Indirect Methods in Nuclear Astrophysics

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TEXAS A&M UNIVERSITY COMMERCE

THE 12TH EUROPEAN SUMMER SCHOOL ON EXPERIMENTAL NUCLEAR ASTROPHYSICS

Aci Trezza, 16-22 June 2024

Nuclear reactions in astrophysics

decay

Abundance changes

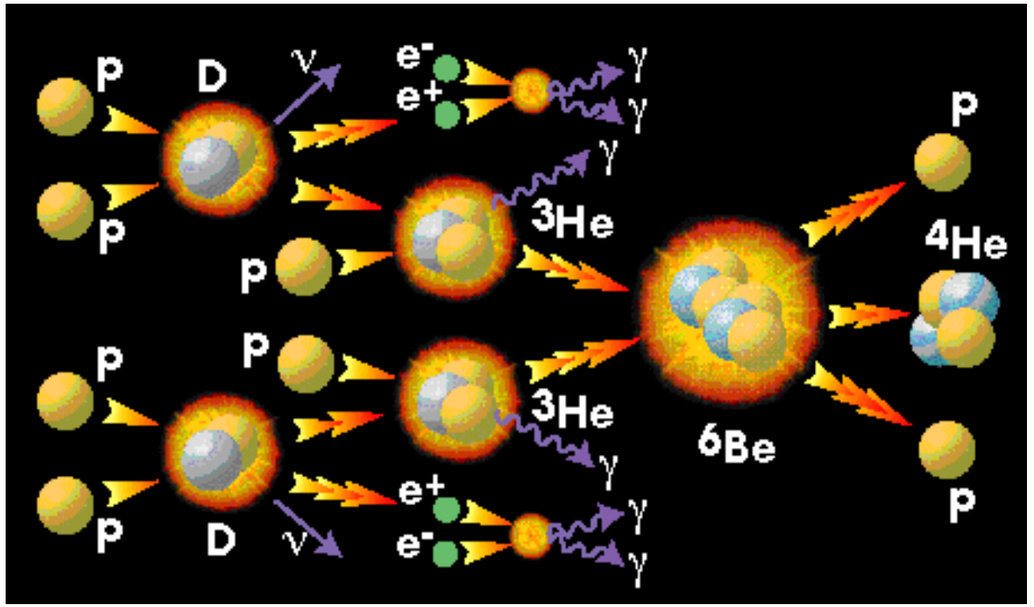
$$\frac{dY_i}{dt} = \sum_j N_j^i \lambda_j Y_j + \sum_{jk} N_{jk}^i \rho N_A \langle \sigma v \rangle Y_j Y_k + \sum_{jkm} N_{jkm}^i \rho^2 N_A^2 \langle \sigma v \rangle Y_j Y_k Y_m + \cdots$$

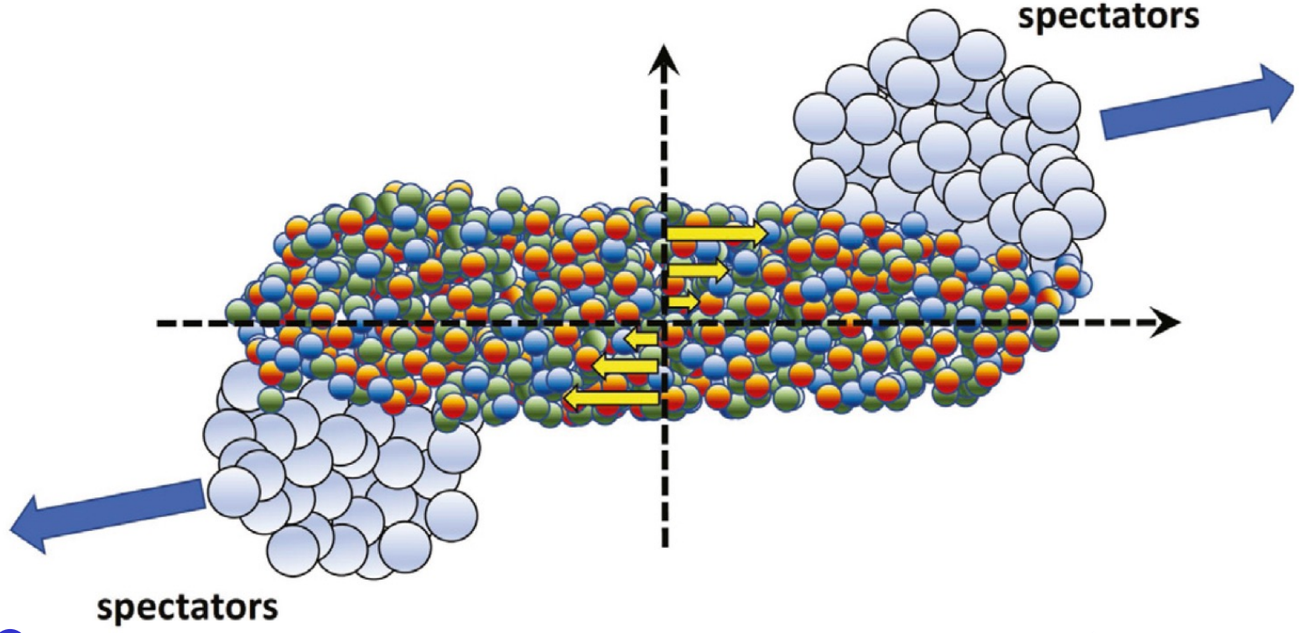
formation

Input:
- density
- temperature
- composition
- nuclear masses
- decay constants
- reaction rates
- . . .

Output:
- nucleosynthesis
- stellar evolution
- bursts, explosions
- neutron stars
- black holes
- . . .

Nuclear reactions





keV

???

Stellar evolution

Elemental abundance

Nuclear many-body problem:

the hardest problem of all physics!

- Interactions are complicated
- Nucleons = composite particles

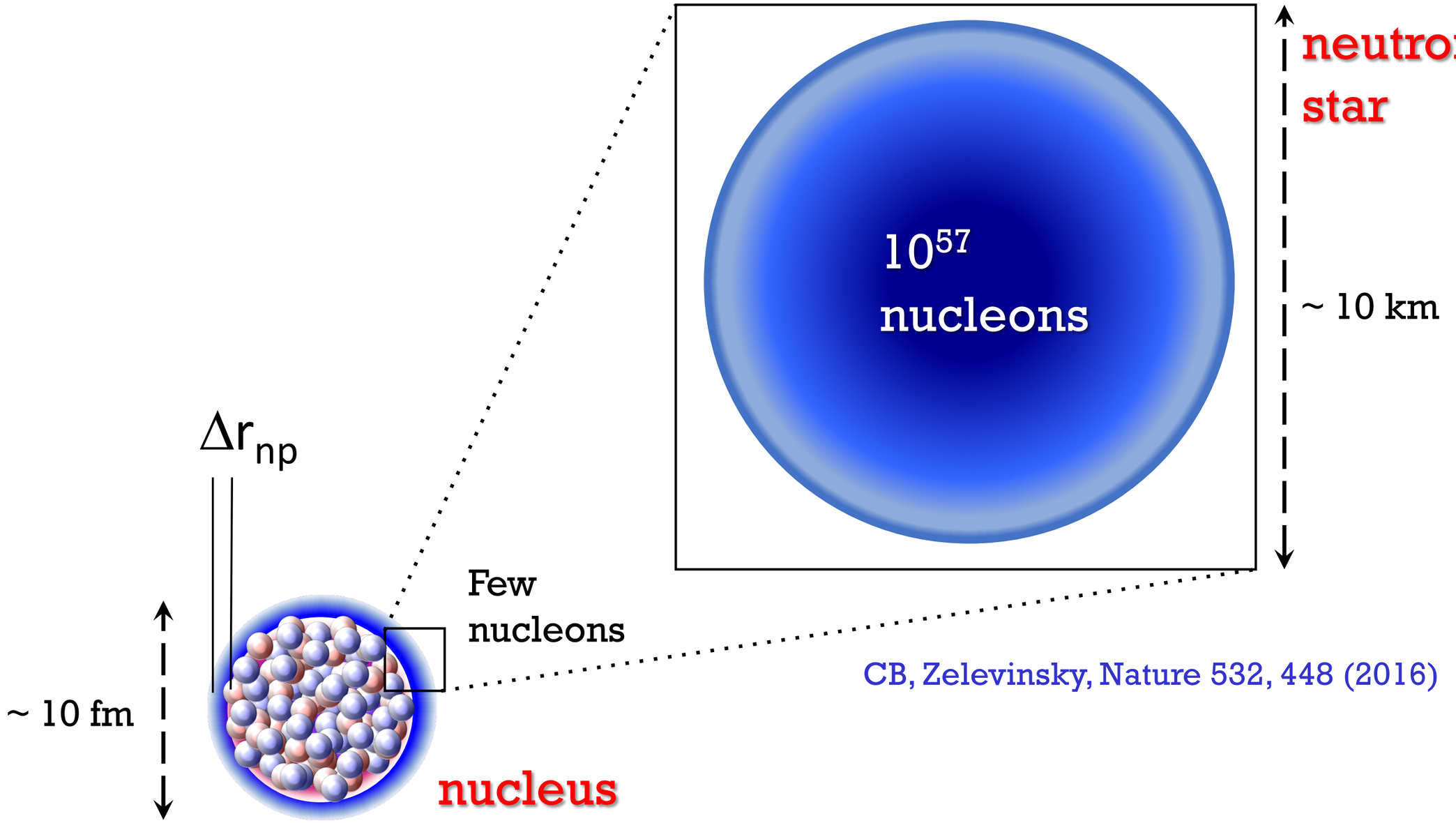
TeV

???

Big bang

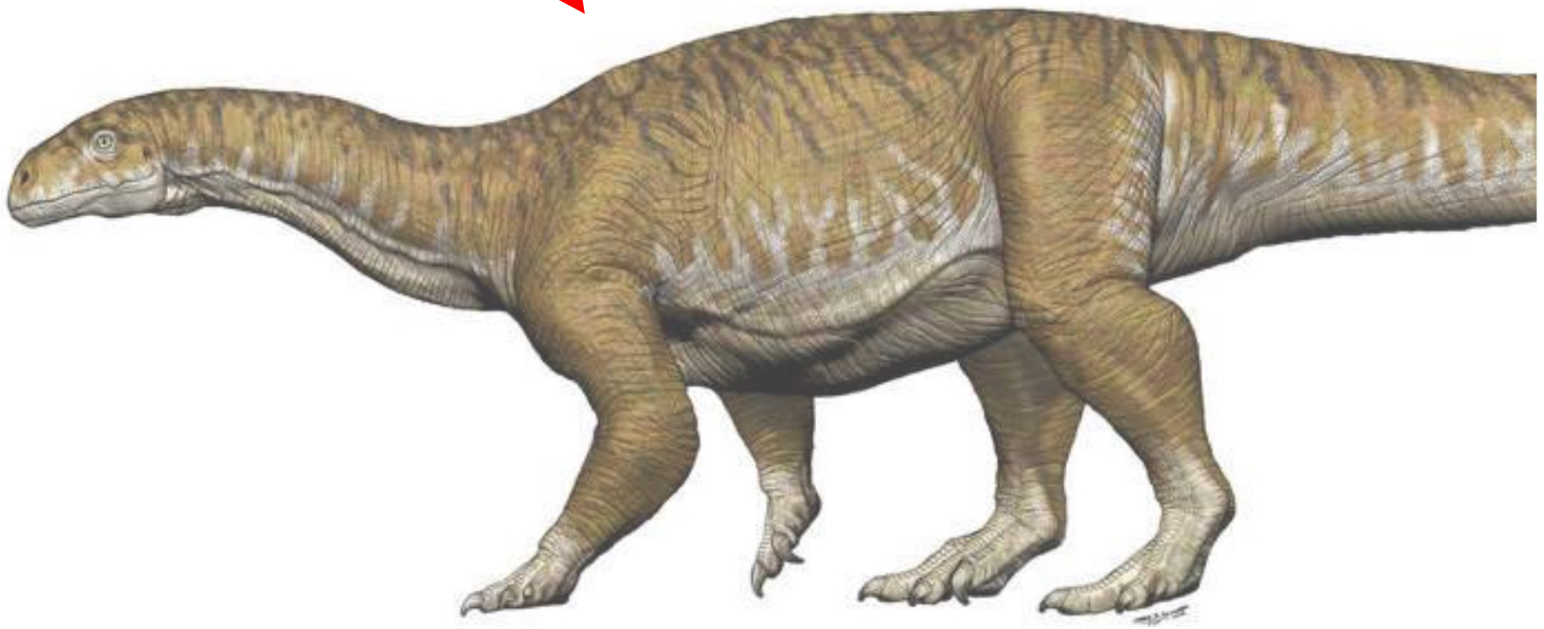
neutron star

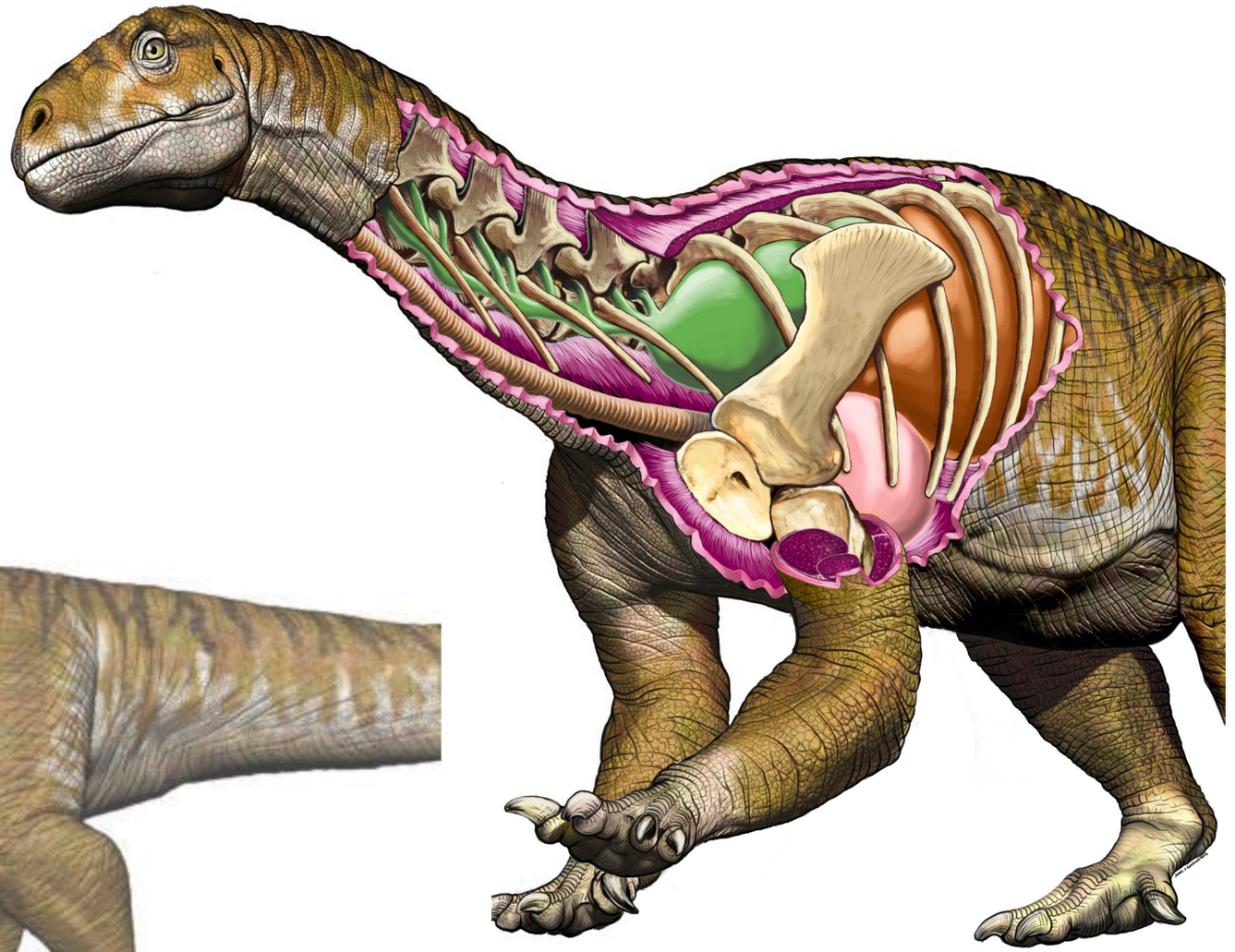
From laboratory experiments to neutron stars



CB, Zelevinsky, Nature 532, 448 (2016)

From bones to dinosaurs





Credit: Nature Ecology & Evolution (2018)

Reconstruction of Ingentia prima from the Late Triassic (205- 210Ma) of Argentina. Total length 8-10 me…

Neutron stars

$$\frac{dP}{dr} = -\frac{G\rho(r)M(r)}{r^2}\left[1+\frac{P(r)}{\rho(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

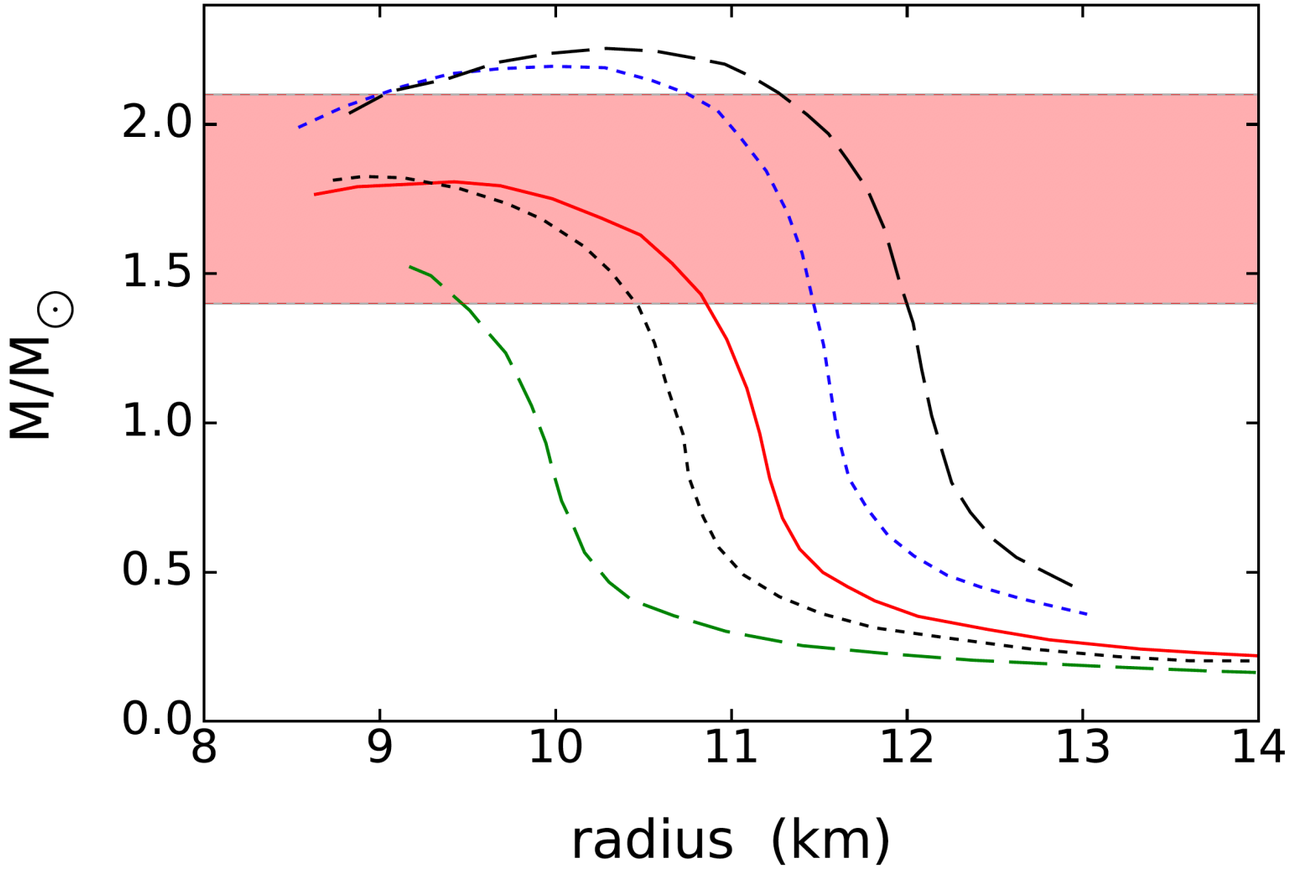
Tolman-Oppenheimer-Volkoff

EOS

$$p[\rho] = \rho^2 \frac{d}{d\rho}\left(\frac{E}{A}[\rho]\right)$$

$$\frac{E}{A}[\rho] = \frac{E}{A}[\rho_0] + \frac{1}{18}K_\infty\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots$$

$$K_\infty = 9\rho^2 \left.\frac{d^2[E/A]}{d\rho^2}\right|_{\rho_0}$$



EOS + symmetry energy

$$\frac{E}{A}[\rho] = \frac{E}{A}[\rho_0] + \frac{1}{18}K_\infty\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + S\left(\frac{\rho_n-\rho_p}{\rho}\right)^2 + \cdots$$

$$S = \frac{1}{2}\left.\frac{\partial^2(E/A)}{\partial\delta^2}\right|_{\delta=0} = J + Lx + \frac{1}{2}K_{sym}x^2 + O(x^3),$$

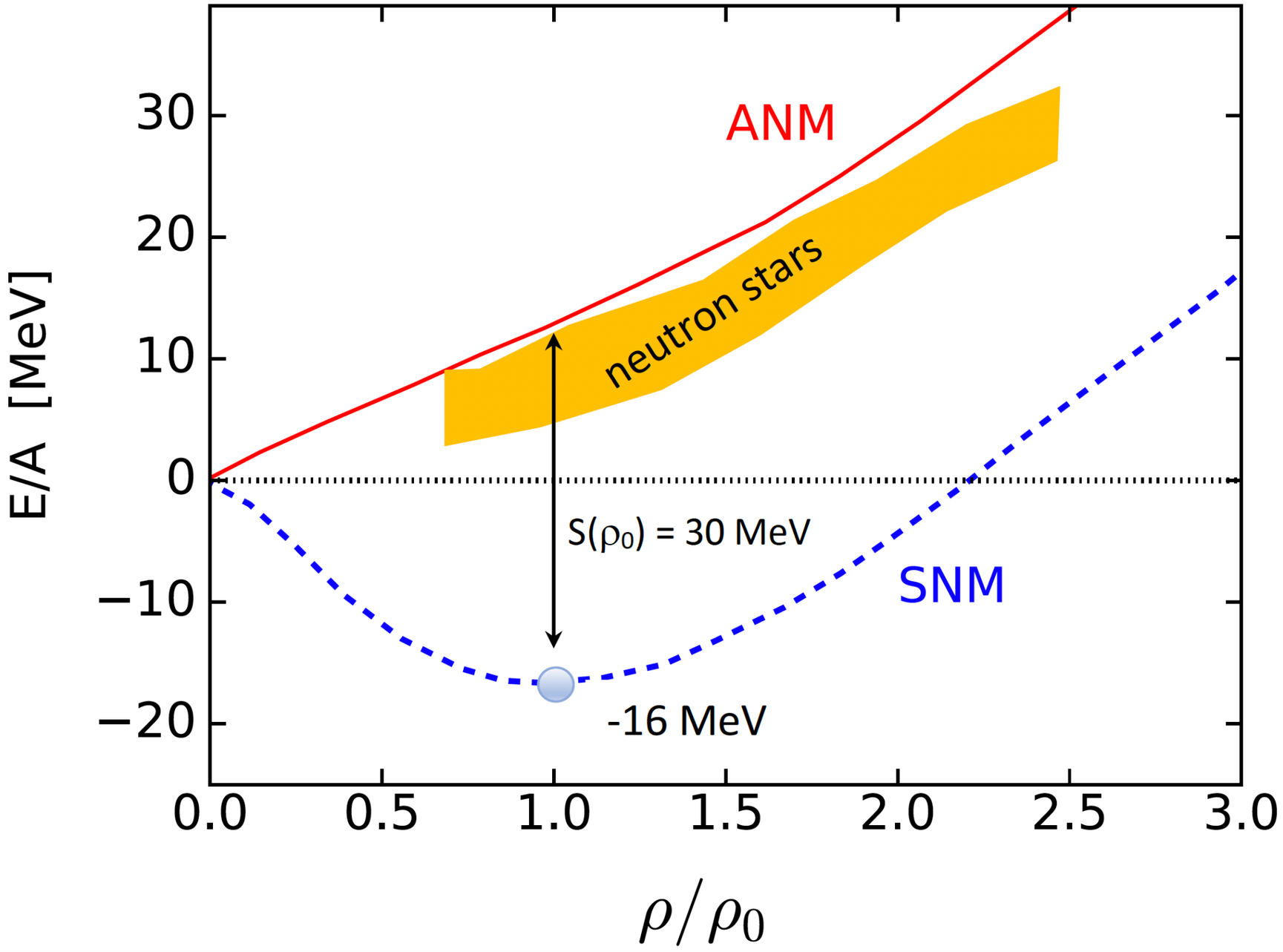
$$L = 3\rho_0\left.\frac{dS(\rho)}{d\rho}\right|_{\rho_0}, \quad \delta = \frac{\rho_n-\rho_p}{\rho}, \quad x = \frac{(\rho-\rho_0)}{3\rho_0}$$

For $\rho \sim \rho_0$ and $\delta \sim 1 \Rightarrow p = \frac{L\rho_0}{3}$

Skyrme	ρ_0	E_0	K_∞	J	L	K_{sym}
SLy5	0.161	-15.99	229.92	32.01	48.15	-112.76
Skxs20	0.162	-15.81	201.95	35.50	67.06	-122.31
SkI5	0.156	-15.82	255.57	36.63	129.27	-155.94

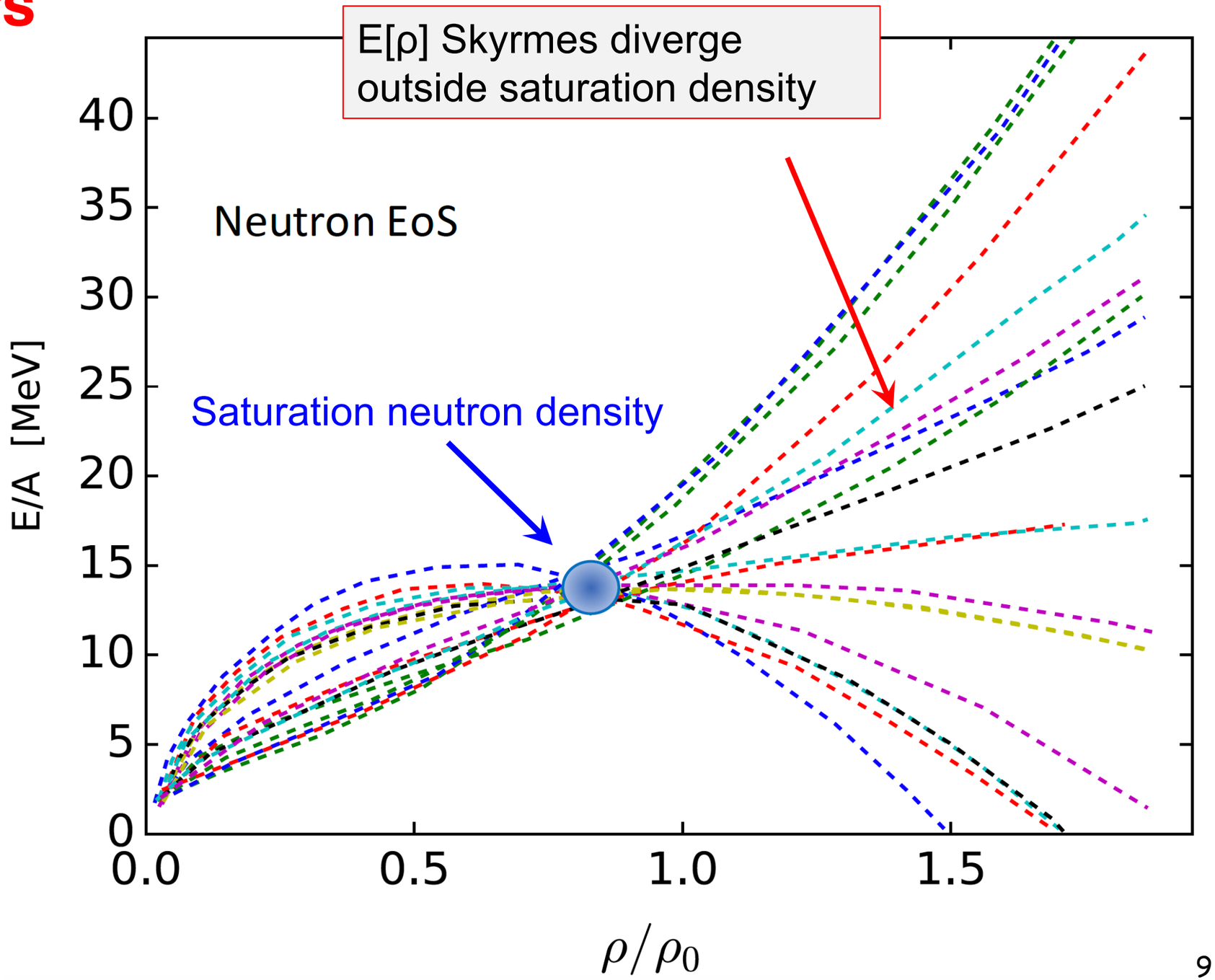
L crucial for neutron matter

EOS of neutron stars



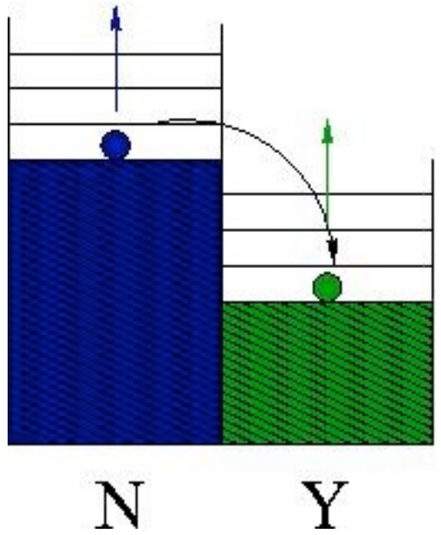
EOS & Neutron stars

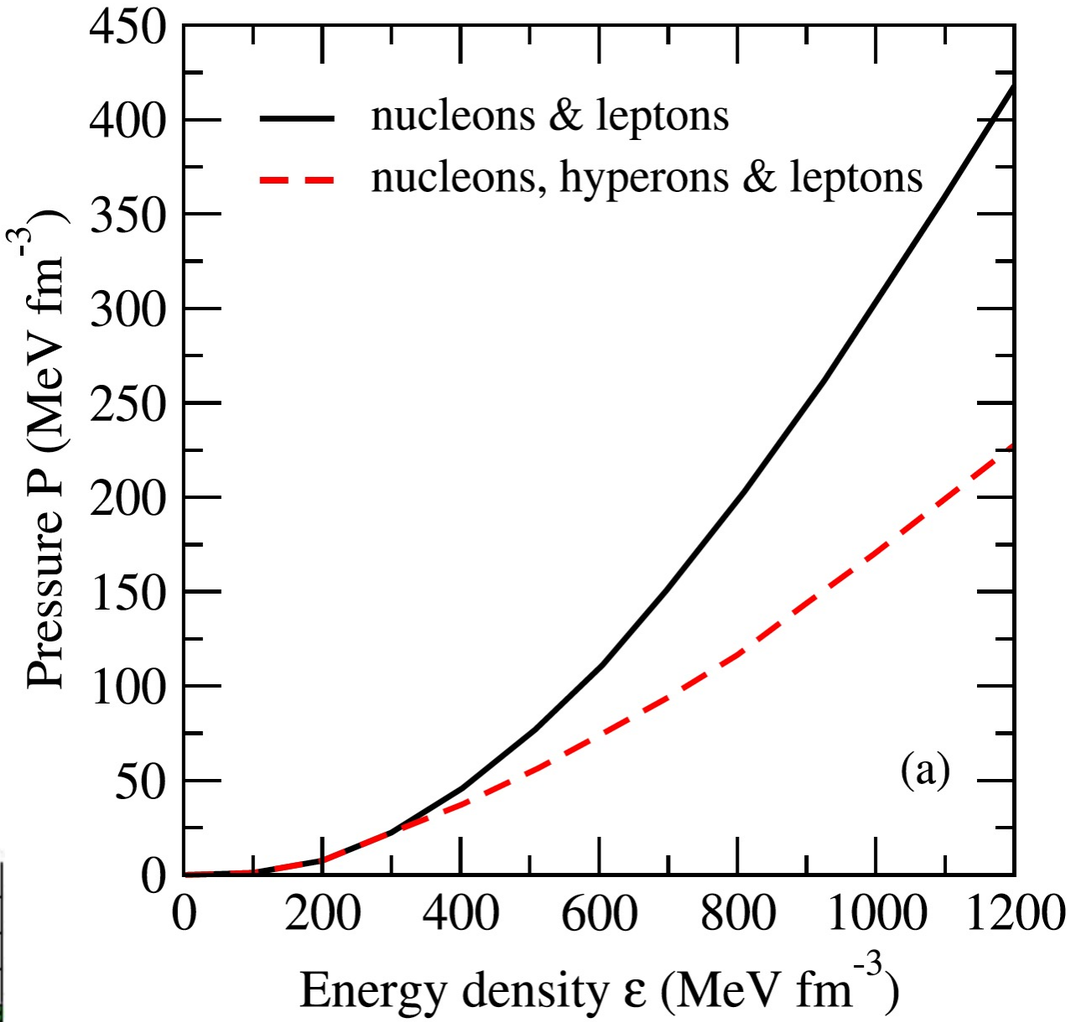
Pethick, Ravenhall,
ARNPS 45 (1995) 429

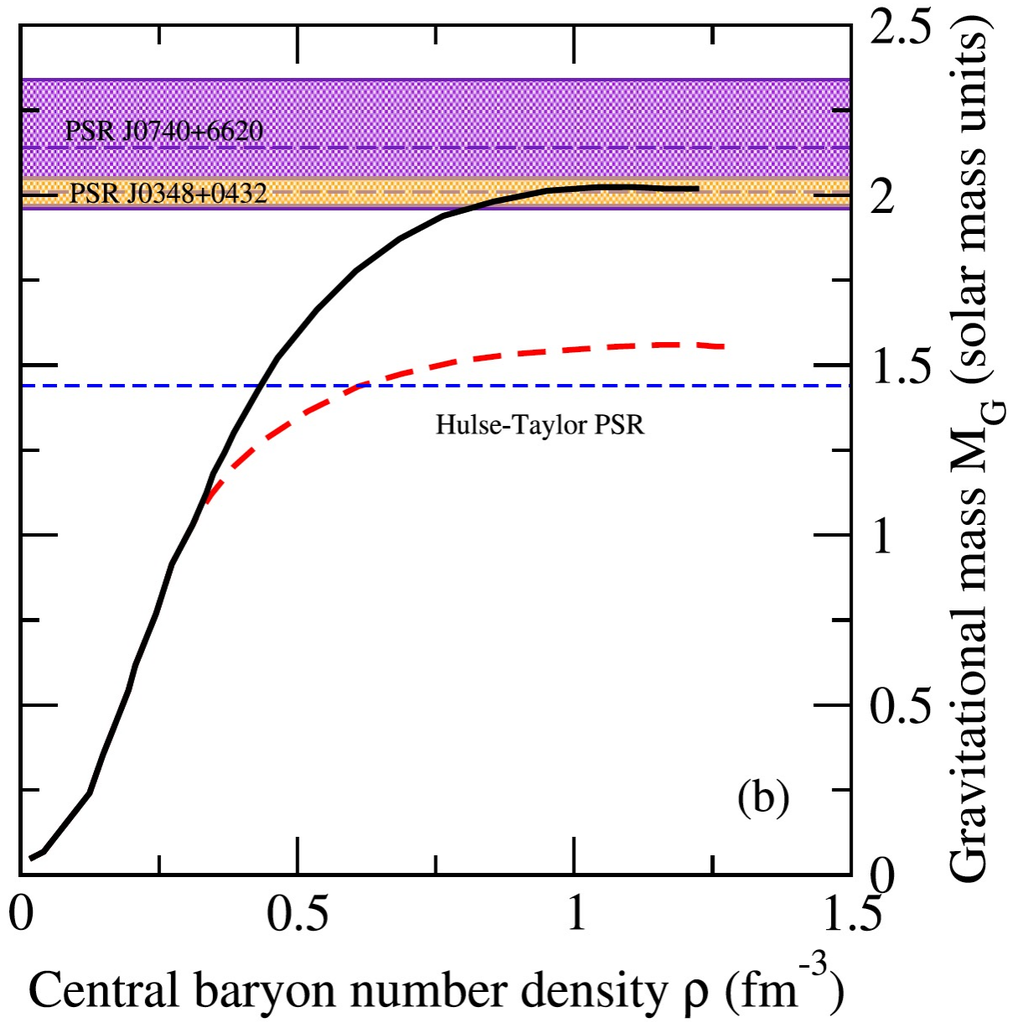


Hyperons and neutron stars

Hyperons make the EoS softer → reduction of the mass



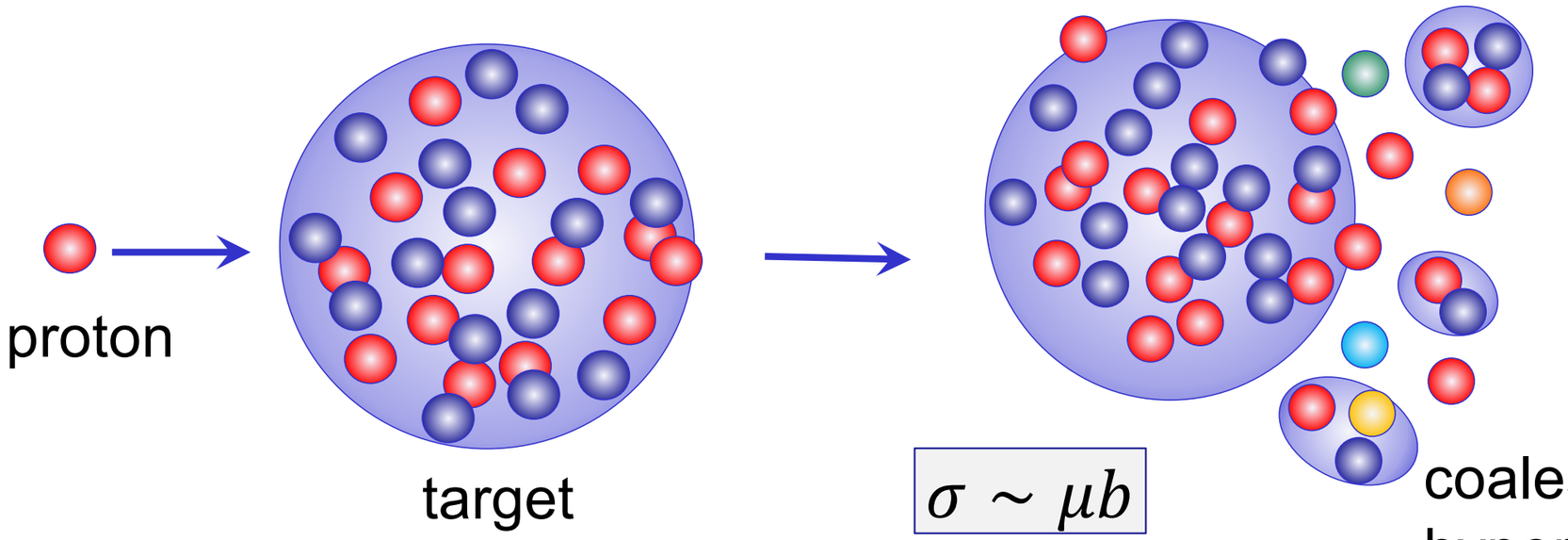




Hyperon "puzzle"

Tolos, Fabbietti, PPNP 112 103770 (2020)

Hypernuclei in the lab



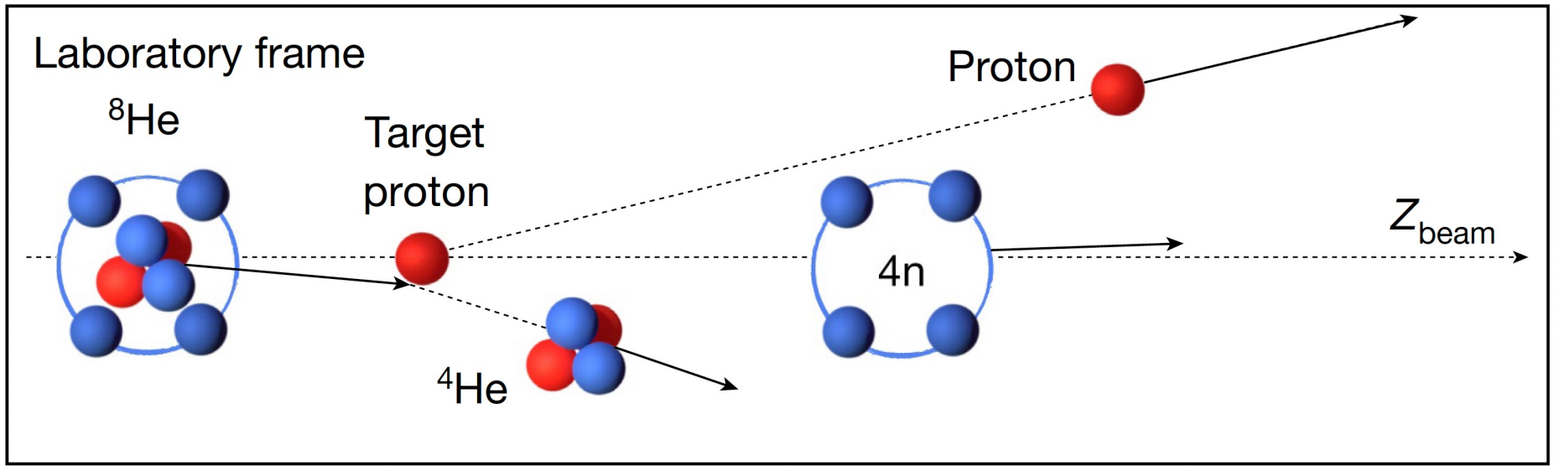
$$C_A = \frac{2J_A + 1}{2^A} \frac{1}{\sqrt{A} m_T^{A-1}} \left(\frac{2\pi}{R^2 + \left(\frac{r_A}{2}\right)^2} \right)^{\frac{3}{2}(A-1)}$$

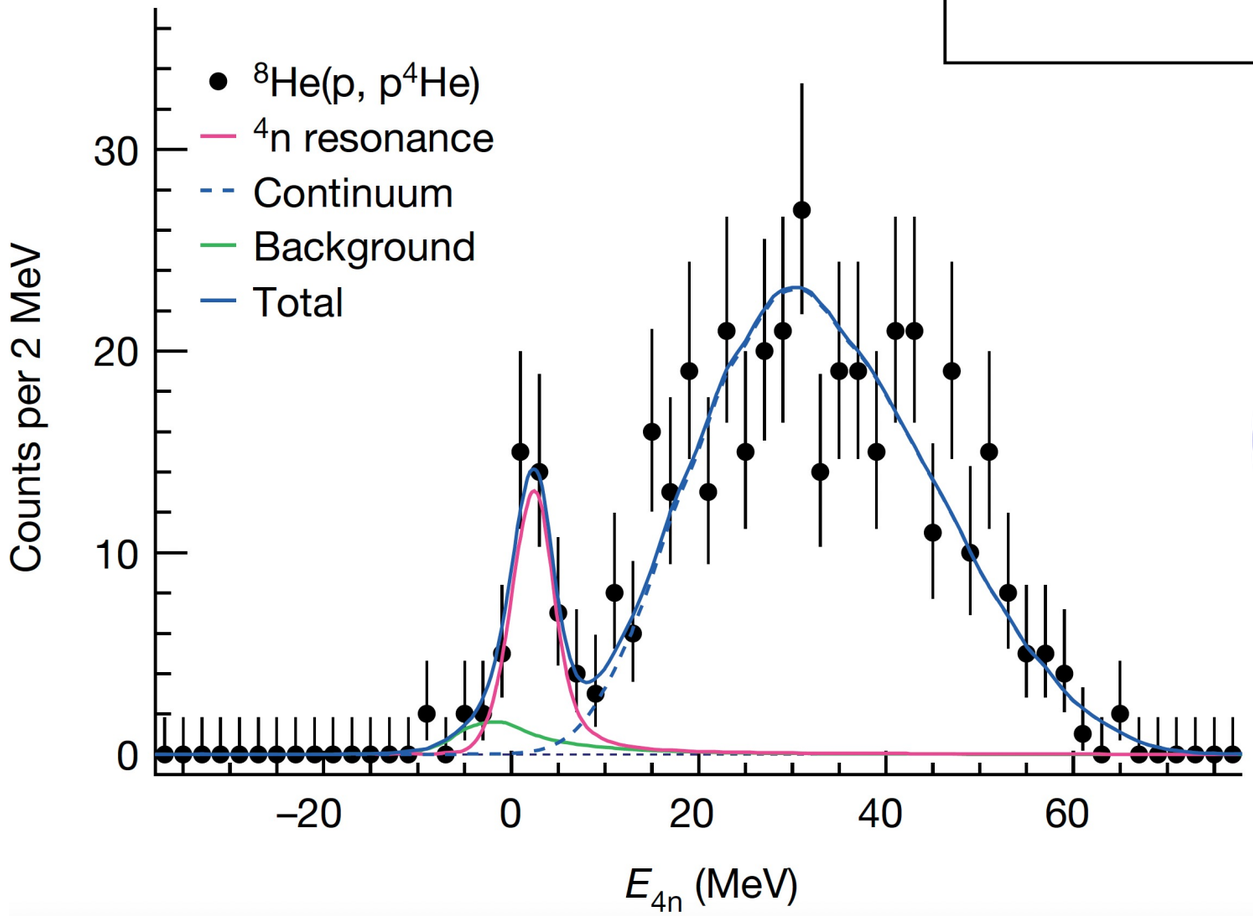
Particles produced coalesce into nuclei if they are close in space and momentum.

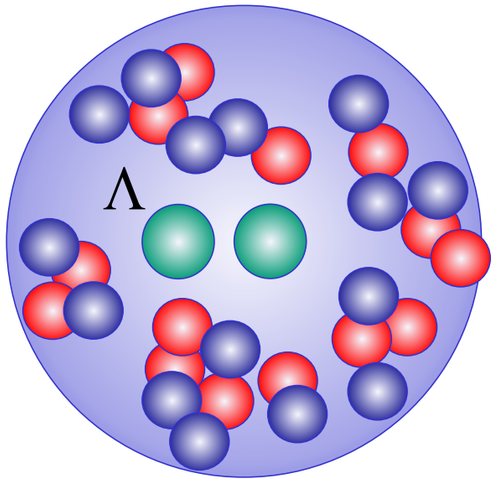
R = source size, r_A = nuclear size
m_T = transverse mass of coalesc. part.

Nucleon clusters and NΛ clusters in the lab

Duer, et al.,
Nature 606, 678 (2022)



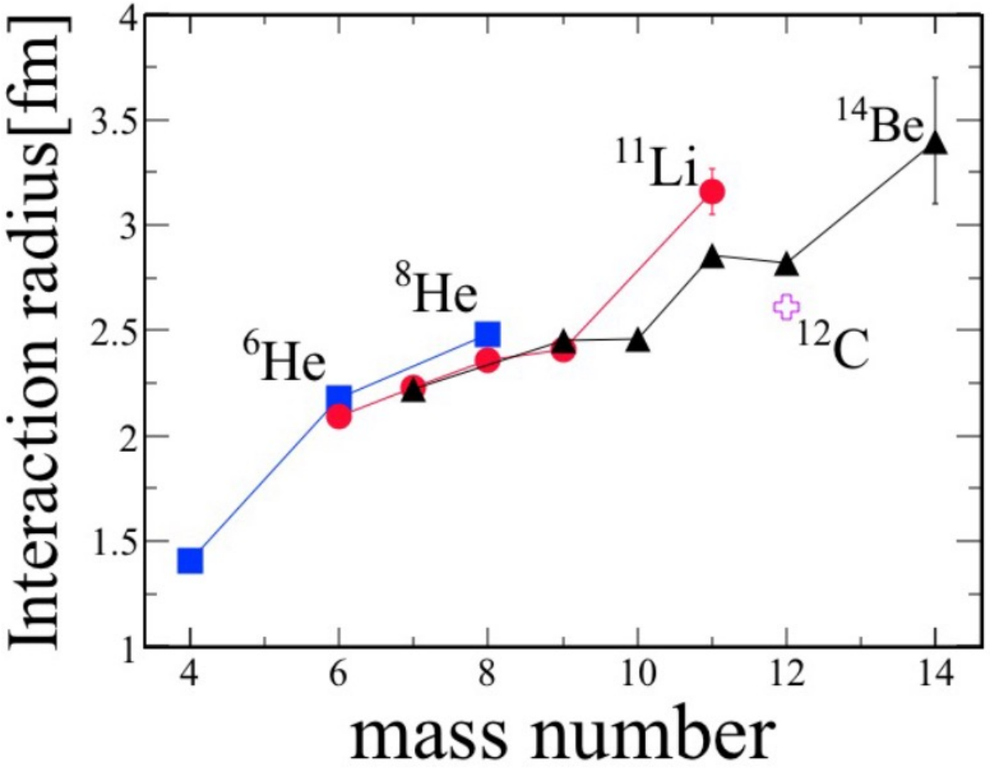


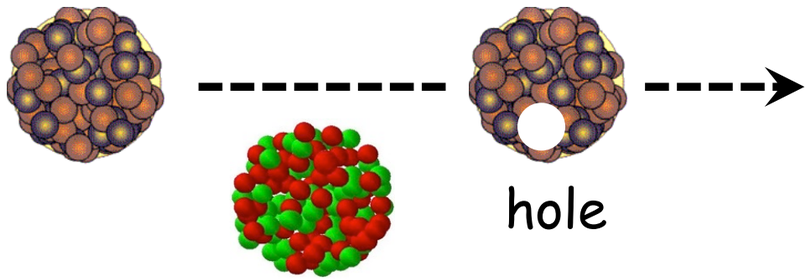


Hypernuclei spectroscopy
→ NΛ interaction

Difficult task

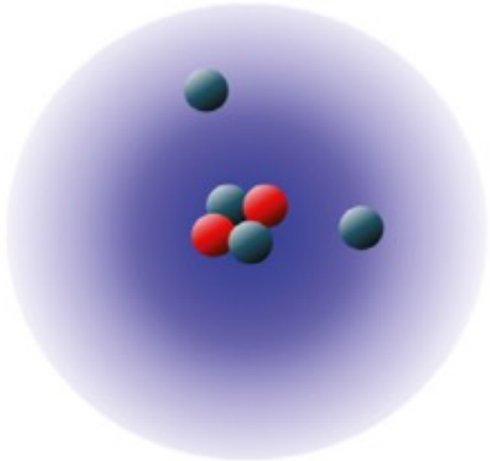
Nuclear halos



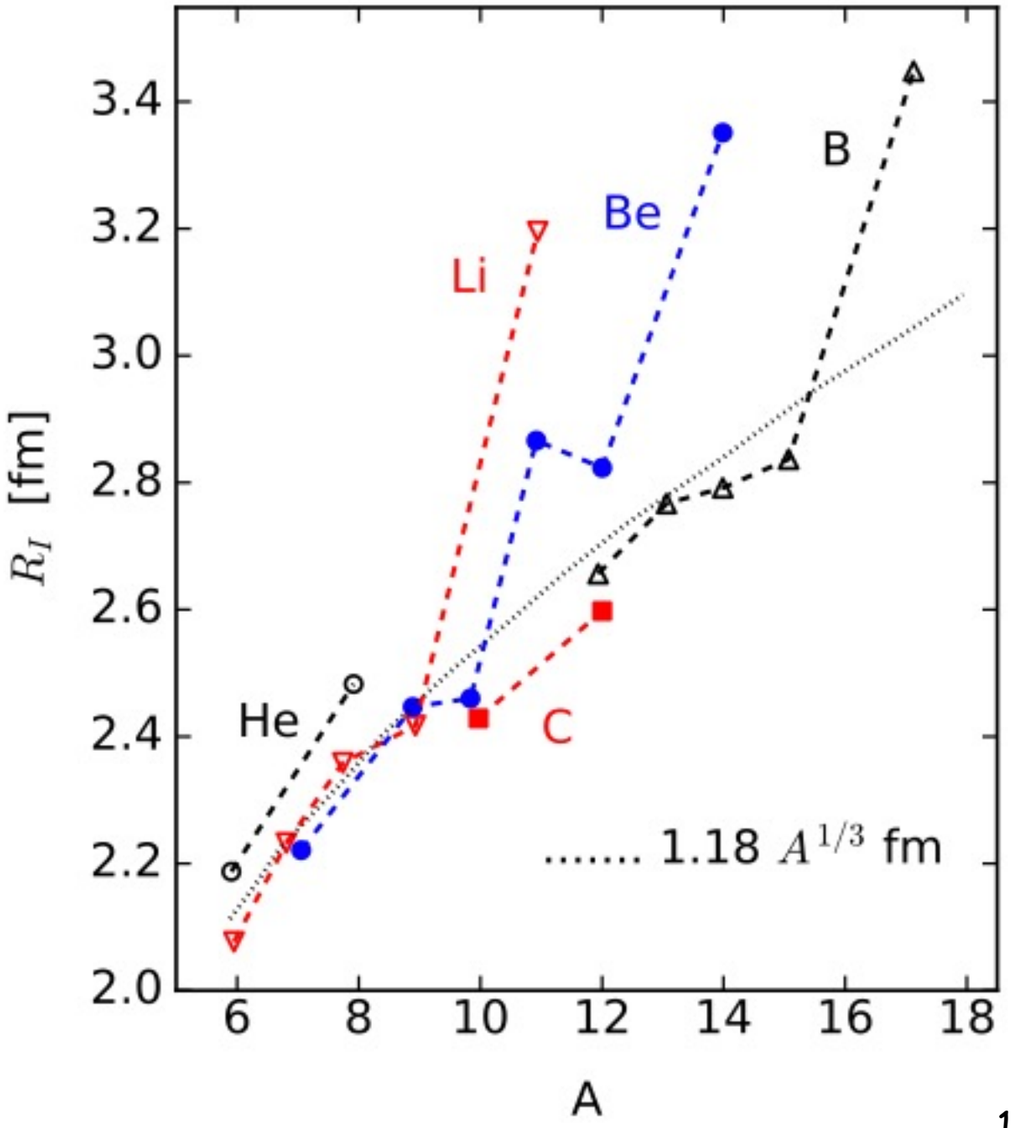


$$\sigma_I = \pi[R_I(P) + R_I(T)]^2$$

^{6}He



Tanihata et al.,
PRL 55, 2676 (1985)



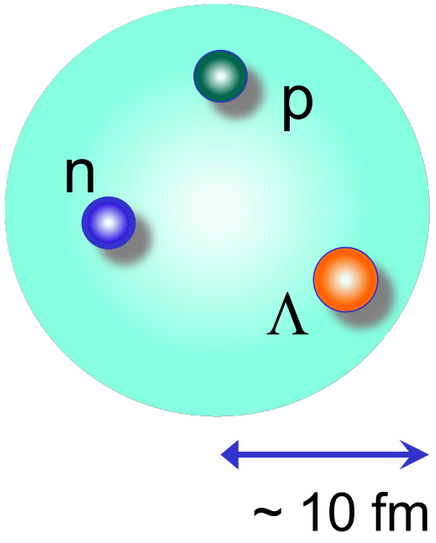
Hypertriton halo

$\tau_{\Lambda} = 263$ ps

$\tau_{{}^{3}_{\Lambda}\mathrm{H}} = 253 \pm 11(\mathrm{stat.}) \pm 6(\mathrm{sys.})$ ps

Philipp Eckert, et al., EPJ Web Conf. 271 (2022) 01006

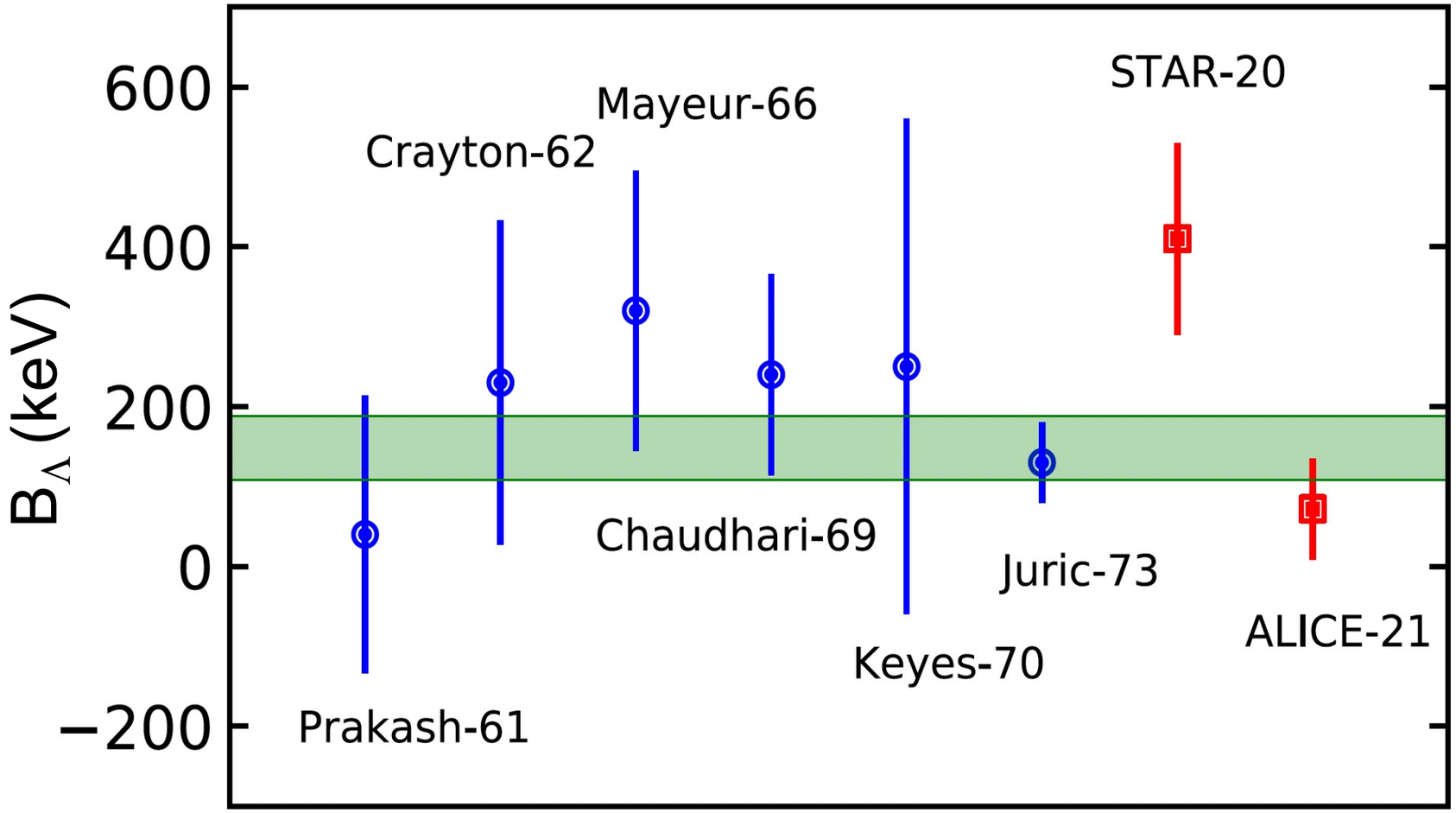
https://hypernuclei.kph.uni-mainz.de



Halo

${}^{6}_{\Lambda}\mathrm{He}$ (S_n = 0.17 MeV)

${}^{7}_{\Lambda}\mathrm{Be}$($S_{2p}$ = 0.67 MeV)



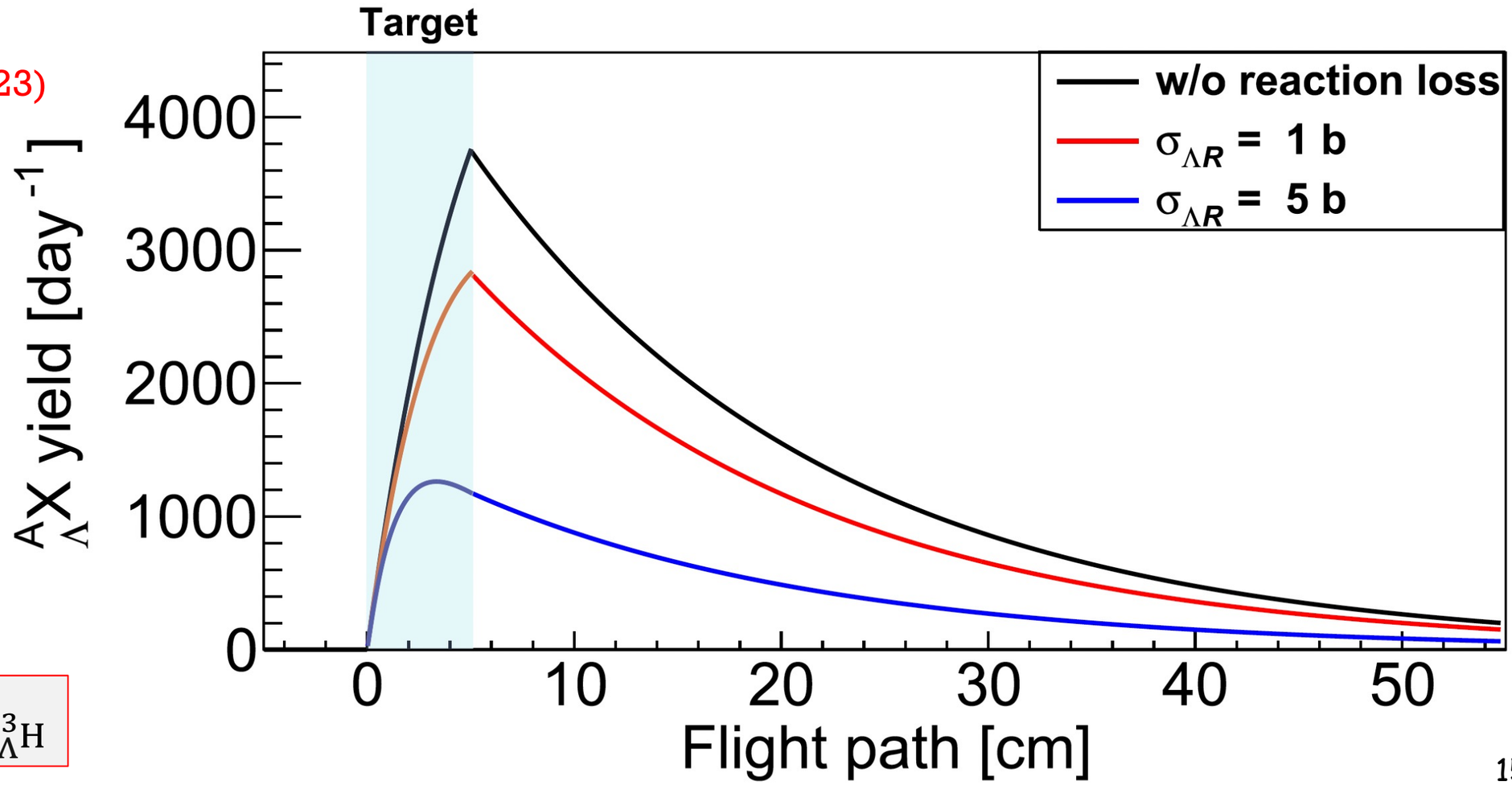
Production & Decay of Hypertriton

Use active target to:
(a) Produce hypertriton
(b) Reconstruct hypertriton by measuring weak decay ($^{3}_{\Lambda}\mathrm{H} \rightarrow \pi^{-} + {}^{3}\mathrm{He}$)
(c) Interaction cross section though meson decay vertex distribution

Velardita et al.,
Eur. Phys. J. A 59,139 (2023)

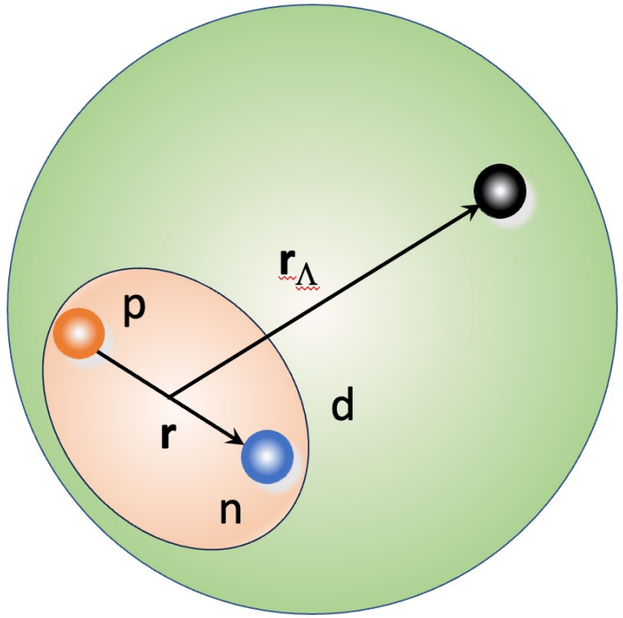
$\sigma_I = \sigma_{\Lambda R}$ ~ 1.8 b

τ ~ 200 ps

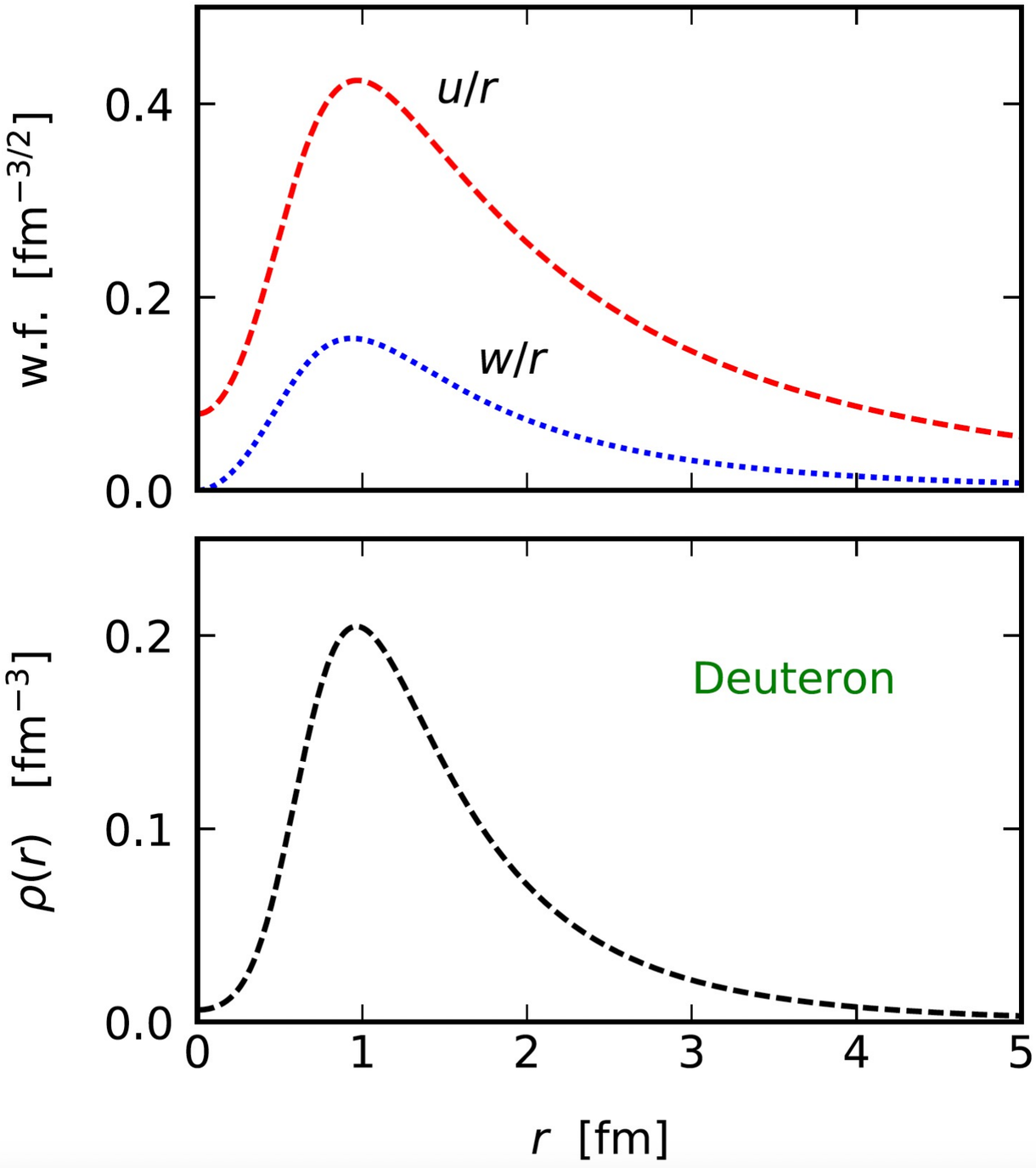


Experiment → σ_I → $R_{^{3}_{\Lambda}\mathrm{H}}$

Hypertriton model

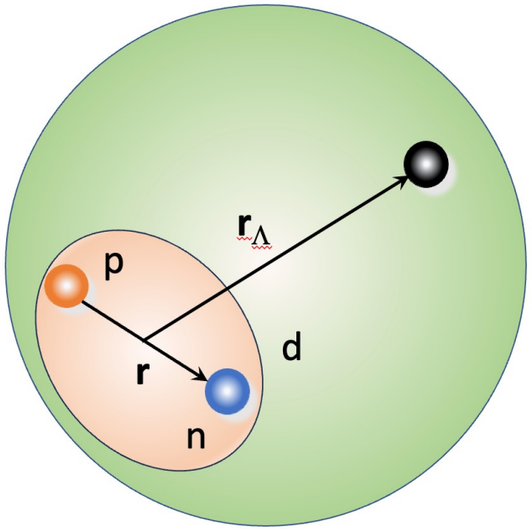


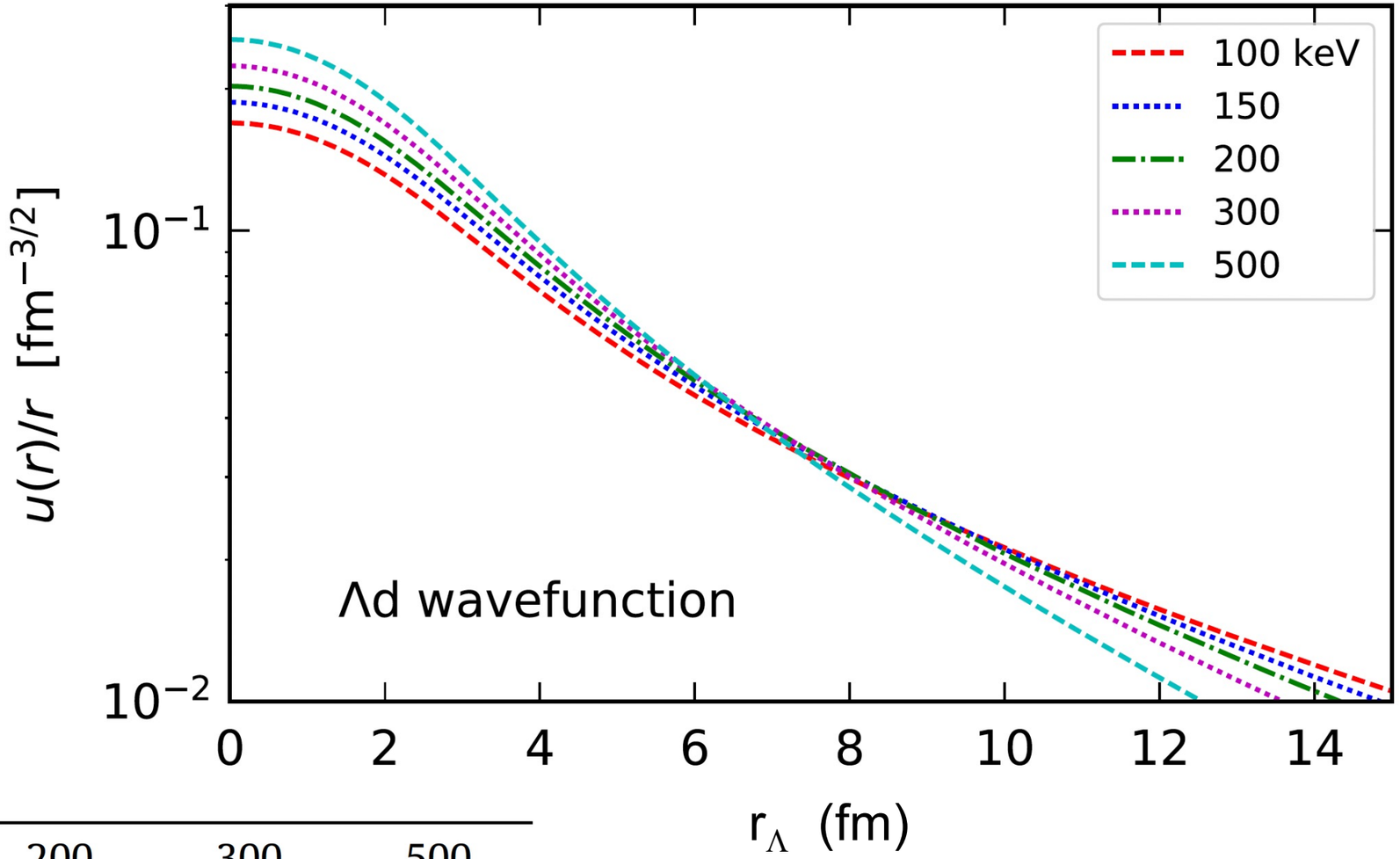
Deuteron radial s-wave, *u(r)/r*, and d-wave, *w(r)/r* as a function of the proton-neutron distance *r* using Av18 interaction.



Hypertriton model

CB, PLB 837, 137639 (2023)



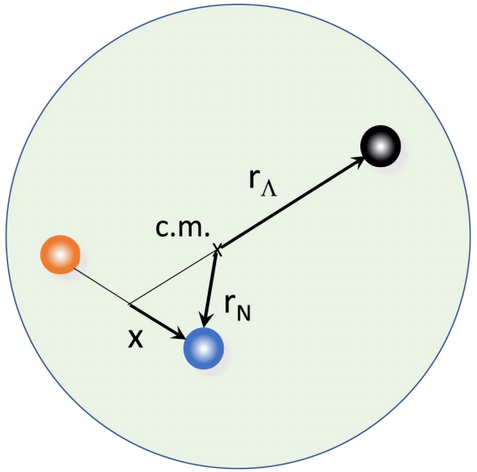


B_Λ (keV)	100	150	200	300	500
V_0 [MeV]	-11.8	12.2	-12.6	-13.2	-14.3

Λ-deuteron wavefunction - Woods-Saxon, radius R = 2.5fm, diffuseness a = 0.65 fm.

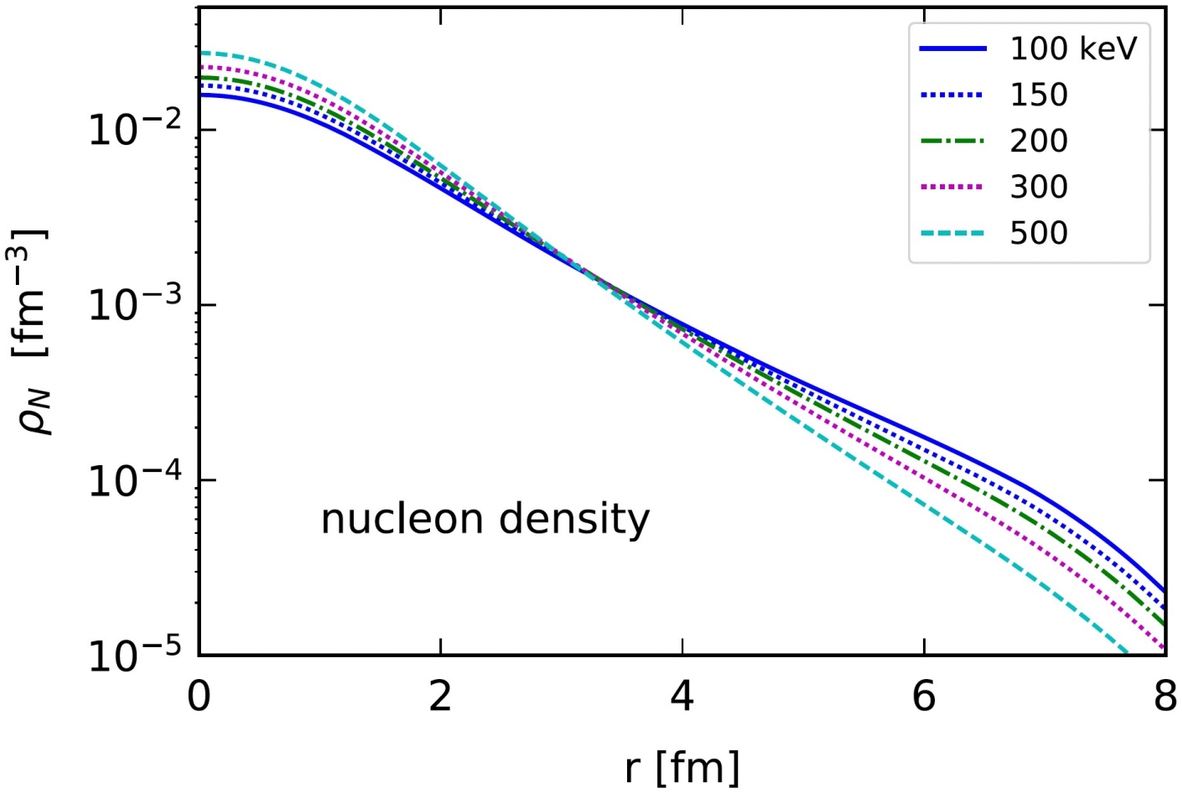
u(r)/r for the s-wave and binding energies B = 100, 150, 200, 300 and 500 keV.

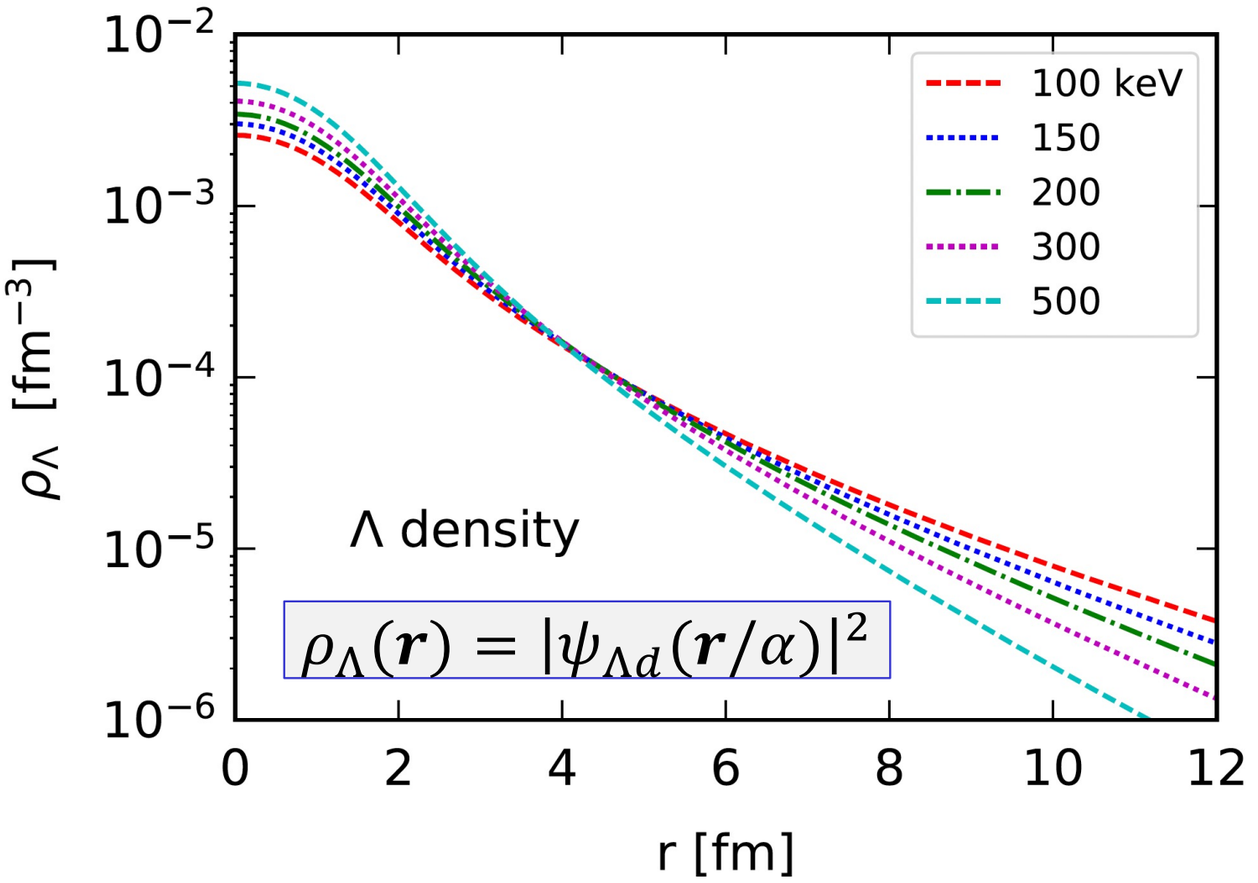
Hypertriton + nucleon density



$$\alpha = \frac{m_d}{m_\Lambda + m_d}$$

$$\beta = \frac{m_\Lambda}{m_\Lambda + m_d}$$





$$\rho_\Lambda(\boldsymbol{r}) = |\psi_{\Lambda d}(\boldsymbol{r}/\alpha)|^2$$

$\rho_1(\boldsymbol{r}_\Lambda) = |\psi_{\Lambda d}(\boldsymbol{r}/\beta)|^2$ Λ dist. within ${}^3_\Lambda\mathrm{H}$

$\rho_2(\boldsymbol{r}_p) = \left|\psi^{free}_{deut}\,(2\boldsymbol{x})\right|^2$ p dist. within deut.

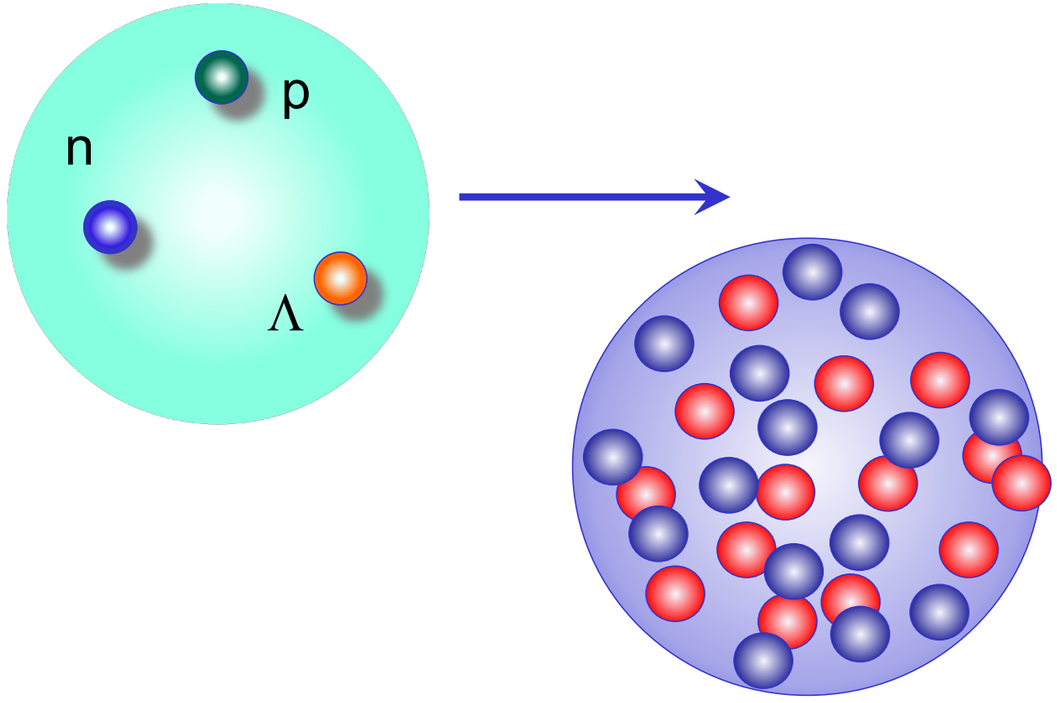
$\rho_N(\boldsymbol{r}_N) = \int \rho_1(|\boldsymbol{r} - \boldsymbol{x}|)\ \rho_2(\boldsymbol{x})$

nucleon distribution within ${}^3_\Lambda\mathrm{H}$

Hypertriton destruction

$$T(b) = T_\Lambda(b)\, T_p(b)\, T_n(b)$$

Transmission probability



$$T_i(b) = \int d^2 s_i dz_i (z_i, \boldsymbol{s}_i - \boldsymbol{b})$$

$$= \exp\left[-\sigma_{pi} Z_T \int dz'\, \rho_p^T(z', \boldsymbol{s})\right]$$

$$= \exp\left[-\sigma_{ni} N_T \int dz'\, \rho_n^T(z', \boldsymbol{s})\right]$$

$i = \Lambda, p,$ or n

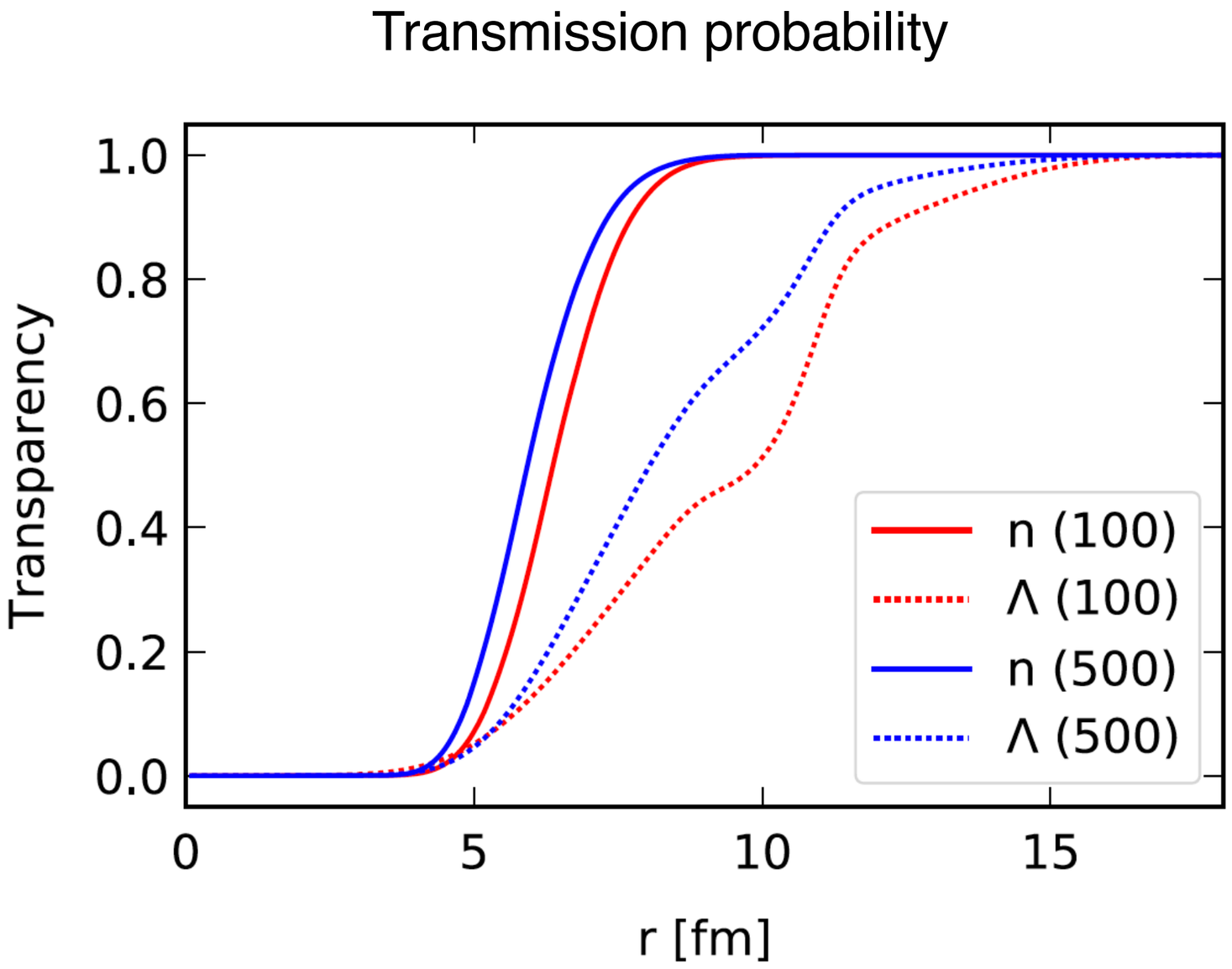
1.5 GeV/nucleon ${}^{3}_{\Lambda}\mathrm{H}$ incident on ${}^{12}\mathrm{C}$, ${}^{120}\mathrm{Sn}$, ${}^{208}\mathrm{Pb}$

$\sigma_{np} = 45.8$ mb

$\sigma_{pp} = 40$ mb

$\sigma_{\Lambda p} = 35$ mb

Hypertriton destruction



Transition from full opaque-ness to full transparency displays changes in the slope due to structure of the deuteron within ${}^{3}_{\Lambda}\mathrm{H}$.

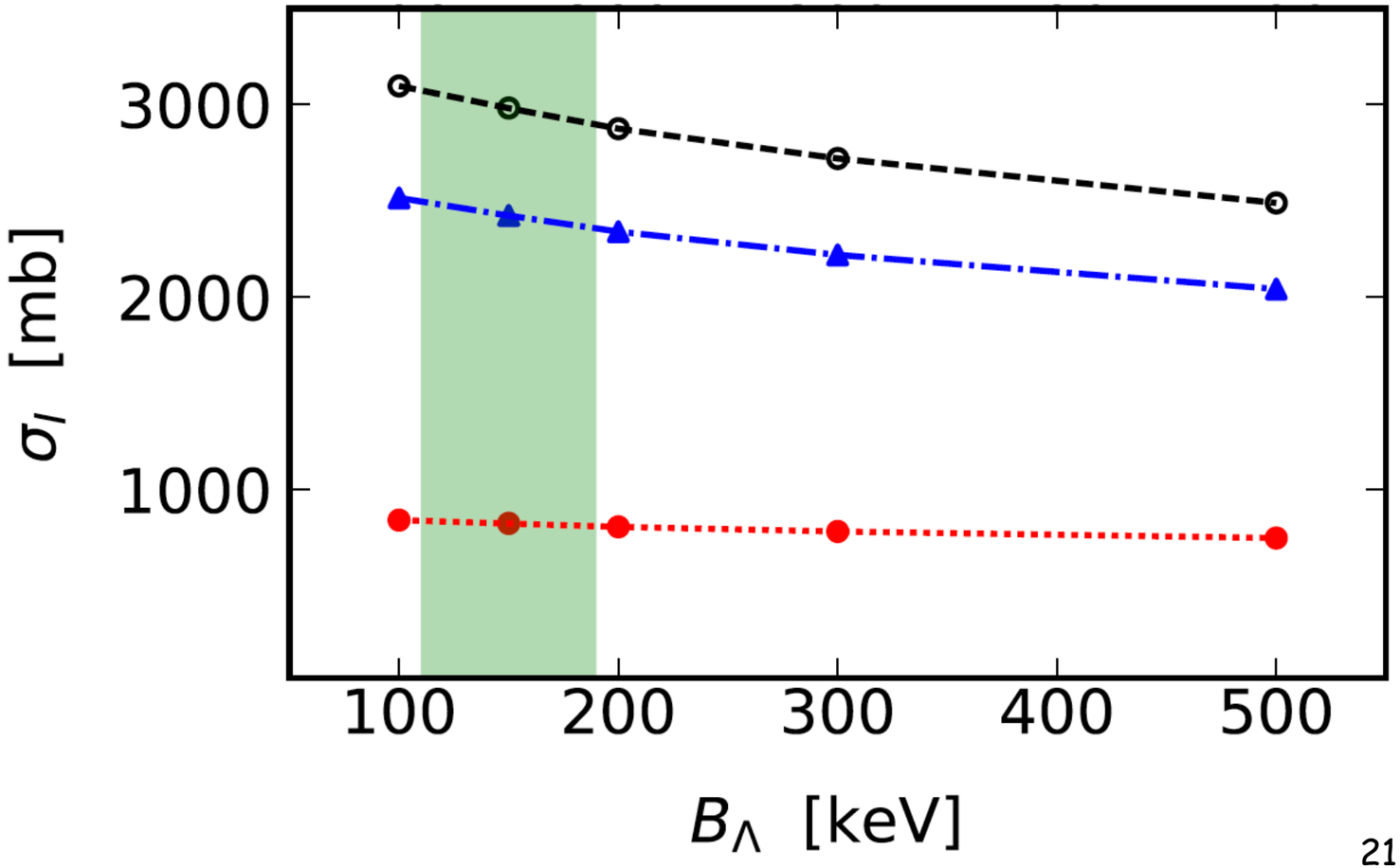
Sensitivity to hypertriton interaction cross sections

1.5 GeV/nucleon ${}^{3}_{\Lambda}\mathrm{H}$ incident on ${}^{12}\mathrm{C}$, ${}^{120}\mathrm{Sn}$, ${}^{208}\mathrm{Pb}$

B_Λ (keV)	σ_I(C)	σ_I(Sn)	σ_I(Pb)
100	842.	2516.	3098.
150	824.	2424.	2982.
200	807.	2341.	2876.
300	783.	2220.	2721.
500	749.	2043.	2490.

For a C target: a 12% reduction of the cross section $\sigma_{\Lambda I}$ from B = 100 MeV to B = 500 MeV.

$B_\Lambda = 148 \pm 40$ keV



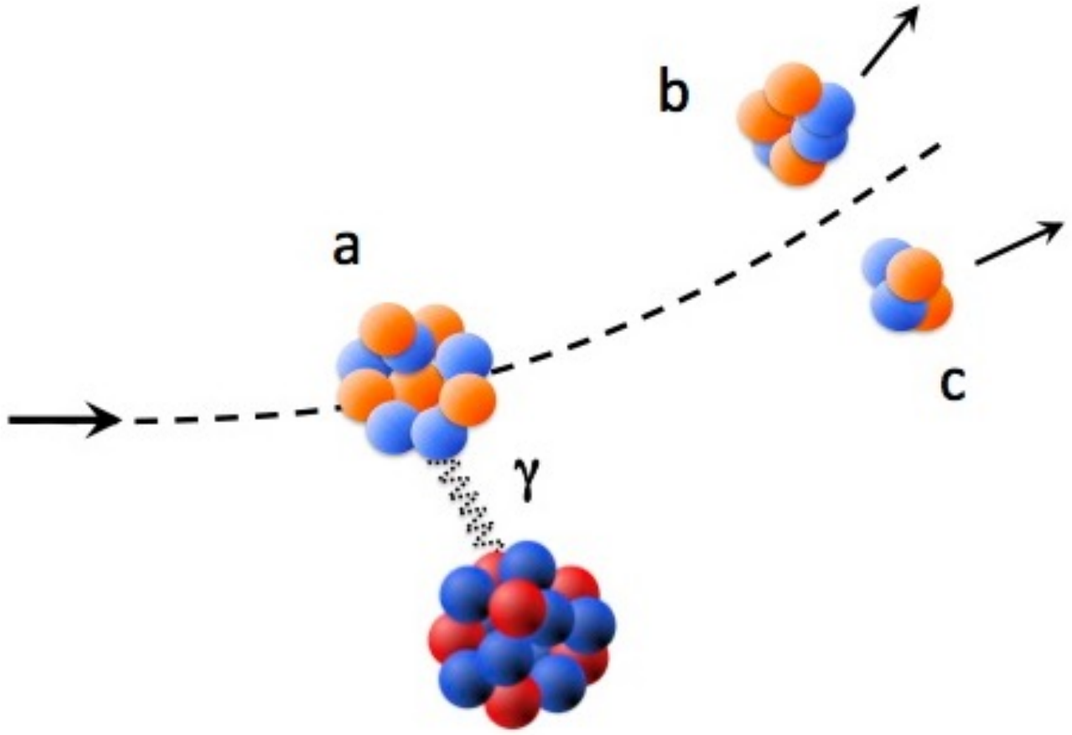
Nuclear astrophysics

Baur, CB, Rebel, NPA (1986)

$$\frac{d\sigma}{dE_\gamma d\Omega} = \frac{1}{E_\gamma} \sum_l \frac{dn_l(E_\gamma,\Omega)}{dE_\gamma d\Omega} \sigma_{\gamma + a \rightarrow b + c}(E_\gamma)$$

Theory

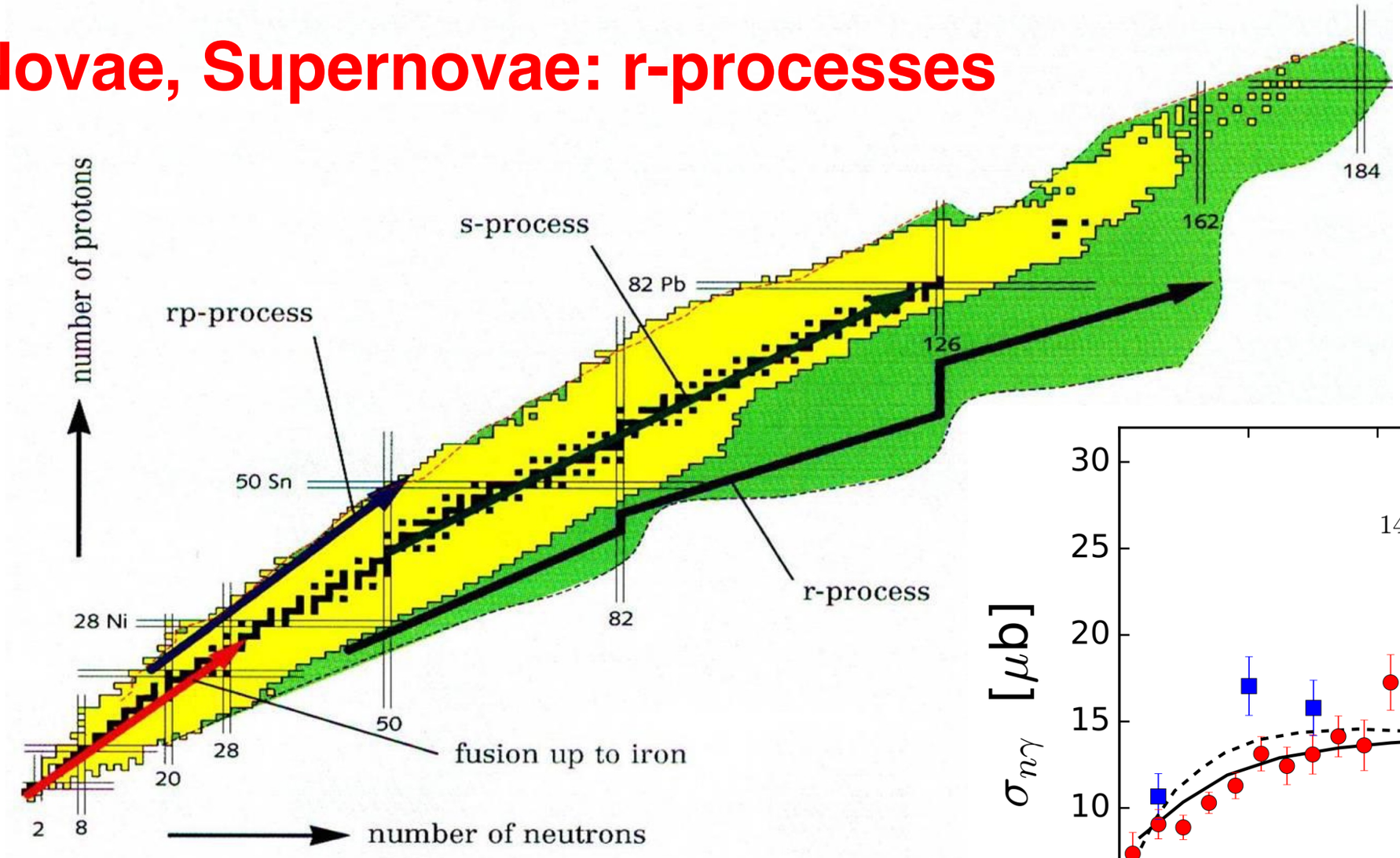
detailed balance

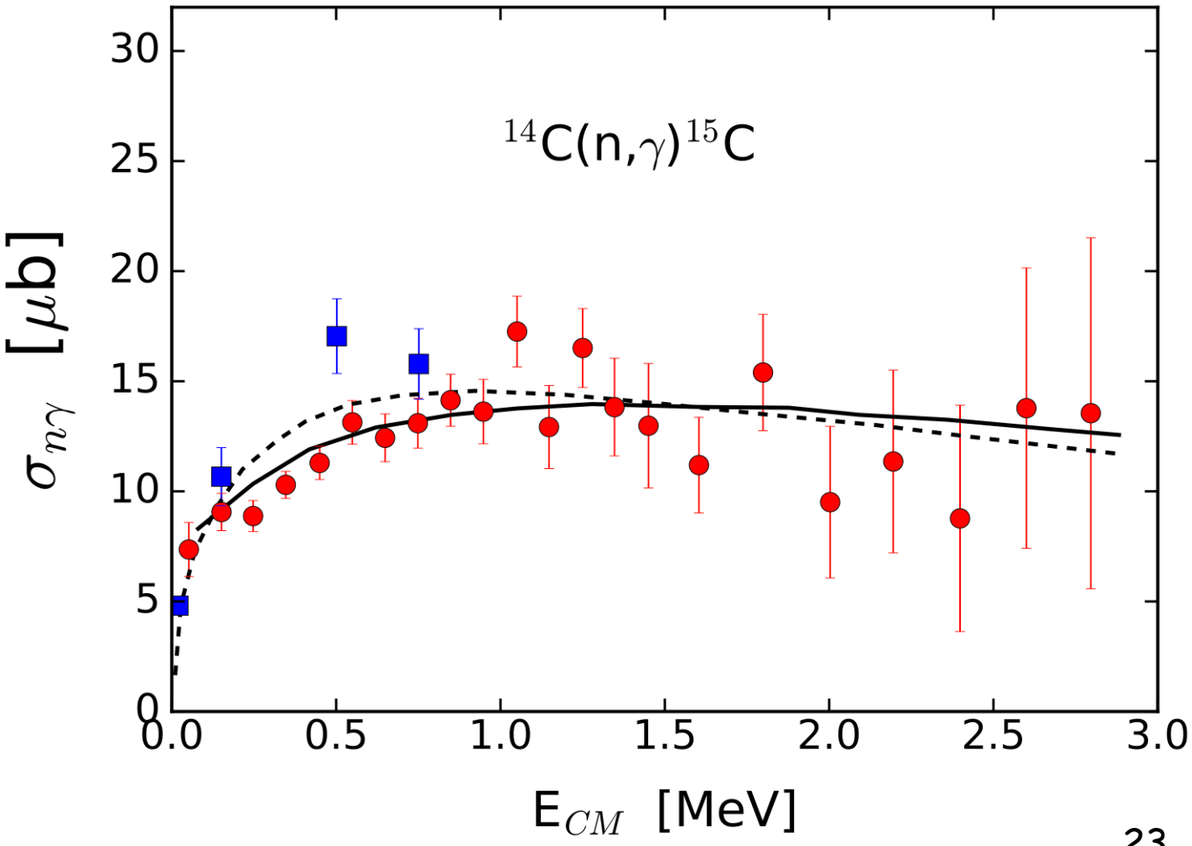


$$\sigma_{b+c\rightarrow a+\gamma} = \frac{2(2j_a+1)}{(2j_b+1)(2j_c+1)} \frac{k_{bc}^2}{k_\gamma^2} \sigma_{\gamma+a\rightarrow b+c}$$

Applications to radiative capture (n,γ) and (p,γ) reactions in nuclear astrophysics

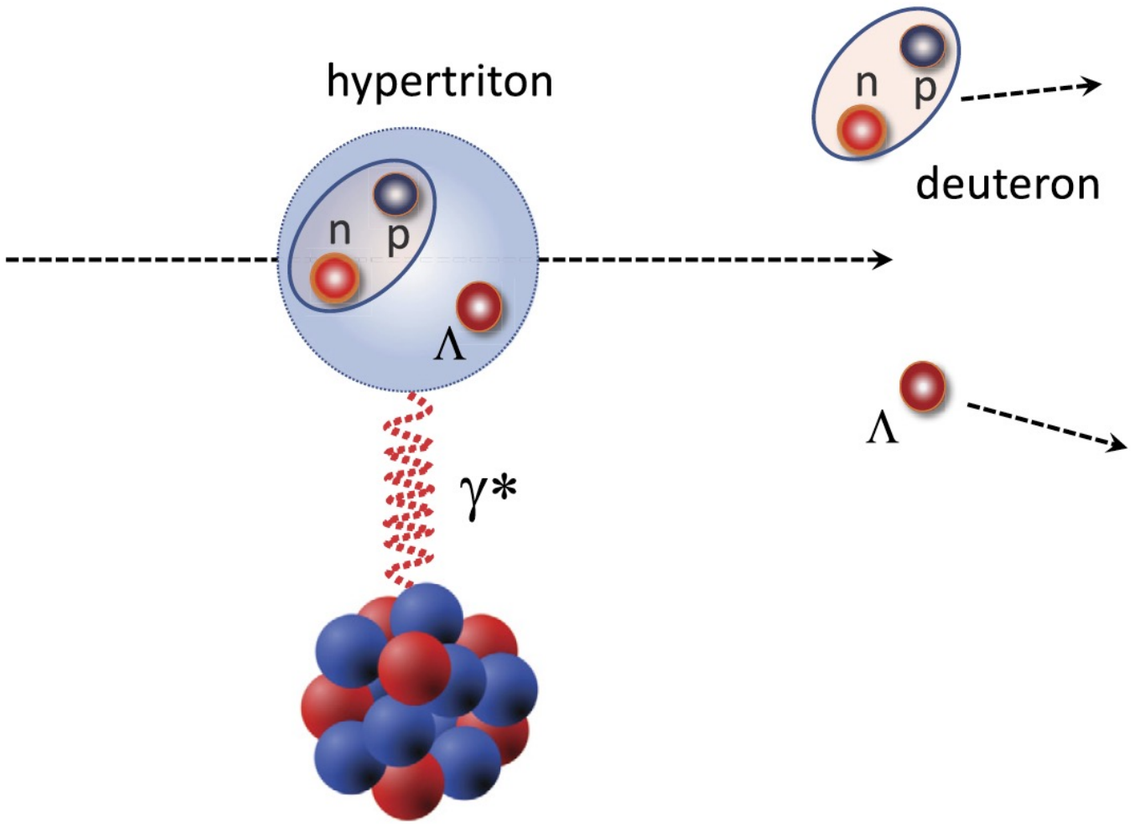
Novae, Supernovae: r-processes





Aumann, Nakamura,
Phys. Scr. T152 (2013) 014012

Electromagnetic response of the hypertriton



$$\frac{d\sigma_C}{dE} = \frac{16\pi^3}{9\hbar c} n(E) \frac{dB(E)}{dE}$$

$$n(E) = \frac{2Z_T^2\alpha}{\pi}\left(\frac{Ec}{\gamma\hbar v^2}\right)^2 \int_0^\infty dbb\left[K_1^2 + \frac{1}{\gamma^2}K_0^2\right]T(b)$$

$$x = Eb/\gamma v$$

First-order perturbation theory

$$\frac{dB(E)}{dE} = \frac{1}{\hbar}\sqrt{\frac{\mu}{2E}}|\langle g.s.||\mathcal{O}_{E1}||E,l\rangle|^2$$

$$\langle g.s.||\mathcal{O}_{E1}||E,l\rangle = (-1)^l \frac{e_{eff}}{\sqrt{4\pi}}\int_0^\infty dr\, r u_{g.s.}(r) u_{E,l}(r)$$

$$u_{E,l}(r) \to \sqrt{2\mu_{\Lambda d}/\pi\hbar^2 k}\, e^{i\delta_l} sin(kr + \delta_l)$$

Electromagnetic response of the hypertriton

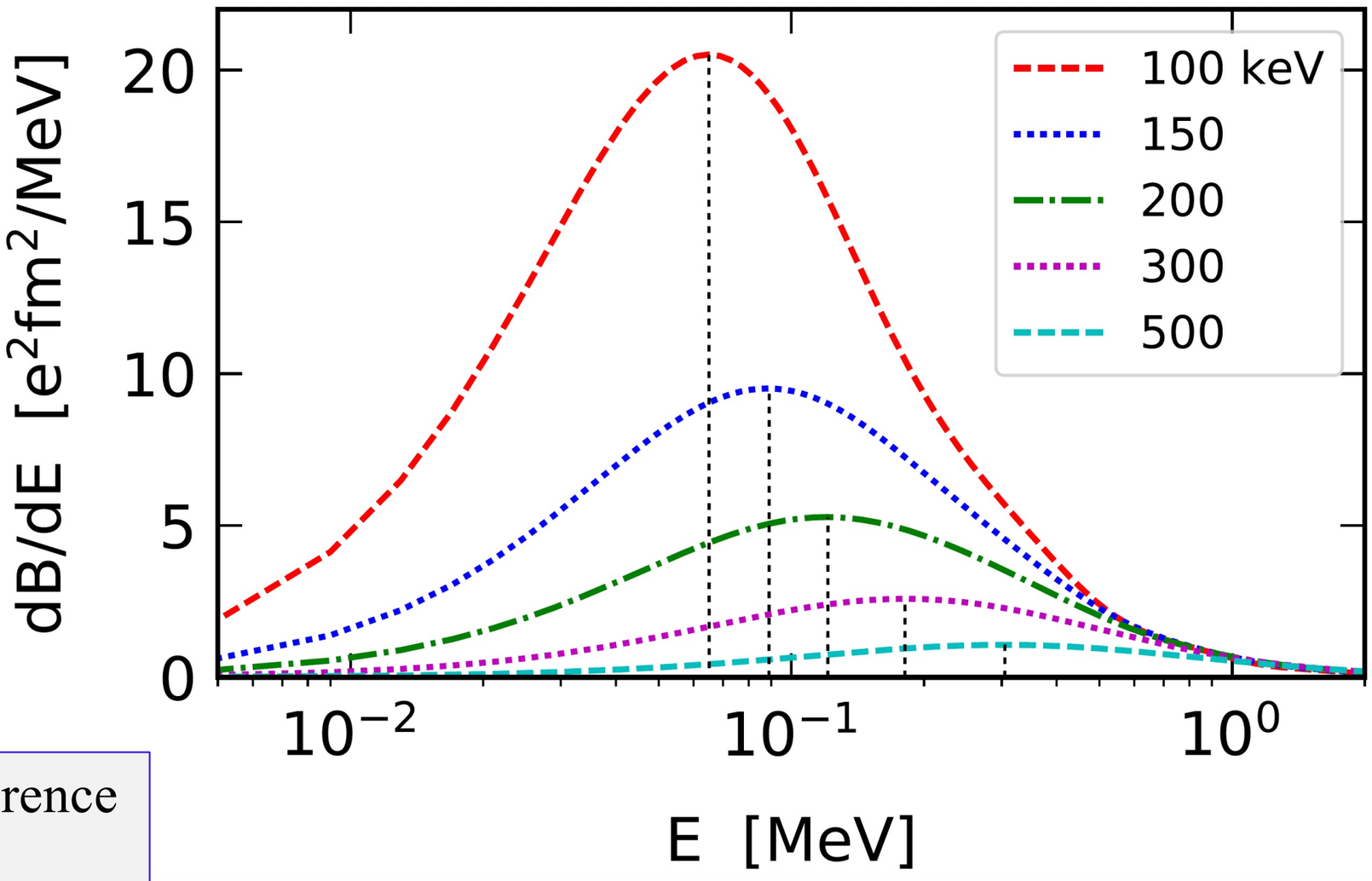
$$\frac{dB(E)}{dE} = \frac{1}{\hbar}\sqrt{\frac{\mu}{2E}}|\langle g.s.||\mathcal{O}_{E1}||E,l\rangle|^2$$

Analytical model:

CB, Sustich
Phys. Rev. C 46 (1992) 2340

$$\frac{dB(E)}{dE} = C\sqrt{B_\Lambda}\frac{E^{3/2}}{(E+B_\Lambda)^4}$$

$$E_{max} = \frac{3}{5}B_\Lambda$$



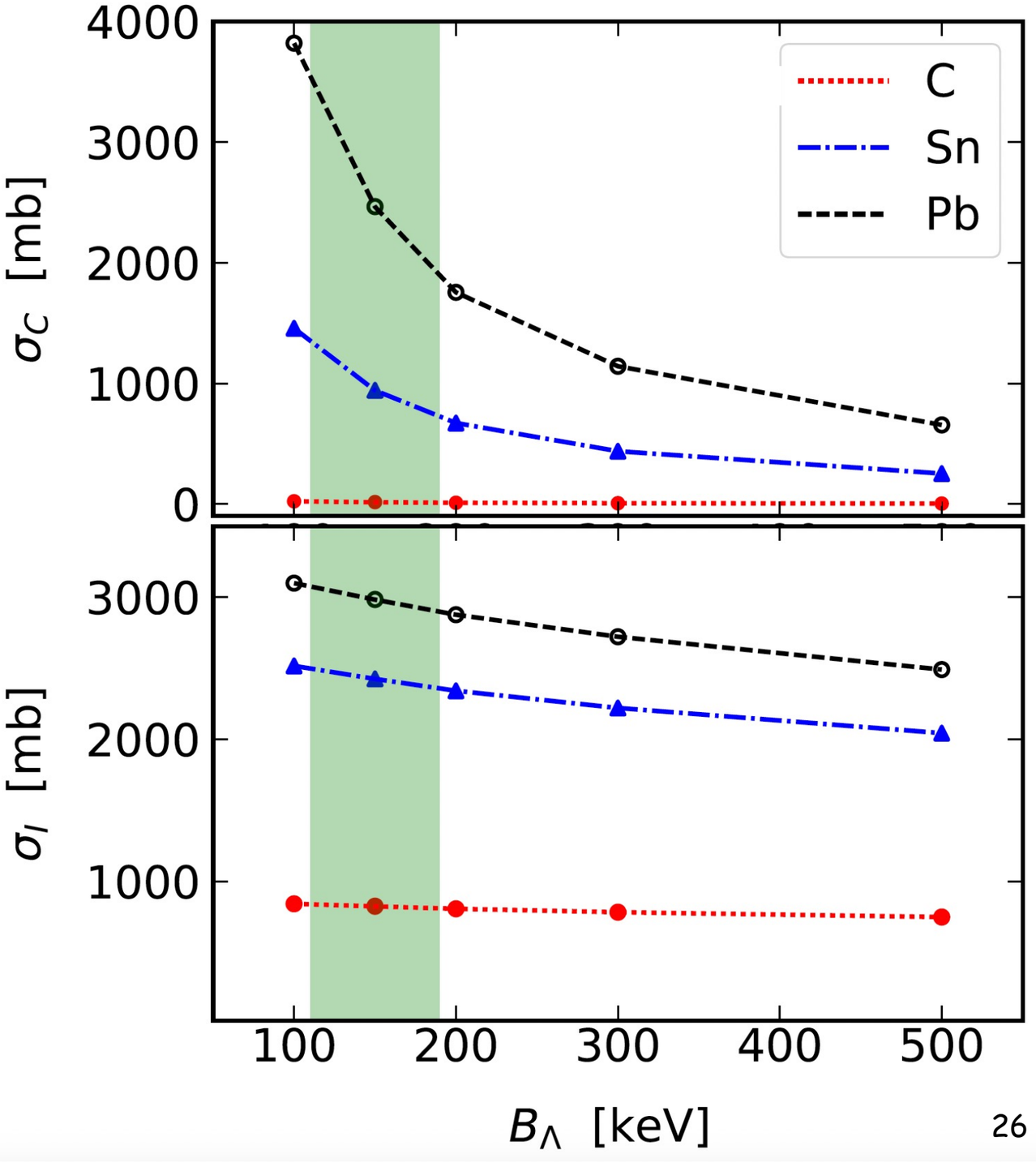
- Excellent agreement < 4% difference
- FSI small

EM response of the hypertriton

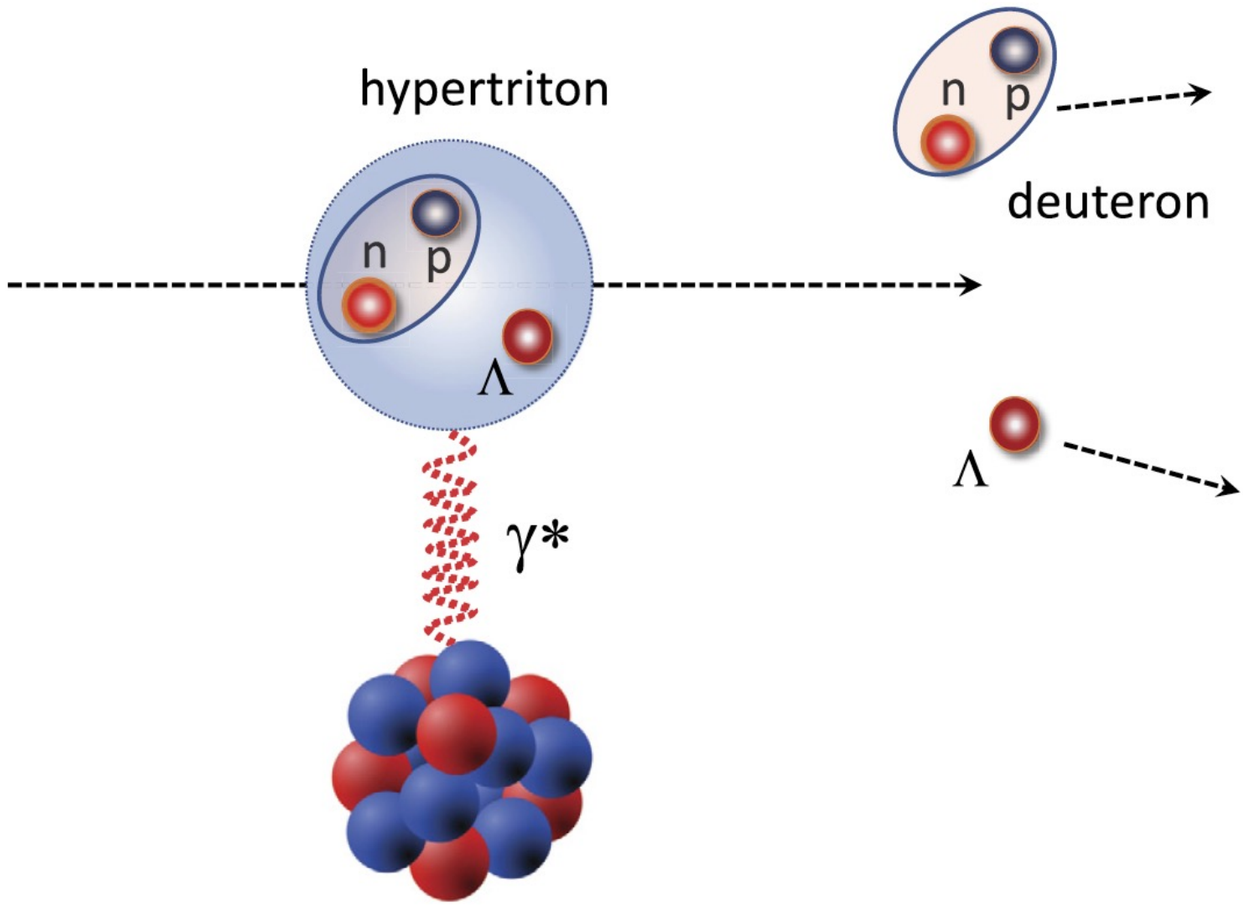
1.5 GeV/nuc. ${}^{3}_{\Lambda}$H incident on ^{12}C, ^{120}Sn, ^{208}Pb

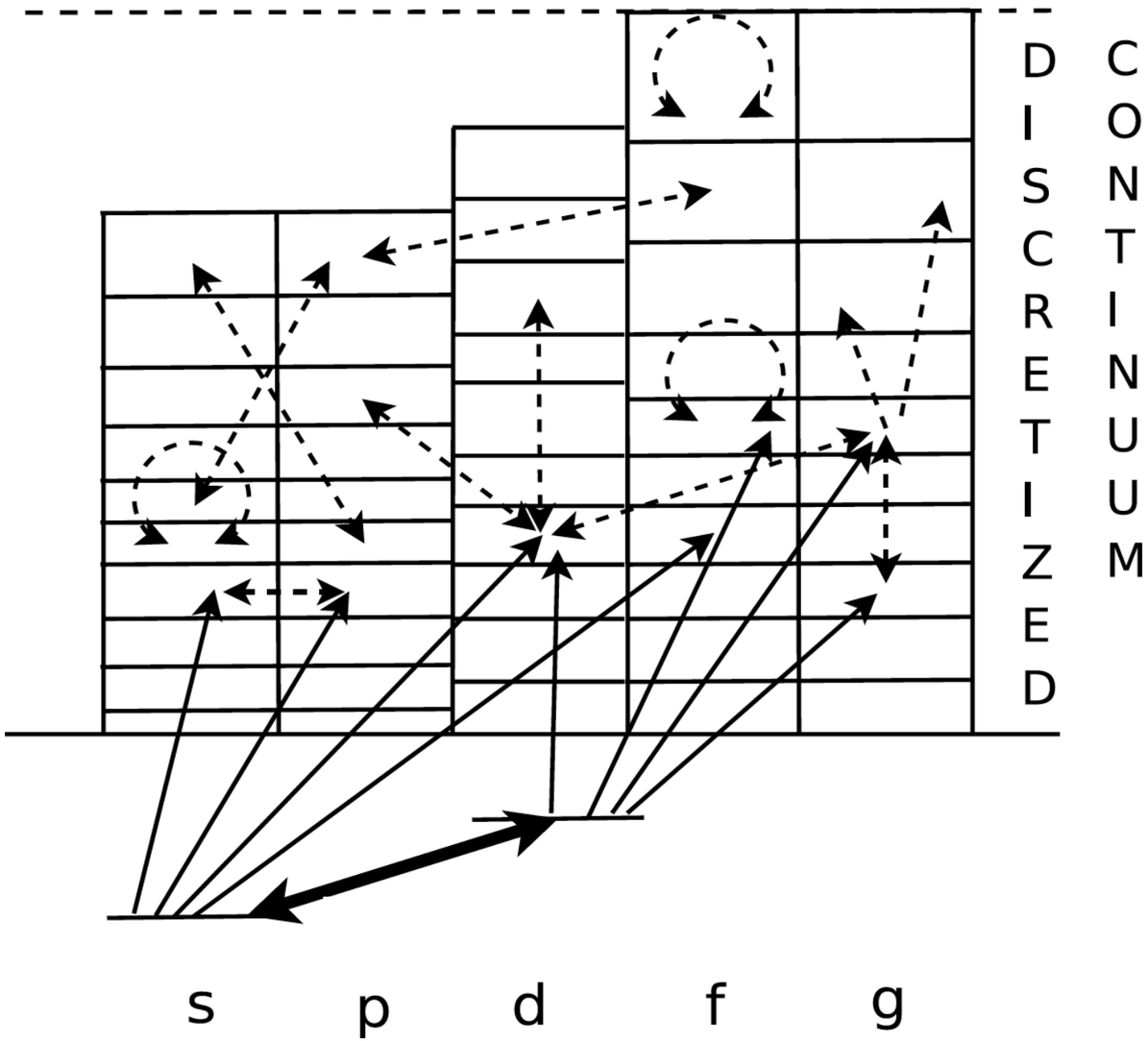
B_Λ (keV)	σ_C (C)	σ_C (Sn)	σ_C (Pb)
100	22.9	1457.	3820.
150	14.9	942.	2464.
200	10.7	672.	1755.
300	7.1	438.	1142.
500	4.1	253.	656.

B_Λ (keV)	σ_I (C)	σ_I (Sn)	σ_I (Pb)
100	842.	2516.	3098.
150	824.	2424.	2982.
200	807.	2341.	2876.
300	783.	2220.	2721.
500	749.	2043.	2490.



EM response of the hypertriton





- Basis expansion for intrinsic ϕ
- Eikonal scattering waves
- Nuclear + EM potentials
- Relativity

- Continuum discretization
- Coupled-channels (relativistic CDCC)

Electromagnetic response of loosely bound nuclei

$$i\hbar v \frac{d}{dz} S_c(\mathbf{b}, z) = \sum_{c'} \langle \Phi_c | H_{int}(\mathbf{b}, z) | \Phi_{c'} \rangle S_{c'}(\mathbf{b}, z) \exp\left[i \frac{E_{cc'} z}{\hbar c} \right]$$

$$f_c(\boldsymbol{q}) = -\frac{ik}{2\pi} \int db \exp(i\boldsymbol{q} \cdot \boldsymbol{b}) \left[S_c(\mathbf{b}, z) - \delta_{0c} \right]$$

$$\frac{d\sigma_c}{d\Omega} = \sum_{M_0, M_c} \left| f_c^{(M_c - M_0)}(\theta, E_c) \right|^2$$

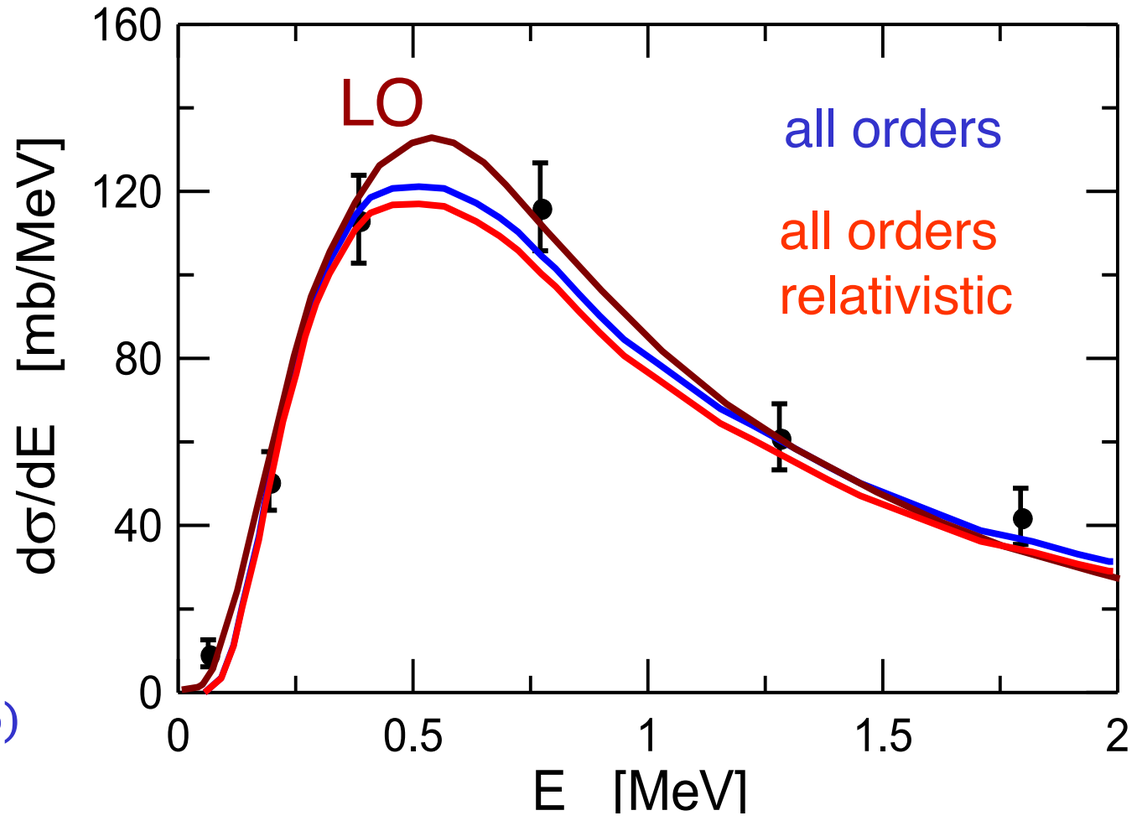
$$\left| E_c \right\rangle = \int dE_c' \, \Gamma(E_c') \left| E_c' \right\rangle$$

$$\Gamma(E_j) = \begin{cases} \dfrac{1}{\Delta E} & \text{if} \quad (j-1)\Delta E < E_c < j\Delta E \\ 0, & \text{otherwise} \end{cases}$$

$H_{int} = H_{EM} = easy$

$H_{int} = H_{nucleus-nucleus}$
= very complicated

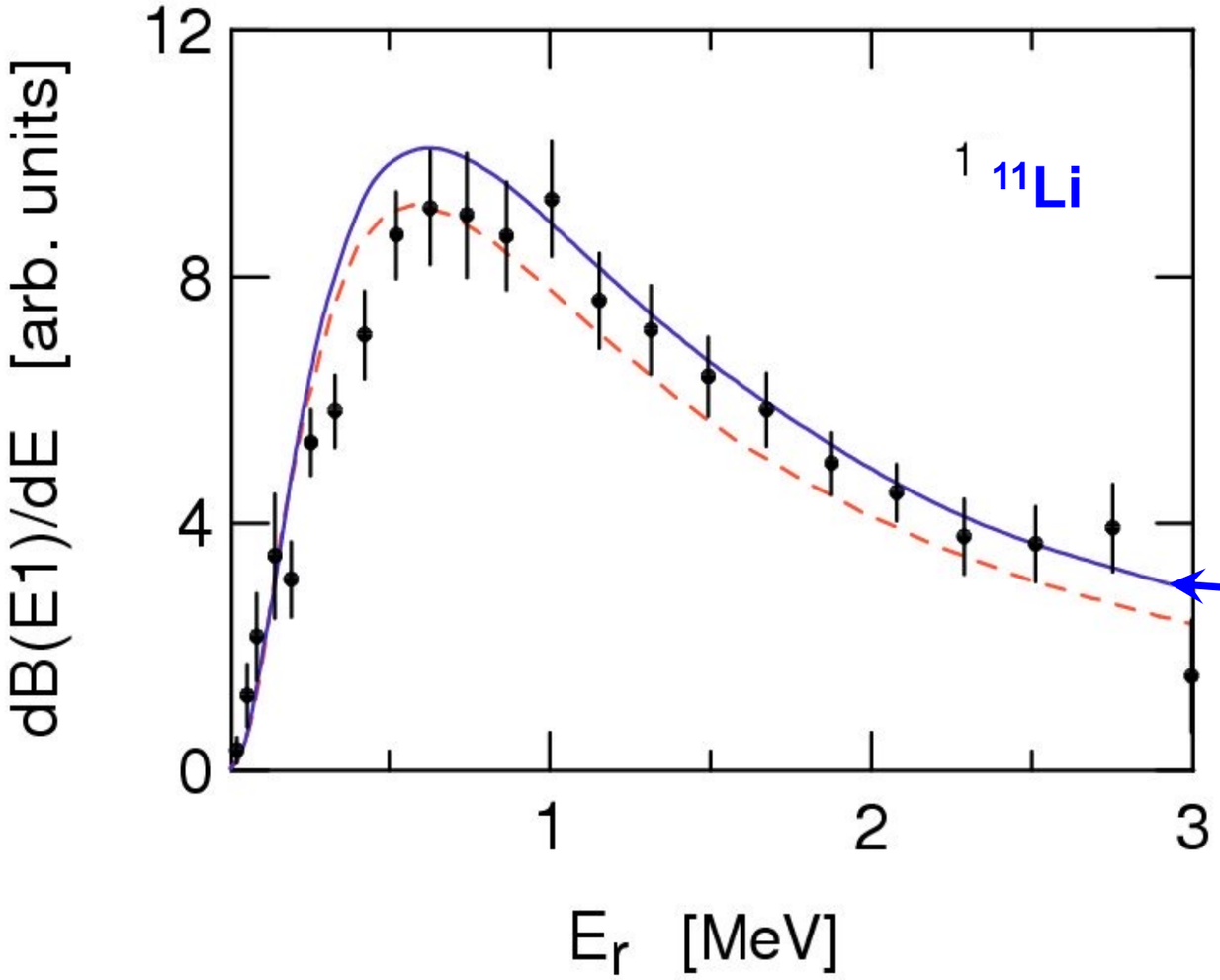
Pb (^{8}B,p^{7}Be) at 100 MeV/nucleon

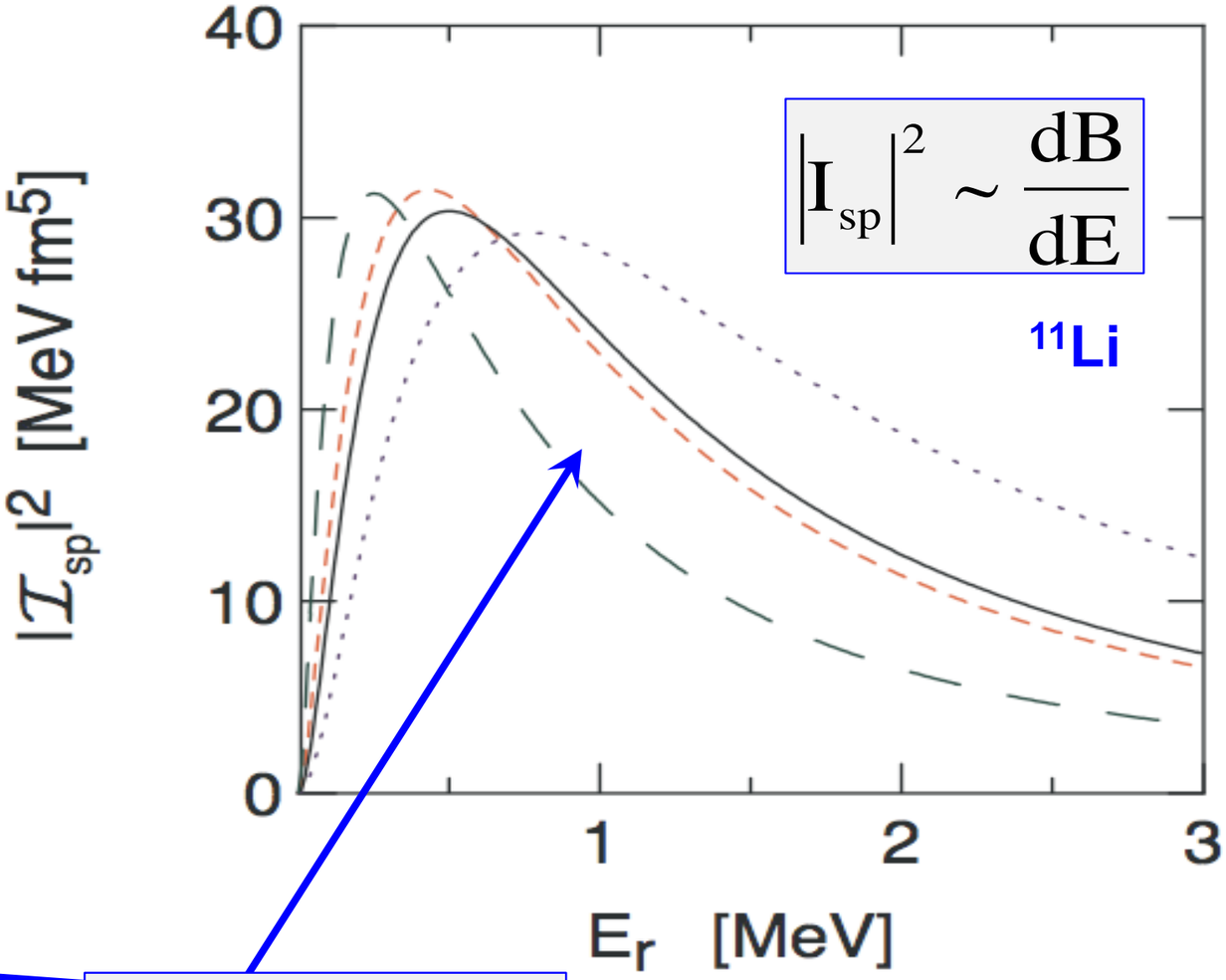


CB, PRL 94, 072701 (2005)

Final state interactions (FSI)

3-body configurations and FSI widen dB(E1)/dE

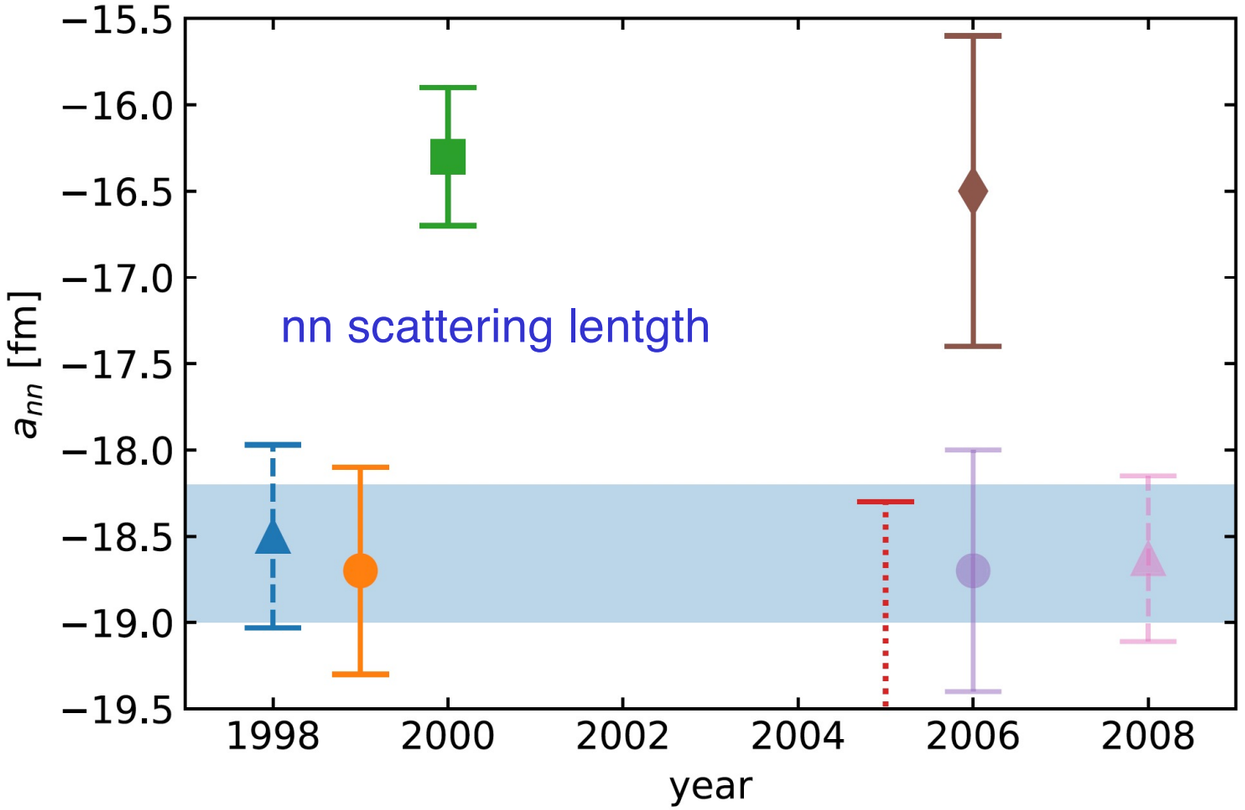


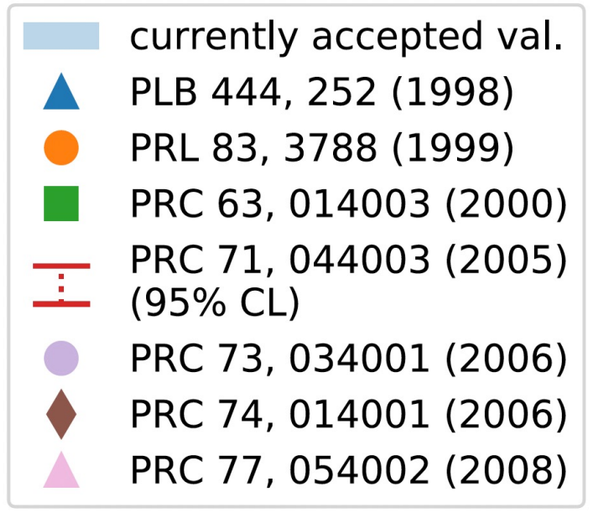


FSI: Different scattering lengths, effective ranges

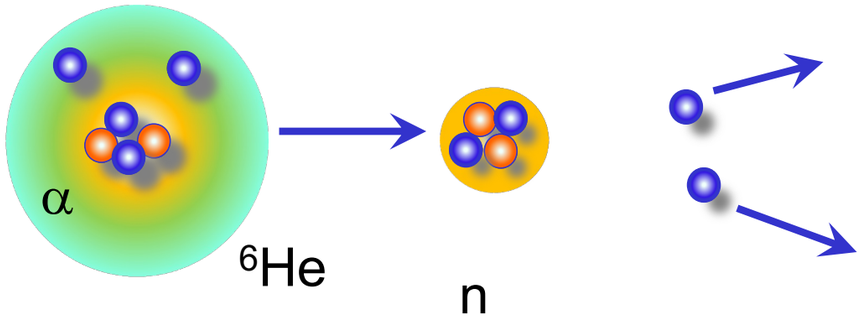
CB, PRC 75, 024606 (2007)

FSI and nn scattering length





dashed: $d + \pi^- \rightarrow \gamma + n + n$
solid: $d + n \rightarrow p + n + n$
dotted: $d + d \rightarrow {}^2\mathrm{He} + n + n$



^{6}He(p,pα)nn reaction in inverse kinematics at high energies

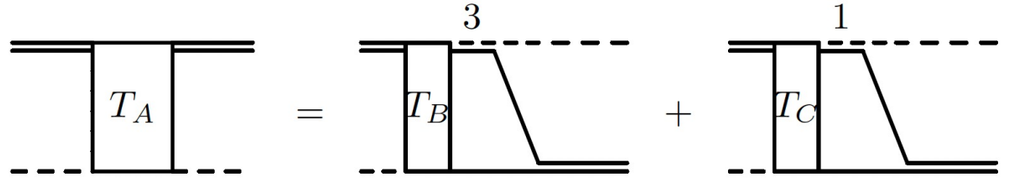
Göbel, et al, PRC 104, 024001 (2021)

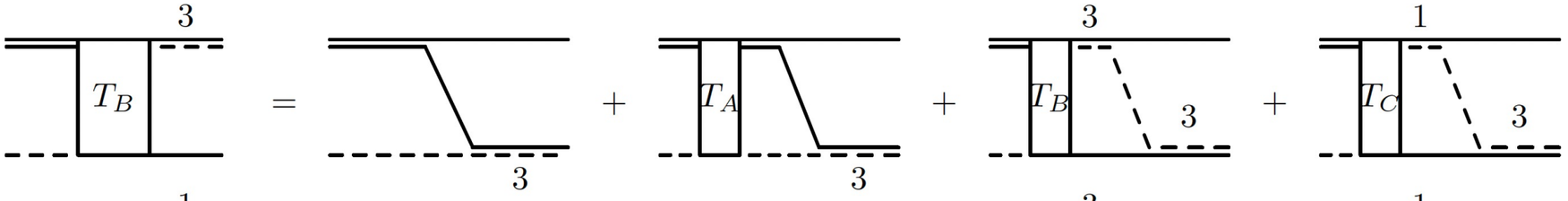
Effective Field Theory (EFT)

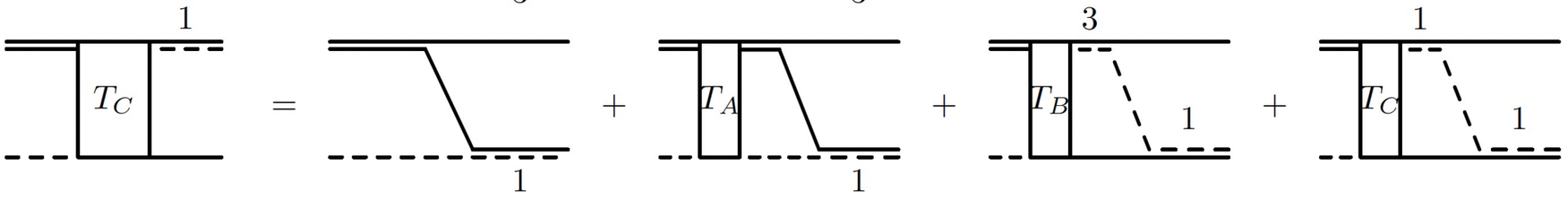
Pionless EFT
(a) Momentum scales < pion mass
(b) Dibaryon field Δ_d
(c) Cutoff parameter Λ_c
(d) Simplified 3-body force H(s_0,Λ_c)
(e) Asymptotic analysis

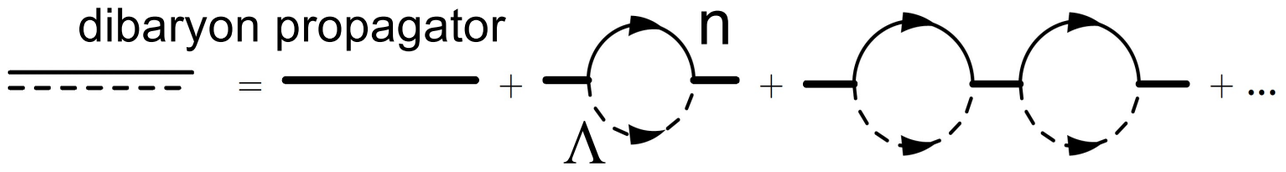
$$\begin{aligned}\mathcal{L} =& N^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) N + \Lambda^\dagger \left(i\partial_t + \frac{\nabla^2}{2M_\Lambda} \right) \Lambda \\ &+ \Delta_d d_l^\dagger d_l - \frac{g_d}{2} \left[d_l^\dagger N^T (i\tau_2) (i\sigma_l\sigma_2) N + \text{H.c.} \right] \\ &+ \Delta_s s_j^\dagger s_j - \frac{g_s}{2} \left[s_j^\dagger N^T (i\tau_j\tau_2) (i\sigma_2) N + \text{H.c.} \right] \\ &+ \Delta_3 \left(u_l^3\right)^\dagger u_l^3 - g_3 \left[i \left(u_l^3\right)^\dagger \Lambda^T (i\sigma_l\sigma_2) N + \text{H.c.} \right] \\ &+ \Delta_1 \left(u^1\right)^\dagger u^1 - g_1 \left[i \left(u^1\right)^\dagger \Lambda^T (i\sigma_2) N + \text{H.c.} \right] + \ldots\end{aligned}$$

Hildenbrand, Hammer,
Phys. Rev. C, 100, 034002 (2019)





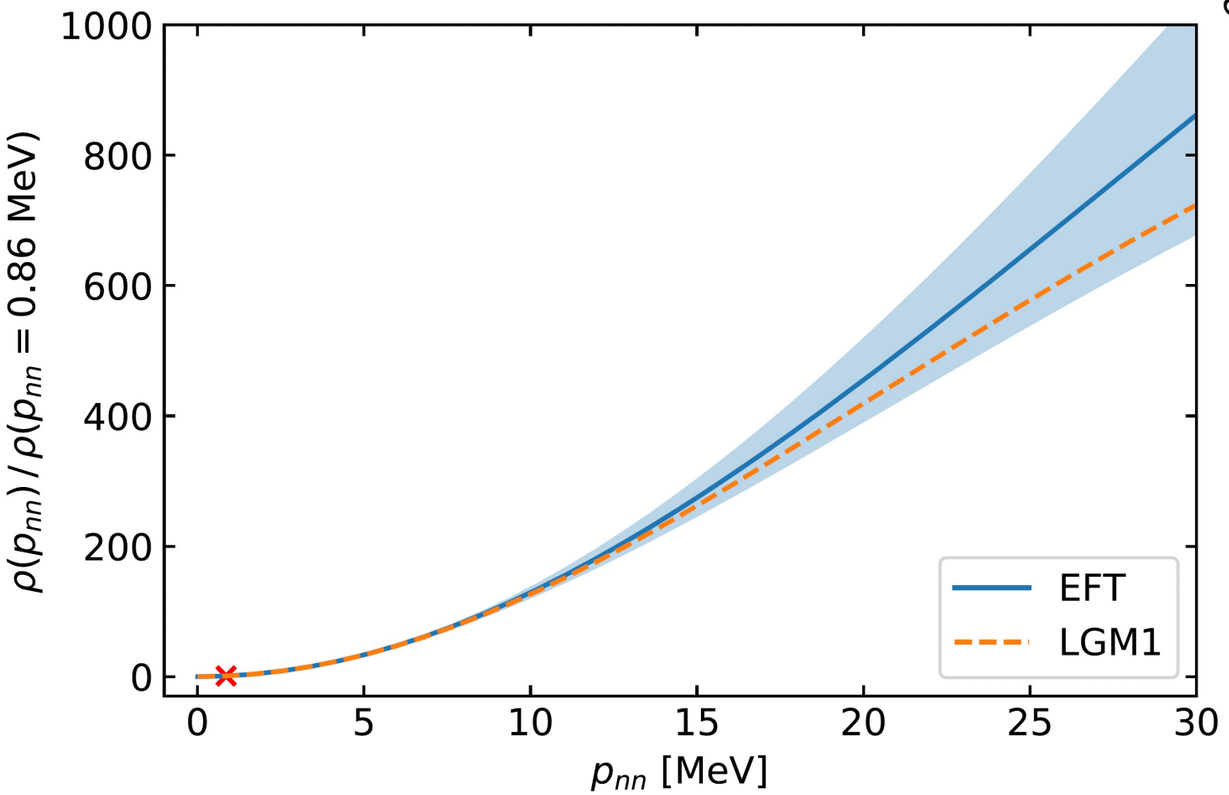


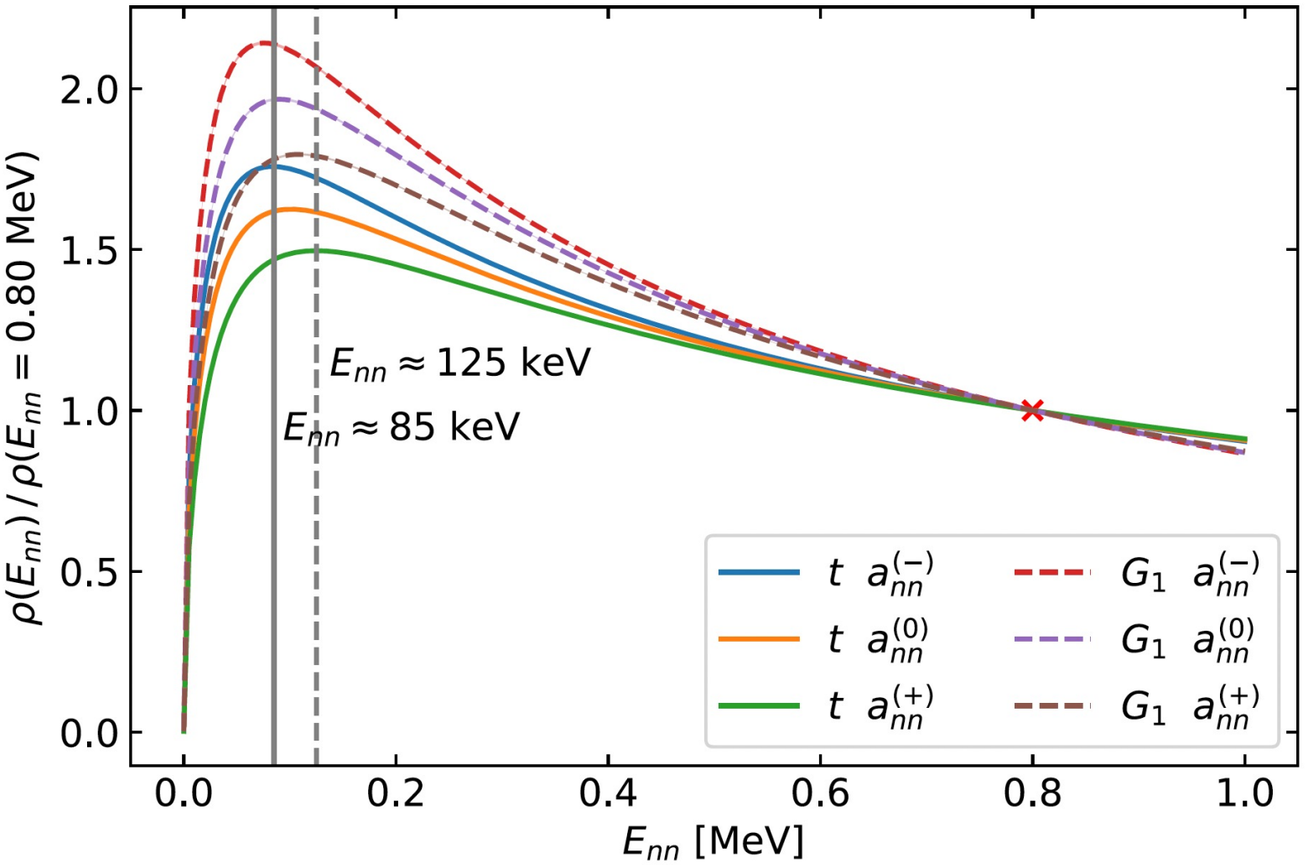


nn scattering length

Göbel, et al, PRC 104, 024001 (2021)

Halo EFT → nn configuration in ^{6}He → E_{nn} density





LGM1 = Faddeev calculations with local Gaussian potentials

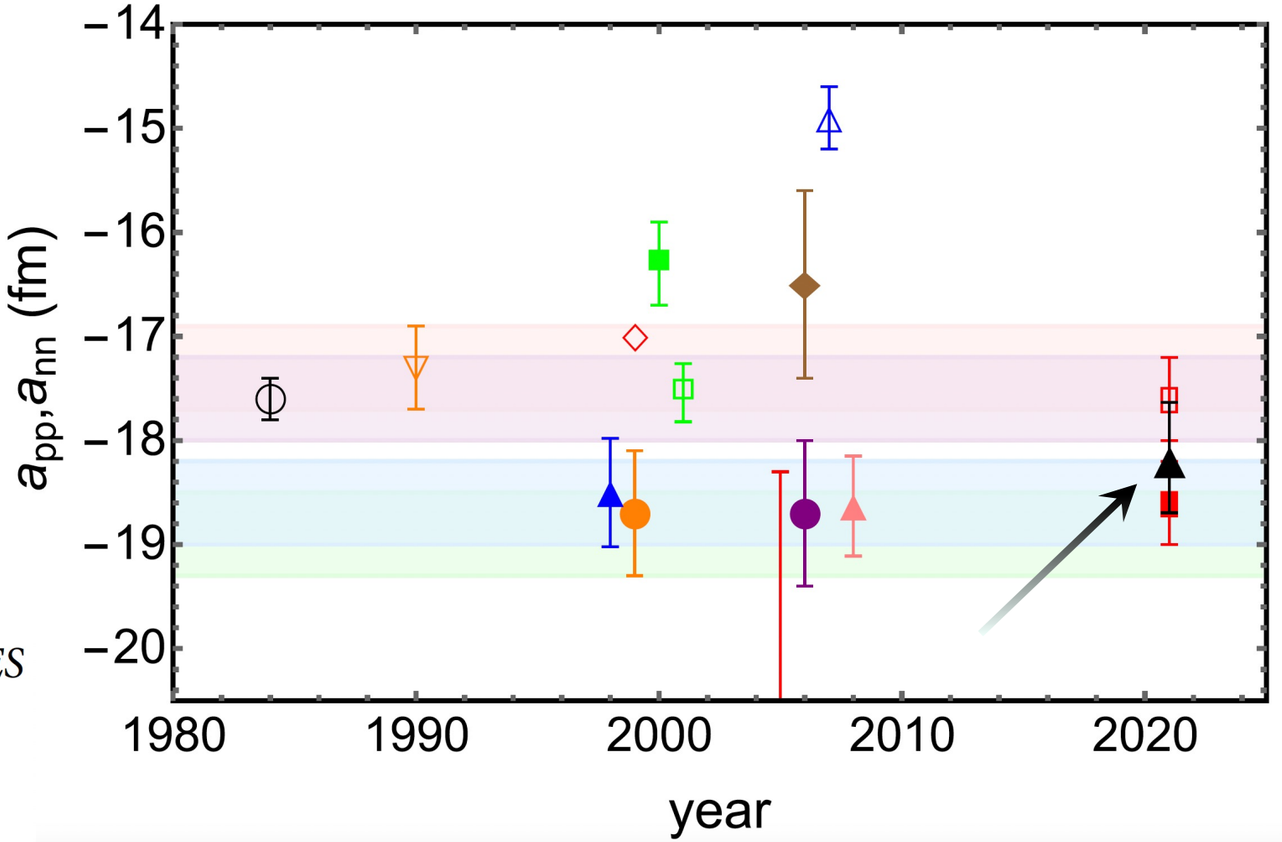
pp scattering lentgth

v_{NN} charge independence violation: $m_{\pi^\pm} \neq m_{\pi^0}$

v_{NN} charge symmetry violation: $m_{down} \neq m_{up}$

p + d ⟶ p + p + n ➔ p + p scattering

Tumino, et al,
Nature Comm. Phys. 6, 106 (2023)



$$\frac{d^3\sigma}{d\Omega_B d\Omega_b dE_B} = (KF)\cdot|\phi(p_{xs})|^2 \cdot \left[\frac{d^2\sigma_{xA\to bB}}{dE_{c.m.}d\Omega}\right]^{HOES}$$

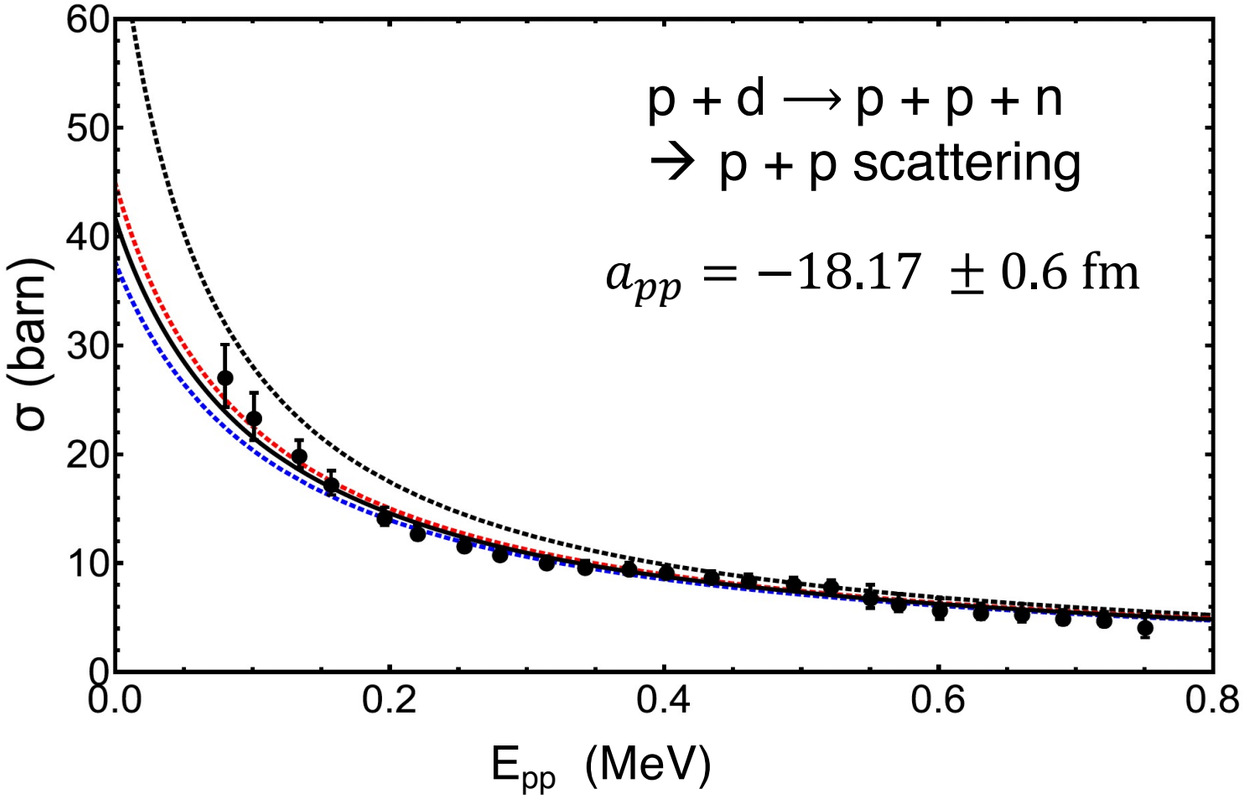
$$\left(\frac{d\sigma}{d\Omega_{c.m.}}\right)^{HOES} = \frac{1}{4k^2}\left(\left|F(\mathbf{p},\mathbf{k}) - 2T_{CN}(p,k)\right|^2 + 3\left|F(\mathbf{p},\mathbf{k})\right|^2\right)$$

$$F(\mathbf{p},\mathbf{k}) = m_p e^2 e^{-\pi\eta}\Gamma(1+i\eta)(p^2-k^2)^{i\eta} g(\mathbf{p},\mathbf{k})$$

$$g(\mathbf{p},\mathbf{k}) = (\mathbf{p}-\mathbf{k})^{-2(1+i\eta)} \pm (\mathbf{p}+\mathbf{k})^{-2(1+i\eta)}$$

Energy differences between mirror nuclei small as compared to experiment (Nolen-Schiffer anomaly)

pp scattering lentgth



NN s-wave phase shift

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 k^2$$

$$\sigma = \frac{4\pi}{\left(\frac{1}{a} - \frac{1}{2} r_0 k^2\right)^2 + k^2}$$

$$V_{NN}(R) = V_0 e^{-\frac{r^2}{r_G^2}} + \frac{e_{NN}^2}{r}$$

disable EM → d from short-range universal window

Table 2 The effect of electromagnetic terms. Low-energy parameters with and without the inclusion of the electromagnetic terms for the two potential models indicated.

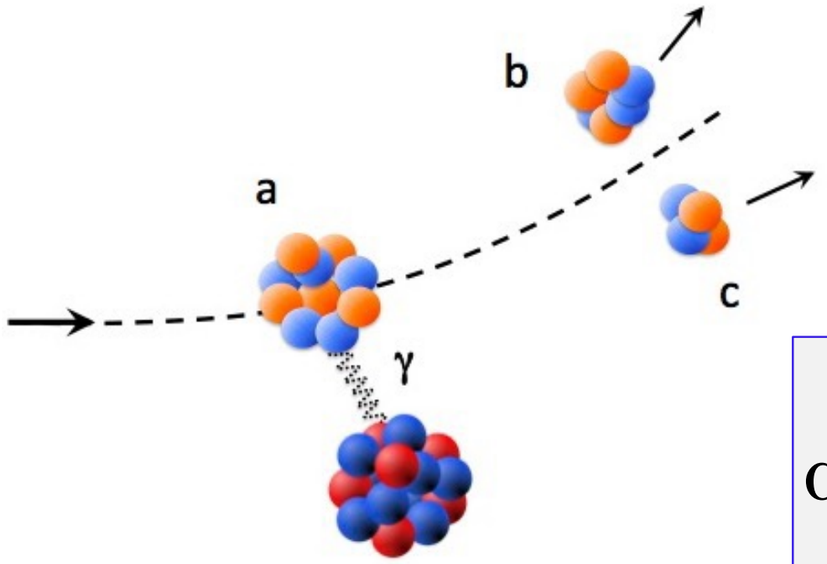
	Argonne v_{18}	**w/o v^{EM}**		**Argonne v_{18}**	**w/o v^{EM}**
$^1a_{nn}$(fm)	−18.487	−18.818	$^1a_{pp}$(fm)	−7.806	−17.164
$^1r_{nn}$(fm)	2.840	2.834	$^1r_{pp}$(fm)	2.788	2.865
	Gaussian	**w/o v^{EM}**		**Gaussian**	**w/o v^{EM}**
$^1a_{nn}$(fm)	−18.487	−18.89 ± 0.02	$^1a_{pp}$(fm)	−7.806	−17.19 ± 0.03
$^1r_{nn}$(fm)	2.85 ± 0.07	2.83 ± 0.07	$^1r_{pp}$(fm)	2.77 ± 0.08	2.86 ± 0.08

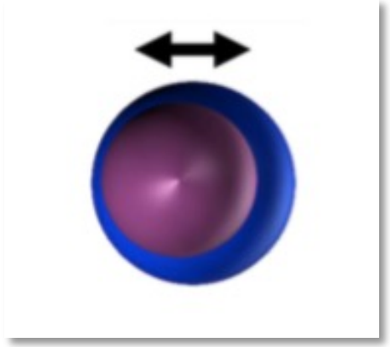
Pigmy resonances & Dipole polarizability

Rossi et al.
PRL 111 (2013) 242503

Wieland et al.
PRL 102, 092502 (2009)

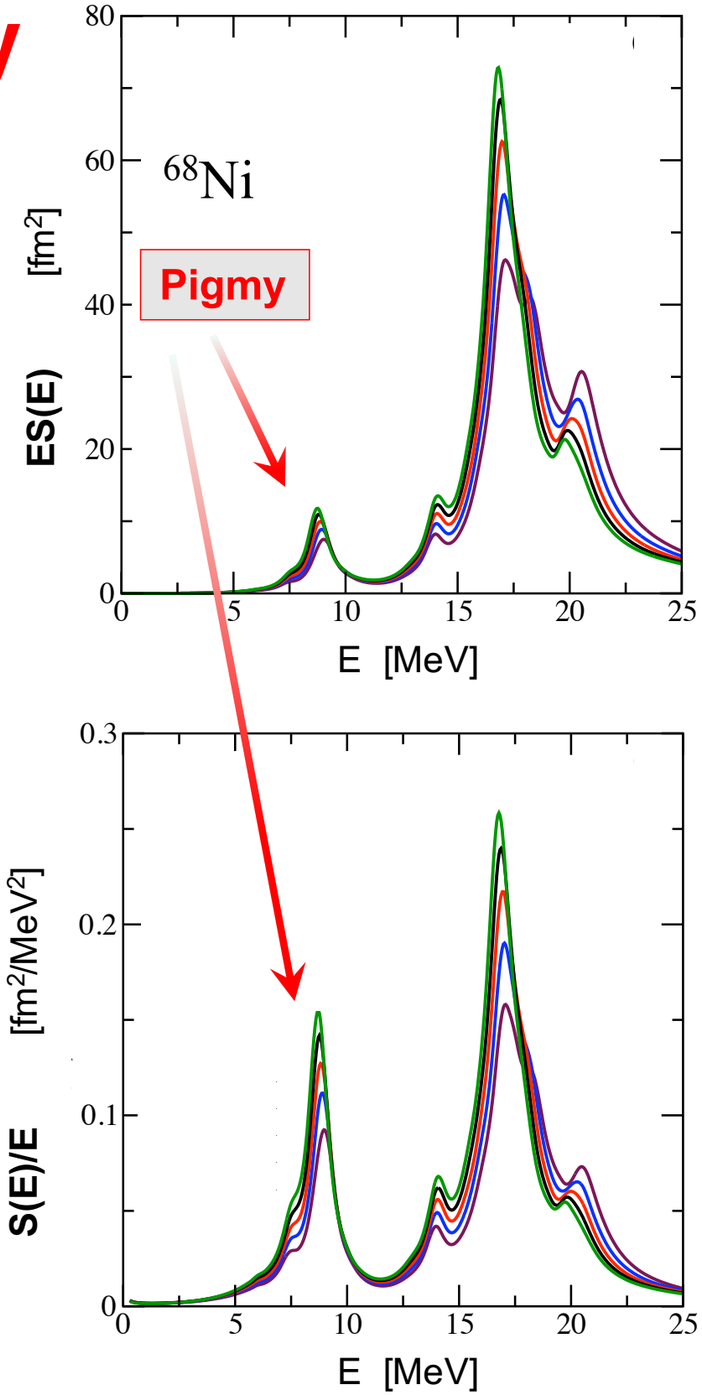
$$\sigma_C \sim (\cdots)\int_0^\infty \frac{\sigma_\gamma(E)}{E^2}dE$$





$$\alpha_D = \frac{\hbar c}{2\pi^2}\int_0^\infty \frac{\sigma_\gamma(E)}{E^2}dE$$

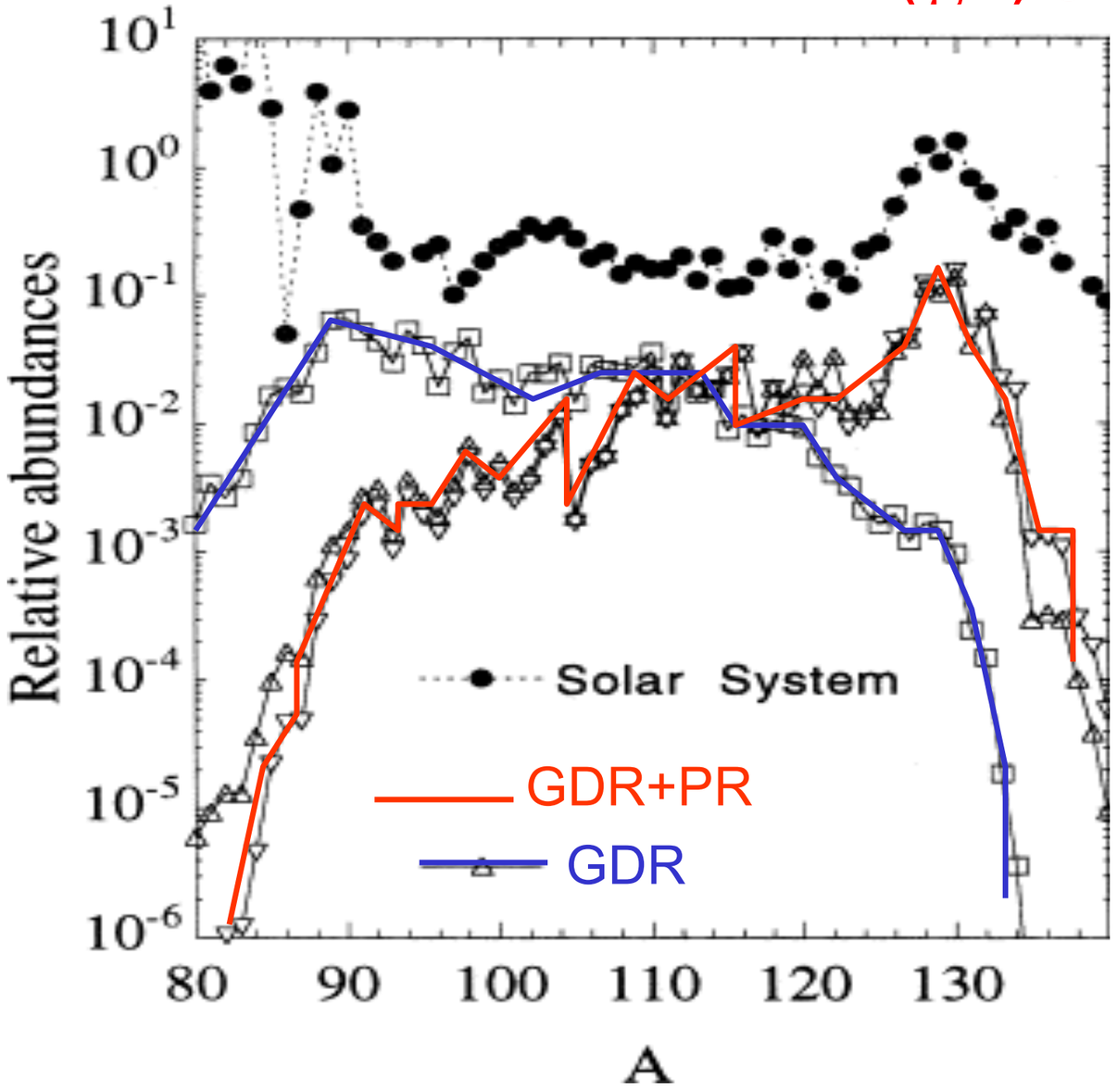
$$= \frac{8\pi}{9}\int \frac{B(E1,E_x)}{E_x}dE_x$$



Impact of pygmies on nucleosynthesis (??!)

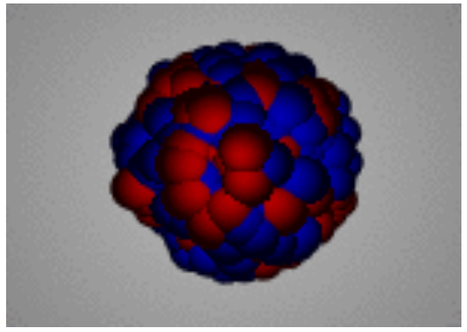
Goriely, PLB 436, 10 (1998)

(γ,n) or (n,γ) cross sections in the r-process

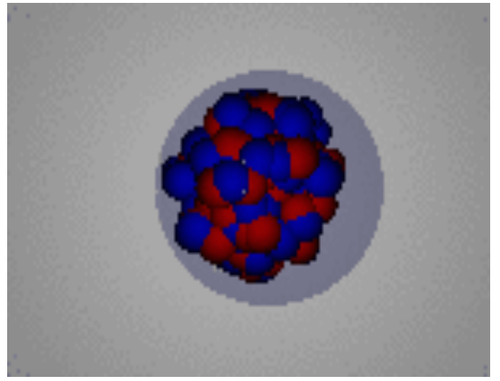


Impact: r-process abundances

- Calculation for
  $T = 10^9$ K, $N_n = 10^{20}$ cm^{-3}, $\tau = 2.3$ s
- Under some conditions, PDR can enhance production in some regions

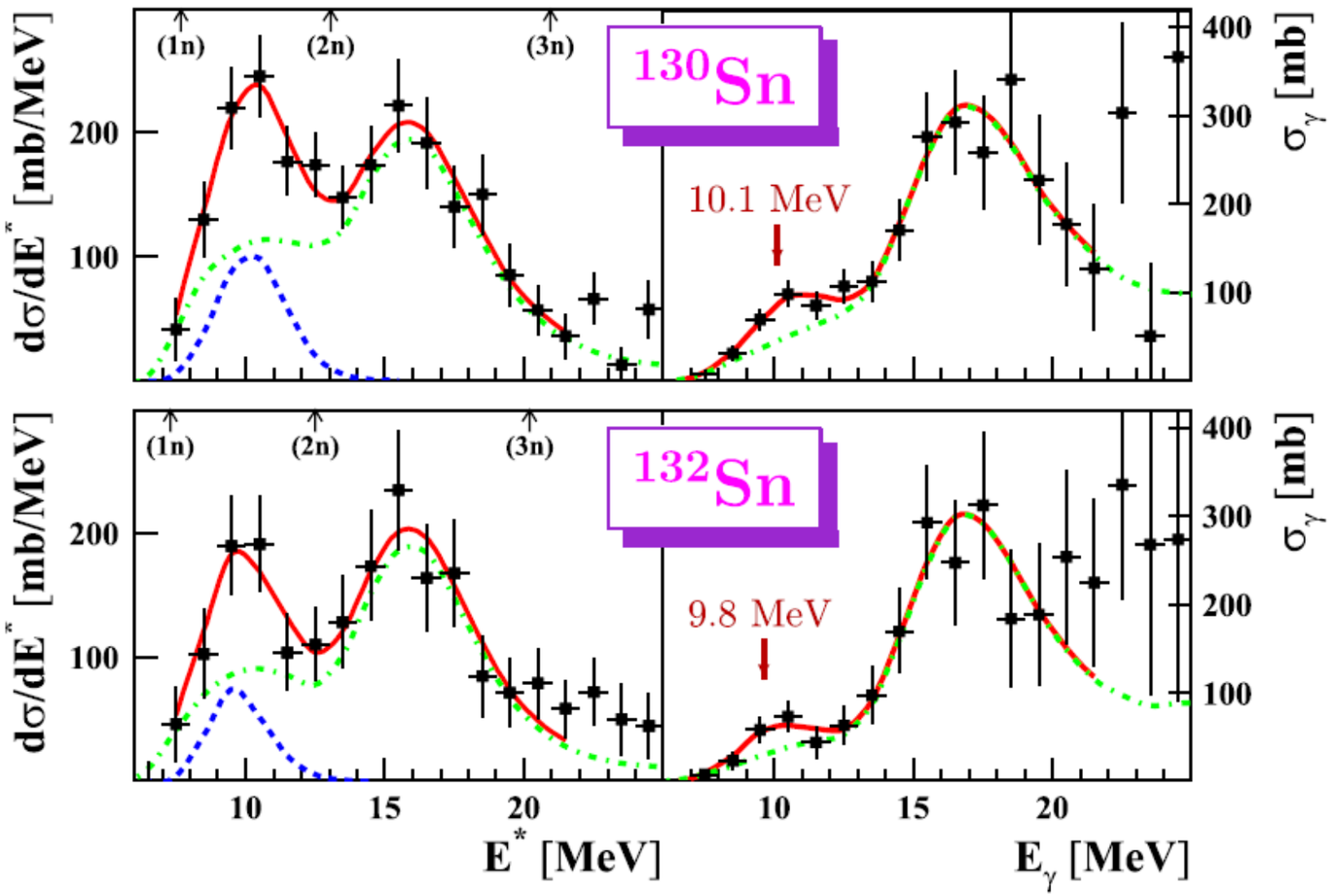


Giant dipole

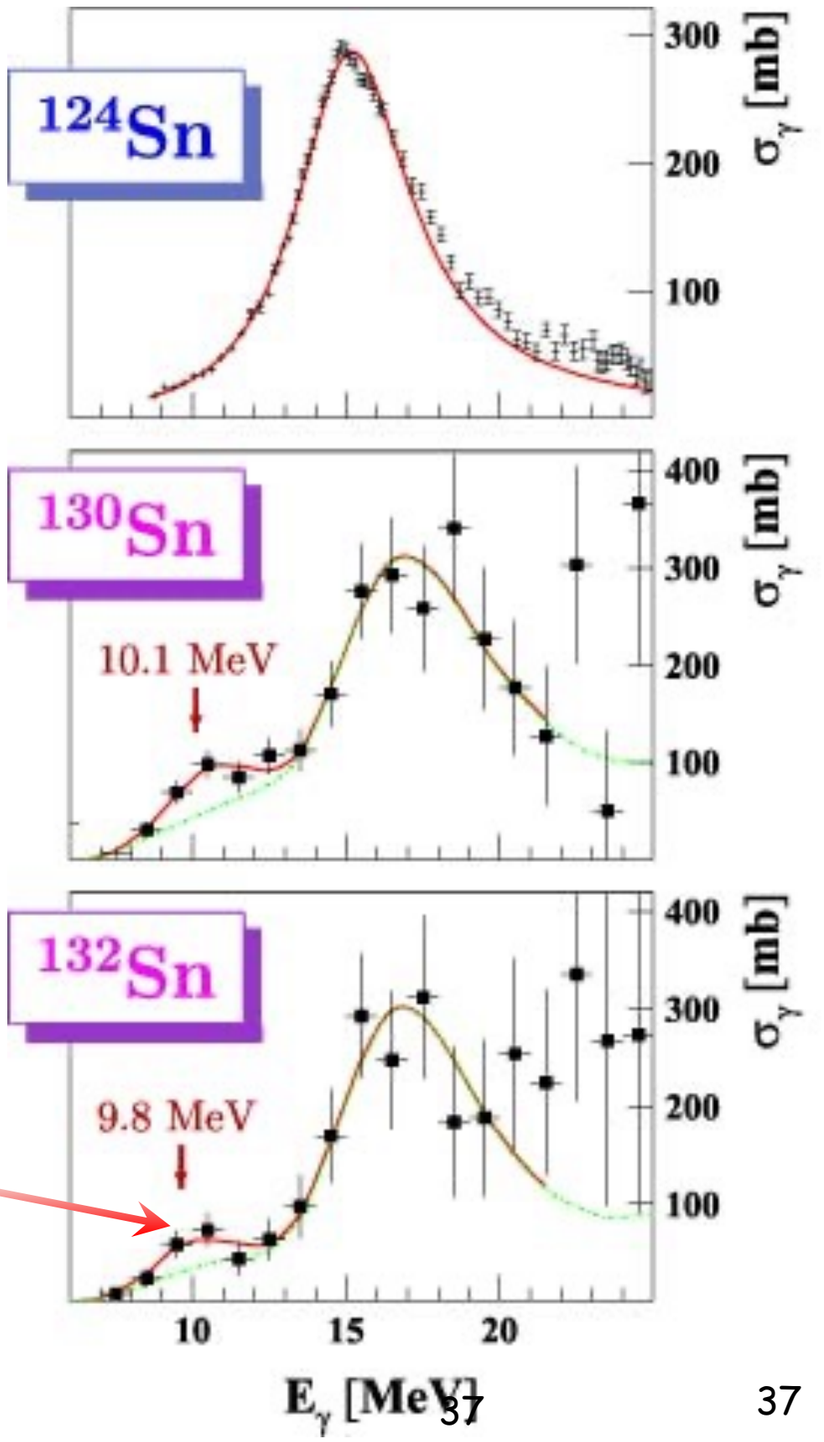


Pygmy dipole

Dipole polarizability



Adrich et al., PRL 95, 132501 (2005)



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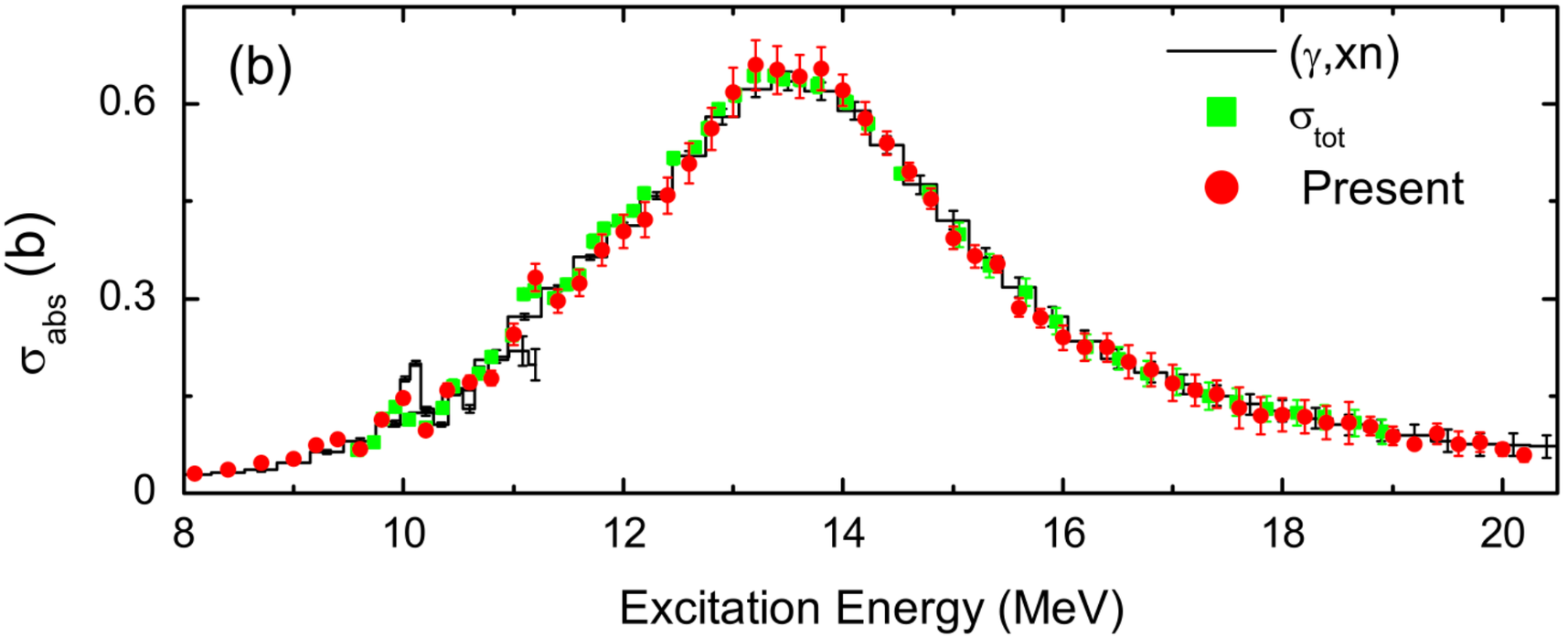
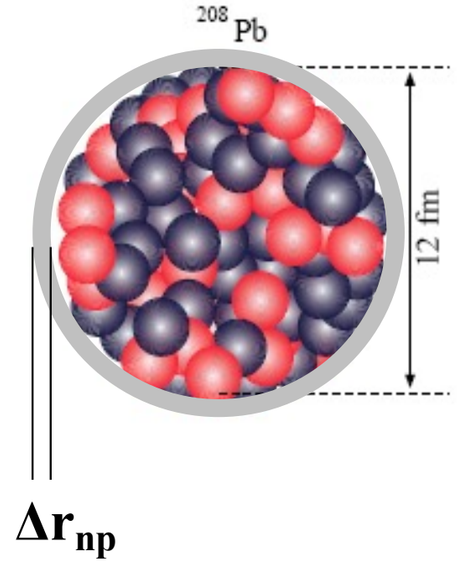
Dipole polarizability & neutron skin



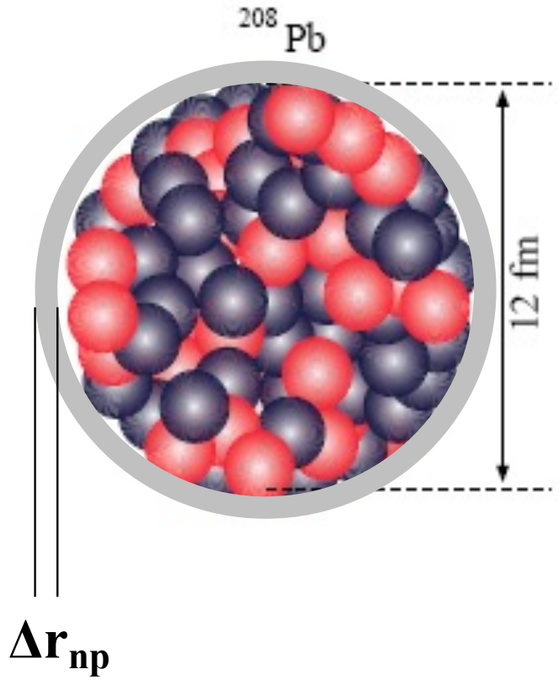


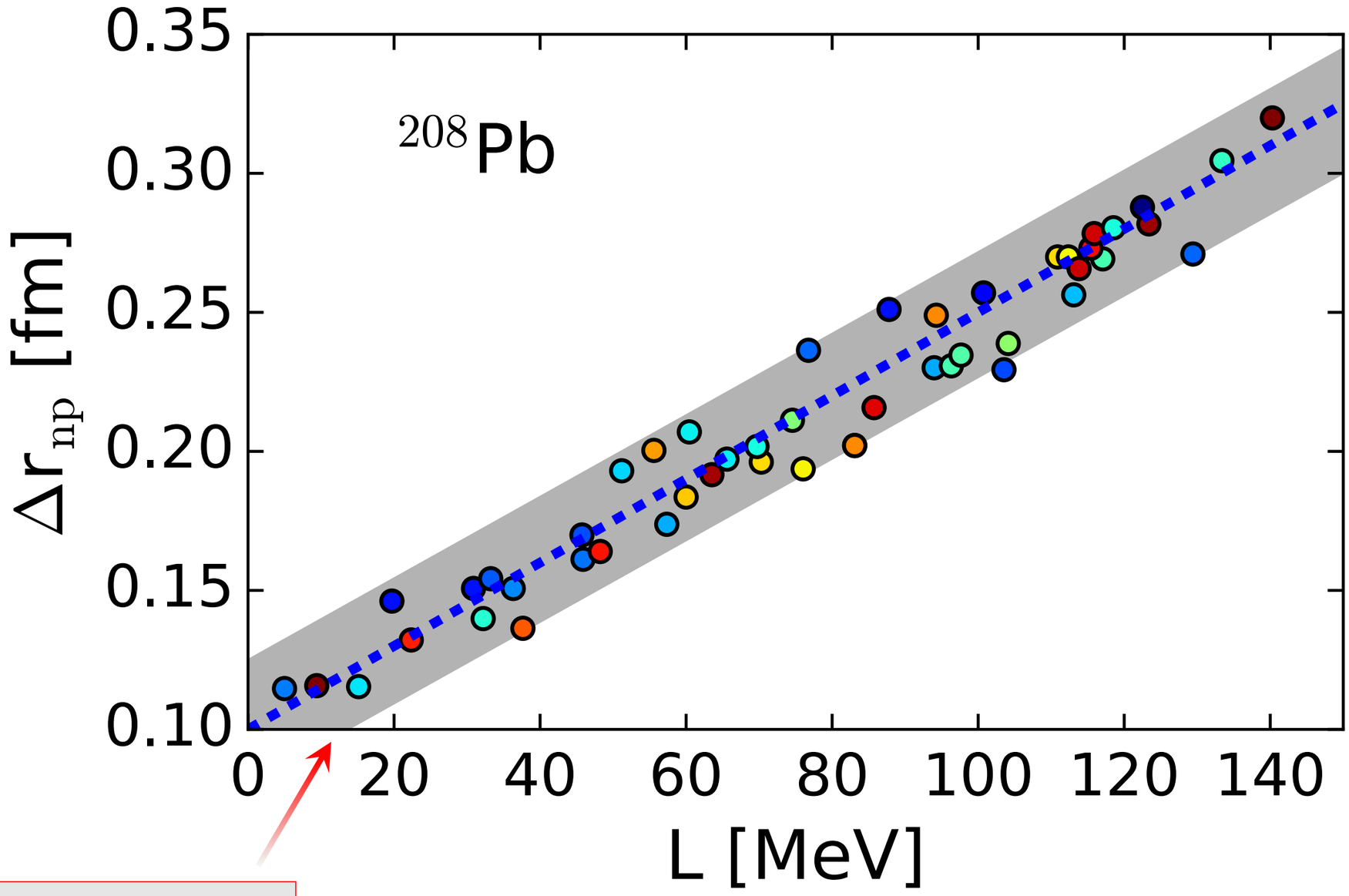
Tamii et al.,
PRL 107, 062502 (2011)

$$\alpha_D = \frac{\hbar c}{2\pi^2}\int_0^\infty \frac{\sigma_\gamma(E)}{E^2}dE$$

$$= \frac{8\pi}{9}\int \frac{B(E1,E_x)}{E_x}dE_x$$

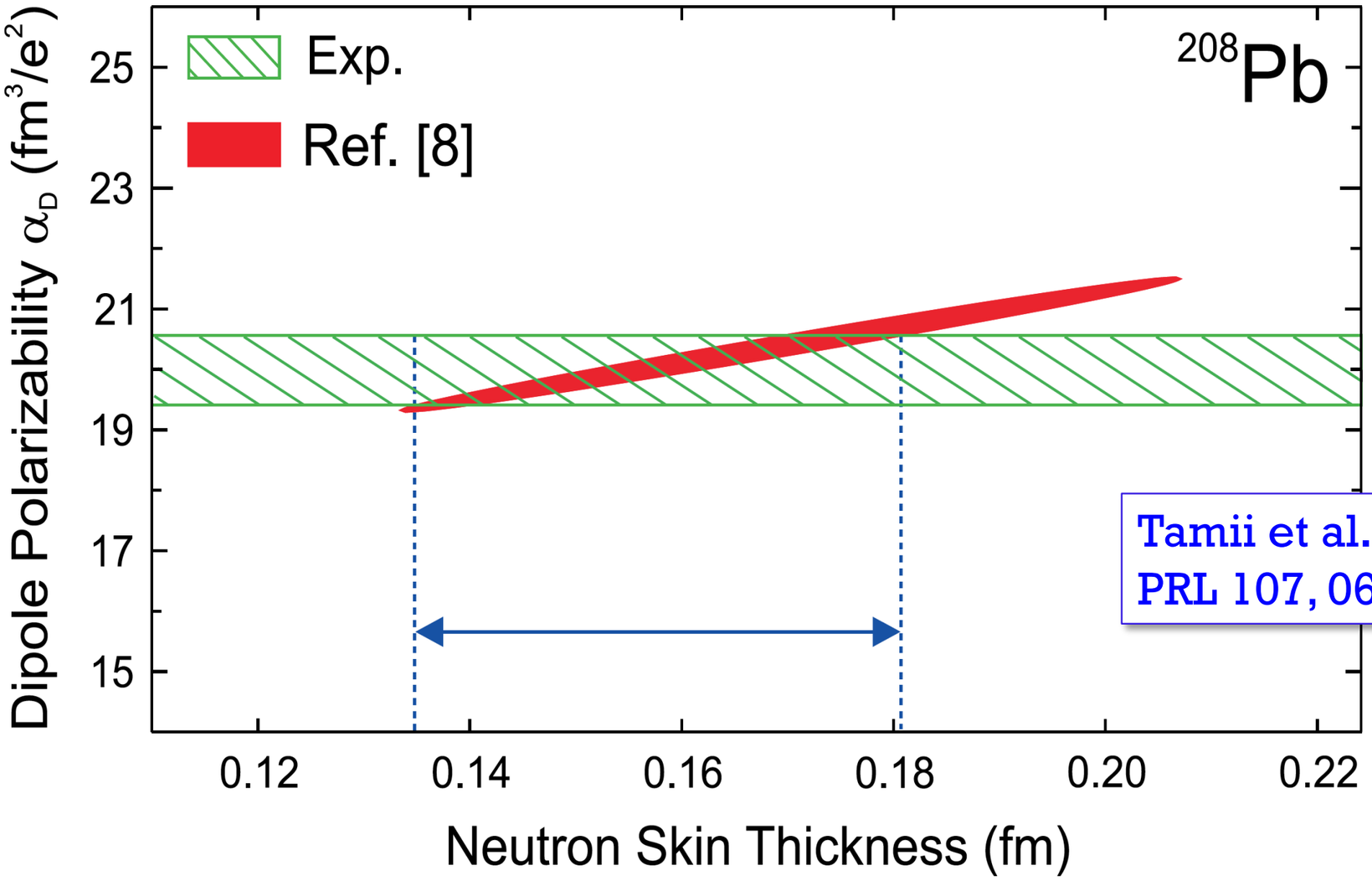
Correlation between symmetry energy & neutron skin





Numerous EDF

Dipole polarizability & neutron skin



Reinhard, Nazarewicz,
PRC 81, 051303(R) (2010)

Tamii et al.,
PRL 107, 062502 (2011)

EFT:

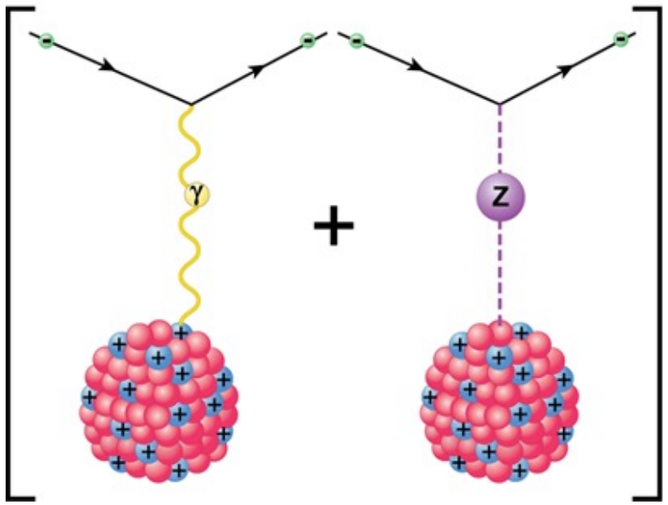
Hebeler et al.,
PRL 105, 161102 (2010)

$\Delta r_{np} \sim 0.17$ fm

Experiment:

$\Delta r_{np} \sim 0.156$ fm

n-skin from PV e- scattering

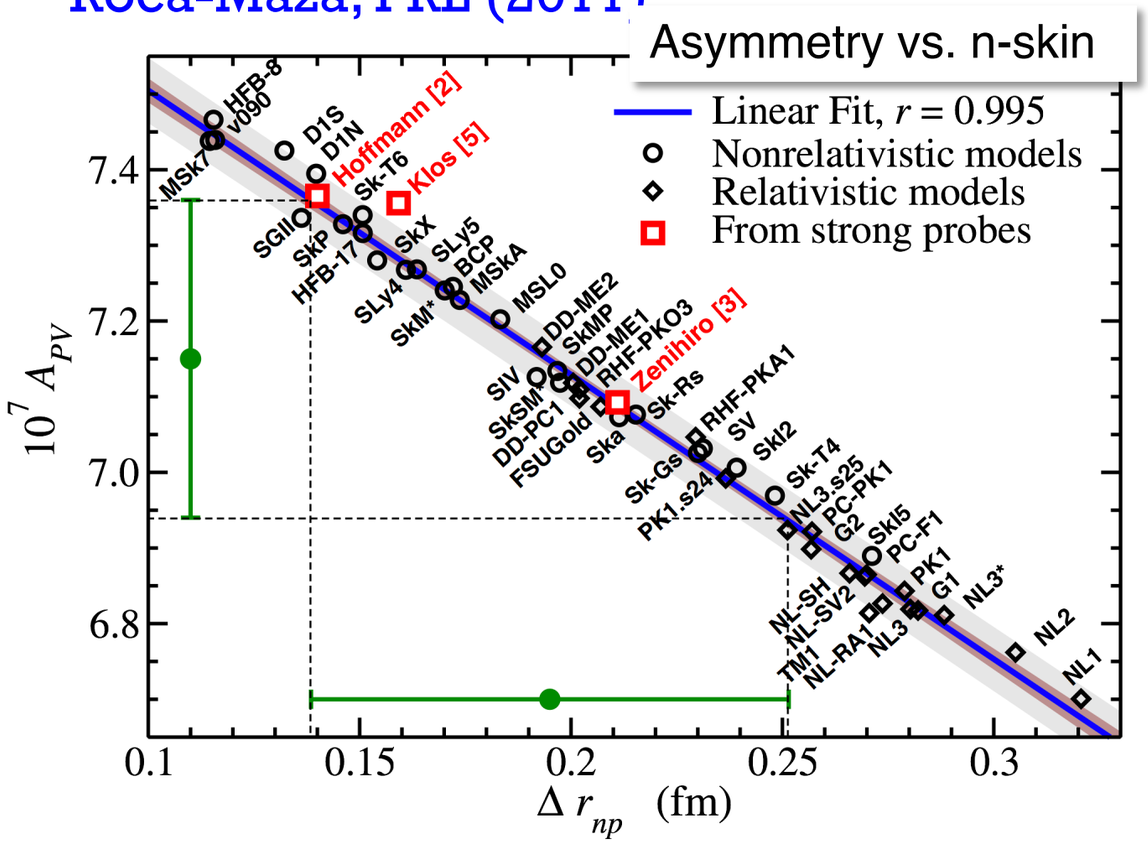


Abrahamyan et al,
PRL 108, 112502 (2012)

Howowitz et al,
PRC 85, 032501(R) (2012)

$$A_{PV} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L}$$

Roca-Maza, PRL (2011)



$$A_{PV}(Q^2) \sim \frac{G_F Q^2}{4\pi e^2 \sqrt{2}} \frac{F_w(Q)}{F_{ch}(Q)}$$

$$F_w = \frac{1}{Q_w} \int dr \frac{sin(Qr)}{Qr} \rho_w(r)$$

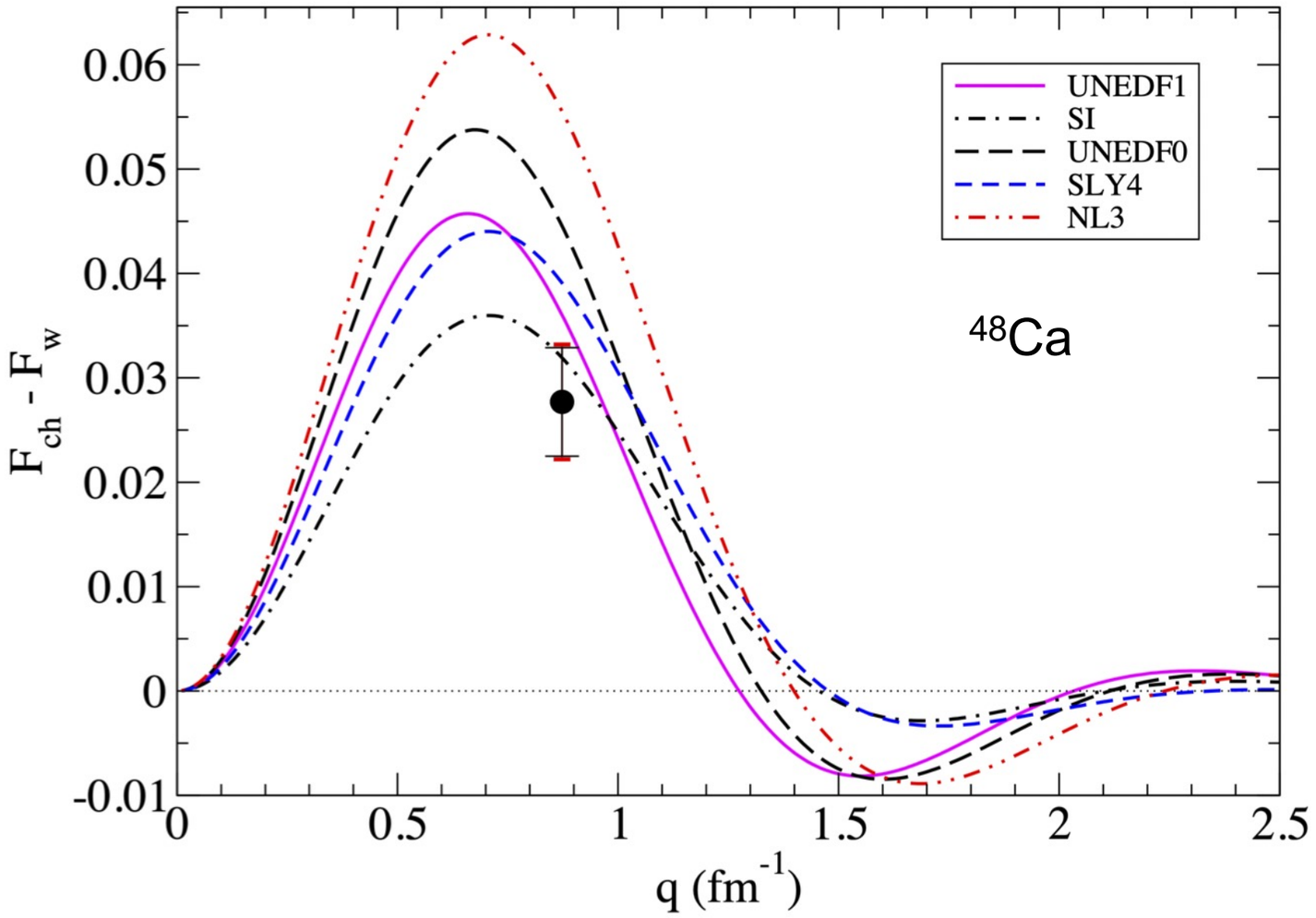
$$\rho_w(r) = q_p\, \rho_{ch}(r) + q_n \int d^3r' \left[G_E^p \rho_n + G_E^n \rho_p \right]$$

$$q_p = 0.0721, \qquad q_n = 0.0721,$$

n-skin from e⁻ PV scattering ^{48}Ca

$$F(Q^2) \sim 1 - \frac{1}{6} q^2 \langle r^2 \rangle$$

$$\langle r^2 \rangle \cong -6 \frac{dF(Q^2)}{dQ^2}$$

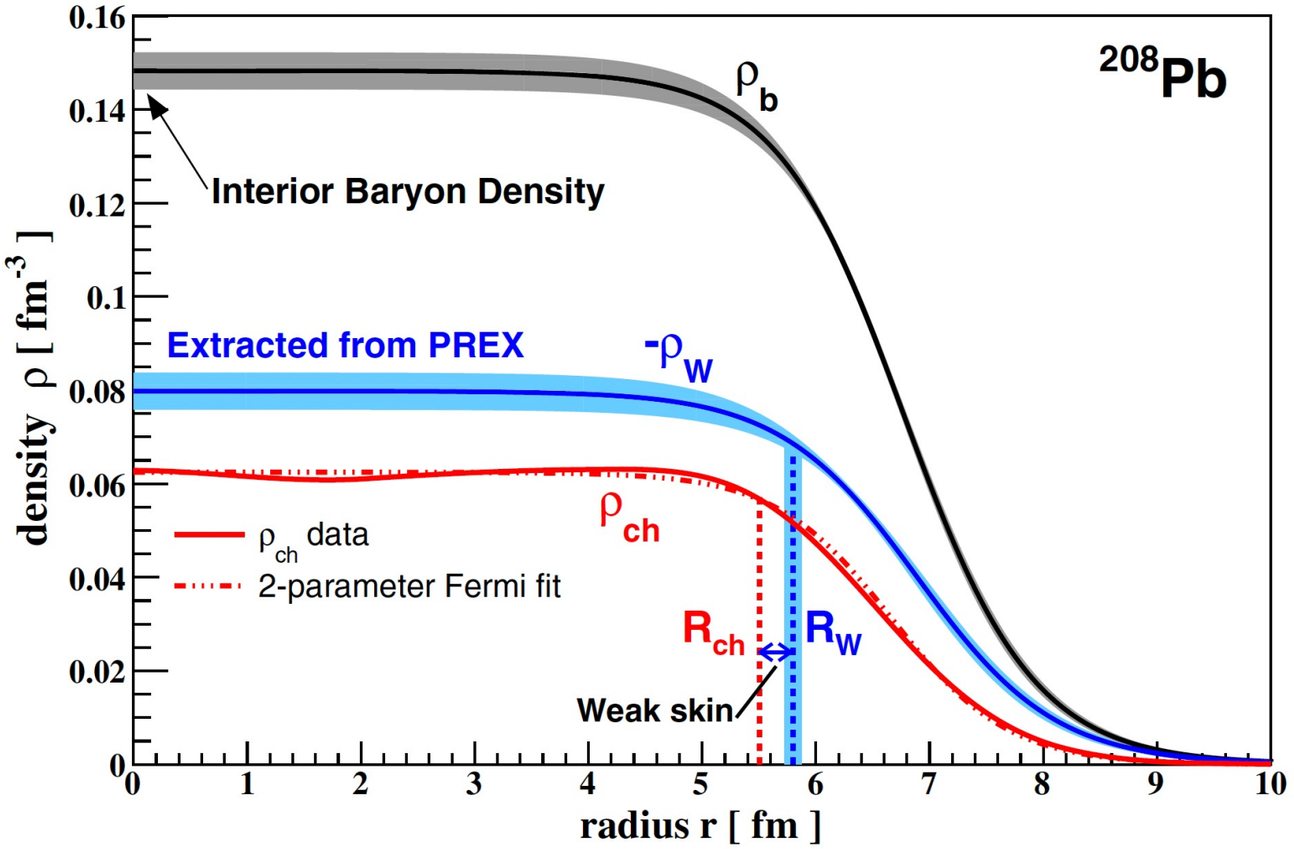


- PREX & CREX: measurement of parity violating asymmetry
- Determine n-skin and/or L by comparison to predictions from DFT

$(R_n - R_p)_{48Ca} = 0.121 \pm 0.025$ fm

Adhikari et al, PRL 129, 042501(2022)

n-skin from e⁻ scattering ^{208}Pb

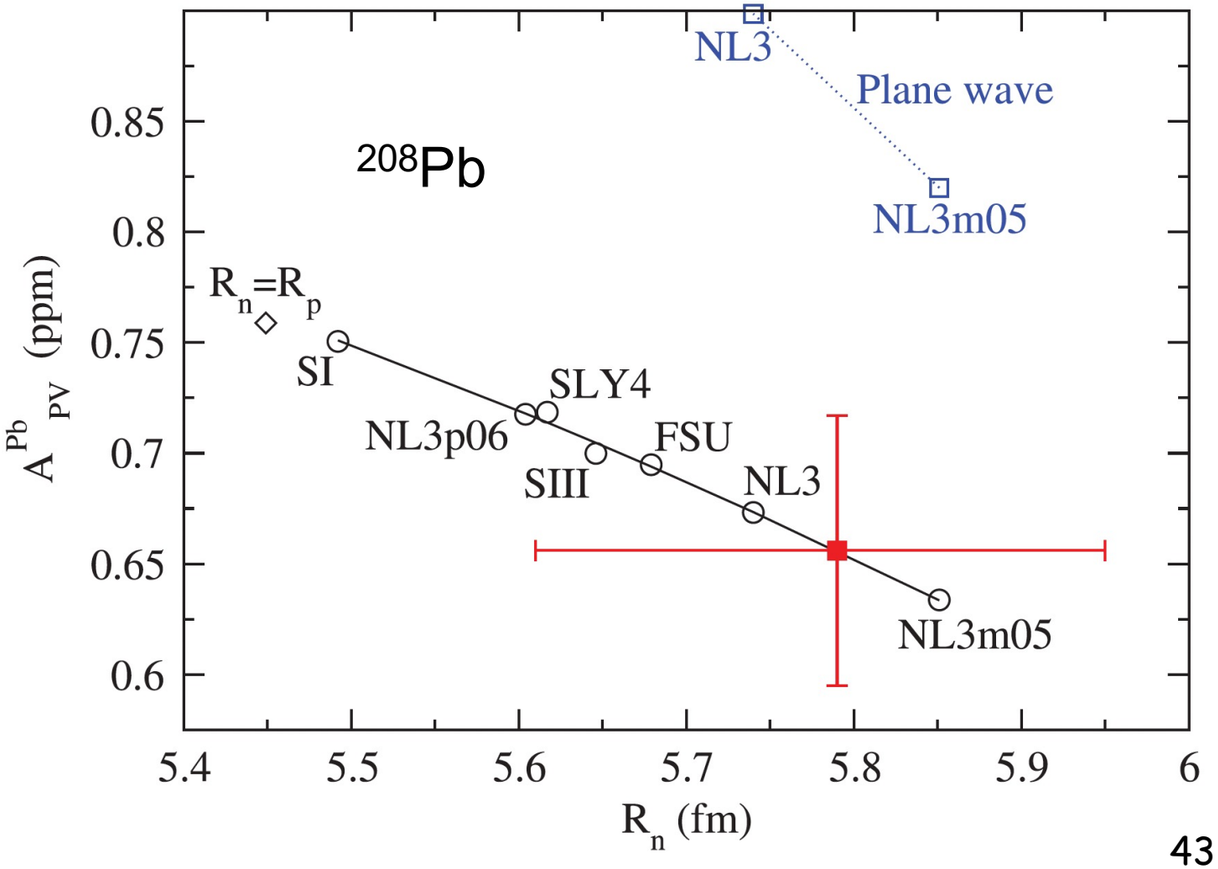


very large value!

PREX-I

$R_n - R_p = 0.33 \pm 0.17$ fm

Adhikari et al, PRL 126, 172502 (2021)



$\Delta r_{np} \sim 0.156$ fm

Dipole polarizability

Tamii et al., PRL 107, 062502 (2011)

Radiative capture, spectroscopic factors

Overlap function $I_{lj(\boldsymbol{r}_n)} = \langle \Psi_{A=B+n}(\boldsymbol{\xi}_B; \boldsymbol{r}_n) | \Psi_B(\boldsymbol{\xi}_B) \rangle$

$$\int d^3 r_n \, |I_{lj}(\boldsymbol{r}_n)|^2 = S(lj)$$

Spectroscopic factor

Asymptotic Normalization Coefficient (ANC)

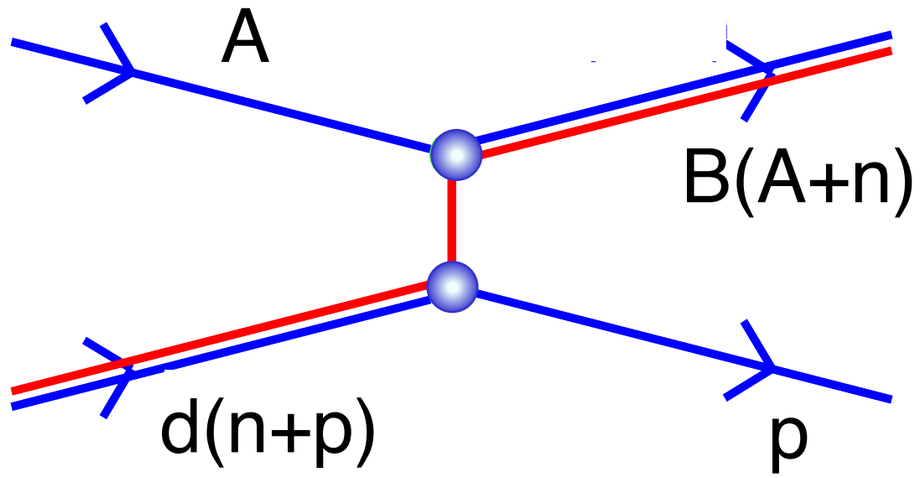
Large **r** $\quad I_{lj}(r) \to C_{lj} h_{ij}^{(1)}(i\kappa r)$

If $I_{lj(r)} = \sqrt{S(lj)} \varphi_{lj}^{s.p.}(r)$ and $\varphi_{lj}^{s.p.}(r) \to b_{lj} h_{ij}^{(1)}(i\kappa r)$

then $S(lj) = \dfrac{C_{lj}^2}{b_{lj}^2}$ $\qquad b_{lj}^2 = \text{s. p. ANC}$

ANC

Assume cross section for *A*(*d*,*p*)*B*



$$\sigma^{DW} = |M|^2 = \left|\langle \psi_f^{(-)} I_{An}^B | V | \phi_{pn} \psi_i^{(+)} \rangle\right|^2$$

With the single particle approximation

$$\sigma^{DW} = S\left|\langle \psi_f^{(-)} \phi_{An}(n_r lj) | V | \phi_{pn} \psi_i^{(+)} \rangle\right|^2$$

If the reaction is peripheral

$$I_{lj}(r) \rightarrow C_{lj} h_{ij}^{(1)}(i\kappa r)$$

$$\frac{d\sigma}{d\Omega} = \left(C^A_{Bpl_A j_A}\right)^2 \left(C^A_{Bpl_d j_d}\right)^2 \frac{\sigma^{DW}_{l_A j_A l_d j_d}}{b^2_{Bpl_A j_A} b^2_{Bpl_d j_d}}$$

Use of ANCs

- Find a peripheral transfer reaction
- Measure angular distribution
- Fit with a DWBA calculation
- Determine ANCs
- Use ANCs to calculate $\sigma_{astrophysics}$.

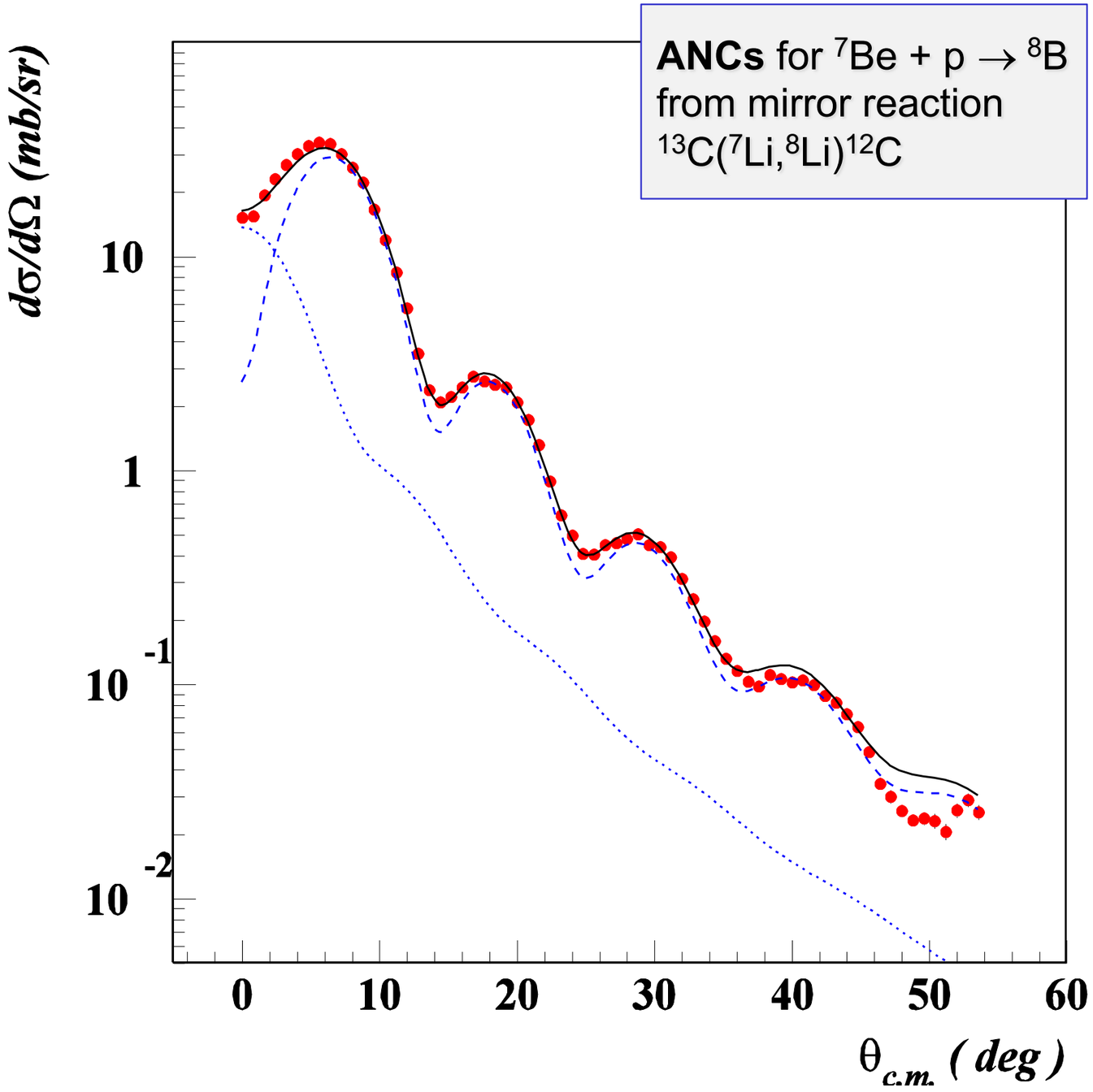
Example: radiative capture reactions

b + c → a + γ

$$\sigma_{E1} \propto |\langle \Psi_a | r Y_1 | \Psi_{b+c} \rangle|^2$$

Only tail of Ψ_a necessary to calculate σ_{E1}

Ψ_{b+c} is rather reliably calculated in a potential model

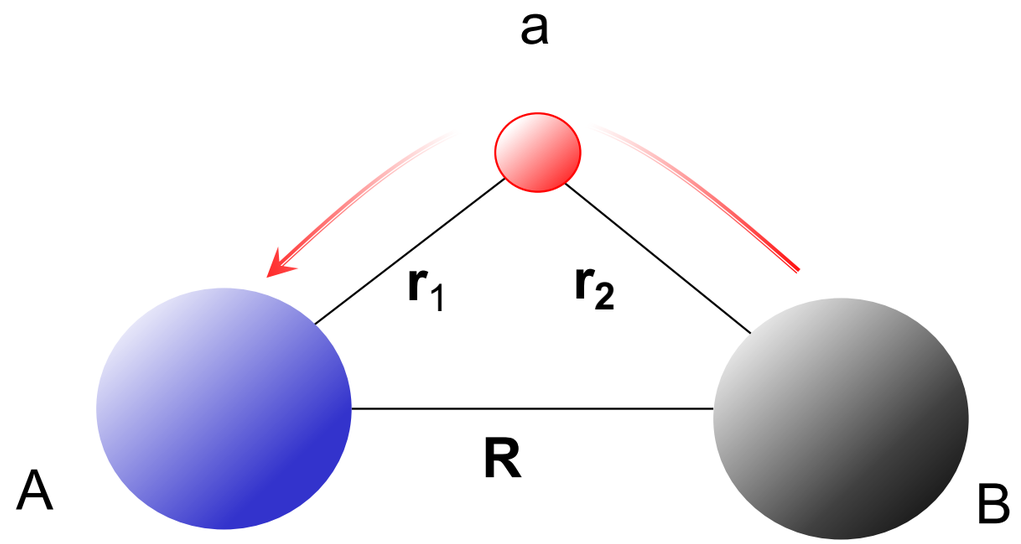


$S_{17}(0) = 17.6 \pm 1.7$ eV.b

Trache, PRC (2003)

Nucleon transfer, in general

S_x = spectroscopic factors
Q = momentum transfer
V_{1A} = transfer interaction



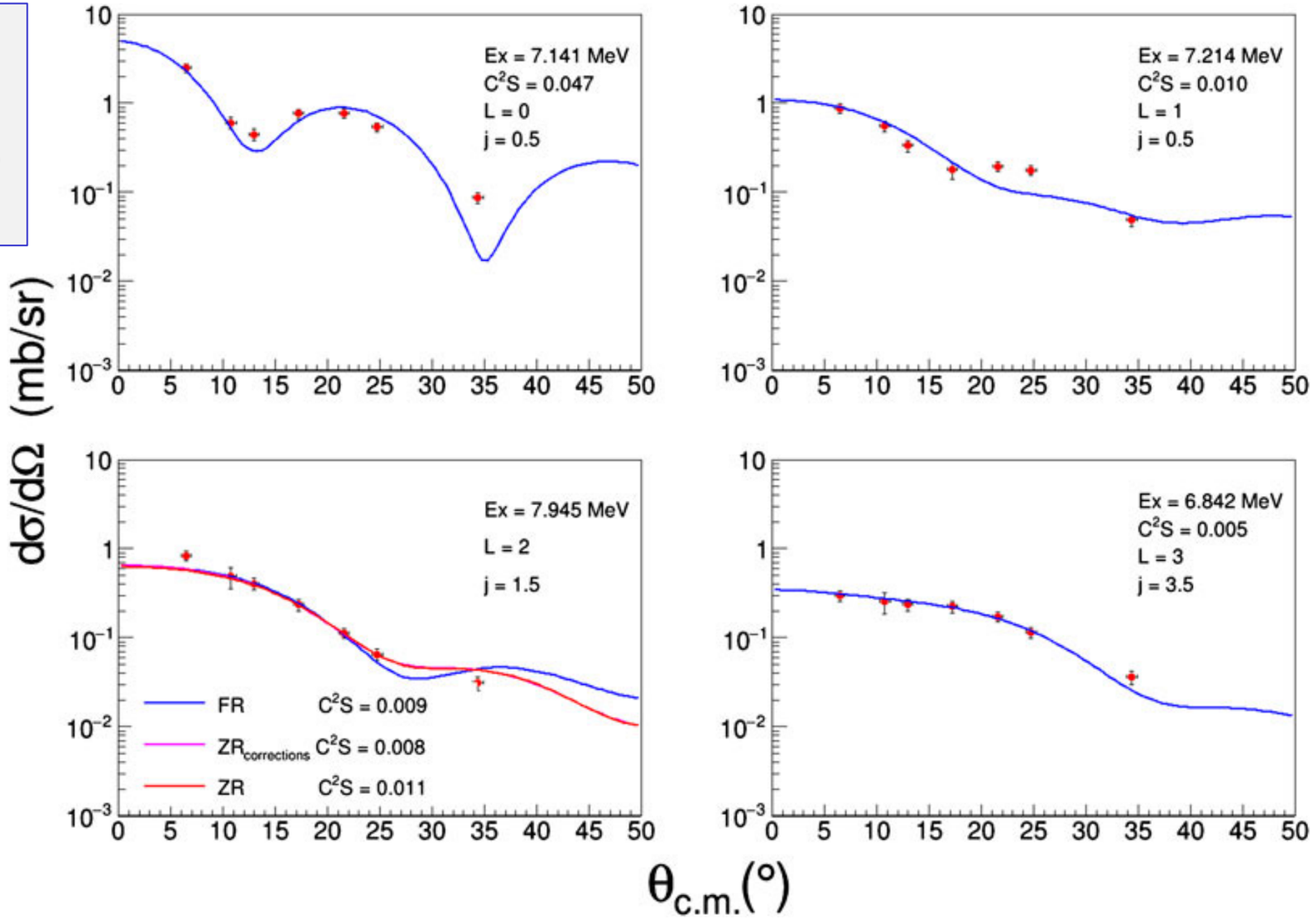
$$\left(\frac{d\sigma}{d\Omega}\right)_{PWBA} \sim S_{aA}S_{aB}\left|\int d^3R F_{PWBA}(\boldsymbol{R})\right|^2$$

$$F_{PWBA}(\boldsymbol{R}) \sim \int d^3r_1 e^{i\boldsymbol{Q}\cdot\boldsymbol{r}_1}\varphi_a^{(A)}(\boldsymbol{R}+\boldsymbol{r}_1)V_{1A}\,\varphi_a^{(B)}(\boldsymbol{r}_1)$$

$\chi_{aA}{}^{*}\chi_{aB}$ = distorted waves ⟶ DWBA

P *enhancement* & Mg *depletion* in globular clusters
Pollution by early generation stars
← importance of $^{30}Si(p,\gamma)^{31}P$

Low E resonances not accessible due to Coulomb barrier

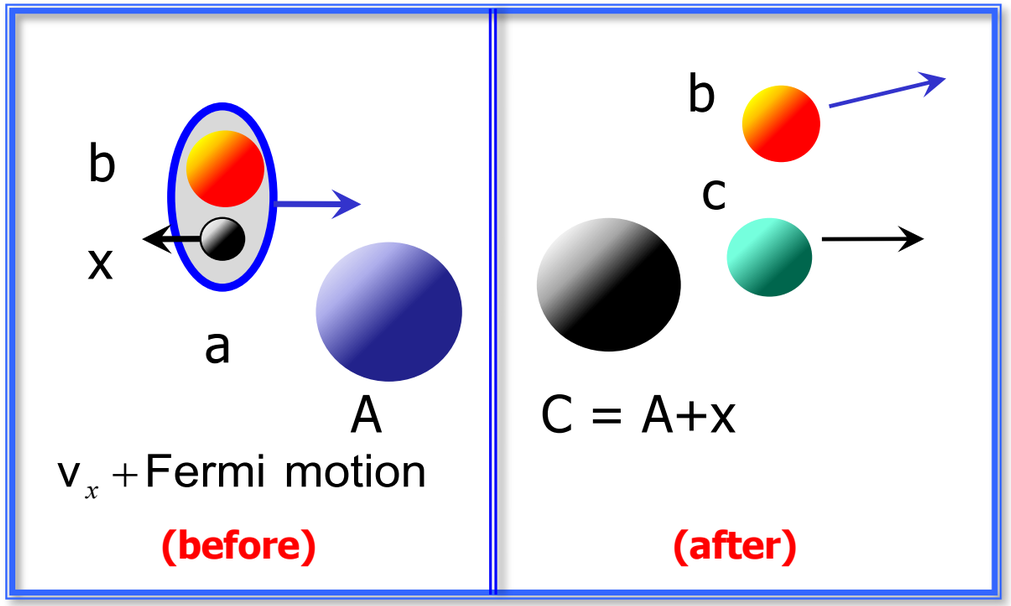


^{31}P states populated with $^{30}Si(^{3}He,d)^{31}P$

Hammache (2021)

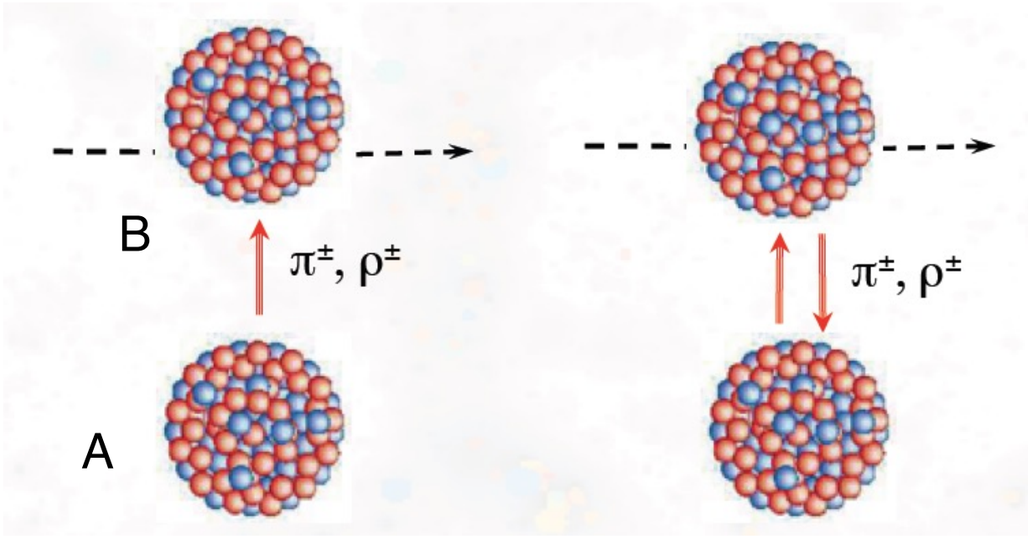
Trojan horse method

+ many other indirect methods



Applications to astrophysical A + x → C + c reactions

Charge exchange

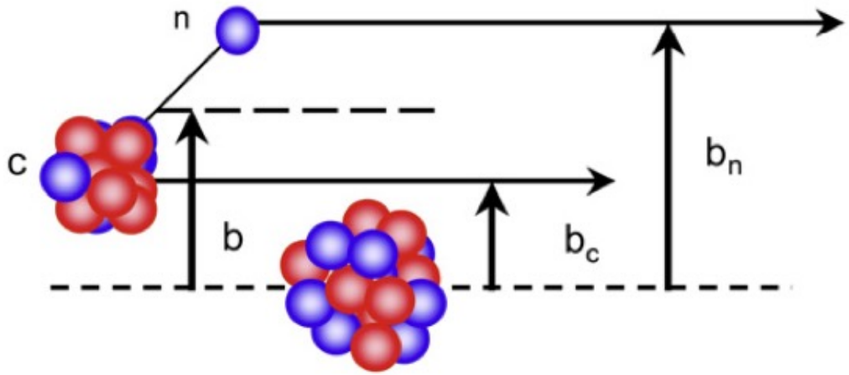


One step

Two step
(meson exchange)

Applications to electron capture and ν induced reactions in stars

(double-beta decay)
NUMEN



Knockout reactions

Distribution of single particle strength in nuclei

Theory for indirect methods for NA

- **Optical potentials:** (a) parameter quantification analysis, (b) microscopic theory
- **Off-shellness:** On-shell energy/momentum of extracted information not well established for most reactions. I.e., extracted NA cross sections not the same as with free particles
- **Relativistic corrections**, 100 MeV/nucleon ~ 10% nucleon mass: (a) kinematics easy, (b) dynamics difficult because of retardation, simultaneity. Problem worsens at large E_{lab}
- **Multinucleon collisions** not well assessed in knockout, (p,2p) or CE reactions
- **Factorization** into structure and reaction parts not consistent (e.g., different interactions, eigen and scattering states obtained with different Hamiltonians)
- **Medium effects**, nuclear polarizabilities, coupled channels, too simplified
- **No statistical theory** exists for nuclear reactions in the nucleus-nucleus continuum