Stellar evolution and standard solar model

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OVERVIEW:

- The Sun: what we know and what we do not know.
- Stellar structure equations and a stellar evolution code.
- The virial theorem: pre-main sequence stars.
- Nuclear reactions in main sequence stars (H burning).
- A standard solar model.
- Solar abundances and helioseismology constraints.
- Solar neutrinos.

http://pos.sissa.it/cgi-bin/reader/conf.cgi?confid=148

https://link.springer.com/article/10.1140/epjp/i2018-12237-1

PoS PROCEEDINGS OF SCIENCE

An introduction to Stellar Evolution

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Stellar evolution theory is among the greatest successes of modern astrophysics. It is a powerful tool to interpret many astronomical observations, such as the Colour-Magnitude diagrams of simple and complex stellar systems or the chemical composition of the stars. Stellar models are required to derive important parameters, like the age of various cosmic components or their distances. In addition, stellar models allow us to investigate the behaviour of matter and radiation in the extreme physical conditions taking place in stellar interiors.

In this lecture, I review the main ingredients needed to *cook* a stellar model. Few examples of the potential of stellar evolution in the interpretation of relevant astronomical observations are illustrated. A final section is devoted to core-He burning stellar models and the interplay between nuclear burning and convection that determines the evolution of many stars.

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On the mass of supernova progenitors

Review | Published: 26 September 2018

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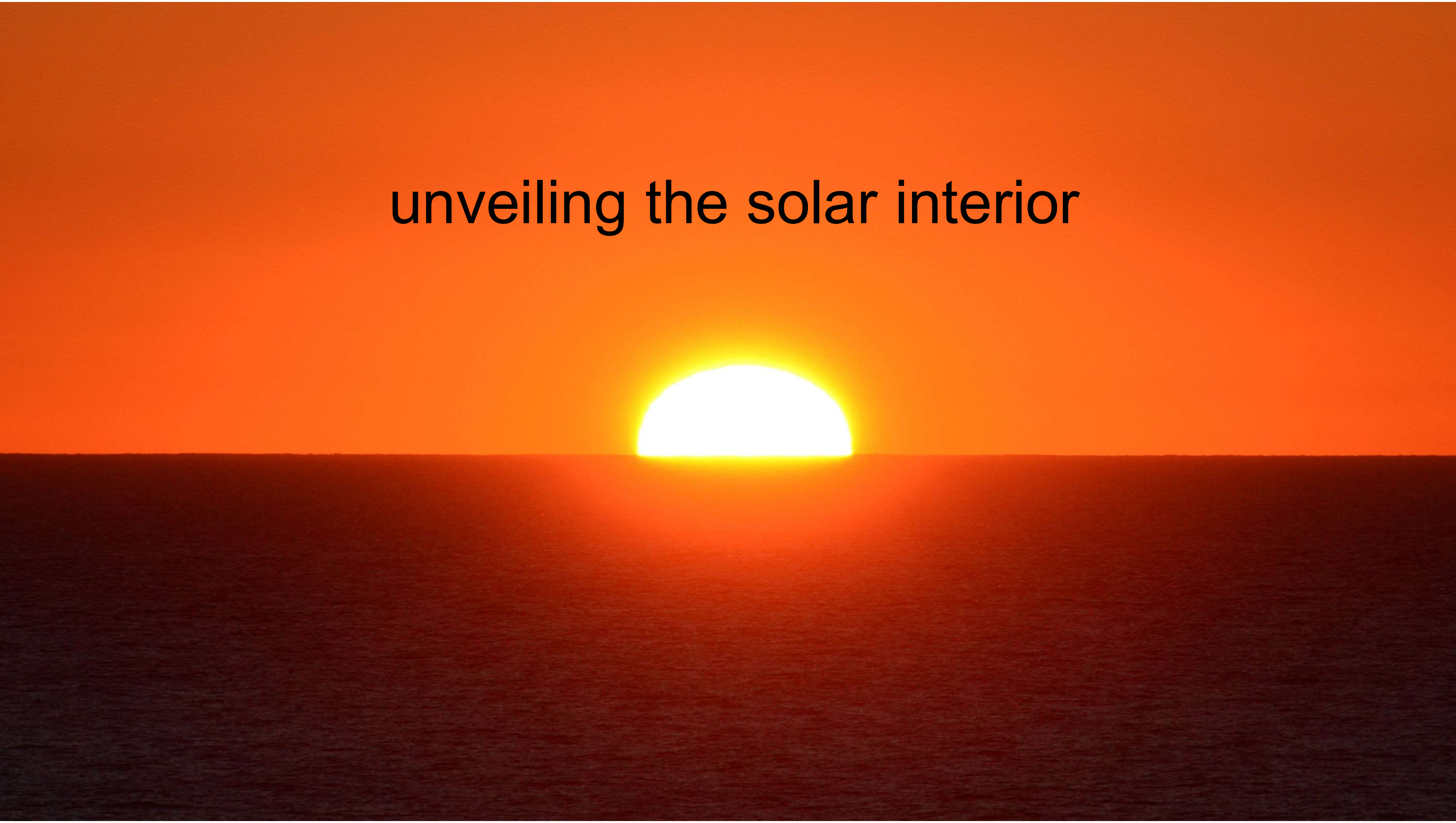

unveiling the solar interior

What we know

- **Age**, from radioactive isotopes.
- **Mass**, from planets orbits
- **Composition (relative to H)**, from spectroscopy/meteorites/solar wind…..
- **Luminosity**, from photometry
- **Radius**, from interferometry/photometry/spectroscopy

the interior (from helioseismology & neutrinos telescope)

- **Depth of the convective envelope**
- **Present-day (absolute) He abundance (in the convective envelope)**
- **Internal rotation velocity**
- **Sound speed profile** $c_S = \sqrt{\gamma \frac{P}{\rho}}$
- **Neutrinos fluxes (at earth)**

What we do not know

- Detailed internal structure (temperature/density/composition profiles)
- Past and future evolution
- Neutrinos luminosities

That are the outputs of a Standard Solar Model (SSM)

How to built a SSM

- We start from a rather cool homogeneous stellar structure (1 M⊙), in hydrostatic and thermal equilibrium. This is the first hydrostatic structure that forms after the collapse of the pre-solar nebula, when the matter becomes optically tick.
- The initial, homogeneous composition is unknown. We assume that the abundance ratios of the pre-solar nebula were those we get by combining spectroscopic analysis (present day atmospheric composition) and pristine meteorites abundances (original pre-solar composition).
- The model of 1 M⊙ is left to evolve up to the present age (4.58 Gyr). WE NEED E STELLAR EVOLUTION CODE.
- The initial helium mass fraction (Y_{ini}), the initial metallicity (Z_{ini}), and the mixing length (α) are FREE PARAMETERs. They are adjusted in order to reproduce the present day L, R and Z/X.

Stellar structure equations
and
stellar evolution code

(plots and numbers in the following have been obtained by means of the last version of the FuNS, full network stellar evolution code, see. eg,. Straniero et al. 2006 Nucl.Phys A)

A stellar model may be as complex as you want: rotation, magnetic fields, dynamical and secular instabilities of various nature, ……..

However, most of our knowledge concerning stellar evolution is based on relatively simple models: Self-gravitating, spherical symmetric stars in thermal and hydrostatic equilibrium.

STELLAR STRUCTURE EQUATIONS:

$$\frac{dP}{dm_r} = -\frac{Gm_r}{4\pi r^4}$$ ***Hydrostatic* equilibrium**

$$\frac{dr}{dm_r} = \frac{1}{4\pi r^2 \rho}$$ **Mass conservation**

$$\frac{dL_r}{dm_r} = \varepsilon_{nucl} - \varepsilon_\nu + \varepsilon_{grav}$$ **Energy conservation**

$$\frac{dT}{dm_r} = -\nabla \frac{Gm_r}{4\pi r^4}\frac{T}{P}$$ **Energy transport:** $\nabla = \left(\frac{dlogT}{dlogP}\right)$

$$\frac{dY_i}{dt} = \left(\frac{dX_i}{dt}\right)_{nucl} + \left(\frac{dX_i}{dt}\right)_{mix}, \quad i = 1, \ldots.., n$$

Chemical evolution=Nuclear reactions + turbulent convection

The overall problem: 4+N differential equations, in 4+N dependent variables:

- r (radius),
- L_r (luminosity),
- P (pressure),
- T (temperature),
- Y_i (chemical composition: abundances)

all depending on the lagrangian mass coordinate:

- $m_r = \int_0^r 4\pi\rho r^2 dr$ (mass within r)

and

- t (time)

***Appropriate boundary conditions needed.**

Numerical solutions: spatial & temporal resolution:

1 stellar model contains about 1000 mesh points: $0 \leq m_r \leq M$

1 evolutionary track contains up to 10^6 stellar models: $0 \leq t \leq$ stellar-lifetime

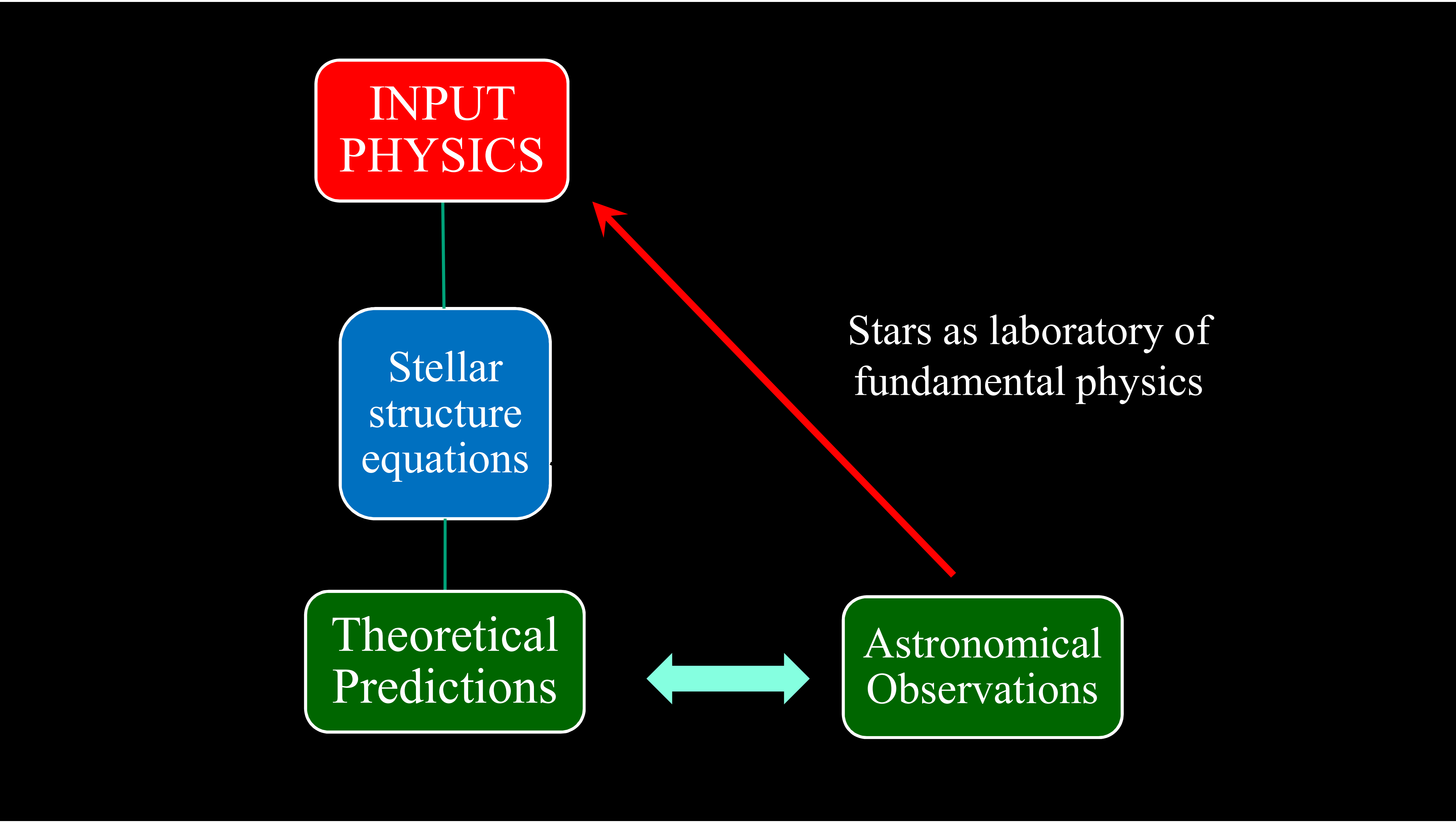
4+n differential equations, in 4+n dependent variables:

- r (radius),
- L (luminosity),
- P (pressure),
- T (temperature),
- X_i i=1….n (abundancies)

all function of the lagrangian mass coordinate:

- $m_r = \int_0^r 4\pi\rho r^2 dr$

*Appropriate boundary conditions needed


INPUT PHYSICS
Stellar structure equations
Theoretical Predictions
Astronomical Observations
Stars as laboratory of fundamental physics

a STAR evolves because:

1. Energy loss, as due to photons and neutrinos emission.

2. Chemical composition changes, as due to the concurrent action of nuclear burning and mixing.

Nevertheless, thermal and hydrostatic equilibrium are maintained, until the lost energy can be replaced by:

1. Conversion of gravitational potential energy

and/or

2. Nuclear power.

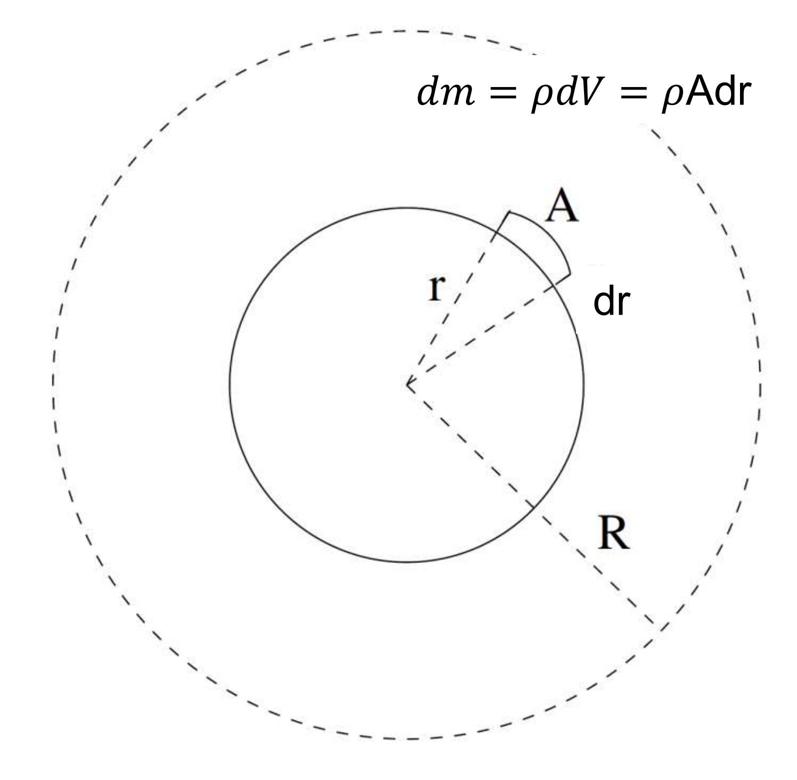
hydrostatic equilibrium equation

Let consider a small gas element, whose mass is dm, located at distance r from the stellar centre, dr its width and A its surface. Then, the pressure difference between the internal and the external boundaries is:

$$dP = P(r+dr) - P(r)$$

which corresponds to a force:

$$F_P = AdP$$

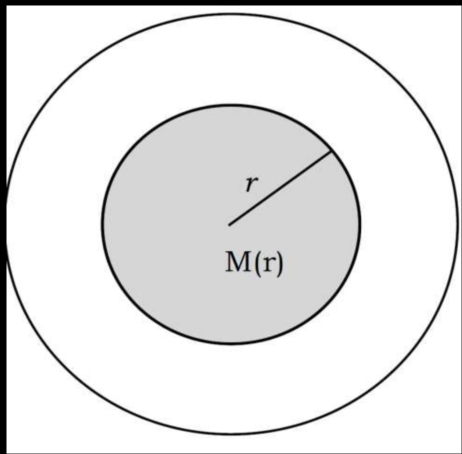


On the other hand, gravity contrasts this pressure force. From the Newton law:

$$F_g = -\frac{Gm(r)dm}{r^2} = -\frac{Gm(r)\rho(r)Adr}{r^2}$$

Being *dm=ρAdr* , the mass of the gas element, Adr its volume and *m(r)* the mass within the radius *r, i.e.:*

$$m(r) = m_r = \int_0^r 4\pi r'^2 \rho dr'$$



At the equilibrium, the total force acting on the gas element should be 0:

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm_r \rho_r}{r^2} \qquad \text{-1-}$$

In addition, the mass within a shell of width dr is:

Mass continuity equation

$$dm = 4\pi r^2 \rho(r) dr$$

$$\frac{dm}{dr} = 4\pi r^2 \rho(r) \qquad \text{-2-}$$

Virial Theorem

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \quad \Rightarrow \quad \int_0^R 4\pi r^3 dP = -\int_0^R \frac{Gm\rho}{r^2} 4\pi r^3 dr$$

$dV = 4\pi r^2 dr$

$dm = 4\pi r^2 \rho dr$

$$3\int_0^R PdV - \left[4\pi r^3 P\right]_0^R = -\int_0^R \frac{Gm}{r} dm = -\Omega_g$$

For a non-relativistic gas

$$P = \frac{2}{3}E$$

$$U = \int EdV$$

$$2U + \Omega_g = 0$$

More in general, the relation between pressure and energy density is:

$$P = (\gamma - 1)\, E$$

Where γ is the specific heat ratio $\gamma = c_P/c_V$

non-rel. $\rightarrow$ *relativistic*

$$\gamma = \frac{5}{3} \rightarrow \gamma = \frac{4}{3}$$

$$3(\gamma - 1)U + \Omega_g = 0$$

$$dm = \rho dV = 4\pi r^2 \rho dr$$

Internal energy:

$$U = \int E dV = \int_0^M \frac{1}{\gamma - 1} \frac{P}{\rho} dm$$

Gravitational energy:

$$\Omega_g = -\int_0^M \frac{Gm}{r} dm$$

also $\Delta\Omega_g = \Omega_g(\infty) - \Omega_g(R) = -\Omega_g(R)$

Some consequence of the Virial Theorem

The star is bound if if the total energy W<0:

$$W = E_i + \Omega_g = -\frac{\Omega_g}{3(\gamma - 1)} + \Omega_g = \frac{(3\gamma - 4)}{(3\gamma - 3)}\Omega_g$$

γ>4/3, W<0 → bound,

γ<4/3, W>0 → unbound

the evolution of a massive star ends, when a density above a few 10^9 g/cm^3 develops in its Fe-core. At such a high density, the fermi energy of the electrons is $\epsilon_F \gg mc^2$, and $\gamma = \frac{4}{3}$. This is a marginal stability condition. Then, electron captures start, causing the drop of the pressure and, in turn, the core collapse.

Note: $\Omega_g < 0$, hence W<0 imp;ies: $\frac{(3\gamma - 4)}{(3\gamma - 3)} > 0$

A proto-star is too cool for nuclear reactions. As photons are emitted from the stellar surface, the star contracts extracting energy from its own gravitational potential pool:

$$L = -\frac{dW}{dt} = -\frac{(3\gamma - 4)}{(3\gamma - 3)}\frac{d\Omega_g}{dt}$$

Since γ=5/3 ⇨

$$L = -\frac{1}{2}\frac{d\Omega_g}{dt}$$

In practice, only the 50% of the released gravitational energy is spent to replace the energy loss, while the rest goes into internal energy.

Kelvin-Helmholtz time-scale

$$\left|\Omega_g\right| = G\int_0^M \frac{m\,dm}{r} = q\frac{GM^2}{R}$$

$$q = \int_0^1 \frac{m}{r}\frac{R}{M}\,d\frac{m}{M} \approx 1$$

q is a “form factor” order of ~1 for a proto-star.

What wold be the lifetime of a star, If there are no other energy sources, except for the gravitational potential pool?

The Kelvin-Helmhotz time-scale for a star whose luminosity is *L* , is:

$$\tau_{\mathrm{KH}} = \frac{G\mathcal{M}^2}{R\mathcal{L}}$$

For the Sun: $\tau_{KH} \sim 10^8$ yr

Since $L \propto M^{3.5} \rightarrow \tau_{KH} \propto M^{-1.5}$

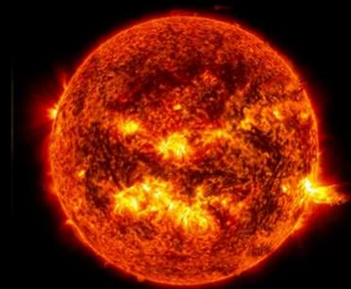
- Star formation begins with the collapse of a cold molecular cloud (Jeans instability).
- When the density increases enough, the matter becomes optically thick, the thermodynamic equilibrium is reached, the collapse ceases and gives way to hydrostatic equilibrium.
- The star continues to lose energy, but more slowly (KH time-scale).
- Meanwhile, the core heats up, according to the virial theorem.
- Eventually, when the temperature is high enough, **nuclear reaction starts**:

Predeictions vs observation: an example

Consider the energy balance:

$$\frac{dL}{dr} = (\epsilon_N + \epsilon_g - \epsilon_\nu) 4\pi r^2 \rho \quad \Rightarrow \quad L = \int_0^R \frac{dL}{dr} dr$$

To be compared to:



The «epsilons» are the energy production rates (erg/gr/s):

$\epsilon_N \rightarrow$ nuclear

$\epsilon_g = -T\frac{dS}{dt} \rightarrow$ gravity

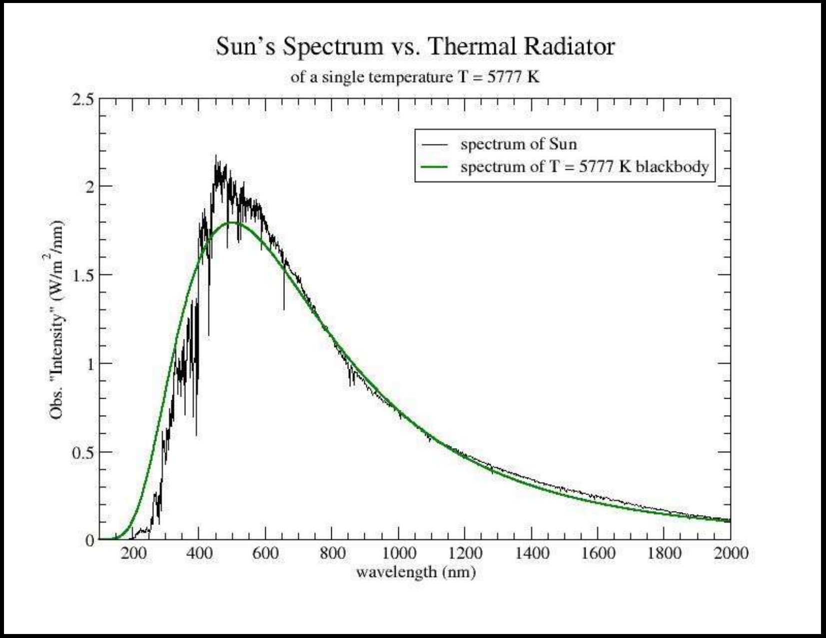
$\epsilon_\nu \rightarrow$ thermal neutrinos

All depend on T, ρ and composition

Effective temperature, solar spectrum and HR diagram

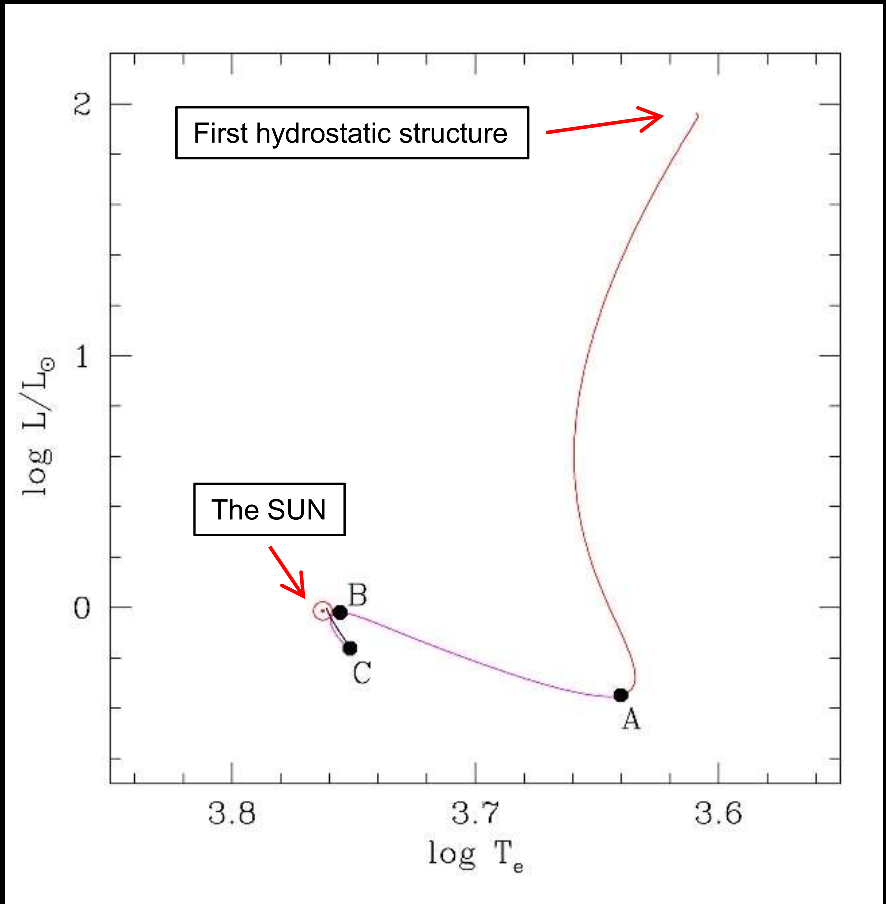
T_e is the temperature of the «equivalent» blackbody:

$$L = 4\pi R^2 \sigma T_e^4$$



- Below the photosphere, the Sun behave as an almost perfect blackbody (T=5770 K).
- Then, the outcoming radiation is partially absorbed and re-emitted when passing through the photosphere.
- Absorption lines reveal the chemical composition of the photosphere.

Evolution of the Sun in the HR diagram



A. 10 Myr: onset of the deuterium burning

B. 28 Myr: ^3He equilibrium

C. 42 Myr: onset of the H burning

$p+p \rightarrow d+e^+ +\nu$
$d+p \rightarrow {}^3He+\gamma$

${}^3He+{}^3He \rightarrow {}^4He+2p$

${}^3He+{}^4He \rightarrow {}^7Be+\gamma$

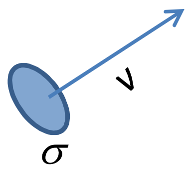
${}^7Be+e^- \rightarrow {}^7Li+\nu$
${}^7Li+p \rightarrow {}^4He+{}^4He$

${}^7Be+p \rightarrow {}^8B+\gamma$
${}^8B \rightarrow {}^8Be+e^+ +\nu$
${}^8Be \rightarrow {}^4He+{}^4He$

4 protons → ^4He + 25 MeV (γ) + 2 ν

Nuclear Reaction Rate

Monocromatic rate (n. of reactions/time/volume):



$$reaction \quad rate = n_{jk}\sigma \mathrm{v} = \frac{n_j n_k}{1+\delta_{jk}}\sigma v$$

Wher n_{jk} is the jk pair density (δ_{jk}=1 in case of identical particles) and σv is the volume crossed during 1 s by a particle whose cross section and velocity are σ and ***v***, respectively.

Total rate:

$$v = \left(\frac{2E}{\mu}\right)^{1/2}$$

$$R_{jk} = \frac{1}{1+\delta_{jk}} n_j n_k \int_0^\infty \sigma(E)\left(\frac{2E}{\mu}\right)^{\frac{1}{2}} f(E)dE$$

***f(E)*=energy distribution function**

$$<\sigma v>_{jk} = \left(\frac{8}{\pi\mu}\right)^{1/2}\left(\frac{1}{KT}\right)^{3/2}\int_0^\infty \sigma(E) E e^{-\frac{E}{KT}} dE$$

Maxwell-Boltzman

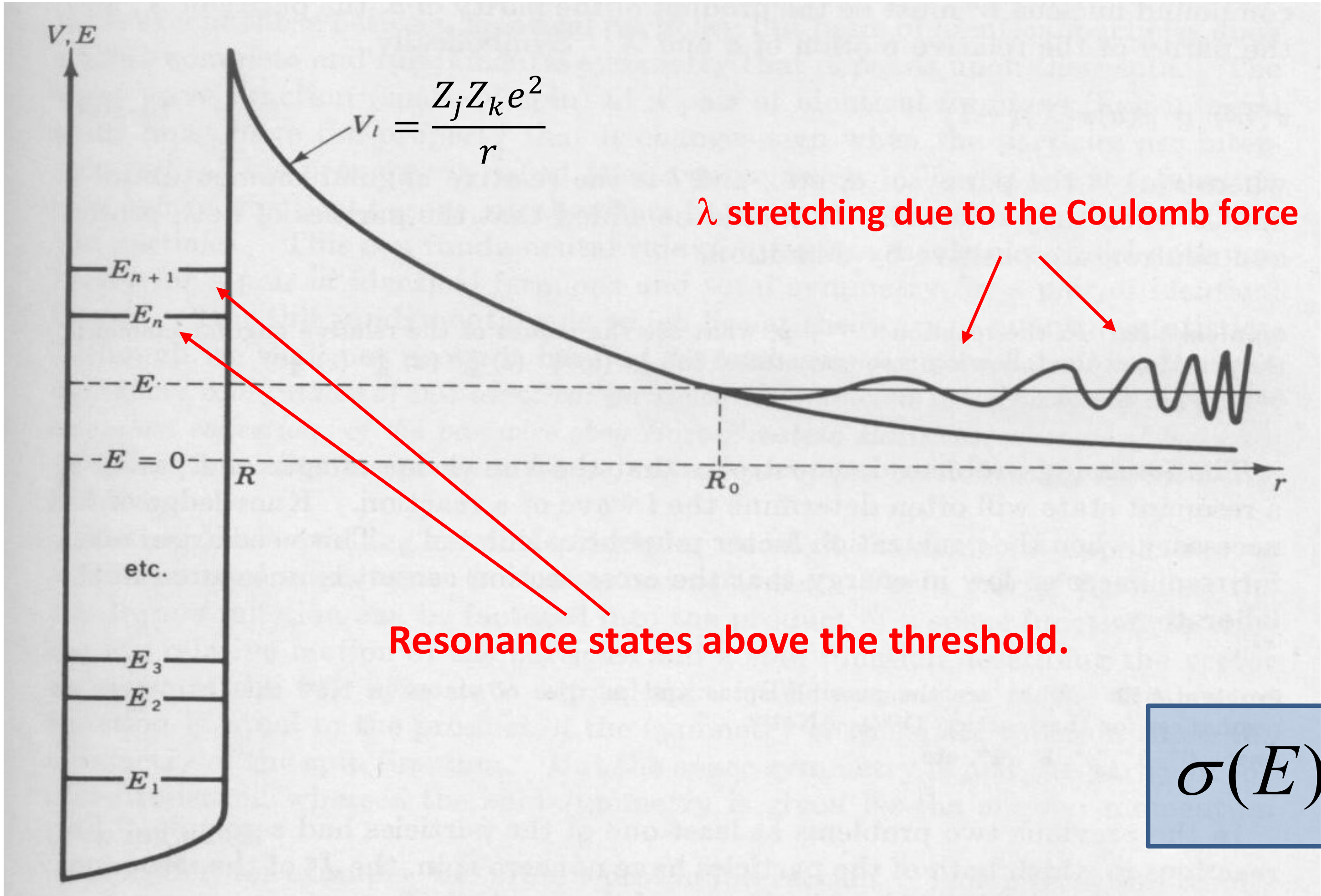
$$n_j = \frac{\rho \mathrm{N}_A X_j}{A_j}$$

X_j, mass fraction; ρ, mass density

$$R_{jk} = \frac{1}{1+\delta_{jk}} \frac{X_j X_k}{A_j A_k} \rho^2 \mathrm{N}_A{}^2 <\sigma v>_{jk}$$

Reaction Rate = n. of reaction/time/volume

CROSS SECTION



$$\sigma(E) = P_{Cou} P_{nuc}$$

The penetrability of the Coulomb barrier:

- The Coulomb barrier is too high, but quantum mechanics allows its penetrability.

The tunnel probability is (George Gamow 1928):

$$P_{Coul} = \exp\left(-\frac{2\pi Z_j Z_k e^2}{\hbar v}\right) = \exp\left(-\frac{\sqrt{2}\pi Z_j Z_k e^2}{\hbar}\left(\frac{\mu}{E}\right)^{1/2}\right)$$

$$P_{Coul} = \exp\left(-\frac{b}{\sqrt{E}}\right)$$

- For non-resonant reactions, $P_{nuc}(E)$ is a slowly decreasing function of E:

$$P_{nuc}(E) \approx \pi \lambda^2 \propto \left(\frac{1}{p}\right)^2 \propto \frac{1}{E} \qquad \lambda = \frac{h}{p}$$

- So, introducing the ***astrophysical factor S(E)***

$$P_{nuc}(E) = \frac{S(E)}{E}$$

For non-resonant reactions, S(E) is a slowly varying function of E.

- The cross section is the product of the **tunnel probability** and a **nuclear probability**:

$$\sigma(E) = \frac{1}{E} S(E) \exp\left(-\frac{b}{E^{1/2}}\right)$$

$$b = \frac{\sqrt{2}\pi Z_j Z_k e^2 \mu^{1/2}}{\hbar} = 0.99 Z_j Z_k \mu^{1/2} \ \ \mathrm{MeV}^{1/2}$$

So providing $X(E)$, whatever it may be, is not rapidly varying with energy

$$\sigma(E) \propto \frac{e^{-2\pi\eta}}{E}$$

which motivates the definition of an "S-factor"

$$S = \sigma(E)\, E\, e^{2\pi\eta} \qquad \eta = 0.1575\, Z_I Z_j \sqrt{\hat{A}/E}$$

$$\hat{A} = \frac{A_I A_j}{A_I + A_j}$$

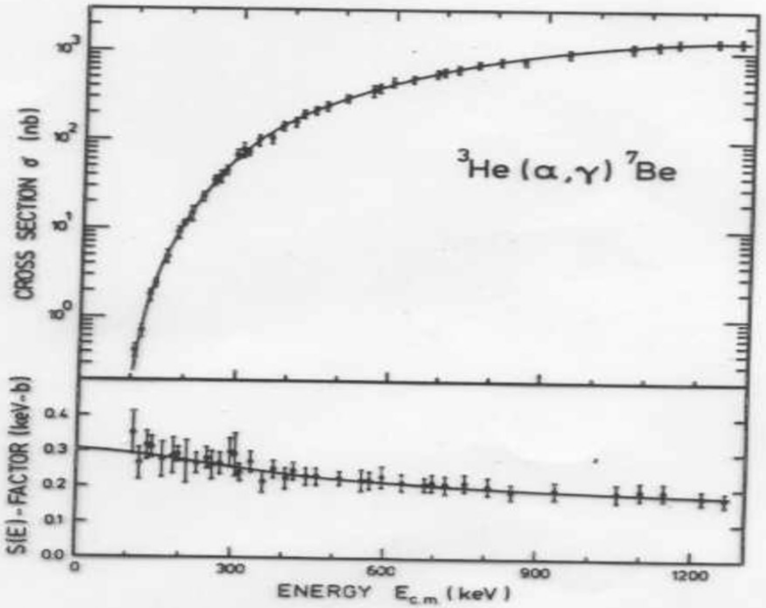
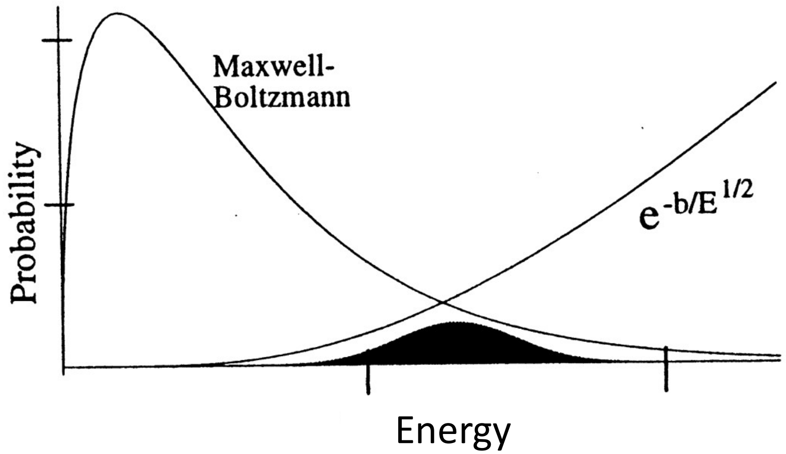


FIGURE 4.4. Energy dependence of the cross section $\sigma(E)$ and the factor $S(E)$ for the $^3He(\alpha,\gamma)^7Be$ reaction (Krä82). The line through the data points represents a theoretical description of the cross section in terms of the direct-capture model. This theory is used to extrapolate the data to zero energy.

The Gamow's peak

- After substitutions:

$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp\left[-\frac{E}{kT} - \frac{b}{E^{1/2}}\right] dE$$



The maximum probability occurs at:

$$\frac{d}{dE}\exp\left(-\frac{E}{KT}-\frac{b}{E^{1/2}}\right)=-\exp\left(-\frac{E}{KT}-\frac{b}{E^{1/2}}\right)*\left(\frac{1}{KT}-\frac{b}{2}E^{-\frac{3}{2}}\right)=0$$

$$E_o=\left(\frac{bKT}{2}\right)^{\frac{2}{3}}$$

GAMOW's Peak

Gamow's peak energy in different nucleosynthesis environments:

T = 1 GK (BBN)	
p+d	100 KeV

T = 16 MK (Sun)	
p+p	5 KeV
3He+3He	20 KeV
3He+4He	21 KeV
14N+p	25 KeV
T = 70 MK (RGB)	
14N+p	65 KeV
T = 200 MK (He-burning)	
12C+α	300 KeV
T = 500 MK (C-burning)	
12C+12C	1.5 MeV

Resonance contribution to σ (Breit-Wigner):

$$A + p1 \rightarrow C^* \begin{cases} A + p1 \\ B + p2 \end{cases}$$

$$\sigma(E) = \pi \lambda\!\!\!^{-2} \quad \omega \quad \frac{\Gamma_1 \Gamma_2}{(E - E_r)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

Usual geometric factor

$$= \frac{0.656}{\hat{A}} \frac{1}{E} \text{barn}$$

Spin factor:

$$\omega = \frac{2J_r + 1}{(2J_1 + 1)(2J_2 + 1)}$$

$\propto \Gamma_1$ Partial width for decay of resonance by emission of particle 1
= Rate for formation of Compund nucleus state

$\propto \Gamma_2$ Partial width for decay of resonance by emission of particle 2
= Rate for decay of Compund nucleus into the right exit channel

Γ = Total width is in the denominator as a large total width reduces the maximum probabilities (on resonance) for decay into specific channels.

Eugene Wigner
(1902-95)
Nobel Prize 1963

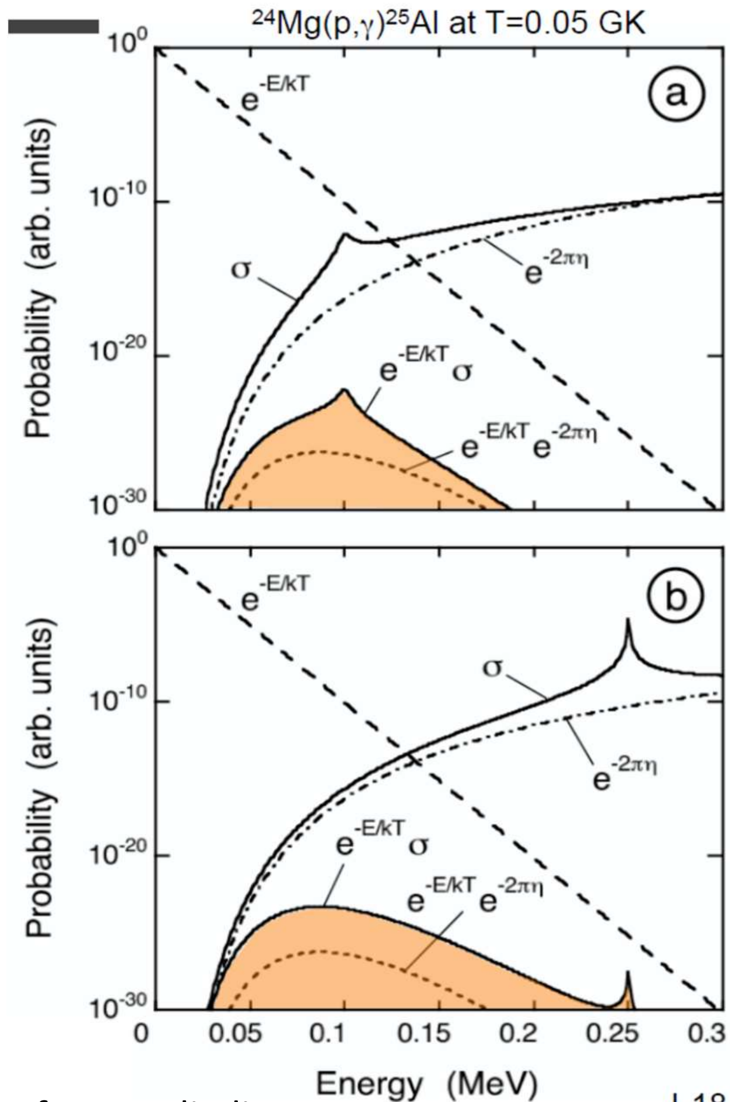
An example of “resonce contribuition” to the cross section:

$$<\sigma v>_{jk} = \left(\frac{8}{\pi\mu}\right)^{1/2}\left(\frac{1}{KT}\right)^{3/2}\int_0^\infty \sigma(E)Ee^{-\frac{E}{KT}}dE$$

- “narrow resonance”, E_r within the Gamow peal.

+

- Low-energy tail of a higher-energy “broad resonance”



Figures from C. Iliadis

A couple of questions:

- How this scheme change in case of neutron captures?

- What about the plasma screening (not only electrons)?

Rate x unit mass

$$A + p1 \rightarrow C^{*} \rightarrow A + p1$$

$$\Re_{jk} = \frac{R_{jk}}{\rho} = \frac{X_j X_k}{A_j A_k \left(1 + \delta_{jk}\right)} N_a^2 \rho \ < \sigma v >_{jk}$$

Rate of energy production ⟹

$$\varepsilon_{Nuclear} = Q_{jK} \Re_{jk}$$

where

$$Q_{jK} = \Delta mc^2 - E_\nu$$

Δm=mass of interacting particles – mass of products

neutrino-energy loss (if any)

Energy transport by radiation and/or convective instabilities

$$\frac{dP}{dm_r} = -\frac{Gm_r}{4\pi r^4}$$

$$\frac{dr}{dm_r} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{dL_r}{dm_r} = \varepsilon_{nucl} - \varepsilon_{\nu} + \varepsilon_{grav}$$

$$\frac{dT}{dm_r} = -\nabla \frac{Gm_r}{4\pi r^4} \frac{T}{P}$$

$$\frac{dY_i}{dt} = \left(\frac{dX_i}{dt}\right)_{nucl} + \left(\frac{dX_i}{dt}\right)_{mix}, \quad i = 1, ..., n$$

Energy transport

$$\nabla = \frac{d \ln T}{d \ln P}$$

opacity

$$\nabla_{rad} = \frac{3\kappa r^2 P}{4acGm_r T^4} F = \frac{3\kappa r^2 P}{4acGm_r T^4} \frac{L_r}{4\pi r^2}$$

$$\nabla_{ad} = \frac{\gamma - 1}{\gamma \chi_T} \qquad \gamma = \frac{C_P}{C_V} \qquad \chi_T = \left(\frac{d \ln P}{d \ln T} \right)_{\rho, Yi}$$

Convective inStability

$$\nabla_{rad} > \nabla_{ad} + \frac{\phi}{\delta}\nabla_{\mu}$$ convection switch on

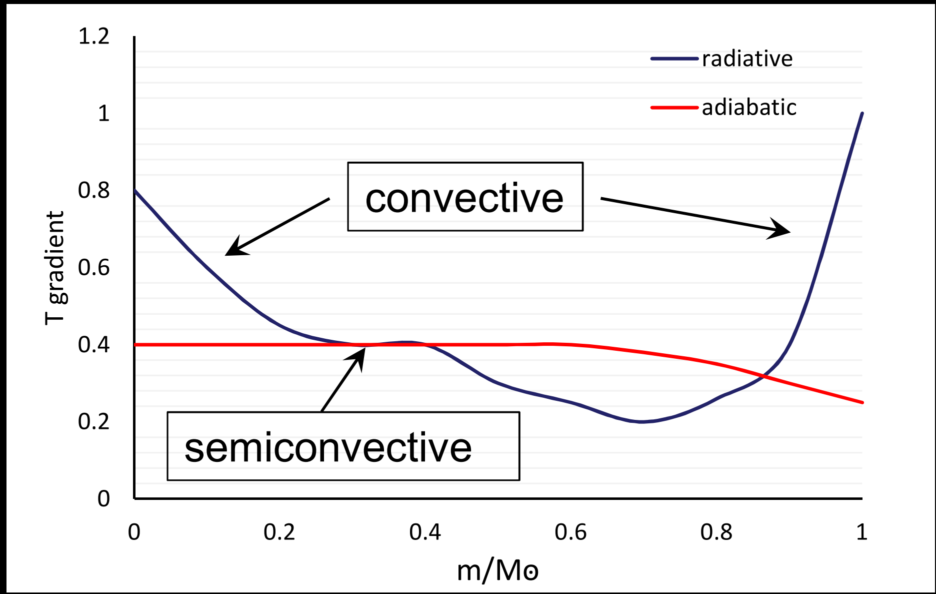
where:

$$\phi = \left(\frac{\partial \ln \rho}{\partial \mu}\right)_{P,T} = 1 \text{ for a perfet gas}$$

$$\delta = \left(\frac{\partial \ln \rho}{\partial T}\right)_{P,\mu} = 1 \text{ for a perfet gas}$$

$$\nabla_{\mu} = \frac{d \ln \mu}{d \ln P} = 0 \text{ for homogeneous layers}$$

Schwarzschild/Ledoux criteria for convective instability



$$\text{conv. dominated } \nabla_{ad} \le \nabla \le \nabla_{rad} \text{ rad. dominated}$$

A standard solar model

How to built a SSM

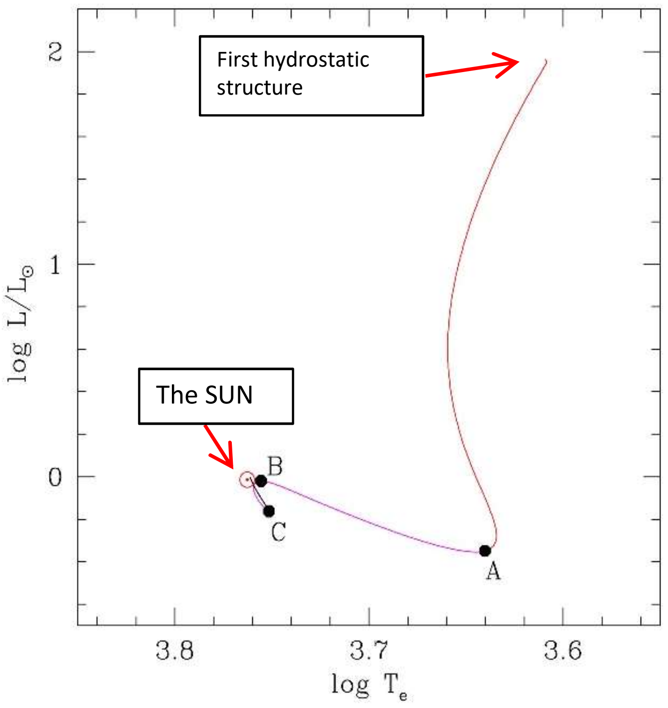
- We start from a rather cool homogeneous stellar structure (1 M⊙), in hydrostatic and thermal equilibrium. This is the first hydrostatic structure that forms after the collapse of the pre-solar nebula, when the matter becomes optically tick.
- Combining spectroscopic and meteoritic abundances*, for each element, except H and He, we have:
  a. its abundance relative to the total metallicity: $\frac{X_j}{Z}$
  b. the isotopic fractions (IUPAC)
- Unknown inputs of the model are:
  1. Initial He (Y)
  2. Initial metallicity (Z)
  3. The mixing length parameter (α). It determines how much adiabatic is the convective energy transport:

$$\text{conv. dominated} \quad \nabla_{ad} \le \nabla \le \nabla_{rad} \quad \text{rad. dominated}$$

$\alpha\uparrow$ $\qquad\qquad$ $\alpha\downarrow$

How to built a SSM

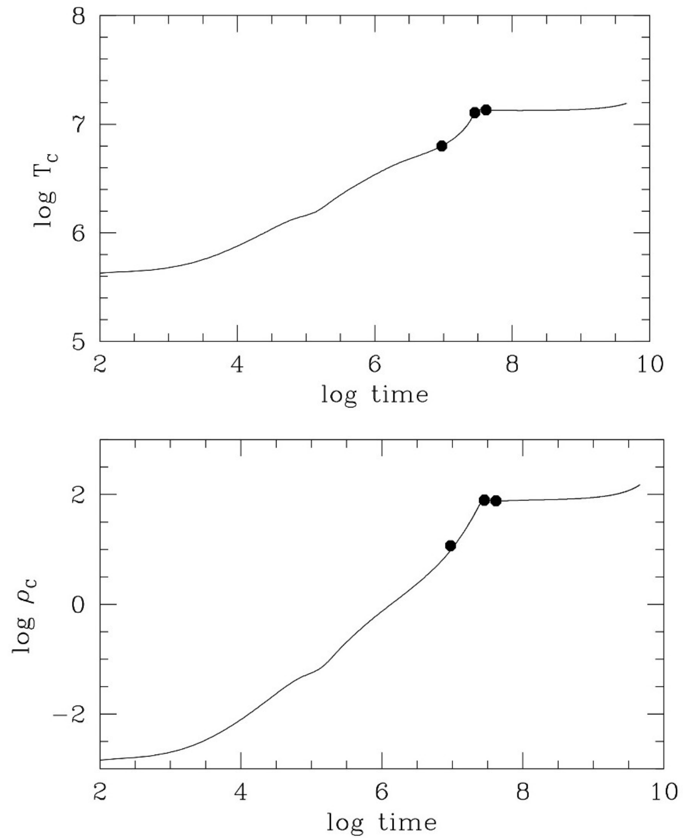
- The initial structure is evolved up to 4.5 Gyr since the start.
- The calculation is iterated by adjusting Y_{ini},Z_{ini},α, until the present-day L, R and Z/X are reproduced.



from the proto-Sun
to
the present

SURFACE

CENTER



Solar abundances and helioseismology constraints

Changing the solar abundances and the nuclear reaction rates.

- Higher C,N,O abundances from Lodders 2009 to Magg+ 2022.
- It implies higher opacity and, in turn a deeper convective envelope.
- Also the final He abundance in the convective envelope higher.
- ***Both in good agreement, now, with helioseismic measurements.***
- Higher initial He and Z. ***Important implication for galactic chemical evolution***

composition	lodders 2009	Magg 2022	Magg 2022	
network	Sol. Fus. II	Sol. Fus. II	Sol. Fus. III	
T_c:	1.55E+07	1.57E+07	1.55E+07	
Ro_c:	1.52E+02	1.54E+02	1.51E+02	
X_in:	7.16E-01	7.05E-01	7.04E-01	
X_end:	7.49E-01	7.38E-01	7.36E-01	
Y_in:	2.68E-01	2.76E-01	2.78E-01	
Y_end:	2.37E-01	2.45E-01	2.47E-01	0.2485+/-0.0034
Z_in:	1.59E-02	1.85E-02	1.84E-02	
Z_end:	1.42E-02	1.67E-02	1.66E-02	
M_ce:	9.79E-01	9.77E-01	9.75E-01	
R_ce:	7.24E-01	7.17E-01	7.15E-01	0.713+/-0.001
T_ce:	2.07E+06	2.15E+06	2.18E+06	
Ro_ce:	1.66E-01	1.81E-01	1.91E-01	

Solar neutrinos

As a by-product of a SSM, we can get the neutrinos flux from the Sun

- The story starts with the Homestake mine experiments (by R. Davis). It was a radiochemical detector: $\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$ (threshold@0.814 MeV). Only neutrinos from ^{8}B decay (PPIII). Results: 1/3 of the expected neutrinos
- Other radiochemical experiments, e.g. GALLEX and SAGE, based on $\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$ (threshold@0.251 MeV), and those based on the Cherenkov radiation emitted by the scattering of neutrinos with electrons $\nu_x + e^- \rightarrow \nu_x + e^-$ (e.g., SNO, Kamiokande, super-Kamiokande, Borexino) confirmed the neutrino deficit at earth.

Homestake	0.34 ± 0.03
Super-K	0.46 ± 0.02
SAGE	0.59 ± 0.06
Gallex and GNO	0.58 ± 0.05

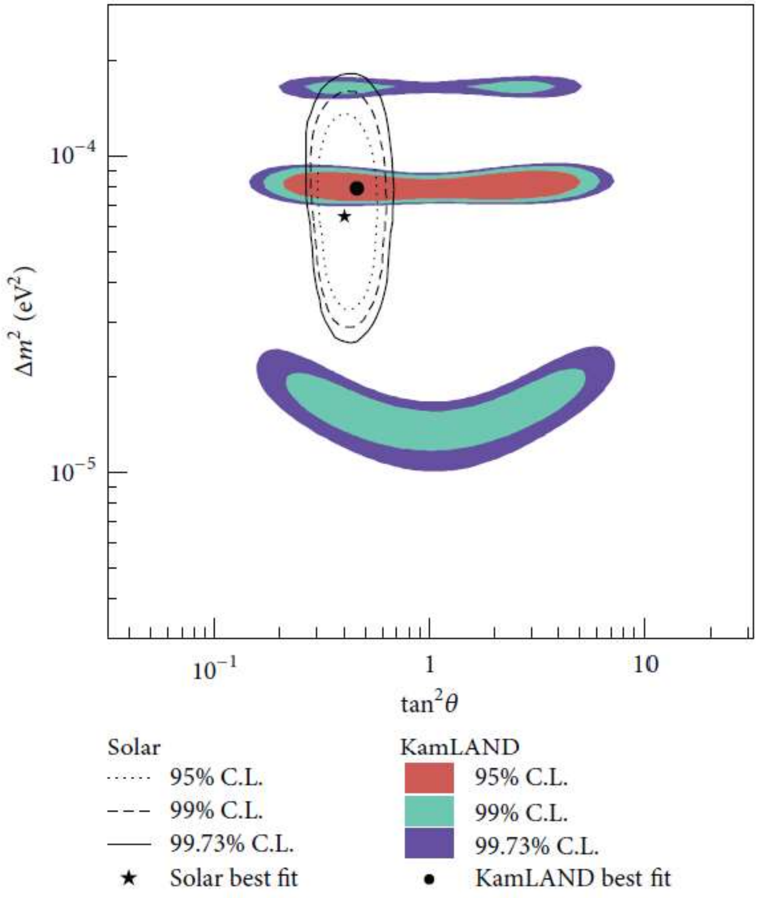
From Antonelli et al. 2013

3 possible solutions:

- The neutrino experiments are wrong
- The SSM predictions are wrong
- NEW PHYSICS.

Massive neutrinos change flavor. In practice, electron neutrinos emitted in the solar interior by nuclear reactions are in part converted into μ (τ) neutrinos, when crossing the solar radius.

KamLAND measured the neutrino flux produced by a reactor, independently confirming the solar neutrinos claim

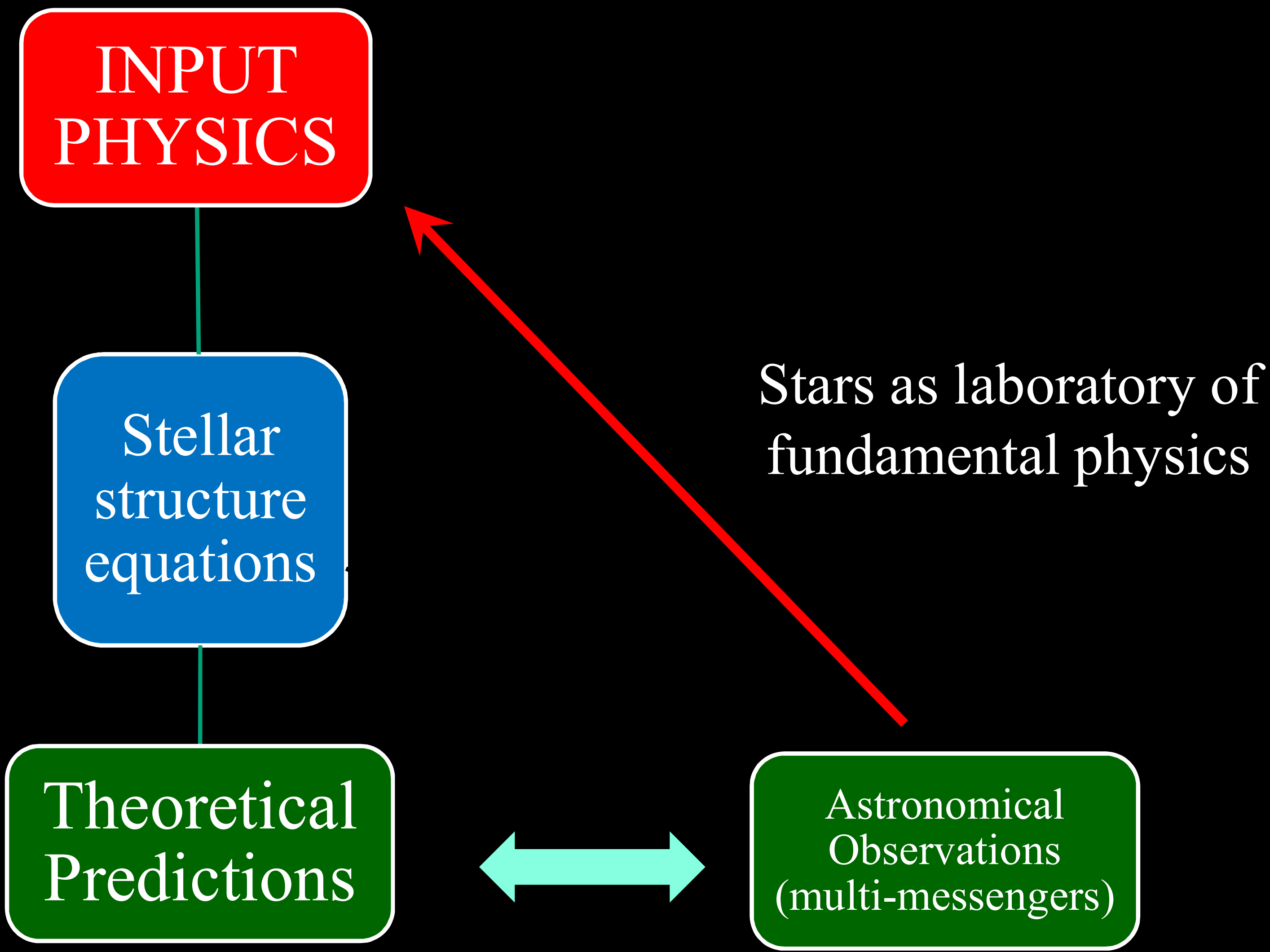


A test of the SSM predictions (the first case of multi-messenger astronomy)

In principle, we may reverse the method, using neutrino fluxes to test the predictions of an SSM. Once the neutrinos parameters (Δm and θ) are known, the neutrino suppression probability can be computed. It depends on the density profile within the Sun, while the neutrino production rates depend on the core temperature. So, by comparing the expected and measured neutrino fluxes at earth we may check the accuracy of a SSM and test the adopted nuclear reaction rates.

Composition	Iodders 2009	Magg 2022	Magg 2022
network	Sol. Fus. II	Sol. Fus. II	Sol. Fus. III
pp	6.02E+10	5.97E+10	6.00E+10
pep	1.47E+08	1.44E+08	1.42E+08
hep	8.30E+03	8.11E+03	8.03E+03
7Be	4.55E+09	4.89E+09	4.65E+09
8Be	4.24E+06	5.13E+06	4.51E+06
13N	1.88E+08	2.97E+08	2.74E+08
15O	1.31E+08	2.15E+08	1.92E+08
17F	3.24E+06	4.08E+06	3.64E+06
18F	4.03E-77	4.06E-77	4.07E-77

Flux	Solar	BP04
pp	$6.05\,(1^{+0.003}_{-0.011})$	$5.94\,(1 \pm 0.01)$
pep	$1.46\,(1^{+0.010}_{-0.014})$	$1.40\,(1 \pm 0.02)$
hep	$18\,(1^{+0.4}_{-0.5})$	$7.8\,(1 \pm 0.16)$
^{7}Be	$4.82\,(1^{+0.05}_{-0.04})$	$4.86\,(1 \pm 0.12)$
^{8}B	$5.00\,(1 \pm 0.03)$	$5.79\,(1 \pm 0.23)$
^{13}N	≤ 6.7	$5.71\,(1 \pm 0.36)$
^{15}O	≤ 3.2	$5.03\,(1 \pm 0.41)$
^{17}F	≤ 5.9	$5.91\,(1 \pm 0.44)$


INPUT PHYSICS
Stellar structure equations
Theoretical Predictions
Astronomical Observations (multi-messengers)
Stars as laboratory of fundamental physics