



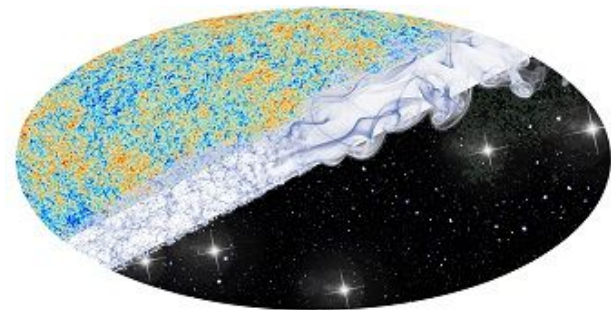
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TAsP Meeting



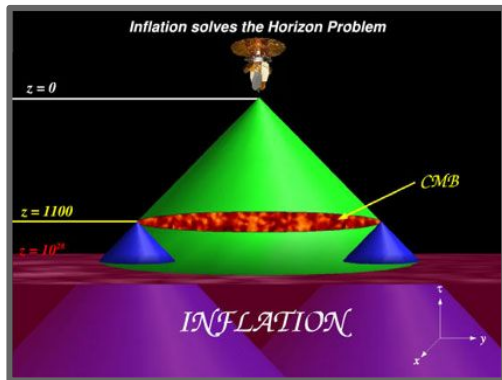
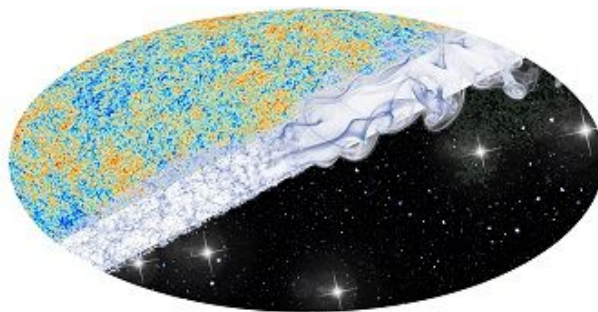
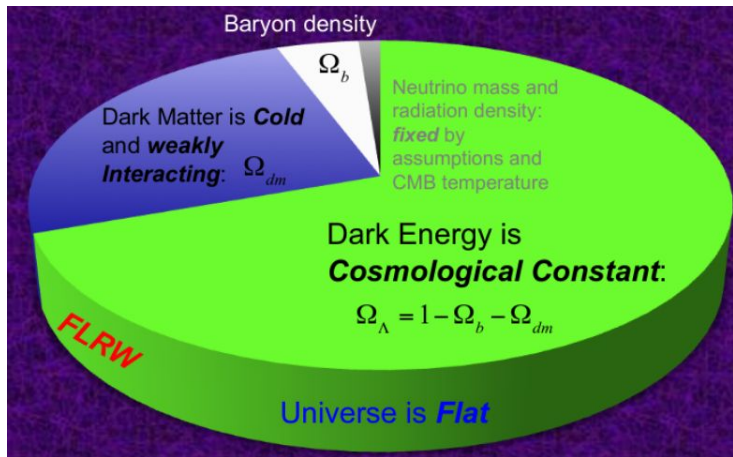
Extended gravity models for inflation in light of the CMB data

Simony Santos da Costa and collaborators



January 18, 2024

The concordance model - Λ CDM: a set of assumptions confirmed by observational data

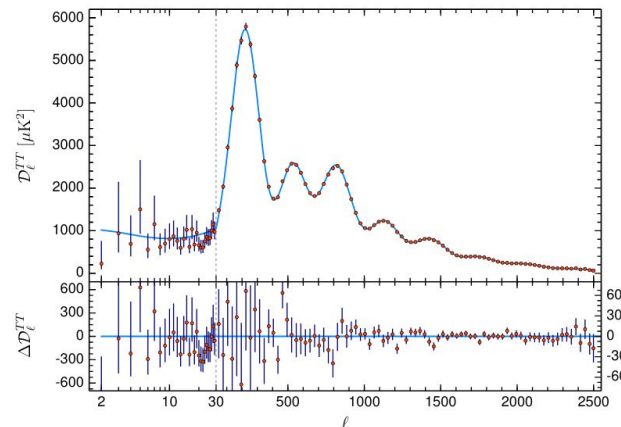


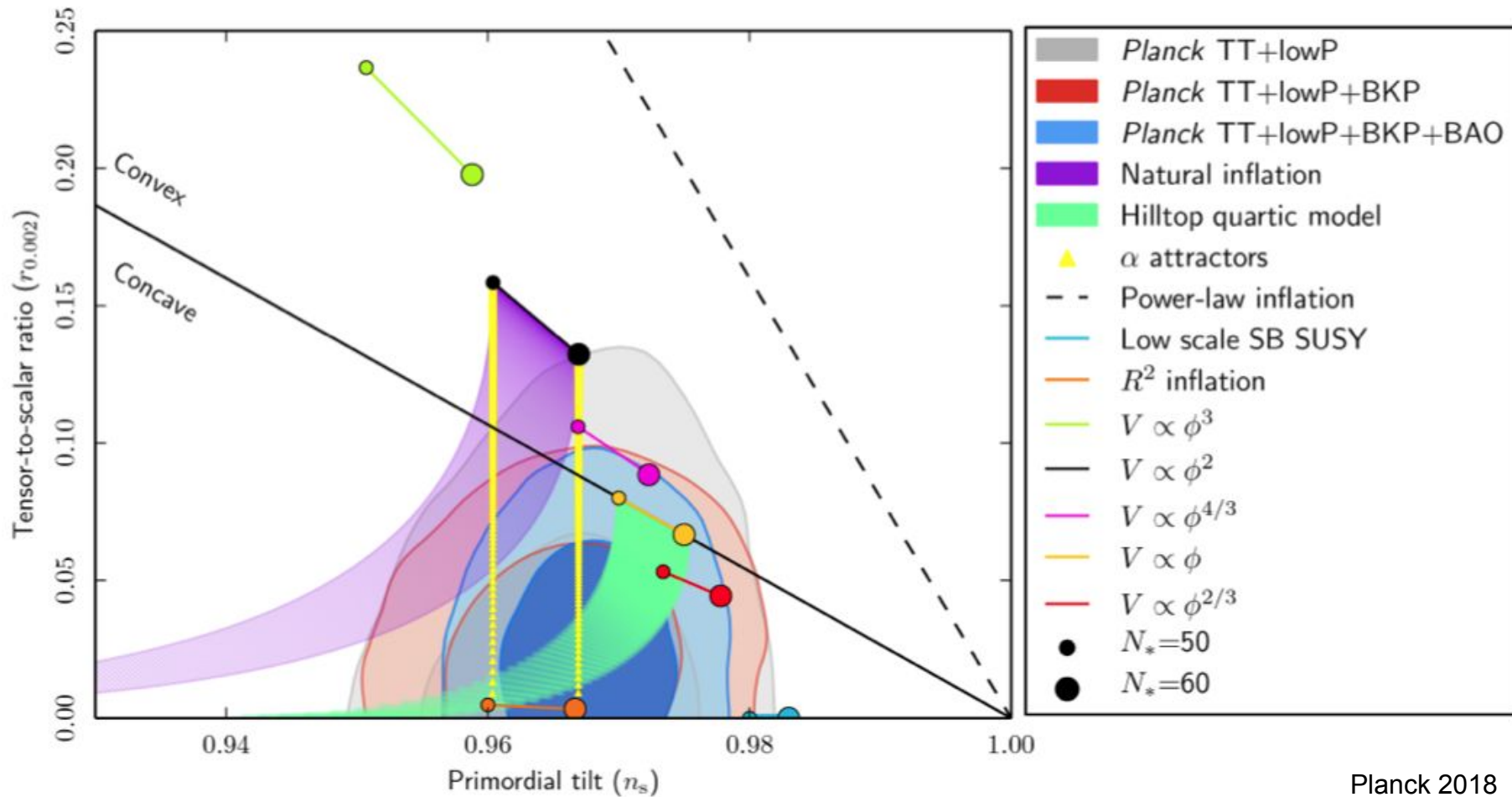
Initial Conditions:
Form of the Primordial
Spectrum is **Power-law**

$$n_s, A_s$$



$$P_R = A_s \left(\frac{k}{k_{pivot}} \right)^{n_s - 1}$$





Planck 2018

Slow-roll

$$H^2 = \frac{1}{3} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right]$$
$$\ddot{\phi} = -3H\dot{\phi} - V'(\phi)$$

$$\dot{H} \sim 0.$$

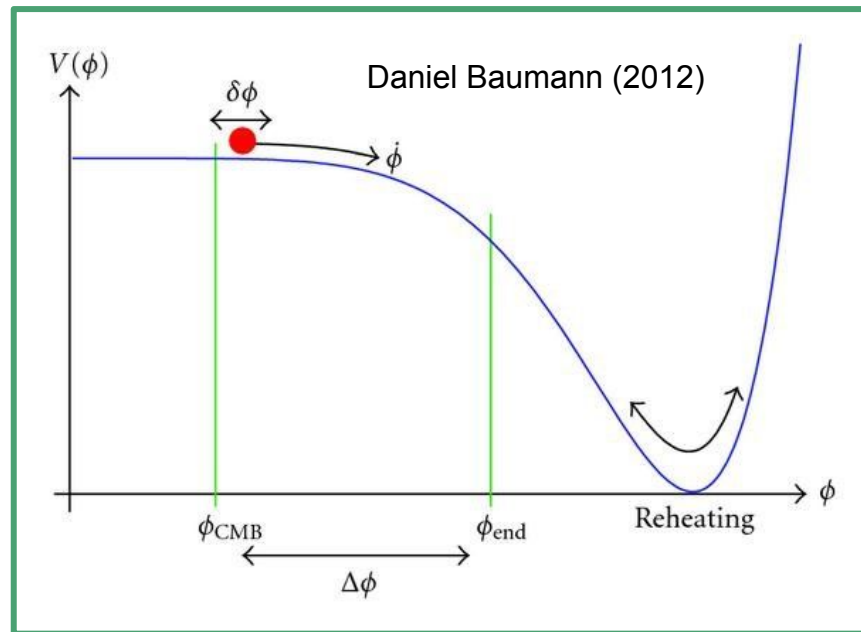
$$\frac{\dot{H}}{H^2} \ll 1,$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2$$

$$\eta = -\frac{1}{H} \frac{\ddot{\phi}}{\dot{\phi}} = M_{Pl}^2 \frac{V''}{V}$$

↓

$$\epsilon \ll 1 \quad \eta \ll 1$$

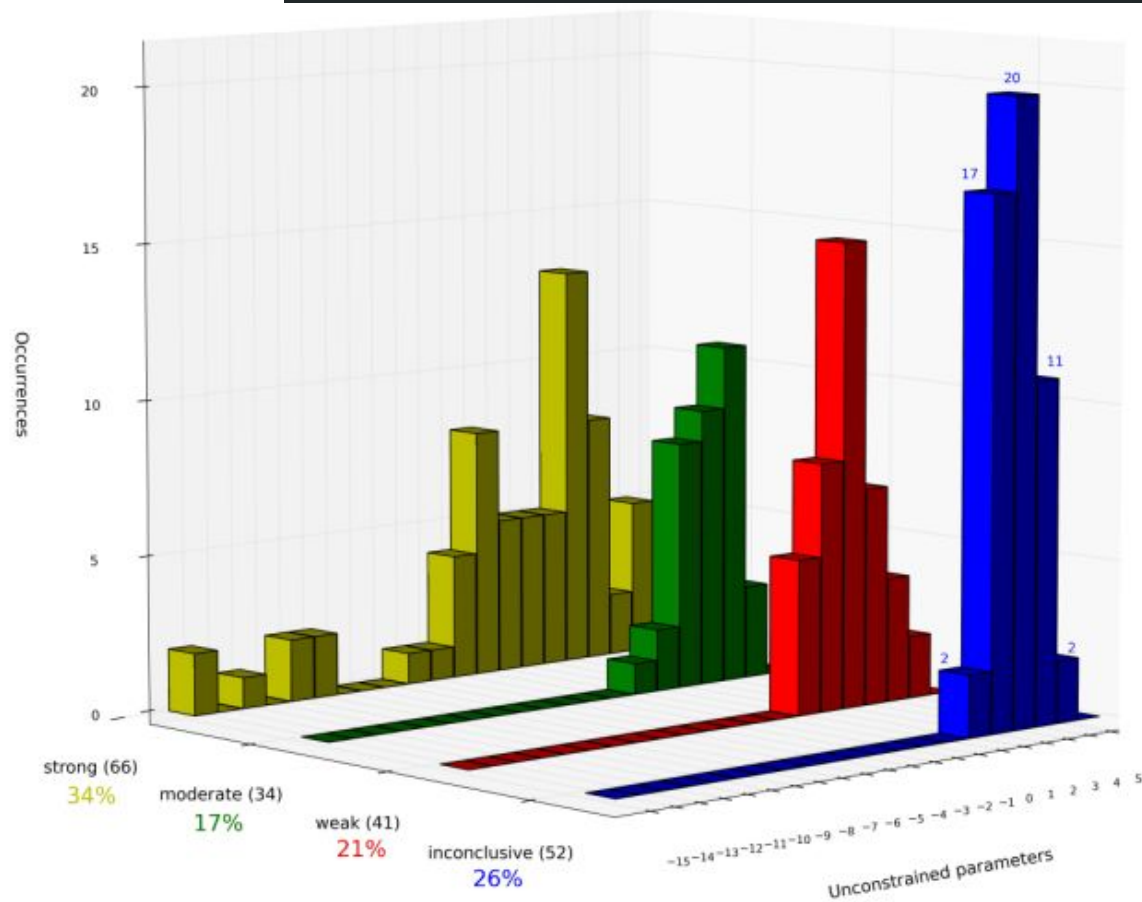


$$N \equiv \int_{a_{ini}}^{a_{end}} d \ln a = \int_{t_{ini}}^{t_{end}} H dt = \int_{\phi_{ini}}^{\phi_{end}} \frac{H}{\dot{\phi}} |d\phi| = \int_{\phi_{end}}^{\phi_{ini}} \frac{|d\phi|}{\sqrt{2\epsilon}}$$

A lot of models

Best performance:

Starobinsky



[Martin *et al.* (2014)]

Theoretical Motivation

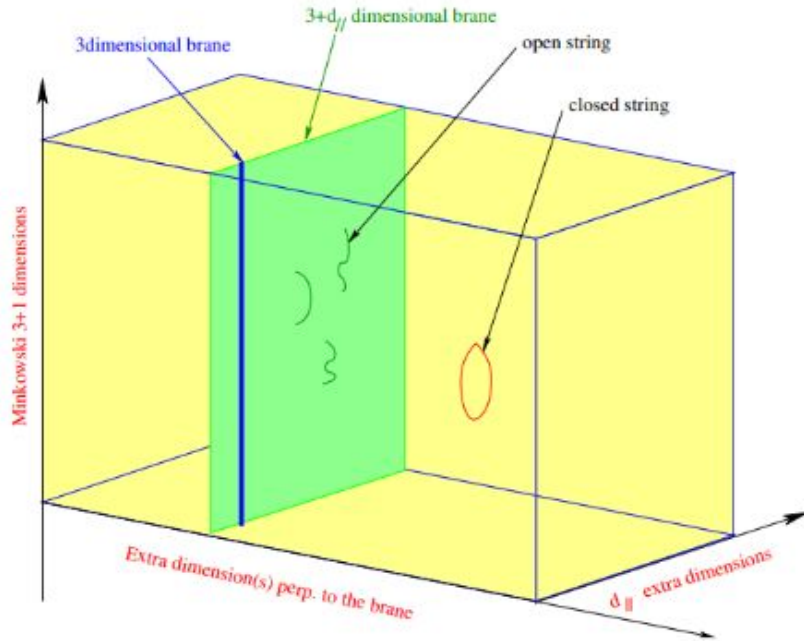
$$V(\phi) = V_0 \left[1 - \exp \left(-\sqrt{\frac{2}{3}} \frac{\phi}{M_{Pl}} \right) \right]^2$$

- Higgs model with a non-minimal coupling to gravity, $\frac{\xi}{2}\phi^2 R - \frac{\lambda}{4}(\phi^2 - v^2)^2$, for $\xi < 0$ in the limit $1 + \xi v^2 \ll 1$ [Linde *et al.* (2011)];
- Higgs inflation in the theory $\frac{\lambda}{4}\phi^4$ with a sufficiently large non-minimal coupling to gravity $\frac{\xi}{2}\phi^2 R$ [Bezrukov *et al.* (2008)];
- Simple locally conformally invariant theory with spontaneous symmetry breaking [Kallosh *et al.* (2013)];
- Superconformal theory and supergravity [Kallosh *et al.* (2013)];
- $f(R)$ theories [Huang (2014)] and higher order polynomial corrections [Artymowski *et al.* (2015)];
- From a wide family of string models by using the Noether Symmetry Approach Capozziello *et al.* (2016)].

Brane inflation and the robustness of the Starobinsky inflationary model

Eur.Phys.J.Plus 136 (2021) 1, 84

S. Santos da Costa¹, M. Benetti^{2,3,4}, R.M.P. Neves⁵, F. A. Brito^{5,6}, R. Silva^{7,8}, and J. Alcaniz¹



Antoniadis , I. (2007)

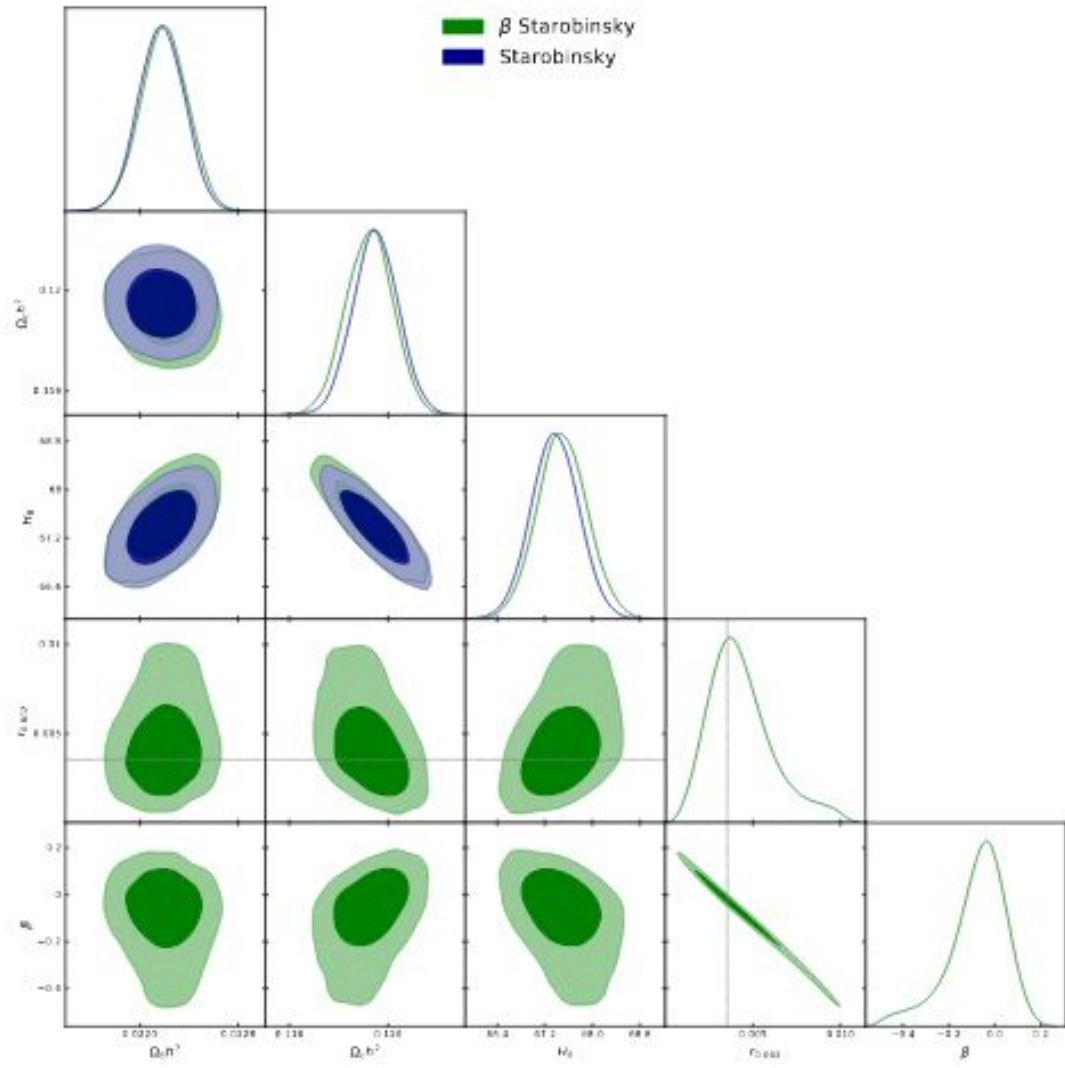
$$S = \int d^D x \sqrt{-g} \left(-\frac{1}{4} R + \frac{1}{2} \partial_M \phi_i \partial^M \phi_i - V(\phi_i) \right),$$

$$V_{\text{eff}}(L) = A_0 (1 - c_1 L)^{\frac{1}{\lambda c_1}} + \frac{1}{2} \sigma,$$

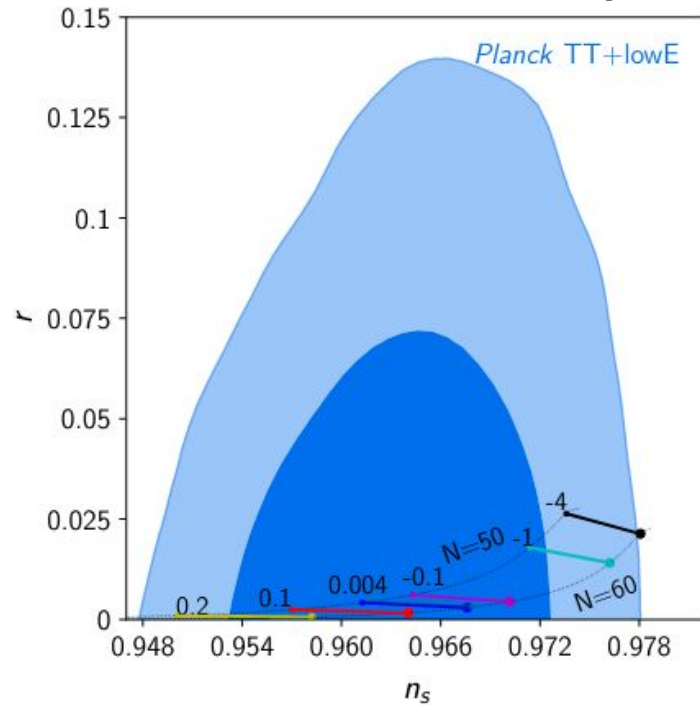
Extended Starobinsky

$$V(\phi) = V_0 \left[1 - \left(1 - \beta \sqrt{\frac{2}{3}} \frac{\phi}{M_{\text{Pl}}} \right)^{\frac{1}{\beta}} \right]^2,$$

$$\exp_{1-\beta}(f) = [1 + \beta f]^{1/\beta}$$



Extended Starobinsky



$$n_s = 1 + \frac{8}{3} \frac{\chi^{\frac{1}{\beta}-2}}{\left(1 - \chi^{\frac{1}{\beta}}\right)^2} \left[\beta \left(1 - \chi^{\frac{1}{\beta}}\right) - 1 - \chi^{\frac{1}{\beta}} \right],$$

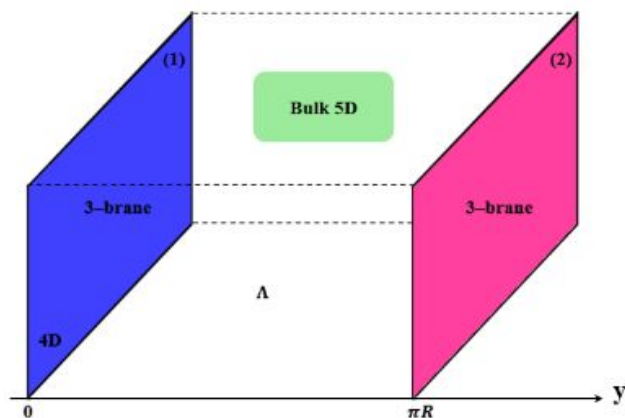
$$r = \frac{64}{3} \chi^{\frac{2}{\beta}-2} \left(1 - \chi^{\frac{1}{\beta}}\right)^{-2}, \quad \chi \equiv 1 - \beta \sqrt{\frac{2}{3}} \frac{\phi}{M_{Pl}}.$$

Brane inflation driven by an arctan potential: CMB constraints and Reheating

JCAP 07 (2022) 07, 024

$$e^{-1} \mathcal{L}_{sugra} = -\frac{1}{4} M_*^3 R_{(5)} + G_{AB} \partial_\mu \phi^A \partial^\mu \phi^B - \frac{1}{4} G^{AB} \frac{\partial W(\phi)}{\partial \phi^A} \frac{\partial W(\phi)}{\partial \phi^B} + \frac{1}{3} \frac{1}{M_*^3} W(\phi)^2. \quad (3.5.40)$$

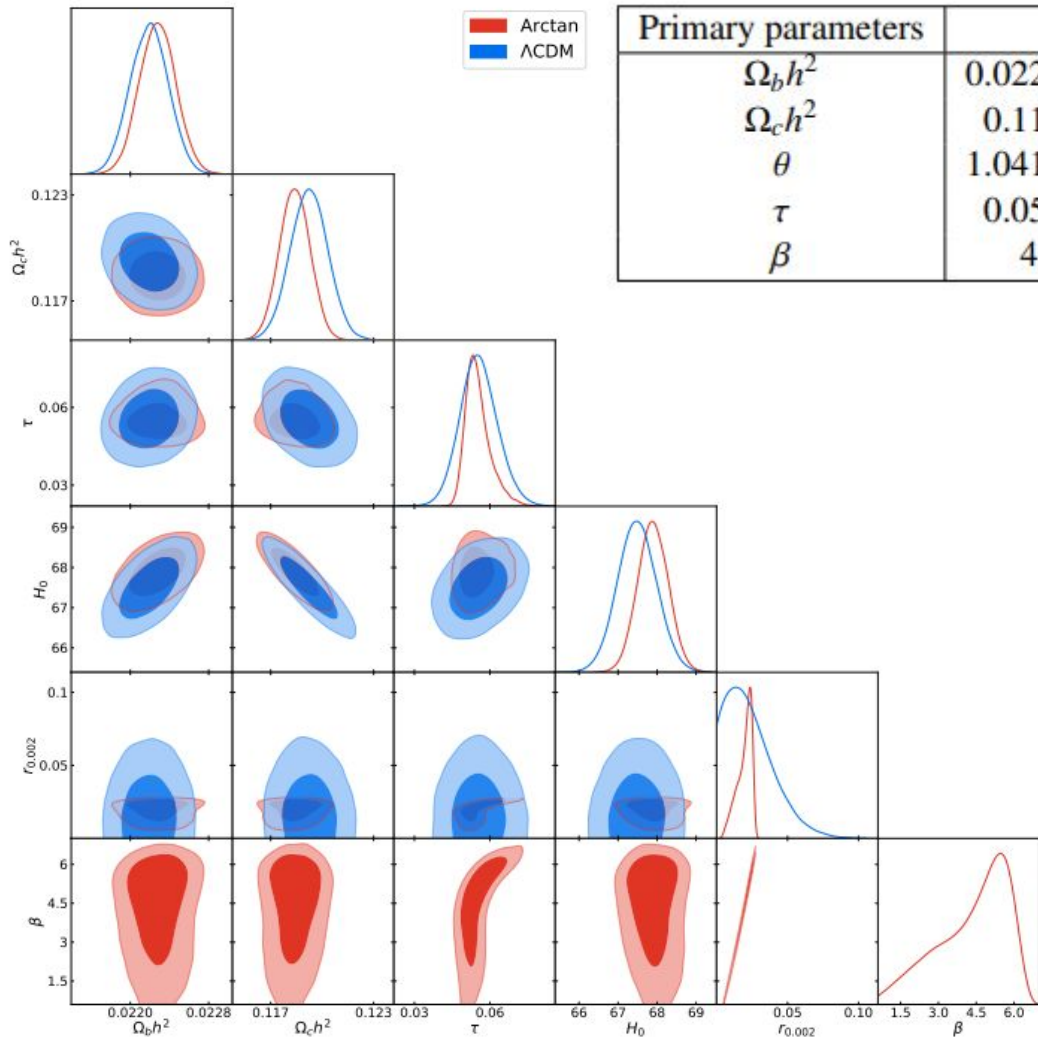
R. M. P. Neves^a S. Santos da Costa^b F. A. Brito^{a,c} J. S. Alcaniz^b



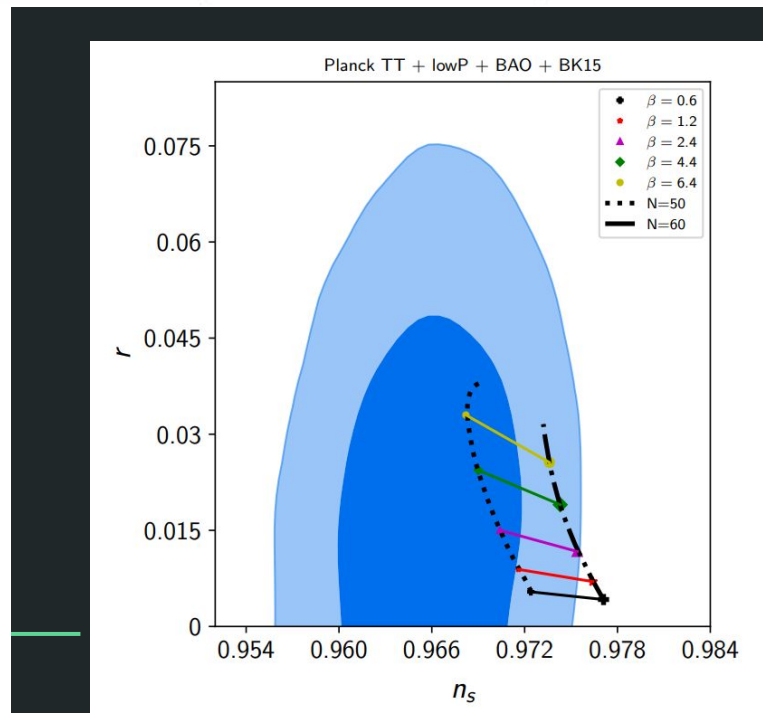
$$V(\phi) = K \beta \arctan \left(\frac{\phi}{\beta} \right),$$

$$n_s = 1 - \frac{4}{\beta^2 \arctan \left(\frac{\phi}{\beta} \right) \left(1 + \frac{\phi^2}{\beta^2} \right)^2} - \frac{3}{2\beta^2 \arctan^2 \left(\frac{\phi}{\beta} \right) \left(1 + \frac{\phi^2}{\beta^2} \right)^2},$$

$$r = \frac{8}{\beta^2 \arctan \left(\frac{\phi}{\beta} \right) \left(1 + \frac{\phi^2}{\beta^2} \right)^2}$$



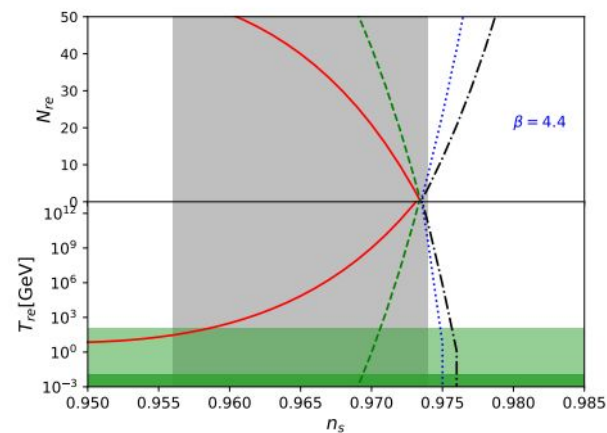
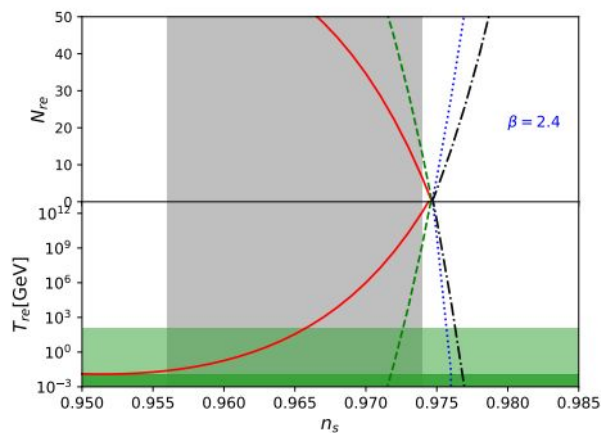
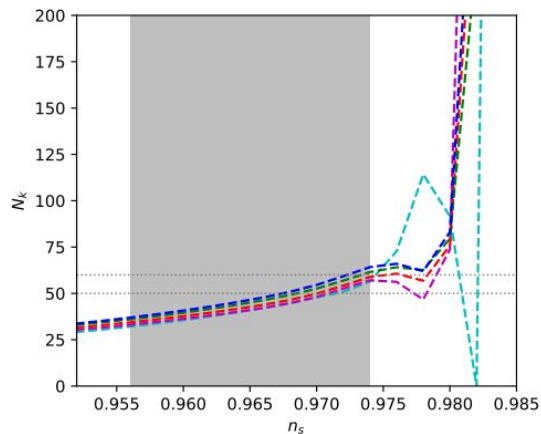
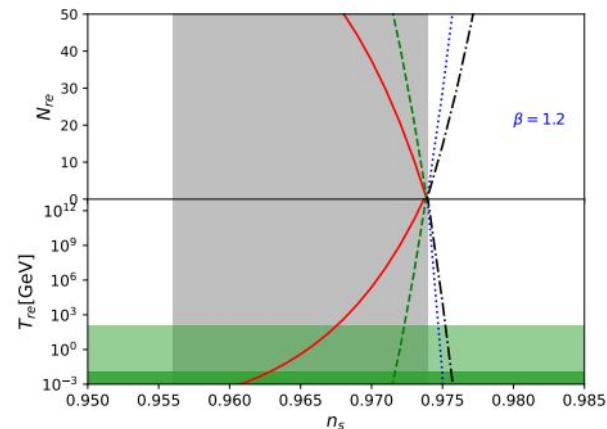
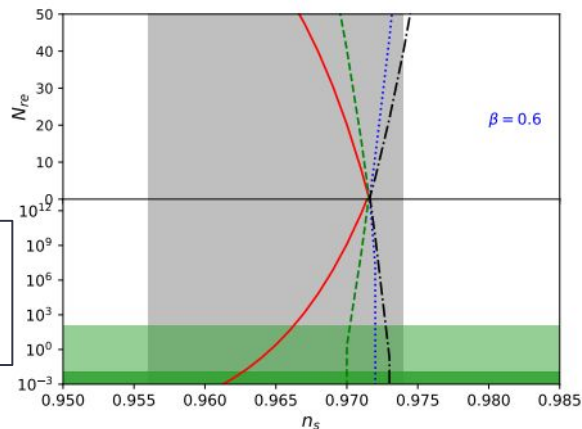
Primary parameters	Derived parameters
$\Omega_b h^2$	H_0
$\Omega_c h^2$	Ω_m
θ	Ω_Λ
τ	n_s
β	$r_{0.002}$
	0.02228 ± 0.00018
	0.1184 ± 0.0009
	1.04107 ± 0.00041
	0.0553 ± 0.0049
	4.37 ± 1.38
	67.89 ± 0.38
	0.307 ± 0.005
	0.693 ± 0.005
	0.9719 ± 0.0562
	0.0196 ± 0.0052



Reheating constraints

$$N_{re} = \frac{4}{1 - 3w_{re}} \left[61.6 - \ln \left(\frac{V_{end}^{\frac{1}{4}}}{H_k} \right) - N_k \right],$$

$$T_{re} = \left[\left(\frac{11g_{re}}{43} \right)^{\frac{1}{3}} \left(\frac{k}{a_0 T_0} \right) H_k e^{-N_k} \left(\frac{45V_{end}}{\pi^2 g_{re}} \right)^{-\frac{1}{3(1+w_{re})}} \right]^{\frac{3(1+w_{re})}{3w_{re}-1}}$$



Observational constraints on α -attractor inflationary models with a Higgs-like potential

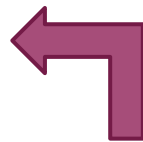
Phys.Lett.B 815 (2021) 136156

J. G. Rodrigues^a, S. Santos da Costa^b, J. S. Alcaniz^b

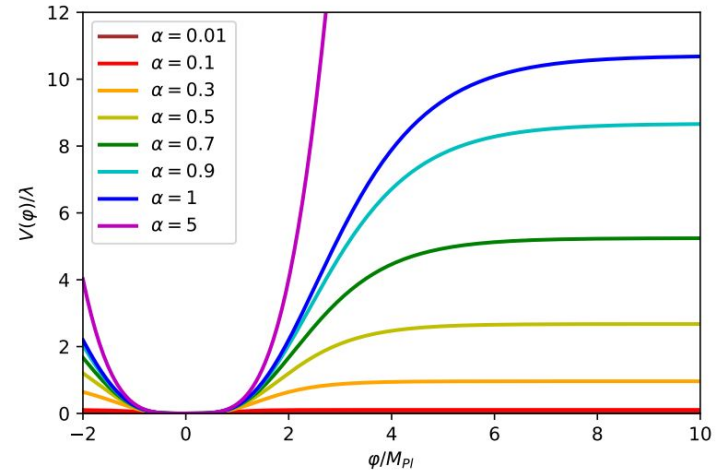
$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2,$$

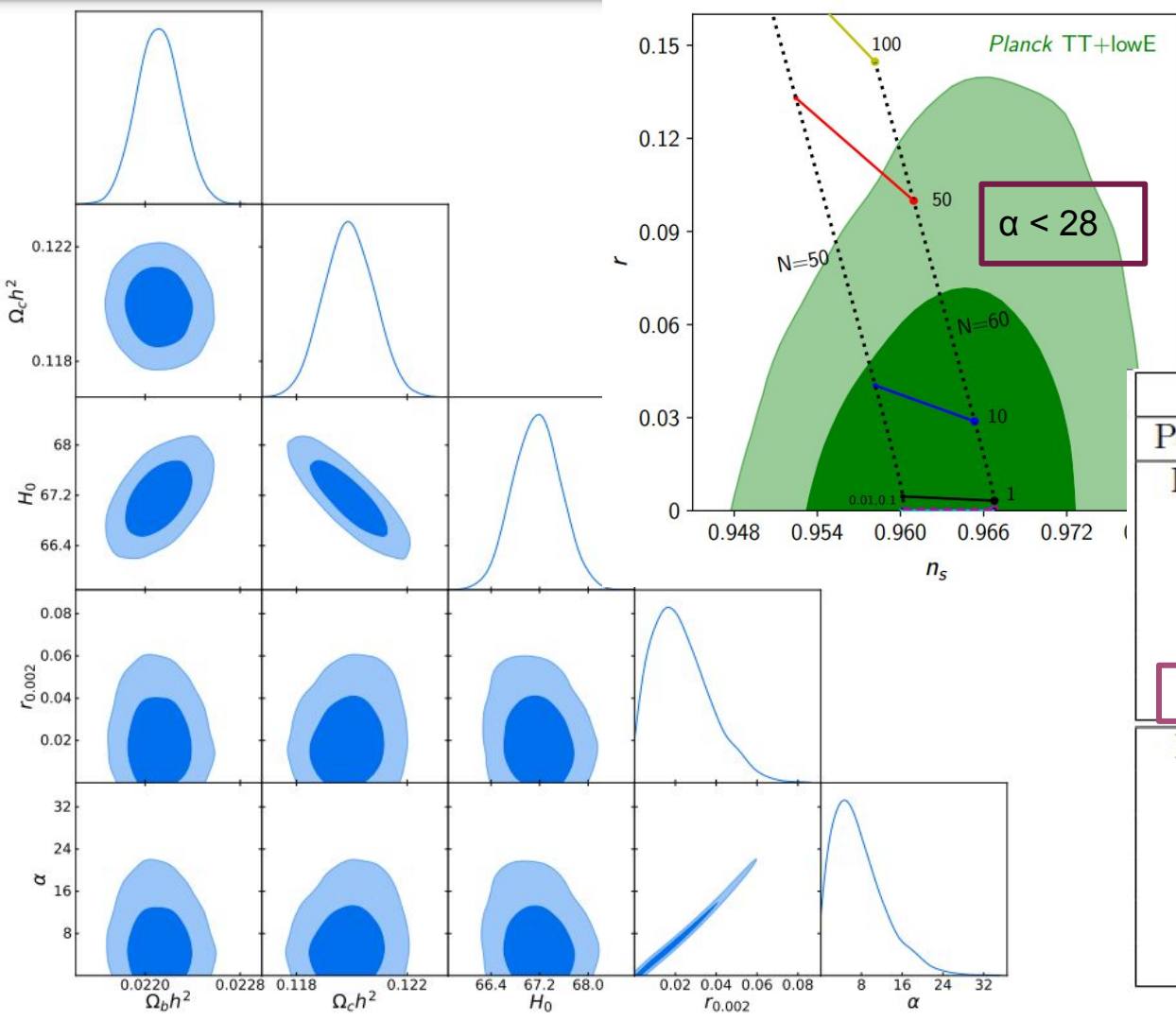
$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2}R - \frac{(\partial\varphi)^2}{2} - V(\sqrt{6\alpha} \tanh \frac{\varphi}{\sqrt{6\alpha}}) \right].$$

$$\phi = \sqrt{6\alpha} \tanh \frac{\varphi}{\sqrt{6\alpha}},$$



$$V(\varphi) = 9\alpha^2\lambda \left(\tanh \frac{\varphi}{\sqrt{6\alpha}} \right)^4,$$





Parameter	Higgs-like	
	mean	best fit
Primary		
$\Omega_b h^2$	0.02212 ± 0.018	0.02208
$\Omega_c h^2$	0.1199 ± 0.0009	0.1196
θ	1.04085 ± 0.00040	1.04108
τ	0.048 ± 0.003	0.048
α	7.56 ± 5.15	9.82
Derived		
H_0	67.16 ± 0.38	67.21
Ω_m	0.316 ± 0.005	0.320
Ω_Λ	0.684 ± 0.005	0.680
n_s	0.963 ± 0.001	0.9621
$r_{0.002}$	0.023 ± 0.014	0.030

Constraining non-minimally coupled β -exponential inflation with CMB data

JCAP 06 (2022) 06, 001

F. B. M. dos Santos^a S. Santos da Costa^b R. Silva^{a,c} M. Benetti,^{d,e} J. S. Alcaniz^b

$$\int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} \hat{R} - \frac{1}{2} F^2(\phi) \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \hat{V}(\phi) \right],$$

$$F^2(\phi) \equiv \frac{1 + \kappa^2 \xi \phi^2 (1 + 6\xi)}{(1 + \kappa^2 \xi \phi^2)^2},$$

$$\hat{V}(\chi) \equiv \frac{V(\phi)}{(1 + \kappa^2 \xi \phi^2)^2}.$$

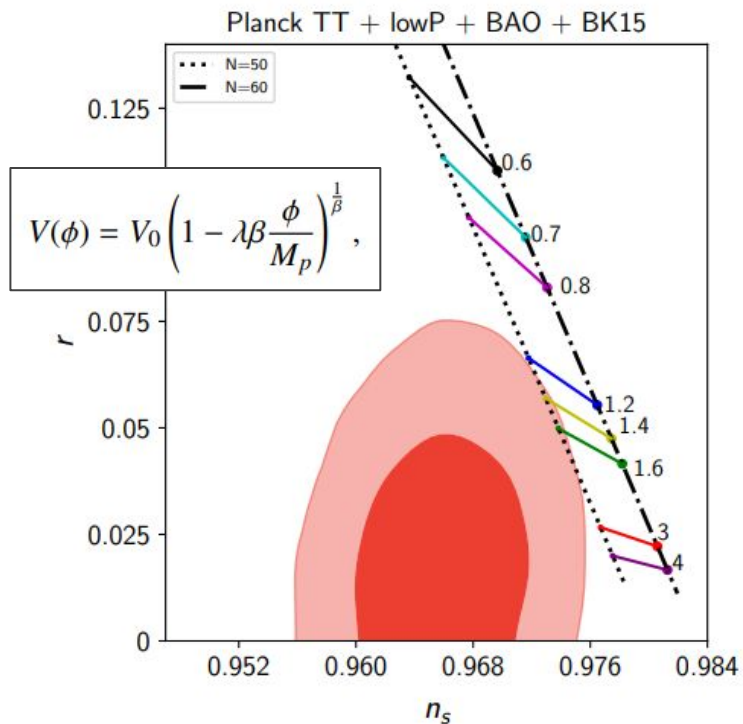


$$S = \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} \hat{R} - \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \hat{V}(\chi) \right],$$

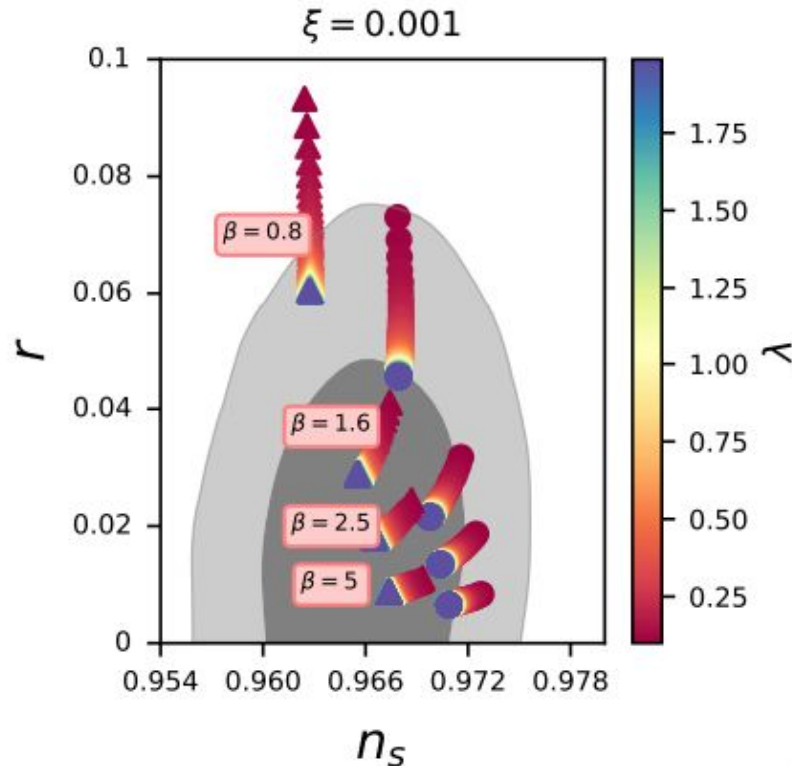
$$\chi_\phi \equiv \frac{d\chi}{d\phi} = \sqrt{\frac{1 + \kappa^2 \xi \phi^2 (1 + 6\xi)}{(1 + \kappa^2 \xi \phi^2)^2}},$$

$$\epsilon = \frac{M_P^2}{2} \left(\frac{V_\phi}{V \chi_\phi} \right)^2, \quad \eta = M_P^2 \left(\frac{V_{\phi\phi}}{V \chi_\phi^2} - \frac{V_\phi \chi_{\phi\phi}}{V \chi_\phi^3} \right), \quad \zeta^2 = M_P^4 \frac{V_\phi}{V^2 \chi_\phi^2} \left(\frac{V_{\phi\phi\phi}}{\chi_\phi^2} - \frac{3V_{\phi\phi} \chi_{\phi\phi}}{\chi_\phi^3} + \frac{3V_\phi \chi_{\phi\phi}^2}{\chi_\phi^4} - \frac{V_\phi \chi_{\phi\phi\phi}}{\chi_\phi^3} \right).$$

The β -exponential potential



Minimally coupled



Non-minimally coupled

$$\hat{V}(\chi(\phi)) = \frac{V_0(1 - \lambda\beta\phi)^{1/\beta}}{(1 + \xi\phi^2)^2}.$$

Cosmological models for $f(R, T) - \Lambda(\phi)$ gravity

¹Joao R. L. Santos,^{*} ²S. Santos da Costa,[†] and ³Romario S. Santos[‡]

$$S = \int d^4x \sqrt{-g} \left(f(R, T) - \frac{\Lambda(\phi)}{2} + \mathcal{L} \right),$$

Phys.Dark Univ. 42 (2023) 101356

First order formalism

$$H \equiv h(\phi); \quad \dot{H} = h_\phi \dot{\phi},$$

$$\dot{\phi} = -W_\phi; \quad W_\phi = \frac{dW(\phi)}{d\phi},$$

$$h_\phi = (1 - 2f')W_\phi.$$

$$V_\phi = \frac{(3hh_\phi - W_\phi h_{\phi\phi} - \rho_{\Lambda\phi})W_\phi}{2h_\phi - W_\phi}.$$

A. First Model - $f_\phi = \alpha T_\phi$

$$W = b_1 \left(-\phi + \frac{\phi^3}{3} \right) + \frac{b_2}{(1 - 2\alpha)}.$$

$$W_\phi = b_1 (\phi^2 - 1); \quad h(\phi) = (1 - 2\alpha) W,$$

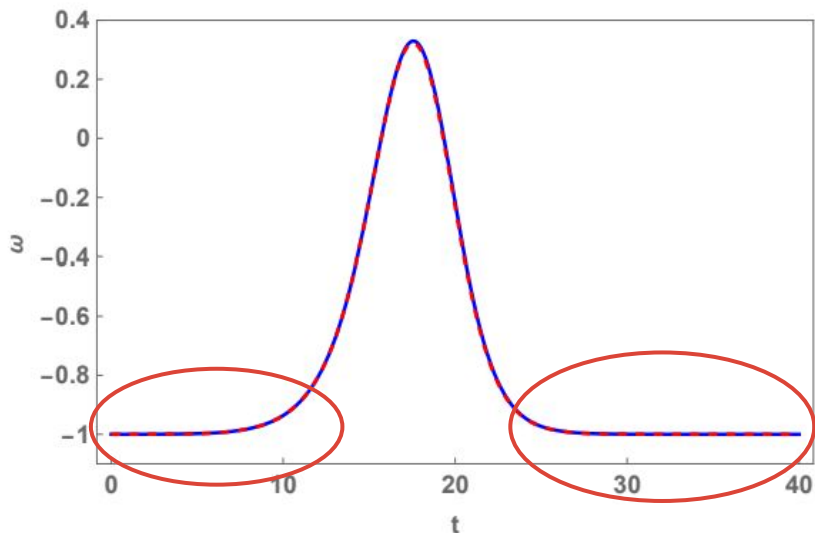
Cosmological models for $f(R, T) - \Lambda(\phi)$ gravity

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Phys.Dark Univ. 42 (2023) 101356

First order formalism



A. First Model - $f_\phi = \alpha T_\phi$

$$W = b_1 \left(-\phi + \frac{\phi^3}{3} \right) + \frac{b_2}{(1 - 2\alpha)}.$$

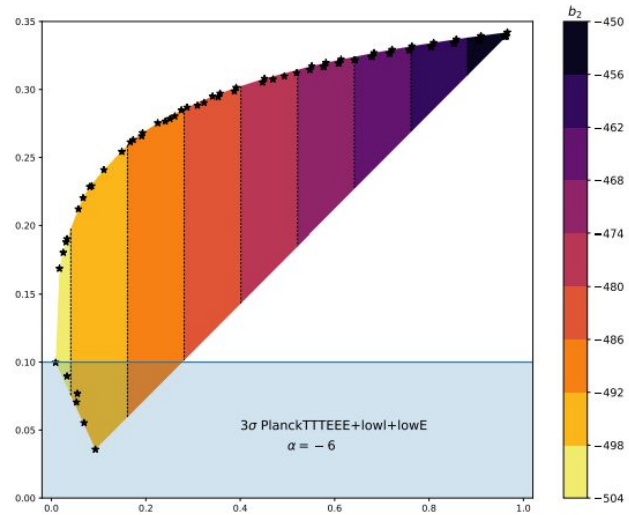
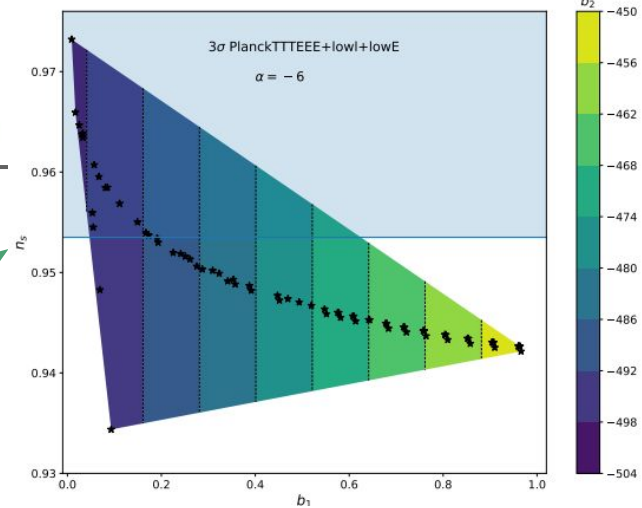
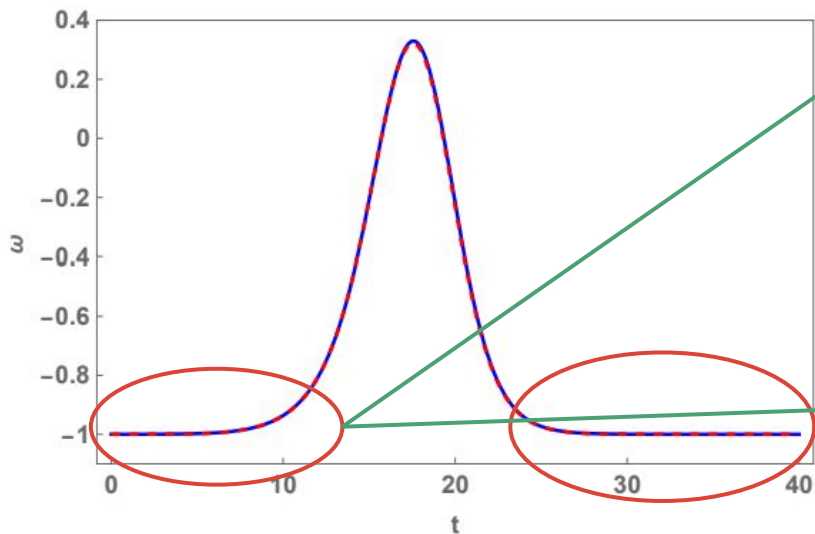
$$W_\phi = b_1 (\phi^2 - 1); \quad h(\phi) = (1 - 2\alpha) W,$$

Cosmological models for $f(R, T) - \Lambda(\phi)$ gravity

¹Joao R. L. Santos,^{*} ²S. Santos da Costa,[†] and ³Romario S. Santos[‡]

Phys.Dark Univ. 42 (2023) 101356

First order formalism



Gauge invariant quantum backreaction in $U(1)$ axion inflation

Davide Campanella Galanti^{a,b}✉, Pietro Conzino^{a,b}✉, Giovanni Marozzi^{a,b}✉ and Simony Santos da Costa^b✉

^aDipartimento di Fisica, Università di Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy

^bIstituto Nazionale di Fisica Nucleare, Sezione di Pisa, Italy

Without matter and metric fluctuations

C. Natural Inflation

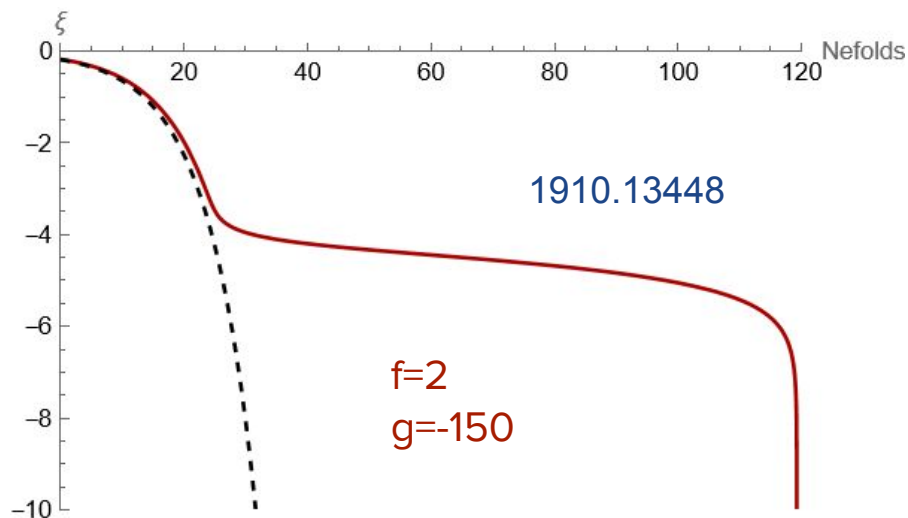
We now compute the gauge invariant backreaction effect to the case of a natural inflation where we have

$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right) \right], \quad (61)$$

$$H^2 = \frac{1}{3m_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) + \frac{\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle}{2} \right],$$

$$\dot{H} = -\frac{1}{2m_{Pl}^2} \left[\dot{\phi}^2 + \frac{2}{3} \langle \mathbf{E}^2 + \mathbf{B}^2 \rangle \right].$$

$$\xi \equiv g\dot{\phi}/(2H). \quad g = \alpha/f,$$



Gauge invariant quantum backreaction in $U(1)$ axion inflation

Davide Campanella Galanti^{a,b,*} Pietro Conzino^{a,b} Giovanni Marozzi^{a,b} and Simony Santos da Costa^b

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^b*Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Italy*

Second order perturbative equations

$$H^2 = \frac{1}{3m_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) + \frac{\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle}{2} \right], \quad \boxed{f \gg M_{Pl}} \quad H_{eff}^2 = H^2 \left[1 - \frac{2}{H} \langle \dot{\psi}^{(2)} \rangle \right]$$
$$\dot{H} = -\frac{1}{2m_{Pl}^2} \left[\dot{\phi}^2 + \frac{2}{3} \langle \mathbf{E}^2 + \mathbf{B}^2 \rangle \right]. \quad = H^2 \left[1 + \frac{1}{3H^2 M_{Pl}^2} \frac{\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle}{2} \right],$$

Full regime

$$H_{eff}^2 = H^2 \left\{ 1 + \frac{1}{M_{Pl}^2} \left[\frac{1}{3H^2} + \left(\frac{M_{Pl}}{f} \right)^4 \frac{1}{\dot{H}} \left(\frac{G(H)}{6} + \frac{1}{144} \right) \right] \frac{\langle \vec{E}^2 + \vec{B}^2 \rangle}{2} \right\}$$

Final remarks

- There is still **space to play** with inflationary models:
 - Advantage >> Theoretically motivated models - foundation of an extended theory of gravity;
- We investigated different braneworld scenarios: extended Starobinsky + ArcTan potentials;
- We have constrained for the first time the parameters associated with the alpha attractor Higgs-like potential;
- We can recover the observational viability of models by coupling the scalar field to the gravity sector;
- Implementation of the *first order formalism* to study the inflationary phase;
- Work in progress with interesting perspectives:
 - **Backreaction in U(1) axion inflation**

