Phenomenology of semi-leptonic B meson decays

Santo Stefano Balbo - Theory group retreat

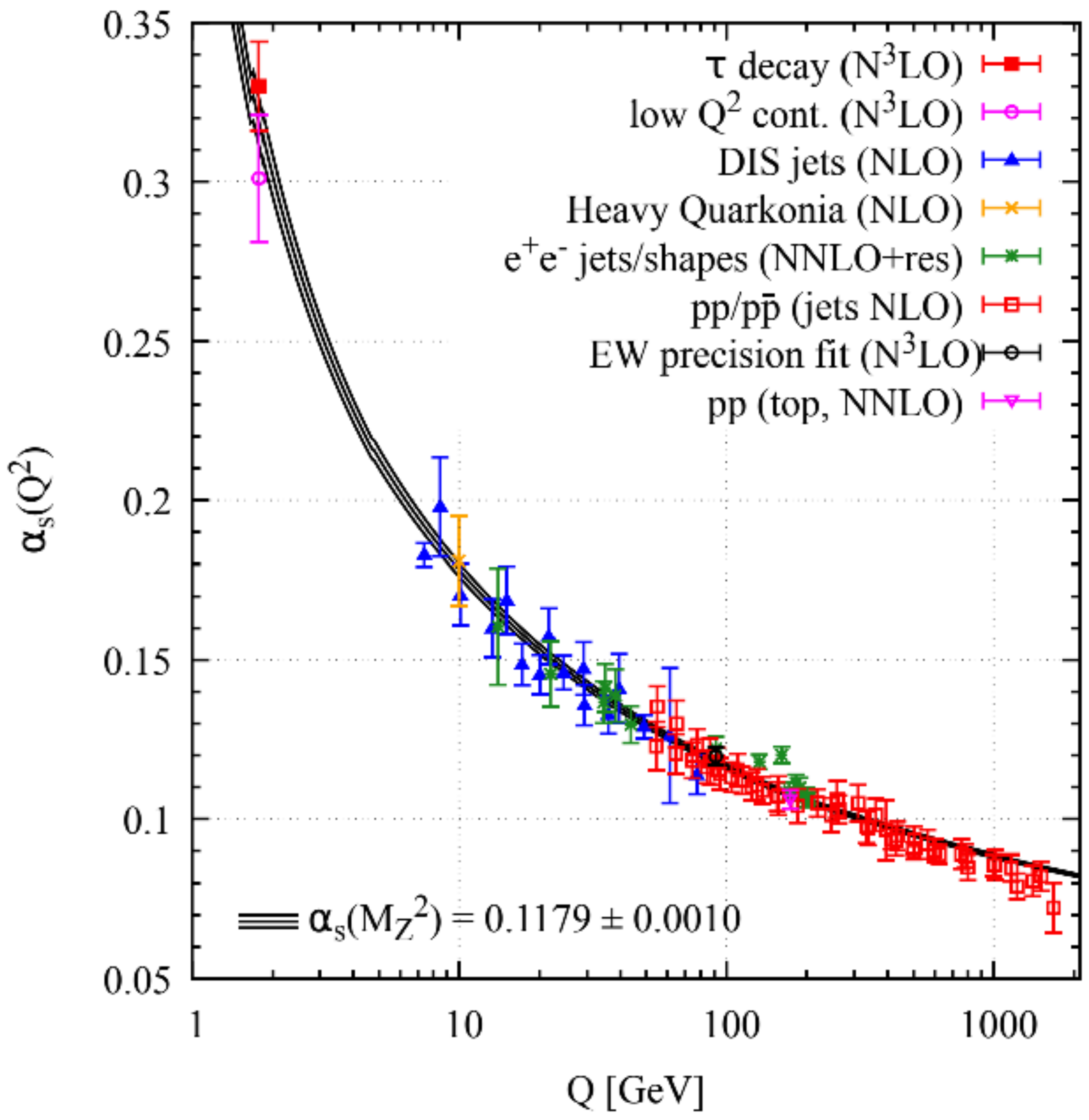
Alexandre Carvunis - 11/11/2023

Testing the Standard Model of particle physics

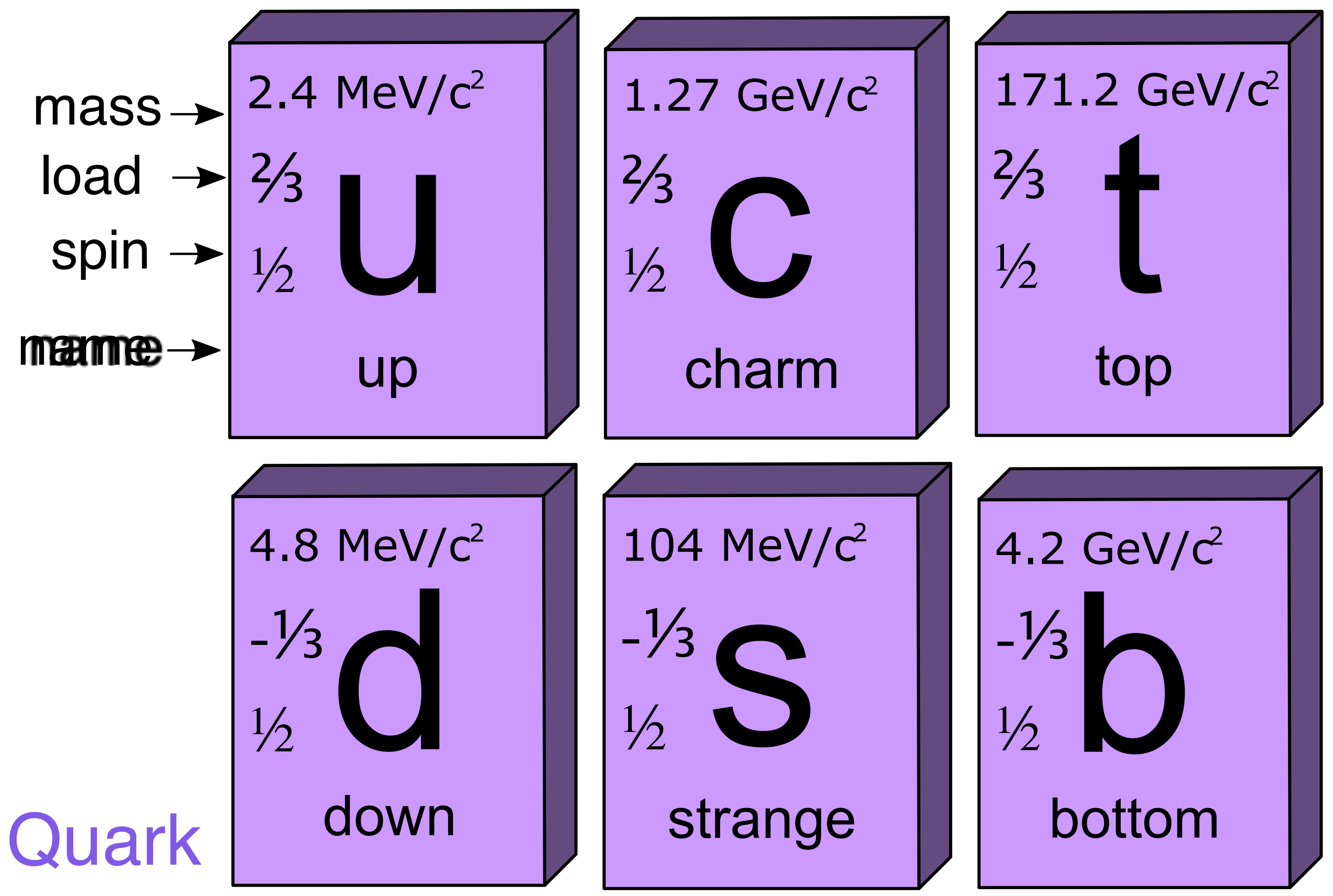
The big picture

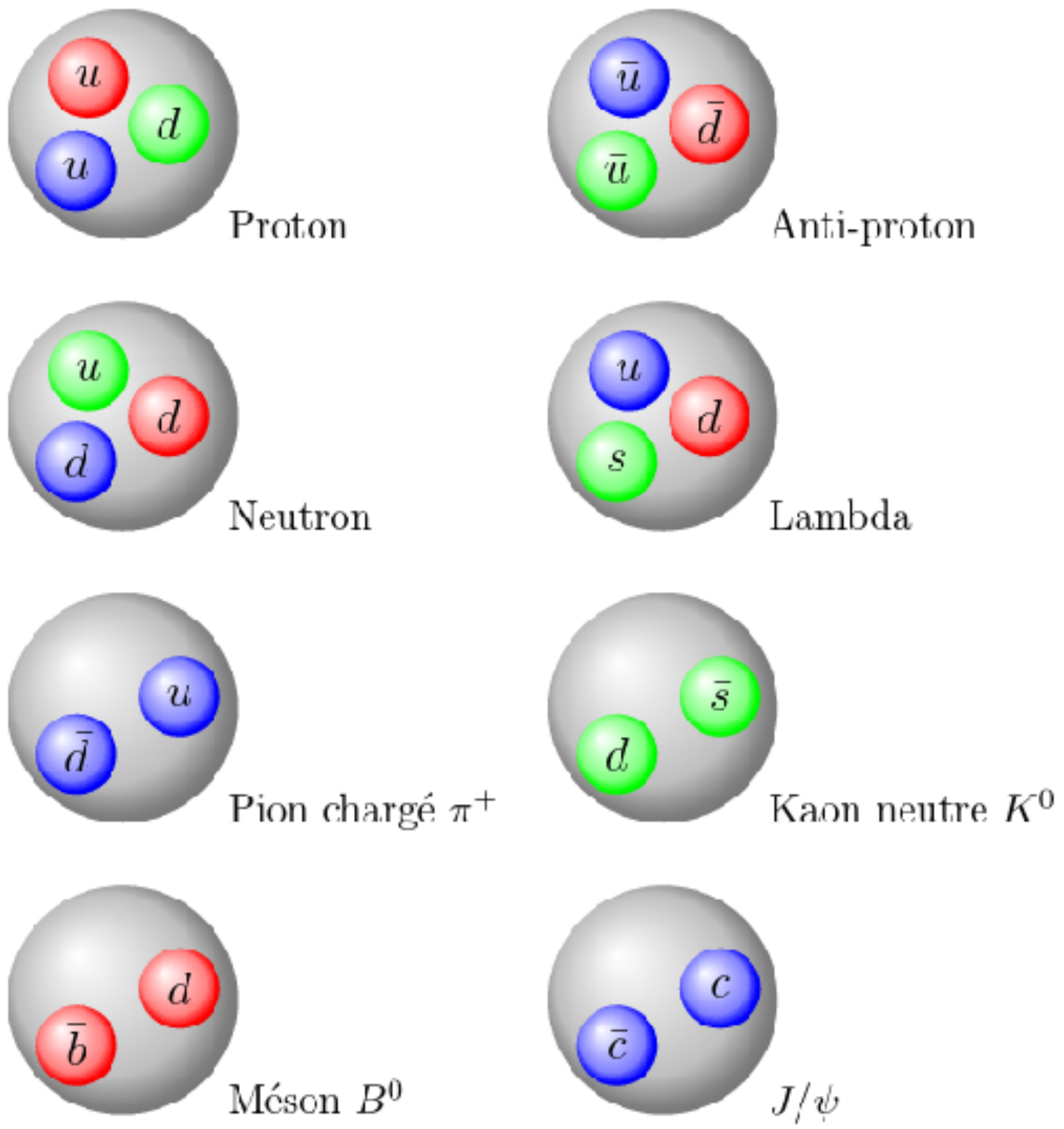
- We are certain that there is something beyond the Standard Model (SM) - dark matter, matter-antimatter asymmetry, quantum gravity, neutrino mass, etc.
- Yet, no (significant) sign of Beyond the SM physics has been observed in collider experiments, in neither direct and indirect searches.
- We reached maximum c.o.m. energy in the collision of the LHC, higher energies will not be probed (in colliders) before decades…
- High luminosity will allow us to measure rare processes and improve the measurement of most observables, and test the SM with a better sensitivity to potential small New Physics effects
- **TH can be the limiting factor for discovery, notably when non-perturbative QCD enters the picture**.

Confinement scale of QCD



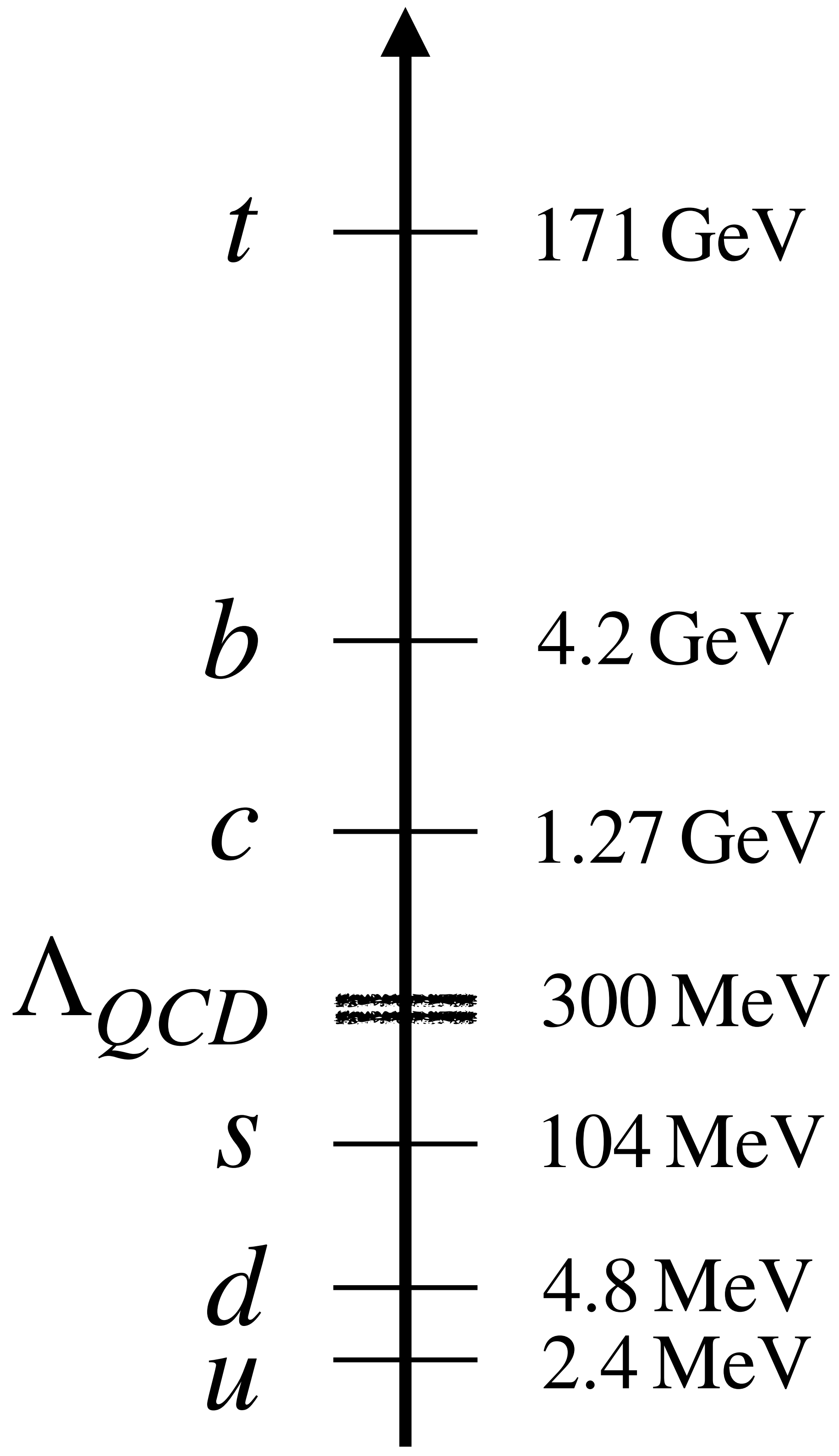
From PDG 2019

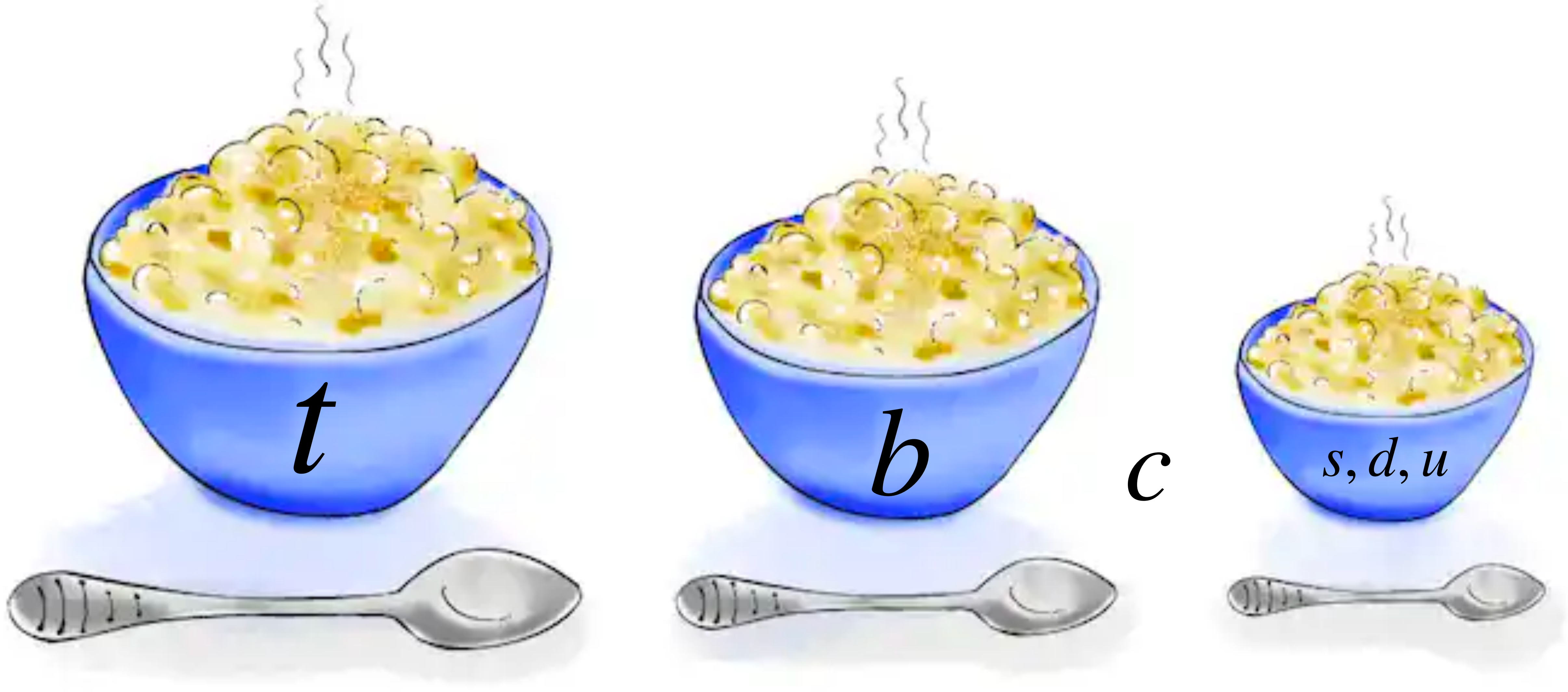




Why B mesons?

- The b quark is the Goldilocks quark for phenomenology, not too light, not too heavy
  - $m_b \gg \Lambda_{QCD}$ allows for perturbative expansion in $1/m_b$ with good results (HQS, HQE, HQET, NRQCD, etc)
  - $m_b \ll m_t\,, m_W$: the physics can be described by 'simple' EFTs
- Simultaneously B mesons are now measured accurately:
  - B^0 ($\bar{b}d$) and B^+ ($\bar{b}u$) can be produced in B factories $e^+e^- \to \Upsilon(4S) \to B\bar{B}$ (small background, full angular reconstruction), BaBar, Belle, Belle II
  - Higher luminosity of B mesons (and more flavors B_s, B_c) are produced and measured at the LHC thanks to the large energy at the center of mass, measurements by ATLAS, CMS, LHCb. (Large background, forward detection only at LHCb)

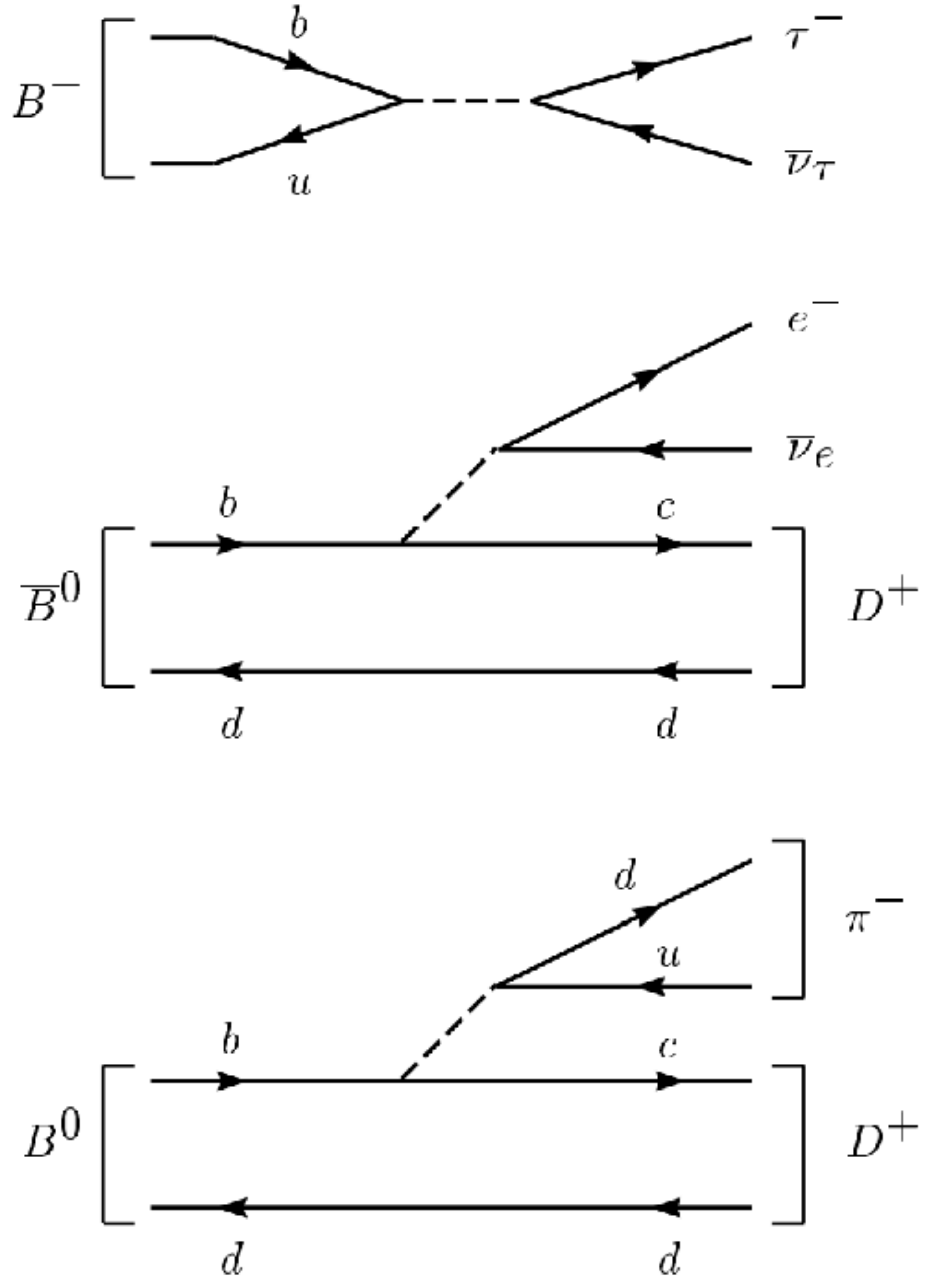



t
b
c
s, d, u

Why semi-leptonic decays?

Decay channel Goldilocks

Rare decay, small TH error

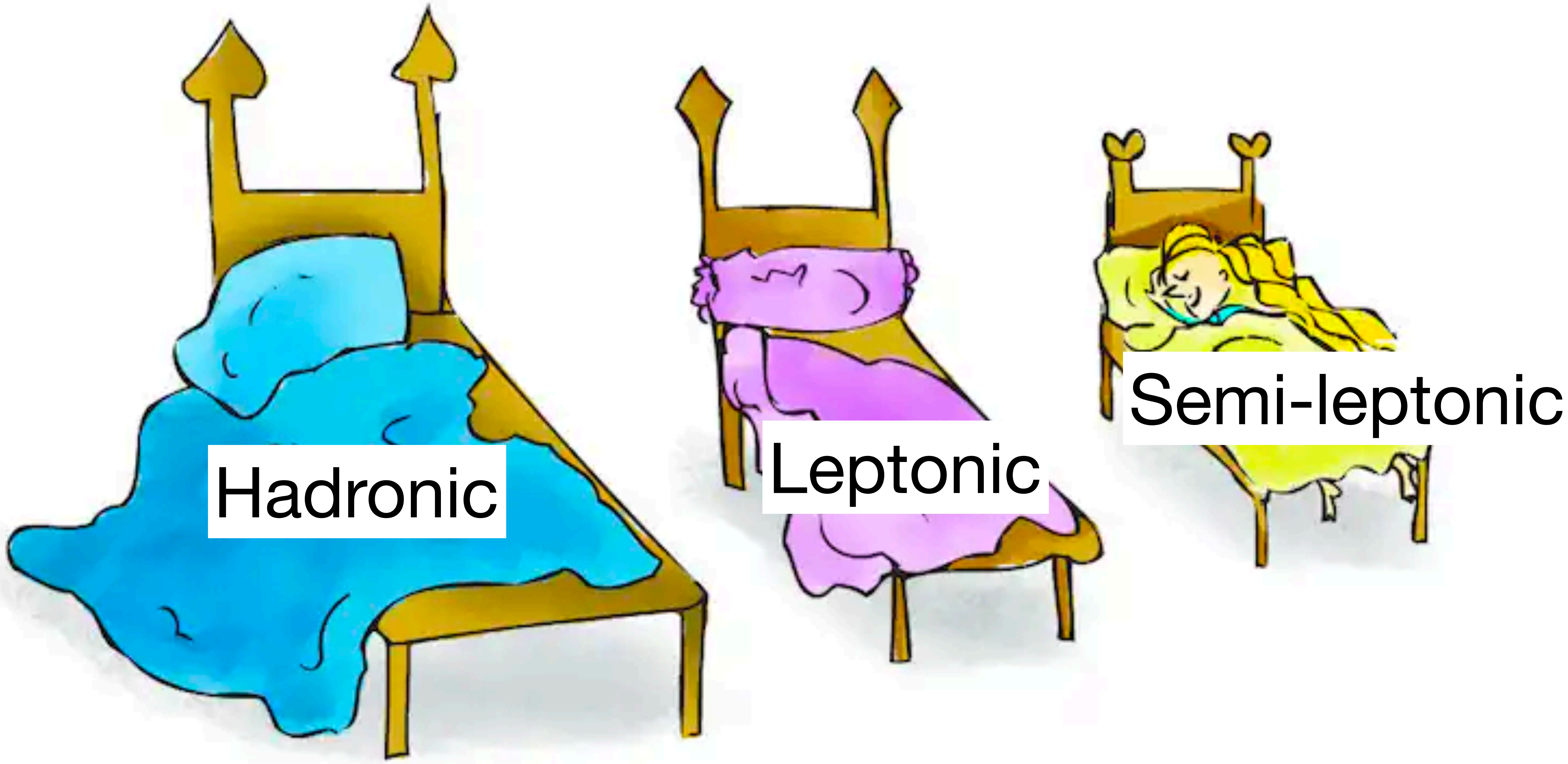


Leptonic

Semi-leptonic

Hadronic

Large BF, large TH error

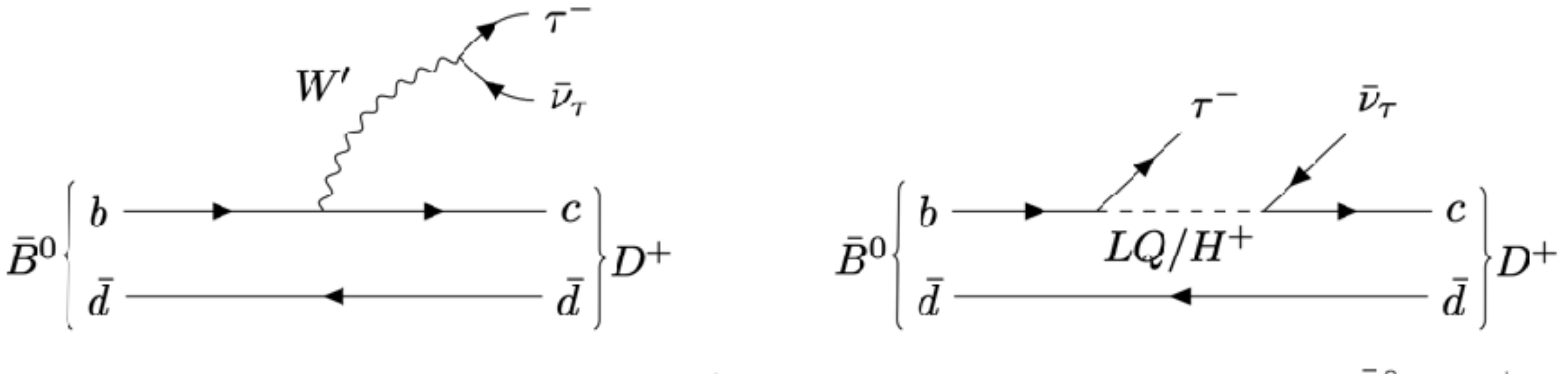

Hadronic
Leptonic
Semi-leptonic

Charged vs. Neutral currents

FCCC vs FCNC

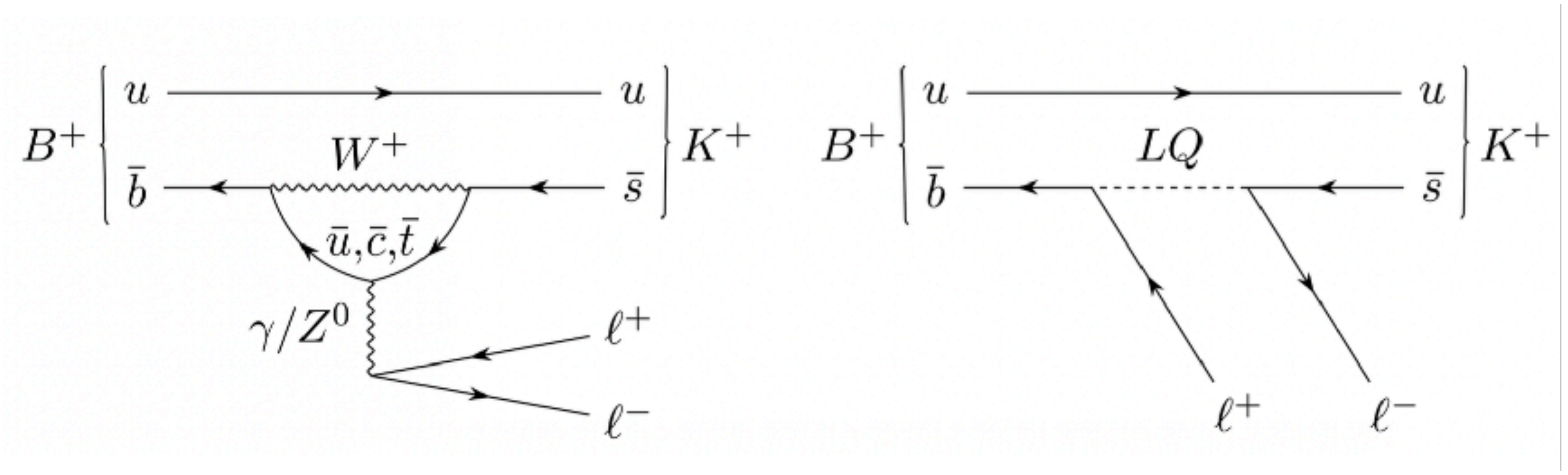
$$b \to c\ell\bar{\nu}$$

- Tree-level in the SM: Large Branching Fraction
- Heavy to heavy meson decay
- E.g. $B \to D^{(*)}, B_c \to J/\psi$
- Amplitude proportional to V_{cb}
- $b \to u$ also measured although CKM suppressed w.r.t. $b \to c$



$$b \to s\ell\ell$$

- Loop only in the SM: small branching fraction
- Sensitive to small BSM contributions
- Heavy to light meson decay
- E.g. $B \to K^{(*)}, B_s \to \phi$



Amplitude of semileptonic B decays in the WET

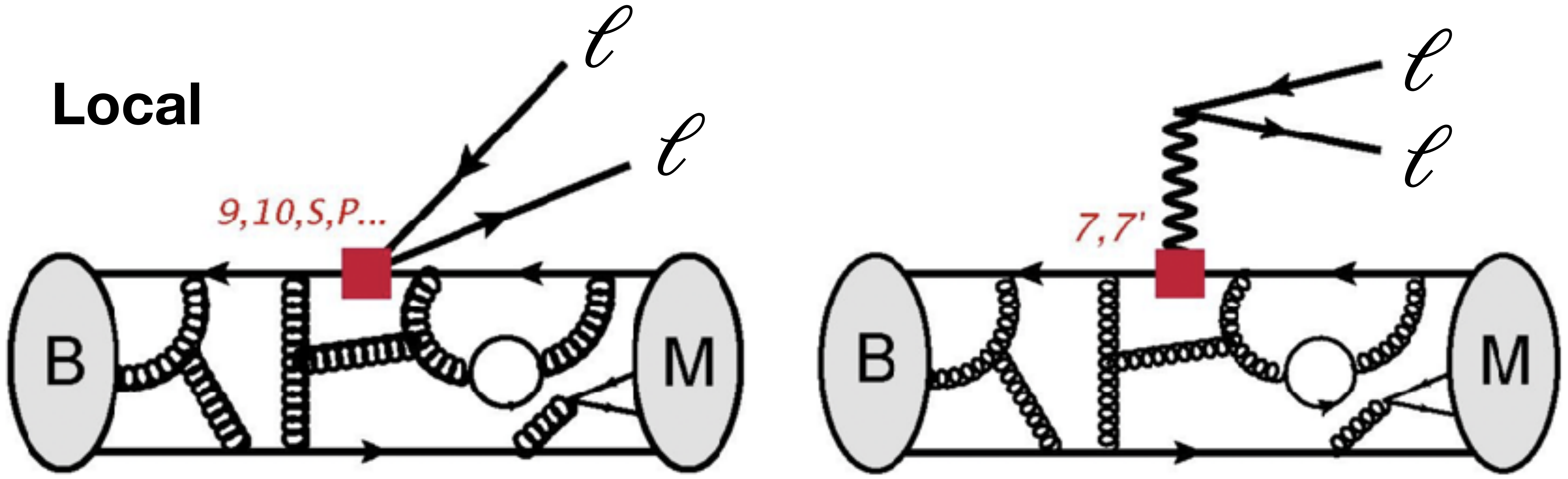
E.g. $\mathrm{b} \to \mathrm{s}\ell\ell$

$$H_{\mathrm{b}\to \mathrm{s}\ell\ell} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=1}^{10} C_i^\ell O_i^\ell$$

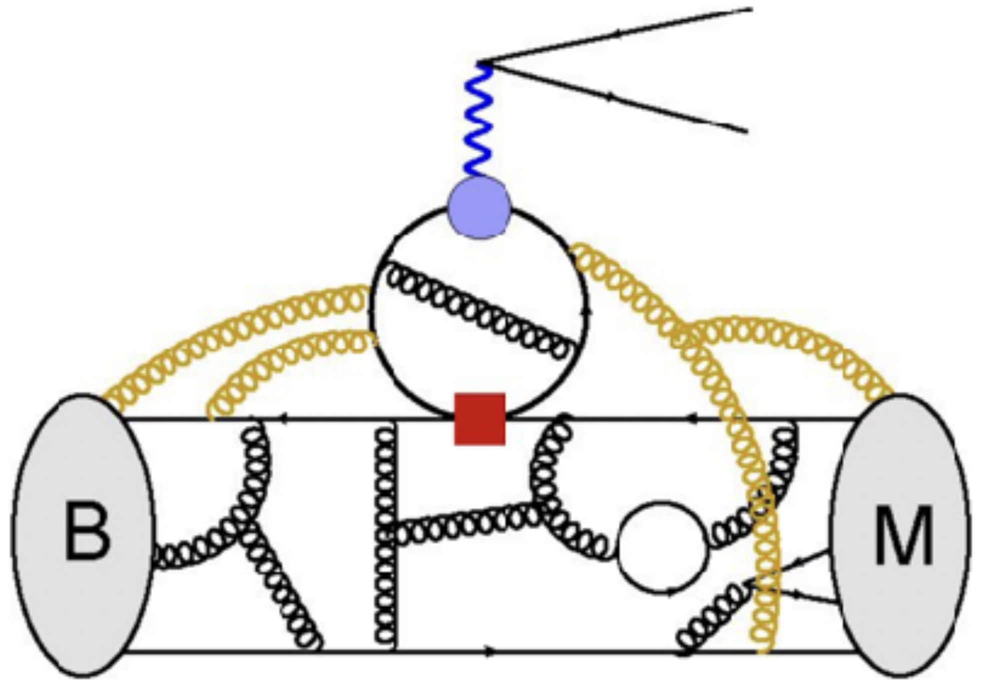
$$C_7^{\mathrm{SM}} \simeq -0.3 \qquad C_9^{\mathrm{SM}} \simeq -4 \quad C_{10}^{\mathrm{SM}} \simeq 4$$

$$O_7 = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}b_R)F^{\mu\nu}, \quad O_{9(10)}^\ell = (\bar{s}\gamma_\mu b_L)(\bar{\ell}\gamma^\mu(\gamma_5)\ell)$$

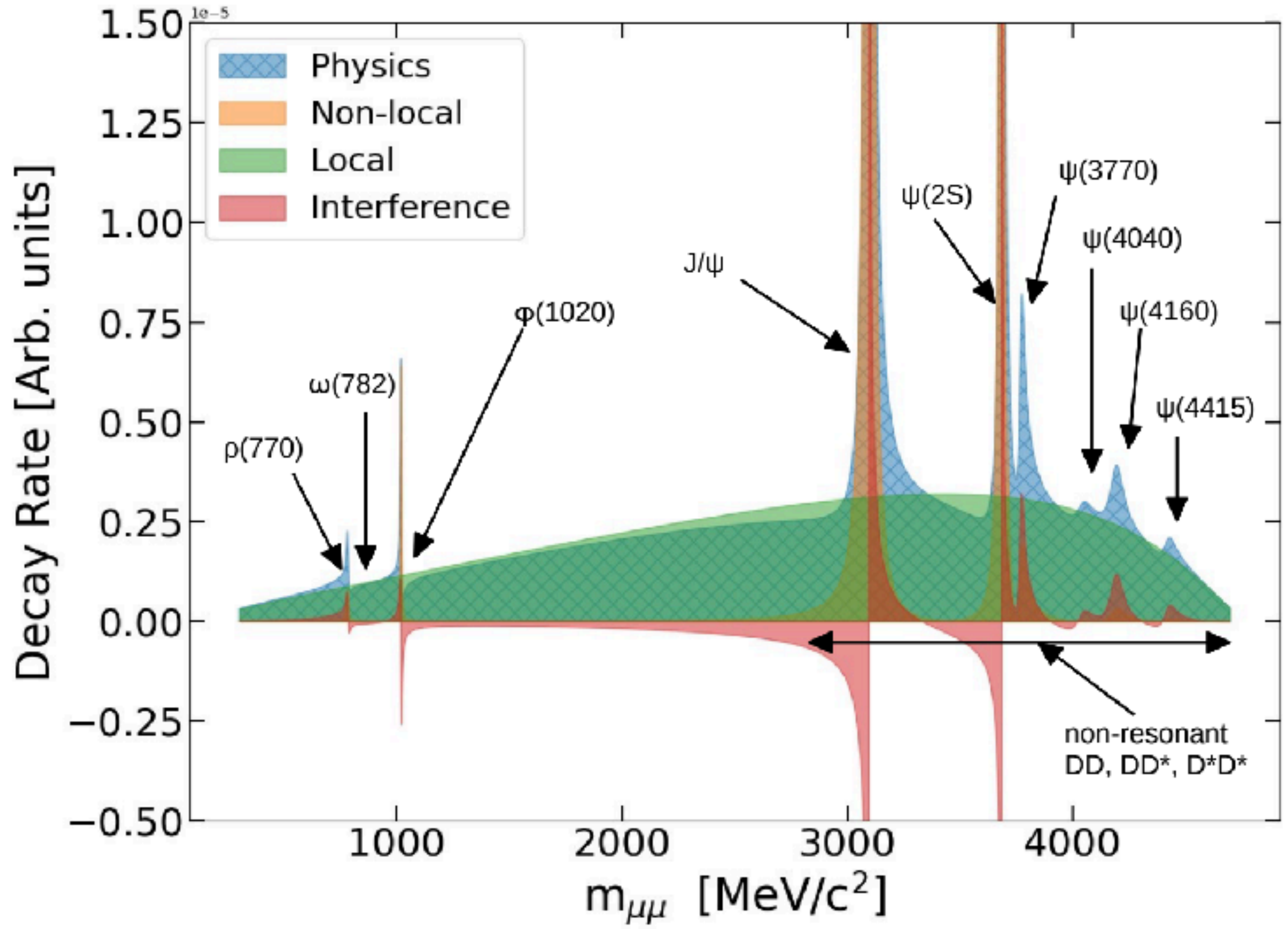
Local



Non-local



Sketches by Javier Virto



Amplitude of $\mathrm{B} \to \mathrm{M}\ell\ell$ decays

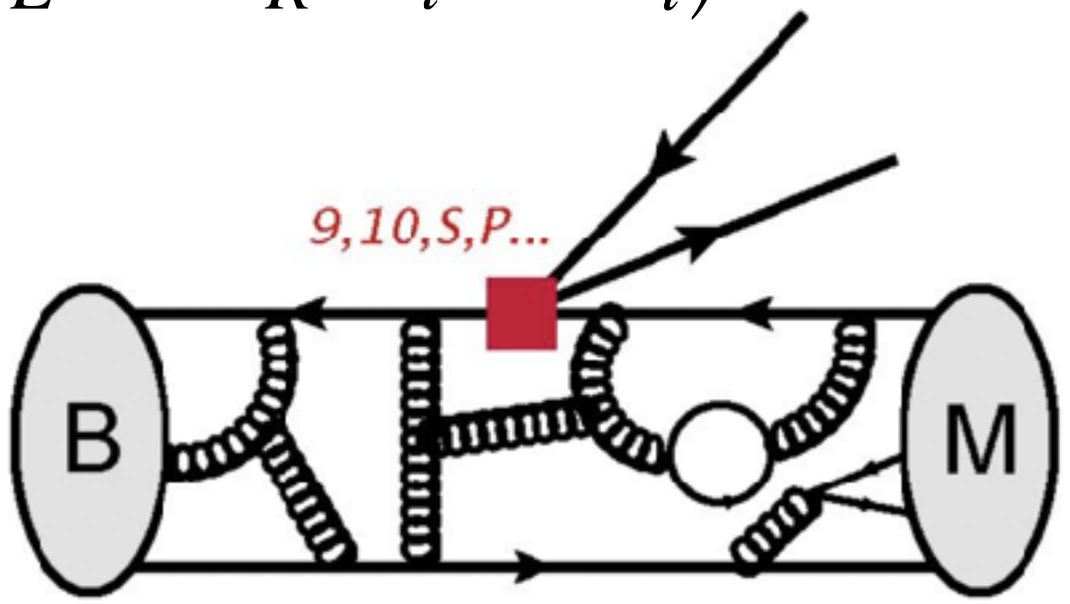
$$\mathcal{M}(B \to M\ell\ell) = \left\langle M\ell\ell \left| H_{\mathrm{b}\to\mathrm{s}\ell\ell} \right| B \right\rangle = \mathcal{N}\left[\left(A_V^\mu + T^\mu\right) \bar{u}_\ell \gamma_\mu v_\ell + A_A^\mu\, \bar{u}_\ell \gamma_\mu \gamma_5 v_\ell + A_S\, \bar{u}_\ell v_\ell + A_P\, \bar{u}_\ell \gamma_5 v_\ell\right]$$

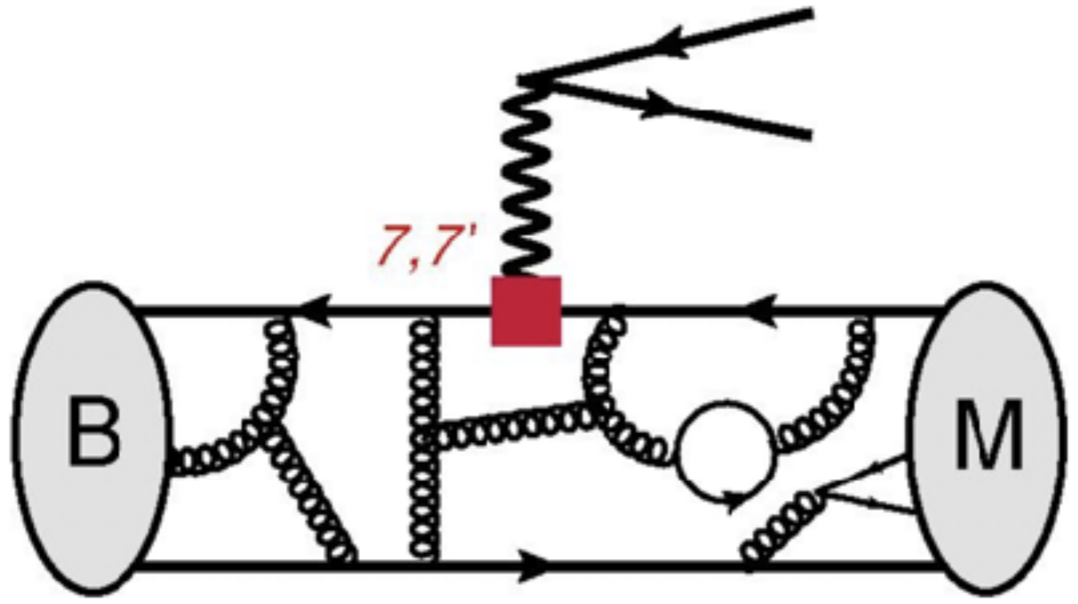
Local contributions

$$A_V^\mu = -\frac{2im_b}{q^2} C_7 \left\langle M \left| \bar{s}\sigma^{\mu\nu} q_\nu P_R b \right| B \right\rangle + C_9 \left\langle M \left| \bar{s}\gamma^\mu P_L b \right| B \right\rangle + \left(P_L \leftrightarrow P_R, C_i \to C_i'\right)$$

$$A_A^\mu = C_{10} \left\langle M \left| \bar{s}\gamma^\mu P_L b \right| B \right\rangle + \left(P_L \leftrightarrow P_R, C_i \to C_i'\right)$$

$$A_{S,P} = C_{S,P} \left\langle M \left| \bar{s} P_R b \right| B \right\rangle + \left(P_L \leftrightarrow P_R, C_i \to C_i'\right)$$





Amplitude of $\mathrm{B} \to \mathrm{M}\ell\ell$ decays

Local contributions - definition of the form factors

- 3 independent f.f. for B to pseudoscalar meson:

$$\left\langle P(k) \left| \bar{q}_1 \gamma^\mu b \right| B(p) \right\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right] f_+^{B\to P} + \frac{m_B^2 - m_P^2}{q^2} q^\mu f_0^{B\to P}$$

$$\left\langle P(k) \left| \bar{q}_1 \sigma^{\mu\nu} q_\nu b \right| B(p) \right\rangle = \frac{i f_T^{B\to P}}{m_B + m_P} \left[q^2 (p+k)^\mu - \left(m_B^2 - m_P^2 \right) q^\mu \right]$$

- 7 independent f.f. for B to vector meson:

$$\left\langle V(k,\eta) \left| \bar{q}_1 \gamma^\mu b \right| B(p) \right\rangle = \epsilon^{\mu\nu\rho\sigma} \eta_\nu^* p_\rho k_\sigma \frac{2 V^{B\to V}}{m_B + m_V}$$

$$\left\langle V(k,\eta) \left| \bar{q}_1 \gamma^\mu \gamma_5 b \right| B(p) \right\rangle = i \eta_\nu^* [g^{\mu\nu} \left(m_B + m_V \right) A_1^{B\to V} - \frac{(p+k)^\mu q^\nu}{m_B + m_V} A_2^{B\to V} - q^\mu q^\nu \frac{2 m_V}{q^2} \left(A_3 - A_0 \right) \Big]$$

$$\left\langle V(k,\eta) \left| \bar{q}_1 i \sigma^{\mu\nu} q_\nu b \right| B(p) \right\rangle = \epsilon^{\mu\nu\rho\sigma} \eta_\nu^* p_\rho k_\sigma 2 T_1^{B\to V}$$

$$\left\langle V(k,\eta) \left| \bar{q}_1 i \sigma^{\mu\nu} q_\nu \gamma_5 b \right| B(p) \right\rangle = i \eta_\nu^* [\left(g^{\mu\nu} \left(m_B^2 - m_V^2 \right) - (p+k)^\mu q^\nu \right) T_2^{B\to V} + q^\nu \left(q^\mu - \frac{q^2}{m_B^2 - m_V^2} (p+k)^\mu \right) T_3^{B\to V} \Big]$$

$$A_3^{B\to V} \equiv \frac{m_B + m_V}{2 m_V} A_1^{B\to V} - \frac{m_B - m_V}{2 m_V} A_2^{B\to V} .$$

Amplitude of $\mathrm{B} \to \mathrm{M}\ell\ell$ decays

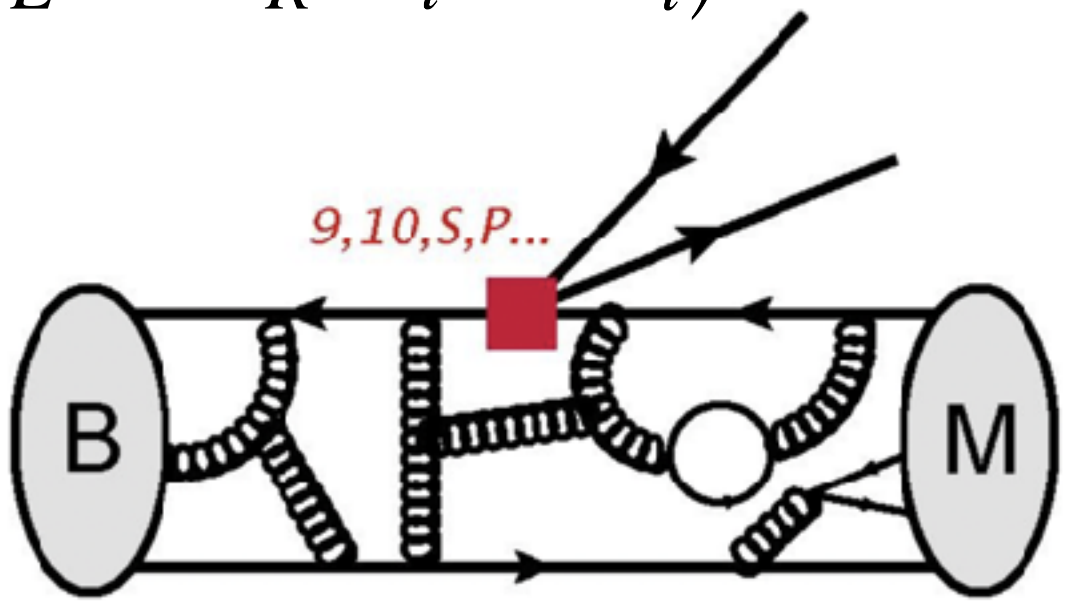
$$\mathscr{M}(B \to M\ell\ell) = \left\langle M\ell\ell \left| H_{\text{eff}} \right| B \right\rangle = \mathscr{N}\left[\left(A_V^\mu + T^\mu\right)\bar{u}_\ell\gamma_\mu v_\ell \,+\, A_A^\mu\,\bar{u}_\ell\gamma_\mu\gamma_5 v_\ell \,+\, A_S\,\bar{u}_\ell v_\ell \,+\, A_P\,\bar{u}_\ell\gamma_5 v_\ell\right]$$

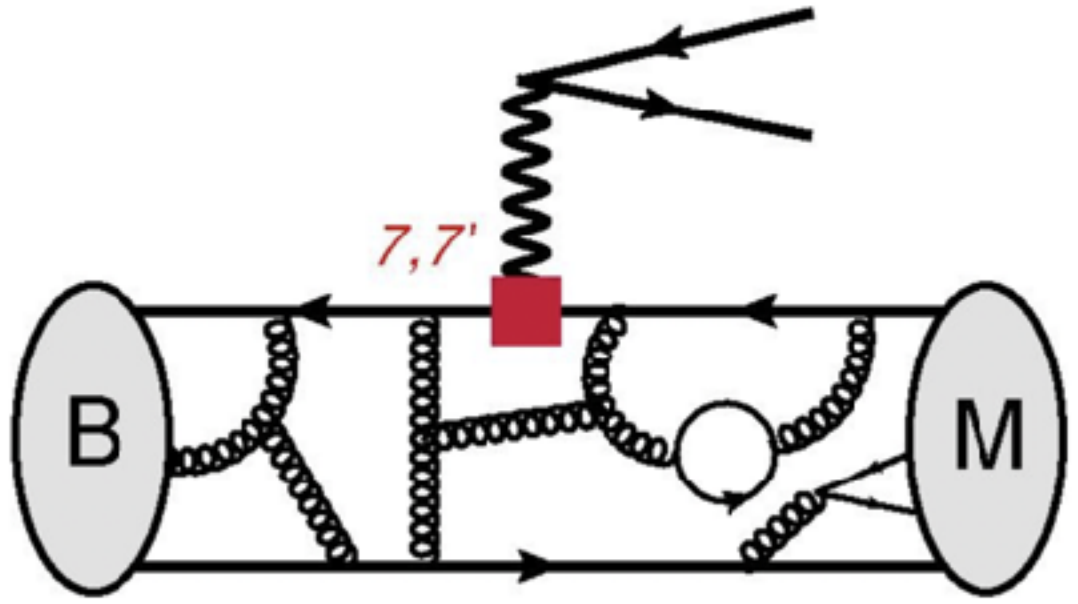
Local contributions

$$A_V^\mu = -\frac{2im_b}{q^2}C_7\left\langle M \left| \bar{s}\sigma^{\mu\nu}q_\nu P_R b \right| B \right\rangle + C_9\left\langle M \left| \bar{s}\gamma^\mu P_L b \right| B \right\rangle + \left(P_L \leftrightarrow P_R, C_i \to C_i'\right)$$

$$A_A^\mu = C_{10}\left\langle M \left| \bar{s}\gamma^\mu P_L b \right| B \right\rangle + \left(P_L \leftrightarrow P_R, C_i \to C_i'\right)$$

$$A_{S,P} = C_{S,P}\left\langle M \left| \bar{s}P_R b \right| B \right\rangle + \left(P_L \leftrightarrow P_R, C_i \to C_i'\right)$$

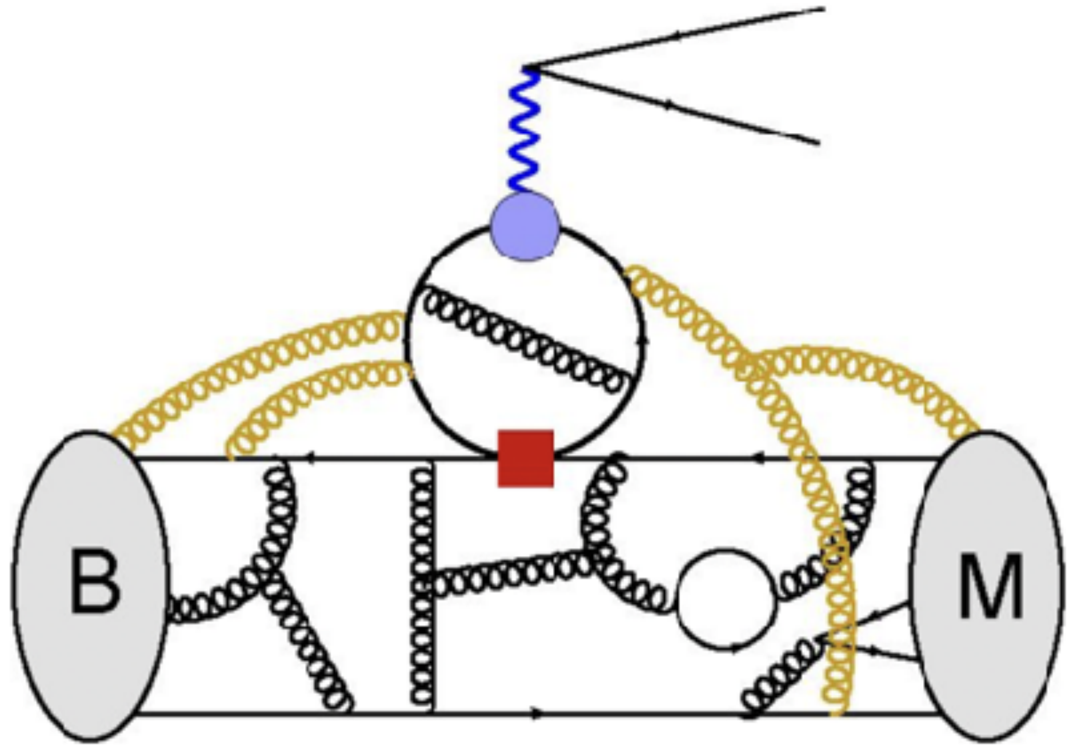




Non-Local contributions

$$T^\mu = \frac{-16i\pi^2}{q^2}\sum_{i=1,\ldots,6,8} C_i \int dx^4 e^{iq\cdot x}\left\langle M \left| T\left\{j_{\text{em}}^\mu(x), O_i(0)\right\} \right| B \right\rangle,$$

$$j_{\text{em}}^\mu = \sum_q Q_q \bar{q}\gamma^\mu q$$



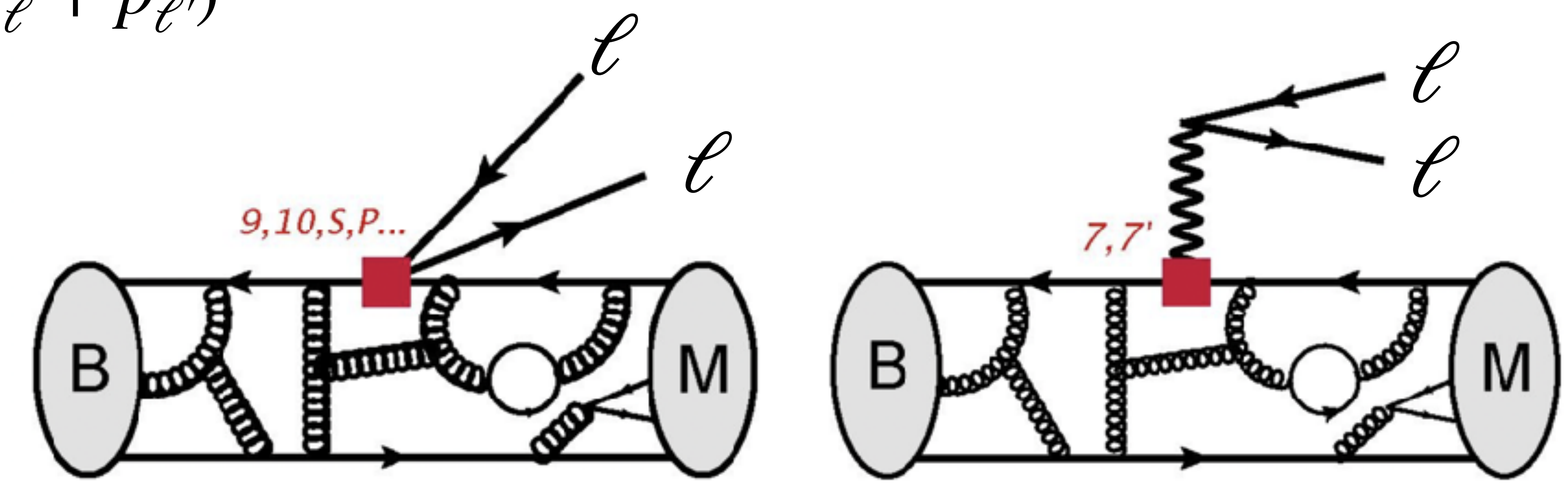
Calculation of the matrix elements

$$\mathcal{M}(B \to M\ell\ell) = \left\langle M\ell\ell \left| H_{\text{eff}} \right| B \right\rangle = \mathcal{N}\left[\left(A_V^\mu + T^\mu\right)\bar{u}_\ell\gamma_\mu v_\ell + A_A^\mu\,\bar{u}_\ell\gamma_\mu\gamma_5 v_\ell + A_S\,\bar{u}_\ell v_\ell + A_P\,\bar{u}_\ell\gamma_5 v_\ell\right]$$

Local contributions

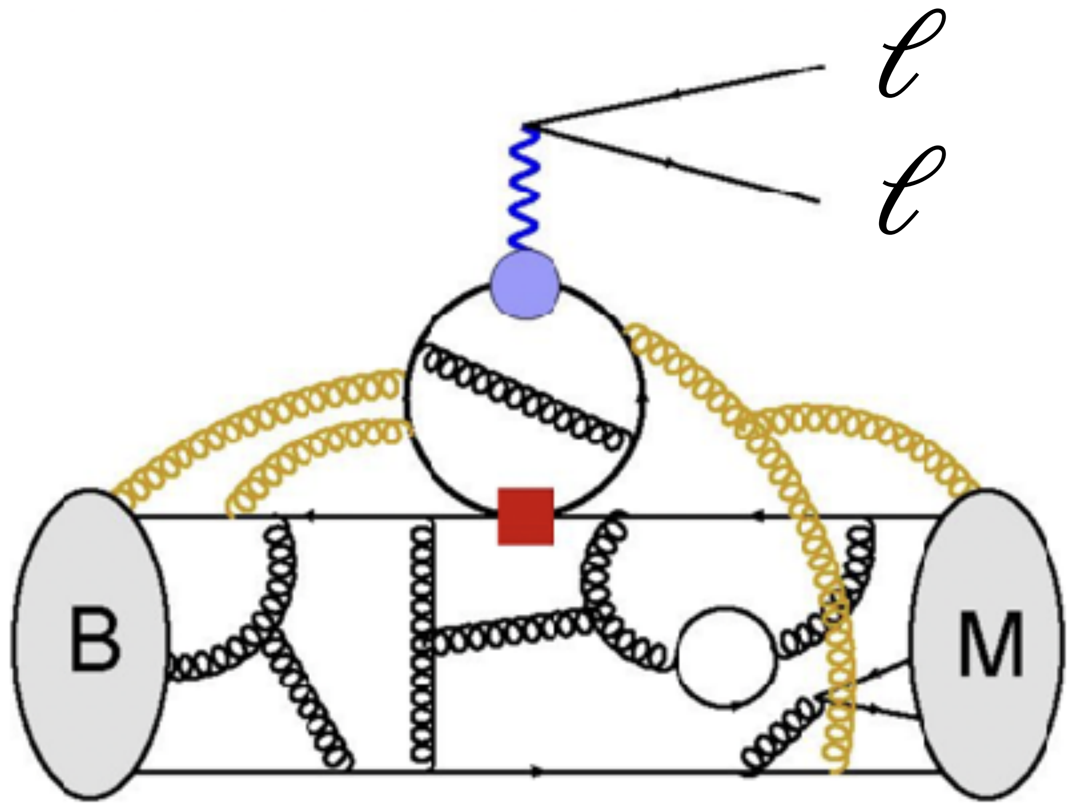
$$q^2 = (p_\ell + p_{\ell'})^2$$

- At high-q2, computed on the lattice
- At low-q2:
  - Lattice available only for certain processes
  - Analytic approach: e.g. Light-Cone Sum Rule (LCSR)



Non-Local contributions

- At low-q2 from QCD factorization (QCDF)
- On the entire kinematic range, we only know ‘dispersive bounds’ GRvDV 22, which are conservative upper bounds



Optimized observables

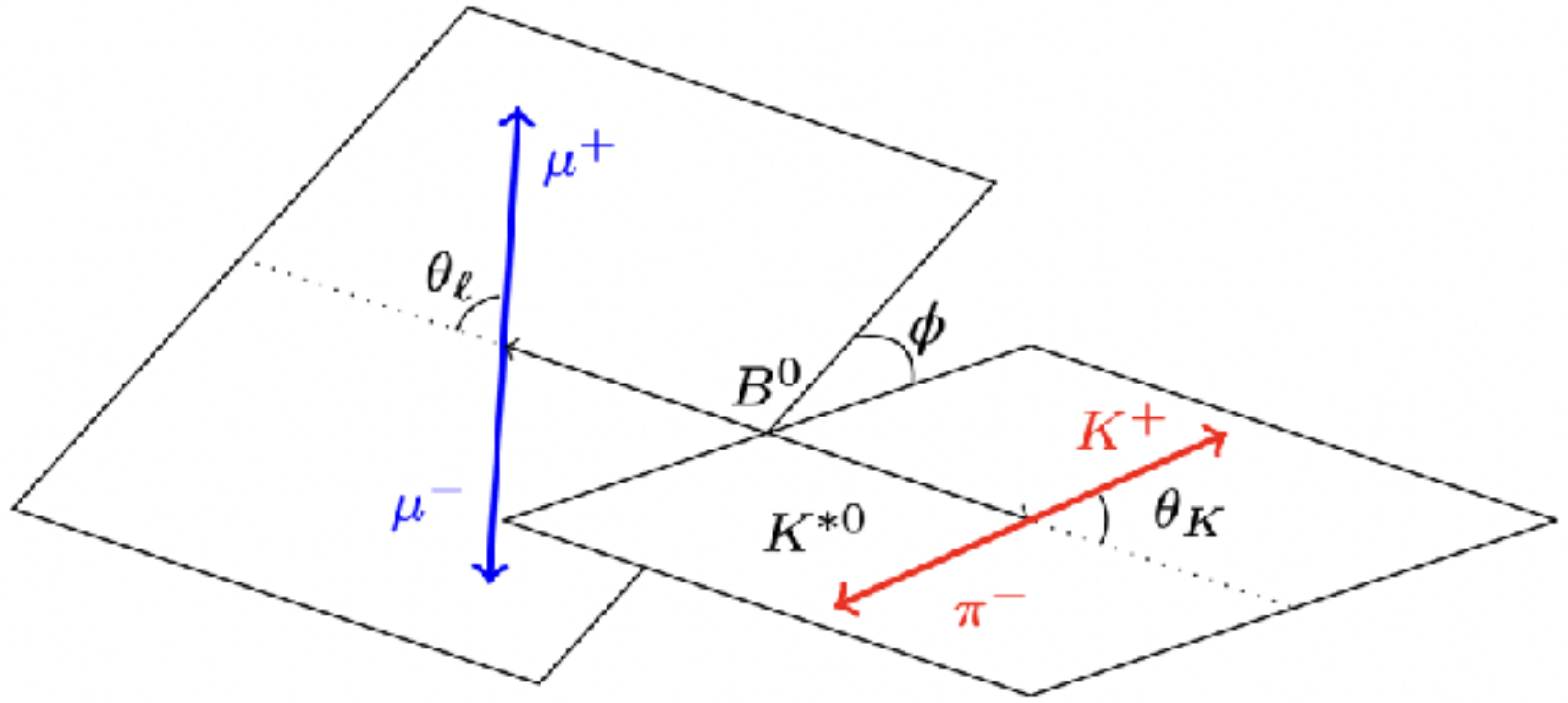
Ratio and angular observables

$$\left.\begin{aligned} R_{K^{(*)}} &= \frac{BR(B \to K^{(*)}\mu\mu)}{BR(B \to K^{(*)}ee)} \\ R_{D^{(*)}} &= \frac{BR(B \to D^{(*)}\tau\nu)}{BR(B \to D^{(*)}\ell\nu)} \end{aligned}\right)$$

Golden observables, robust to TH uncertainties
Test of LFU, deviations is a smoking gun for NP

$\sim 1\,\%$ TH uncertainty

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\hat{\phi}} = \frac{9}{16\pi}\Big[F_L\cos^2\theta_K + \frac{3}{4}F_T(1-\cos^2\theta_K) - F_L\cos^2\theta_K(2\cos^2\theta_\ell - 1)$$
$$+\frac{1}{4}F_T(1-\cos^2\theta_K)(2\cos^2\theta_\ell - 1) + \frac{1}{2}P_1F_T(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\cos 2\hat{\phi}$$
$$+2P_2F_T(1-\cos^2\theta_K)\cos\theta_\ell - P_3F_T(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\sin 2\hat{\phi}\Big]\frac{d\Gamma}{dq^2}\,, \quad (4)$$



$$P_5' = \frac{S_5}{\sqrt{F_T F_L}}$$

From 1207.2753 Descotes-Genon, Matias, Ramon, Virto

Status of the B-anomalies

$$q^2 = (p_\ell + p_{\ell'})^2$$

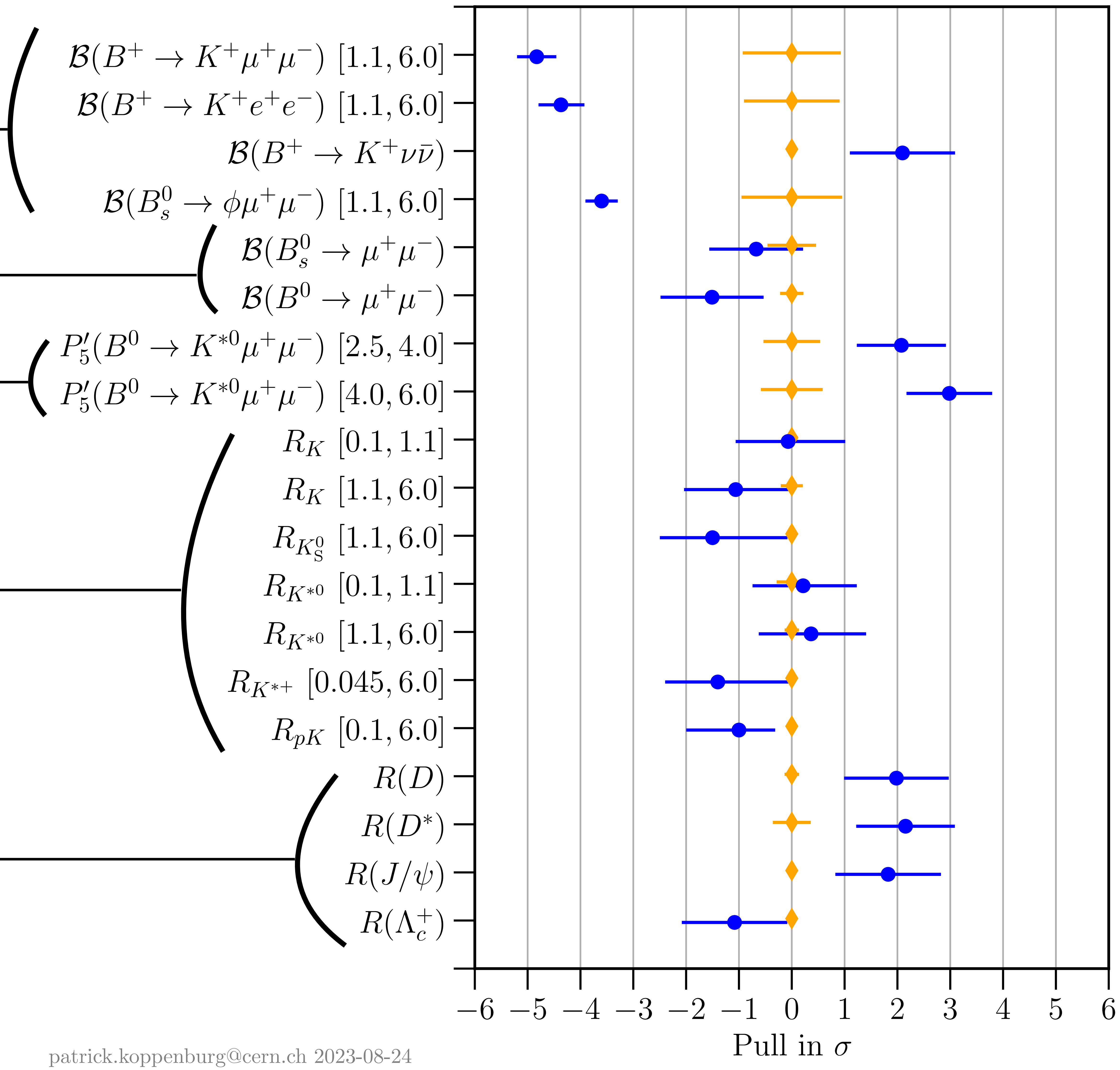
$BR(b \to s\ell\ell)$
$B \to Kee$ obtained from $B \to K\mu\mu$ and R_K
Pull very TH dependent

Leptonic B decays, TH clean, tension gone after LHCb 2021
Exp upper bound only for $B^0 \to \mu\mu$

Optimized angular observables
$P_5'(B \to K^*\mu\mu)$ (LHCb 2021) also P_2 and Q_5 (Belle 2017) still standing

Ratio observables for $b \to s\ell\ell$ at low q2, TH clean,
no more anomalie after LHCb 2022

Ratio observables for $b \to c\ell\nu$, TH clean,
Anomalies in $R(D^{(*)})$ remain after many measurements



Global fits for $b \to c\tau\bar{\nu}$ observables

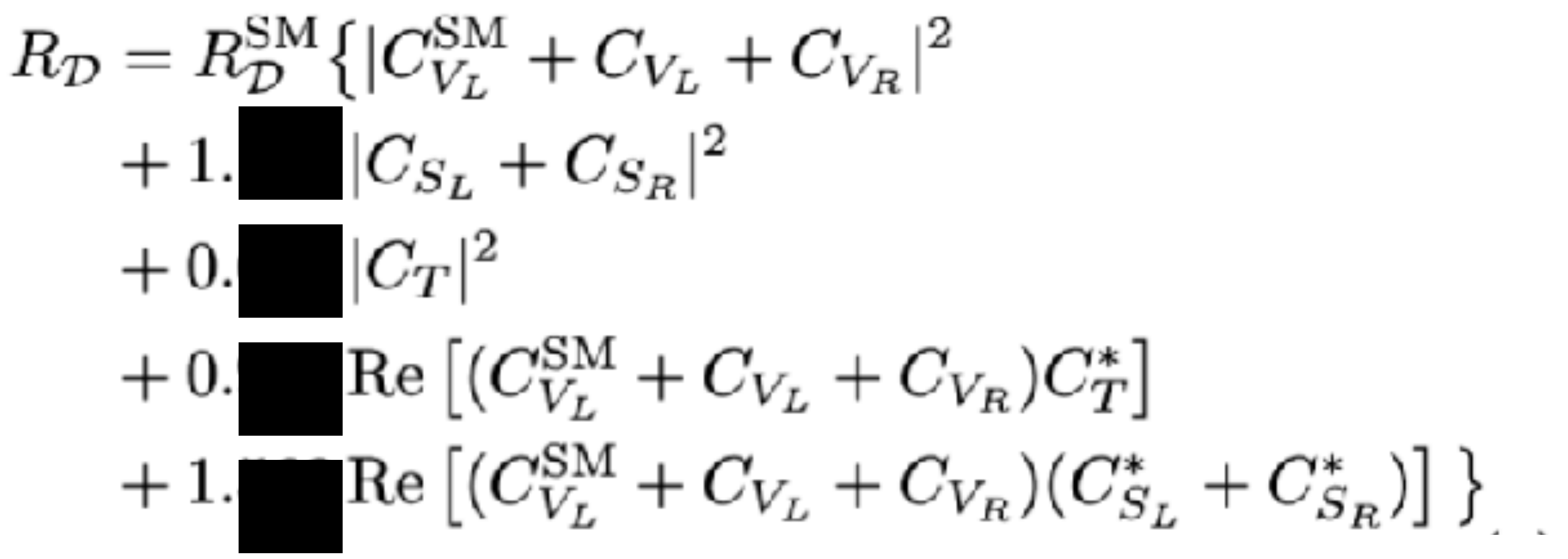
State-of-the-art global fit of $b \to c\tau\bar{\nu}$ observables

EFT assuming NP in the tau sector only

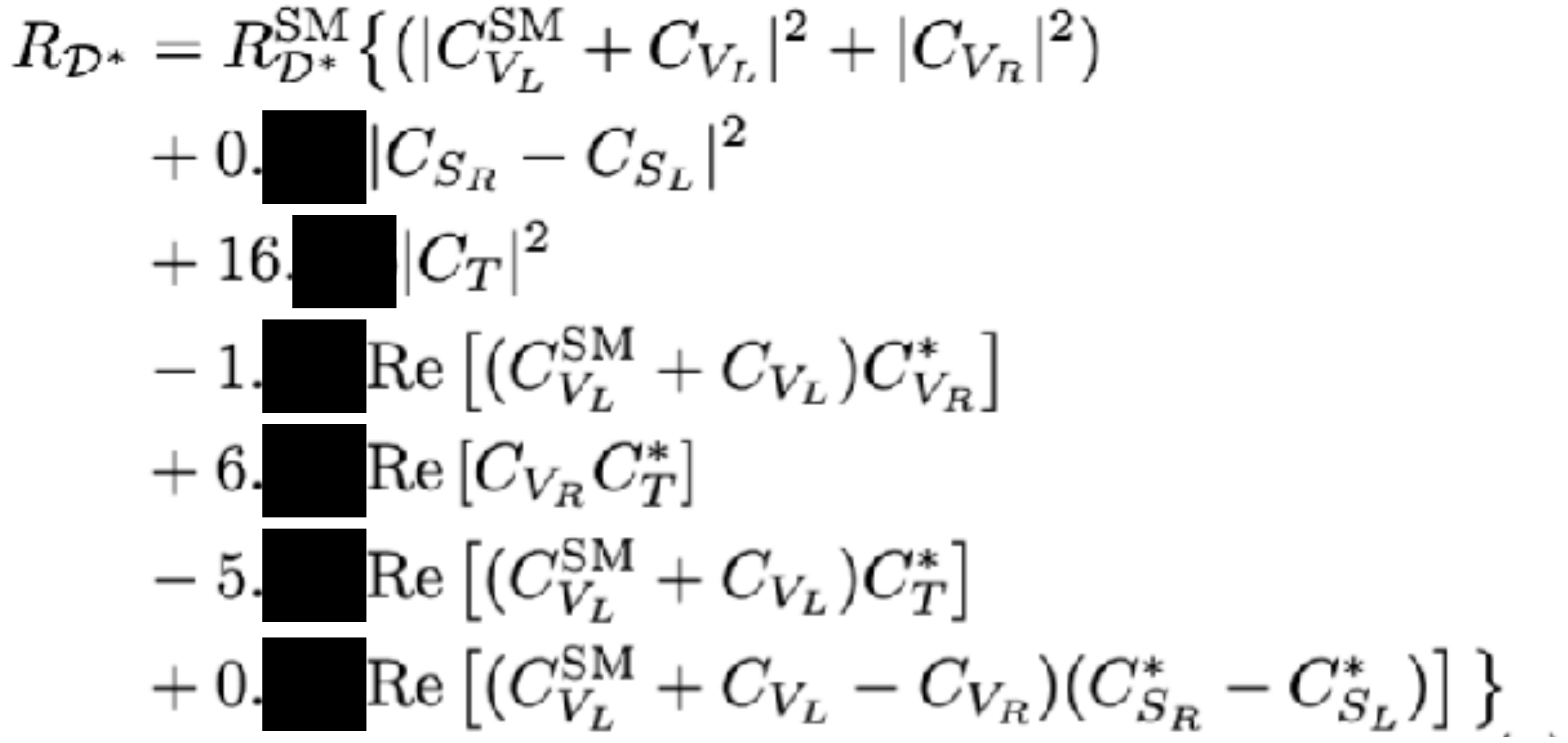
$$\mathscr{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb}\Big[(1 + C_{V_L})\, O^{\tau}_{V_L} + C_{V_R} O^{\tau}_{V_R}$$
$$+ C_{S_R} O^{\tau}_{S_R} + C_{S_L} O^{\tau}_{S_L} + C_T O^{\tau}_T\Big],$$

$$O^{\tau}_{V_{L,R}} = (\bar{c}\gamma^{\mu} P_{L,R} b)(\bar{\tau}\gamma_{\mu} P_L \nu_{\tau})$$
$$O^{\tau}_{S_{L,R}} = (\bar{c} P_{L,R} b)(\bar{\tau} P_L \nu_{\tau}),$$
$$O^{\tau}_T = (\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\tau}\sigma_{\mu\nu} P_L \nu_{\tau}) .$$

Observables can conveniently be expressed in polynomials of WCs



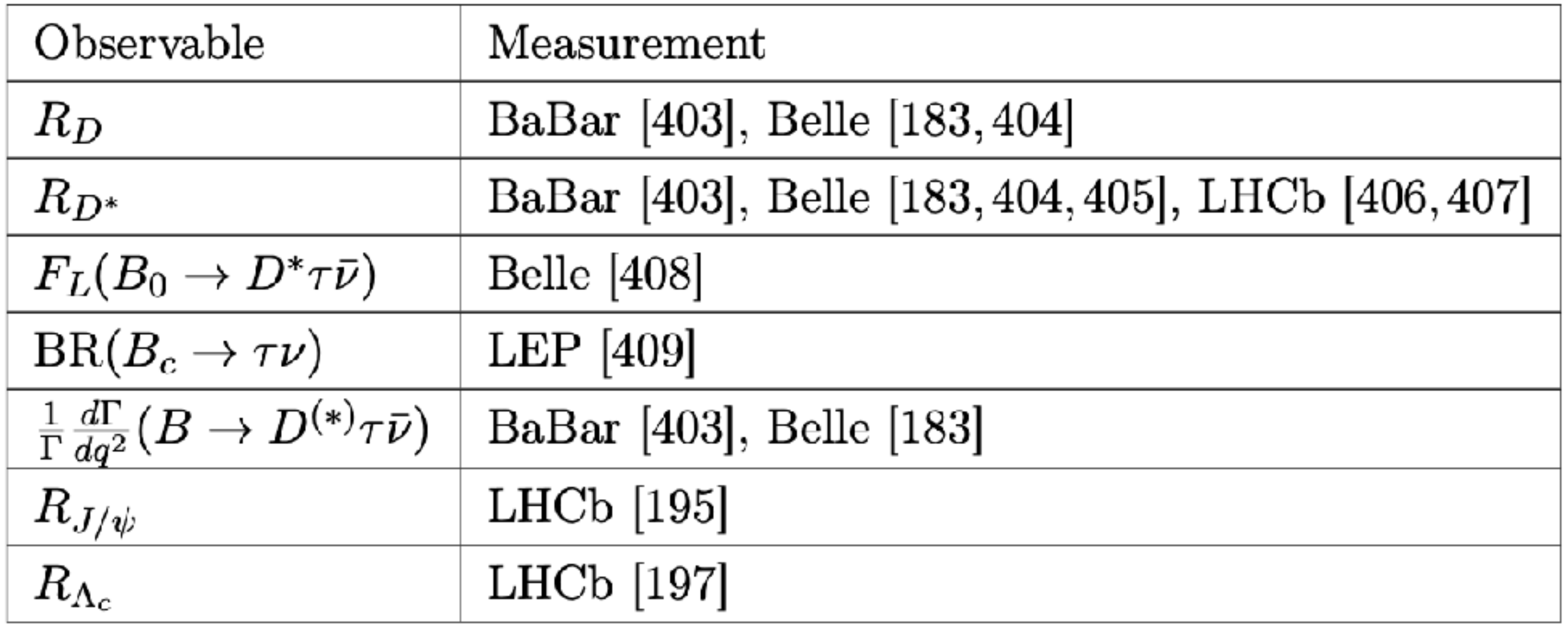
$$\begin{aligned} R_{\mathcal{D}} = R_{\mathcal{D}}^{\text{SM}}\big\{&|C_{V_L}^{\text{SM}} + C_{V_L} + C_{V_R}|^2 \\ &+ 1.\text{[redacted]}\, |C_{S_L} + C_{S_R}|^2 \\ &+ 0.\text{[redacted]}\, |C_T|^2 \\ &+ 0.\text{[redacted]}\, \text{Re}\left[(C_{V_L}^{\text{SM}} + C_{V_L} + C_{V_R}) C_T^*\right] \\ &+ 1.\text{[redacted]}\, \text{Re}\left[(C_{V_L}^{\text{SM}} + C_{V_L} + C_{V_R})(C_{S_L}^* + C_{S_R}^*)\right] \big\} \end{aligned}$$



$$\begin{aligned} R_{\mathcal{D}^*} = R_{\mathcal{D}^*}^{\text{SM}}\big\{&(|C_{V_L}^{\text{SM}} + C_{V_L}|^2 + |C_{V_R}|^2) \\ &+ 0.\text{[redacted]}\, |C_{S_R} - C_{S_L}|^2 \\ &+ 16.\text{[redacted]}\, |C_T|^2 \\ &- 1.\text{[redacted]}\, \text{Re}\left[(C_{V_L}^{\text{SM}} + C_{V_L}) C_{V_R}^*\right] \\ &+ 6.\text{[redacted]}\, \text{Re}\left[C_{V_R} C_T^*\right] \\ &- 5.\text{[redacted]}\, \text{Re}\left[(C_{V_L}^{\text{SM}} + C_{V_L}) C_T^*\right] \\ &+ 0.\text{[redacted]}\, \text{Re}\left[(C_{V_L}^{\text{SM}} + C_{V_L} - C_{V_R})(C_{S_R}^* - C_{S_L}^*)\right] \big\} \end{aligned}$$

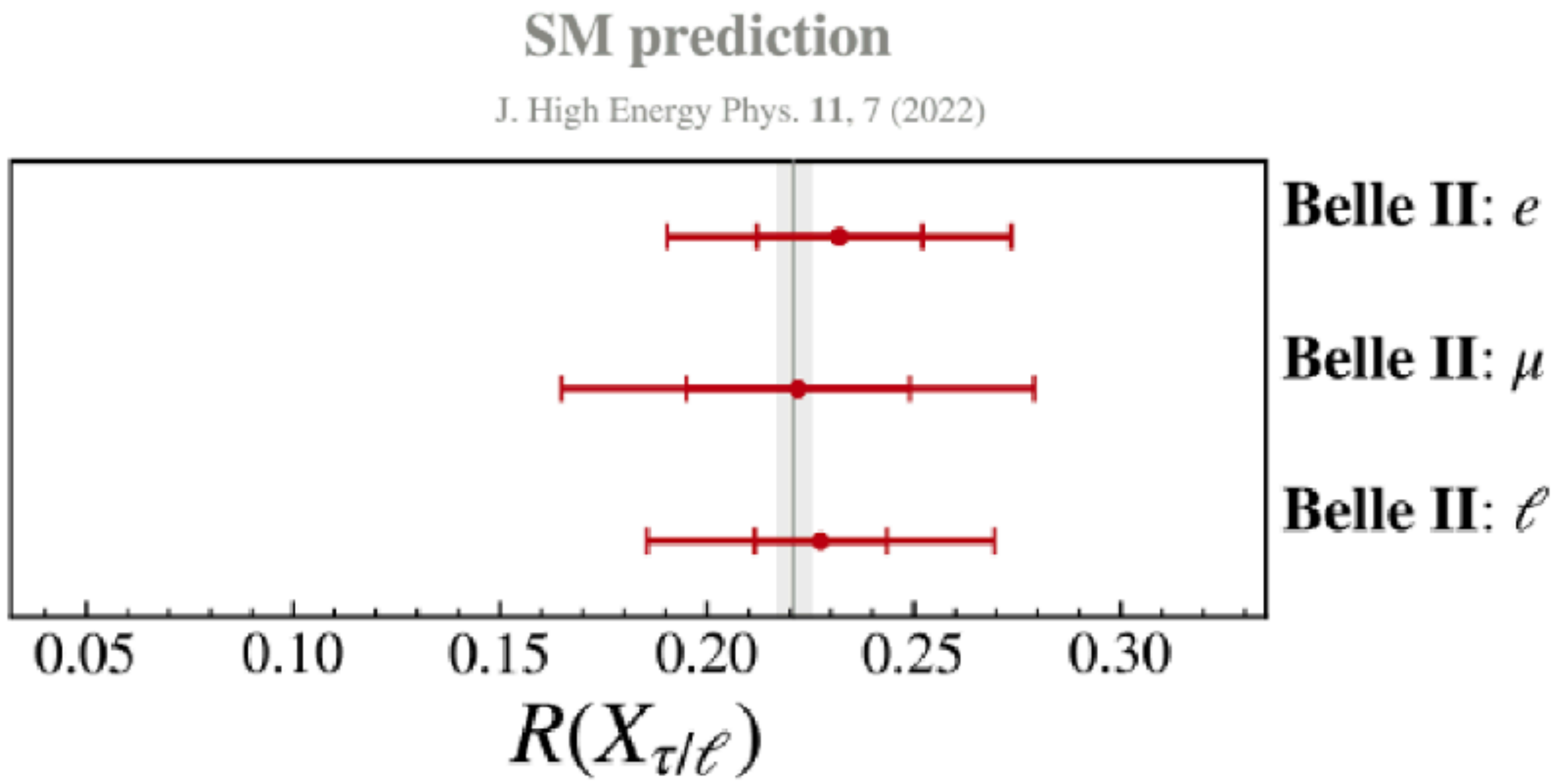
State-of-the-art global fit of $b \to c\tau\bar{\nu}$ observables

Available data



Observable	Measurement
R_D	BaBar [403], Belle [183, 404]
R_{D^*}	BaBar [403], Belle [183, 404, 405], LHCb [406, 407]
$F_L(B_0 \to D^*\tau\bar{\nu})$	Belle [408]
$\text{BR}(B_c \to \tau\nu)$	LEP [409]
$\frac{1}{\Gamma}\frac{d\Gamma}{dq^2}(B \to D^{(*)}\tau\bar{\nu})$	BaBar [403], Belle [183]
$R_{J/\psi}$	LHCb [195]
R_{Λ_c}	LHCb [197]

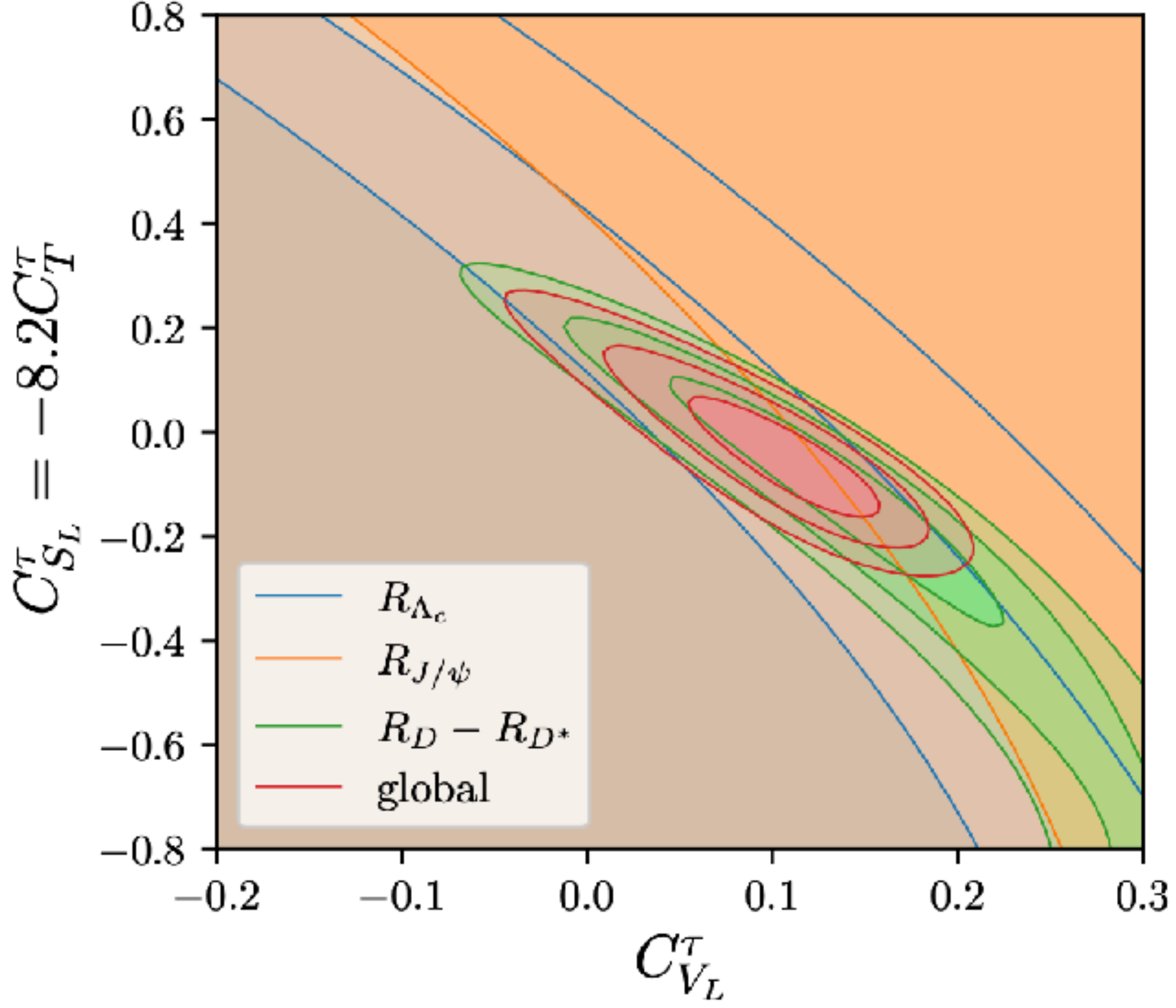
+ 2023 data: CMS R(J/Psi), Belle II R(Xc), R(D*)



State-of-the-art global fit of $b \to c\tau\bar{\nu}$ observables

Preliminary results (not up to date) - using flavio + smelli

S_1 leptoquark



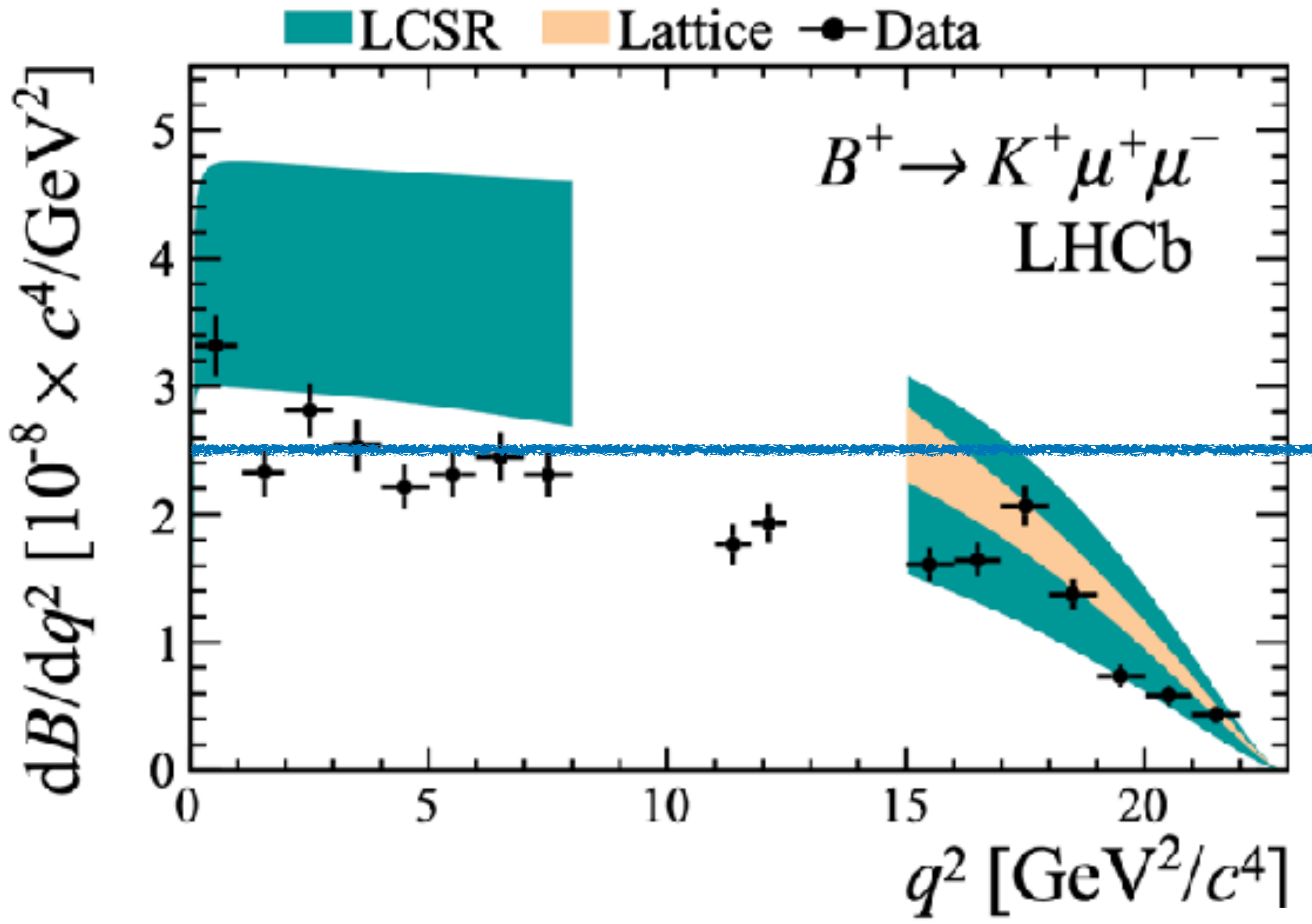
LQ	WC	SM pull	$\chi^2/N_{\rm dof}$
S_3, U_3	$C_{VL} = 0.088$	4.90σ	$53.8/64$
U_1	$C_{VL} = 0.097$ $C_{SR} = -0.035$	4.54σ	$53.6/63$
S_1	$C_{VL} = 0.111$ $C_{SL} = -8.2C_T$ $= -0.058$	4.57σ	$53.3/63$

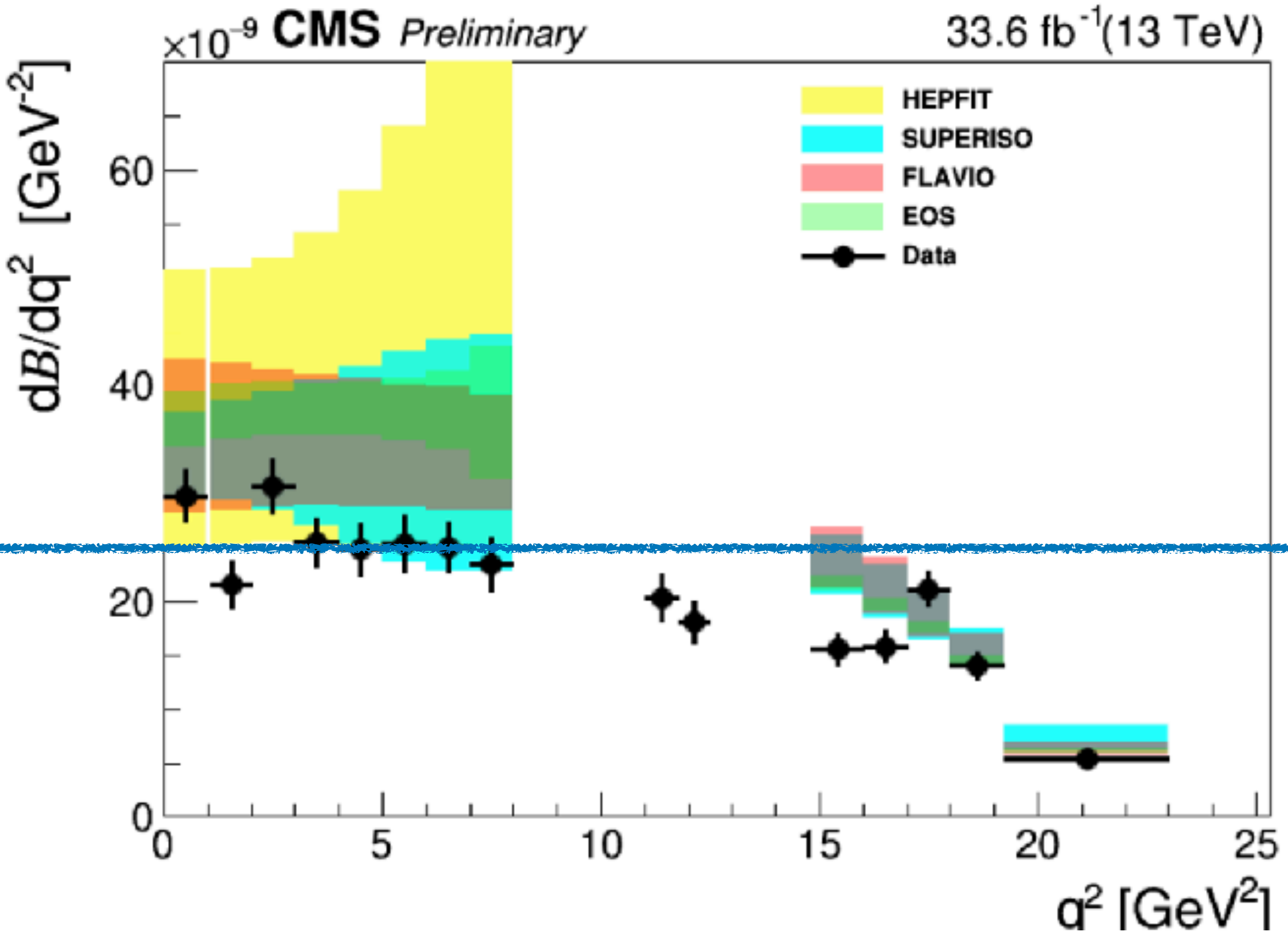
Running and matching from $M_{LQ} = 1.5TeV$ to $\mu = m_b$

Other bounds are available from e.g. EW precision obs.

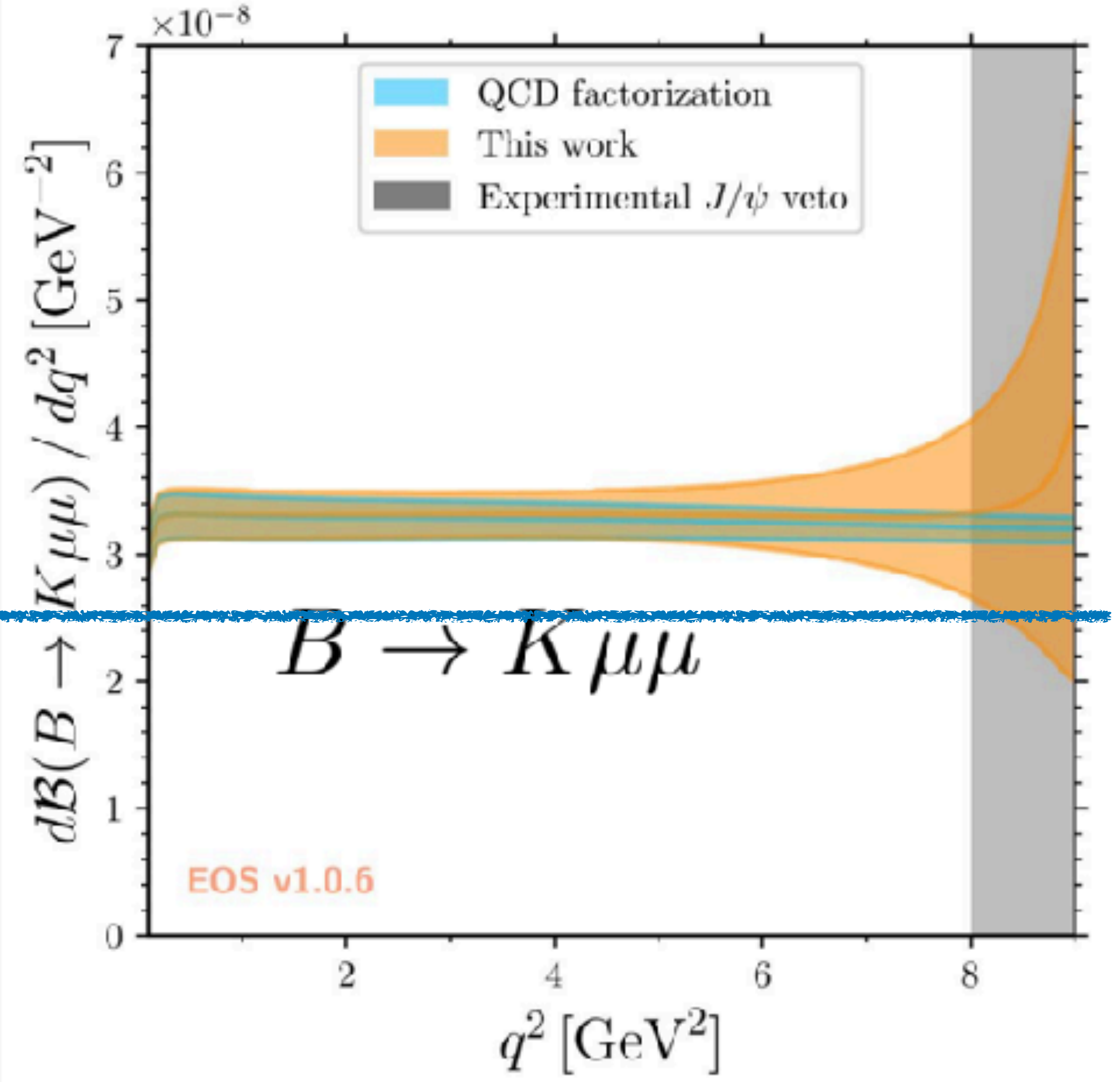
$B \to K$ local form factor from light-cone sum rules with B-meson LC distribution amplitude

$B^+ \to K^+\mu\mu$ in 2023

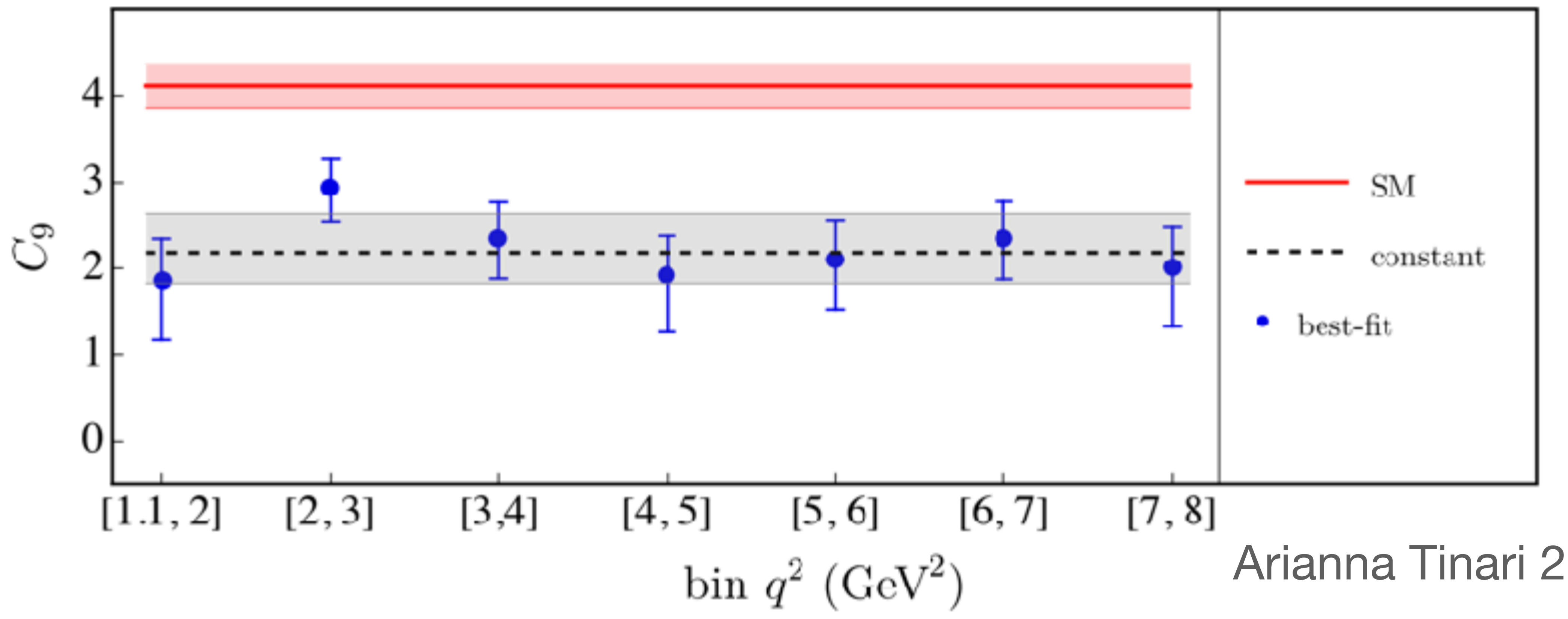




GRvDV 2206.03797



Using local FF from LQCD
HPQCD, Parrot, Bouchard, Davies (2207.13371)

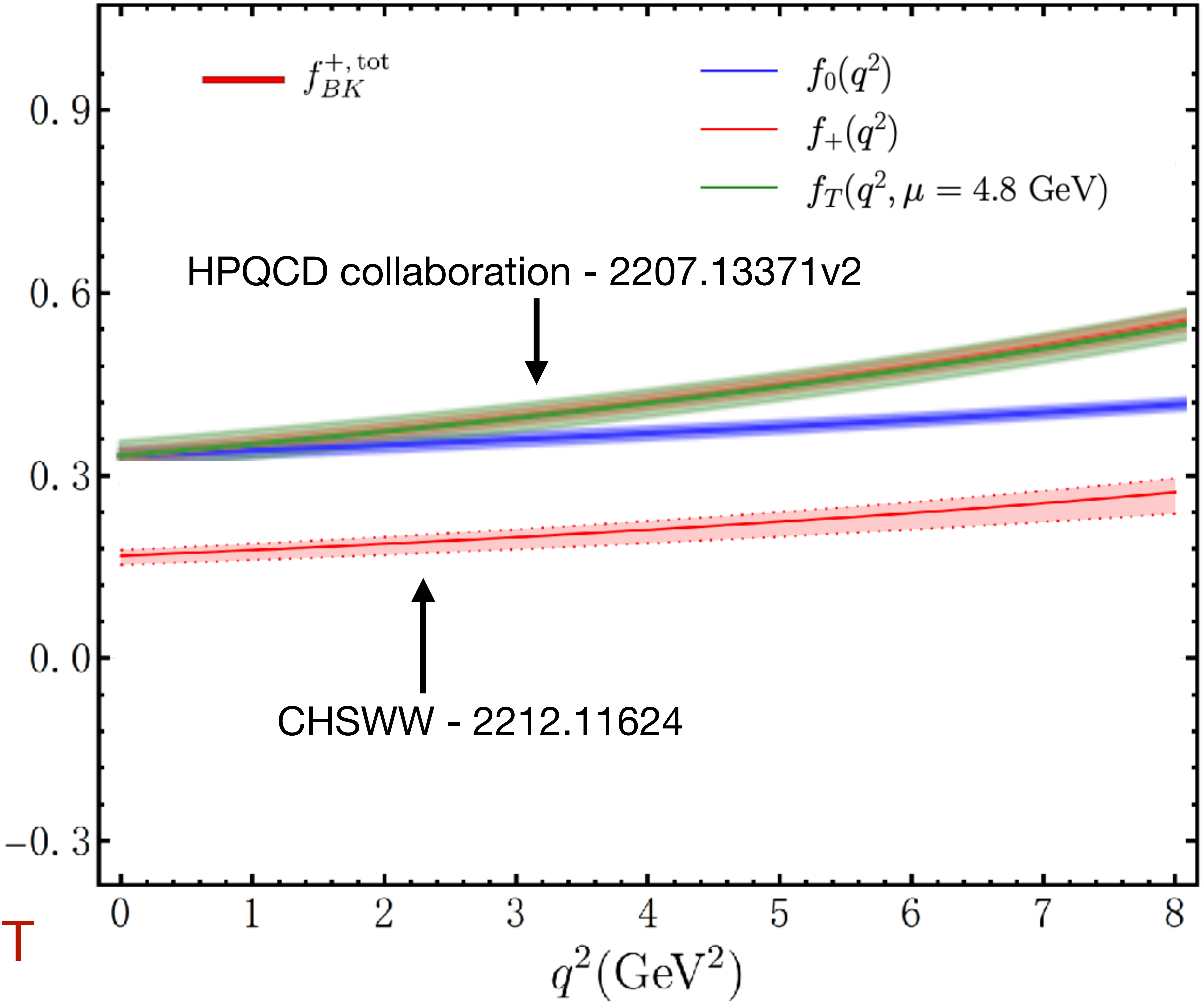


Arianna Tinari 2311.00782

LCSR vs Lattice QCD $B \to K$ form factor

- Low-q2 range has long been out of reach of lattice, HPQCD's result has not been reproduced yet
- HPQCD generally agrees with LCSR predictions (e.g. Khodjamirian 2017) and comes with smaller uncertainty
- LCSR with B-meson DA's has been computed including DA expansion up to twist-5 and $\mathcal{O}(\alpha_s)$ corrections in SCET by CHSWW, they find significantly smaller FFs.

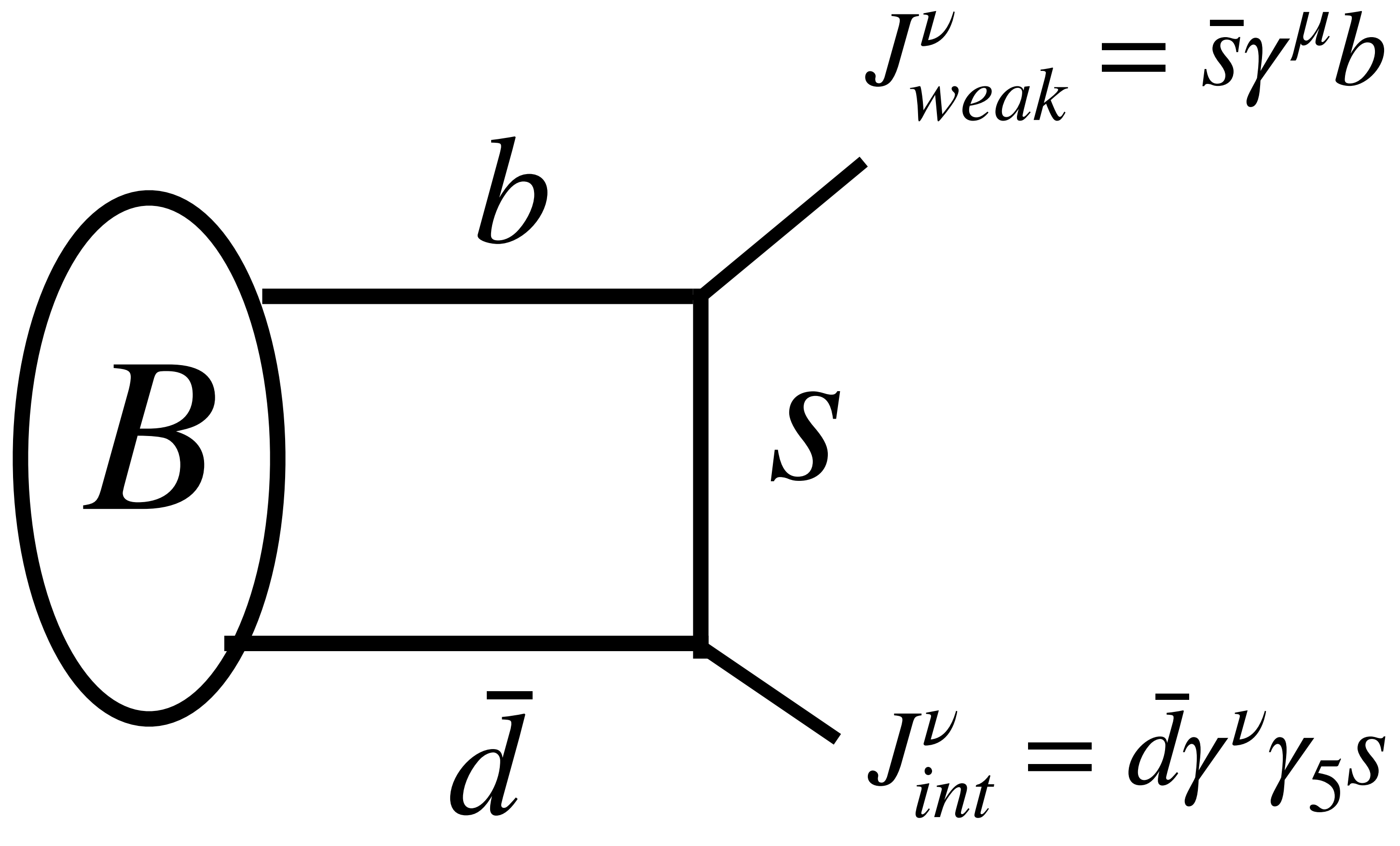
Goal: expand CHSWW's result to HQET



HPQCD collaboration - 2207.13371v2

CHSWW - 2212.11624

Procedure for Light Cone Sum Rules



$$\boxed{\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0 \;\; TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0) \;\; \bar{B}(P_B = q+k) >}$$

Hadronic unitarity relation
&
Dispersion relation

HQET - heavy m_b limit

∝ Light hadron decay constant

What we want to compute

$$\Pi^{\mu\nu}(q^2,k^2) = \frac{\langle 0 \;\; j_{\nu} \;\; M(k)\rangle\langle M(k) \;\; j_{\mu} \;\; B\rangle}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{\infty} ds\frac{\rho^{\mu\nu}}{s-k^2}$$

$$\Pi^{\mu\nu} = \int d^4x T(x)\Phi(x)$$

Perturbative piece

Non local matrix element

Integral dominated by contributions on the light cone $x^2 \ll 1/\Lambda^2_{QCD}$

Near the LC: Φ expandable in twists
(Twist = dimension - spin)
Using **LC B-meson distribution amplitudes**

$$\boxed{K^{(F)}\frac{FF\left(q^2\right)}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\frac{\rho(s)}{s-k^2} = f_B m_B\int_0^{+\infty} ds\sum_{n=1}^{+\infty}\frac{I_n(s)}{(s-k^2)^n}}$$

Then, quark hadron duality…

NLO correction to LCSR with B-meson DA's

$$\Pi_{\mu\nu} = \Pi^0_{\mu\nu} + \frac{\alpha_s}{4\pi}\Pi^1_{\mu\nu} + \ldots \qquad \Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0 \;\; T J^\nu_{int}(x) J^\mu_{\text{weak}}(0) \;\; \bar{B}(P_B = q+k) >$$

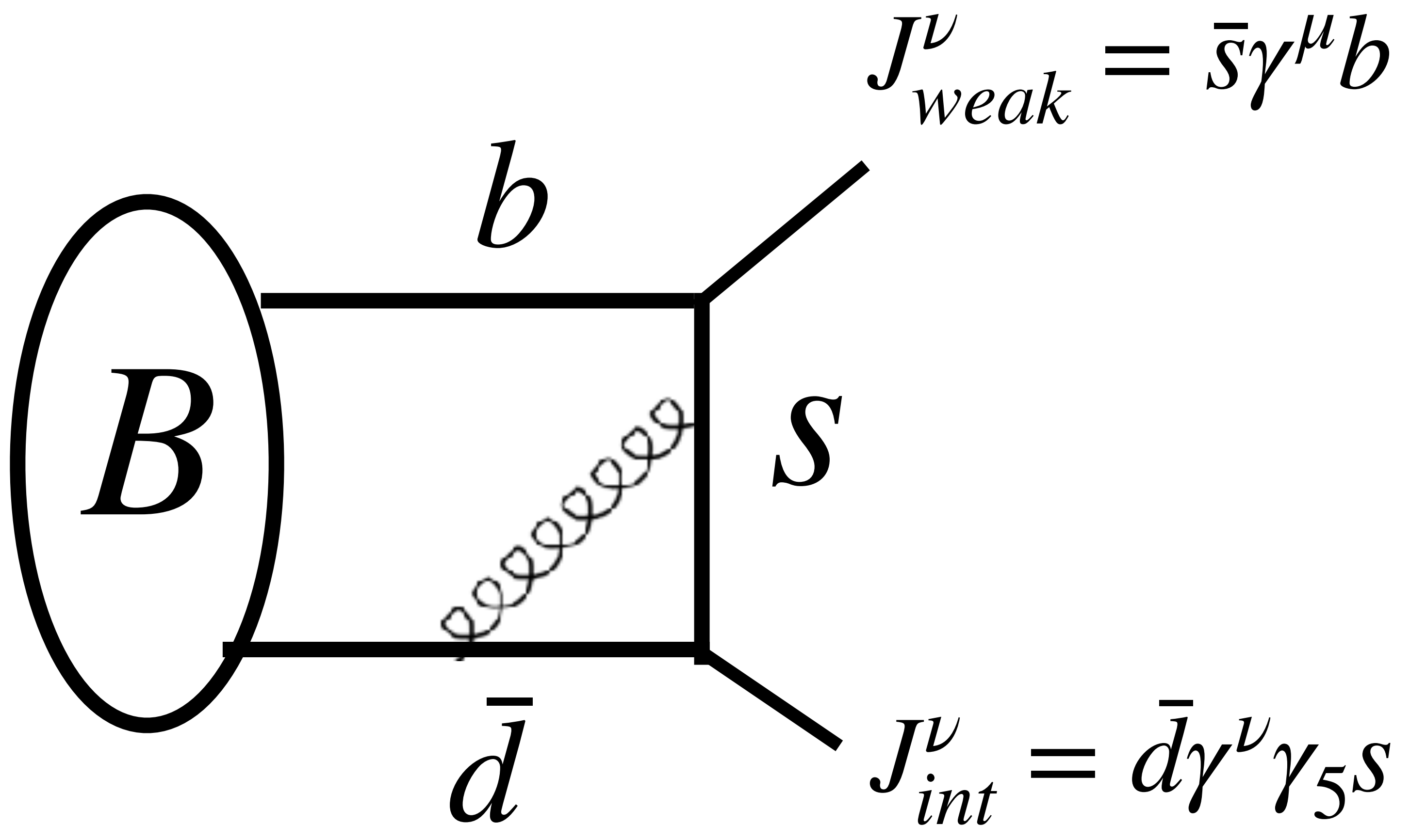
$$\Pi^0_{\mu\nu} = T^0_{\mu\nu} \otimes \Phi^0$$

See e.g. GKvD 1811.00983

$$\Pi^1_{\mu\nu} = T^1_{\mu\nu} \otimes \Phi^0 + T^0_{\mu\nu} \otimes \Phi^1$$

Unknown

Known



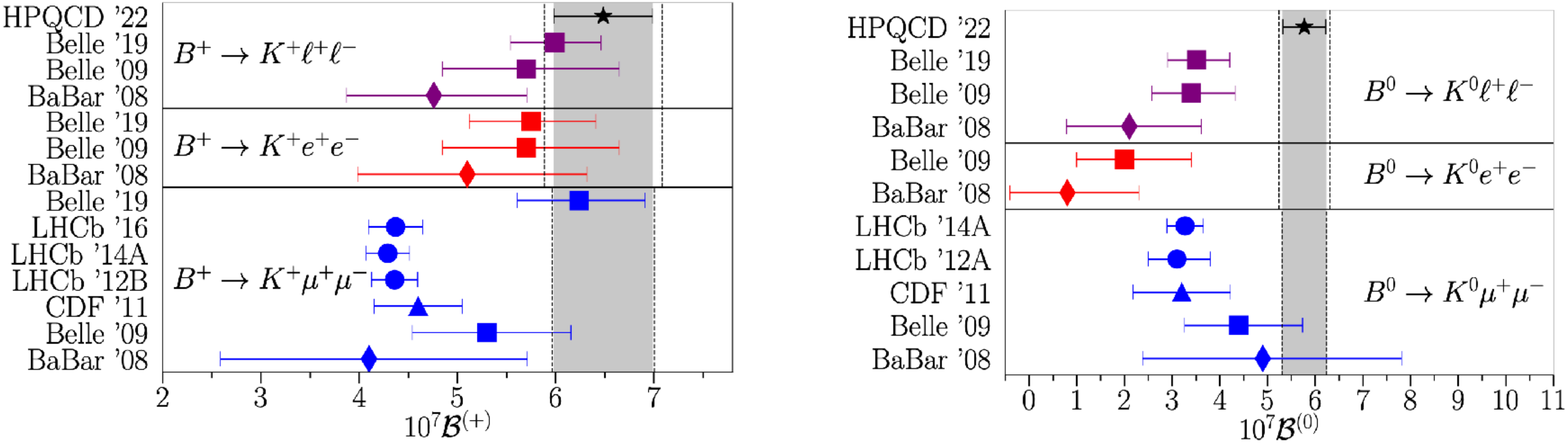
Trick: T^i is independent of the long distance physics, to compute them we can go to the limit where the external states are partonic where Π is directly computable, and replace Φ with $\tilde{\Phi} \equiv$ light cone wave function at tree-level

Summary and prospects

- Deviations in charged current B decays subsists in $R(D^{(*)})$
  - Global fits of EFTs provide precious information about the nature of the putative NP
  - Theory papers used need to be cited properly when using public codes (not always easy!)
- Deviations in neutral current B decays subsist in BR's and angular observables
  - In BR, TH uncertainty is as large or larger than EXP and is dominated by local form factor uncertainties
  - For $B \to K$, LQCD and LCSR with B-meson DA's in SCET with NLO give incompatible FFs, we are computing these FFs in HQET
  - Using B-meson DA's lets us compute many different FF: $B \to K^{(*)}, \pi, \rho, D^{(*)}, \ldots$

Thank you!

Backup

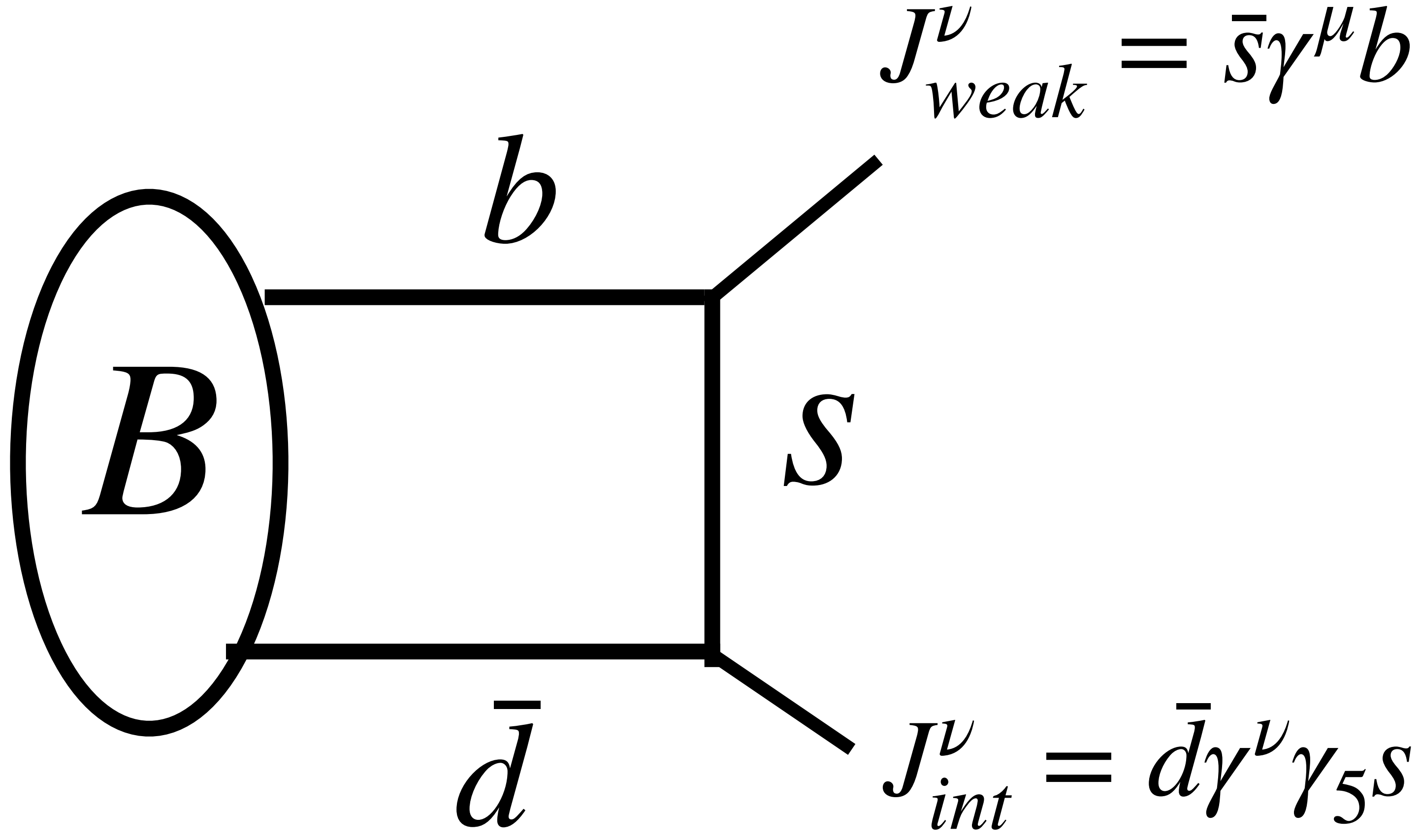


HPQCD Collaboration - 2207.13371

Procedure for Light Cone Sum Rules

$$\boxed{\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0 \ \ TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0) \ \ \bar{B}(P_B = q+k) >}$$

Correlation function of B to vacuum
(also possible with final meson to vacuum)



1) Express Π in function of the non-perturbative quantities that we want to calculate

2) Compute Π perturbatively

3) 1) = 2) + use of quark-hadron duality

Procedure for Light Cone Sum Rules

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0 \ \ TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0) \ \ \bar{B}(P_B = q+k) >$$

Hadronic unitarity relation
&
Dispersion relation

Density of excited states
of the final meson

∝ Light hadron decay constant

What we want to compute

$$\Pi^{\mu\nu}(q^2,k^2) = \frac{\langle 0 \ \ j_\nu \ \ M(k)\rangle\langle M(k) \ \ j_\mu \ \ B\rangle}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{\infty} ds \frac{\rho^{\mu\nu}}{s-k^2}$$

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$$K^{(F)}\frac{F\left(q^2\right)}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho(s)}{s-k^2}$$

Procedure for Light Cone Sum Rules

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0 \;\; T J^{\nu}_{int}(x) J^{\mu}_{\text{weak}}(0) \;\; \bar{B}(P_B = q+k) >$$

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HQET - heavy m_b limit

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What we want to compute

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$$\Pi^{\mu\nu} = \int d^4x \int \frac{d^4p'}{(2\pi)^4} e^{i(k-p').x} \left[\Gamma_2^{\nu} \frac{\not{p}' + m_1}{m_1^2 - p'^2}\Gamma_1^{\mu}\right]_{\alpha\beta} < 0 \;\; \bar{q}_2^{\alpha}(x) h_v^{\beta}(0) \;\; \bar{B}(v) >$$

$$K^{(F)}\frac{F\left(q^2\right)}{m_M^2-k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho(s)}{s-k^2}$$

Procedure for Light Cone Sum Rules

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Integral dominated by terms
on the light cone $x^2 \ll 1/\Lambda_{QCD}^2$

Near the LC: Expansion in twists
(Twist = dimension - spin)
In terms of **LC B-meson distribution amplitudes**

$$\boxed{K^{(F)}\frac{F\left(q^2\right)}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\frac{\rho(s)}{s - k^2} = f_B m_B\int_0^{+\infty} ds\sum_{n=1}^{+\infty}\frac{I_n(s)}{(s - k^2)^n}}$$

$k^2 \ll \Lambda_{\text{had}}^2$
$\tilde{q} \leq m_b^2 + m_b k^2/\Lambda_{\text{had}}$

Quark-Hadron Duality at leading order in twist

$$K^{(F)}\frac{F\left(q^2\right)}{m^2-k^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\frac{\rho(s)}{s-k^2} = \Pi = f_B m_B \int_0^{+\infty} ds\,\frac{I_1(s)}{s-k^2}$$

$$\mathscr{B}_{M^2} f(k^2) = \lim_{\substack{-k^2,\, n\to\infty \\ \frac{-k^2}{n}=M^2}} \frac{(-q^2)^{n+1}}{n!}\left(\frac{d}{dk^2}\right)^n f(k^2)$$

$\Downarrow$ Borel transform

$$K^{(F)}F\left(q^2\right)e^{-m^2/M^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\rho(s)\,e^{-s/M^2} = \Pi = f_B m_B\int_0^{+\infty} ds\,I_1(s)e^{-s/M^2}$$

M^2 : Borel parameter

s_0 : Duality threshold

Semi-global quark hadron duality: there is a s_0 such that

$$\frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\,\rho(s)\,e^{-s/M^2} \simeq \int_{s_0}^{+\infty} ds\,\mathrm{Im}\,\Pi^{\mathrm{pert}}(\mathrm{q}^2,\mathrm{s})\,e^{-s/M^2} \simeq \mathrm{f_B m_B}\int_{s_0}^{+\infty} ds\,I_1(s)e^{-s/M^2}$$

unknown systematic error

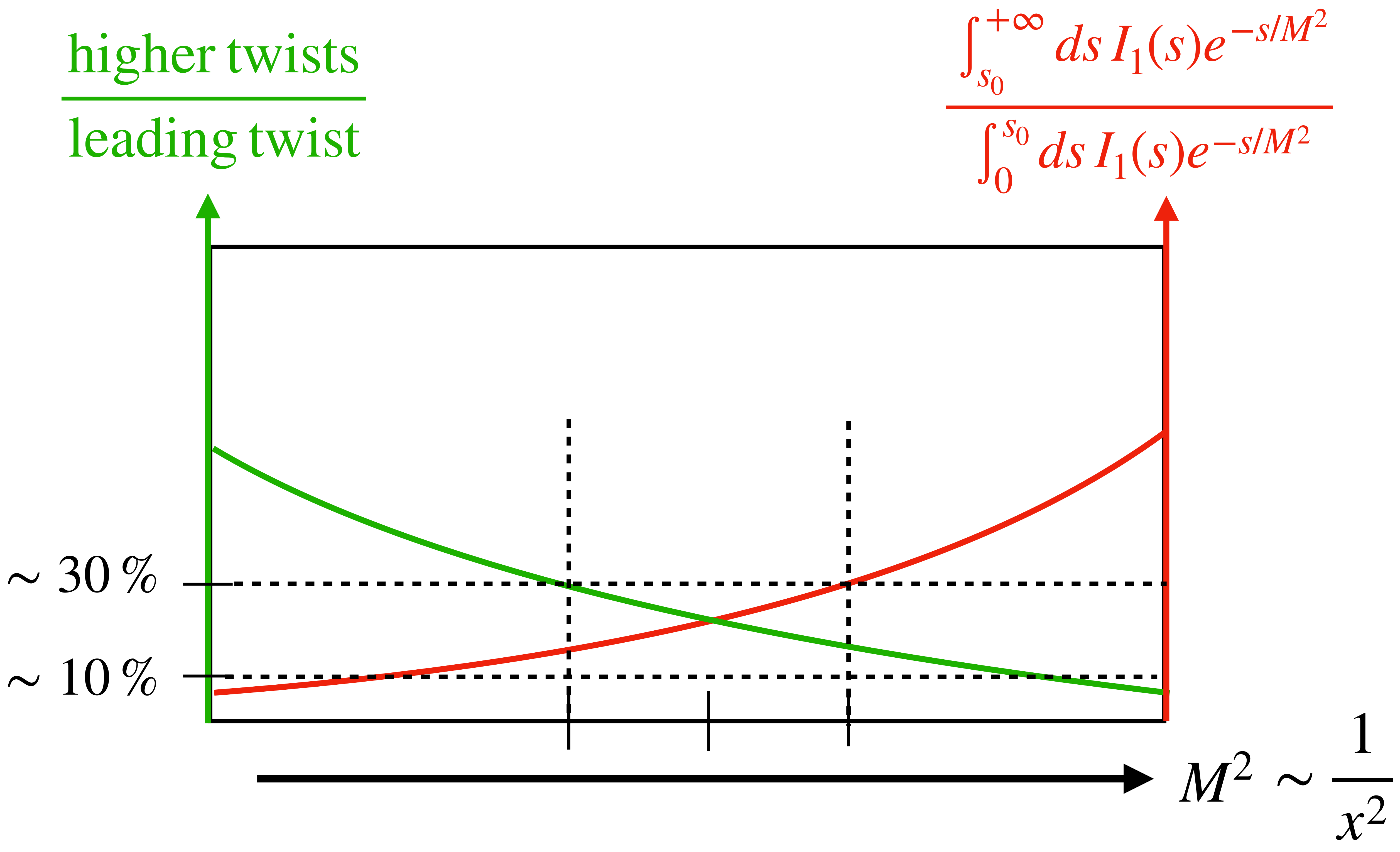
$$\boxed{F\left(q^2\right) = \frac{f_B m_B}{K^{(F)}}\int_0^{s_0} ds\,I_1(s)\,e^{\frac{-s+m^2}{M^2}}}$$

How to determine the threshold parameter s_0

$$F\left(q^2\right) = \frac{f_B m_B}{K^{(F)}} \int_0^{s_0} ds\, I_1(s)\, e^{\frac{-s+m^2}{M^2}}$$

Threshold s_0 can be determined by looking for independence wrt M^2

Daughter sum rule: $\frac{d}{dM^2} F(q^2) = 0$



Range of the Borel parameter

E.g. for $B \to K$: $M^2 \in [0.5, 1.5]\,\text{GeV}^2$