Present and future perspectives in Hadron Physics

OVERVIEW
ON
GPDs AND TMDs

BARBARA PASQUINI

University of Pavia and INFN Pavia

UNIVERSITÀ
DI PAVIA

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INFN
Istituto Nazionale
di Fisica Nucleare

Emergent phenomena in QCD

“the whole is more than the sum of its parts”

``What proton is depends on how you look at it, or rather on how hard you hit it''

A. Cooper-Sarkar, CERN Courier, June, 2019

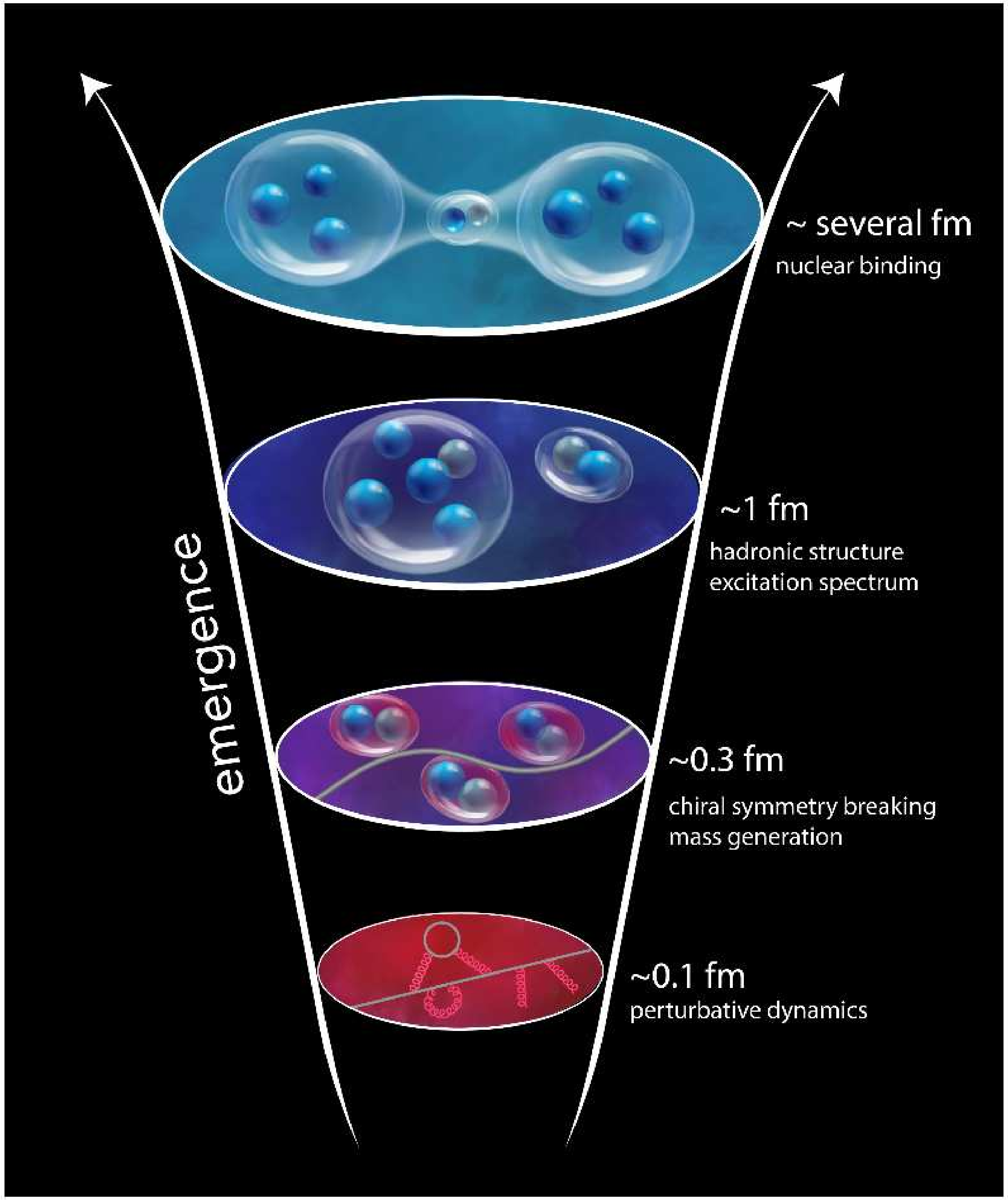
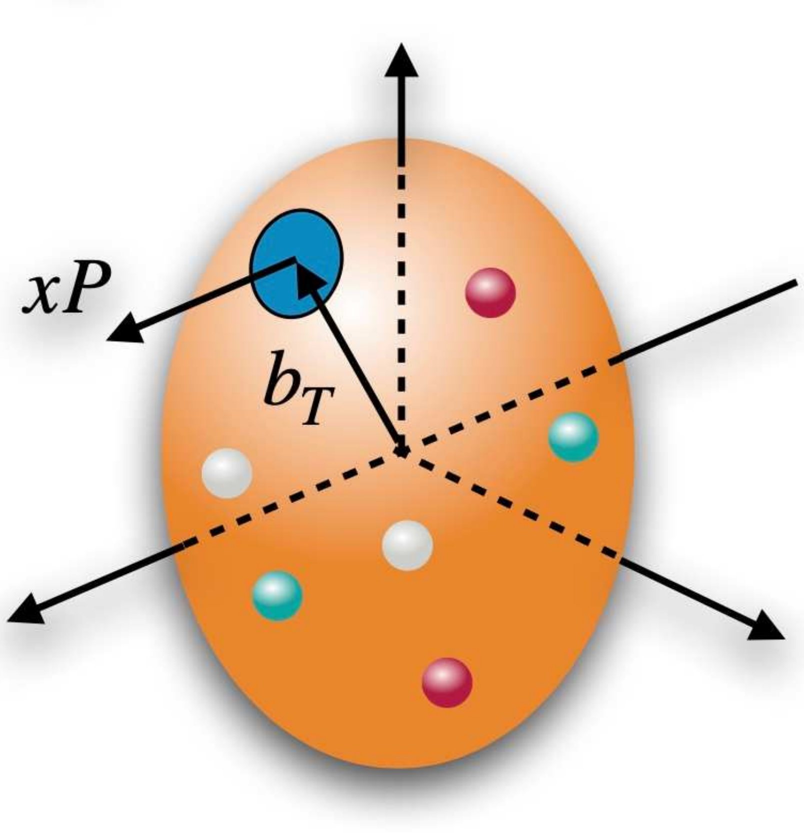


Fig. from arXiv: 2306.09360

Two-scale processes:
length resolution scale
soft momentum scale to probe the emergent regimes at different resolution scales

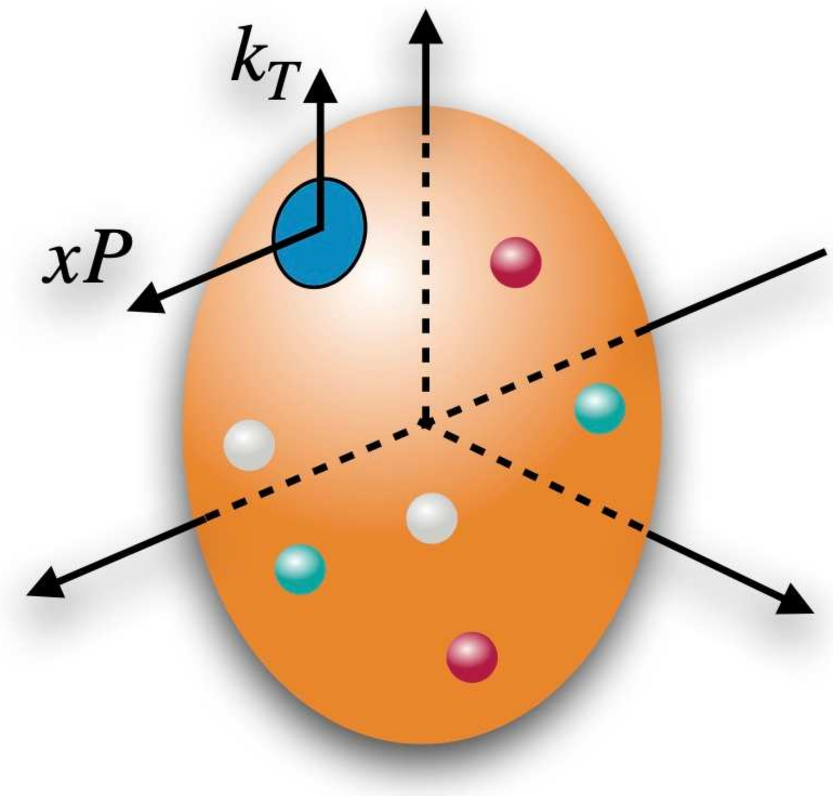
GPDs

Generalized Parton Distributions



TMDs

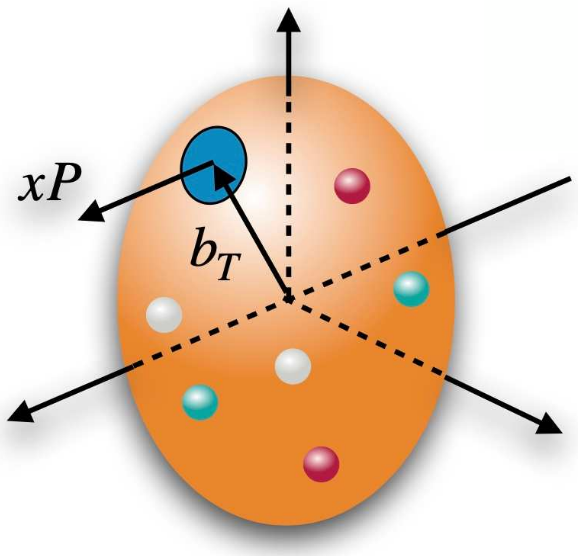
Transverse Momentum Dependent Distributions



Resolution scale $1/Q^2 <<$ $\longrightarrow$ Parton degrees of freedom

Emergence from QCD $\longrightarrow$ 3D structure of the nucleon in space and in momentum

Key information from GPDs



- Multidimensional picture of the proton in the 1+2D

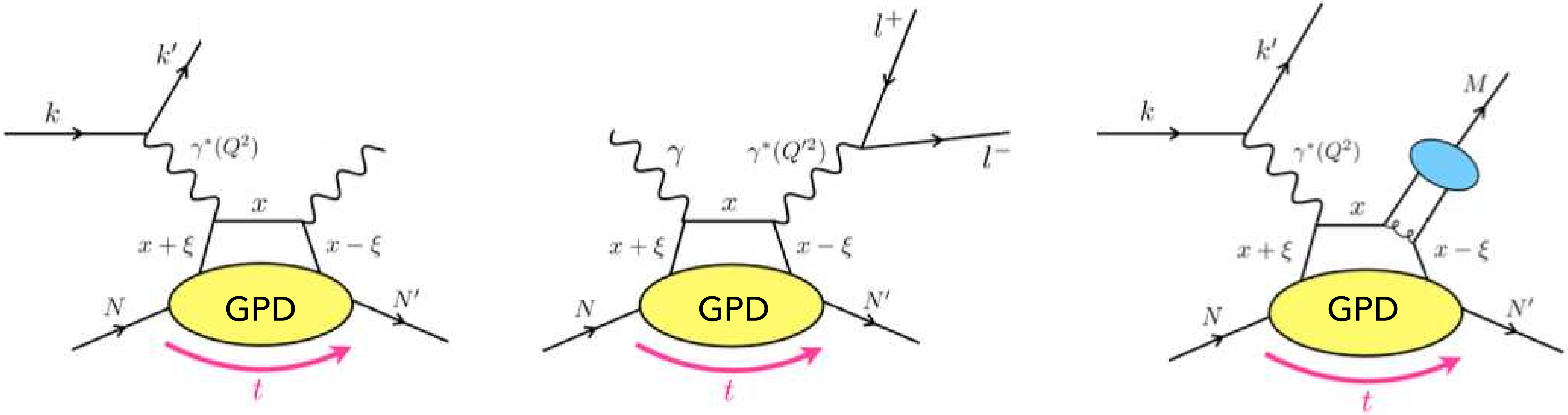
- Access to Form Factors of Energy Momentum Tensor

  ⟶ “mechanical” properties of the nucleon

  ⟶ quark and gluon contribution to mass of the nucleon

- Sum rule for Angular Momentum

How to measure GPDs



- factorization for large Q^2, $|t| << Q^2$, s

- accessible in a variety of exclusive reactions: universality of GPDs

- amplitudes depending on 3 variables: $x,\ \xi,\ t = \Delta^2$ and a scale μ^2

$$x = \frac{(k+k')^+}{(p+p')^+} = \frac{\bar{k}^+}{P^+}$$

average fraction of the longitudinal momentum carried by partons

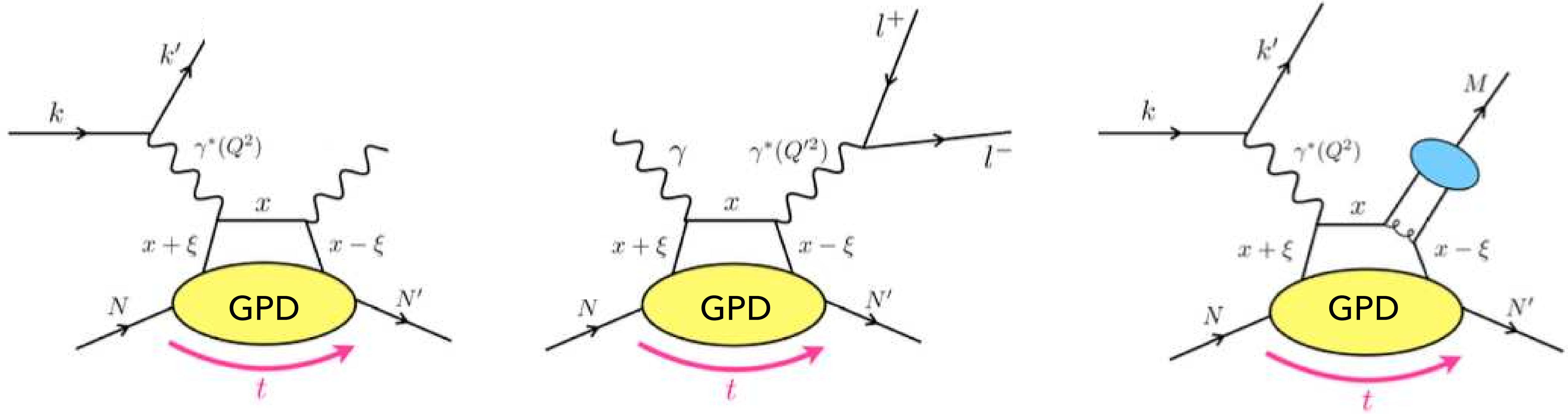
$$\xi = \frac{(p-p')^+}{(p+p')^+} = -\frac{\Delta^+}{2P^+}$$

skewness parameter: fraction of longitudinal momentum transfer

$$t = (p-p')^2 \equiv \Delta^2$$

t-channel momentum transfer squared

How to measure GPDs



- factorization for large Q², |t|<< Q², s
- accessible in a variety of exclusive reactions: universality of GPDs
- amplitudes depending on 3 variables: $x,\, \xi,\, t = \Delta^2$ and a scale μ^2
- Compton form factors $\mathrm{Im}\mathcal{H} \overset{\mathrm{LO}}{=} H(\xi,\xi,t)$ $\mathrm{Re}\mathcal{H} \overset{\mathrm{LO}}{=} \mathcal{P}\int_{-1}^{1} \mathrm{d}x \frac{H(x,\xi,t)}{x-\xi}$

Deconvolution problem: V. Bertone et al., PRD103, 114019 (2021)

GPD table: leading twist

quark polarization

nucleon polarization

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T, \ \tilde{H}_T$

*similar classification for gluon GPDs

GPDs in **black** survive in the collinear limit and reduce to the PDFs

GPDs in **red** vanish if there is no quark orbital angular momentum

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$$(\text{at } \xi = 0) \qquad \vec{\Delta}_\perp \overset{\text{FT}}{\longleftrightarrow} \vec{b}_\perp$$

Impact Parameter Distributions

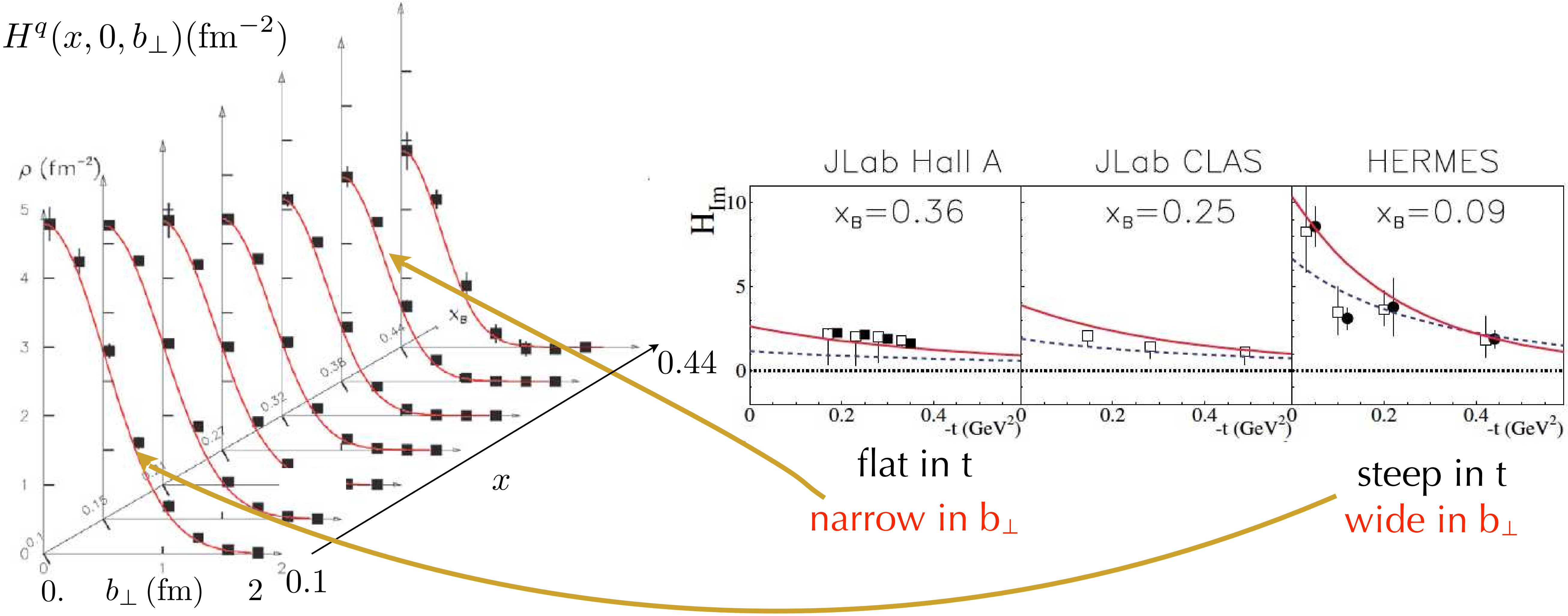
density interpretation: 1D in momentum space (x) and 2D in transverse coordinate space $(\vec{b}_\perp)$

Burkardt., IJMA 18 (2003)173

The unpolarized GPD H

$$H^q(x,0,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} \underset{\downarrow}{H^q(x,0,t)} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \qquad (t = -\vec{\Delta}_\perp^2)$$

No exp. data at $\xi = 0$: extrapolation (thus model dependence) necessary



flat in t

narrow in $b_\perp$

steep in t

wide in $b_\perp$

Dupré et al., PRD95(2017)011501

Guidal et al., Rep. Prog. Phys. ***76*** *(2013) 066202*

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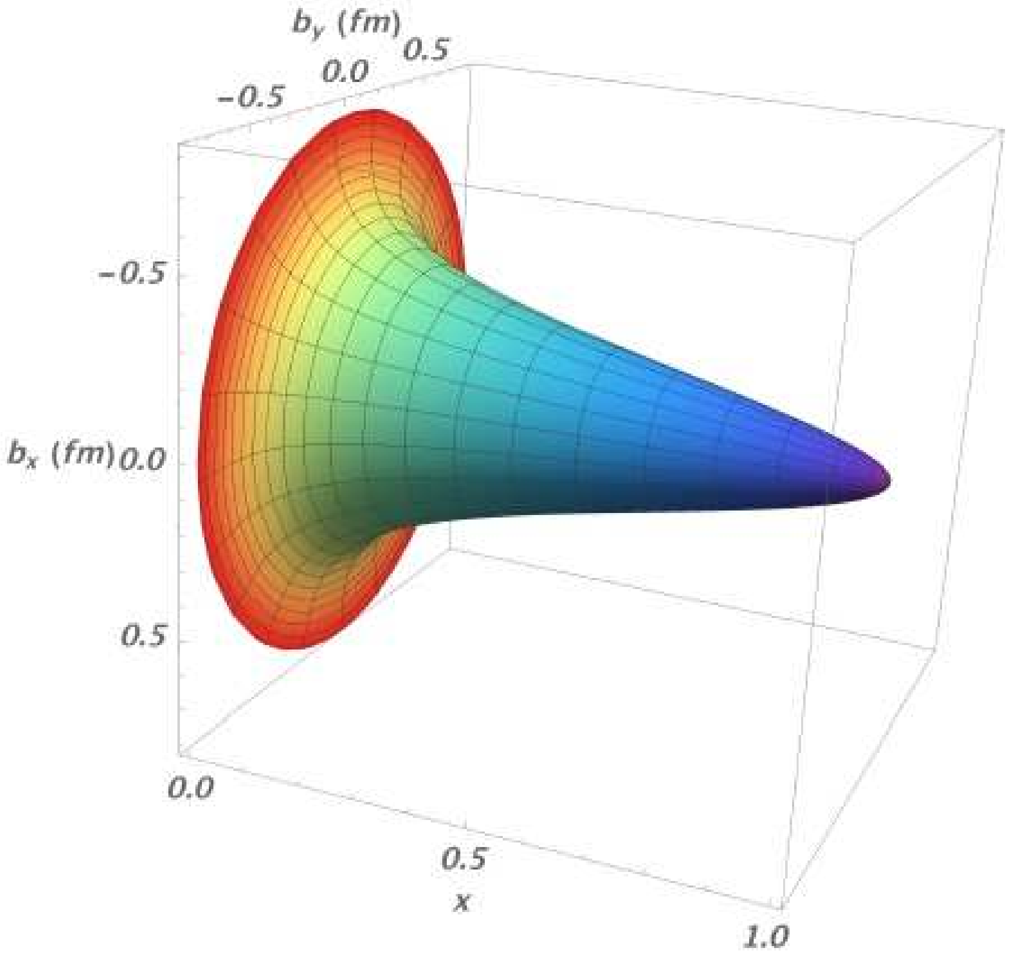
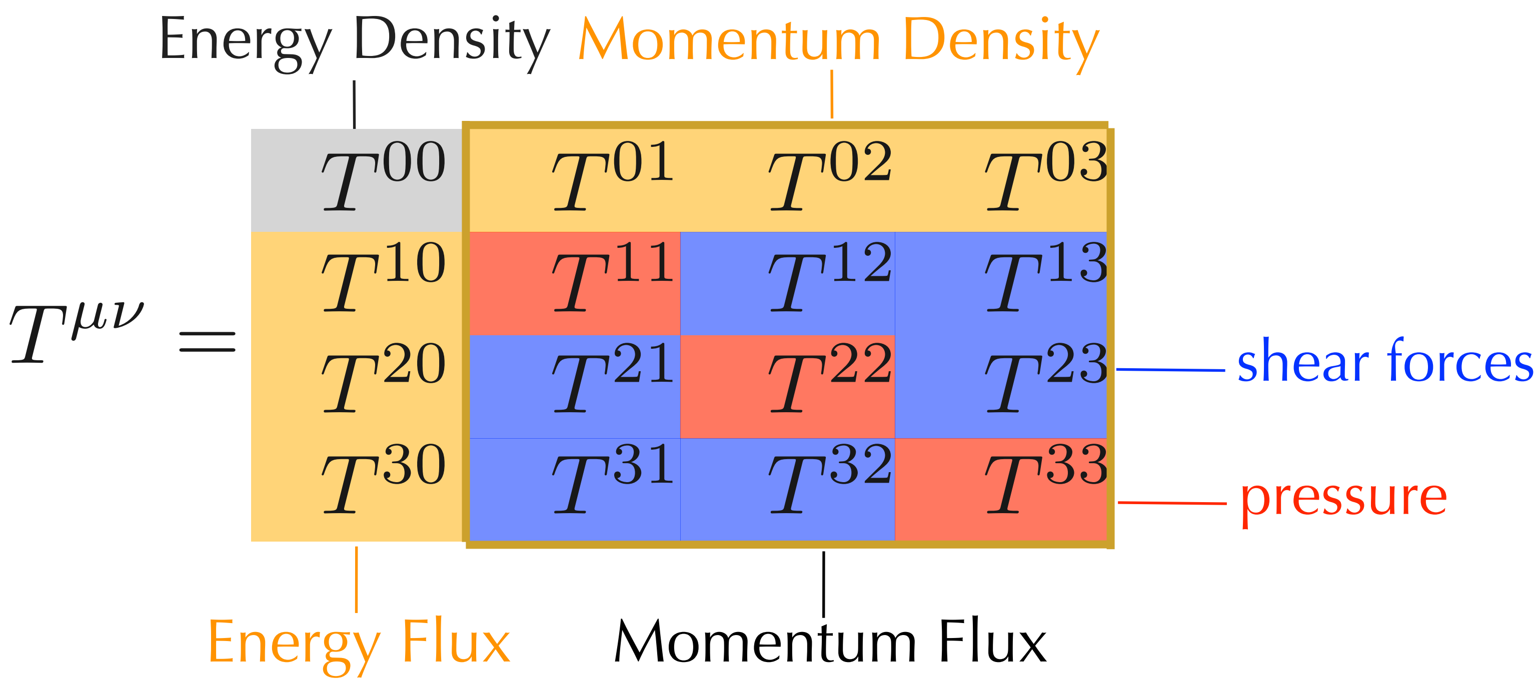
$$\langle \vec{b}_\perp^2(x)\rangle = \frac{\int \mathrm{d}^2\vec{b}_\perp\, \vec{b}_\perp^2\, H(x,0,b_\perp)}{\int \mathrm{d}^2\vec{b}_\perp\, H(x,0,b_\perp)}$$



As x ⟶ 1, the active parton carries all the momentum and represents the centre of momentum

Dupré et al., PRD95(2017)011501

Form Factors of Energy Momentum Tensor



Energy Density

Momentum Density

$$T^{\mu\nu} = \begin{array}{cccc} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{array}$$

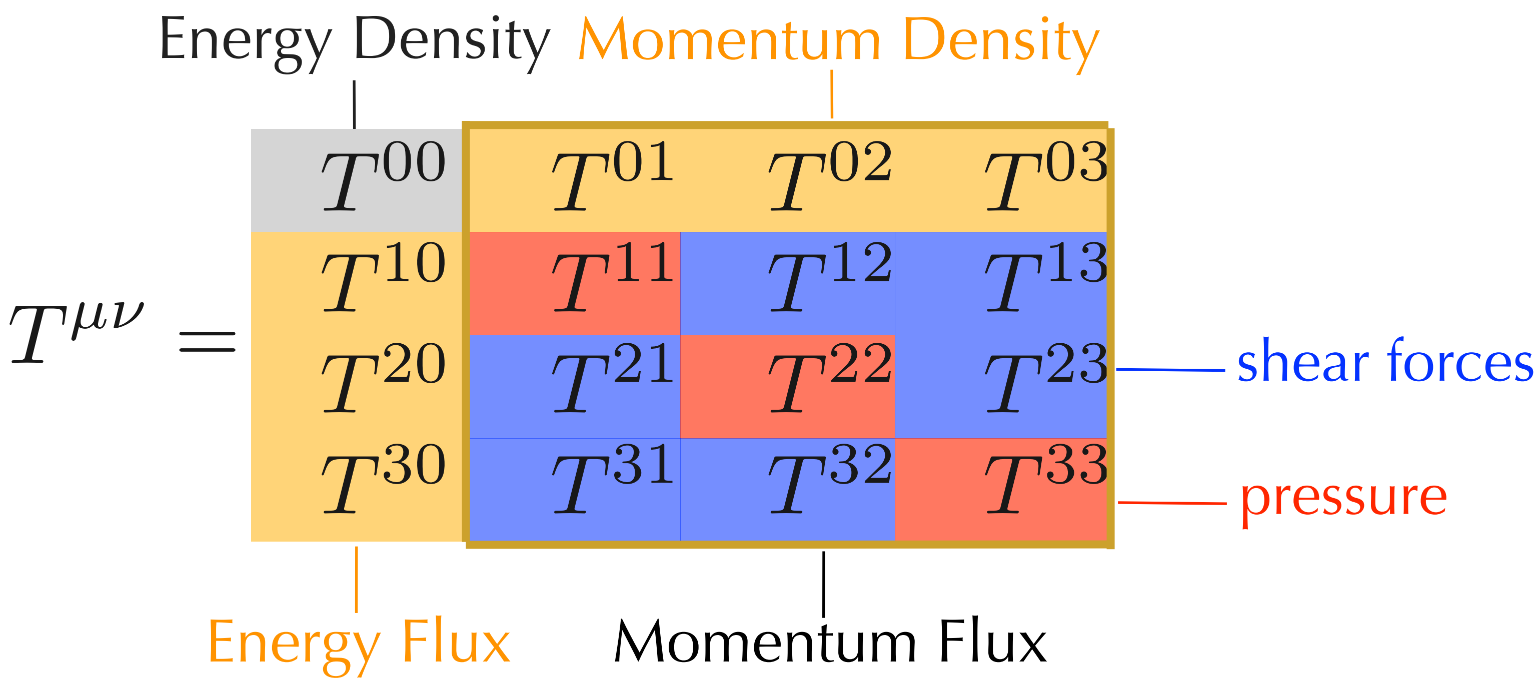
shear forces

pressure

Energy Flux

Momentum Flux

Form Factors of Energy Momentum Tensor



Energy Density

Momentum Density

$$T^{\mu\nu} = \begin{matrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{matrix}$$

shear forces

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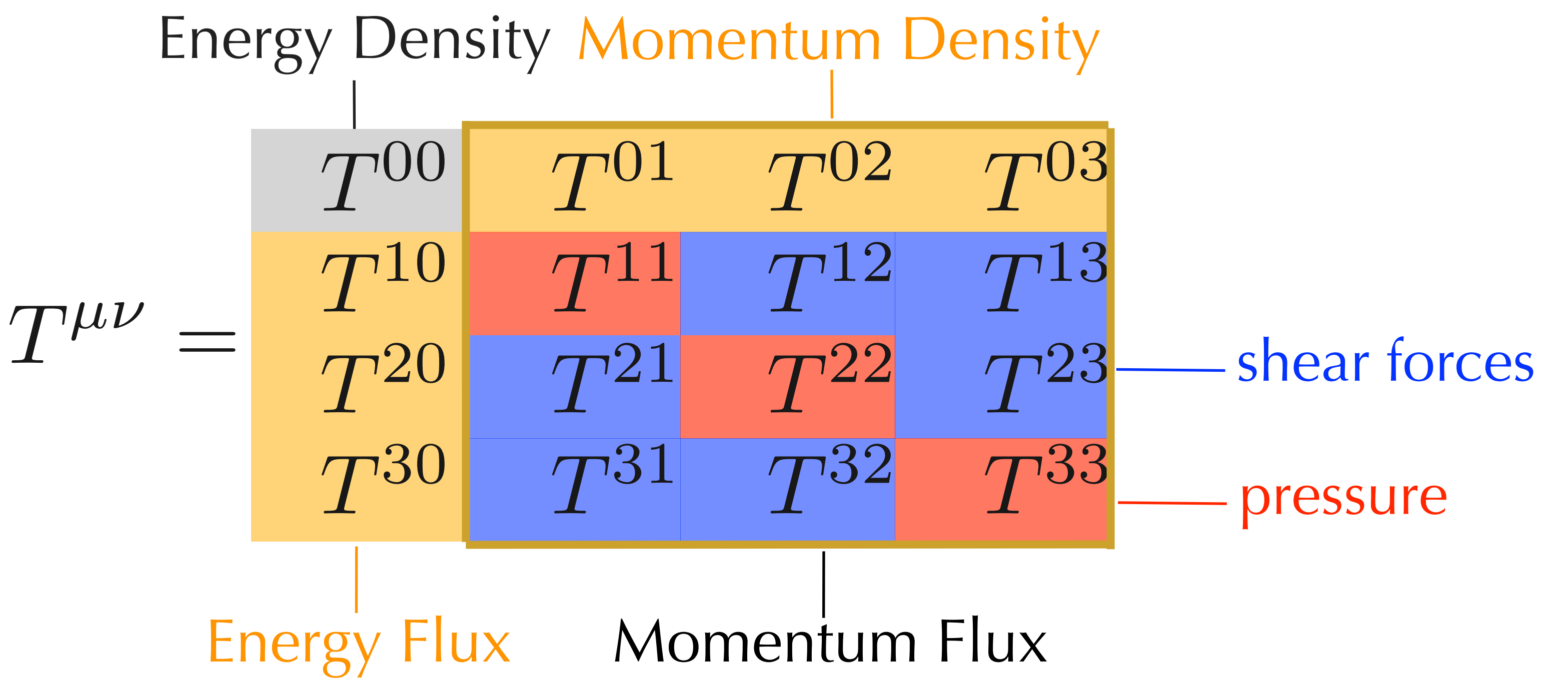
Energy Flux

Momentum Flux

$$\langle p|\, T^{Q,G}_{\mu\nu}\, |p'\rangle = \bar{u}(p')\left[M_2^{Q,G}(t)\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t) g_{\mu\nu}\right] u(p)$$

Form Factors of Energy Momentum Tensor



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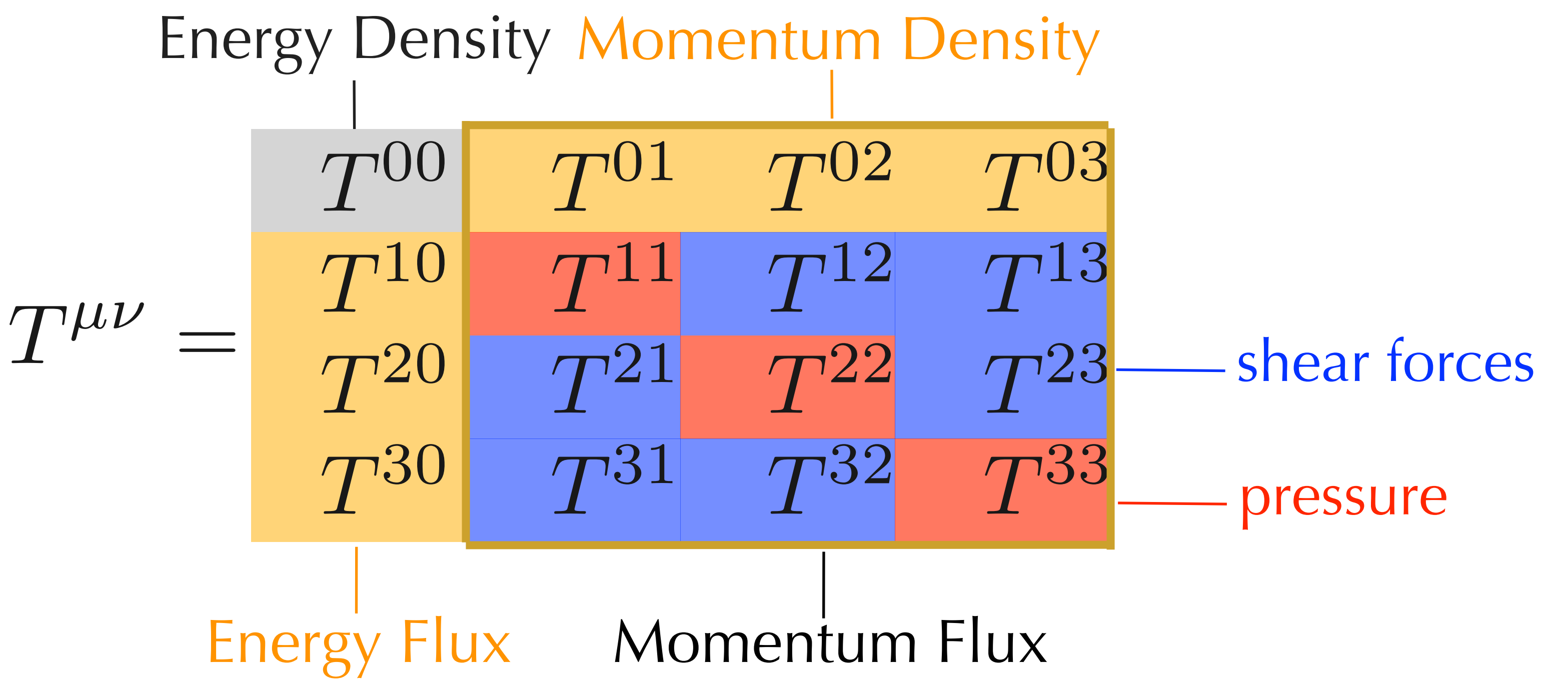
Relation with second-moments of GPDs:

$$\sum_q \int \mathrm{d}x\, x\, H^q(x,\xi,t) = M_2^Q(t) + \frac{4}{5}\, d_1^Q(t)\xi^2$$

$$\sum_q \int \mathrm{d}x\, x\, E^q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5}\, d_1^Q(t)\xi^2$$

Form Factors of Energy Momentum Tensor



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Momentum Flux

$$\langle p| T^{Q,G}_{\mu\nu} |p'\rangle = \bar{u}(p') \left[M_2^{Q,G}(t) \frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t) g_{\mu\nu} \right] u(p)$$

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"Charges" of the EMT Form Factors at t=0

$M_2(0)$	nucleon momentum carried by parton
$J(0)$	angular momentum of partons
$d_1(0)$	D-term ("stability" of the nucleon)

D(t) form factor from data

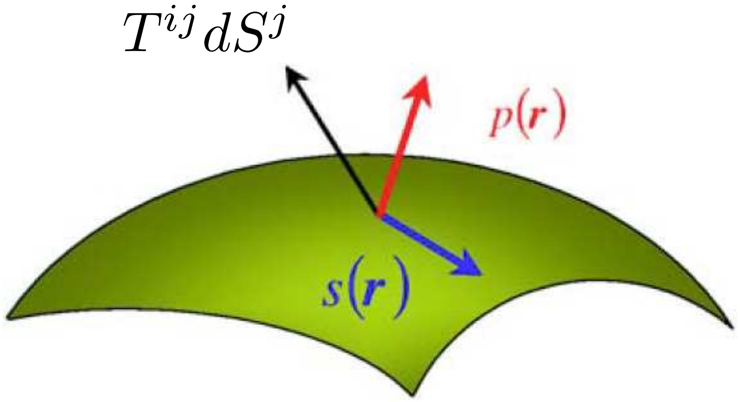
➡ Fourier transform in coordinate space

$$T^Q_{ij}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\,\delta_{ij}$$

$s(\vec{r})$ ↓ shear forces

$p(\vec{r})$ ↓ pressure

“mechanical properties” of nucleon

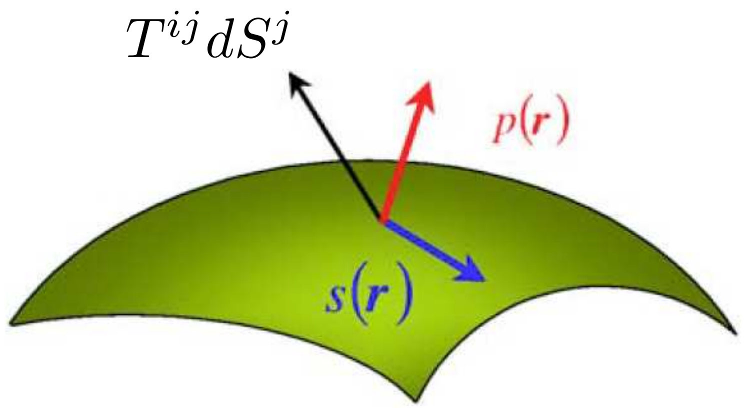


D(t) form factor from data

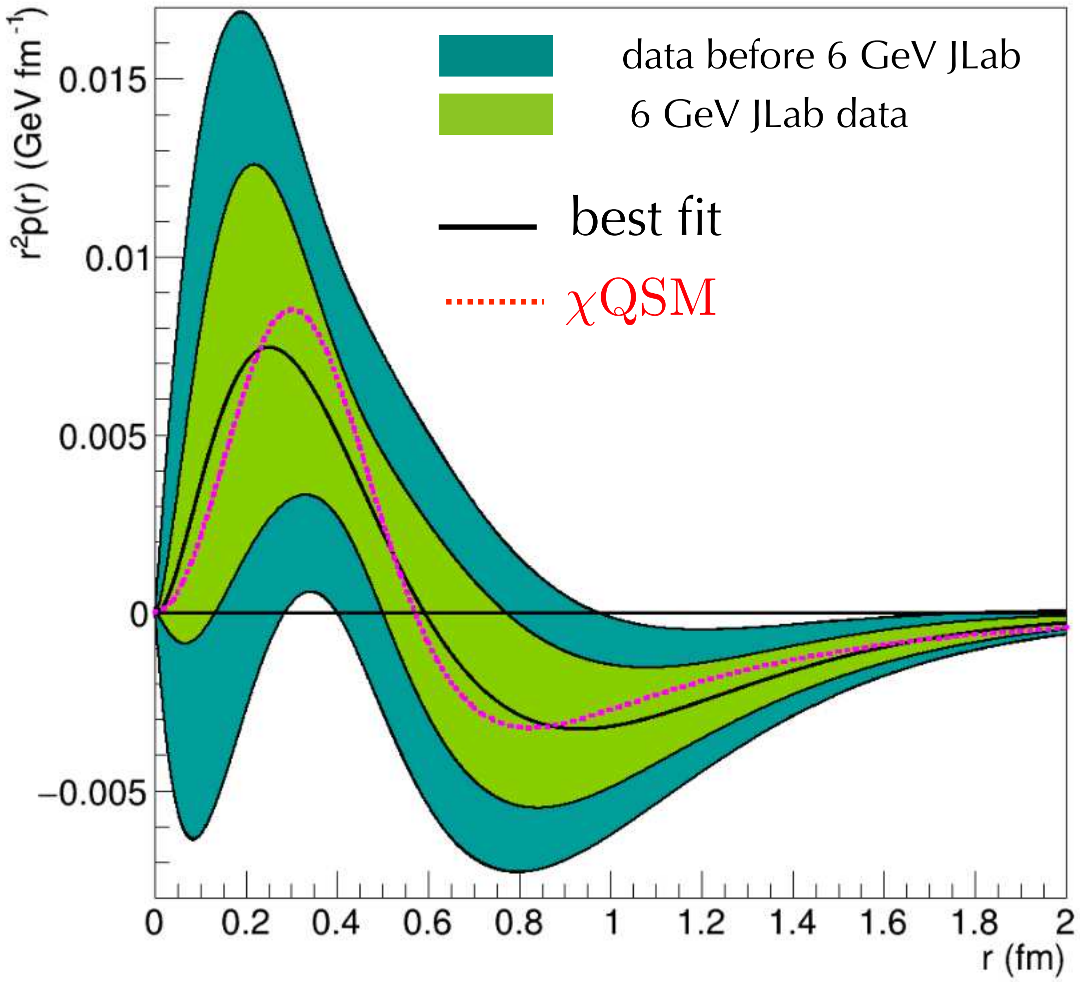
→ Fourier transform in coordinate space

$$T^{Q}_{ij}(\vec{r}) = \underset{\substack{\downarrow \\ \text{shear forces}}}{s(\vec{r})} \left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij} \right) + \underset{\substack{\downarrow \\ \text{pressure}}}{p(\vec{r})}\, \delta_{ij}$$

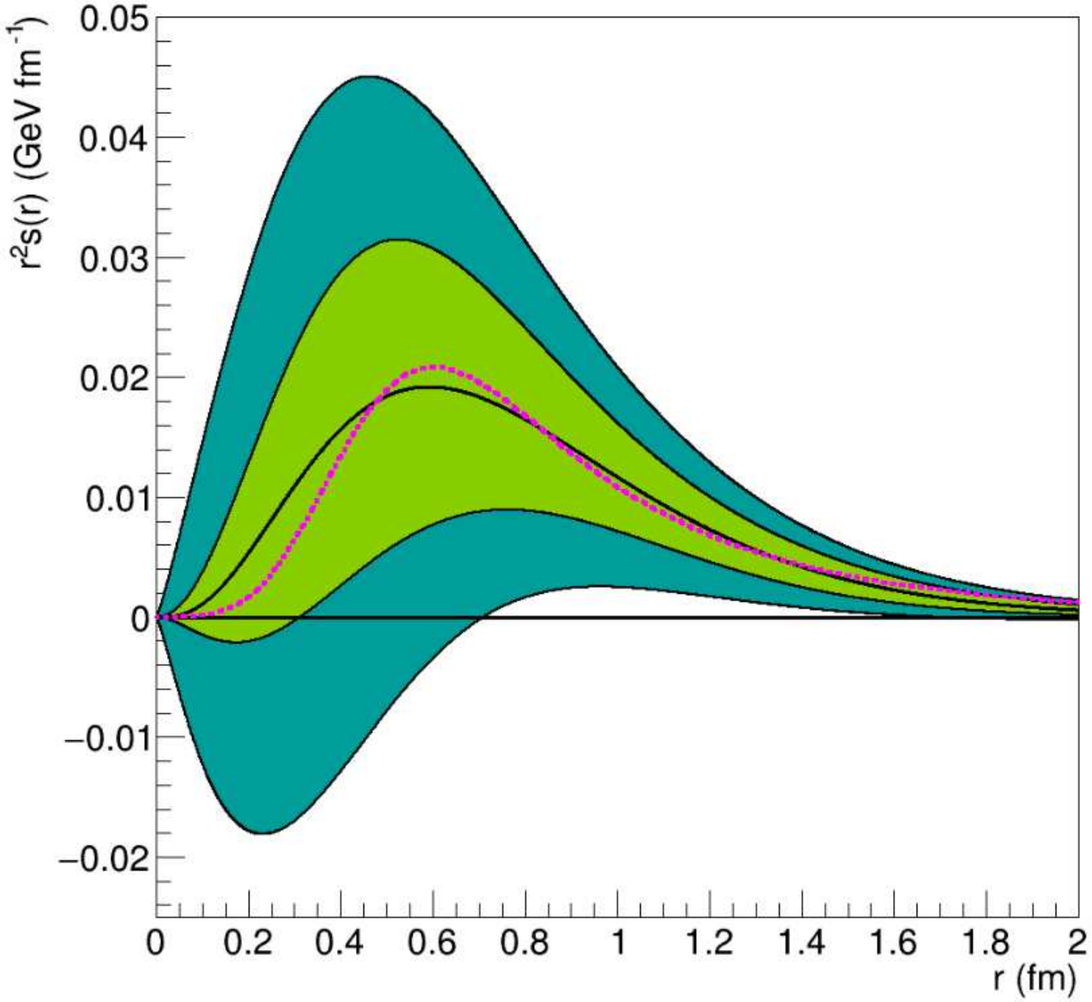
“mechanical properties” of nucleon



$$p(r) = \frac{1}{6M_N} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} D(r)$$



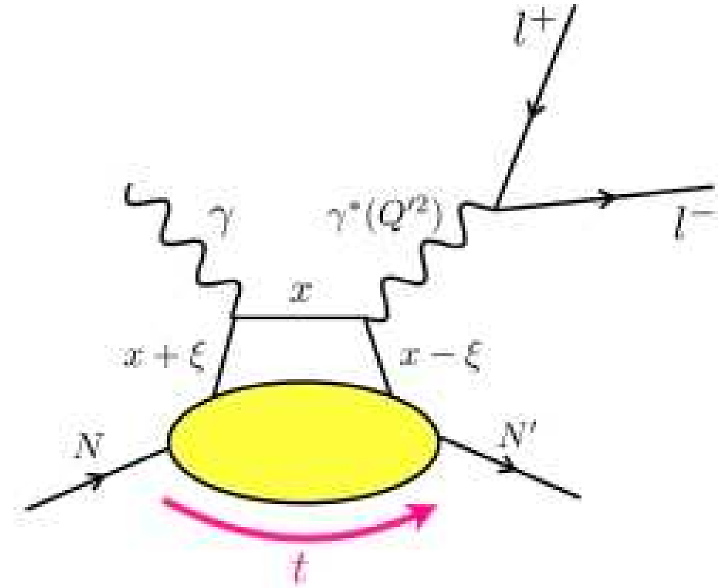
$$s(r) = -\frac{1}{4M_N} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} D(r)$$



Girod, Elouadrhiri,Burkert, Nature 557 (2018) 7705; Polyakov, Schweitzer, IJMA 33 (2018) 1830025

Timelike Compton scattering

Chatagnon et al. (CLAS12 Coll.), PRL127, 262501(2021)

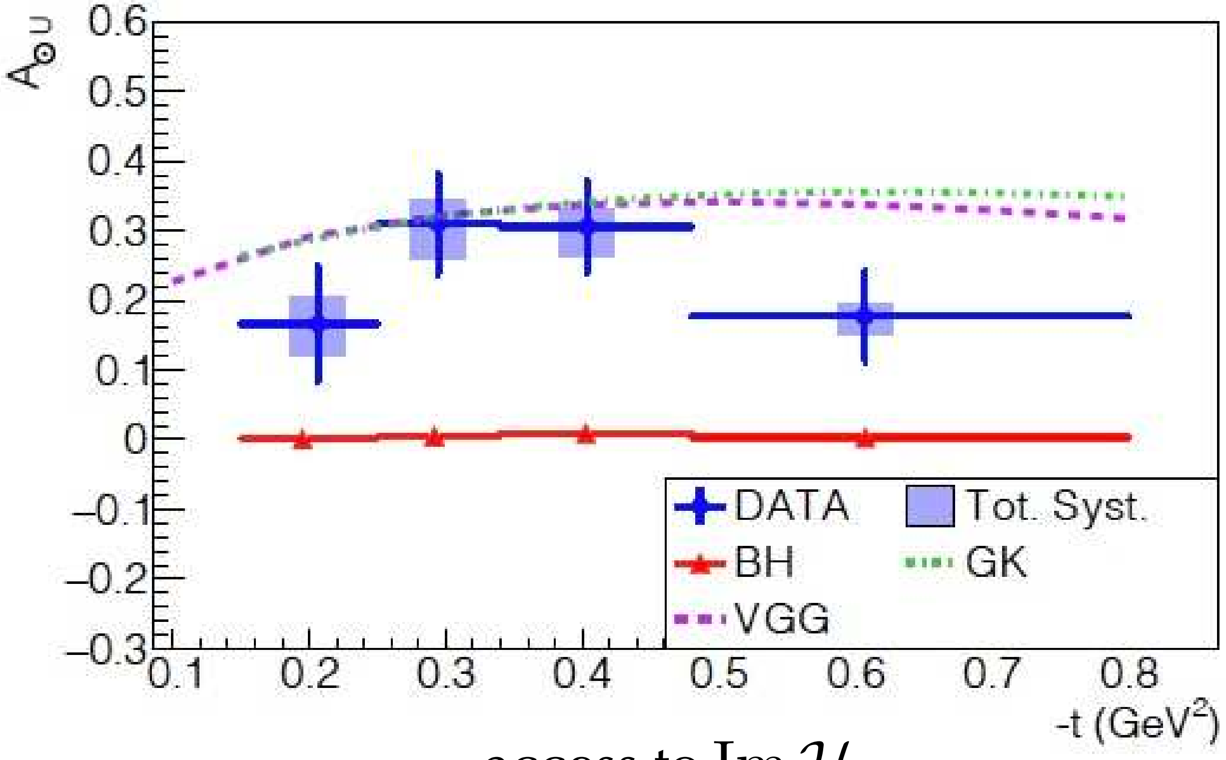


photon polarization asymmetry

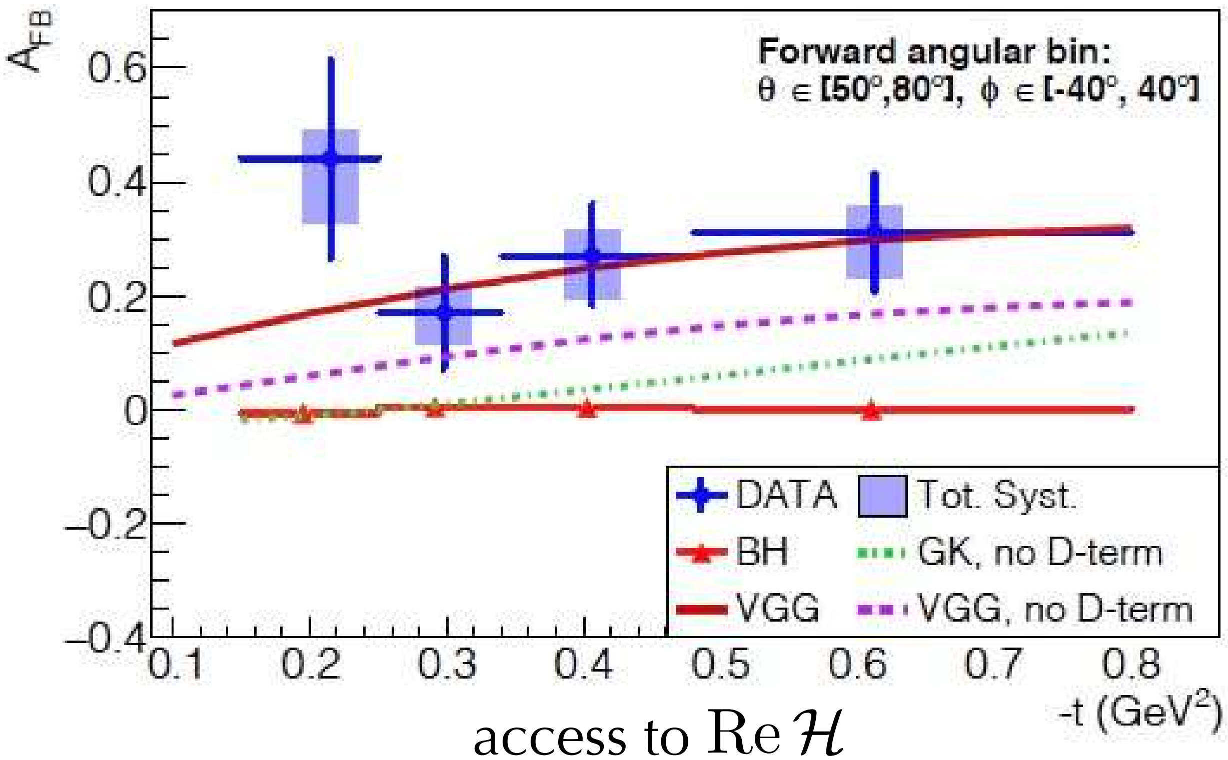
$$A_{\odot U} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

forward-backward asymmetry

$$A_{FB} = \frac{d\sigma(\theta,\phi) - d\sigma(180^\circ - \theta, 180^\circ + \phi)}{d\sigma(\theta,\phi) + d\sigma(180^\circ - \theta, 180^\circ + \phi)}$$



access to $\mathrm{Im}\,\mathcal{H}$



access to $\mathrm{Re}\,\mathcal{H}$

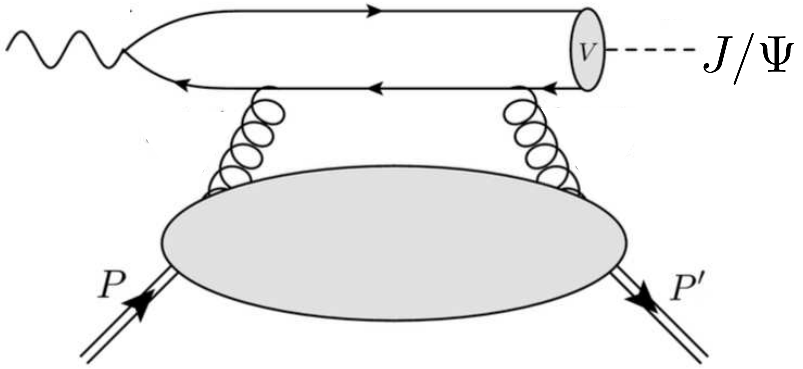
✓ Test of the universality of GPDs

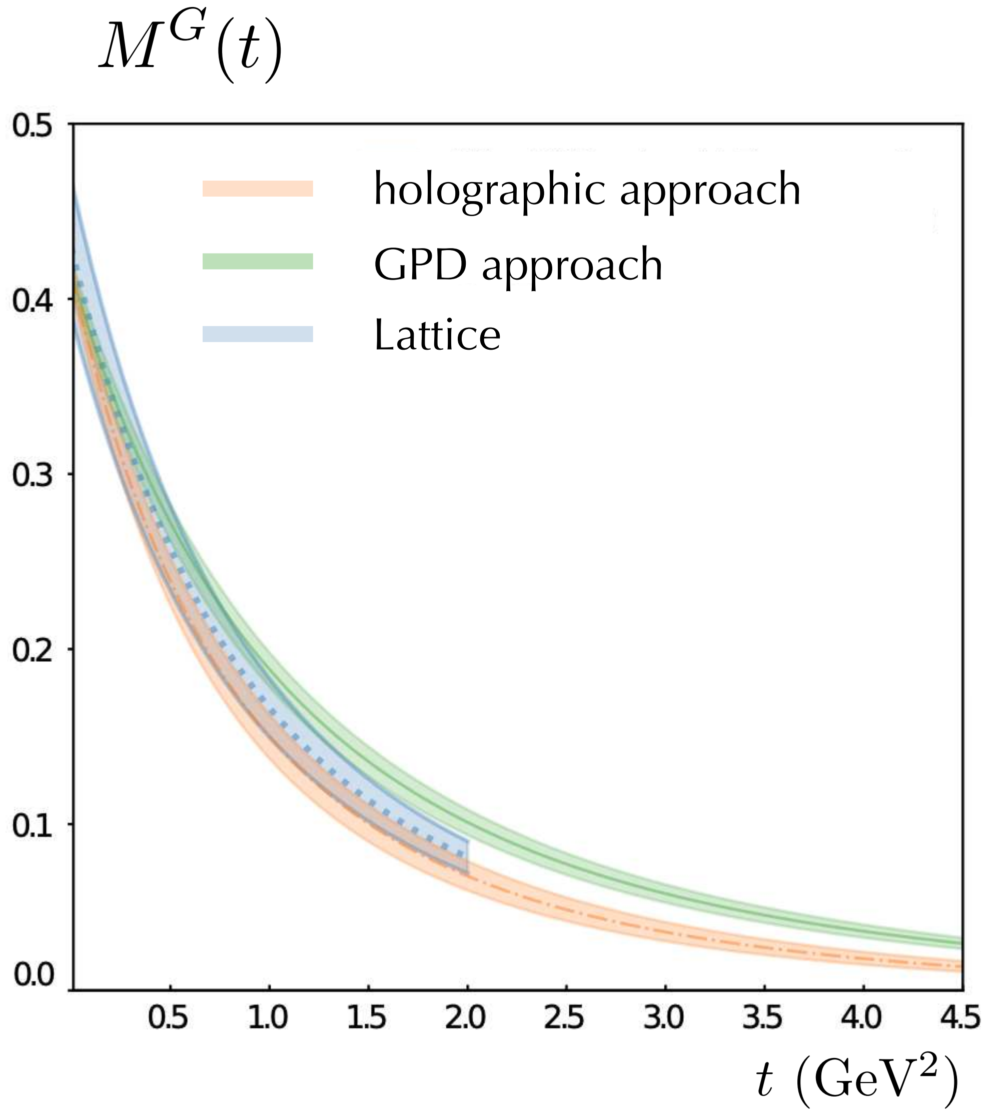
✓ Further data from JLab12 and future EIC

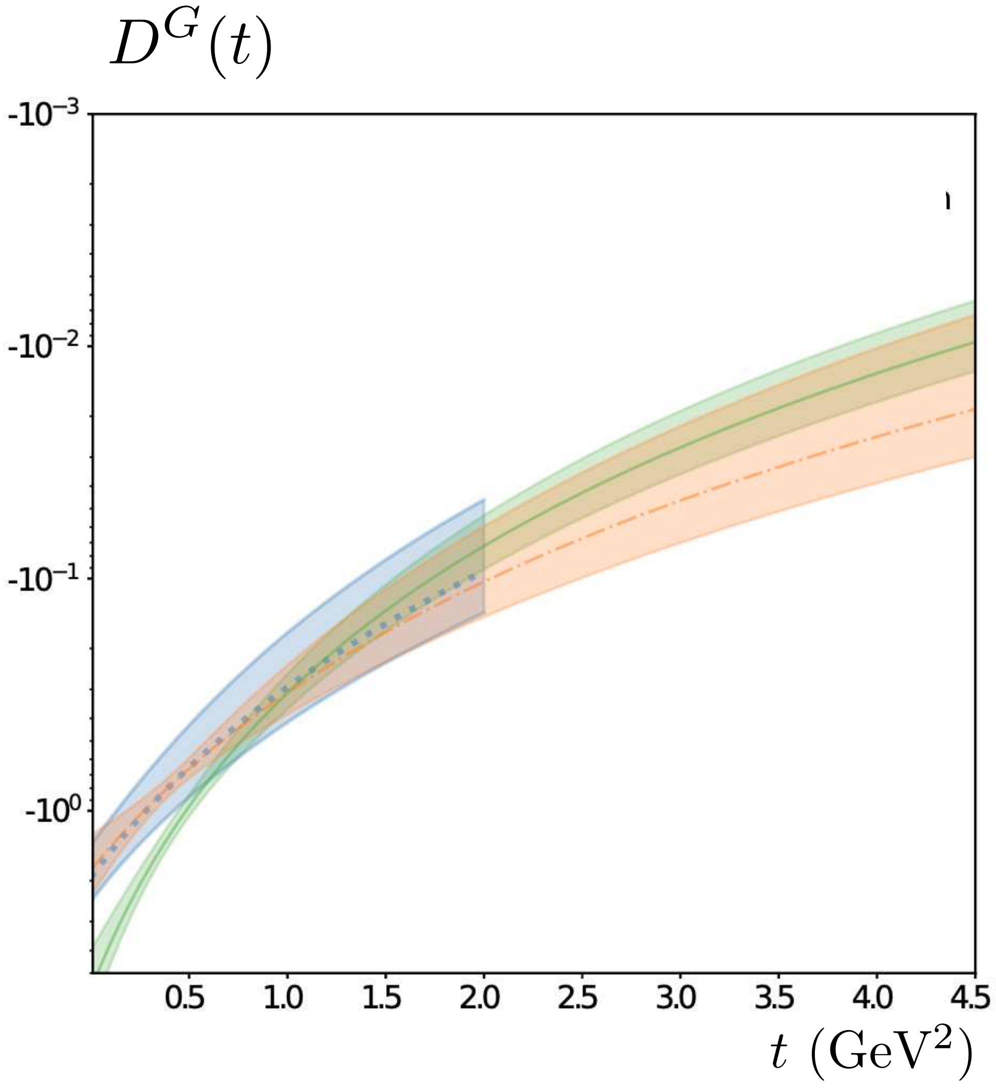
✓ New promising path towards the extraction of $\mathrm{Re}\,\mathcal{H}$ and then the D-term (also with positron beam)

Gluonic EMT Form Factors

Duran et al., Nature 615 (2023) 7954



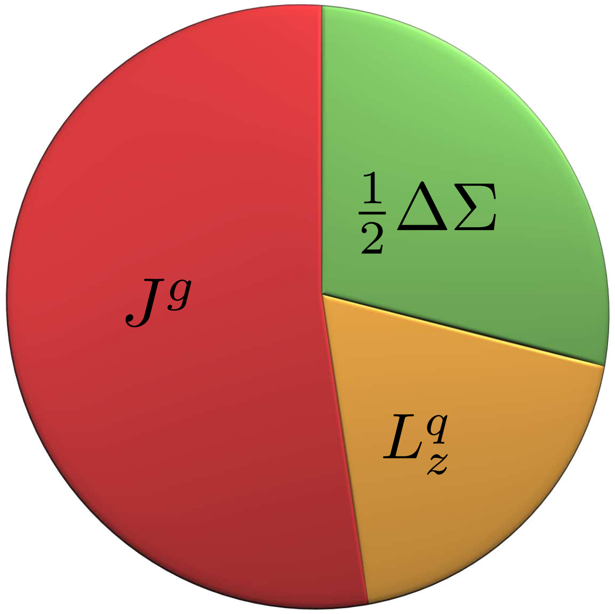




- proof of concept of feasibility to extract gluonic structure
- further measurements planned with SOLID at JLab
- JLab22 crucial for these measurements: high luminosity and leverage in t
- EIC: complementary measurements for Υ photo- and electro-production, but require L=100 fb^{-1}

Angular Momentum Relation

X. Ji, PRL 78 (1997) 610



$$\frac{1}{2} = J^q + J^g \qquad\qquad L_z^q = J^q - \frac{1}{2}\Delta\Sigma$$

$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} dx\, x\, \left(\underset{\substack{\downarrow \\ \text{at } \xi=0 \text{ unpolarized PDF}}}{H^{q,g}(x,\xi,0)} + \underset{\substack{\downarrow \\ \text{not directly accessible}}}{E^{q,g}(x,\xi,0)}\right)$$

- Requires extrapolation to t=0
- Requires spanning x at fixed values of ξ ($\xi = 0$ is the most convenient)
- $J^{q,g}(x) \neq \frac{1}{2}[xH^{q,g}(x,0,0) + E^{q,g}(x,0,0)] \longrightarrow$ not angular momentum density

Angular momentum of the proton from GPD measurements

$J^q = \frac{1}{2}\int_{-1}^{1} dx\, x\, (H^q(x,\xi,0) + E^q(x,\xi,0))$ $L^q = J^q - \frac{1}{2}\Delta\Sigma$

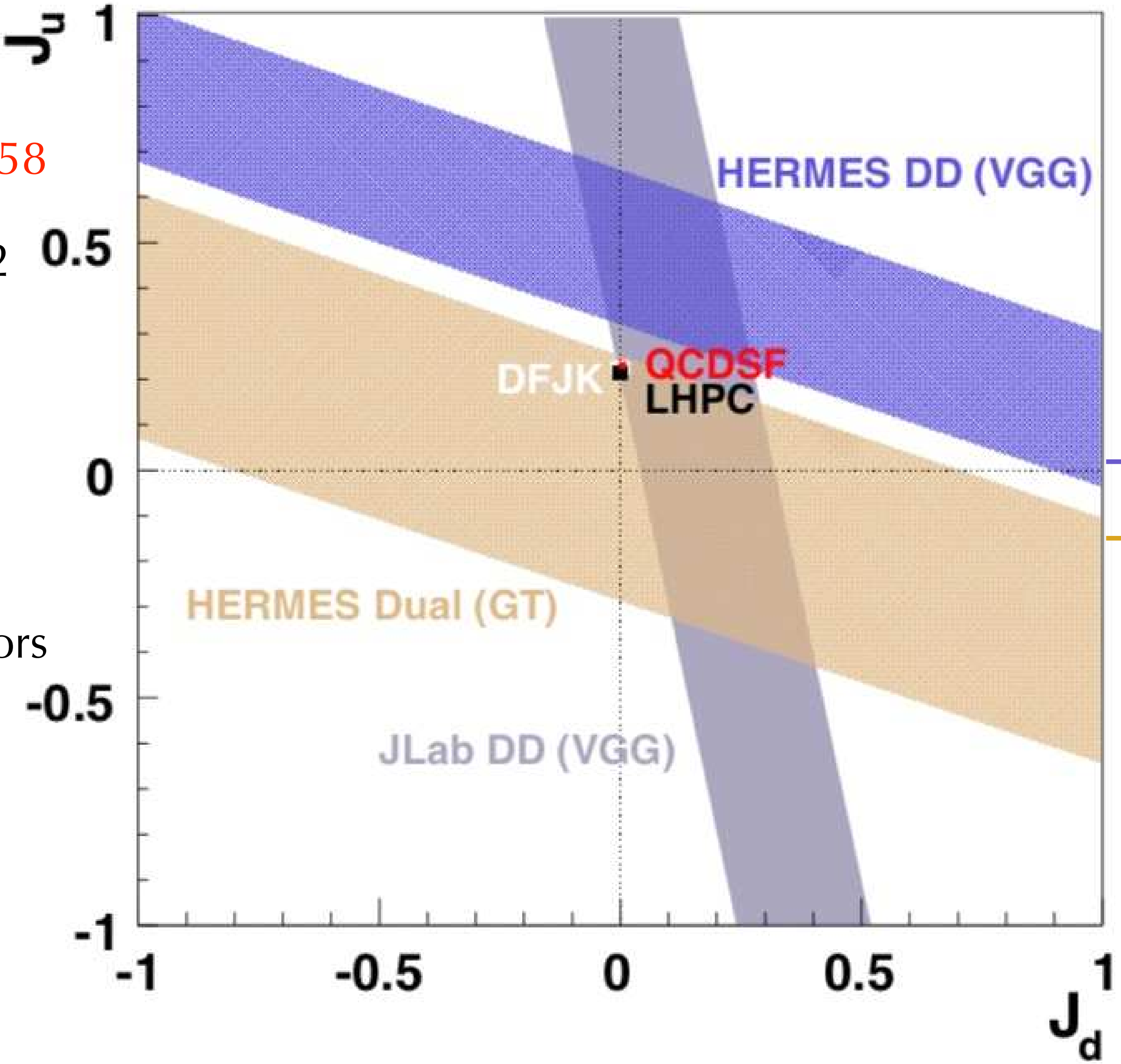
Lattice results

QCDSF: PoS (Lattice 2007) 158

LHPC: PRD77 (2008) 094502

GPDs extracted from form factors

DFJK, EPJC39 (2005) 1



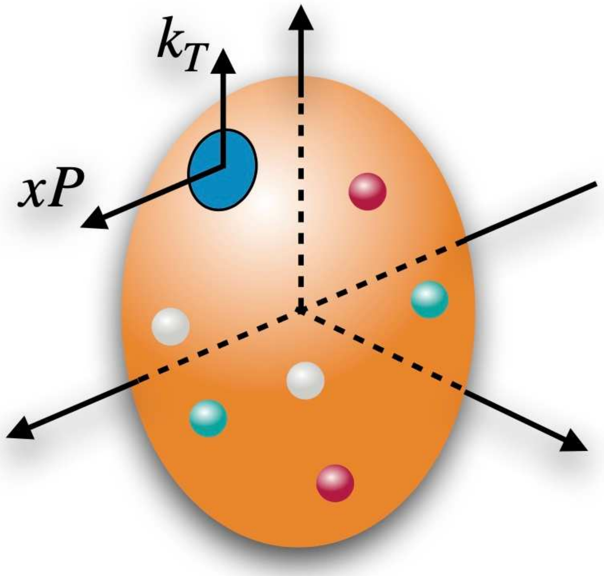
extractions from HERMES data using two different models

JLab Hall A, Phys. Rev. Lett. 99 (2007) 242501

Hermes Coll., JHEP 06 (2008) 066

Improved accuracy with JLab12 and future EIC measurements!

Key information from TMDs

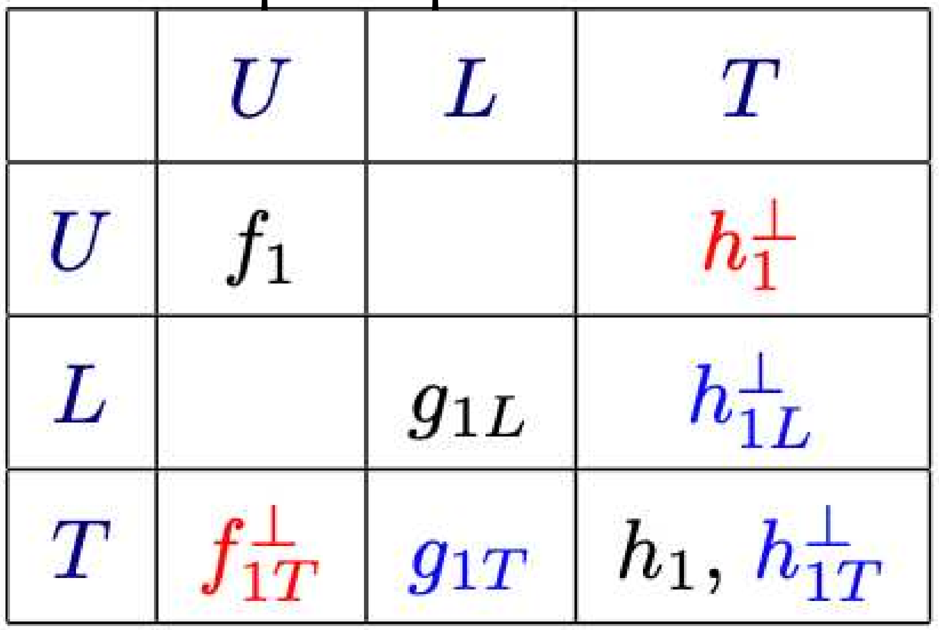


- Complete momentum spectrum of single particle
- Transverse momentum size as function of x (3D map) at different Q^2
- Spin-Spin and Spin-Orbit Correlations of partons
- Information on parton orbital angular momentum
  (no direct model-independent relation)

TMD table: leading twist

quark polarization

nucleon polariz.



	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

*similar classification for gluon TMDs

TMDs in **black** survive integration over transverse momentum and reduce to the PDFs

TMDs in **blue** and **red** vanish if there is no quark orbital angular momentum

TMDs in **red** are time-reversal odd

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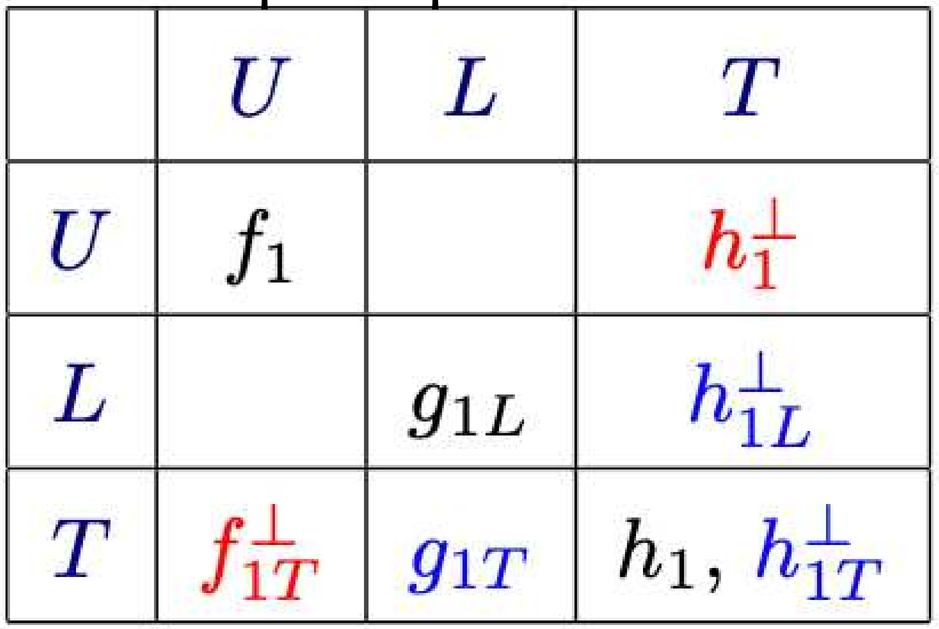

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- Very good knowledge of x dependence of f_1 and g_{1L}

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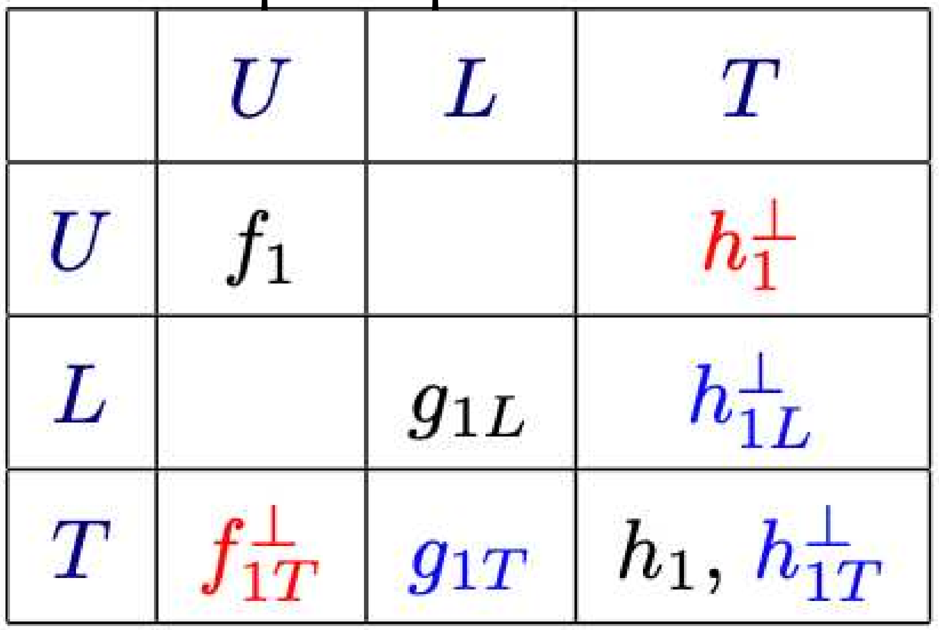

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- Good knowledge of the k_T dependence of f_1 (also for the pion)

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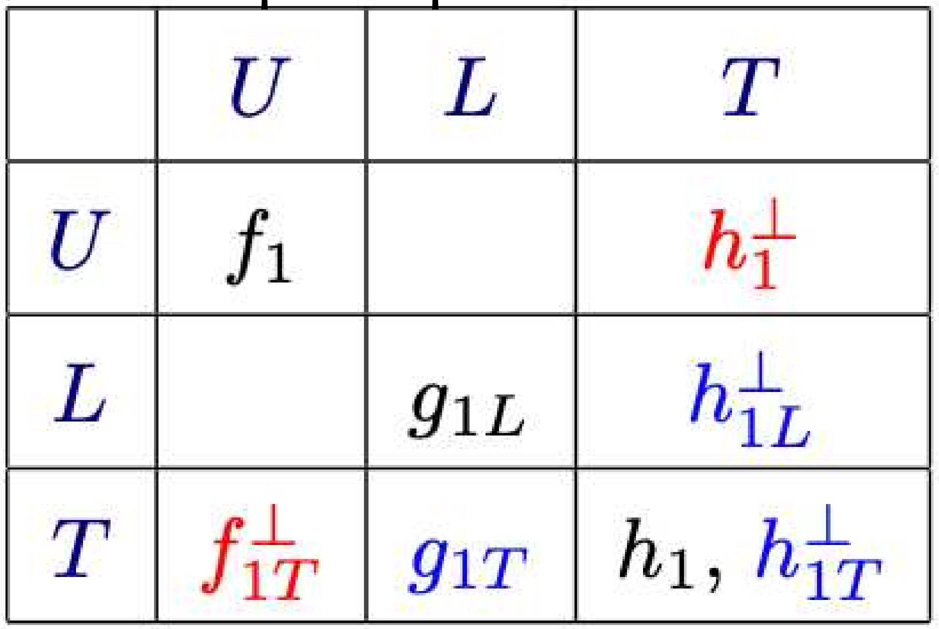

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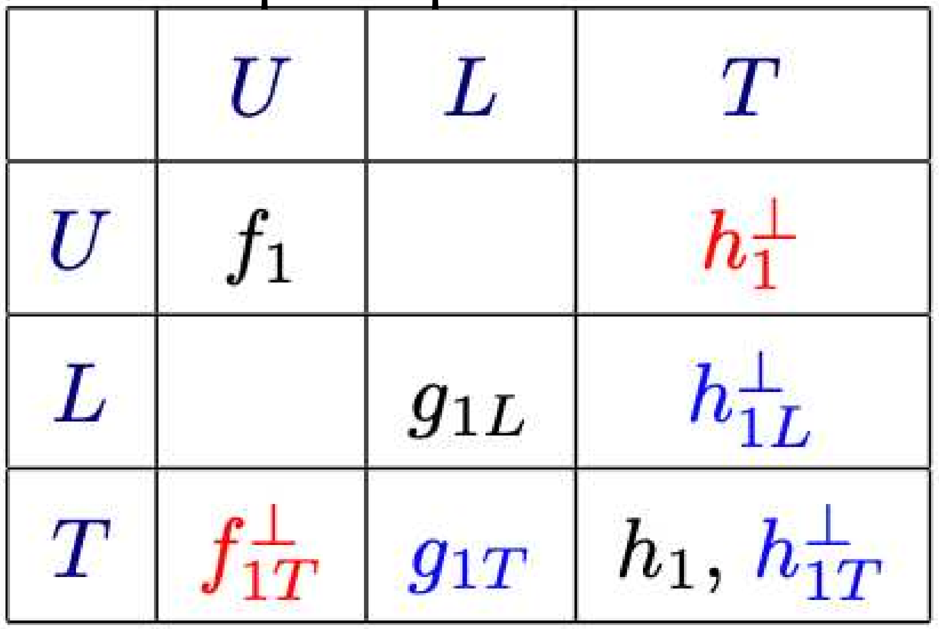
- Fair knowledge of the Sivers and transversity (mainly x dependence)

TMD table: leading twist



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nucleon polariz.

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T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

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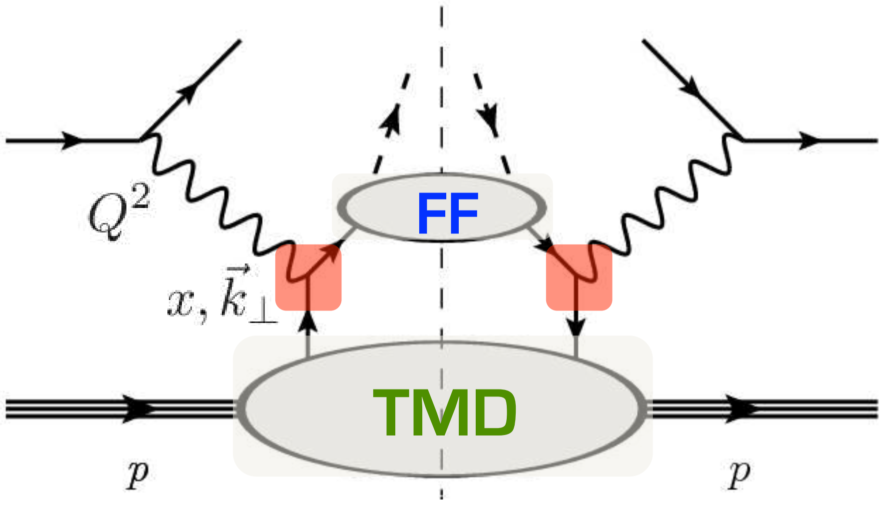
- Very good knowledge of x dependence of f_1 and g_{1L}
- Good knowledge of the k_T dependence of f_1 (also for the pion)
- Fair knowledge of the Sivers and transversity (mainly x dependence)
- Some hints about all other

How to measure TMDs

SIDIS

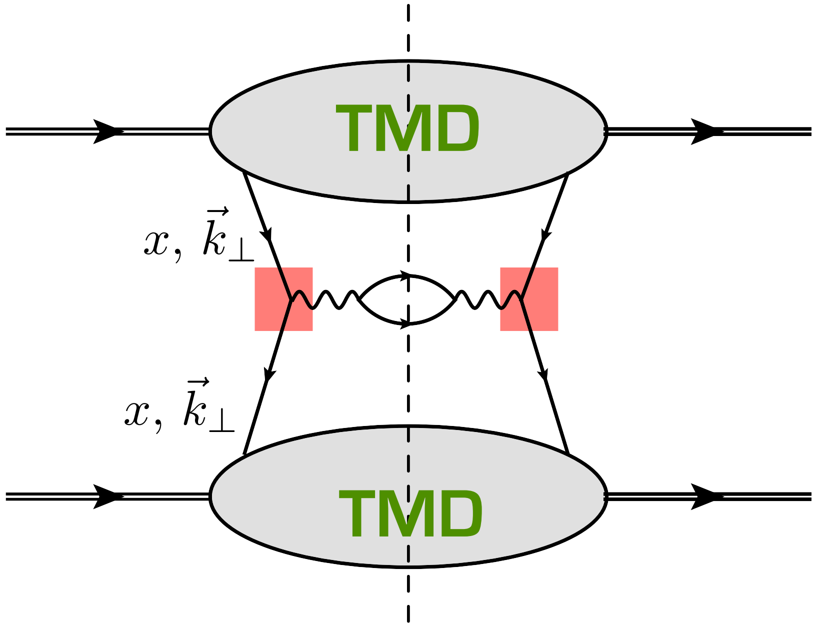
$\ell(l) + N(P) \to \ell(l') + h(P_h) + X$



$$d\sigma \sim \sum \mathrm{TMD}(x,\vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \mathrm{FF}(z,\vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

Drell-Yan

$h(P_1) + h(P_2) \to \ell^+(l) + \ell^-(l')$



$$d\sigma \sim \sum \mathrm{TMD}(x,\vec{k}_\perp) \otimes \overline{\mathrm{TMD}}(x,\vec{k}_\perp) \otimes d\hat{\sigma}_{hard}$$

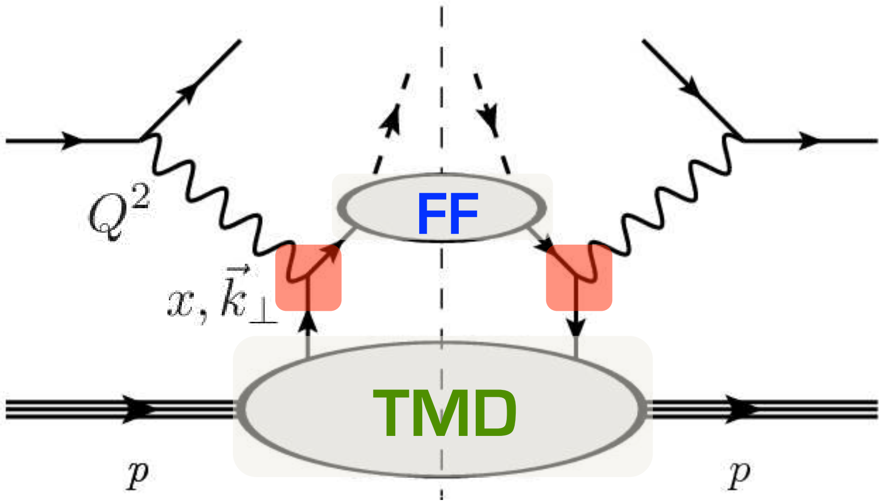
✓Factorization

✓Universality

How to measure TMDs

SIDIS

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X$$

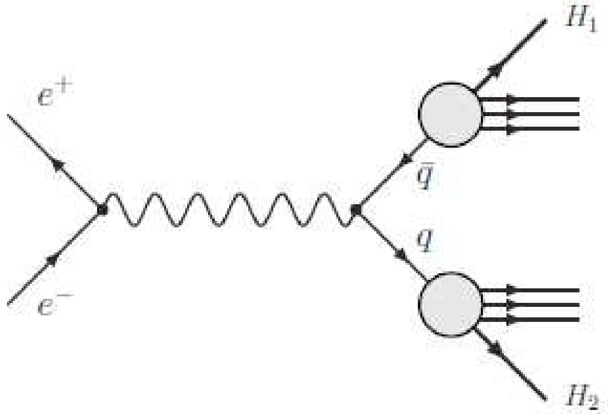


$$d\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \mathrm{FF}(z, \vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

$\downarrow$

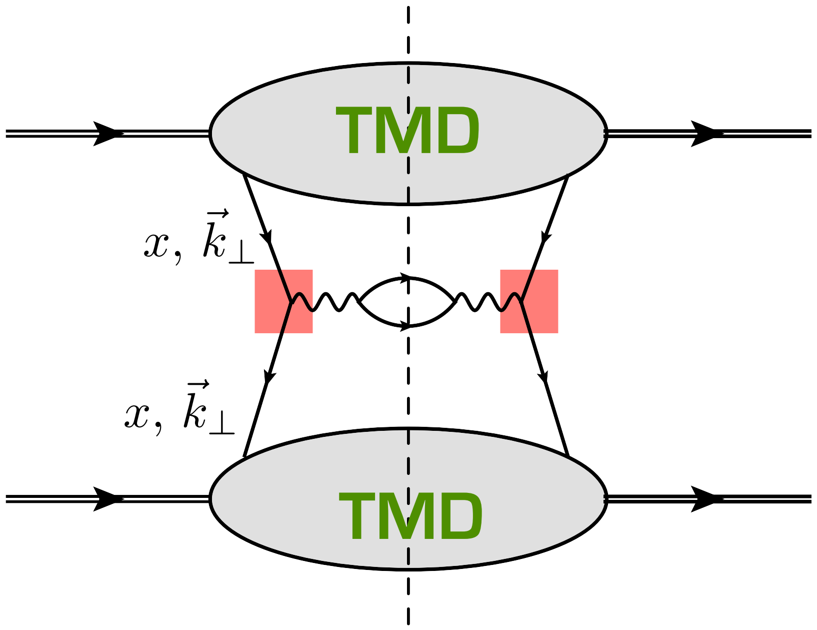
Fragmentation Functions

$e^+e^- \to hh'X$



Drell-Yan

$$h(P_1) + h(P_2) \to \ell^+(l) + \ell^-(l')$$



$$d\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes \overline{\mathrm{TMD}}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard}$$

✓Factorization

✓Universality

Quark unpolarized TMD extractions
flavor independent

	Framework	HERMES	COMPASS	DY	Z Production	N of points
BSV 2019 arXiv:1902.08474	NNLL	✗	✗	✓	✓	457
Pavia 2019 arXiv:1912.07550	N3LL	✗	✗	✓	✓	353
SV 2019 arXiv:1912.06532	N3LL	✓	✓	✓	✓	1039
MAP 2022 arXiv:2206.07598	N3LL	✓	✓	✓	✓	2031
ART 2023 arXiv:2305.07473	N4LL	✗	✗	✓	✓	627

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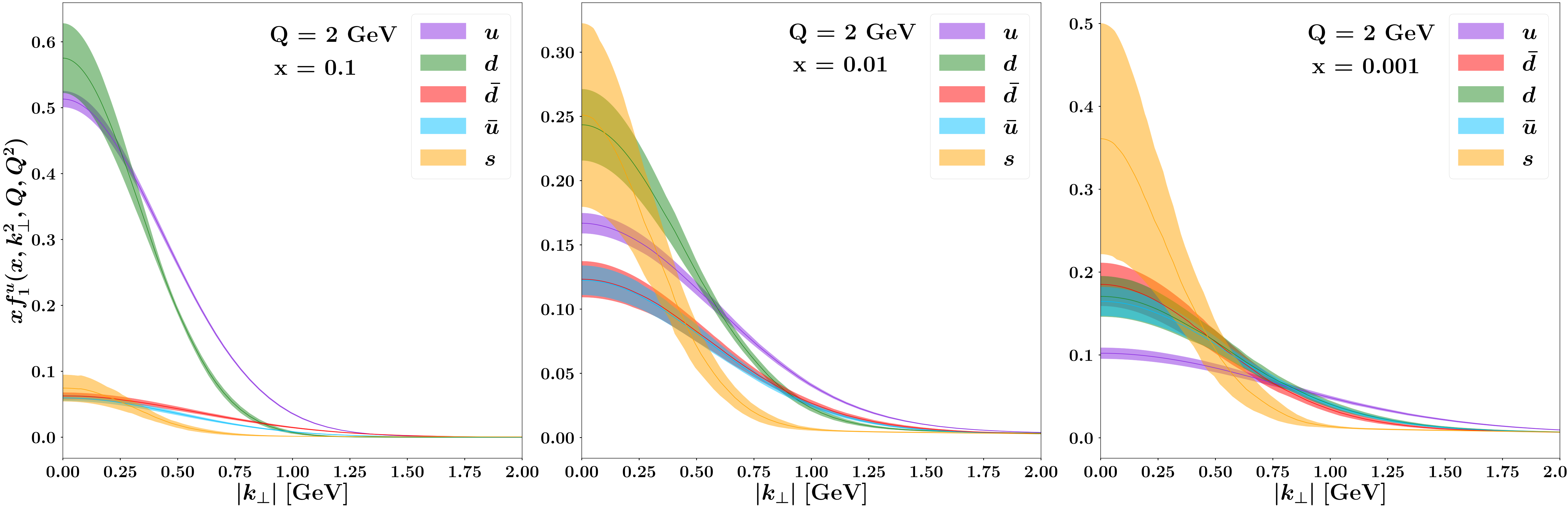
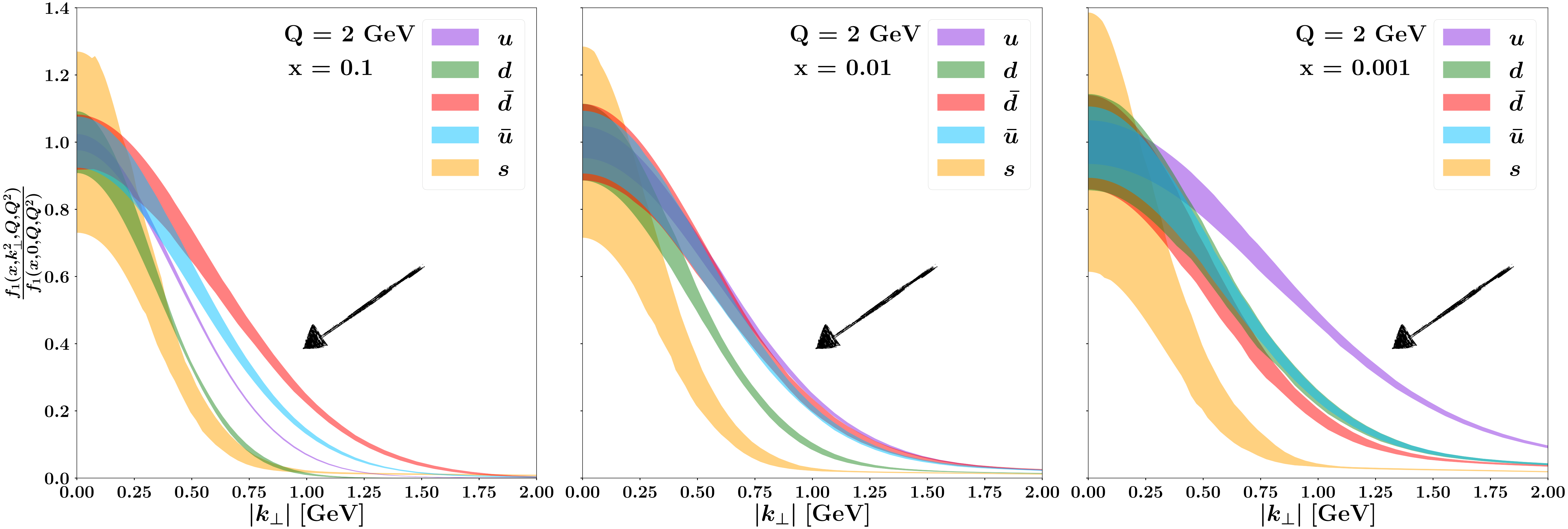
MAP 2024 first flavor dependent extraction from a global fit at N3LL order
arXiv:2405.13833

MAPTMD24 extraction



Evidence of different behavior for different flavors

MAPTMD24 extraction

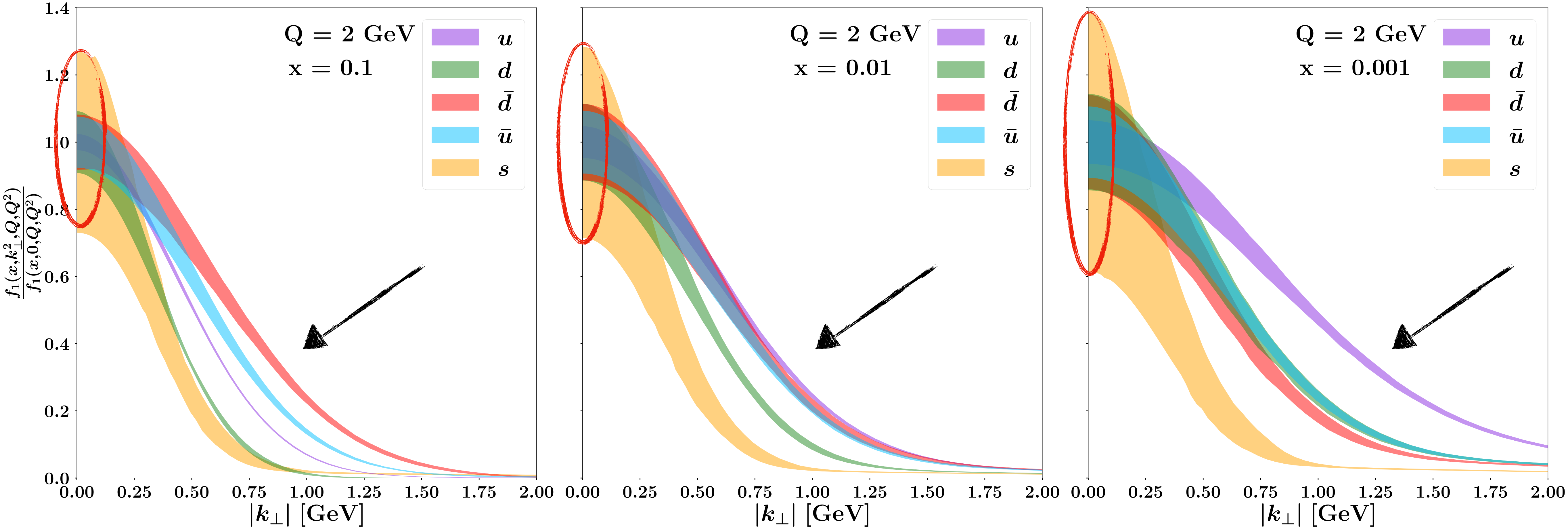


Very different $k_\perp$- behavior!

It changes also by varying x

MAPTMD24 extraction

The sea is the least constrained



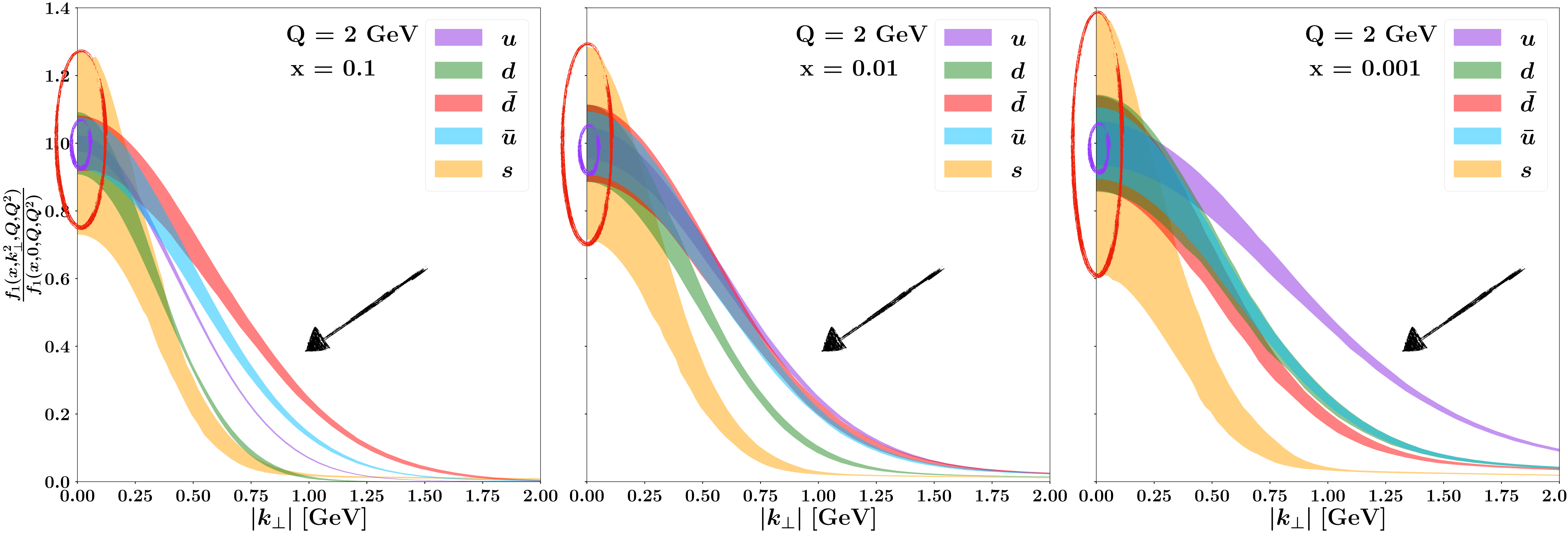
Very different $k_\perp$- behavior!

It changes also by varying x

MAPTMD24 extraction

The sea is the least constrained

The up quark is the most one

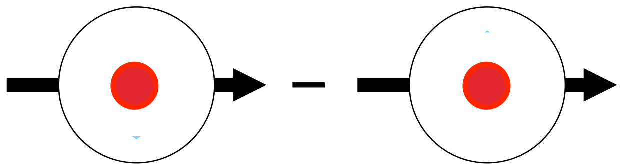


Very different $k_\perp$- behavior!

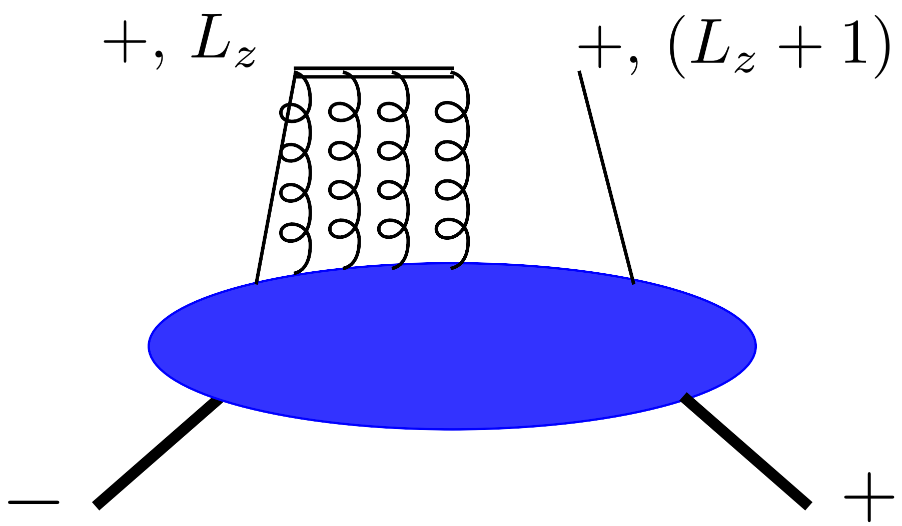
It changes also by varying x

Sivers function

$f_{1T}^{\perp} =$



unpolarized quarks in ⊥ pol. nucleon



the helicity mismatch requires orbital angular momentum (OAM)

non trivial correlation between quark OAM and nucleon transverse spin

no counterpart in IPD and PDF case

non-zero ONLY with final(initial)-state interaction

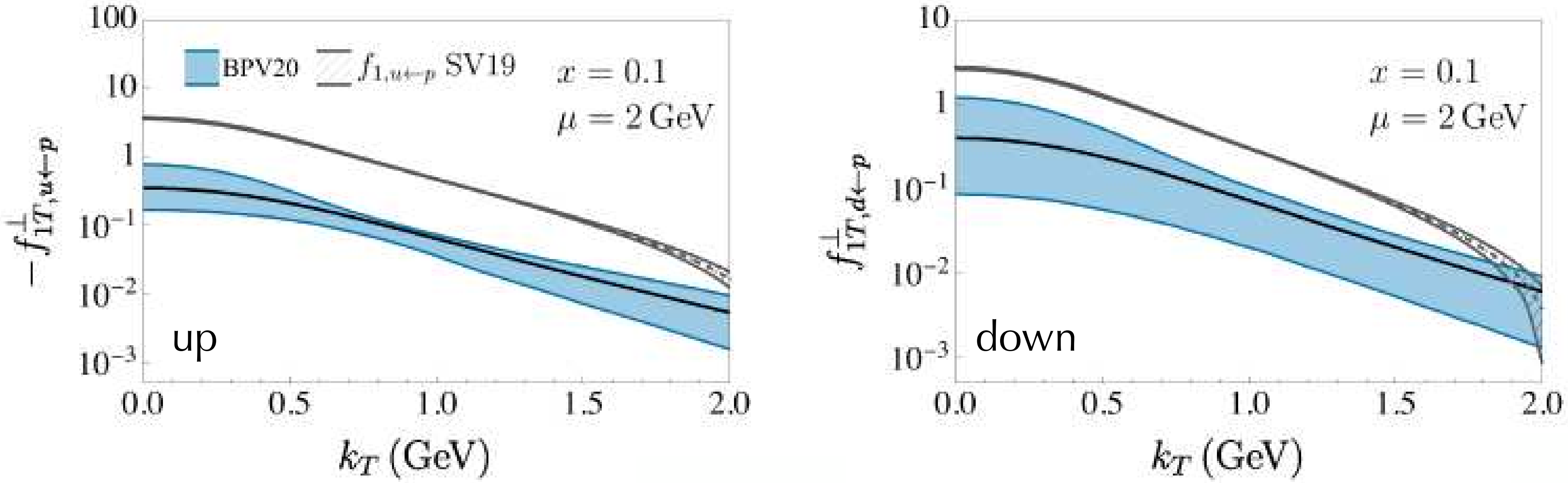
$$f_{1T}^{\text{SIDIS}}(x, k_\perp) = -f_{1T}^{\text{DY}}(x, k_\perp)$$

first hints of sign change from STAR and COMPASS data

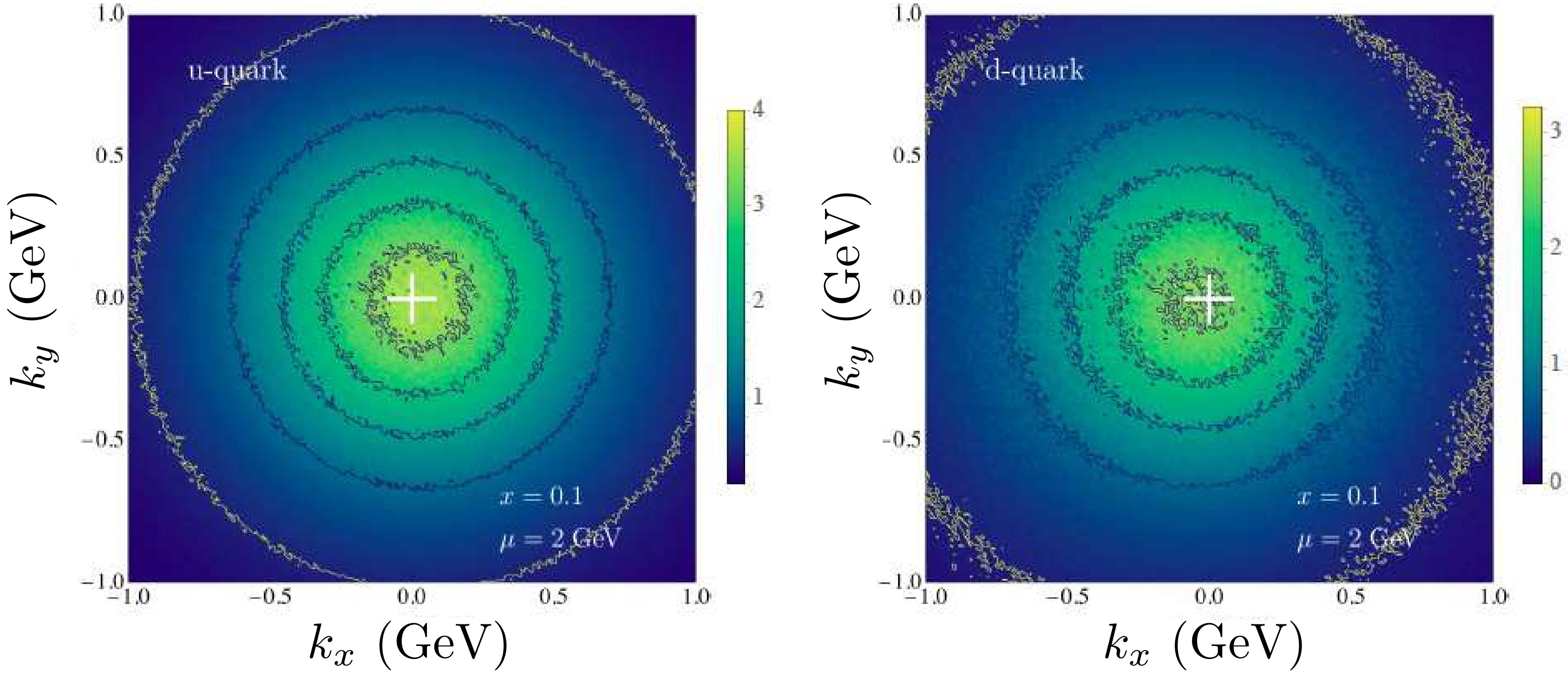
Global fit to SIDIS, DY, $W^{\pm}/Z$ boson production

f_1

$f_{1T}^{\perp}$



$$\rho_{UT_y}(x, \vec{k}_\perp, S_y) = f_1(x, k_\perp) - \frac{k_x}{M} f_{1T}^{\perp}(x, k_\perp)$$



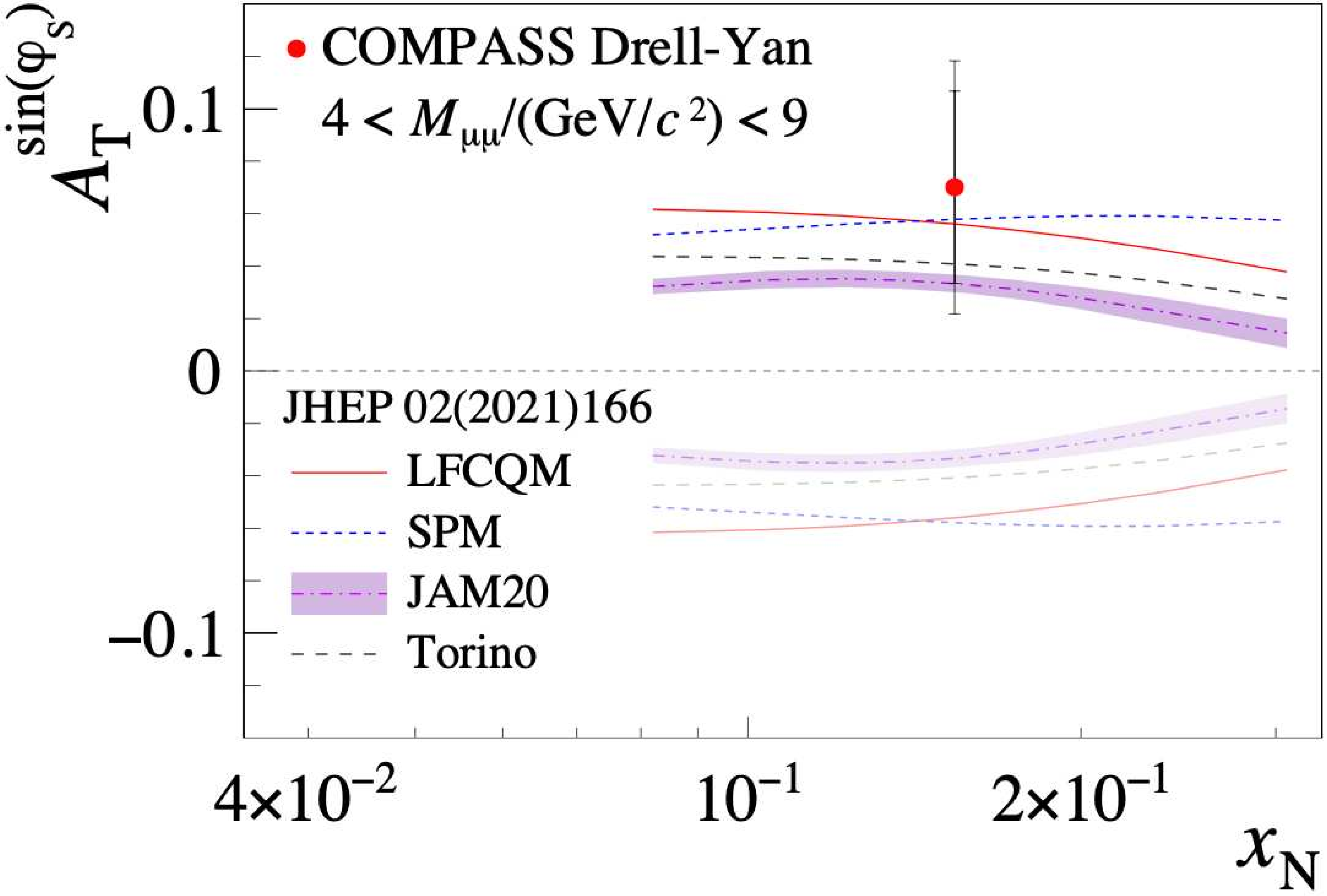
M. Bury, A. Prokudin, A. Vladimirov, JHEP 05 (2021) 151

See also extraction also from MAP Coll., JAM20 Coll., Echevarria et al.

Sign change of Sivers function

Final results for pion-induced Drell-Yan @COMPASS
arXiv: 2312.17379 (in print on PRL)

$$A_T^{\sin\phi_s} \propto f_{1,\pi}^q \otimes f_{1T,p}^{\perp q}$$



sign change

no sign change

Data favor the sign change scenario
of the Sivers TMD PDF, between SIDIS and DY

Talk of V. Bertone

Library and tools for collinear parton distributions

LHAPDF

lhapdf.hepforge.org

xFitter

www.xfitter.org

Dedicated Softwares to study GPDs

PARTONS

partons.cea.fr

PARtonic
Tomography
Of
Nucleon
Software

GeParD

Dedicated software to study and fit TMDs

arTeMiDe

teorica.fis.ucm.es/artemide

TMD lib and TMD Plotter

tmdlib.hepforge.org

NangaParbat

MapCollaboration/NangaParbat

Next Efforts: combine different inputs to understand PDFs, TMDs and GPDs in an unified framework

Conclusions

- The field of ``3D structure of the Hadrons" is continuously advancing and is prominently included in the strategic plans worldwide

- Our knowledge of GPD and TMD keeps increasing, but still many challenging questions

- PDFs, GPDs, TMDs are sensitive to various aspects of the parton structure of hadrons: putting together direct, indirect, and model inspired information from different sides it is our best hope to make quantitative assessments