ALL ORDERS IN GAUGE THEORIES AT HADRON COLLIDERS

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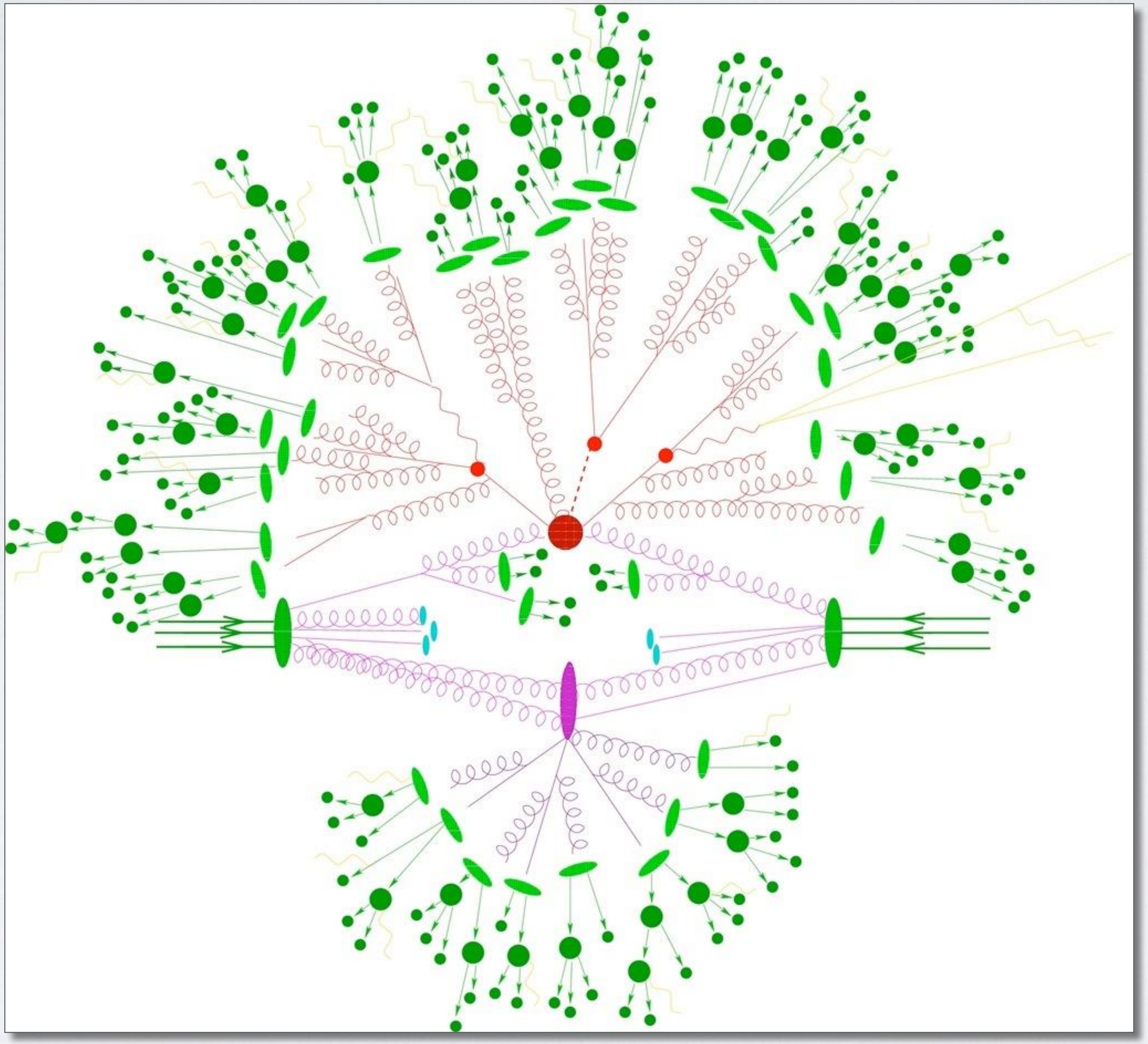
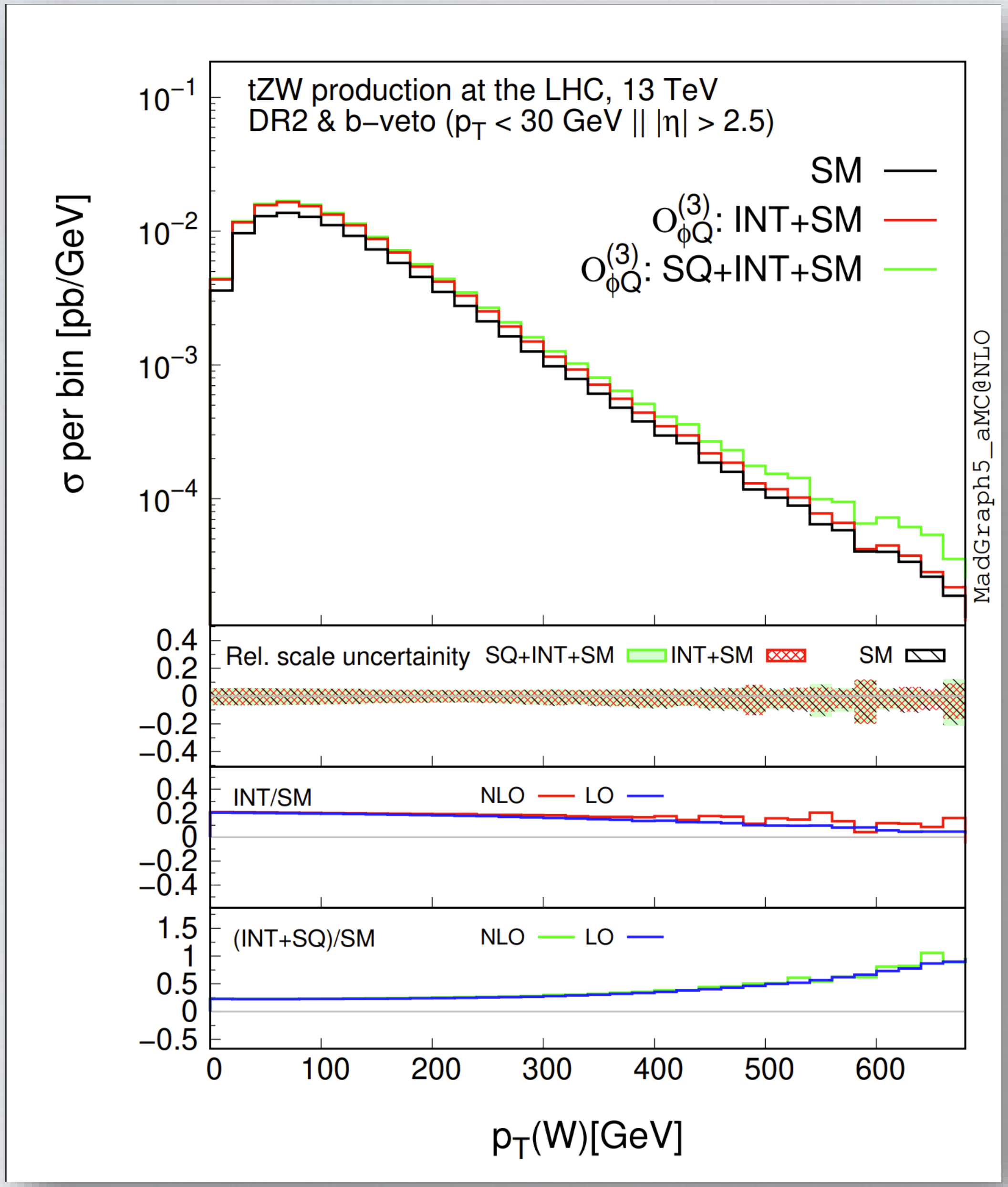
Theory Retreat, Santo Stefano Belbo, 12/11/2023

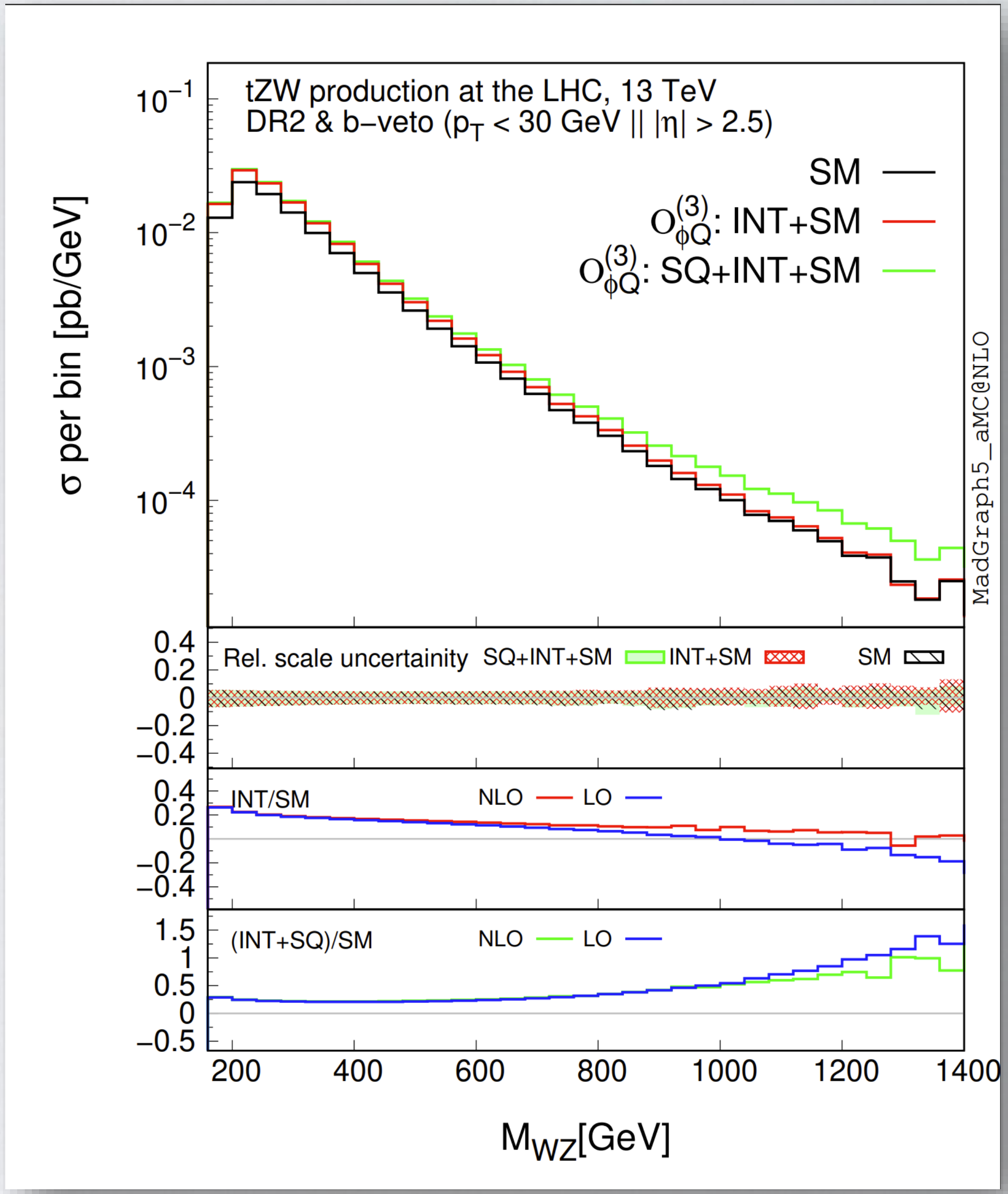
PRECISION IN PARTICLE PHYSICS AT HADRON COLLIDERS



PRECISION FOR COLLIDER PHENOMENOLOGY

- Precision in particle physics offers a valid path to find New Physics, in the form of small deviations from predictions made within the Standard Model.

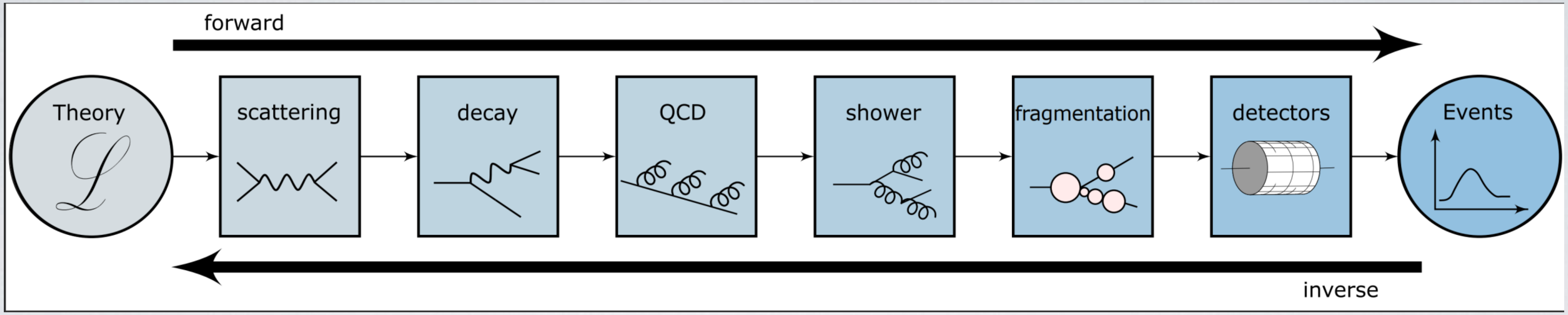




El Faham, Maltoni, Mimasu, Zaro, 2021

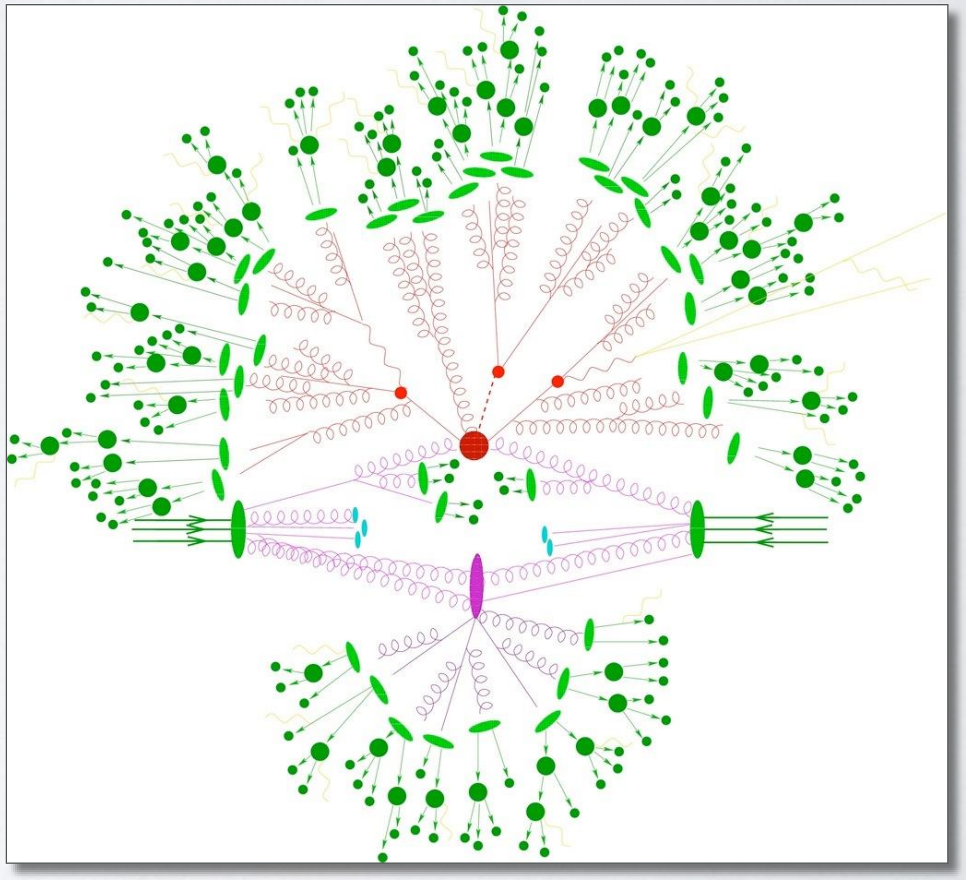
- Highly non-trivial task! Several ingredients are necessary.

PRECISION FOR COLLIDER PHENOMENOLOGY



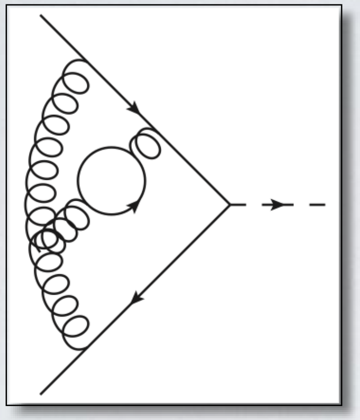
(Figure from 2203.07460)

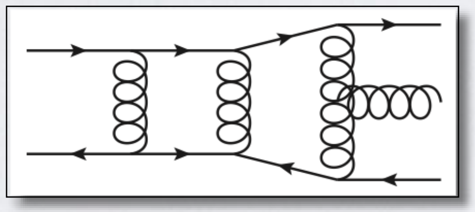
- Here we focus (mostly) on the first step in this chain: the perturbative calculation of hard scattering kernels. This task alone involves already several different topics:
- QCD corrections
- Mixed QCD-EW correction
- Multi-loop and multi-leg processes
- Large logarithms
- SM vs SMEFT
- …

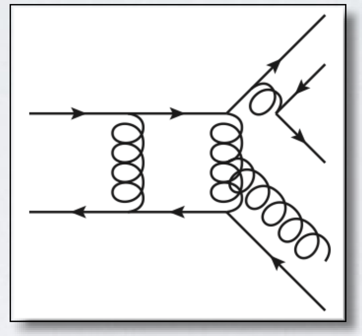


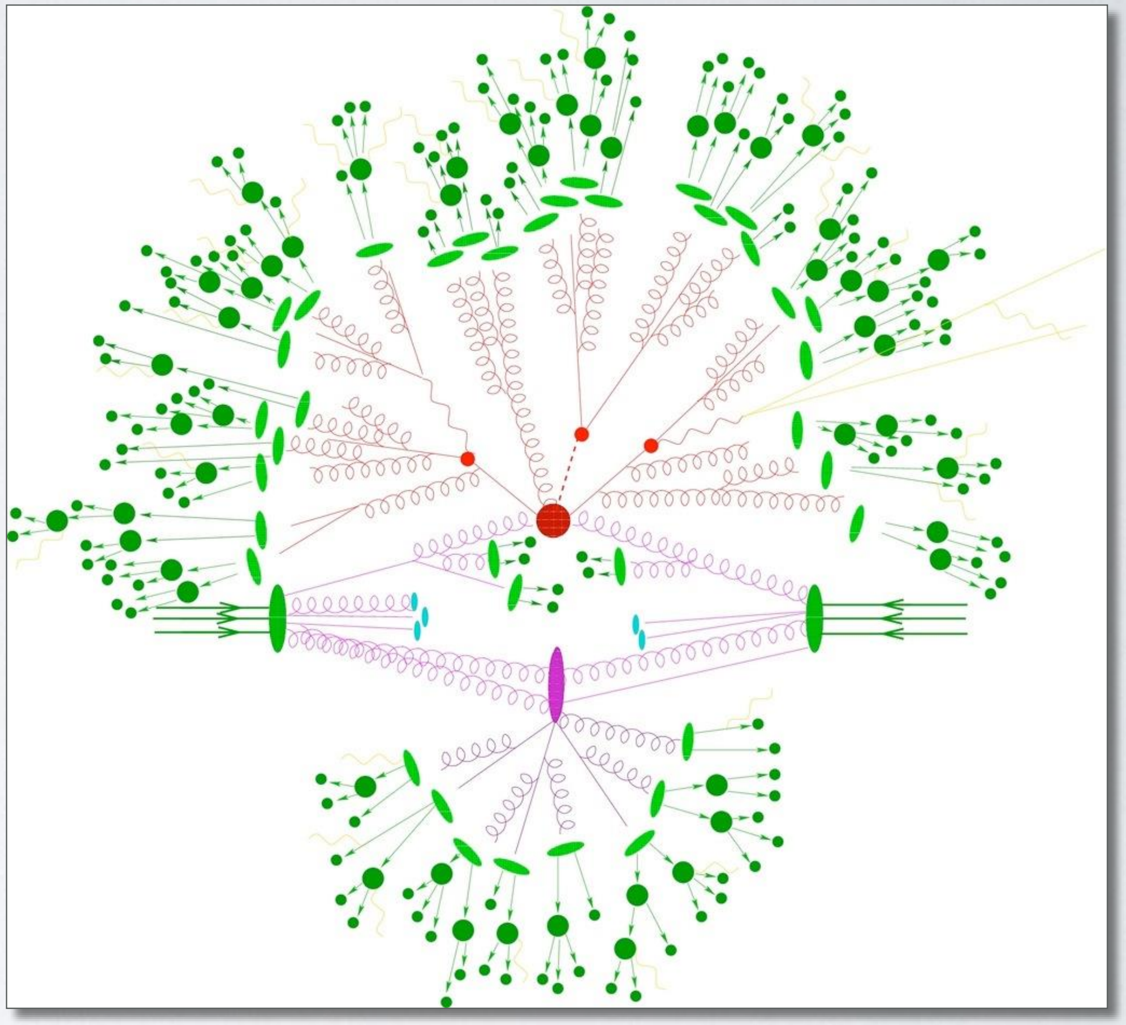
PRECISION FOR COLLIDER PHENOMENOLOGY

- Hard scattering processes are calculated in perturbation theory.









- Going beyond NNLO and N3LO is difficult, yet necessary to match the precision of current and forthcoming experiments!
- Loop and phase space integrals:
  - Analytic vs numerical evaluation
  - Space of functions
  - Infrared divergences
  - Large logarithms

PRECISION FOR COLLIDER PHENOMENOLOGY

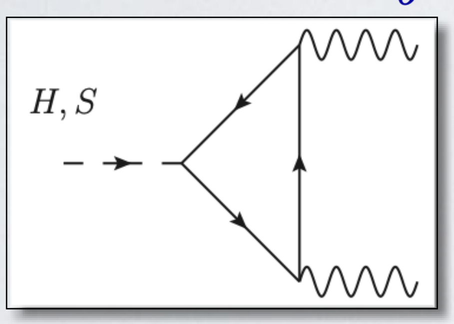
- The presence of largely different scales gives rise to large logarithms:

$$d\sigma \sim 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \dots$$

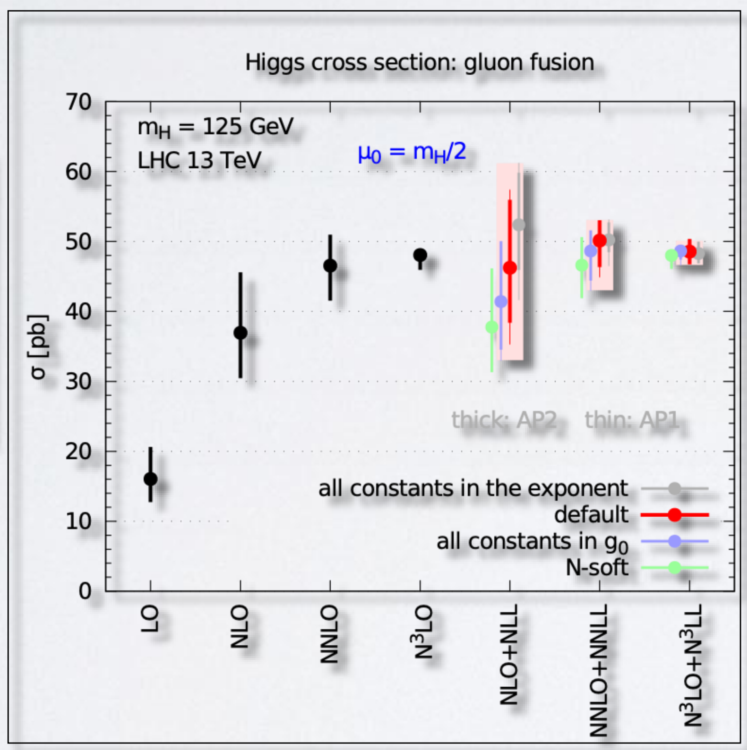
or

$$d\sigma \sim 1 + \alpha_s(L + 1) + \alpha_s^2(L^2 + L + 1) + \dots$$

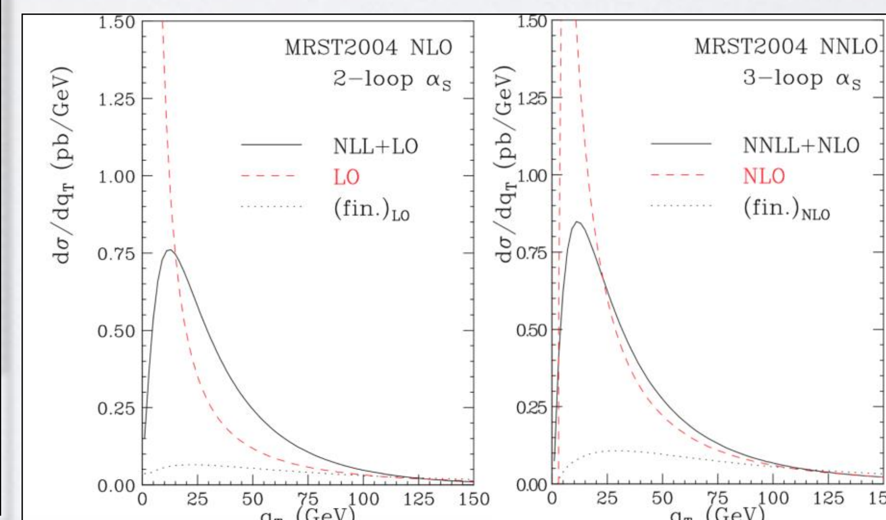
$$\sim \log^2 \frac{m_H^2}{m_b^2}$$



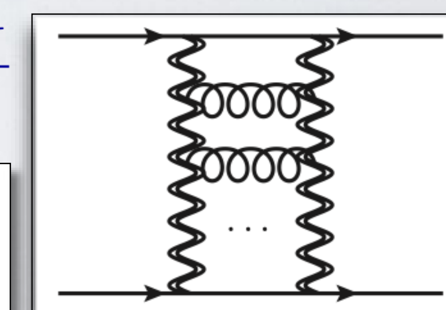
$$\sim \log^2(1-z)$$



$$\sim \log \frac{m_H^2}{p_T^2}$$



$$\sim \log \frac{s}{-t}$$



- Determine logarithms to all order:
  - → improve the convergence of the perturbative series;
  - → study perturbative corrections in a simplified kinematic regime!

PRECISION FOR COLLIDER PHENOMENOLOGY

- My work mostly focus on developing new calculation techniques for resummation.
- Interesting task: it requires to understand all order properties of gauge theories.
- As such, it feeds into several aspects of quantum field theory, providing also important results for fixed order perturbation theory and effective field theories.
- I will illustrate these aspects focusing on two cases:

Scattering in the high-energy limit

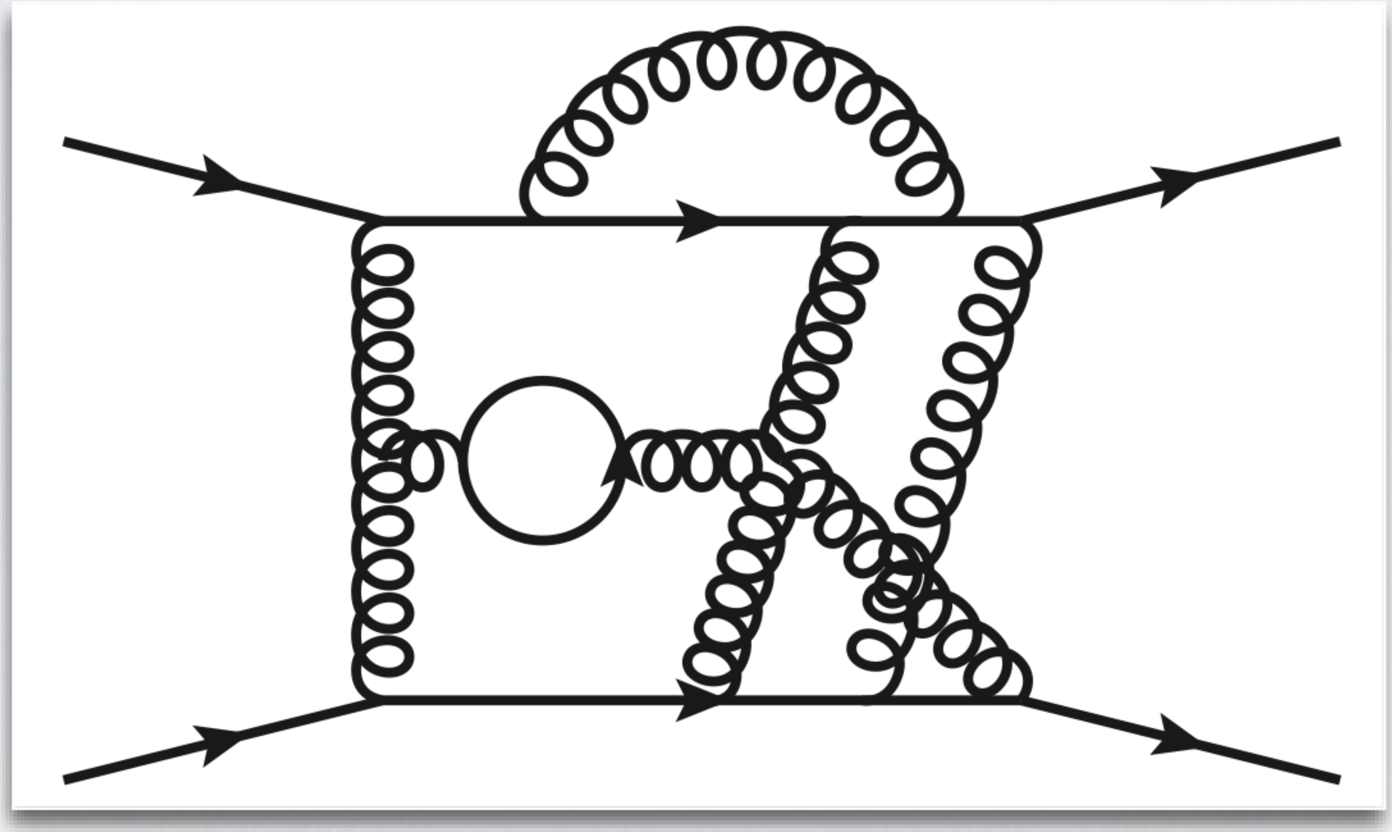
implications for fixed order PT:
→ Infrared fivergences
→ Analytic structure

Scattering near threshold

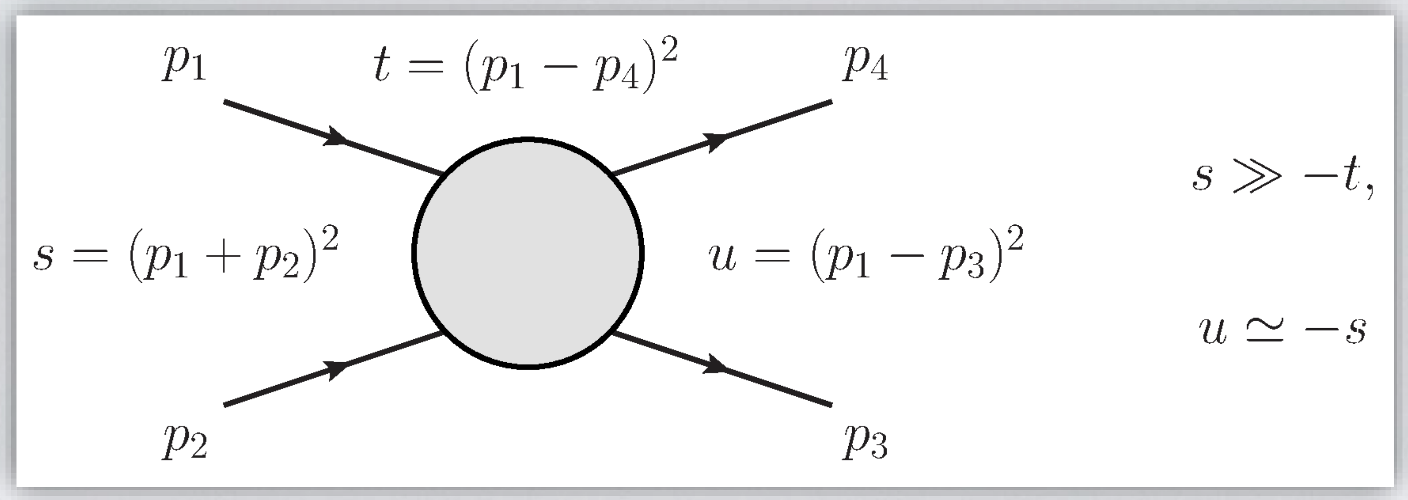
implications for phenomenology and EFTs

- We have developed methods which allows to systematically calculate large logs;
- In turn, we have been able to clarify/solve long standing problems.

SCATTERING IN THE HIGH-ENERGY LIMIT



TWO-PARTON SCATTERING AMPLITUDES



- Expansion in the strong coupling and in towers of (large) logarithms:

$$\mathcal{M}_{ij\to ij} = \mathcal{M}^{(0)} + \frac{\alpha_s}{\pi}\log\frac{s}{-t}\mathcal{M}^{(1,1)} + \frac{\alpha_s}{\pi}\mathcal{M}^{(1,0)} + \left(\frac{\alpha_s}{\pi}\right)^2\log^2\frac{s}{-t}\mathcal{M}^{(2,2)} + \left(\frac{\alpha_s}{\pi}\right)^2\log\frac{s}{-t}\mathcal{M}^{(2,1)} + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{M}^{(2,0)} + \ldots$$

LL NLL NNLL

- Goal: develop a theory to calculate systematically the tower of logarithms at any order in the strong coupling expansion.

HIGH-ENERGY LIMIT

- Very interesting theoretical problem:
  - toy model for full amplitude, yet
    - → retain rich dynamic in the 2D transverse plane,
    - → non-trivial function spaces;
  - Understand the high-energy QCD asymptotic in terms of Regge poles and cuts;
  - predict amplitudes and other observables in overlapping limits:
    - → soft limit, infrared divergences.

MRK in N=4 SYM:
Dixon, Pennington, Duhr, 2012;
Del Duca, Dixon, Pennington, Duhr, 2013;
Del Duca, Druc, Drummond, Duhr, Dulat, Marzucca, Papathanasiou, Verbeek 2019

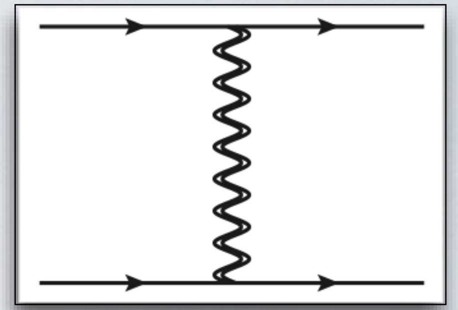
- Relevant for phenomenology at the LHC and future colliders:
  - perturbative phenomenology of forward scattering, e.g.
    - → Deep inelastic scattering/saturation (small x = Regge, large Q^2 = perturbative),
    - → Mueller-Navelet: pp → X+2jets, forward and backward.

See e.g. Andersen, Smillie, 2011; Andersen, Medley Smillie, 2016; Andersen, Hapola, Maier, Smillie, 2017; …

TWO-PARTON SCATTERING AMPLITUDES

- LL tower: one-Reggeon exchange in the t-channel (Regge pole in the complex angular momentum plane)

$$\frac{1}{t} \to \frac{1}{t}\left(\frac{s}{-t}\right)^{\frac{\alpha_s C_A}{\pi}\frac{r_\Gamma}{\epsilon}}$$



Regge, Gribov ~ 1960;
Lipatov; Fadin,Kuraev,Lipatov 1976

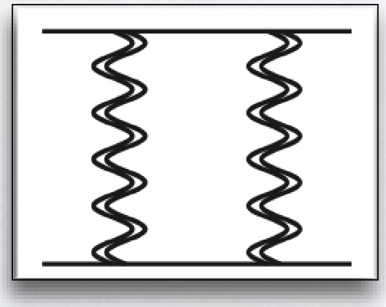
- LL amplitude

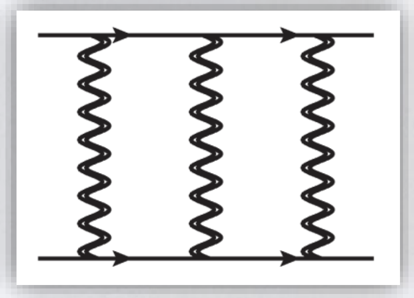
$$\mathcal{M}^{\rm LL} = e^{\frac{\alpha_s C_A L}{\pi}\frac{r_\Gamma}{\epsilon}}\mathcal{M}^{(0)}, \qquad r_\Gamma = e^{\epsilon\gamma_E}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}.$$

- Real amplitude at NLL: described by BFKL:

Fadin,Kuraev,Lipatov 1975-77; Balitsky,Lipatov 1978

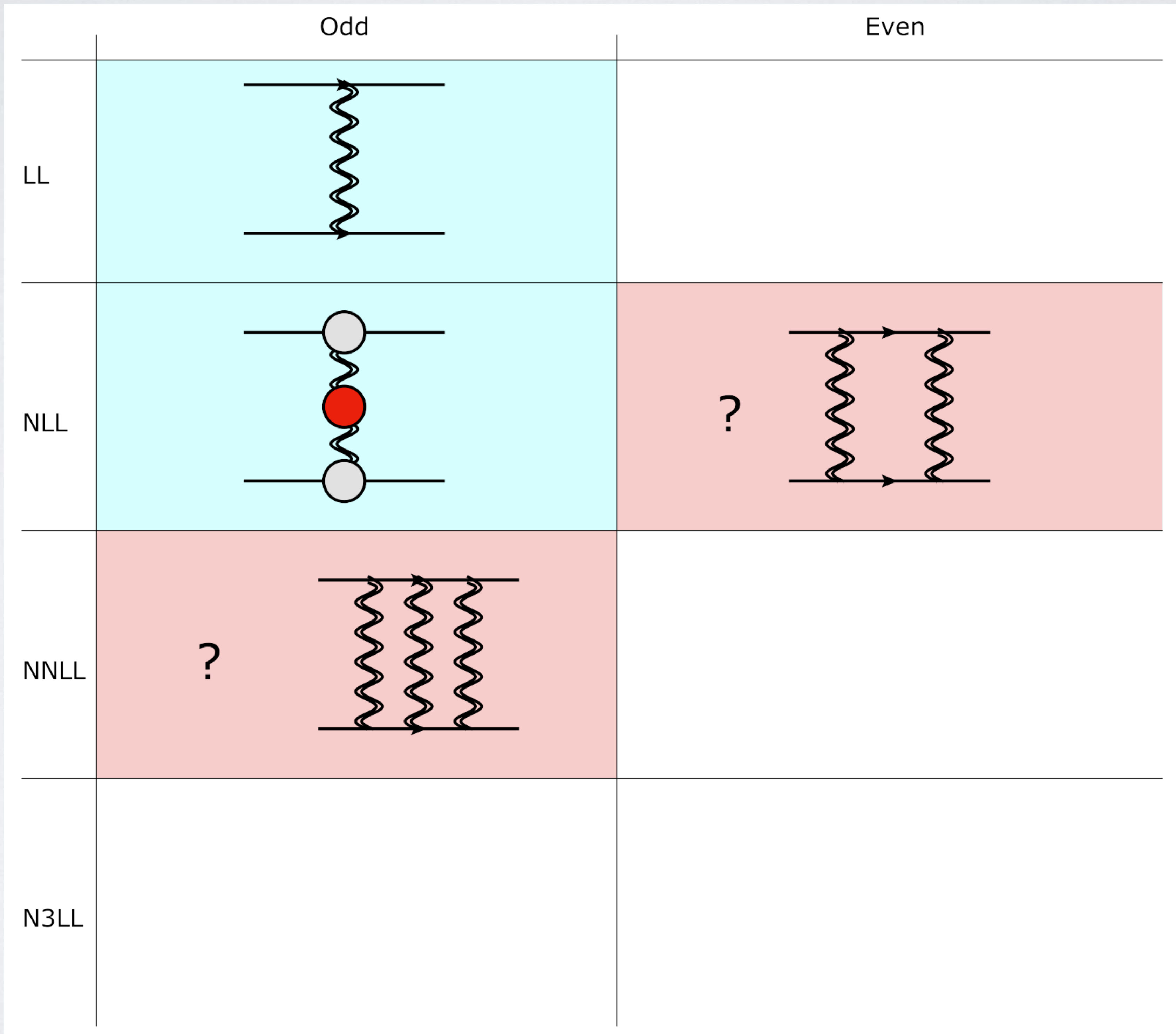
- Beyond real NLL: compound states of multiple-Reggeon exchanges.





TWO PARTON SCATTERING AMPLITUDES

- Status pre ~ 2014:



SCATTERING IN THE HIGH-ENERGY LIMIT

- Multiple Reggeon exchange contribution in scattering amplitudes elusive, until recently.
- First evidence of violation of Regge-pole factorization in

  Del Duca, Glover 2001;

- Interplay with the infrared factorization theorem investigated in

  Del Duca, Duhr, Gardi, Magnea, White 2011; Del Duca, Falcioni, Magnea, LV, 2013, 2014;

- High-energy scattering via Wilson lines:

  Korchemskaya, Korchemsky, 1994,1996; Balitsky 1995; Babansky, Balitsky 2002;

- Two-parton scattering from rapidity evolution of Wilson lines

  Caron-Huot, 2013; Caron-Huot, Gardi, LV, 2017; Caron-Huot, Gardi, Reichel, LV, 2017, 2020; Falcioni, Gardi, Milloy, LV, 2020; Falcioni, Gardi, Maher, Milloy, LV, 2021,2022.

  → This talk

- SCET-based formulation in

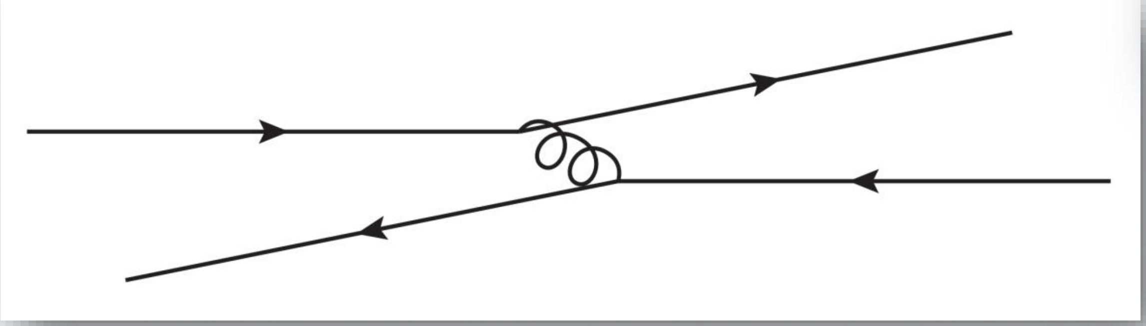
  Rothstein, Stewart 2016; Ridgway, Moult, Stewart, 2019, 2020.

- Calculation of multiple Reggeon exchanges within QCD also obtained in

  Fadin, Lipatov 2017; Fadin 2019, 2020.

FROM BALITSKY-JIMWLK TO AMPLITUDES

- The physical picture: high-energy limit = forward scattering:



Korchemskaya, Korchemsky, 1994, 1996; Babansky, Balitsky, 2002; Caron-Huot, 2013

- To leading power, the fast projectile and target described in terms of Wilson lines:

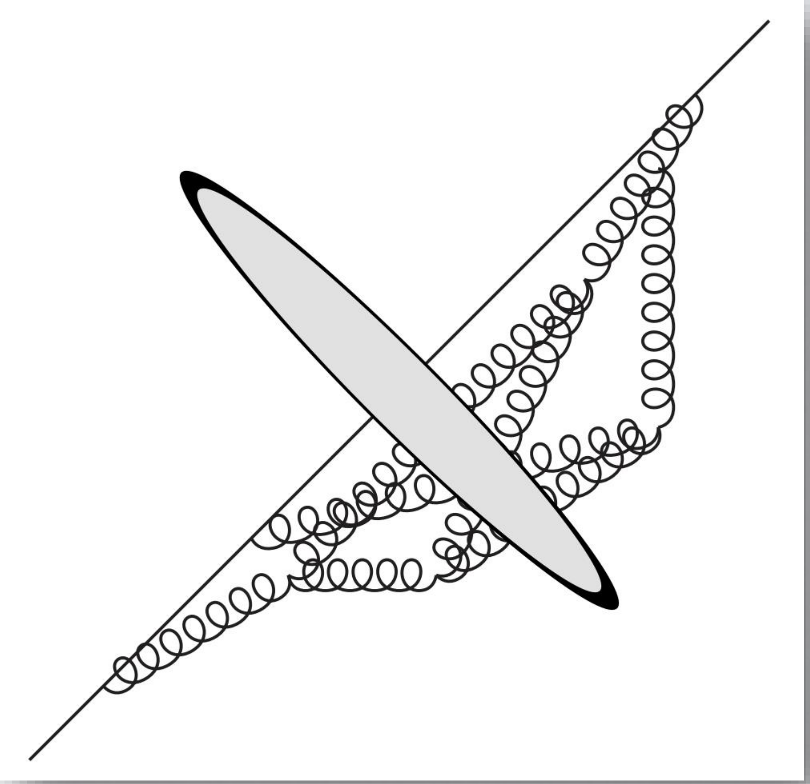
$$U(z_\perp) = \mathcal{P}\exp\left[ig_s\int_{-\infty}^{+\infty} A_+^a(x^+, x^- {=} 0, z_\perp)dx^+ T^a\right].$$

- Upon evolution in energy (rapidity), emitted radiation gives additional Wilson lines!

$$\eta = L \equiv \log\left|\frac{s}{t}\right| - i\frac{\pi}{2}.$$

- Non-linear Balitsky-JIMWLK evolution equation:

$$\frac{d}{d\eta}UU \sim g_s^2\int d^2z_0\, K(z_0, z_1, z_2)\left[U(z_0)UU - UU\right].$$



FROM BALITSKY-JIMWLK TO AMPLITUDES

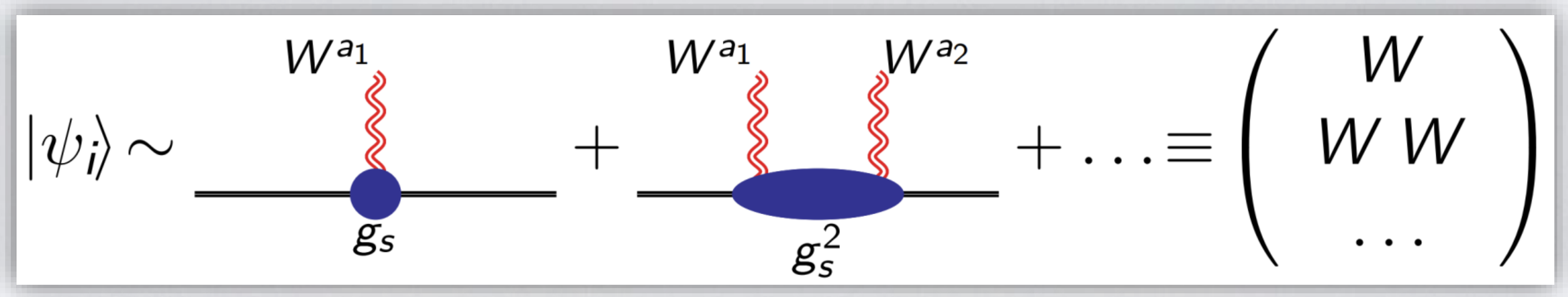
- For scattering amplitudes, we can consider the dilute regime: expand Wilson lines around unity in an effective degree of freedom dubbed as "Reggeon":

$$U^\eta(z_\perp) = \mathcal{P}\exp\left[ig_s\,\mathbf{T}^a\int_{-\infty}^{+\infty}dx^+\,A_+^a(x^+,x^-=0,z_\perp)\right] \equiv e^{ig_s\,\mathbf{T}^a\,W^a(z_\perp)}\,.$$

Caron-Huot, 2013

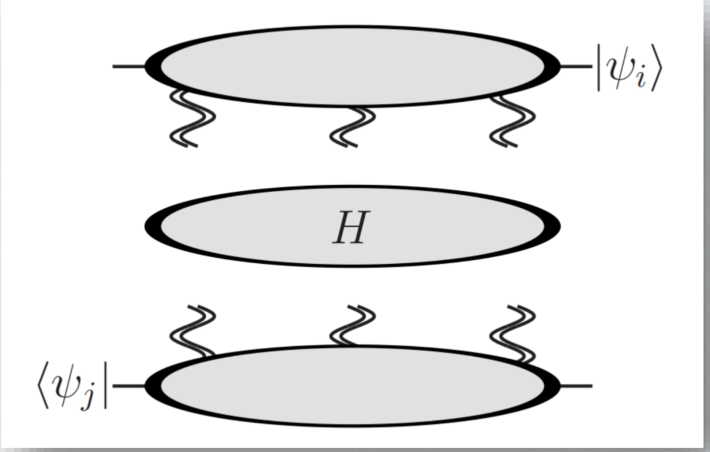
(T^a group generator in the parton representation, $\eta = L$ (implicit) cutoff)

- Scattering states (target and projectile) are expanded in Reggeon fields W^a:



- Scattering amplitude: expectation value of Wilson lines evolved to equal rapidity.

$$\frac{i}{2s}\frac{1}{Z_iZ_j}\mathcal{M}_{ij\to ij} = \langle\psi_j|e^{-LH}|\psi_i\rangle.$$



(Z_i = collinear poles)

Caron-Huot, 2013, Caron-Huot, Gardi, LV, 2017

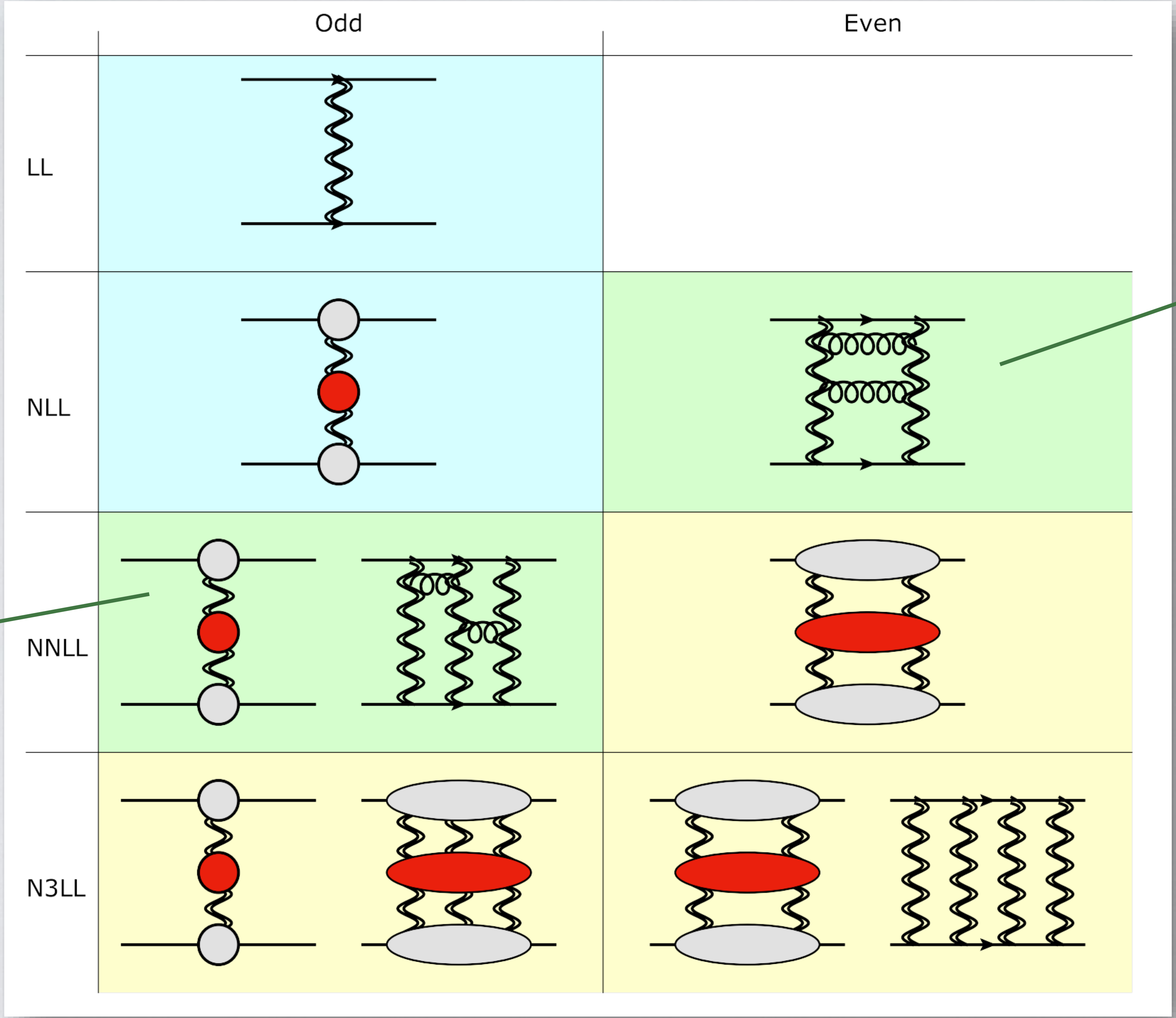
TWO PARTON SCATTERING AMPLITUDES

- Developed a framework for the calculation of amplitudes in the high-energy limit;
- Systematic relation between logarithmic accuracy and number of Reggeons.

Analysed to 2 loops in Del Duca, Falcioni, Magnea, LV 2014;

Calculated to 3 loops in Caron-Huot, Gardi, LV, 2017;

Calculated to 4 loops in Falcioni, Gardi, Milloy, LV, 2020; Falcioni, Gardi, Maher, Milloy, LV, 2021.



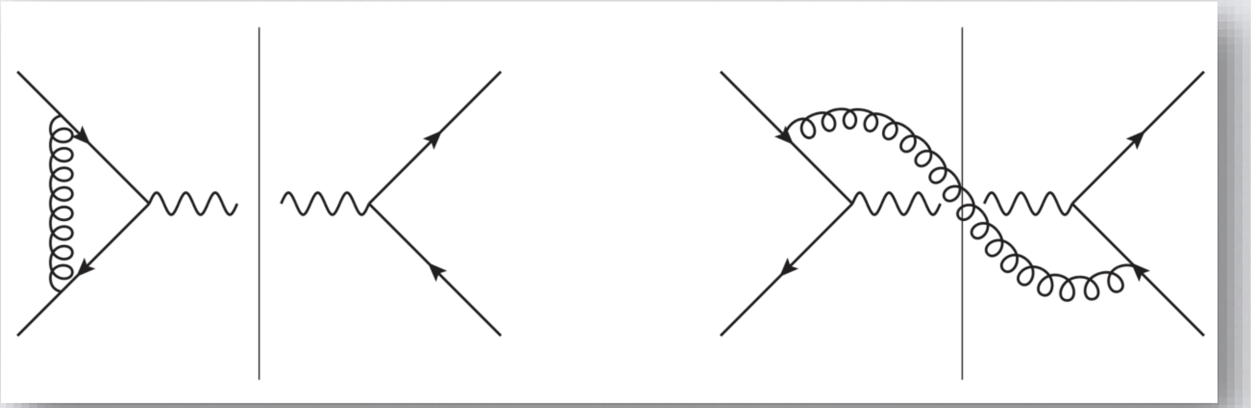
IR rivergences calculated to all orders in Caron-Huot, Gardi, Reichel, LV, 2017;

Finite terms calculated to 13 loops in Caron-Huot, Gardi, Reichel, LV, 2020.

APPLICATION: INFRARED DIVERGENCES

- Individual terms of matrix element squared are infrared divergent;
- Infrared divergences cancel in the sum over equivalent final (and initial) states.

$$\frac{d\sigma_{\rm NLO}}{dX} = \int d\Phi_n \, V \, \delta_n(X) + \int d\Phi_{n+1} \, R \, \delta_{n+1}(X).$$



See for instance
Agarwal, Magnea, Signorile-Signorile, Tripathi, 2021.

- In practice, need to construct counterterms for both terms.

$$\frac{d\sigma_{\rm NLO}}{dX} = \int d\Phi_n \Big(V + I \Big) \delta_n(X) + \int \Big(d\Phi_{n+1} \, R \, \delta_{n+1}(X) - d\widehat{\Phi}_{n+1} \, \overline{K} \, \delta_n(X) \Big), \qquad I = \int d\widehat{\Phi}_{\rm rad} \, \overline{K}.$$

- Structure of infrared divergences is universal: depends on features of soft and collinear radiation in a gauge theory. A lot of work has been devoted to constraint it.

APPLICATION: INFRARED DIVERGENCES

- The infrared divergences of amplitudes are controlled by a renormalization group equation:

$$\mathcal{M}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right) = \mathbf{Z}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right)\mathcal{H}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right),$$

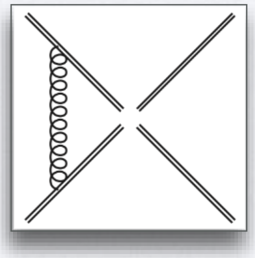
- where $\boldsymbol{Z}_n$ is given as a path-ordered exponential of the soft-anomalous dimension:

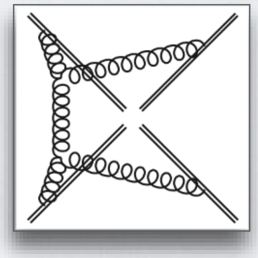
$$\mathbf{Z}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right) = \mathcal{P}\exp\left\{-\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\,\mathbf{\Gamma}_n\left(\{p_i\},\lambda,\alpha_s(\lambda^2)\right)\right\},$$

Becher, Neubert, 2009; Gardi, Magnea, 2009

- The soft anomalous dimension for scattering of massless partons is an operator in color space given by

$$\mathbf{\Gamma}_n\left(\{p_i\},\lambda,\alpha_s(\lambda^2)\right) = \mathbf{\Gamma}_n^{\text{dip.}}\left(\{p_i\},\lambda,\alpha_s(\lambda^2)\right) + \mathbf{\Delta}_n\left(\{\rho_{ijkl}\}\right).$$

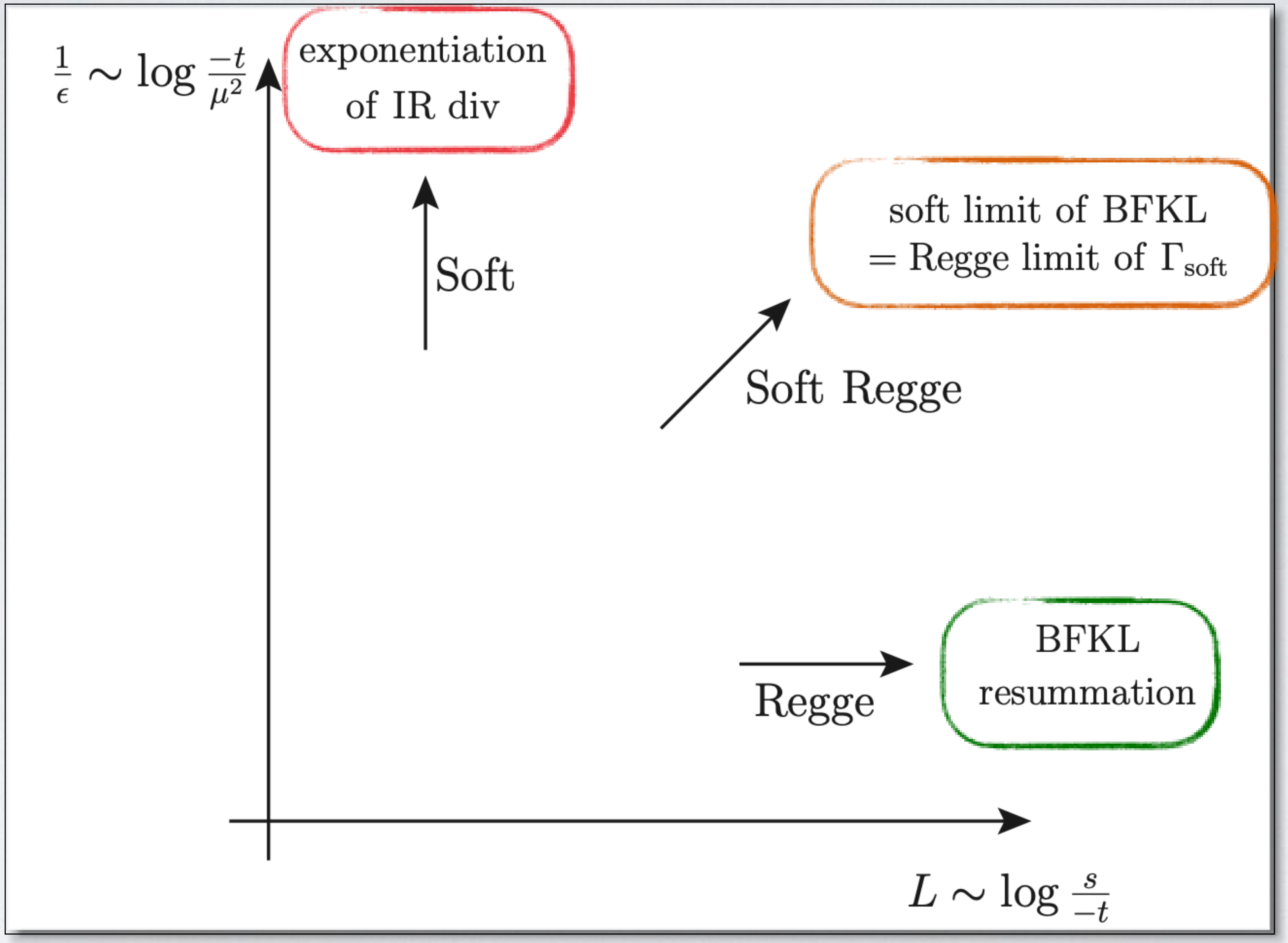




- In the past years a lot of work has been devoted to calculate/constrain $\boldsymbol{\Delta}_n$.

Dixon, Gardi, Magnea, 2009; Del Duca, Duhr, Gardi, Magnea, White, 2011; Neubert, LV, 2012; Caron-Huot, 2013; Almelid, Duhr, Gardi, 2015, 2016; Caron-Huot, Gardi, LV, 2017; Almelid, Duhr, Gardi, McLeod, White, 2017; Becher, Neubert, 2019; Magnea 2021; Falcioni, Gardi, Maher, Milloy, Vernazza 2021.

APPLICATION: INFRARED DIVERGENCES

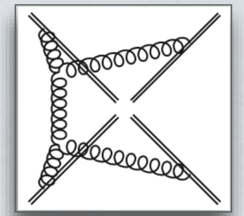


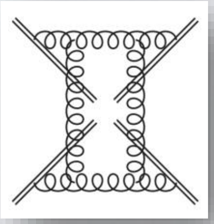
- Use amplitudes calculated in the high-energy limit to extract the soft anomalous dimension in that limit;
- Bootstrap the result to constrain the structure of infrared divergences in general kinematic.

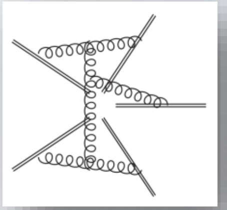
APPLICATION: INFRARED DIVERGENCES

- Structure of the soft anomalous dimension in general kinematic up to four loops:

$$
\begin{aligned}
\mathbf{\Gamma}_n\big(\{s_{ij}\},\mu,\alpha_s(\mu^2)\big) = &-\frac{\gamma_K(\alpha_s)}{4}\sum_{(i,j)}\mathbf{T}_i\cdot\mathbf{T}_i\,\log\frac{-s_{ij}}{\mu^2}+\sum_i\gamma_i(\alpha_s)\\
&+f(\alpha_s)\sum_{(i,j,k)}\mathcal{T}_{iikj}+\sum_{(i,j,k,l)}\mathcal{T}_{ijkl}\,\mathcal{F}(\beta_{ijlk},\beta_{iklj};\alpha_s)\\
&-\sum_R\frac{g^R(\alpha_s)}{2}\Bigg[\sum_{(i,j)}\big(\mathcal{D}^R_{iijj}+2\mathcal{D}^R_{iiij}\big)\ln\frac{-s_{ij}}{\mu^2}+\sum_{(i,j,k)}\mathcal{D}^R_{ijkk}\ln\frac{-s_{ij}}{\mu^2}\Bigg]\\
&+\sum_R\sum_{(i,j,k,l)}\mathcal{D}^R_{ijkl}\mathcal{G}^R(\beta_{ijlk},\beta_{iklj};\alpha_s)+\sum_{(i,j,k,l)}\mathcal{T}_{ijkli}\mathcal{H}_1(\beta_{ijlk},\beta_{iklj};\alpha_s)\\
&+\sum_{(i,j,k,l,m)}\mathcal{T}_{ijklm}\mathcal{H}_2(\beta_{ijkl},\beta_{ijmk},\beta_{ikmj},\beta_{jiml},\beta_{jlmi};\alpha_s)+\mathcal{O}(\alpha_s^5).
\end{aligned}
$$







- From the Regge limit we obtain constrains, useful for a bootstrap approach:

Gardi, Falcioni, Maher, Milloy, LV, 2021.

Signature even				Signature odd			
	L^3	L^2	L^1 (conj.)		L^3	L^2	L^1
$\mathcal{F}_A^{(+,4)}$	0	$-\frac{C_A}{8}\zeta_2\zeta_3$	0	$\mathcal{F}_A^{(-,4)}$	$i\pi\frac{C_A}{24}\zeta_3$	?	?
$\mathcal{F}_F^{(+,4)}$	0	0	0	$\mathcal{F}_F^{(-,4)}$	0	?	?
$\mathcal{G}_A^{(+,4)}$	0	$\frac{1}{2}\zeta_2\zeta_3$	$\frac{1}{6}g_A^{(4)}$				
$\mathcal{G}_F^{(+,4)}$	0	0	$\frac{1}{6}g_F^{(4)}$				
$\mathcal{H}_1^{(+,4)}$	0	0	0	$\mathcal{H}_1^{(-,4)}$	0	?	?
				$\tilde{\mathcal{H}}_1^{(-,4)}$	0	?	?

See e.g. Almelid, Duhr, Gardi, McLeod, White, 2017

APPLICATION: REGGE POLE AND CUT

- Before the development of QCD and perturbation theory, scattering amplitudes have been studied as an analytic function in the complex angular momentum plane.
- In this context, the amplitude is expected to be given in terms of Regge pole and cut:

$$A_{LL} \propto \underbrace{\frac{s^{\alpha_g(t)}}{t}}_{\text{“Regge pole”}}\,, \qquad A_{\rm NLL} \propto \underbrace{\int d\nu\, c(\nu)\, s^{E(\nu)}}_{\text{“Regge cut”}}\,.$$

Regge, Gribov ~ 1960; Lipatov; Fadin, Kuraev, Lipatov 1976.

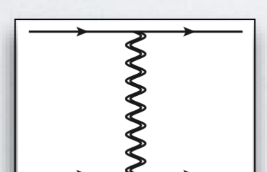
- While the Regge cut arises exclusively due to MR contributions to the amplitude, MR exchanges do contribute also to the Regge pole.

Eden, Landshoff, Olive, Polkinghorne, 1966; P. D. B. Collins, 2009

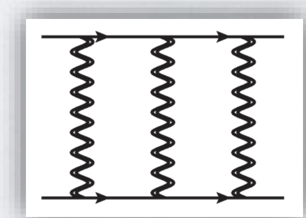
- This is evident in the large-Nc limit, where it is known that the amplitude only features a Regge pole, and yet, MR contributions are present.

Mandelstam 1963; P. D. B. Collins 2009

- It is also known that Regge cuts only arise due to nonplanar diagrams: the Regge cut should be identified as the nonplanar part of the MR contribution, while the Regge pole corresponds to SR plus the planar MR contributions:



$$\mathcal{M}^{(-)}_{ij\to ij} = \underbrace{\mathcal{M}^{(-)\,{\rm SR}}_{ij\to ij} + \mathcal{M}^{(-)\,{\rm MR}}_{ij\to ij}\Big|_{\rm planar}} + \underbrace{\mathcal{M}^{(-)\,{\rm MR}}_{ij\to ij}\Big|_{\rm nonplanar}}$$
$$= \mathcal{M}^{(-)\,{\rm pole}}_{ij\to ij} + \mathcal{M}^{(-)\,{\rm cut}}_{ij\to ij}\,.$$



APPLICATION: REGGE POLE AND CUT

- With this definition we are able to extract unambiguously the Regge trajectory at three loops, matching our calculation of the Regge-cut contribution with the recent calculations of two-parton scattering at three loops in QCD:

Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi, 2021

$$\mathcal{M}^{(-)}_{ij\to ij} = Z_i(t)\,\bar{D}_i(t)\,Z_j(t)\,\bar{D}_j(t)\left[\left(\frac{-s}{-t}\right)^{C_A\tilde{\alpha}_g(t)} + \left(\frac{-u}{-t}\right)^{C_A\tilde{\alpha}_g(t)}\right]\mathcal{M}^{\rm tree}_{ij\to ij} + \sum_{n=2}^{\infty}\frac{\alpha_s}{4\pi}L^{n-2}\mathcal{M}^{(-,n,n-2)\,\rm cut}_{ij\to ij},$$

with

$$\hat{\tilde{\alpha}}_g^{(3)} = K^{(3)} + C_A^2\left(\frac{297029}{93312} - \frac{799\zeta_2}{1296} - \frac{833\zeta_3}{216} - \frac{77\zeta_4}{192} + \frac{5}{24}\zeta_2\zeta_3 + \frac{\zeta_5}{4}\right) + C_A n_f\left(\frac{103\zeta_2}{1296} + \frac{139\zeta_3}{144} - \frac{5\zeta_4}{96} - \frac{31313}{46656}\right)$$
$$+ C_F n_f\left(\frac{19\zeta_3}{72} + \frac{\zeta_4}{8} - \frac{1711}{3456}\right) + n_f^2\left(\frac{29}{1458} - \frac{2\zeta_3}{27}\right) + \mathcal{O}(\epsilon),$$

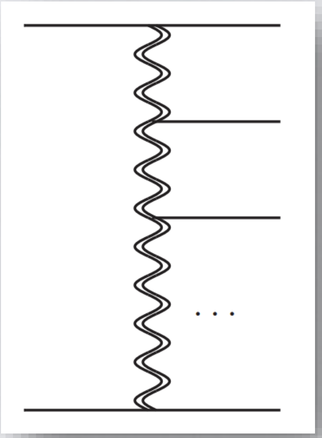
$$K_{\rm cusp}(\alpha_s(\mu^2)) \equiv -\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\Gamma_A^{\rm cusp}(\alpha_s(\lambda^2)).$$

Gardi, Falcioni, Maher, Milloy, LV, 2021.

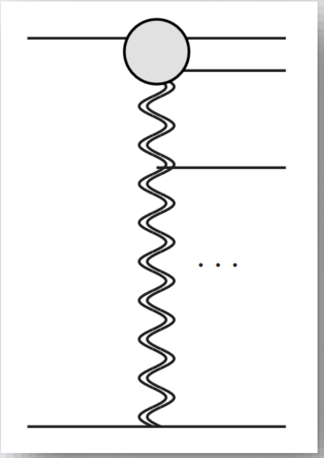
- The Regge-pole contribution is universal among all two-parton scattering processes, but theory dependent (i.e. different in N=4 SYM, QCD, etc);
- The Regge–cut contribution is different for each channel but depends only on the action of color operators in the gauge theory considered.

HIGH ENERGY LIMIT: PERSPECTIVE

- Complete NNLL calculation of two-parton scattering amplitudes;
- Extend the shockwave formalism to Multi-Regge kinematics:



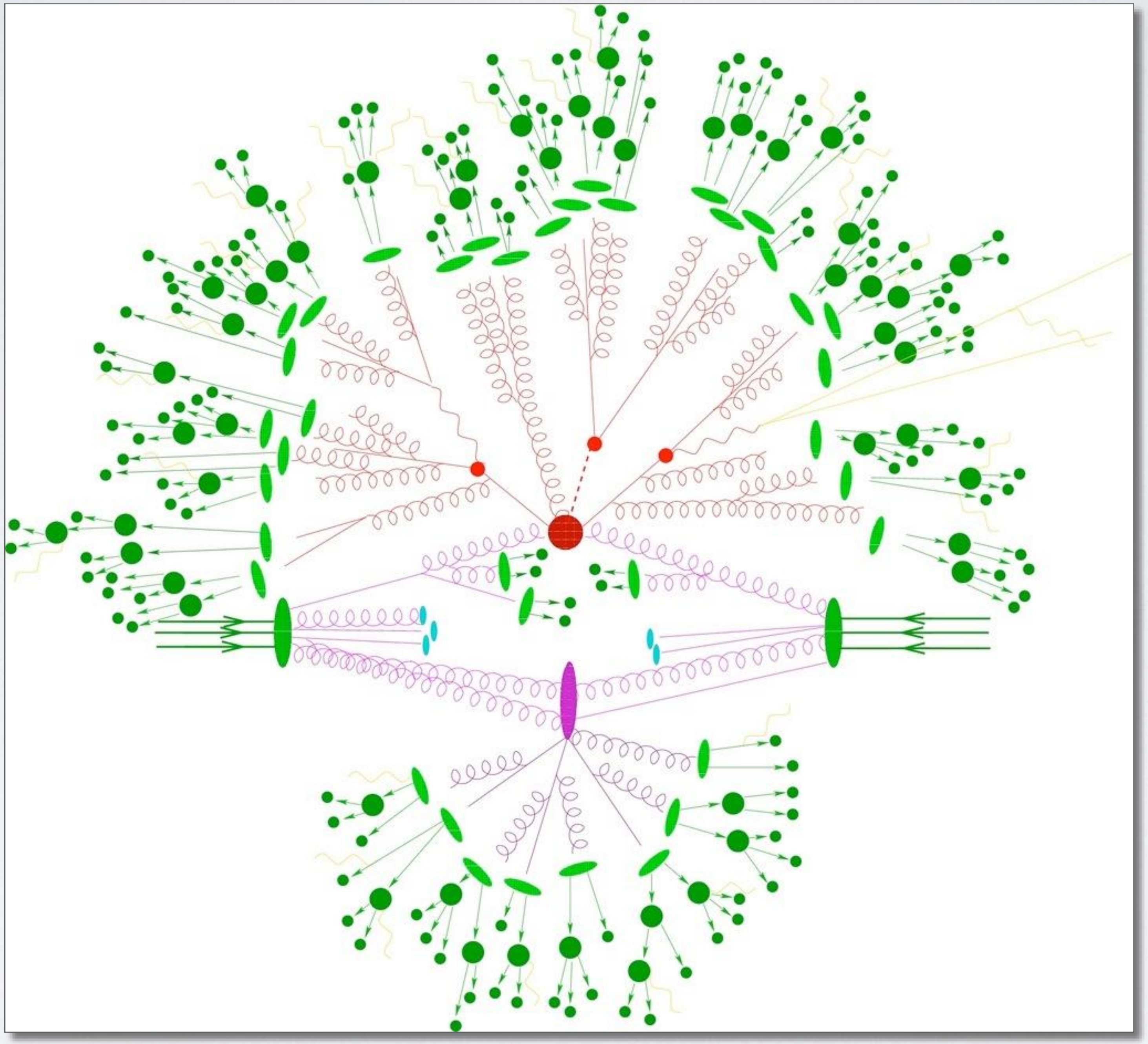
(See for instance Caron-Huot, Chicherin, Henn, Zhang, Zoia, 2020).



(See for instance Canay,Del Duca, 2021).

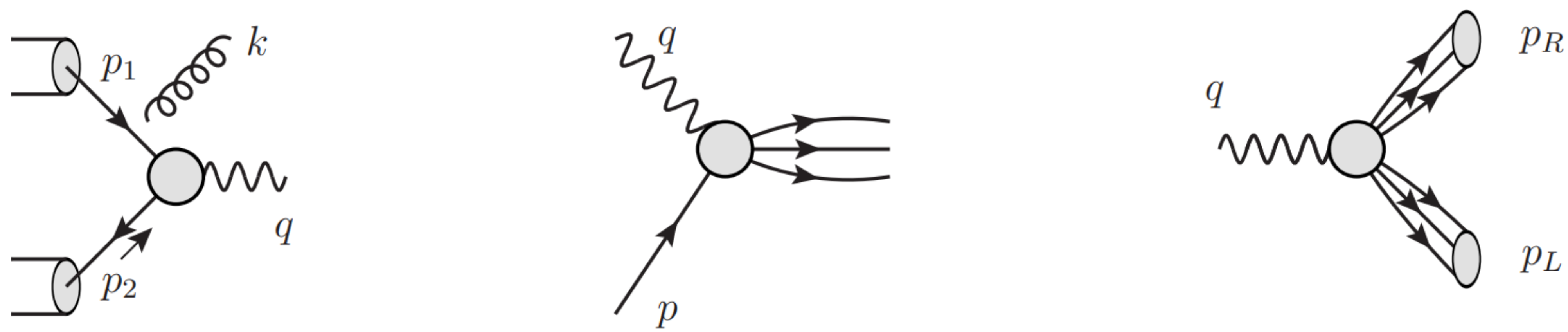
- Provides useful input for the perturbative calculation of multi-leg processes;
- Further constrain the soft anomalous dimension;
- Phenomenology …

PARTICLE SCATTERING NEAR THRESHOLD



PARTICLE SCATTERING NEAR KINEMATIC LIMITS

- Consider Drell-Yan, DIS near partonic threshold and Thrust in the back-to-back jet limit:



$(p_1 + p_2)^2 \equiv \hat{s}, \quad z \equiv \frac{q^2}{\hat{s}} \to 1$ $\qquad Q^2 = -q^2, \quad x \equiv \frac{Q^2}{2p\cdot q} \to 1$ $\qquad q^2 \gg p_L^2, p_R^2,$ $\quad T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \to 1$

- The partonic cross section has singular expansion

$$\Delta_{ab}(\xi) \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[c_n \delta(1-\xi) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-\xi)}{1-\xi} \right]_+ + d_{nm} \ln^m(1-\xi) \right) + \dots \right],$$

LP **NLP**

with $\xi = z$ for DY, $\xi = x$ for DIS, and $\xi = T$ for Thrust.

- Resummation of large logarithms relevant for precision phenomenology.
  - → well understood at LP (up to N3LL),
  - → progress toward resummation at NLP, yet no systematic approach so far.

PARTICLE SCATTERING NEAR KINEMATIC LIMITS

- Subject of intense work in the past few years!
- Within SCET:

  Beneke, Campanario, Mannel, Pecjak, 2004; Larkoski, Neill, Stewart, 2014; Kolodrubetz, Moult, Stewart, 2016; Feige, Kolodrubetz, Moult, Stewart, 2017; Beneke, Garny, Szafron, Wang, 2017-2019; Moult, Rothen, Stewart, Tackmann, Zhu, 2016/17; Boughezal, Liu, Petriello, 2016/17; Moult, Stewart, Vita, Zhu, 2018; Moult, Stewart, Vita, 2019; Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019; Beneke, Broggio, Jaskiewicz, LV, 2019; Broggio, Jaskiewicz, LV, 2021/22; Beneke, Bobeth, Szafron, 2017; Alte, König, Neubert, 2018; Moult et al., 2019; Liu, Neubert, 2019; Wang, 2019; Liu, Mecaj, Neubert, Wang, 2020; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020; Liu, Neubert, Schnubel, Wang, 2021; Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 2018; Moult, Vita, Yan, 2019; Beneke, Hager, Szafron, 2021; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang 2020; Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022 + …

- And "diagrammatic" methods:

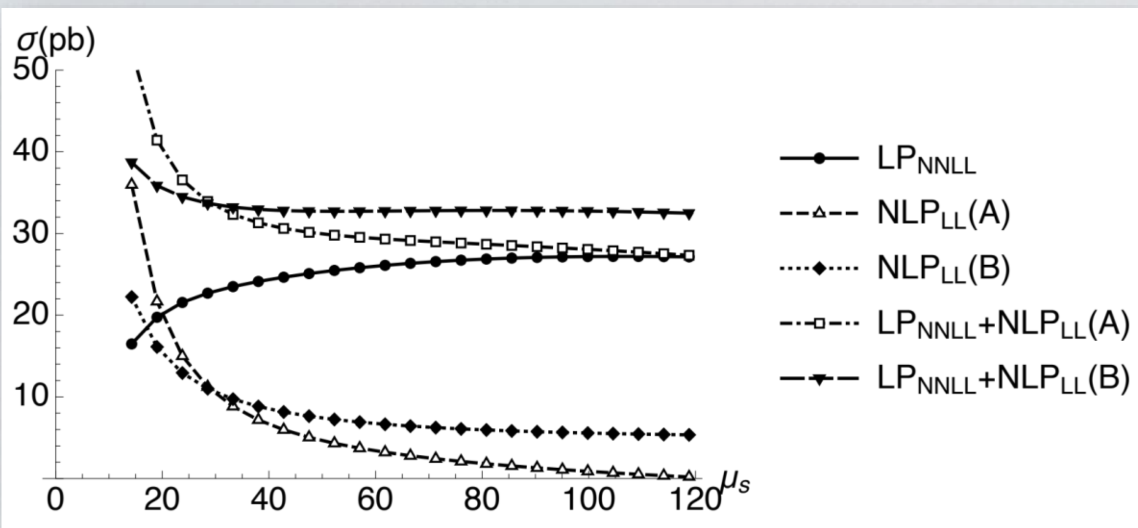
  Del Duca, 1990; Laenen, Magnea, Stavenga, 2008, Laenen, Stavenga, White, 2008; Laenen, Magnea, Stavenga, White, 2010; Bonocore, Laenen, Magnea, LV, White, 2014, 2015, 2016; Bahjat-Abbas, Sinninghe Damsté, LV, White, 2018; Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019; Liu, Penin, 2017/18; Anastasiou, Penin, 2020; Cieri, Oleari, Rocco, 2019; Oleari, Rocco 2020; van Beekveld, Beenakker, Laenen, White, 2019; van Beekveld, Laenen, Sinninghe Damsté, LV, 2021; + …

- Several topics considered:

  LBKD theorem, operator bases, renormalization, N-jettiness subtraction, thrust distribution, Drell-Yan and Higgs production near threshold, DIS for $x \to 1$, QED effects in B decays, New Physics decay, Higgs decay through bottom loops, TMD factorization, energy-energy correlation in $N = 4$ SYM, gravitation, …

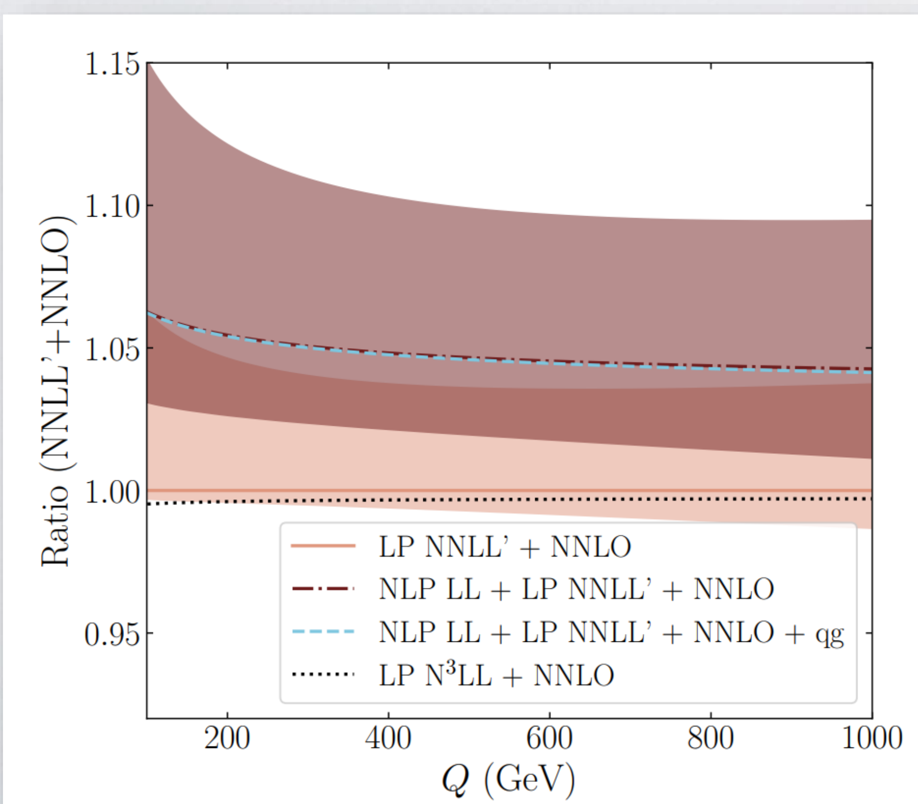
PARTICLE SCATTERING NEAR THRESHOLD

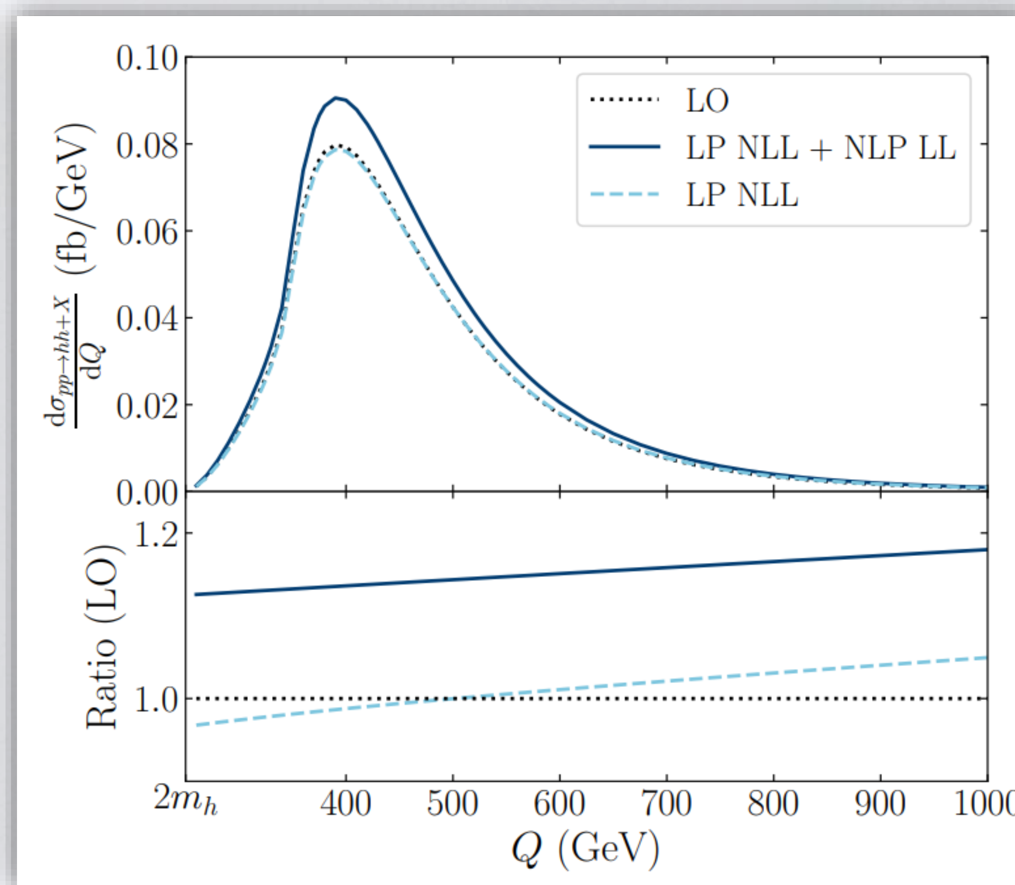
- Phenomenological analyses have shown LLs at NLP to be competitive with NNLLs at LP: relevant for precision physics.



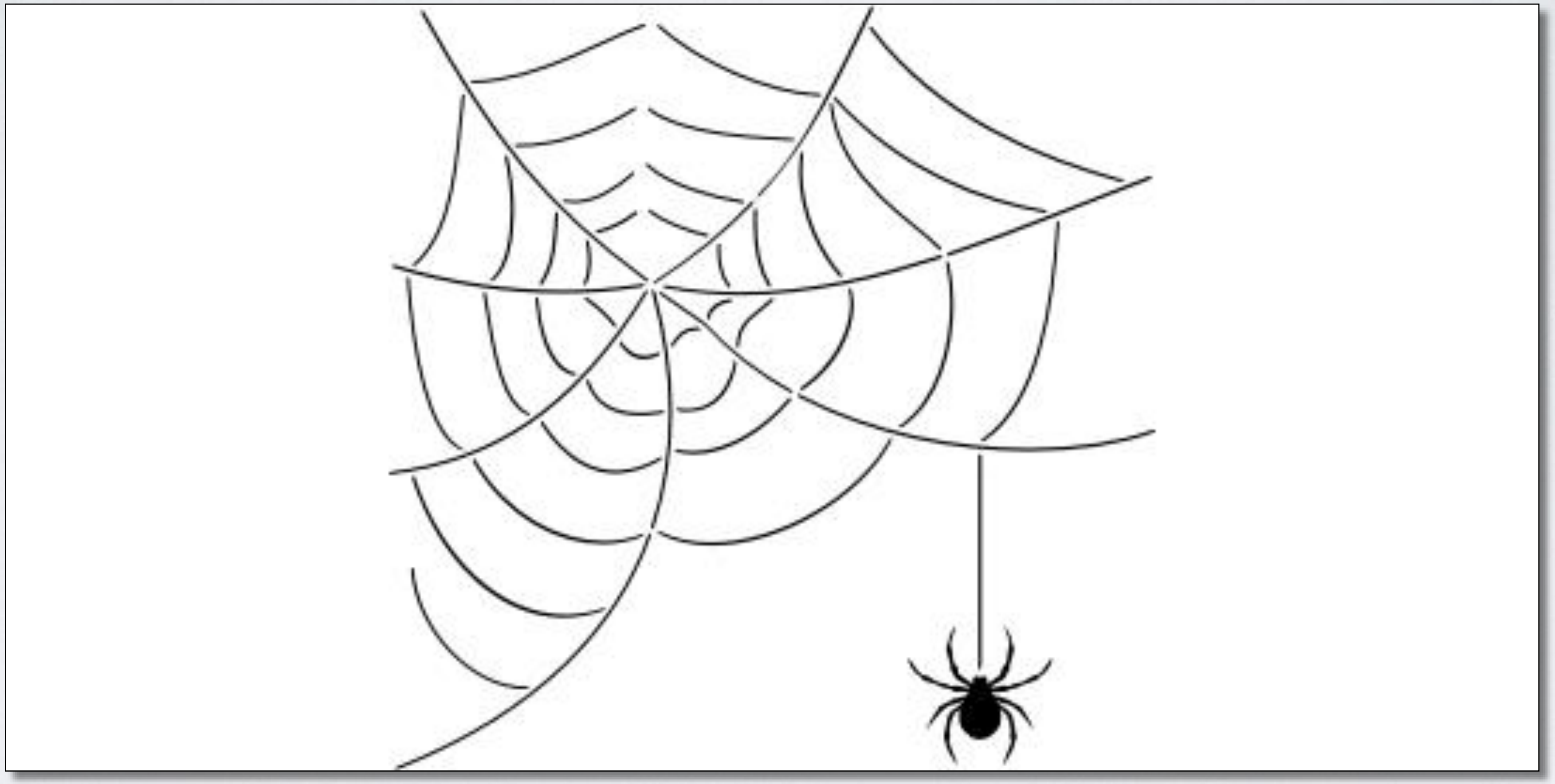
Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019

van Beekveld, Laenen, Sinninghe Damsté, LV, 2021.



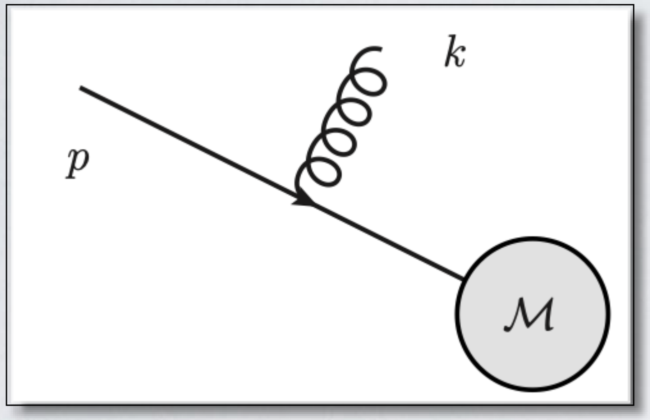


SCATTERING NEAR THRESHOLD: LP VS NLP



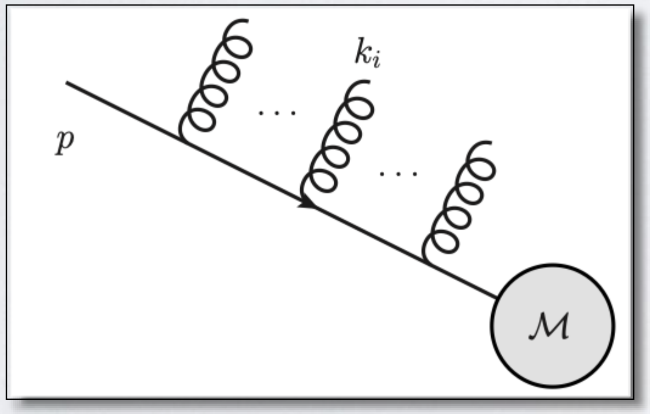
FACTORIZATION OF SOFT GLUONS AT LP

- Emission of soft gluons from an energetic parton (quark):



$$= \mathcal{M}\,\frac{\not{p}-\not{k}}{2p\cdot k}\,\gamma^\mu\,T^A\,u(p) \;\sim\; \mathcal{M}\,\frac{p^\mu}{p\cdot k}\,T^A\,u(p).$$

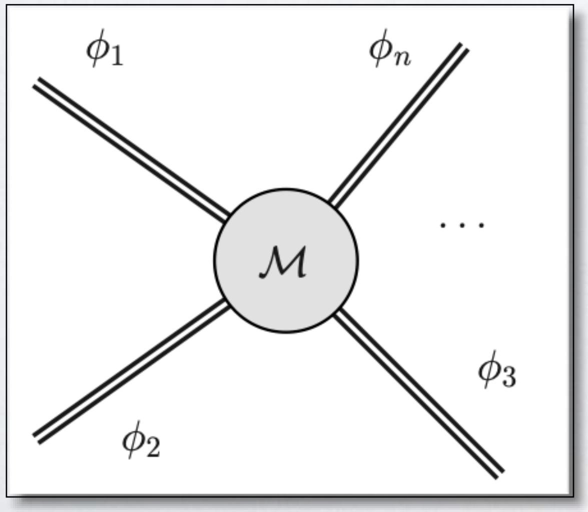
- Emission of multiple soft gluons factorises:



$$\sim \mathcal{M}\,\mathcal{S}\,u(p), \qquad \mathcal{S} = \langle 0|\Phi_\beta(-\infty,0)|0\rangle,$$

$$\Phi_\beta(\lambda_1,\lambda_2) = \mathcal{P}\exp\left\{i\,g_s\int_{\lambda_1}^{\lambda_2} d\lambda\;\beta\cdot A(\lambda\beta)\right\}.$$

- In general



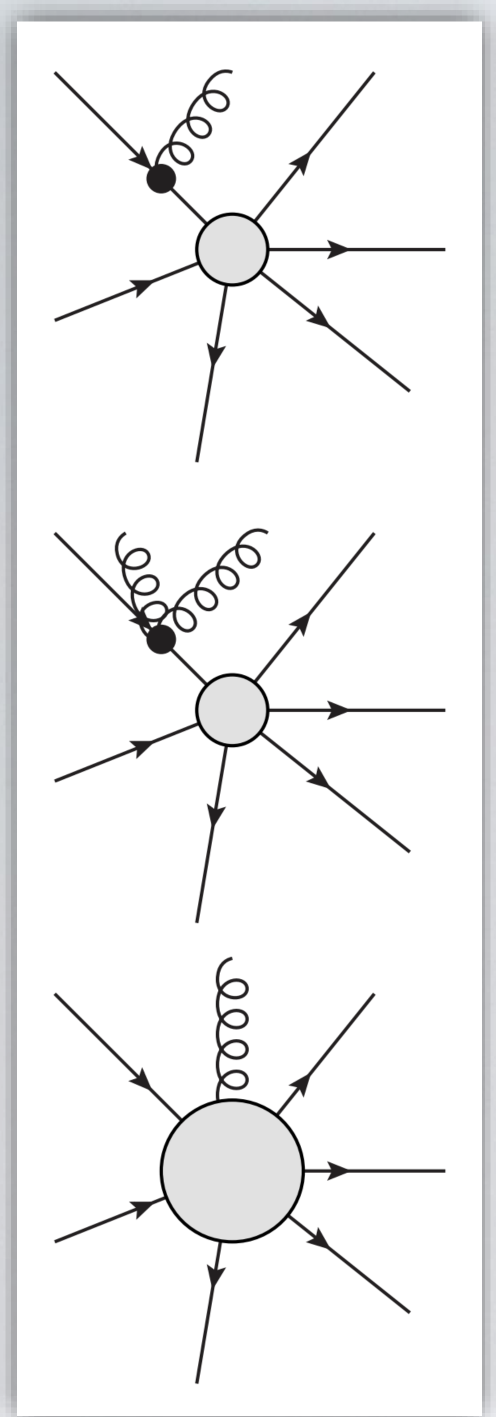
$$\sim \mathcal{M}\,\mathcal{S}\,u(p_1)\bar{v}(p_2)\ldots\bar{u}(p_n),$$

$$\mathcal{S} = \langle 0|\Phi_1\ldots\Phi_n|0\rangle \sim e^{\mathcal{W}_E}.$$

Collins, Soper,Sterman, 1989;

Gardi, Laenen, Stavenga, White, 2010;
Gardi, Smillie, White, 2013

FACTORIZATION OF SOFT GLUONS BEYOND LP

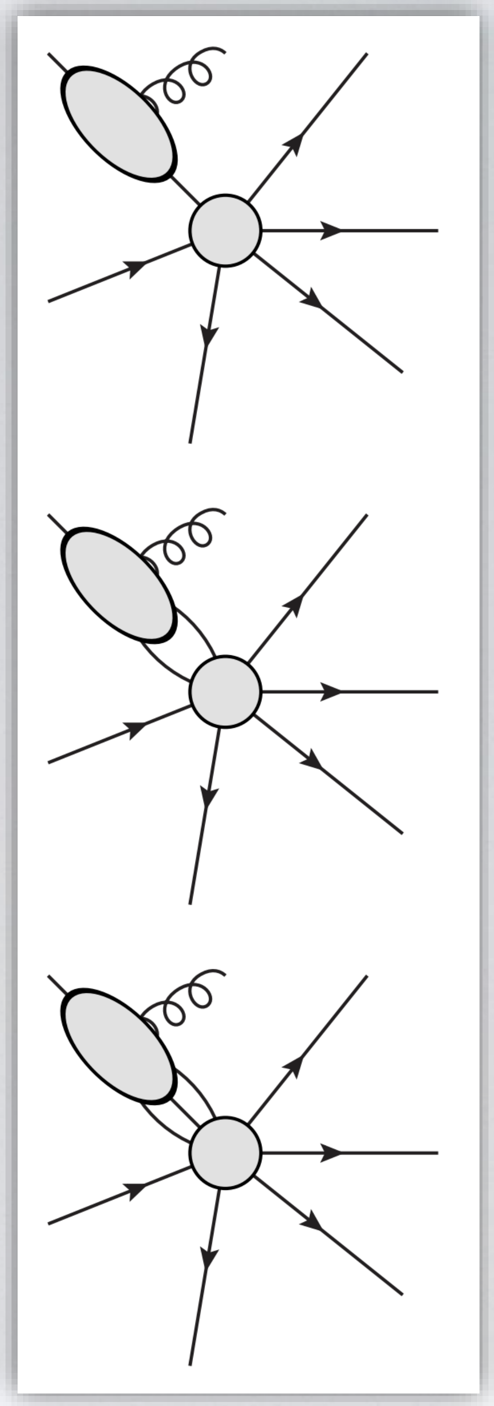


- Emission of soft gluons beyond the eikonal approximation, for instance sensitive to the spin of the emitting particle

Laenen, Magnea, Stavenga, White, 2009, 2010; Bonocore, Laenen, Magnea, LV, White, 2016.

- The soft emission resolve the hard interaction (LBK theorem)

Low 1958, Burnett,Kroll 1968



- Emission of soft gluons from a cluster of collinear particles: one finds several types of "radiative jets".

Del Duca 1990;

Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016;

Gervais 2017;

Laenen, Sinninghe-Damsté, LV, Waalewijn, Zoppi, 2020

Factorize soft and collinear radiation from the hard interaction:

→ by means of Soft-Collinear Effective Field Theory (SCET);
→ by means of a diagrammatic approach in QCD.

DIAGRAMMATIC APPROACH

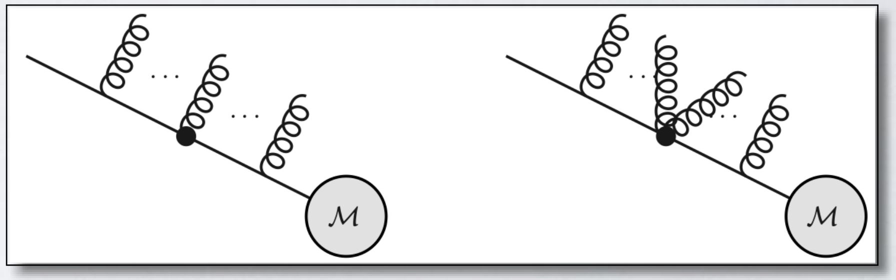
- Describe momentum regions in terms of universal functions in QCD:
- For instance, for Drell-Yan we have

Low 1958, Burnett,Kroll 1968

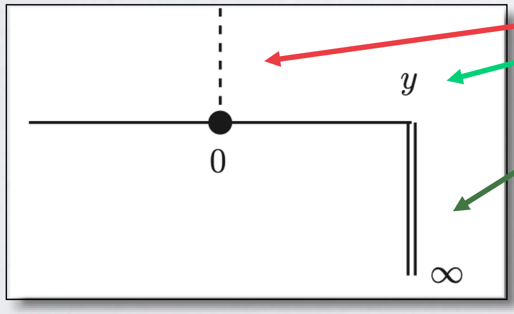
  - "Derivative" of the non-radiative amplitude,

$$\frac{k^\nu}{p_l \cdot k}\left[p_{l,\nu}\frac{\partial}{\partial p_l^\mu} - p_{l,\mu}\frac{\partial}{\partial p_l^\nu}\right]\mathcal{A}(p_j)$$

  - "Generalized" soft function,
  - "Radiative" jet function,



$$J_{\mu,a}(p,n,k)\, u(p) = \int d^d y \; \mathrm{e}^{-\mathrm{i}(p-k)\cdot y} \, \langle 0 \,|\, \Phi_n(\infty, y)\, \psi(y)\, j_{\mu,a}(0)\,|\, p\rangle \, .$$



Del Duca 1990,

Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016

$$F_p(-\infty,0) = \mathcal{P}\exp\left[g\int\frac{d^dk}{(2\pi)^d}A_\mu(k)\left(-\frac{p^\mu}{p\cdot k}+\frac{k^\mu}{2p\cdot k}-k^2\frac{p^\mu}{2(p\cdot k)^2} - \frac{\mathrm{i}k_\nu\Sigma^{\nu\mu}}{p\cdot k}\right)+\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dl}{(2\pi)^d}A_\mu(k)A_\nu(l)\left(\frac{\eta^{\mu\nu}}{2p\cdot(k+l)}+\ldots\right)\right].$$

Laenen, Magnea, Stavenga, White, 2009, 2010

Bonocore, Laenen, Magnea, LV, White,2016

- One has

$$\mathcal{A}_{\mu,a}(p_j,k) = \sum_{i=1}^{2}\left(\frac{1}{2}\,\widetilde{\mathcal{S}}_{\mu,a}(p_j,k) + g\,\mathbf{T}_{i,a}\,G^\nu_{i,\mu}\,\frac{\partial}{\partial p_i^\nu} + J_{\mu,a}(p_i,n_i,k)\right)\mathcal{A}(p_j) - \mathcal{A}^{\widetilde{\mathcal{J}}}_{\mu,a}(p_j,k)\,,$$

for $n_1 = p_2$, $n_2 = p_1$.

(Removes soft-collinear overlap in the radiative jet)

SOFT-COLLINEAR EFFECTIVE FIELD THEORY

- Effective Lagrangian and operators made of collinear and soft fields.

$$\mathcal{L}_{\mathrm{SCET}} = \sum_i \mathcal{L}_{c_i} + \mathcal{L}_s,$$

Bauer, Fleming, Pirjol, Stewart, 2000,2001; Beneke, Chapovsky, Diehl, Feldmann, 2002; Hill, Neubert 2002.

$$\mathcal{O}_n = \int dt_1 \dots dt_n \, \mathcal{C}(t_1, \dots, t_n) \, \phi_1(t_1 n_{1+}) \dots \phi_n(t_n n_{n+}).$$

- Constructed to reproduce a scattering process as obtained with the method of regions.
- The cross section factorizes into a hard scattering kernel, and matrix elements of soft and collinear fields.

$$\sigma \sim \mathcal{H} \otimes \mathcal{J}_1 \otimes \dots \otimes \mathcal{J}_n \otimes \mathcal{S}.$$

Hard matching coefficient

Jet functions – matrix elements of collinear fields

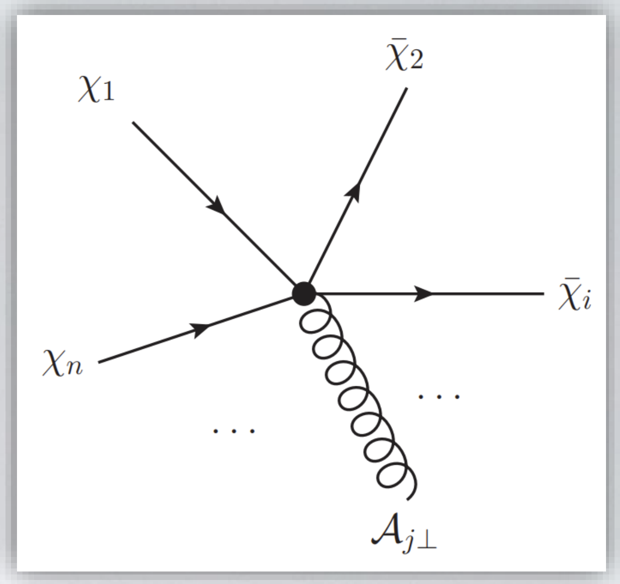
Soft function – matrix element of soft fields

- Renormalize UV divergences of EFT operators and obtain renormalization group equations.
- Each function depends on a single scale: solving the RGE resums large logarithms.

See e.g. Becher, Neubert 2006

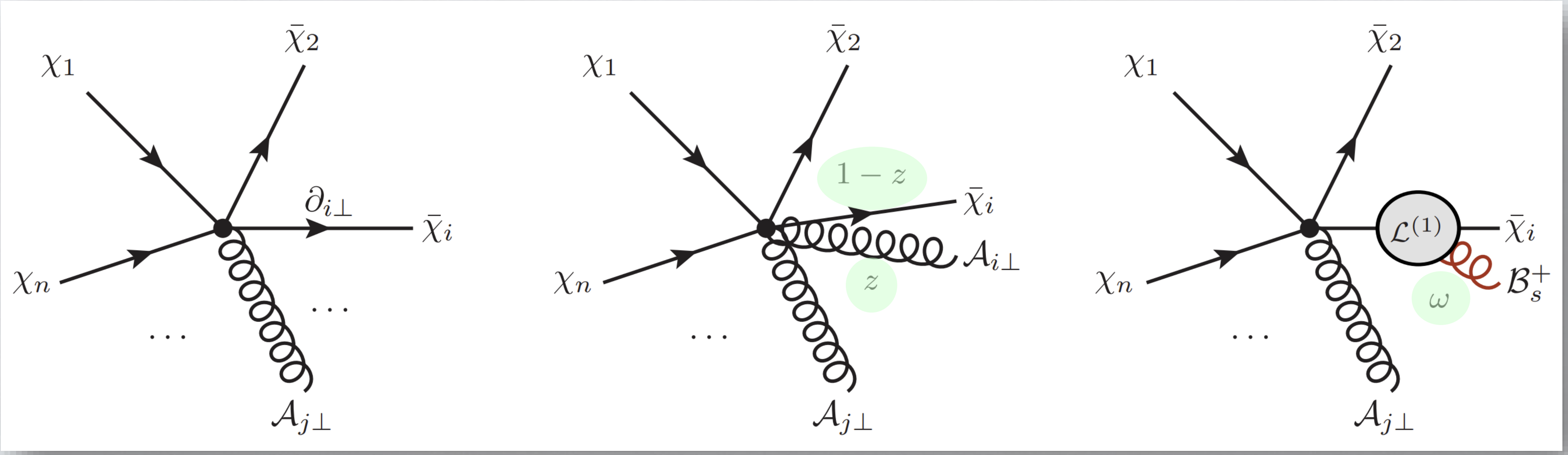
FACTORIZATION IN SCET: LP VS NLP

- Leading power (LP):
  - N-jet operators;
  - Soft-collinear decoupling.



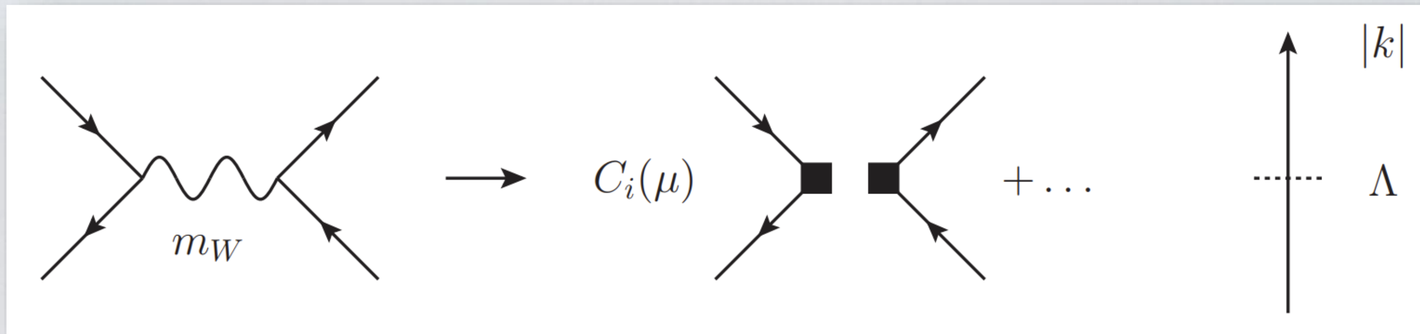
- Next-to-leading power (NLP):
  - Kinematic suppression;
  - Multi-particle emission along the same collinear direction;
  - No soft-collinear decoupling.

Beneke, Garny, Szafron, Wang, 2017,2018

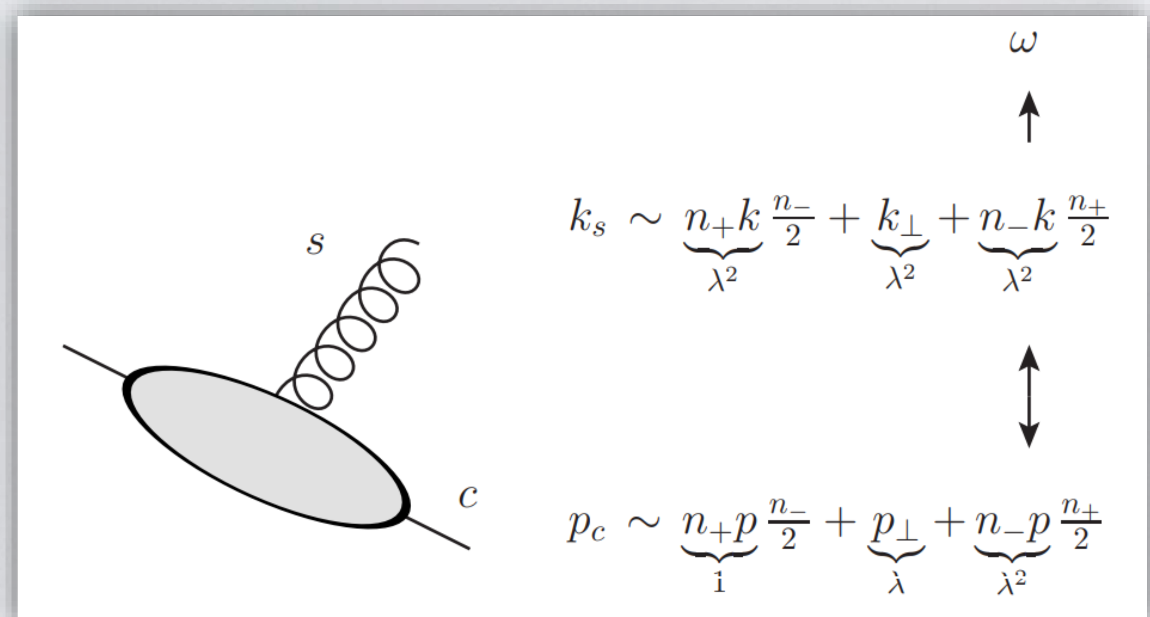


FACTORIZATION IN SCET: NLP

- "Standard" EFTs:



- Non-local EFTs:



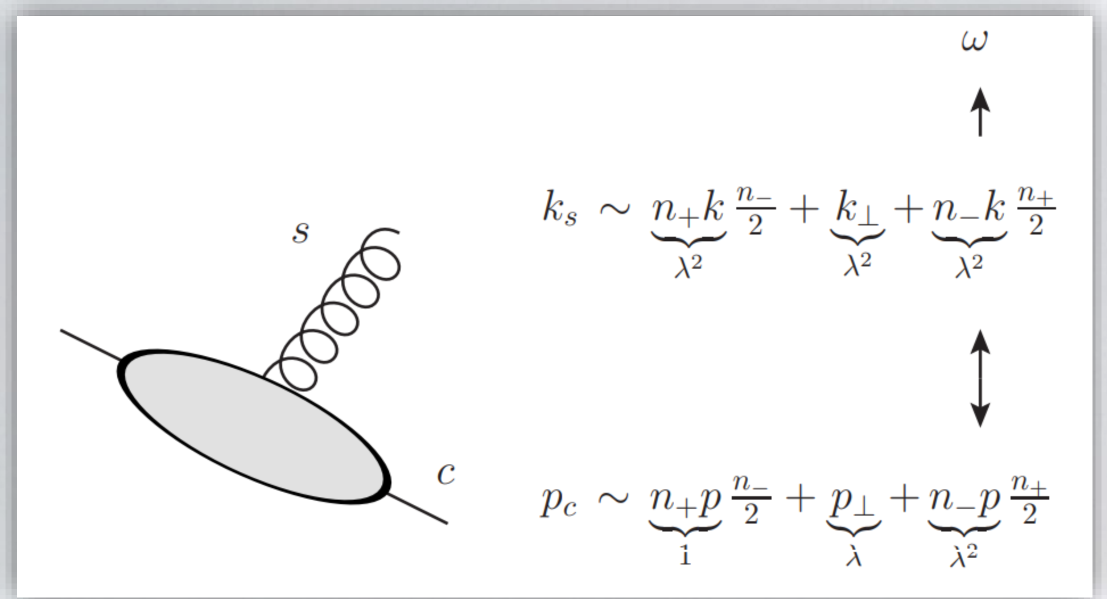
$$\longrightarrow \int d\omega\, J(\omega)\, S(\omega),$$

- At LP convolutions become trivial thanks to the "decoupling transformation": soft-collinear interactions decouple at LP.
- Beyond LP this does not occur, and convolutions are unavoidable.

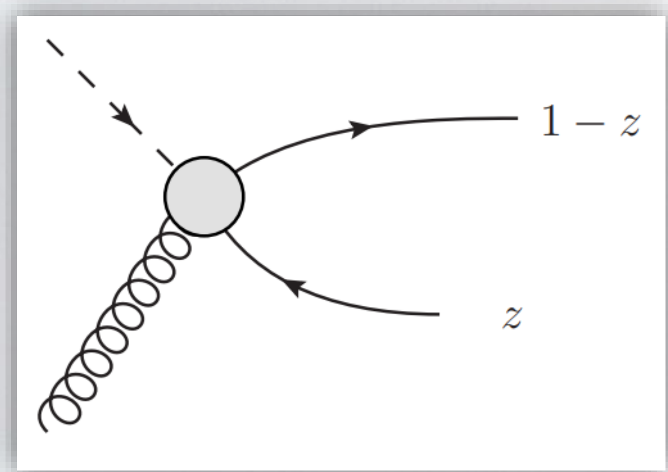
FACTORIZATION IN SCET: NLP

- Convolutions are divergent in $d = 4$!

First observed in Beneke, LV 2008;
Liu, Mecaj, Neubert, Wang, 2019-2020;
Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018
Beneke, Broggio, Jaskiewicz, LV, 2019



$$\longrightarrow \int_0^\Omega d\omega\, \underbrace{\left(n_+p\,\omega\right)^{-\epsilon}}_{\text{collinear piece}}\, \underbrace{\frac{1}{\omega^{1+\epsilon}}\frac{1}{(\Omega-\omega)^\epsilon}}_{\text{soft piece}},$$

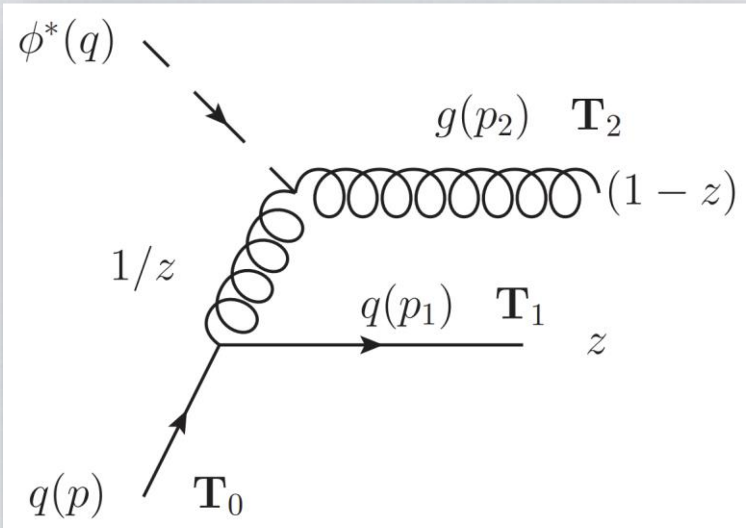


$$\longrightarrow \int_0^1 dz\, \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^\epsilon \frac{\alpha_s C_F}{2\pi}\frac{(1-z)^2}{z}\Big|_{s_{qg}=Q^2\frac{1-x}{x}}\,.$$

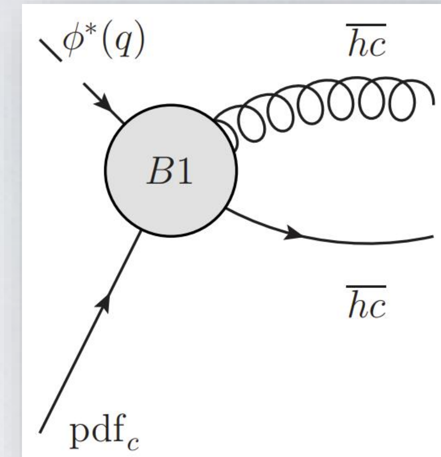
- Cannot apply the standard RGE methods directly to the collinear and soft functions.

BREAKDOWN OF FACTORIZATION NEAR THE ENDPOINT

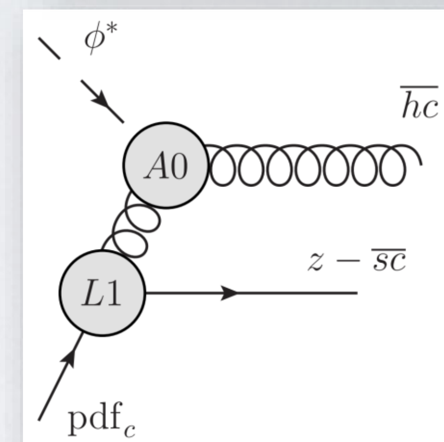
- What happens for $z \to 0$?



For z ~ 1 intermediate propagator is hard



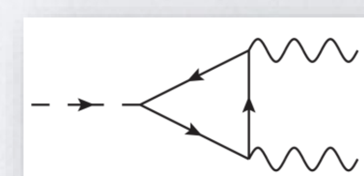
For z « 1 intermediate propagator cannot be integrated out



- Dynamic scale: zQ^2.
- In the endpoint region new counting parameter, $\lambda^2 \ll z \ll 1$.
- New modes contribute: z-softcollinear.
- Need re-factorization:

$$\underbrace{C^{B1}(Q,z)}_{\text{multi-scale function}}\, J^{B1}(z) \overset{z\to 0}{\longrightarrow} C^{A0}(Q^2) \int d^4x\, \mathbf{T}\Big[J^{A0}, \mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = \underbrace{C^{A0}(Q^2) D^{B1}(zQ^2,\mu^2)}_{\text{single-scale functions}}\, J^{B1}_{z-\overline{sc}}.$$

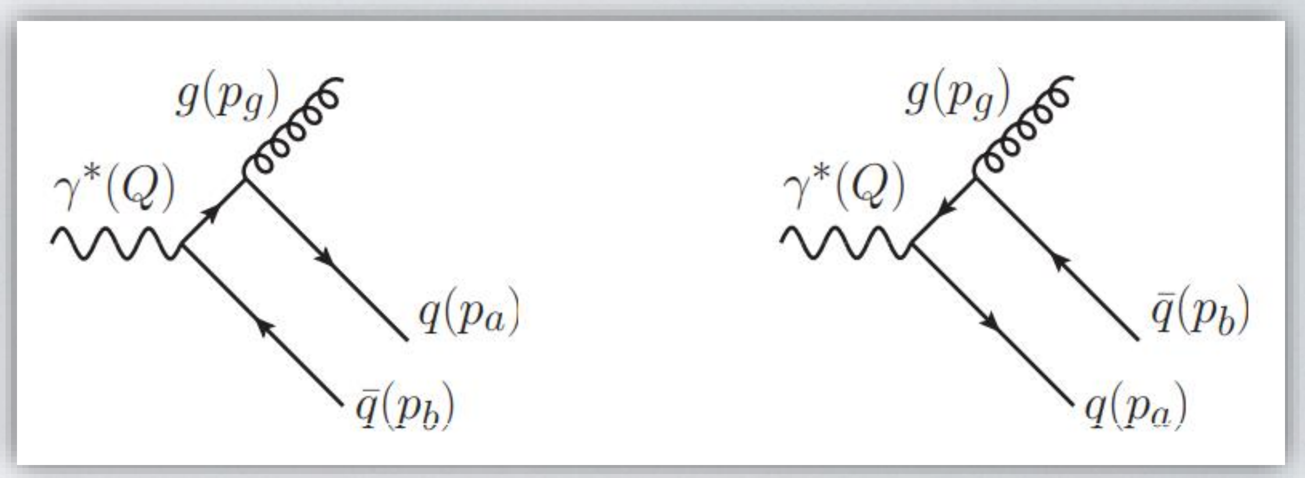
- Similar re-factorization proven in Liu, Mecaj, Neubert, Wang 2020.



OFF-DIAGONAL "GLUON" THRUST

- Consider the power-suppressed contribution to Thrust in the two-jet region:

$$e^+e^- \to \gamma^* \to [g]_c + [q\bar{q}]_{\bar{c}}.$$



- Within SCET one has two contributions:
  "Direct" term (B-type) and time-ordered product soft-quark term (A-type):

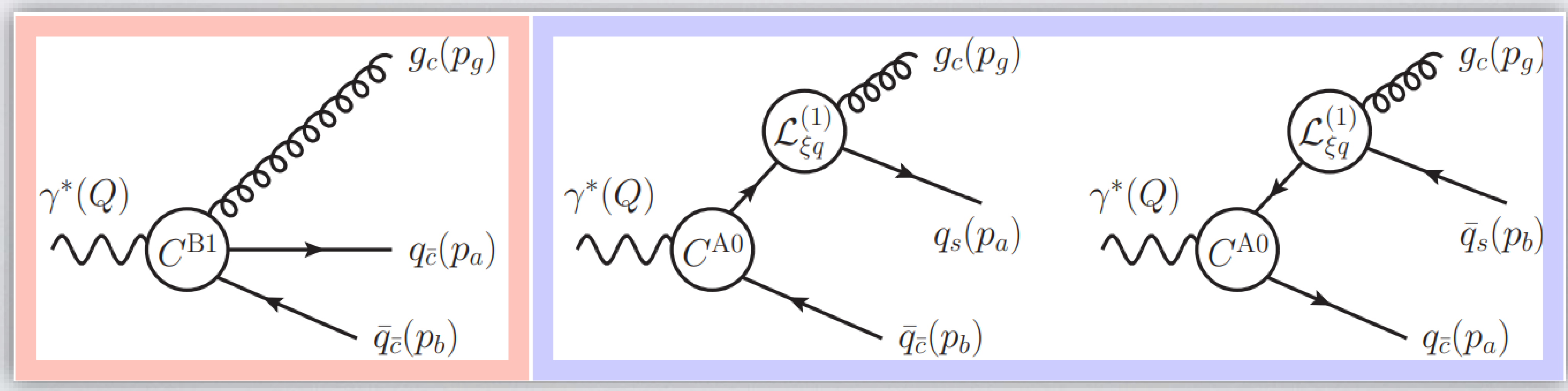
$$\bar{\psi}\gamma_\perp^\mu\psi(0) = \int dt d\bar{t}\, \widetilde{C}^{\rm A0}(t,\bar{t})\, \bar{\chi}_c(tn_+)\gamma_\perp^\mu \chi_{\bar{c}}(\bar{t}n_-) + (c \leftrightarrow \bar{c})$$

$$+ \sum_{i=1,2} \int dt d\bar{t}_1 d\bar{t}_2\, \widetilde{C}_i^{\rm B1}(t,\bar{t}_1,\bar{t}_2)\, \bar{\chi}_{\bar{c}}(\bar{t}_1 n_-)\Gamma_i^{\mu\nu} \mathcal{A}_{c\perp\nu}(tn_+)\chi_{\bar{c}}(\bar{t}_2 n_-) + \ldots$$

"Soft quark Sudakov" in Moult, Stewart, Vita, Zhu, 2019

$$\mathcal{L}_{\xi q}(x) = \bar{q}_s(x_-)\not{\mathcal{A}}_{c\perp}(x)\chi_c(x) + \text{h.c.}\,.$$

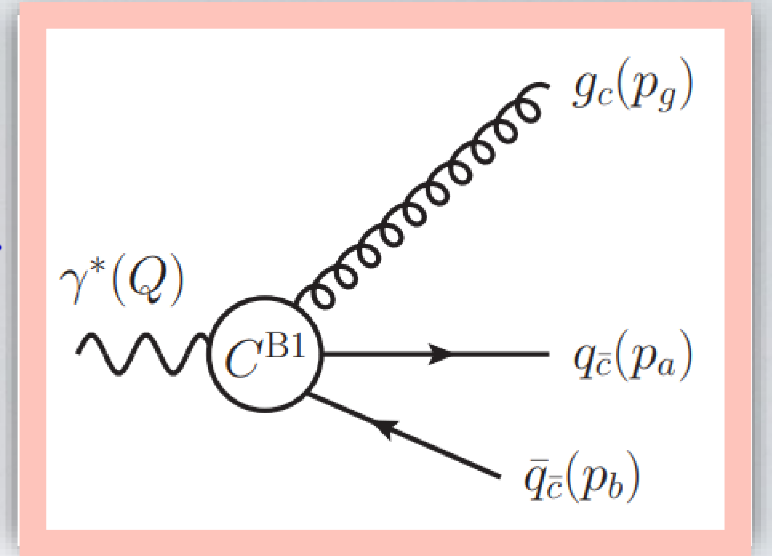
Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022



OFF-DIAGONAL "GLUON" THRUST

- "Direct" B-type term expressed in hard, (anti-)collinear and soft function:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\mathrm{B}} \sim \int_0^1 dr\, dr'\, C^{B1}(r)\, C^{B1}(r') \times \mathcal{J}_{\bar{c}}^{(q\bar{q})}(r,r') \otimes \mathcal{J}_c^{(g)} \otimes S^{(g)}.$$

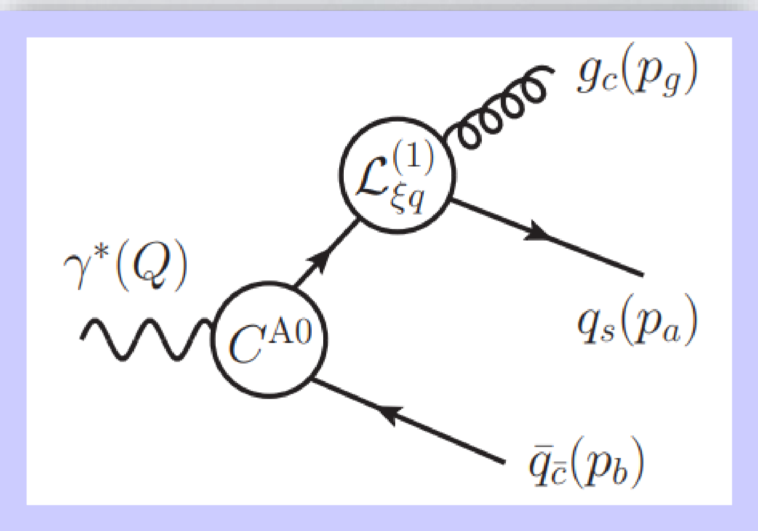


- It develops endpoint divergences when the quark *(r→0)* or anti-quark *(r→1)* become soft:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\mathrm{B}} \propto \int_0^1 dr \left[\frac{1}{r^{1+\epsilon}} + \frac{1}{(1-r)^{1+\epsilon}}\right].$$

- Time-ordered product A-type term expressed in hard, (anti-)collinear and soft function:

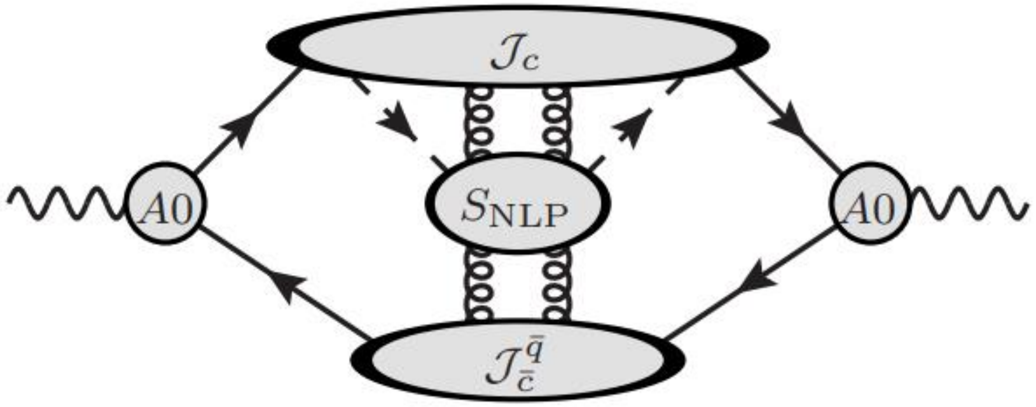
$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\mathrm{A}} \sim \int_0^\infty d\omega\, d\omega'\, |C^{A0}|^2 \times \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes \mathcal{J}_c(\omega,\omega') \otimes S_{\mathrm{NLP}}(\omega,\omega').$$



- It develops endpoint divergences when the soft quark or anti-quark become energetic *(ω→∞)*:

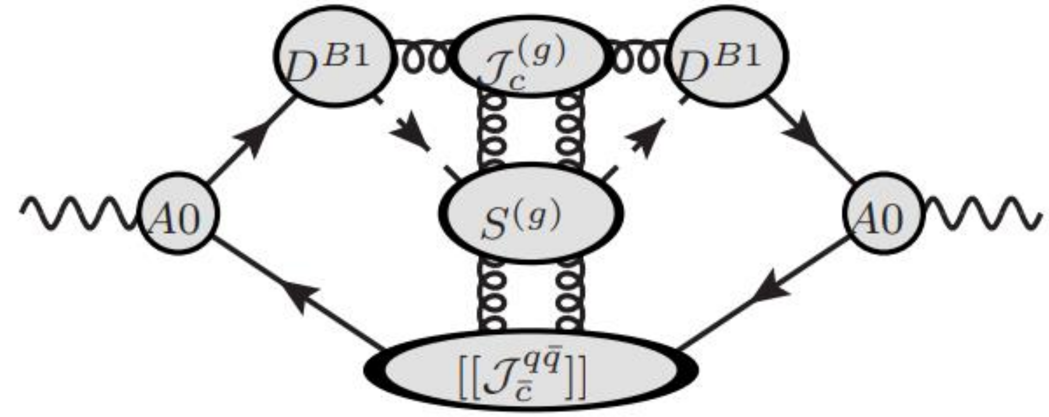
$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\mathrm{A}} \propto 2\int_{M_R^2/Q}^\infty d\omega\, \frac{1}{\omega^{1+\epsilon}}.$$

OFF-DIAGONAL “GLUON” THRUST



$$\omega, \omega' \to \infty$$

$$[[\mathcal{J}_c\,(p^2, \omega, \omega')]] \to \mathcal{J}_c^{(g)}(p^2)\,\frac{D^{\rm B1}(\omega Q)}{\omega}\,\frac{D^{\rm B1\,*}(\omega' Q)}{\omega'}$$

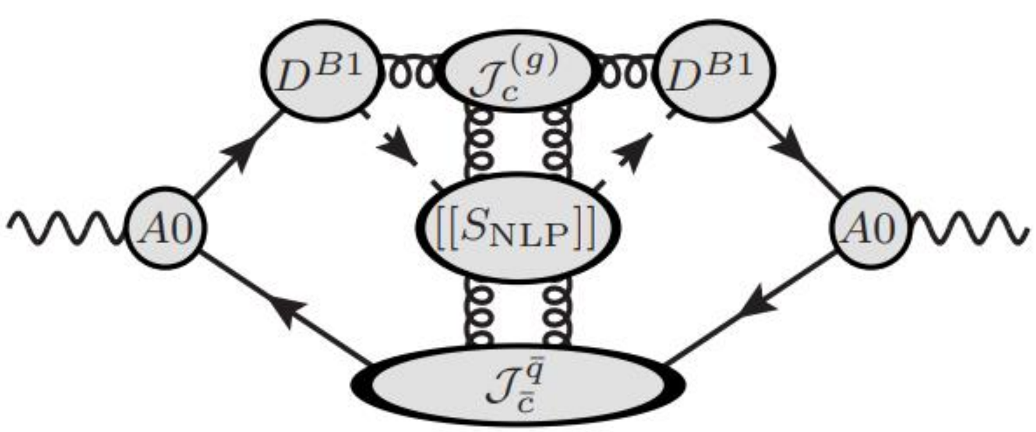


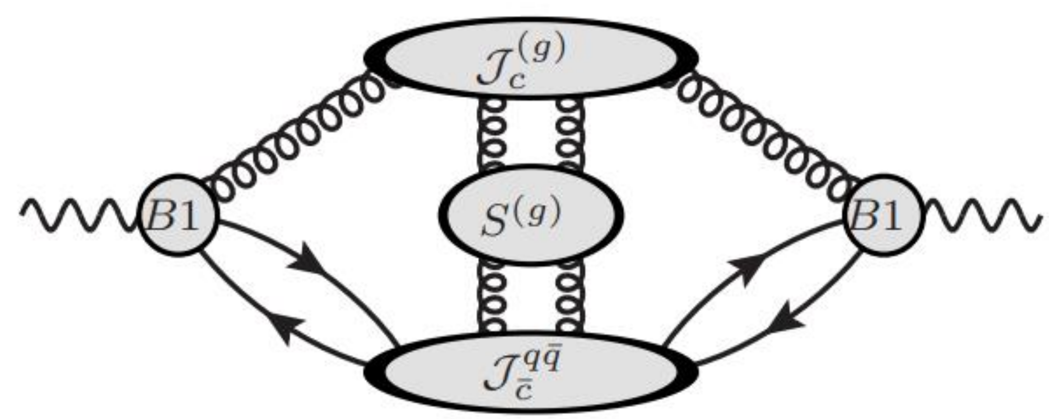
Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022

$$Q\,\widetilde{\mathcal{J}}_{\bar c}^{(\bar q)}(s_R)\,[[\widetilde{S}_{\rm NLP}(s_R, s_L, \omega, \omega')]] \to [[\widetilde{\mathcal{J}}_{\bar c}^{q\bar q(8)}(s_R, r, r')]]\widetilde{S}^{(g)}(s_R, s_L)$$

$$[[C_1^{\rm B1}(Q^2, r)]] \to C^{\rm A0}(Q^2) \times \frac{D^{\rm B1}(rQ^2)}{r}$$

$$r, r' \to 0$$





OFF-DIAGONAL "GLUON" THRUST

- Refactorized factorization formula:

Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022

$$\frac{1}{\sigma_0}\frac{d\widetilde{\sigma}}{ds_R ds_L}|_{\mathrm{A-type}} = \frac{2C_F}{Q}\, f(\epsilon)\, |C^{\mathrm{A0}}(Q^2)|^2\, \widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R) \int_0^\infty d\omega d\omega' \times \Big\{ \widetilde{\mathcal{J}}_c(s_L,\omega,\omega')\, \widetilde{S}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega') - \theta(\omega-\Lambda)\theta(\omega'-\Lambda)\, [\![\widetilde{\mathcal{J}}_c(s_L,\omega,\omega')]\!]\, [\![\widetilde{S}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega')]\!] + \widetilde{\widetilde{\mathcal{J}}}_c(s_L,\omega,\omega')\, \widetilde{\widetilde{S}}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega') \Big\},$$

$$\frac{1}{\sigma_0}\frac{d\widetilde{\sigma}}{ds_R ds_L}|_{\substack{\mathrm{B-type}\\ \mathrm{i=i'=1}}} = \frac{2C_F}{Q^2}\, f(\epsilon)\, \widetilde{\mathcal{J}}_c^{(g)}(s_L)\, \widetilde{S}^{(g)}(s_R,s_L) \int_0^\infty dr dr' \times \Big[\theta(1-r)\theta(1-r')\, C_1^{\mathrm{B1}*}(Q^2,r') C_1^{\mathrm{B1}}(Q^2,r)\, \widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}(s_R,r,r') - [1-\theta(r-\Lambda/Q)\theta(r'-\Lambda/Q)] \times [\![C_1^{\mathrm{B1}*}(Q^2,r')]\!]_0\, [\![C_1^{\mathrm{B1}}(Q^2,r)]\!]_0\, [\![\widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}(s_R,r,r')]\!]_0 \Big].$$

- *Λ* dependence cancels between the two terms. Each separately independent of *dim reg μ.*
- In principle valid to any log accuracy. At LL only the subtraction terms contribute do to the extra log from large ω/small r.

NEXT-TO-LEADING POWER: PERSPECTIVE

- The resummation of large leading logarithms at NLP is now under control, both in the diagonal (quark-antiquark, gluon-gluon) and off-diagonal (quark-gluon) channels, in electroweak annihilation processes (Drell-Yan, Higgs production, etc) and DIS.

- The next step is to formalize the refactorization process, such as to allow for a systematic resummation at NLP, beyond leading logarithmic accuracy.

- These result will be applied to produce phenomenological analysis of relevant processes for the LHC;

- On the other hand, knowledge gained in understanding the structure of large logarithms at NLP near threshold will be useful to extend resummation at NLP to other kinematic limits (small pT, small β, etc).

EXTRA SLIDES

EXAMPLE: THE TWO-REGGEON CUT

- The amplitude reads

$$\hat{\mathcal{M}}^{(+,\ell)}_{\rm NLL} = -i\pi \frac{(B_0)^\ell}{(\ell-1)!} \int [{\rm D}k]\, \frac{p^2}{k^2(k-p)^2}\, \Omega^{(\ell-1)}(p,k)\, \mathbf{T}^2_{s-u}\, \mathcal{M}^{(0)}, \quad B_0 = e^{\epsilon\gamma_{\rm E}} \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}.$$

- One rung = apply once the BFKL kernel on the "target averaged wave function":

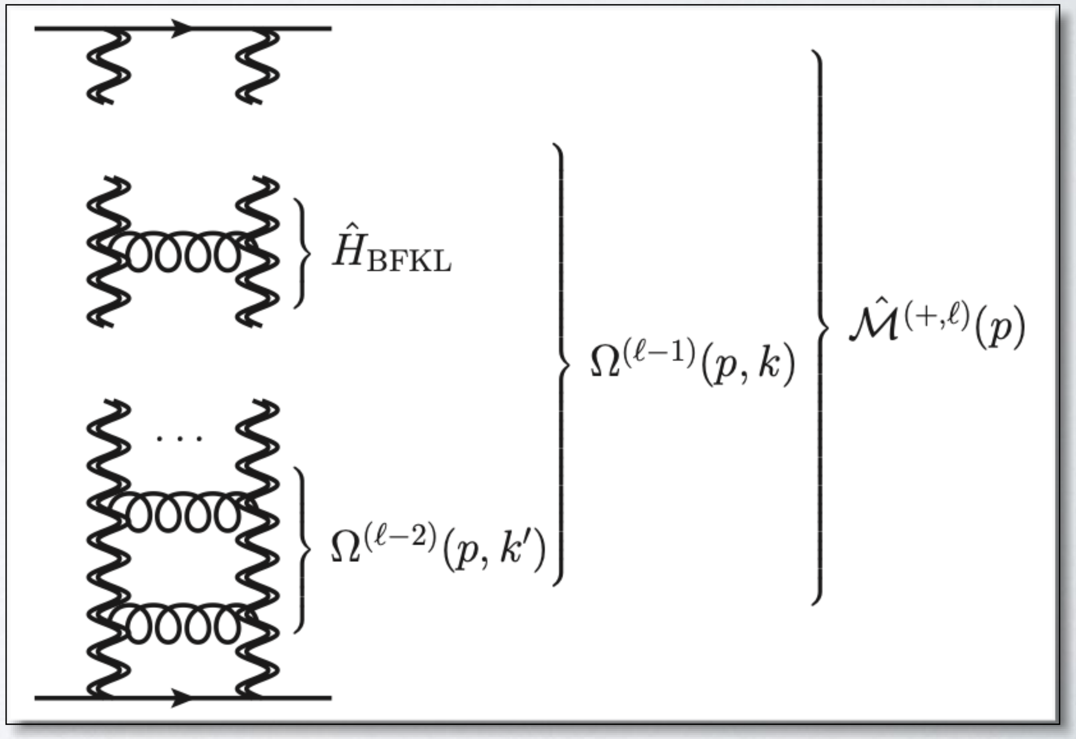
$$\Omega^{(\ell-1)}(p,k) = \hat{H}\, \Omega^{(\ell-2)}(p,k), \qquad \hat{H} = (2C_A - \mathbf{T}_t^2)\, \hat{H}_{\rm i} + (C_A - \mathbf{T}_t^2)\, \hat{H}_{\rm m}$$

- "Integration" part:

Caron-Huot, Gardi, Reichel, LV, 2017,2020

$$\hat{H}_{\rm i}\, \Psi(p,k) = \int [{\rm D}k']\, f(p,k,k') \Big[\Psi(p,k') - \Psi(p,k)\Big],$$

$$f(p,k',k) = \frac{k'^2}{k^2(k-k')^2} + \frac{(p-k')^2}{(p-k)^2(k-k')^2} - \frac{p^2}{k^2(p-k)^2}.$$



- "Multiplication" part:

$$\hat{H}_{\rm m}\Psi(p,k) = \frac{1}{2\epsilon}\left[2 - \left(\frac{p^2}{k^2}\right)^\epsilon - \left(\frac{p^2}{(p-k)^2}\right)^\epsilon\right]\Psi(p,k).$$

- Initial condition

$$\Omega^{(0)}(p,k) = 1.$$

APPLICATION: INFRARED DIVERGENCES

Re	L^0	L^1	L^2	L^3	L^4	L^5	L^6
α_s^1	$\frac{1}{4}\widehat{\gamma}_K^{(1)} \ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i + \sum_{i=1}^4 \gamma_i^{(1)}$	$\frac{1}{2}\widehat{\gamma}_K^{(1)}\mathbf{T}_t^2$					
α_s^2	$\frac{1}{4}\widehat{\gamma}_K^{(2)} \ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i + \sum_{i=1}^4 \gamma_i^{(2)}$	$\frac{1}{2}\widehat{\gamma}_K^{(2)}\mathbf{T}_t^2$	0				
α_s^3	$\frac{1}{4}\widehat{\gamma}_K^{(3)} \ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i + \sum_{i=1}^4 \gamma_i^{(3)} + \Delta^{(+,3,0)}$	$\frac{1}{2}\widehat{\gamma}_K^{(3)}\mathbf{T}_t^2$	0	0			
α_s^4			$\Delta^{(+,4,2)}$	0	0		
α_s^5					0	0	
α_s^6						0	0

Im	L^0	L^1	L^2	L^3	L^4	L^5	L^6
α_s^1	$\frac{1}{2}\widehat{\gamma}_K^{(1)} i\pi\mathbf{T}_{s-u}^2$	0					
α_s^2	$\frac{1}{2}\widehat{\gamma}_K^{(2)} i\pi\mathbf{T}_{s-u}^2$	0	0				
α_s^3	$\frac{1}{2}\widehat{\gamma}_K^{(3)} i\pi\mathbf{T}_{s-u}^2 + \Delta^{(-,3,0)}$	$\Delta^{(-,3,1)}$	0	0			
α_s^4				$\Delta^{(-,4,3)}$	0		
α_s^5					$\Delta^{(-,5,4)}$	0	
α_s^6						$\Delta^{(-,6,5)}$	0

Caron-Huot, Gardi, LV, 2017; Caron-Huot, Gardi, Reichel, LV, 2017; Gardi, Falcioni, Milloy, LV, 2020; Gardi, Falcioni, Maher, Milloy, LV, 2021.

APPLICATION: NUMBER THEORY

$$\hat{\mathcal{M}}_{\rm h}^{(11)} = \frac{i\pi}{8!}\Big\{C_2^2 C_A^8\Big(-\frac{44253\, g_{533}}{5120} - \frac{652795\zeta_3^2\zeta_5}{2048} - \frac{818318277\zeta_{11}}{327680}\Big)$$
$$+ C_2^3 C_A^7\Big(\frac{510873\, g_{533}}{5120} + \frac{10645591\zeta_3^2\zeta_5}{2048} + \frac{14761239427\zeta_{11}}{1966080}\Big)$$
$$+ \ldots + C_2^8 C_A^2\Big(-\frac{2158233\, g_{533}}{5120} - \frac{852453151\zeta_3^2\zeta_5}{2048} - \frac{1295244371839\zeta_{11}}{655360}\Big)$$
$$+ C_2^9 C_A\Big(\frac{6979863\, g_{533}}{5120} + \frac{2225183081\zeta_3^2\zeta_5}{2048} + \frac{741771390019\zeta_{11}}{655360}\Big)$$
$$+ C_2^{10}\Big(\frac{1094181\, g_{533}}{2560} + \frac{2638860059\zeta_3^2\zeta_5}{1024} + \frac{4498262900131\zeta_{11}}{655360}\Big)\Big\}.$$

Brown, 2004, 2013; Schnetz, 2013

- Hard region: only odd ζ_n, consistent with 2D wavefunction made of SVHPLs.
- Finite (hard) amplitude contains g_{533} at 11 loops:

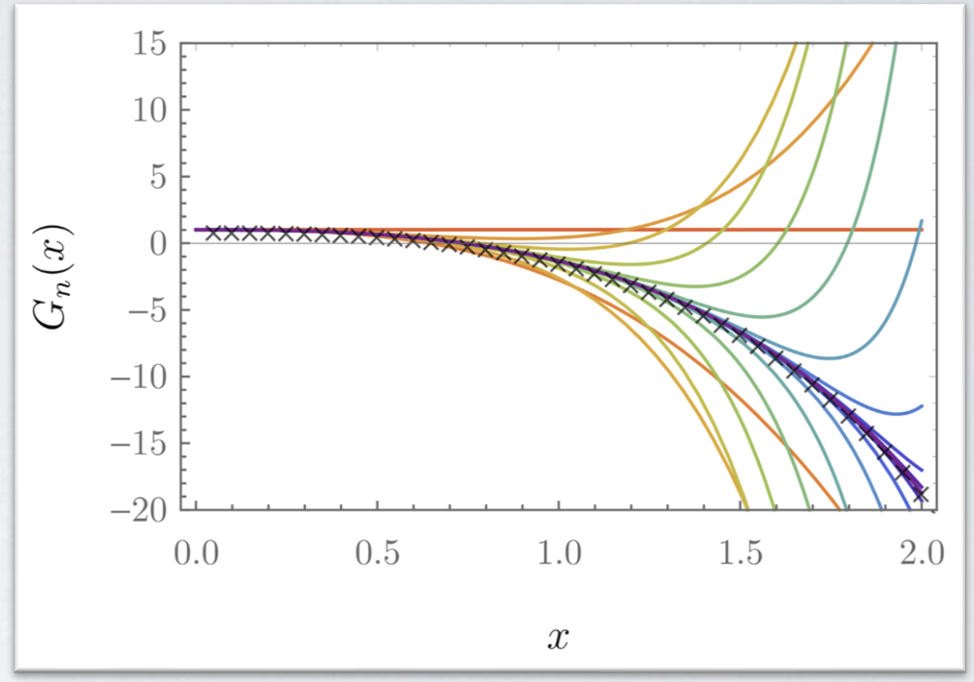
Caron-Huot, Gardi, Reichel, LV, 2020

$$g_{5,3,3} = -\frac{4}{7}\zeta_2^3\zeta_5 + \frac{6}{5}\zeta_2^2\zeta_7 + 45\zeta_2\zeta_9 + \zeta_{5,3,3}\,.$$

→ no exponentiation in terms of Γ functions.

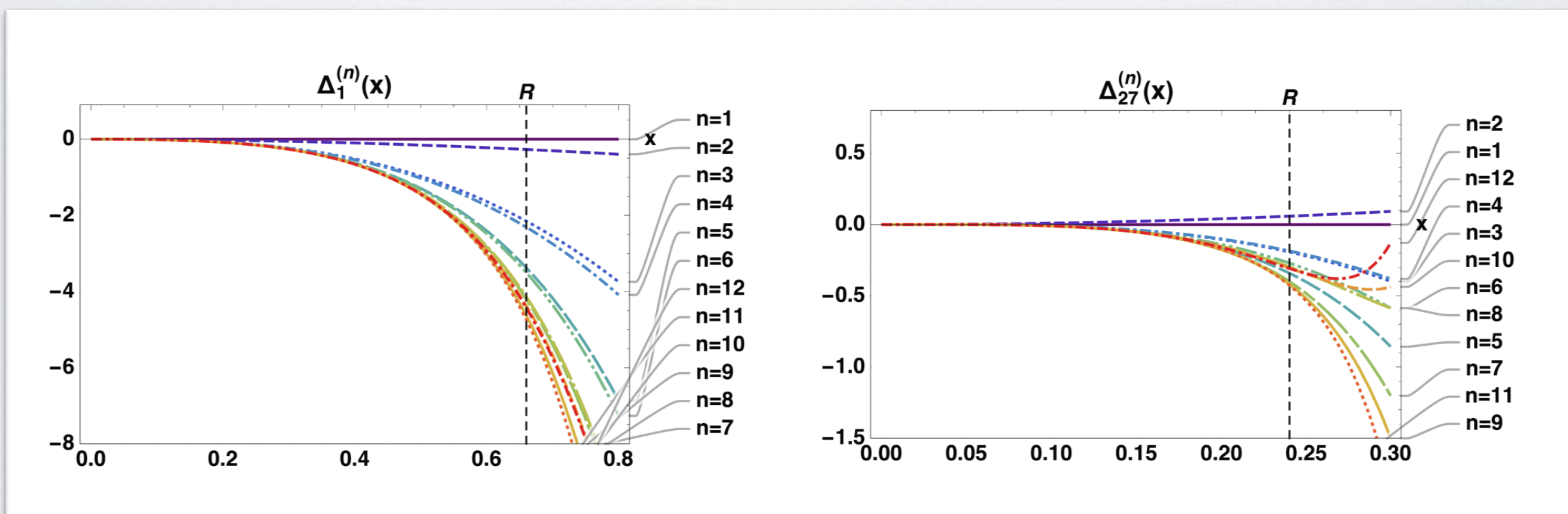
APPLICATION: NUMERICAL STUDIES

- The soft anomalous dimension has an infinite radius of convergence: entire function, free of singularities for any finite $x=\alpha_s/\pi\, L$.



Caron-Huot, Gardi, Reichel, LV, 2017, 2020

- The finite amplitude is an alternating series, whose coefficients grows geometrically:



- Finite radius of convergence in $\alpha_s/\pi\, L$ *that* stabilises to $|R| \simeq 0.66$ for singlet, $|R| \simeq 0.24$ for 27 representation, by means of a Padé approximant (pole at $-|R|$).

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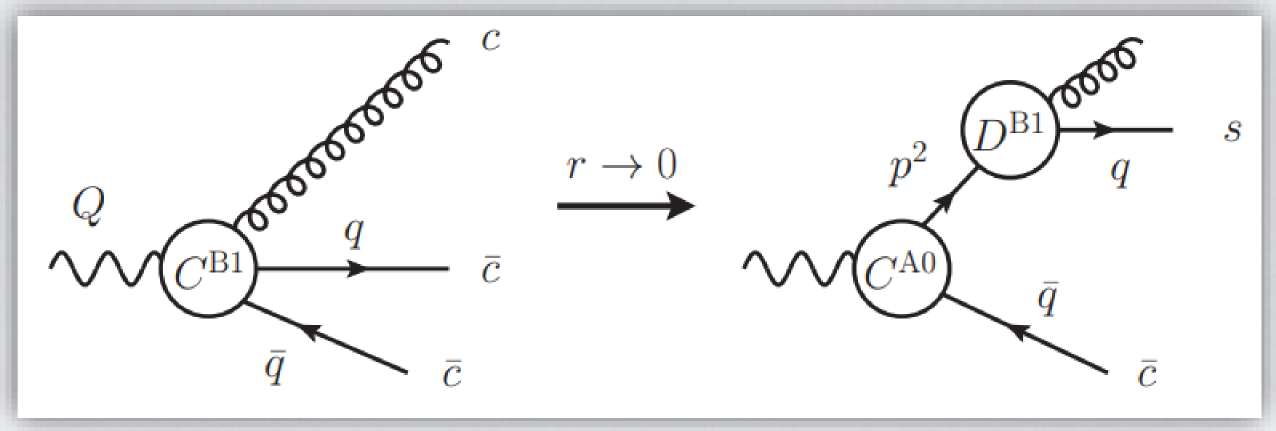
- As for DIS, in the *r→0* (or *r→1*) limit, the B1 coefficient is a two-scale object, which refactorizes, since the intermediate state develops an on-shell pole:



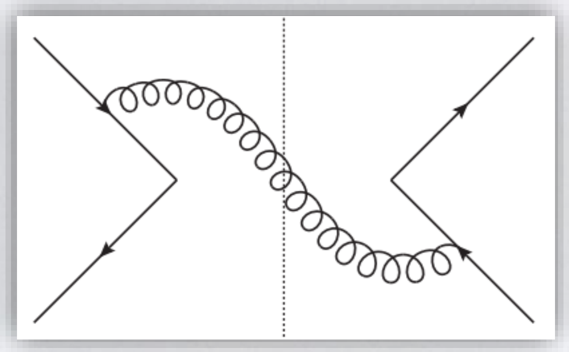
$$C_1^{\mathrm{B1}}(Q^2, r) = C^{\mathrm{A0}}(Q^2) \times \frac{D^{\mathrm{B1}}(rQ^2)}{r} + \mathcal{O}(r^0)\,.$$

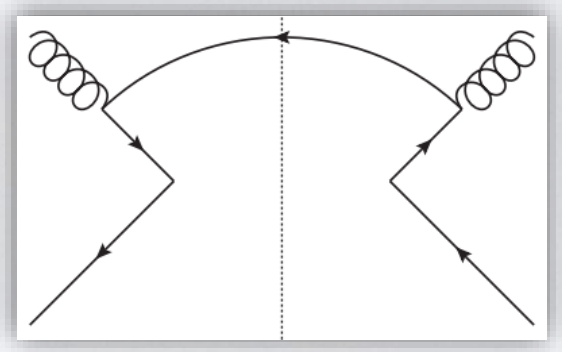
- In *d* dimensions the *1/ε* poles from the divergent convolution integrals cancels. The integrands of A and B match in the asymptotic limits *ω,ω′→∞* (A-type) and r,r′→0(1) (B-type).

- This allows a rearrangement between the terms that makes them separately finite, provided two additional refactorization conditions hold for the soft and jet functions:

DRELL YAN AT NLP

- D^{B1} is an example of new universal functions appearing at NLP.
- In general, these functions are more involved to compute compared to their LP counterparts, as they depend on more variables.
- On the other hands, refactorization conditions imposes additional constraints, as we have seen in case of thrust.
- Important to collect data on these functions. In this respect, in the past few years we have completed the calculation of all terms contributing to Drell-Yan at NLP, up to NNLO: this includes jet functions at NLO, and soft functions at NNLO.





$$\Delta^{dyn}_{q\bar{q},g\bar{q}}(z)|_{\rm NLP} \sim \sum_i \int \{d\omega\}\, J_i(\{\omega\})\, S_i(\{\omega\}).$$

qqb: J at one loop in
Beneke, Broggio, Jaskiewicz, LV, 2019

qg: J at one loop in
Broggio, Jaskiewicz, LV, 2023

qqb: S at two loops in
Broggio, Jaskiewicz, LV, 2021

qg: S at otwo loops in
Broggio, Jaskiewicz, LV, 2023