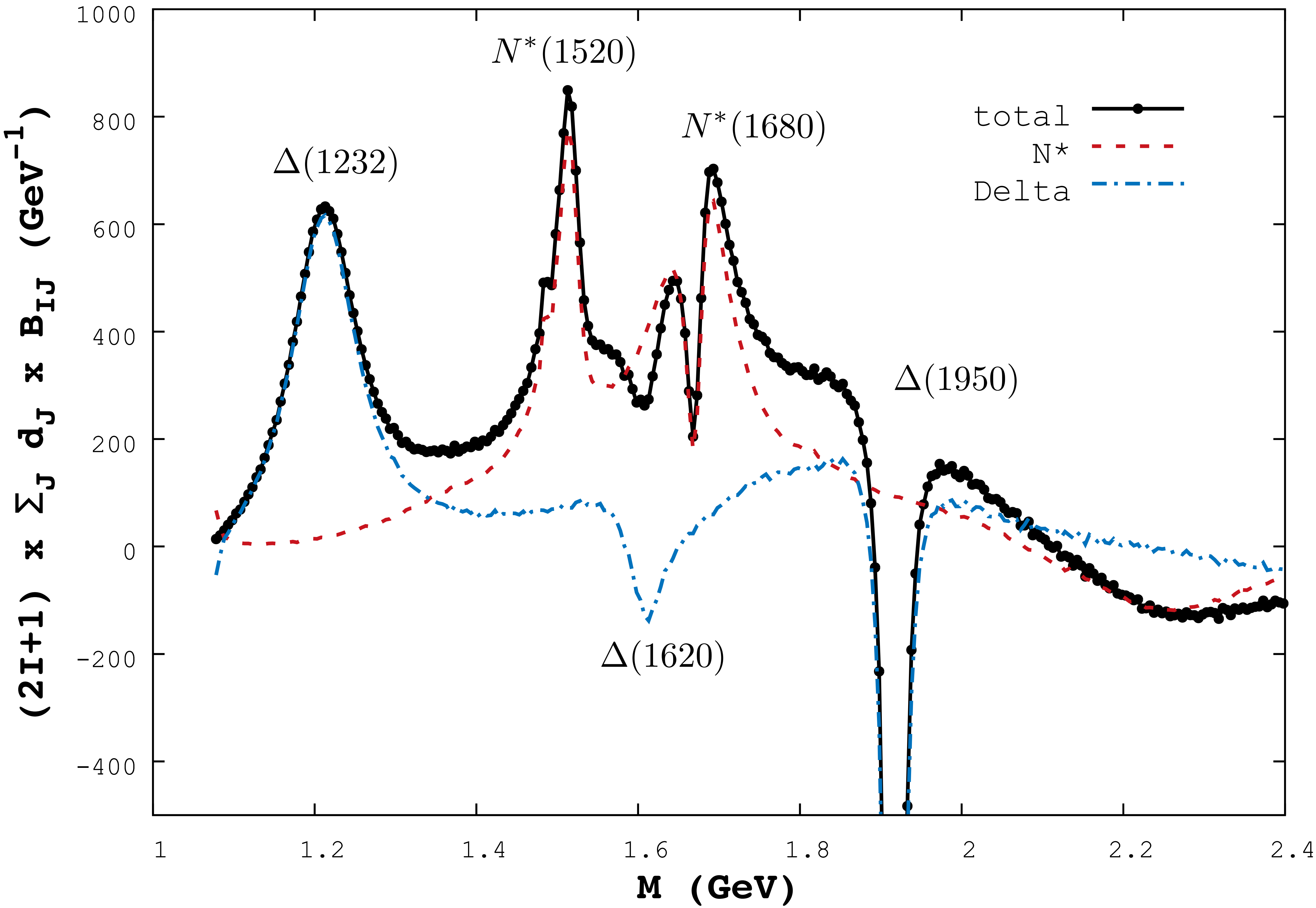
N-body & Coupled-Channel problem

multi (coupled) channel $\frac{1}{\pi} \frac{d}{dE} \mathcal{Q}$

$$\mathcal{Q} = \frac{1}{2} \mathrm{ImTr} \, \ln S = \sum_{\mathrm{channels}} \lambda_i$$

SINGLE CHANNEL -> MULTI-CHANNEL

- Instead of phase shifts -> on-shell S/T-matrix
  - N-channels, inelasticity, etc.
- Tricks of hadron physics
  - Non-resonant interactions
  - Quasi-elastic scattering (N > 2)
  - Unitarity BGs
  - Coupled-channel techniques



$N^*(1520)$
$N^*(1680)$
$\Delta(1232)$
$\Delta(1950)$
$\Delta(1620)$
total
N*
Delta
(2I+1) x Σ_J d_J x B_{IJ} (GeV^{-1})
M (GeV)
1000
800
600
400
200
0
-200
-400
1
1.2
1.4
1.6
1.8
2
2.2
2.4

PHASE SHIFT FROM PWA

```
---------------------------------------------------------------------------------------

Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

---------------------------------------------------------------------------------------
---------------------------------------------------------------------------------------

Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)
Igor V. Danilkin (Jefferson Lab)
Vincent Mathieu (Indiana University)
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosphy. If your
parameters do not make sense, the output will not make sense either.
2 - You can use, share and modify this code under your own responsability.
3 - This code is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
4 - No PhD students or postdocs were severely damaged during the development
of this project.

-----------------------------------------------------------------------
```

- 1 → $\bar{K}N$,
- 2 → $\pi\Sigma$,
- 3 → $\pi\Lambda$,
- 4 → $\eta\Lambda$,
- 5 → $\eta\Sigma$,
- 6 → $\bar{K}_1 N$,
- 7 → $\left[\bar{K}_3 N\right]_-$,
- 8 → $\left[\bar{K}_3 N\right]_+$,
- 9 → $[\pi\Sigma^*]_-$,
- 10 → $[\pi\Sigma^*]_+$,
- 11 → $\left[\bar{K}\Delta\right]_-$,
- 12 → $\left[\bar{K}\Delta\right]_+$,
- 13 → $[\pi\Lambda(1520)]_-$,
- 14 → $[\pi\Lambda(1520)]_+$,
- 15 → $\pi\pi\Lambda$,
- 16 → $\pi\pi\Sigma$.

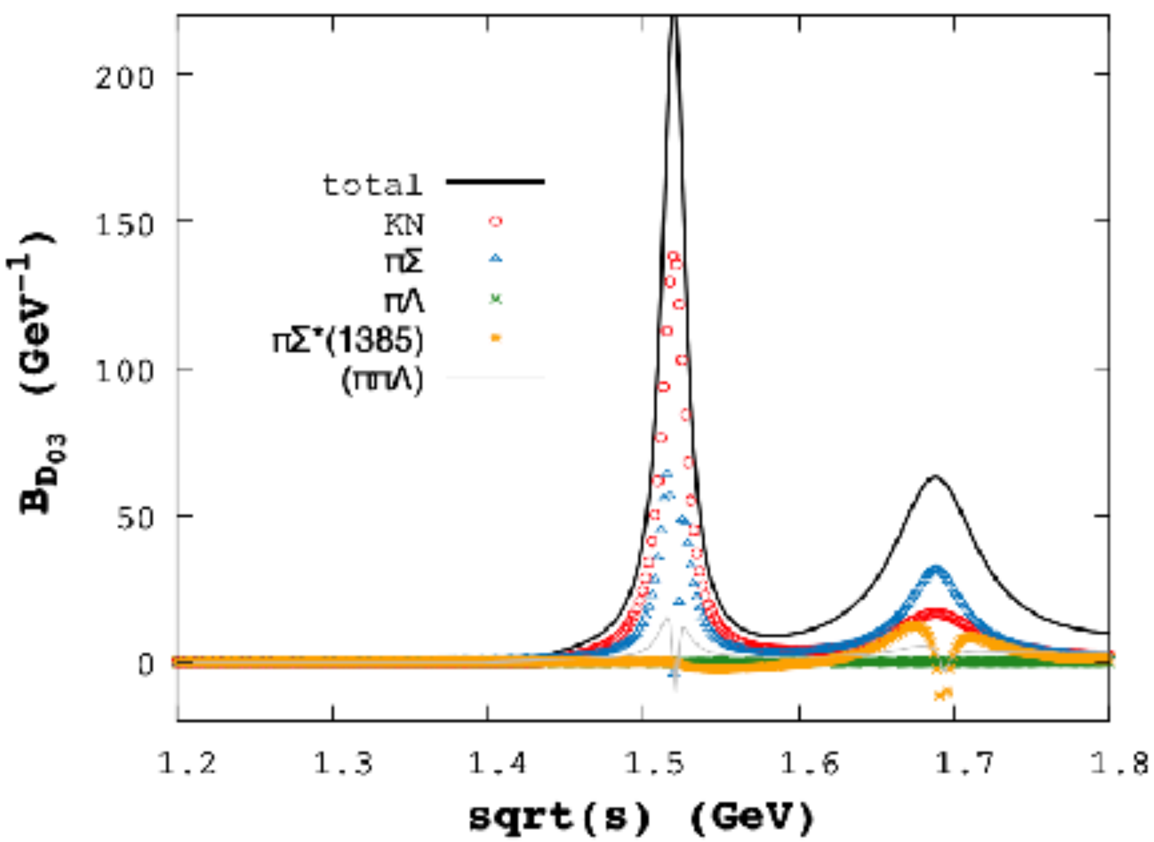
elastic scatterings (elementary)

Effective elementarity

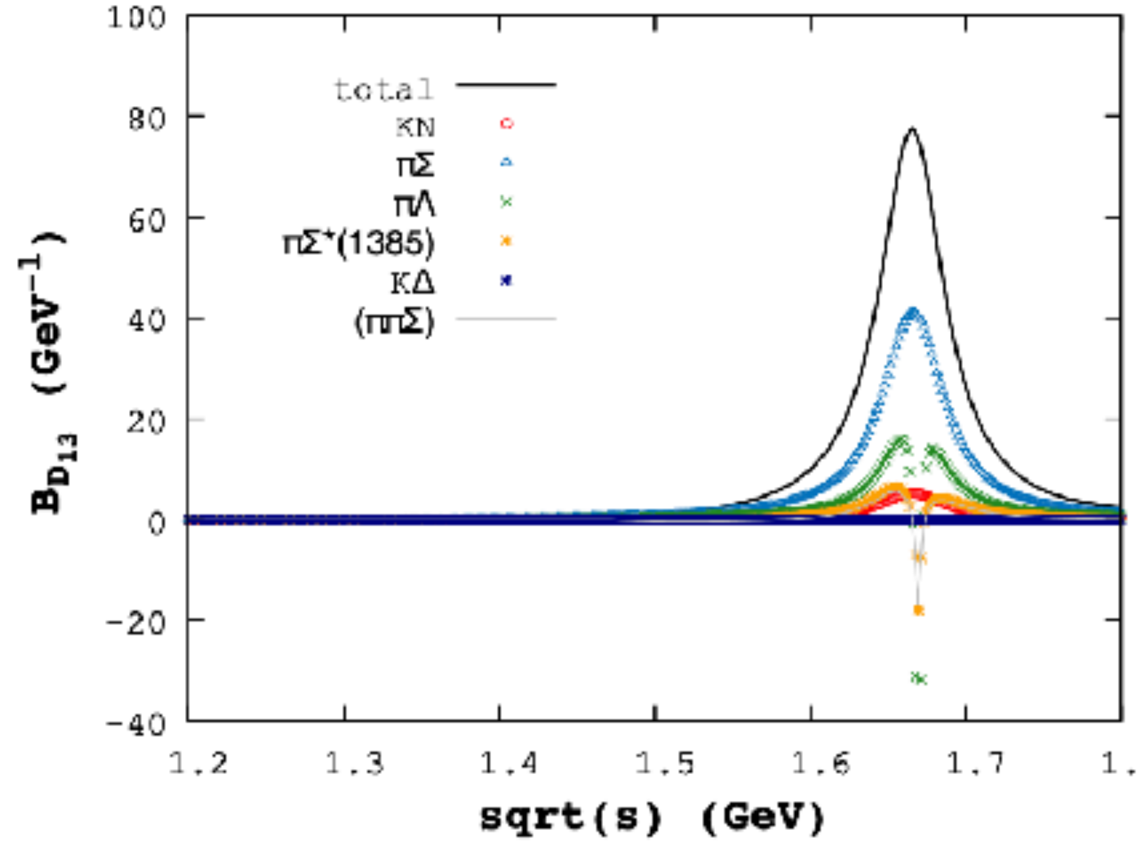
quasi elastic scatterings

unitarity background

1520, 1690



1670



$\Lambda(1520)\ 3/2^-$ $I(J^P) = 0(\frac{3}{2}^-)$

Mass $m = 1519.5 \pm 1.0$ MeV [d]
Full width $\Gamma = 15.6 \pm 1.0$ MeV [d]
$p_{beam} = 0.39$ GeV/c $4\pi\lambda\!\!\!^{-2} = 82.8$ mb

Λ(1520) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	45 ± 1%	243
$\Sigma\pi$	42 ± 1%	268
$\Lambda\pi\pi$	10 ± 1%	259
$\Sigma\pi\pi$	0.9 ± 0.1%	169
$\Lambda\gamma$	0.85 ± 0.15%	350

$\Lambda(1690)\ 3/2^-$ $I(J^P) = 0(\frac{3}{2}^-)$

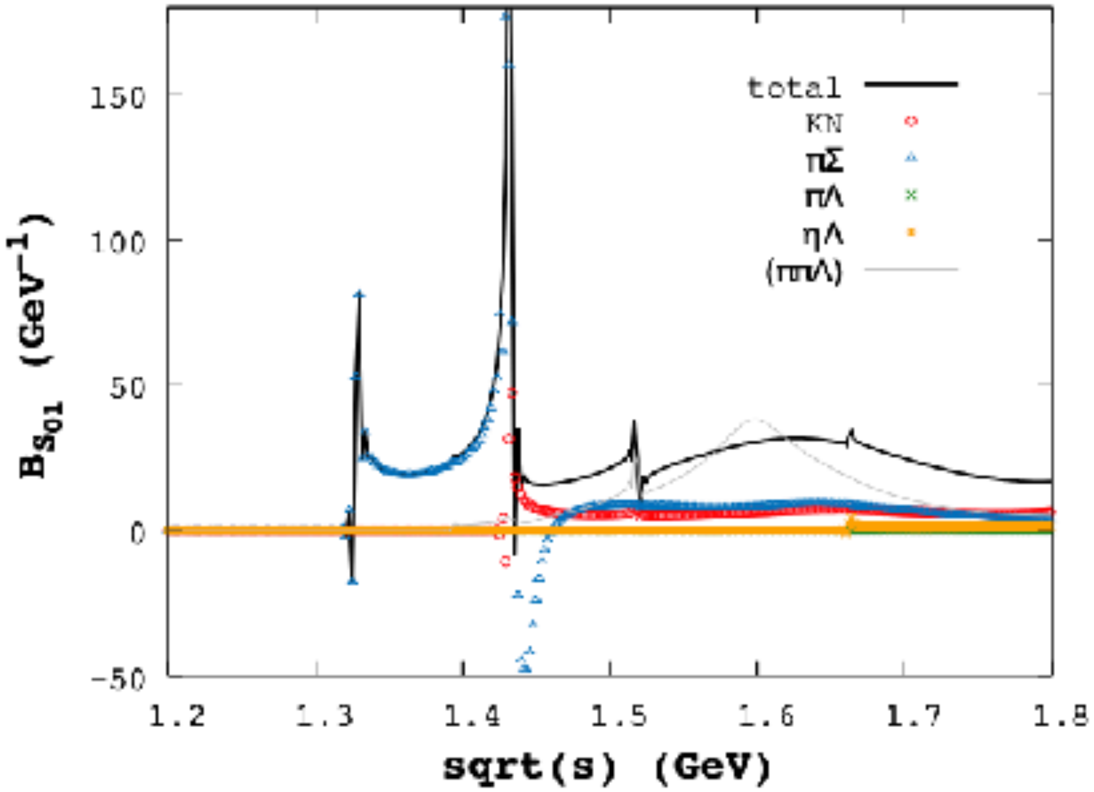
Mass $m = 1685$ to 1695 (≈ 1690) MeV
Full width $\Gamma = 50$ to 70 (≈ 60) MeV
$p_{beam} = 0.78$ GeV/c $4\pi\lambda\!\!\!^{-2} = 26.1$ mb

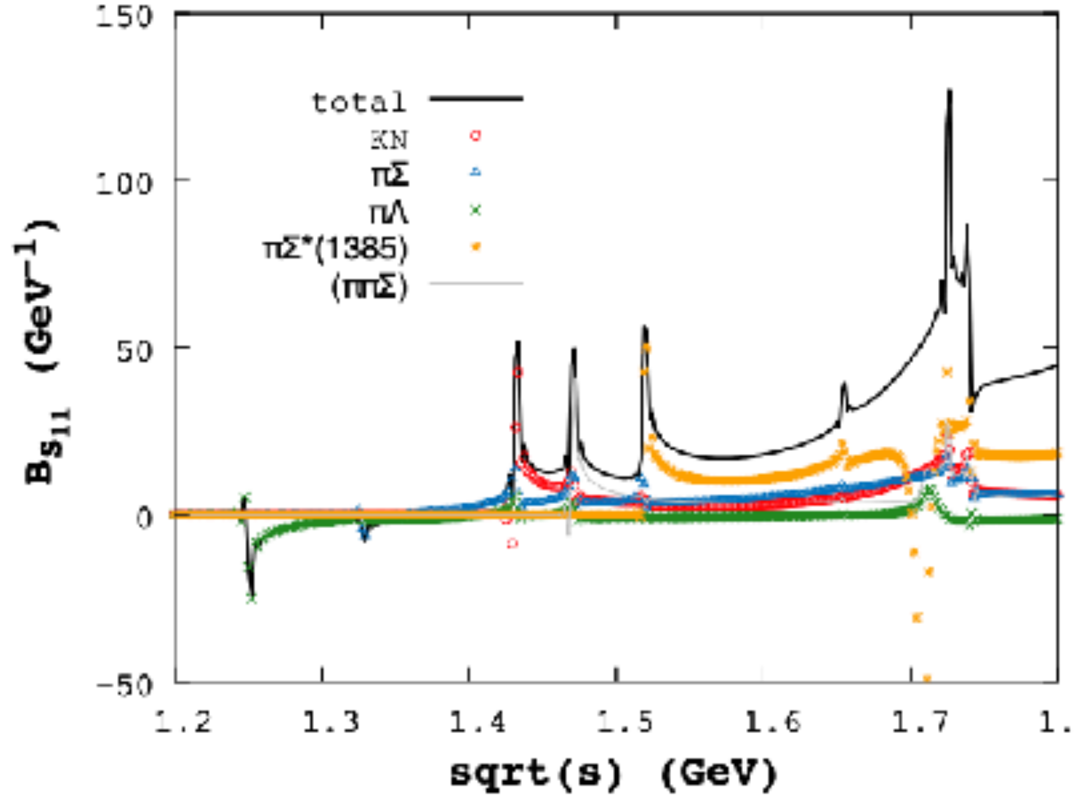
Λ(1690) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	20–30 %	433
$\Sigma\pi$	20–40 %	410
$\Lambda\pi\pi$	~ 25 %	419
$\Sigma\pi\pi$	~ 20 %	358

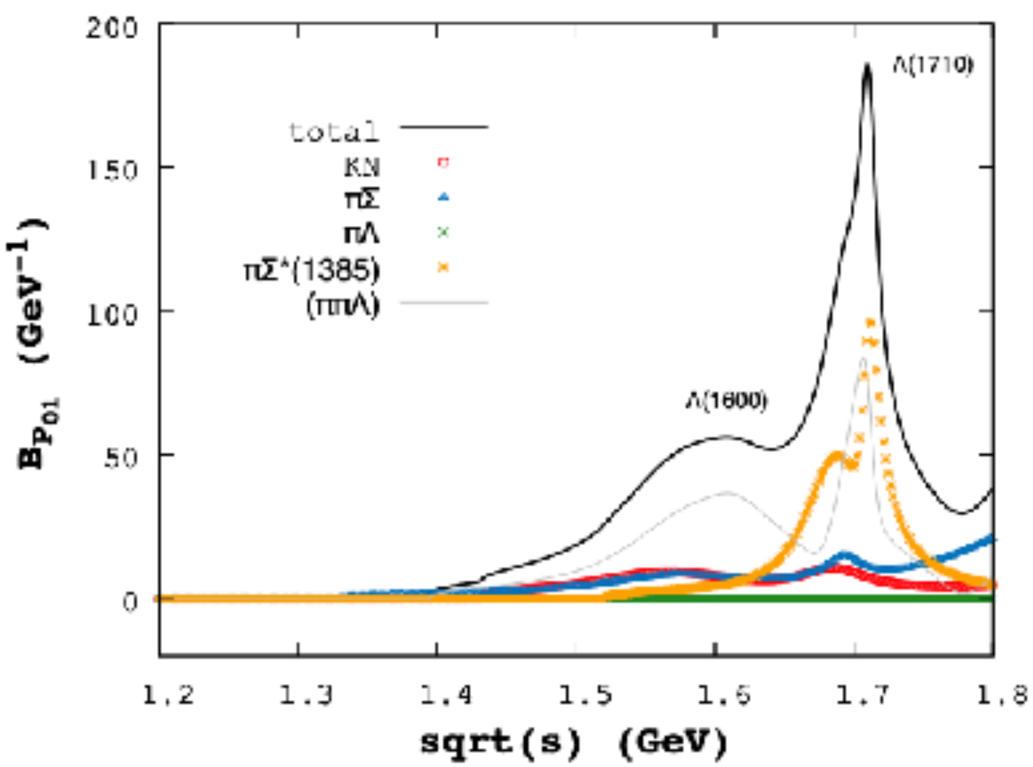
$\Sigma(1670)\ 3/2^-$ $I(J^P) = 1(\frac{3}{2}^-)$

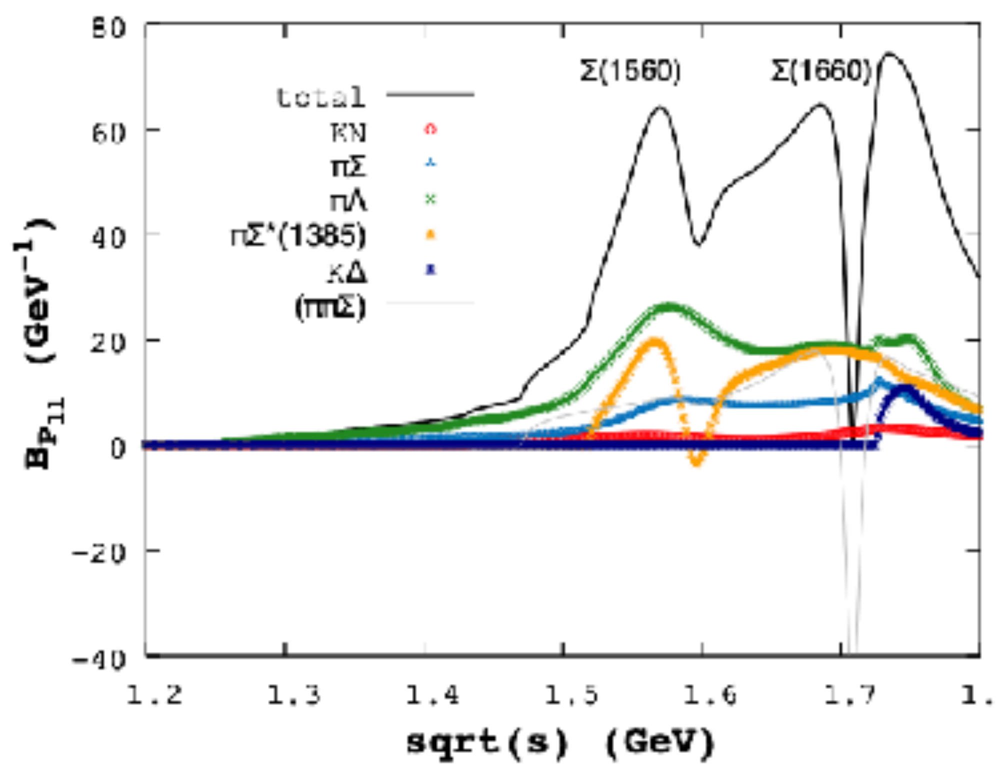
Mass $m = 1665$ to 1685 (≈ 1670) MeV
Full width $\Gamma = 40$ to 80 (≈ 60) MeV
$p_{beam} = 0.74$ GeV/c $4\pi\lambda\!\!\!^{-2} = 28.5$ mb

Σ(1670) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	7–13 %	414
$\Lambda\pi$	5–15 %	448
$\Sigma\pi$	30–60 %	394

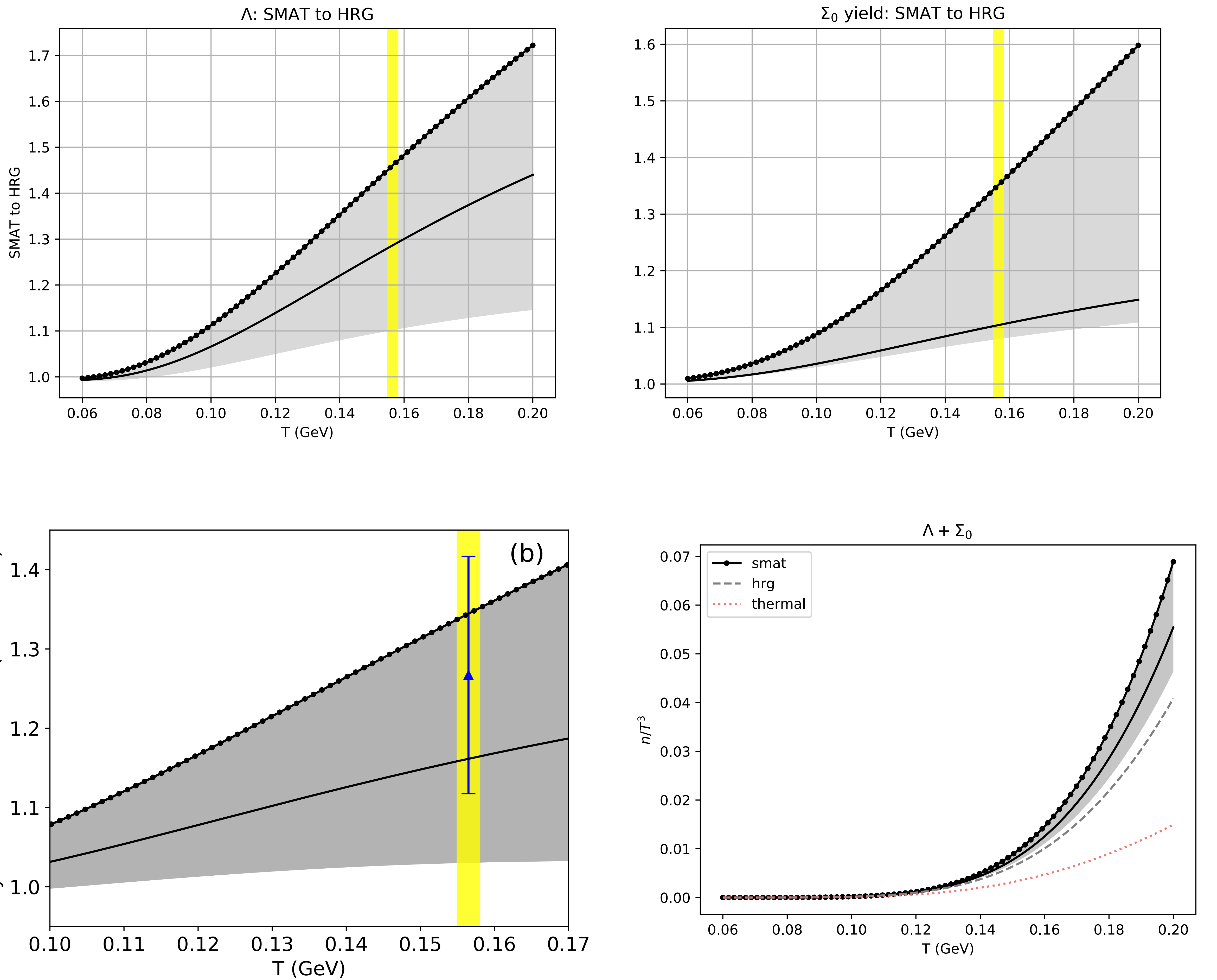




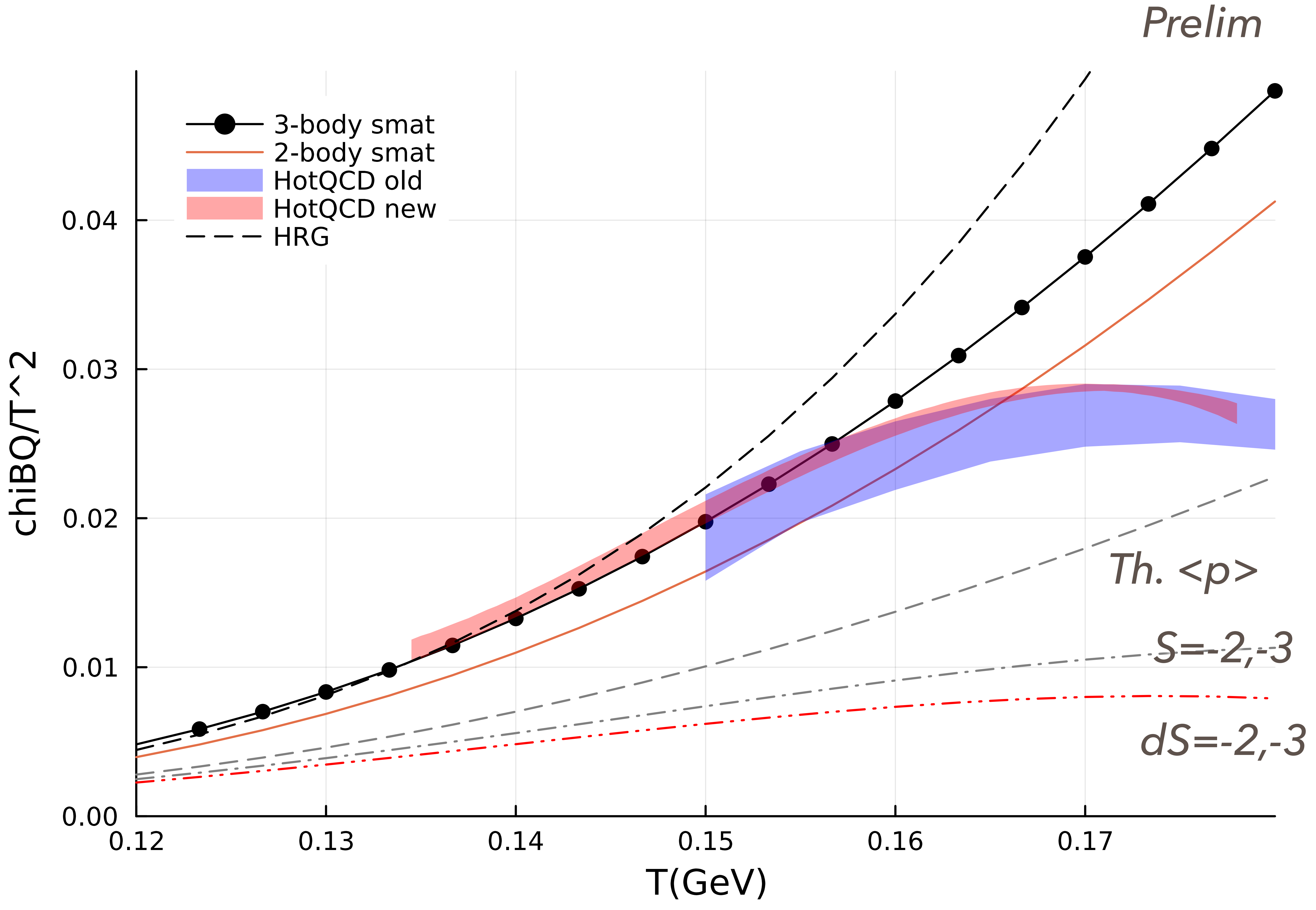




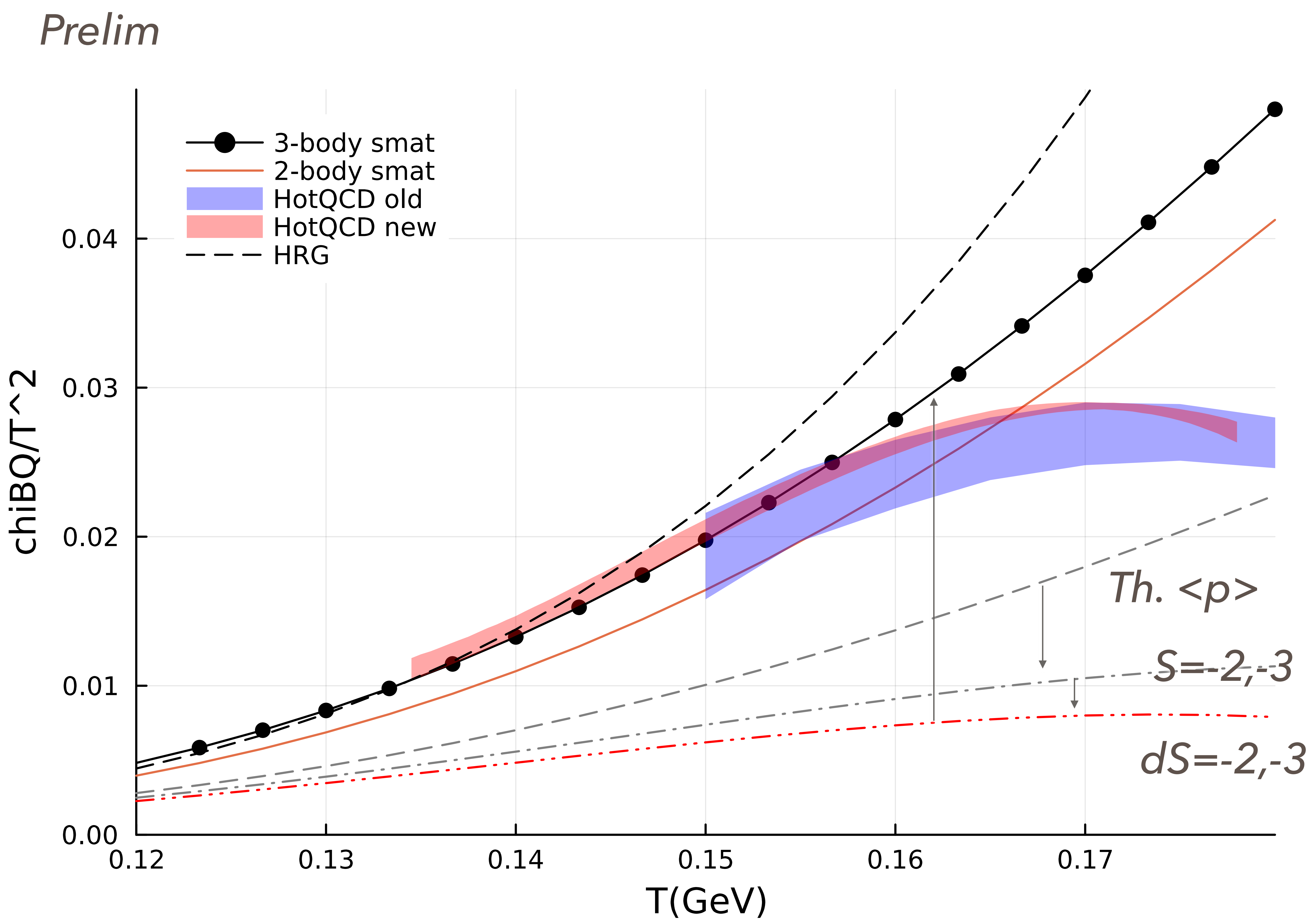
branching ratio?



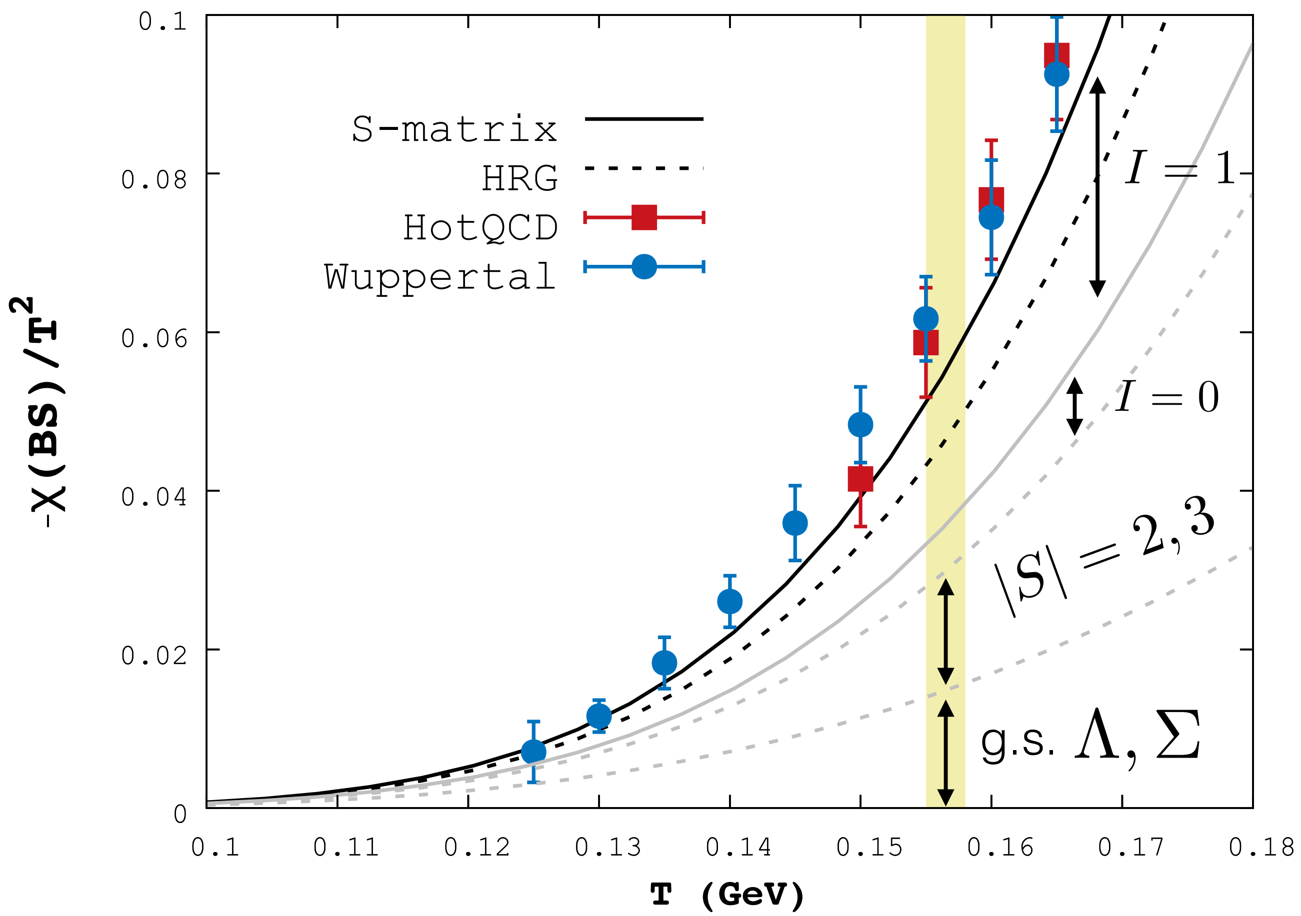
Λ: SMAT to HRG
SMAT to HRG
T (GeV)
Σ_0 yield: SMAT to HRG
T (GeV)
(b)
Λ yield: S-matrix to HRG (with width)
T (GeV)
$\Lambda + \Sigma_0$
smat
hrg
thermal
n/T^3
T (GeV)



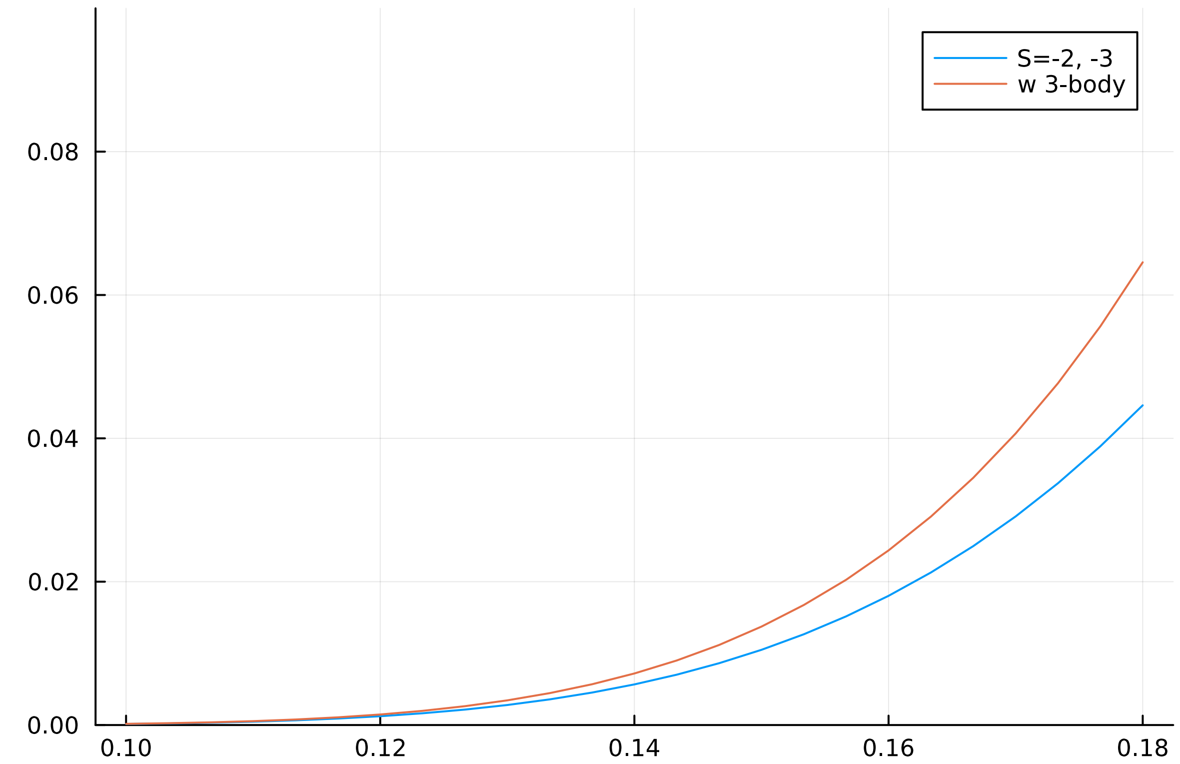
Prelim
3-body smat
2-body smat
HotQCD old
HotQCD new
HRG
chiBQ/T^2
0.00
0.01
0.02
0.03
0.04
0.12
0.13
0.14
0.15
0.16
0.17
T(GeV)
Th. <p>
S=-2,-3
dS=-2,-3



Prelim
3-body smat
2-body smat
HotQCD old
HotQCD new
HRG
0.00
0.01
0.02
0.03
0.04
chiBQ/T^2
0.12
0.13
0.14
0.15
0.16
0.17
T(GeV)
Th. <p>
S=-2,-3
dS=-2,-3

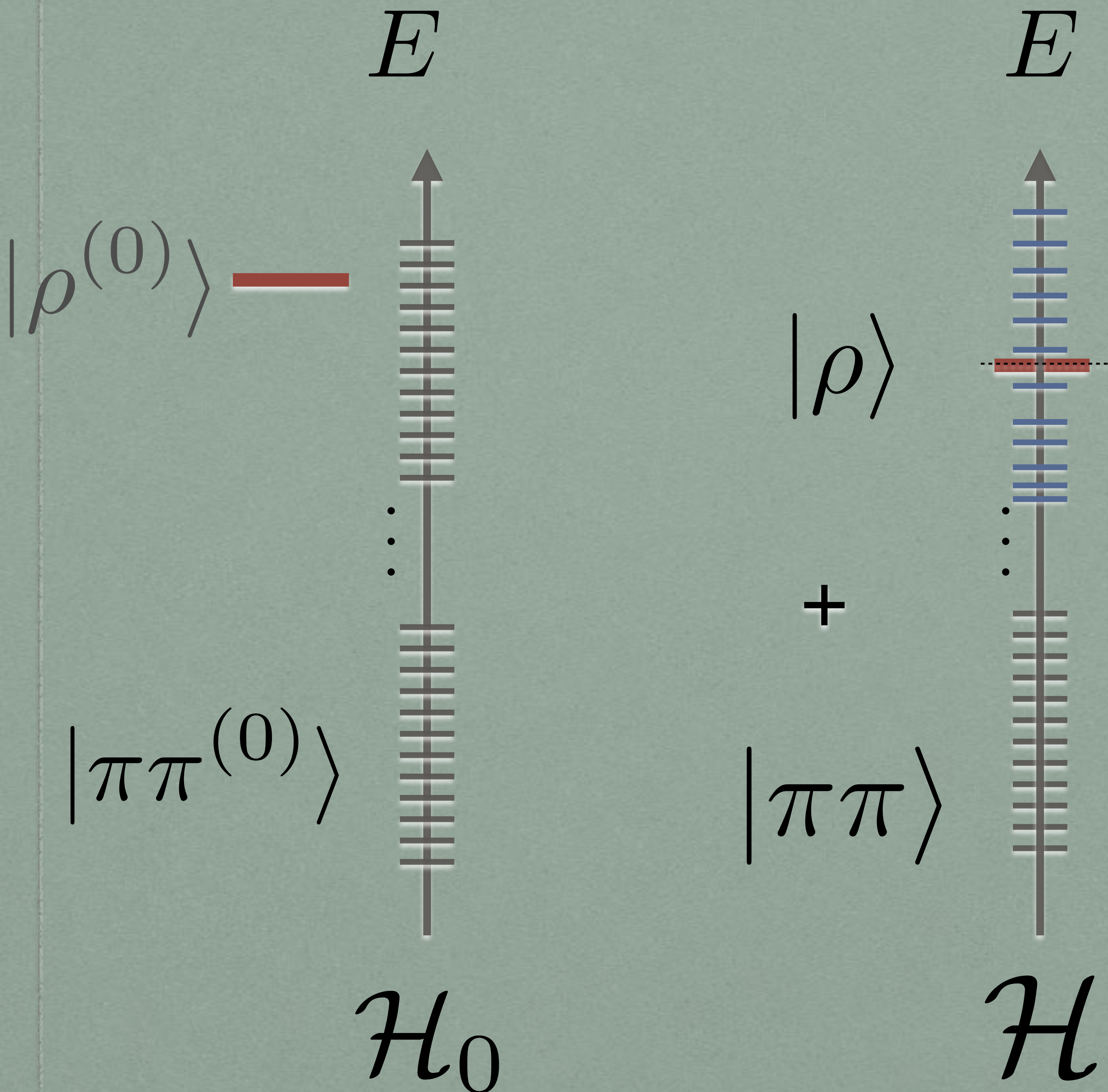


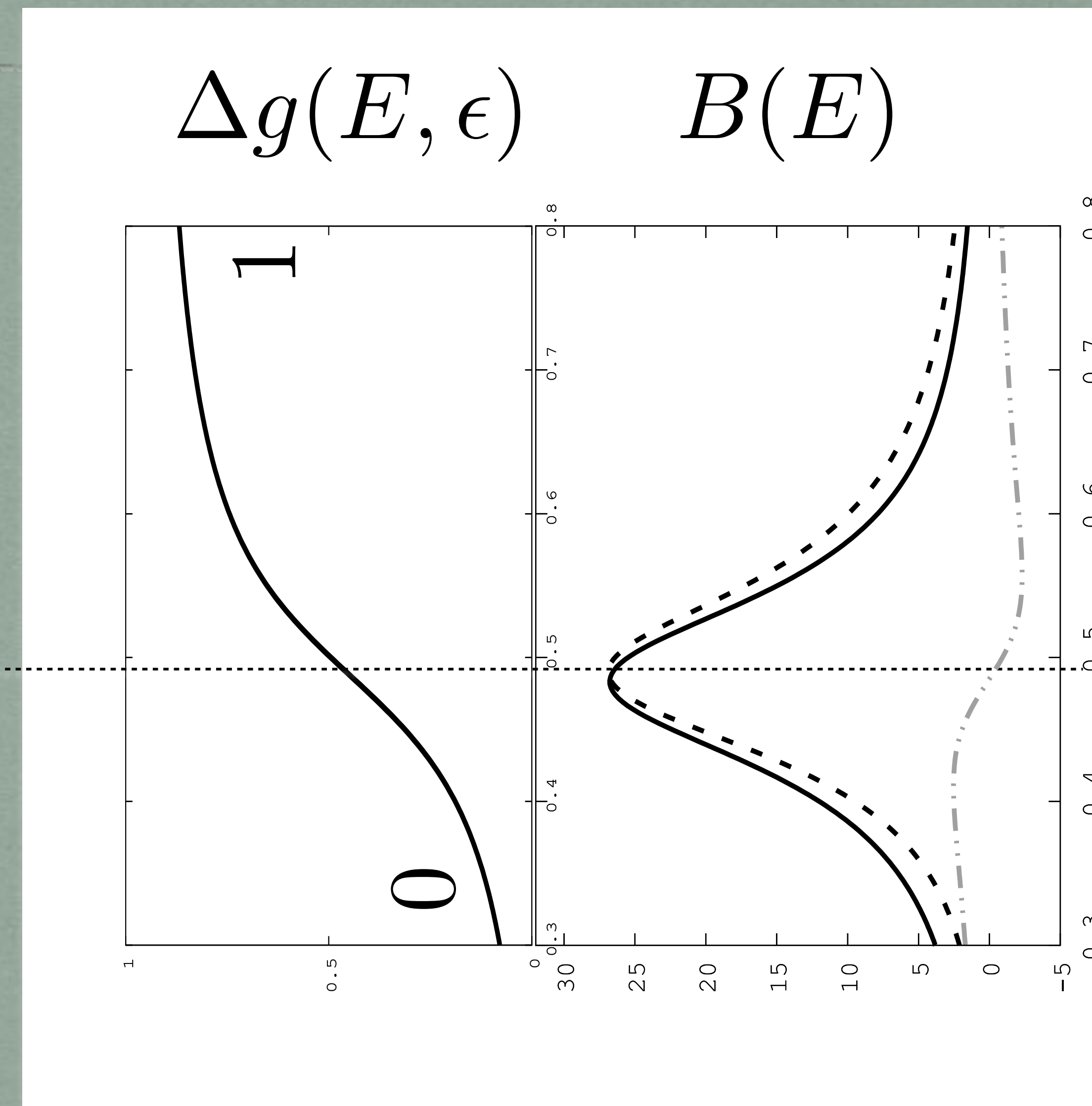
0.1
0.08
0.06
0.04
0.02
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
T (GeV)
-χ(BS)/T²
S-matrix
HRG
HotQCD
Wuppertal
I = 1
I = 0
|S| = 2,3
g.s. Λ, Σ



S=-2, -3
w 3-body
0.08
0.06
0.04
0.02
0.00
0.10
0.12
0.14
0.16
0.18

IN-MEDIUM EFFECTS





$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE}\Delta g(E,\epsilon)$$

$$= A_\rho + \Delta A_{\pi\pi}$$

$\mathrm{Tr}\, e^{-\beta \mathcal{H}_0}$ *vs* $\mathrm{Tr}\, e^{-\beta \mathcal{H}}$

PML, Francesco Giocosa EPJC **79** (2019)

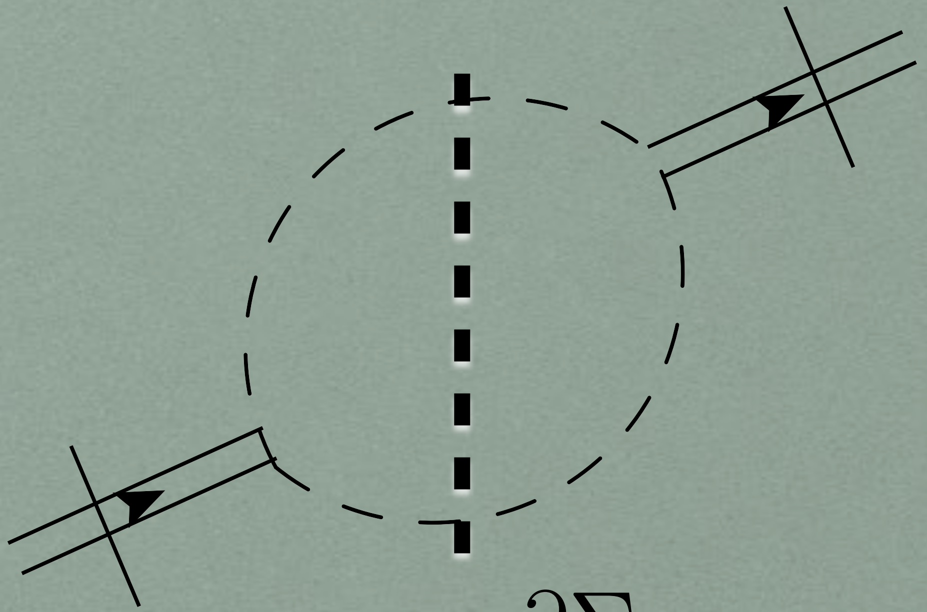
PHYSICS OF B

$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

physical interpretation:

$$B = 2\,\frac{\partial}{\partial E}\delta$$
$$= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1}$$
$$= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial \Sigma_\rho}{\partial E}G_\rho]$$
$$= A_\rho(E) + \Delta A_{\pi\pi}$$

contribution from correlated pi pi pair



$$\frac{\partial \Sigma_\rho}{\partial E}$$

$$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$$ pipi -> pipi

PHYSICS OF B

to rho or not to rho?
that's out of the question!

resonance's picture:

$$B(E) = A_\rho(E) + \Delta A_{\pi\pi}$$

rho

scattering picture:

$$B_1 = \frac{\partial}{\partial E}\,\mathrm{Tr}\,\hat{t}_{\mathrm{re}}$$

$$B_2 = \frac{1}{2}\,\mathrm{Im}\,\mathrm{Tr}\,\hat{t}^\dagger\,\overleftrightarrow{\partial_E}\,\hat{t}$$

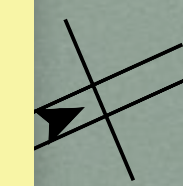
pipi -> pipi

$$\delta = -\mathrm{Im}\,\mathrm{T}$$

on:

n

air



$$B = 2\,\frac{\partial}{\partial E}\delta$$

$$= -2\,\mathrm{Im}\,\frac{\partial}{\partial E}$$

$$= -2\,\mathrm{Im}[G$$

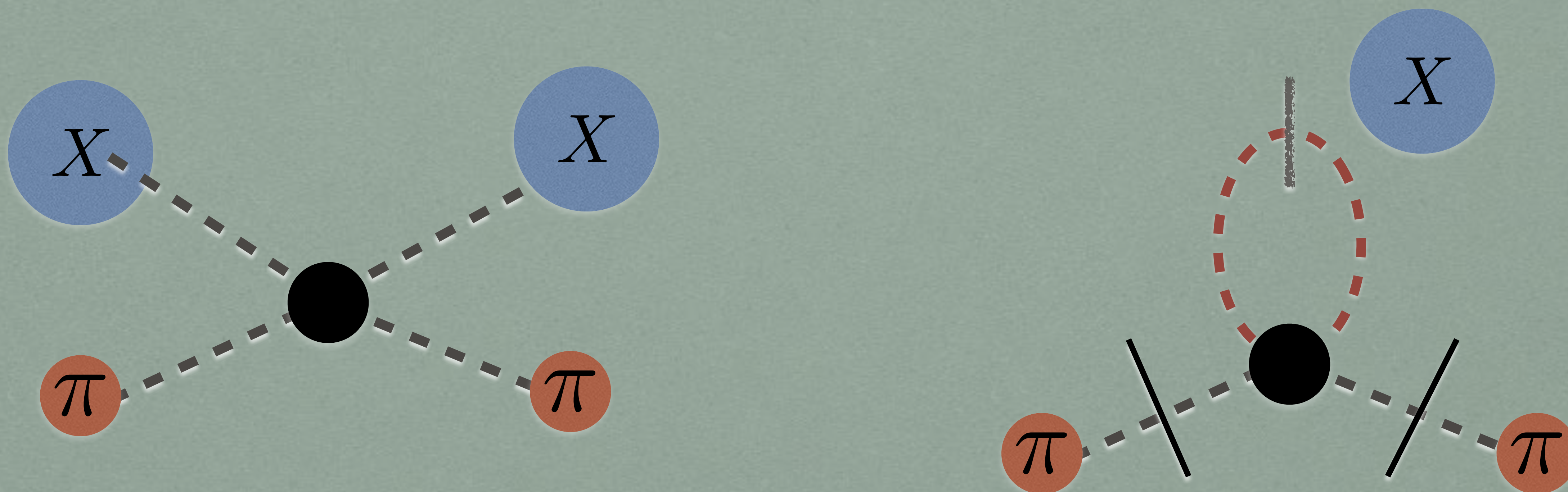
$$= A_\rho(E) + \Delta A_{\pi\pi}$$

$$\frac{\partial \Sigma_\rho}{\partial E}$$

$$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$$

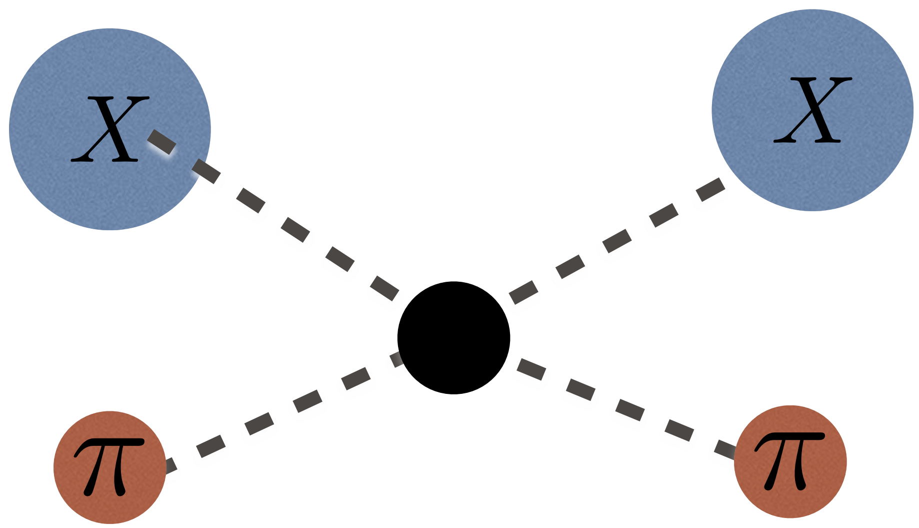
pipi -> pipi

IN-MEDIUM EFFECTS FROM S-MATRIX



A. Schenk NPB 363(1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)



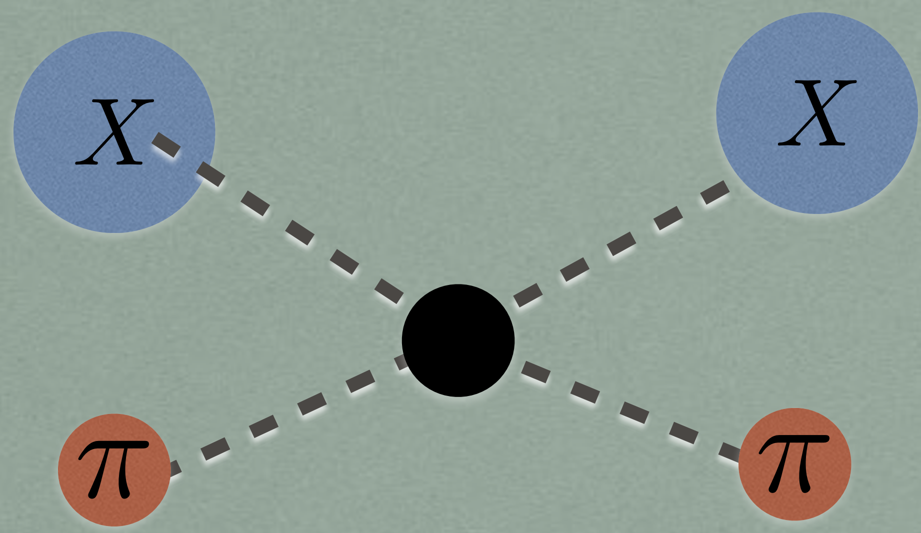
$$T_{\rm nr} \approx -\frac{4\pi f}{2m_{\rm red}}.$$

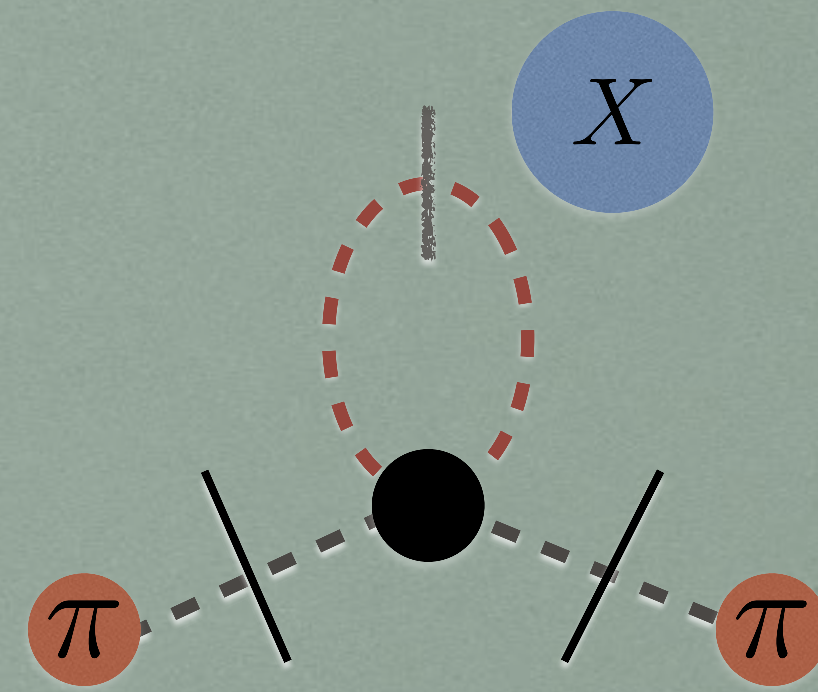
$$\begin{aligned}\Delta P &\approx \int \frac{d^3P}{(2\pi)^3}\,\frac{dE'}{(2\pi)}\,e^{-\beta(m_{\rm tot}+\frac{P^2}{2m_{\rm tot}}+E')}2Q(E') \\ &= \int \frac{d^3P}{(2\pi)^3}\,\frac{d^3q}{(2\pi)^3}\,e^{-\beta(m_{\rm tot}+\frac{P^2}{2m_{\rm tot}}+\frac{q^2}{2m_{\rm red}})}\,(-T_{\rm nr}) \\ &\approx N_{\rm th}^A\,N_{\rm th}^B \times (-T_{\rm nr}).\end{aligned}$$

$$\begin{aligned}\Delta P &\approx T\int \frac{d^3p_A}{(2\pi)^3}\,e^{-\beta(m_A+\frac{p_A^2}{2m_A})}\,(-\beta\Delta m_A) \\ &= -\Delta m_A N_{\rm th}^A \\ &= N_{\rm th}^A\,N_{\rm th}^B \times \frac{4\pi f}{2m_{\rm red}}.\end{aligned}$$

Change of pressure to due “Dressed mass”

IN-MEDIUM EFFECTS FROM S-MATRIX





$$\Delta P = N_{\rm th}^A\, N_{\rm th}^B \times \frac{4\pi f}{2m_{\rm red}}.$$

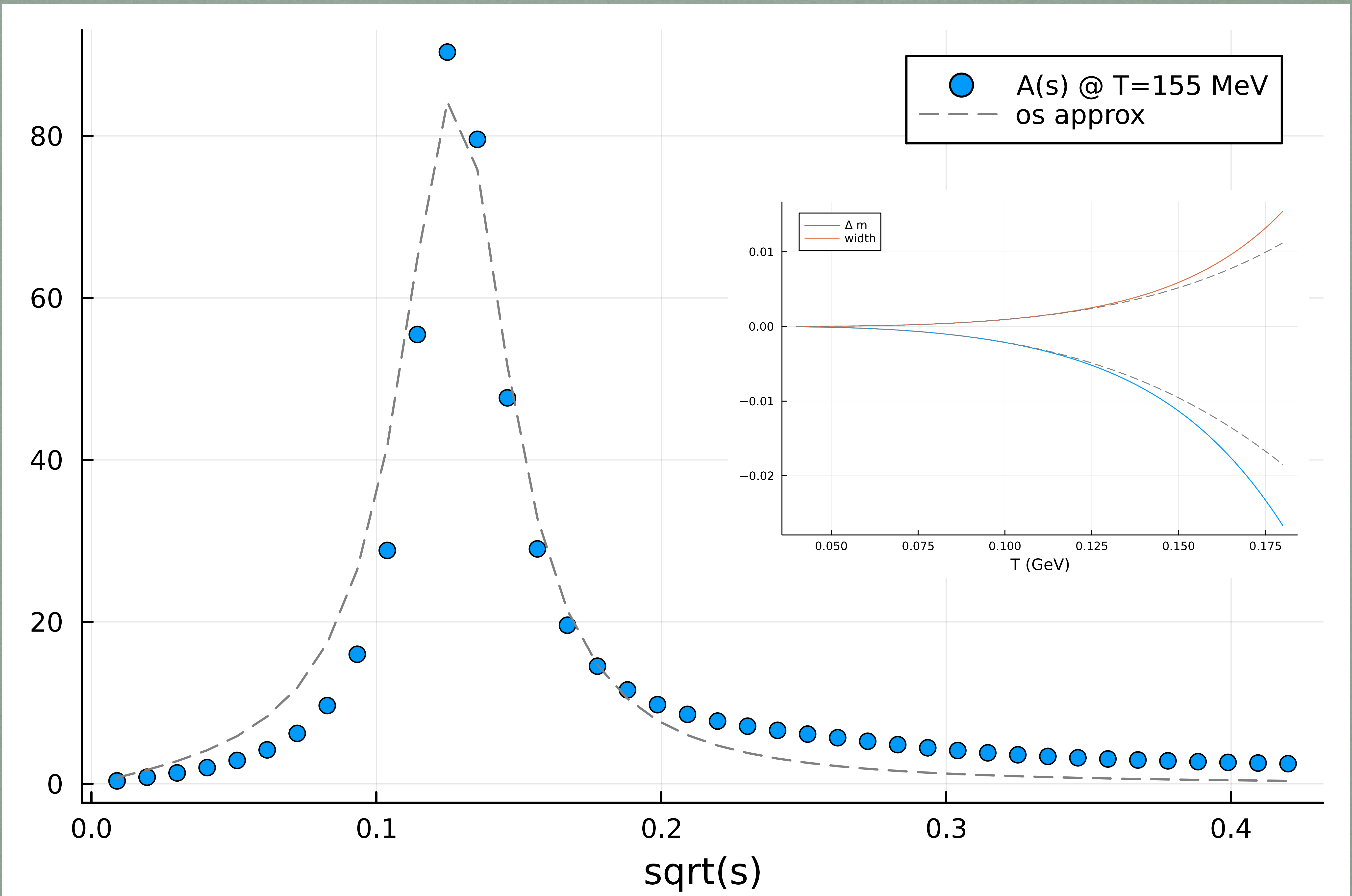
$$\Sigma_A(E_A) = \int \frac{d^3k_B}{(2\pi)^3}\frac{1}{2E_B}\, n_{\rm th}(E_B)\, T(AB\to AB).$$

$$\Delta m_A = \frac{1}{2E_A}\,{\rm Re}\,\Sigma_A(p)$$

$$\approx N_{\rm th}^B \times \frac{-4\pi f}{2m_{\rm red}}.$$

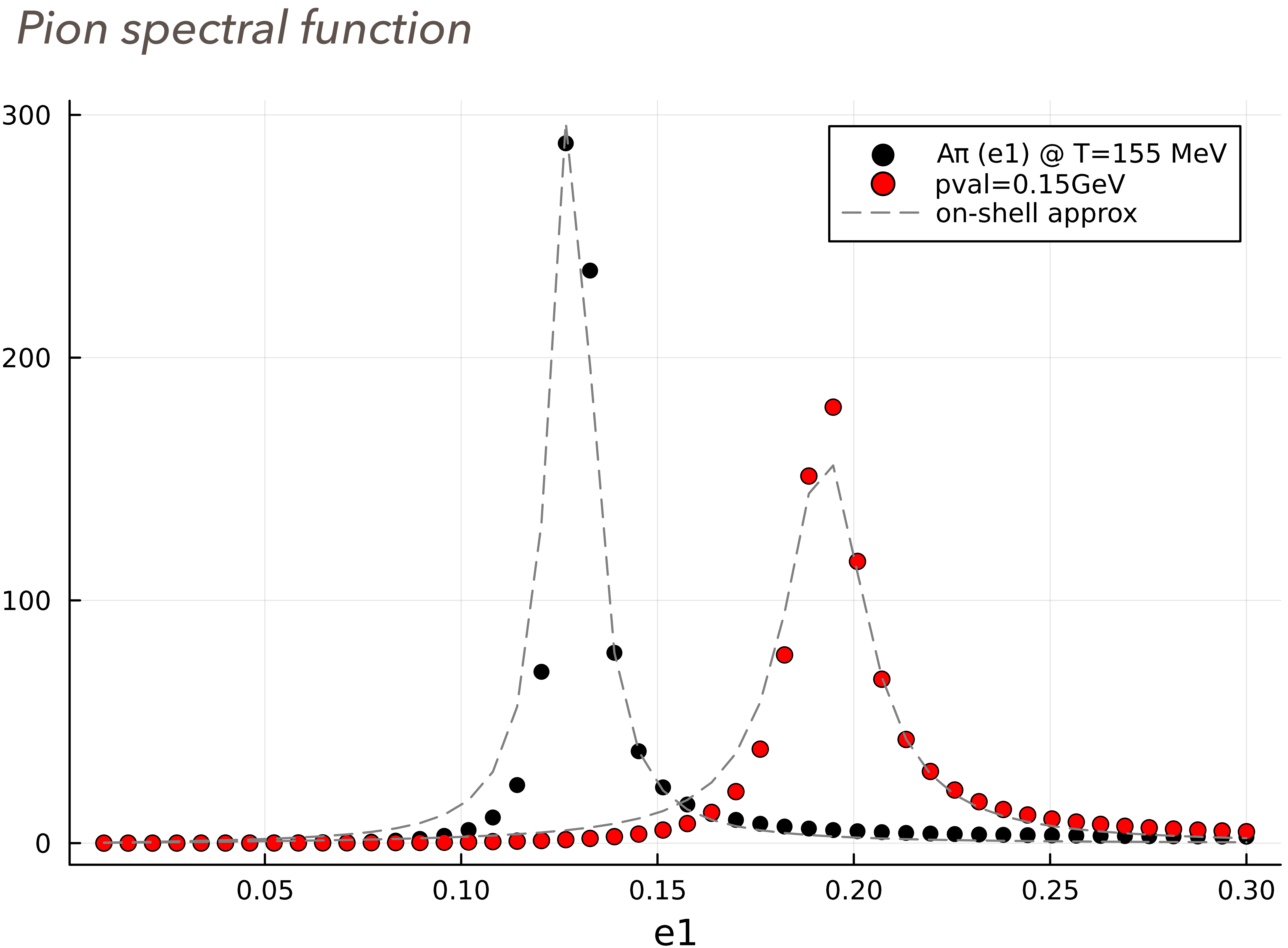
A. Schenk NPB 363(1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)

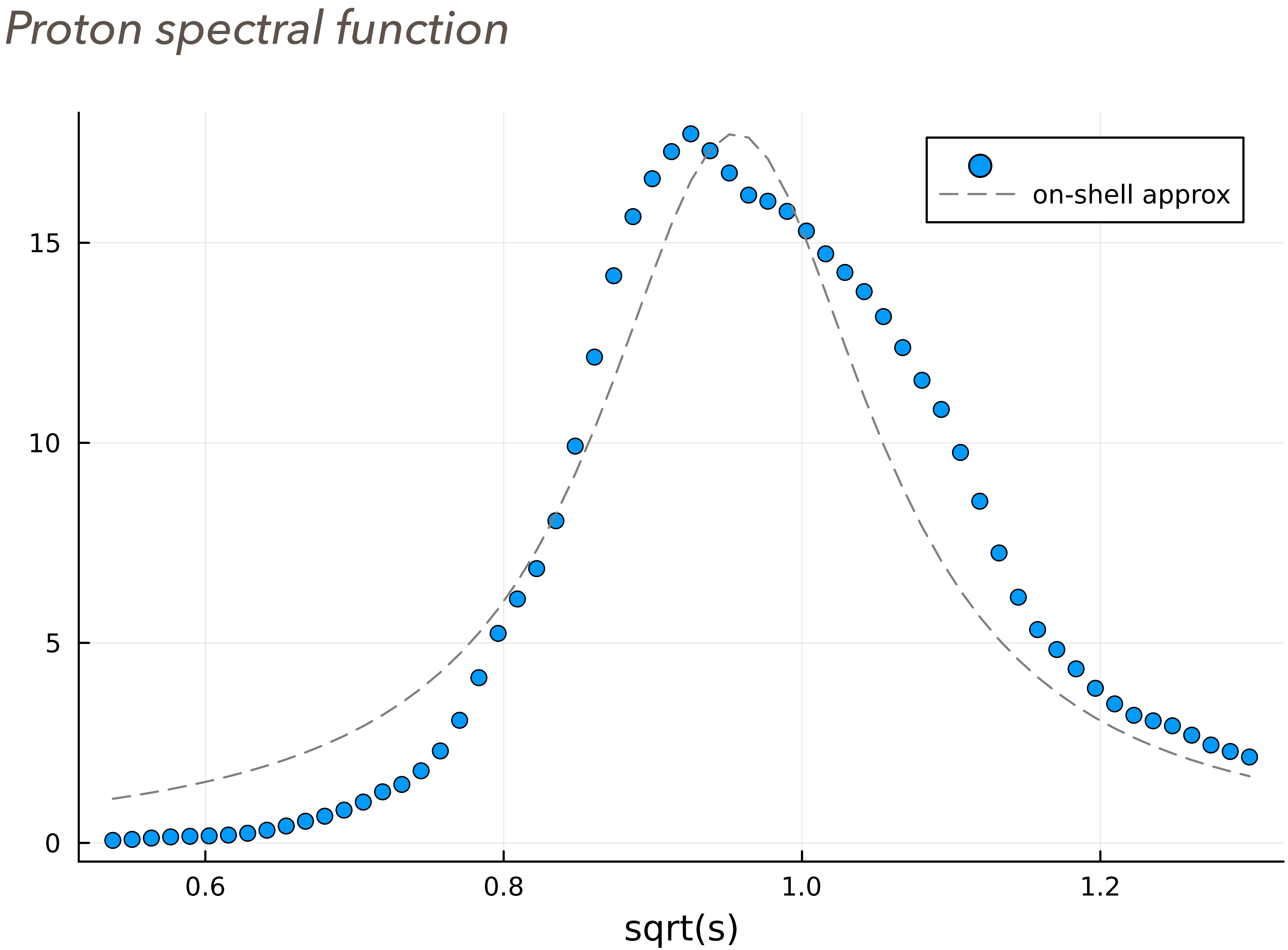


A. Schenk NPB 363 (1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)

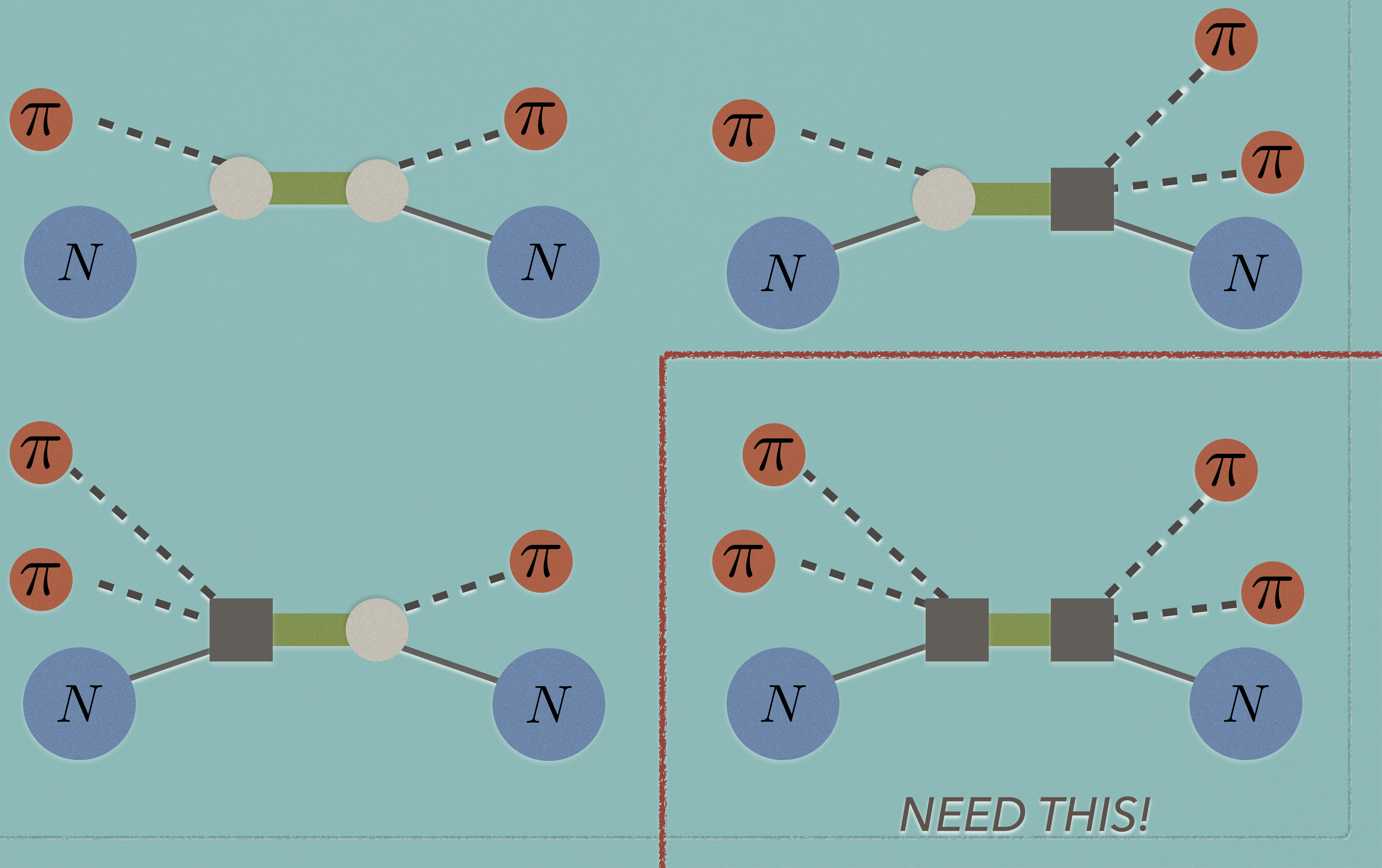


Pion spectral function
Aπ (e1) @ T=155 MeV
pval=0.15GeV
on-shell approx
300
200
100
0
0.05
0.10
0.15
0.20
0.25
0.30
e1

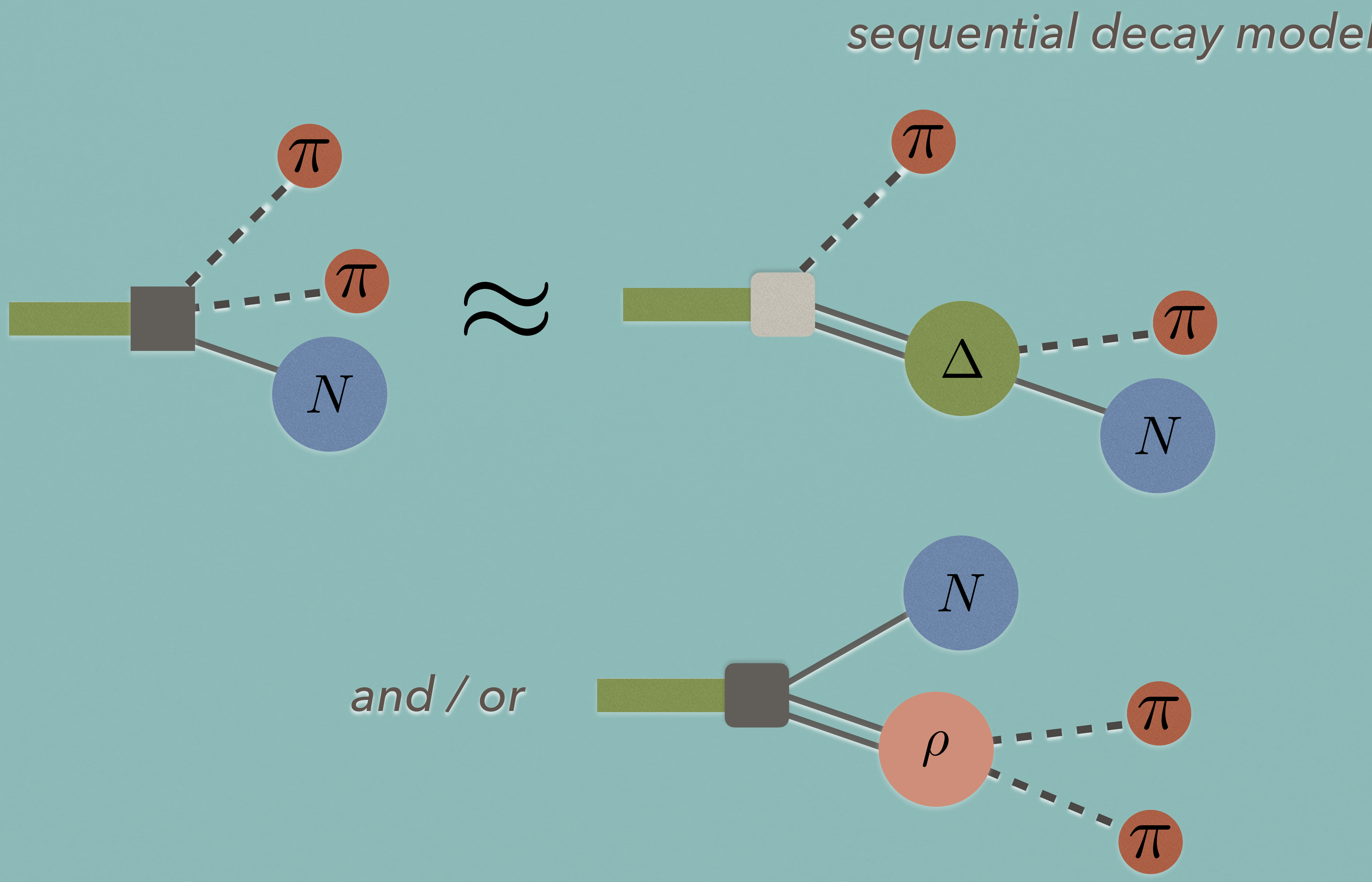


Proton spectral function
15
10
5
0
0.6
0.8
1.0
1.2
sqrt(s)
on-shell approx

ISOBAR MODEL



ISOBAR MODEL



LET'S WORK IT OUT

- Scattering and Thermodynamics
  bulk
  thermal production
- B VS A
  resonance, R-sheets, virtual states
- In-medium & N>2-body prototype
- HIC phenols: invar. mass spec. & femto
- Virial expansion approach to dense(r) matter

THANK YOU