Gravitational waves from holographic phase transitions

Enrico Morgante - U. Trieste & INFN

TAsP meeting — Turin

19/01/2024

Enrico Morgante
Trieste U. & INFN

UNIVERSITÀ DEGLI STUDI DI TRIESTE

Motivation

- Composite sectors…
- Mirror sectors…
- String theory

→ **Strongly interacting dark sector** →

First-order confinement PT

Gravitational waves

EM, Nicklas Ramberg, Pedro Schwaller (U. Mainz)

`Gravitational Waves from Dark SU(3) Yang-Mills Theory [2210.11821, PRD]`

`+ ongoing…`

see talks by
Jacopo Nava
Jun'ya Kume

Enrico Morgante
Trieste U. & INFN

Strong coupling

- Strongly coupled theory —> ?
- Lattice QCD: precious information, but no real-time analysis
- Effective models (Polyakov loop / NJL)
- Holography (bottom-up)

➡ AdS_5 Einstein+dilaton gravity ↔ 4D CFT

Focus on dark SU(3) → SU(N), w/ no fermions

- simple first step
- lattice data well available and understood
- good holographic model

Improved holographic QCD

Gürsoy, Kiritsis, Mazzanti, Nitti

0707.1324, 0707.1349, 0812.0792, 0903.2859, …

Enrico Morgante
Trieste U. & INFN

Improved Holographic QCD

$$\mathcal{S}_5 = -M_P^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\Phi)^2 + V(\Phi) \right] + 2M_P^3 N_c^2 \int_{\partial M} d^4x \sqrt{h} K$$

- Radial fifth-dim coordinate r <—> RG scale
- Scalar field $\lambda = \exp \Phi$ <—> 't Hooft coupling $\lambda_t = N_c g_{\rm YM}^2$
- Scale factor $b(r)$ <—> Energy scale $E = E_0 b(r)$
- Solutions of EOM <—> phases of $\mathrm{SU}(N_c)$
- Φ fluctuations <—> scalar glueballs
- Scalar potential $V(\Phi) \to$ fit SU(3) thermodyn. and spectrum

Enrico Morgante
Trieste U. & INFN

Choice of the potential

UV: $\lambda \to 0$

- Asymptotic freedom
- β function <—> potential

IR: $\lambda \to \infty$

- Confinement
- Linear glueball spectrum $m_n^2 \sim n$
- Thermodynamics
- Screened singularity at $r \to \infty$

Ansatz:

$$V(\lambda) = \frac{12}{\ell^2} \left\{ 1 + V_0 \lambda + V_1 \lambda^{4/3} [\log(1 + V_2 \lambda^{4/3} + V_3 \lambda^2)]^{1/2} \right\}$$

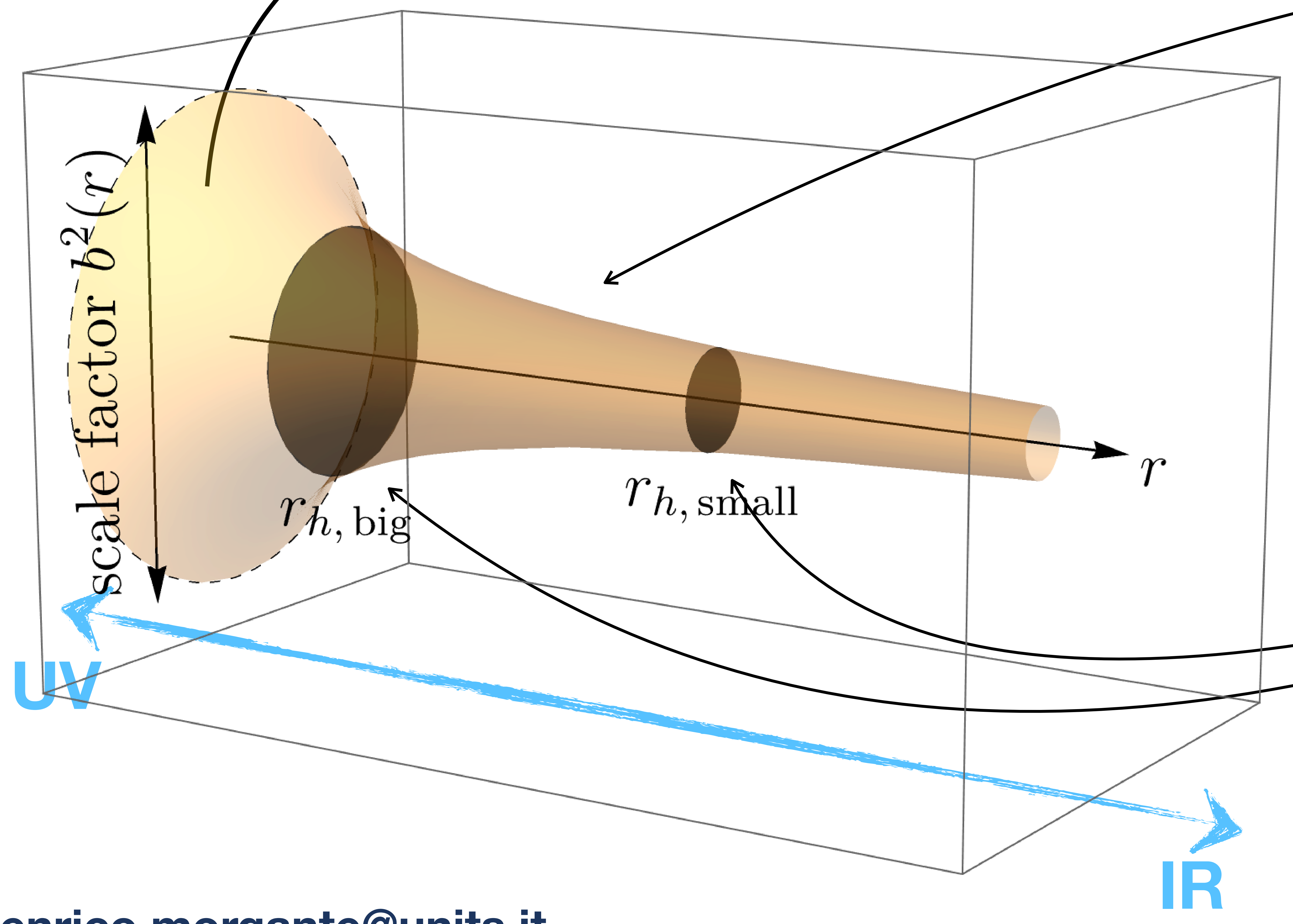
Solutions at $T \neq 0$

Enrico Morgante
Trieste U. & INFN

Solutions

asymptotically AdS

$$\begin{cases} b(r) \approx \frac{\ell}{r}\left[1 + \frac{4}{9}\frac{1}{\log r\Lambda} + \dots\right] \\ \lambda(r) \sim -\frac{1}{\log r\Lambda} \end{cases}$$



Thermal gas <—> confined

$$ds^2 = b^2(r)(dr^2 - dt^2 + d\vec{x}^2)$$

$$\Phi = \Phi_0(r)\,, \quad r \in (0,\infty)$$

Black Hole <—> deconfined

$$ds^2 = b^2(r)\left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2\right]$$

$f(r_h) = 0$ —> BH horizon

$$\Phi = \Phi(r)\,, \quad r \in (0, r_h)$$

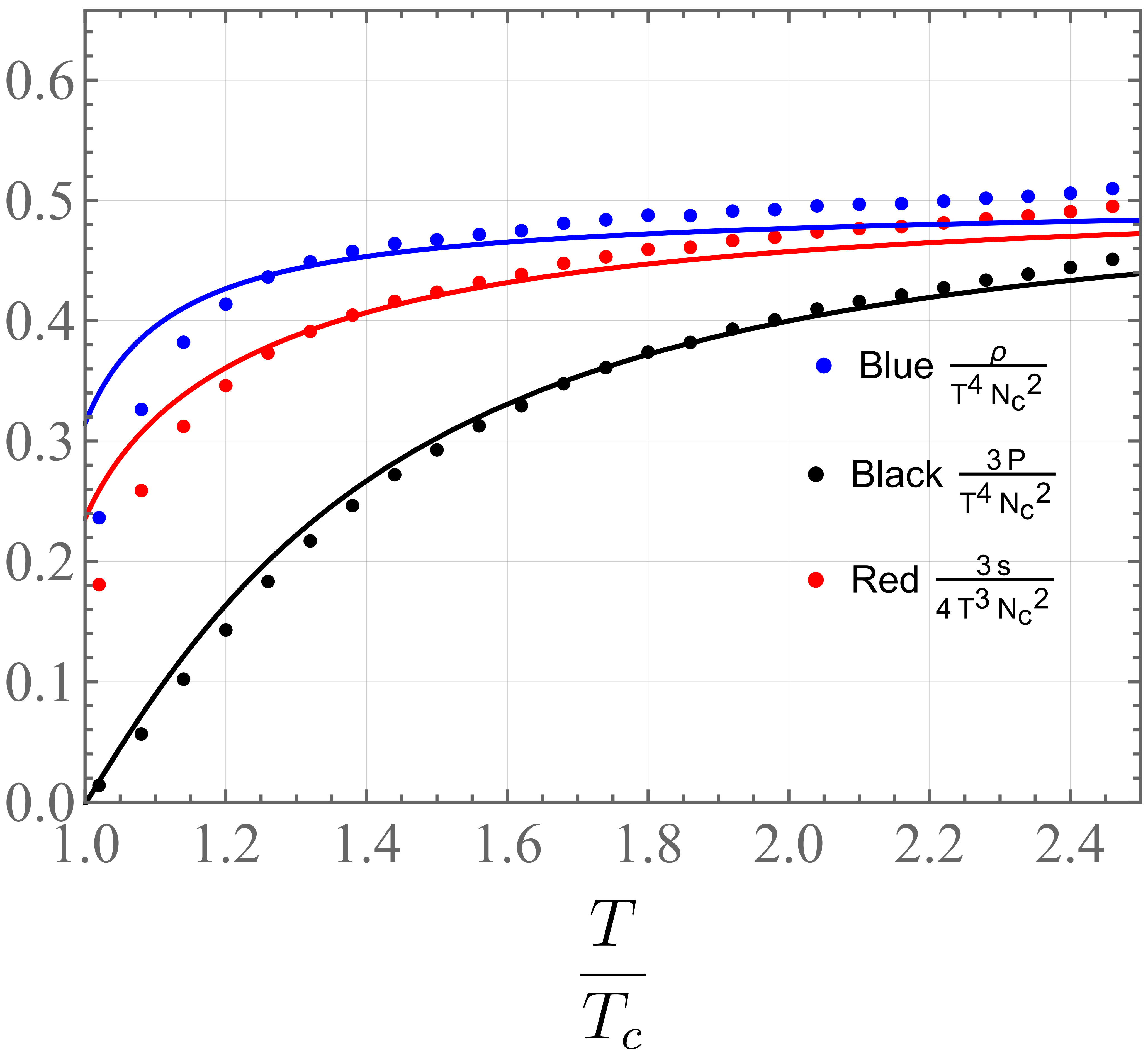
Enrico Morgante
Trieste U. & INFN

UNIVERSITÀ DEGLI STUDI DI TRIESTE

Thermodynamics

thermodynamics of 4D theory

geometry of 5D dual



Phase transition

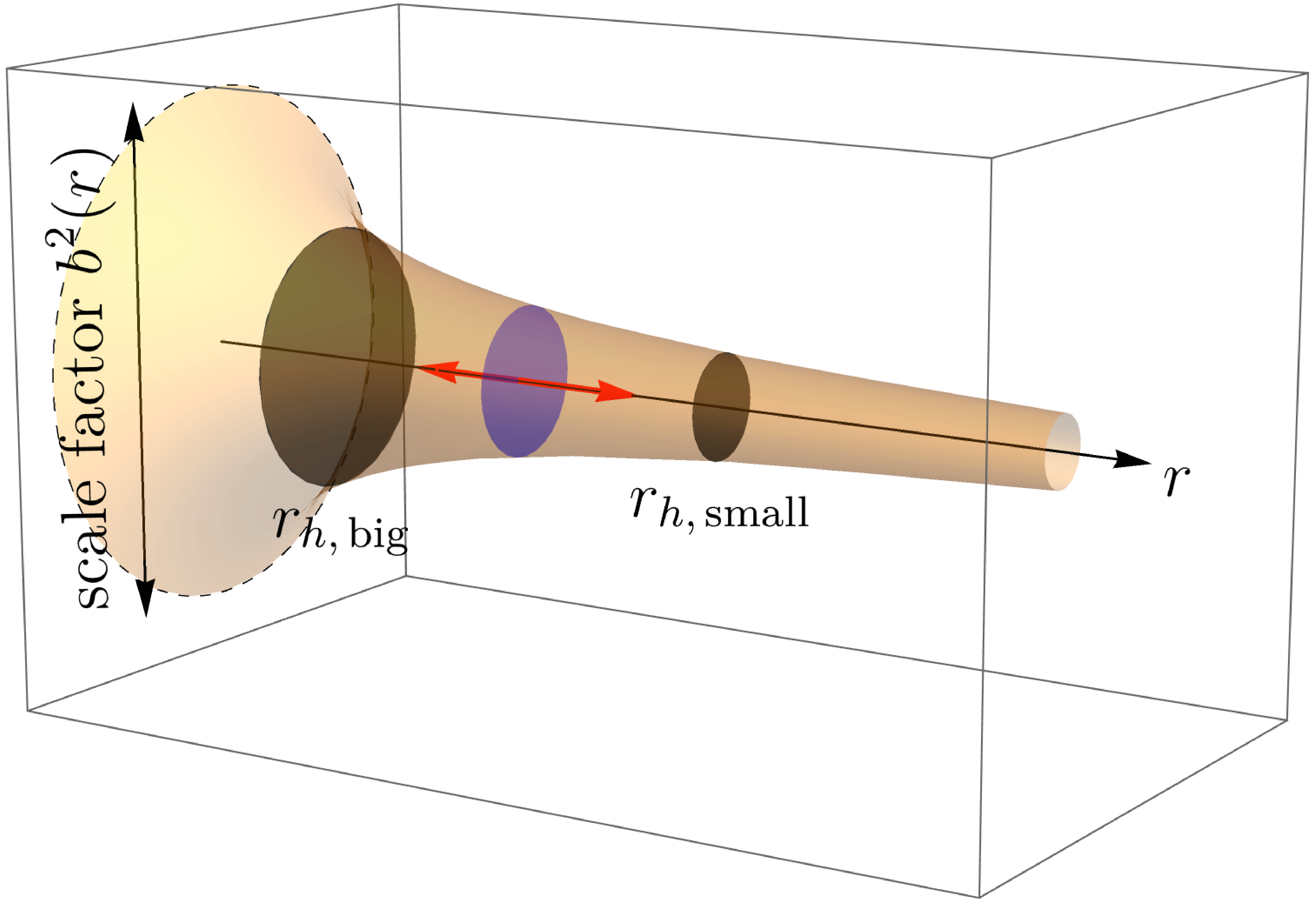
Enrico Morgante
Trieste U. & INFN

Confinement phase transition

- Hawking - Page phase transition [Hawking&Page'83]
- Small BH acts as an instanton

Enrico Morgante
Trieste U. & INFN

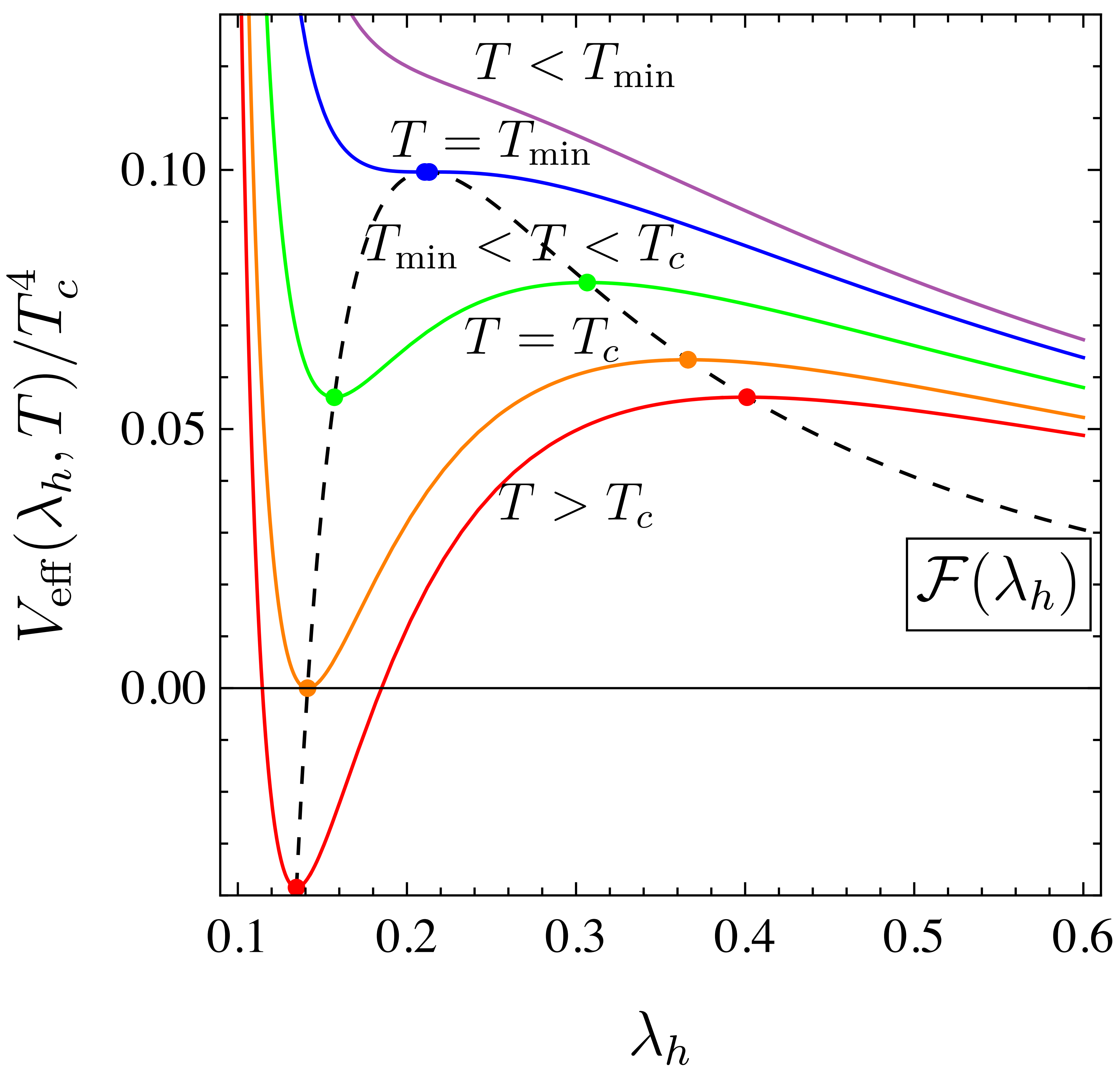
Effective action for tunnelling



- Interpolate between big and small BH
- Choose an order parameter (r_h or λ_h)
- Violate the condition $T_h = T$
  - ➡ BH not thermal eq.
  - ➡ Conical singularity
- Regularize the metric and compute the contribution to the action

Enrico Morgante
Trieste U. & INFN

Effective potential



$$V_{\rm eff} = \mathcal{F}(\lambda_h) - 4\pi M_p^3 N_c^2 b(\lambda_h)^3 \left(1 - \frac{T_h}{T}\right)$$

- Stationary points <—> regular solutions:
    - Big BH: min
    - Small BH: max
- $T > T_c$ —> Big BH are stable
- $T < T_{\rm min}$ —> No BH solution (no deconfined phase)

Enrico Morgante
Trieste U. & INFN

UNIVERSITÀ DEGLI STUDI DI TRIESTE

Bubble nucleation

- O(3) invariant bounce solutions

$$\mathcal{S}_{\rm eff} = \frac{4\pi}{T}\int d\rho\rho^2\left[c\frac{N_c^2}{16\pi^2}(\partial_r\lambda_h(r))^2 + V_{\rm eff}(\lambda_h(r))\right]$$

- Tunnelling rate:

$$\Gamma = T^4\left(\frac{\mathcal{S}_B}{2\pi}\right)^{3/2} e^{-\mathcal{S}_B}$$

- Nucleation: $\Gamma \approx H^4$
- Percolation: Universe ~ filled with confined phase bubbles

Enrico Morgante
Trieste U. & INFN

Phase transition parameters

- $T_n \approx T_p \approx 0.99 T_c$ —> fast PT, no supercooling
- Inverse PT duration (controls the source duration)

$$\frac{\beta}{H} = T \left(\frac{d\mathcal{S}_B}{dT} \right)_{T_p} \approx 10^5$$

- PT strength (energy released)

$$\alpha = \frac{4}{3} \frac{\Delta\theta}{w_+} = \frac{\Delta\rho - 3\Delta p}{w_+} \approx 0.34$$

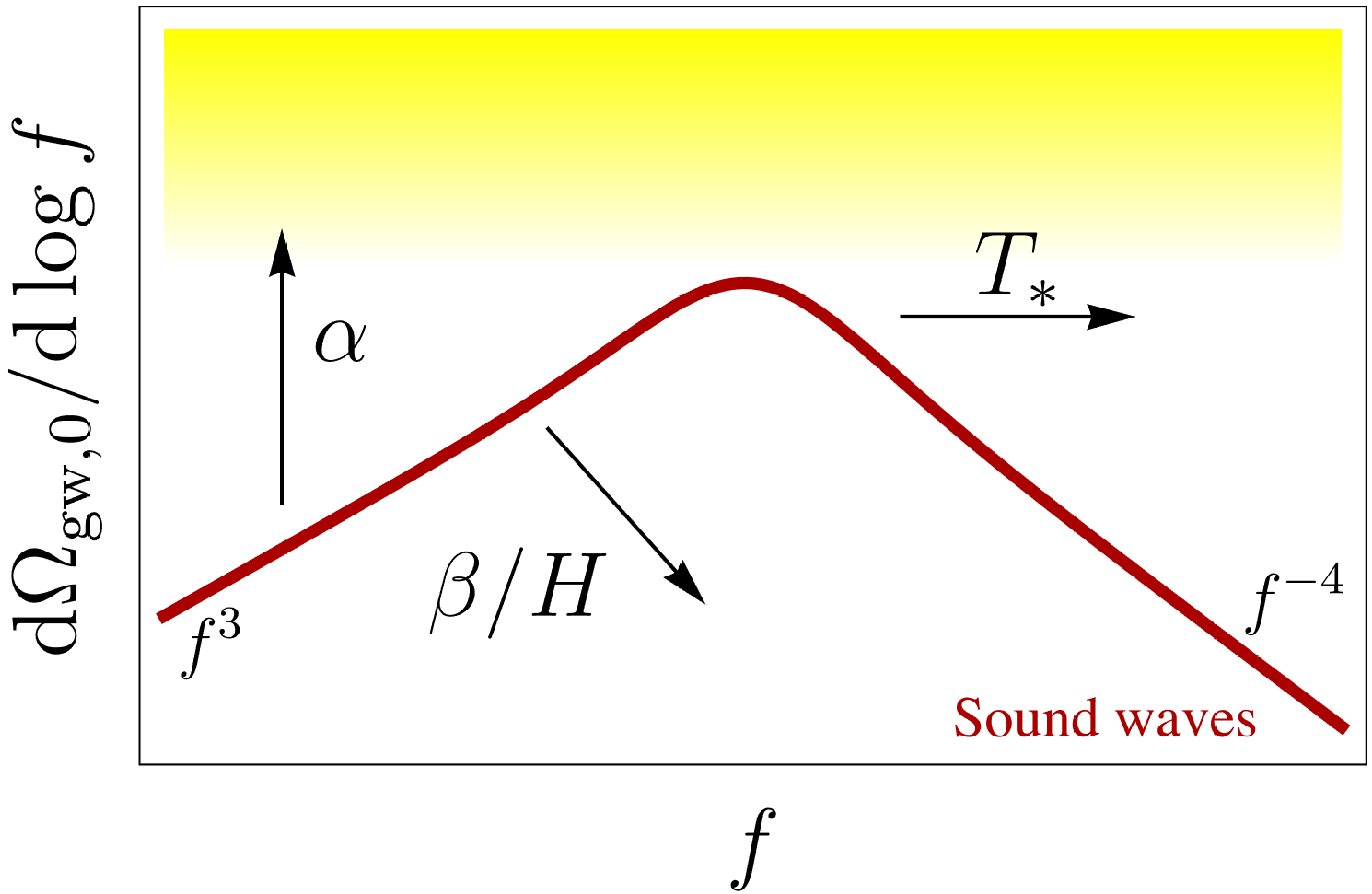
- Source duration (sound waves) —> signal suppression

$$\tau_{\rm sw} H \sim 10^{-4}$$

Enrico Morgante
Trieste U. & INFN

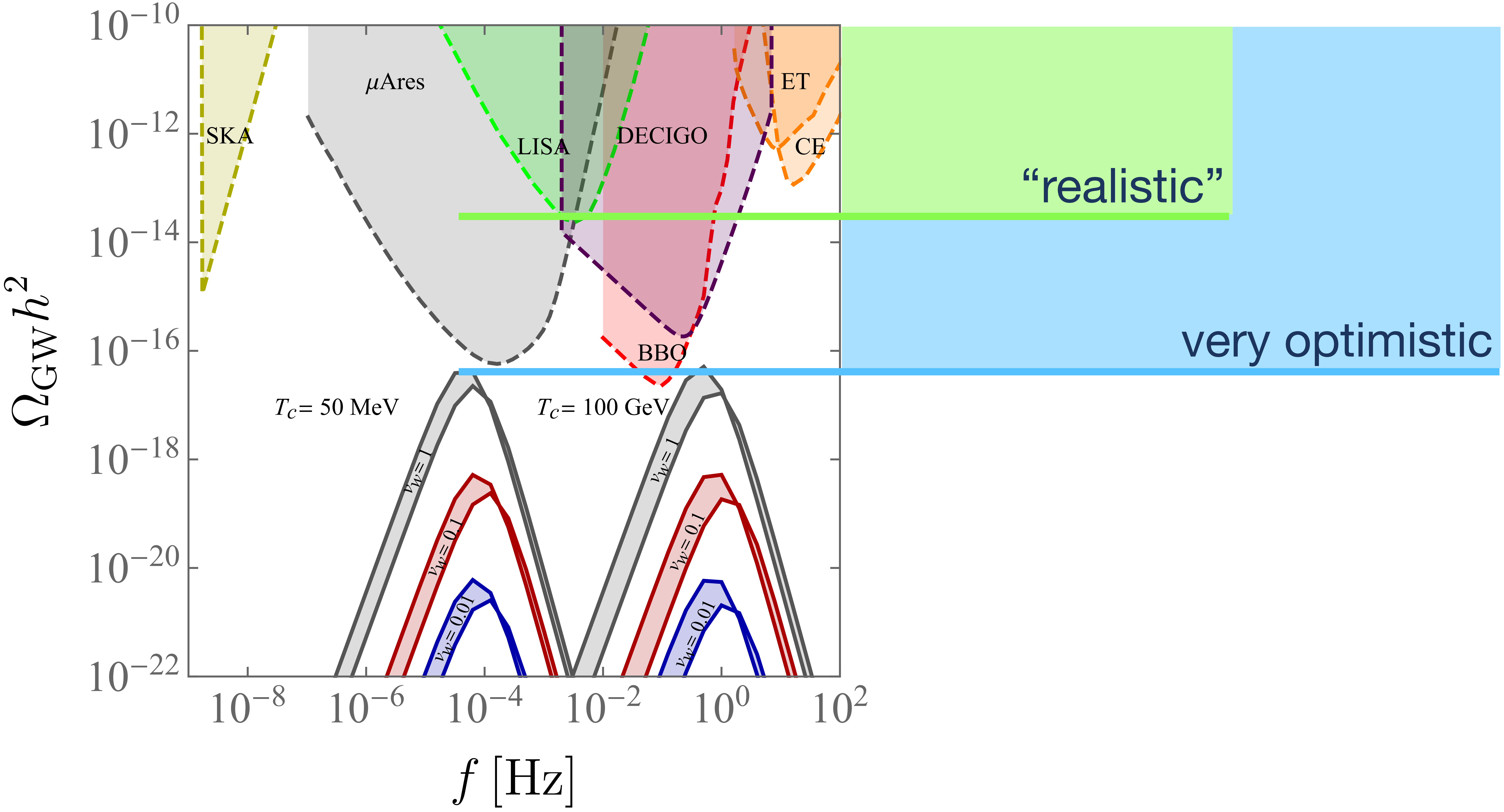
GW - sound waves

1910.13125 LISA Cosmology Working Group



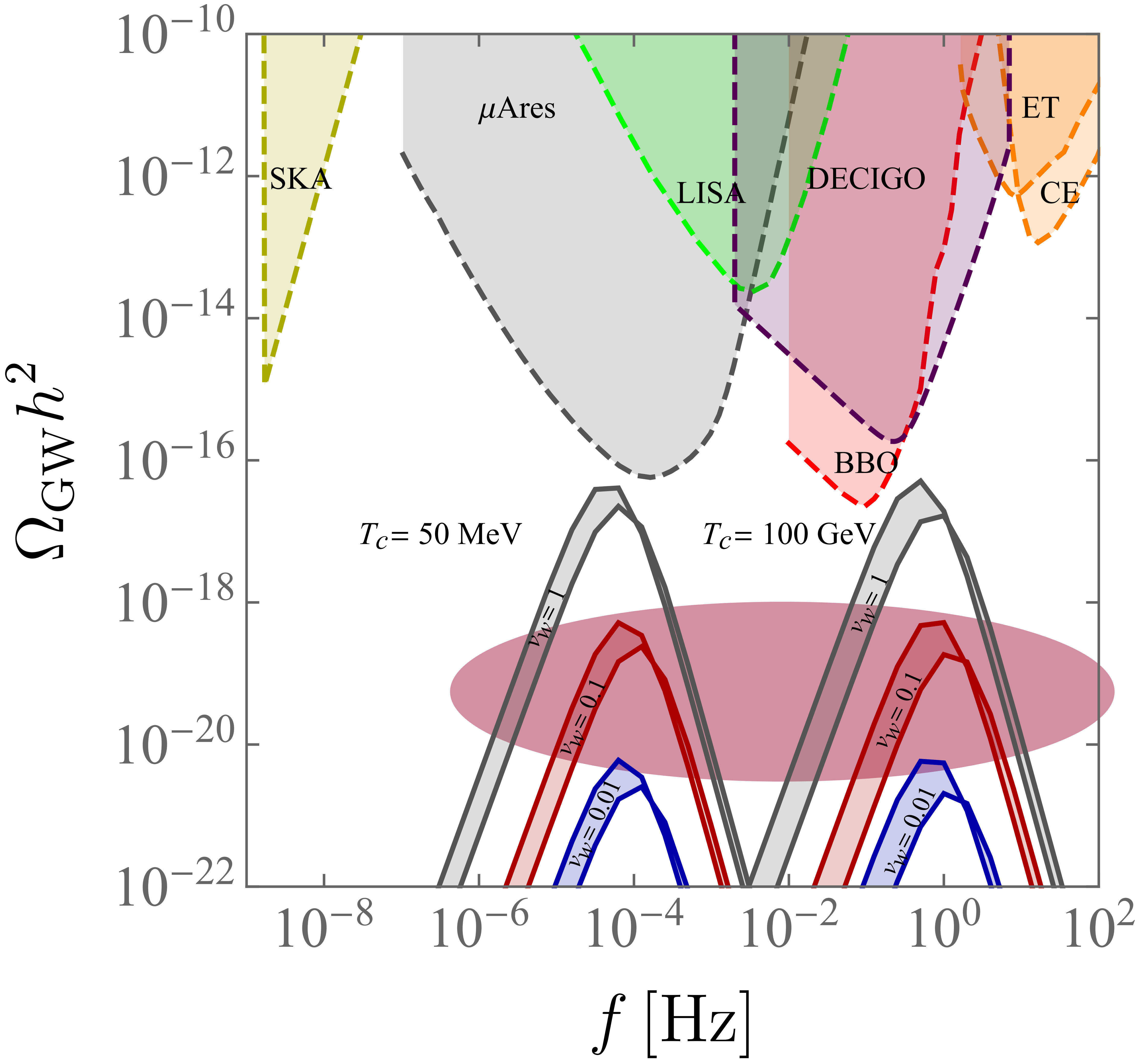
Enrico Morgante
Trieste U. & INFN

GW spectra



Enrico Morgante
Trieste U. & INFN

GW spectra



wall velocity

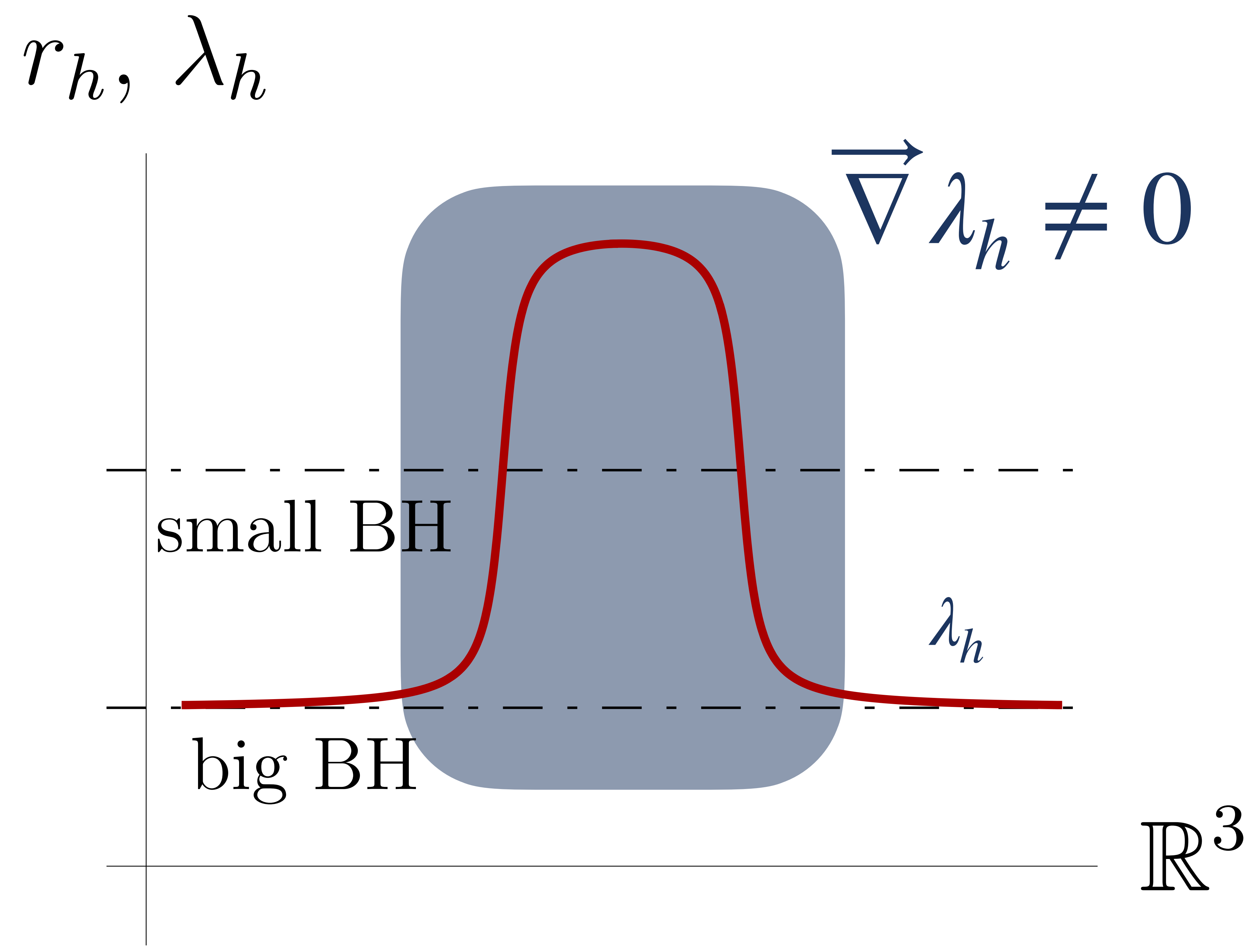
- often a free parameter
- can be estimated $v_w \ll 1$ (eg 2104.05708,2104.12817)
- for our model: 2312.09964 Sanchez-Garitaonandia+

$v_w \simeq 0.07$

Enrico Morgante
Trieste U. & INFN

Kinetic term

PRELIMINARY



$$\mathcal{S}_{\rm eff} = \frac{4\pi}{T}\int d\rho\rho^2\left[c\frac{N_c^2}{16\pi^2}(\partial_r\lambda_h(r))^2 + V_{\rm eff}(\lambda_h(r))\right]$$

Is $c \sim \mathcal{O}(1)$ a good approximation?

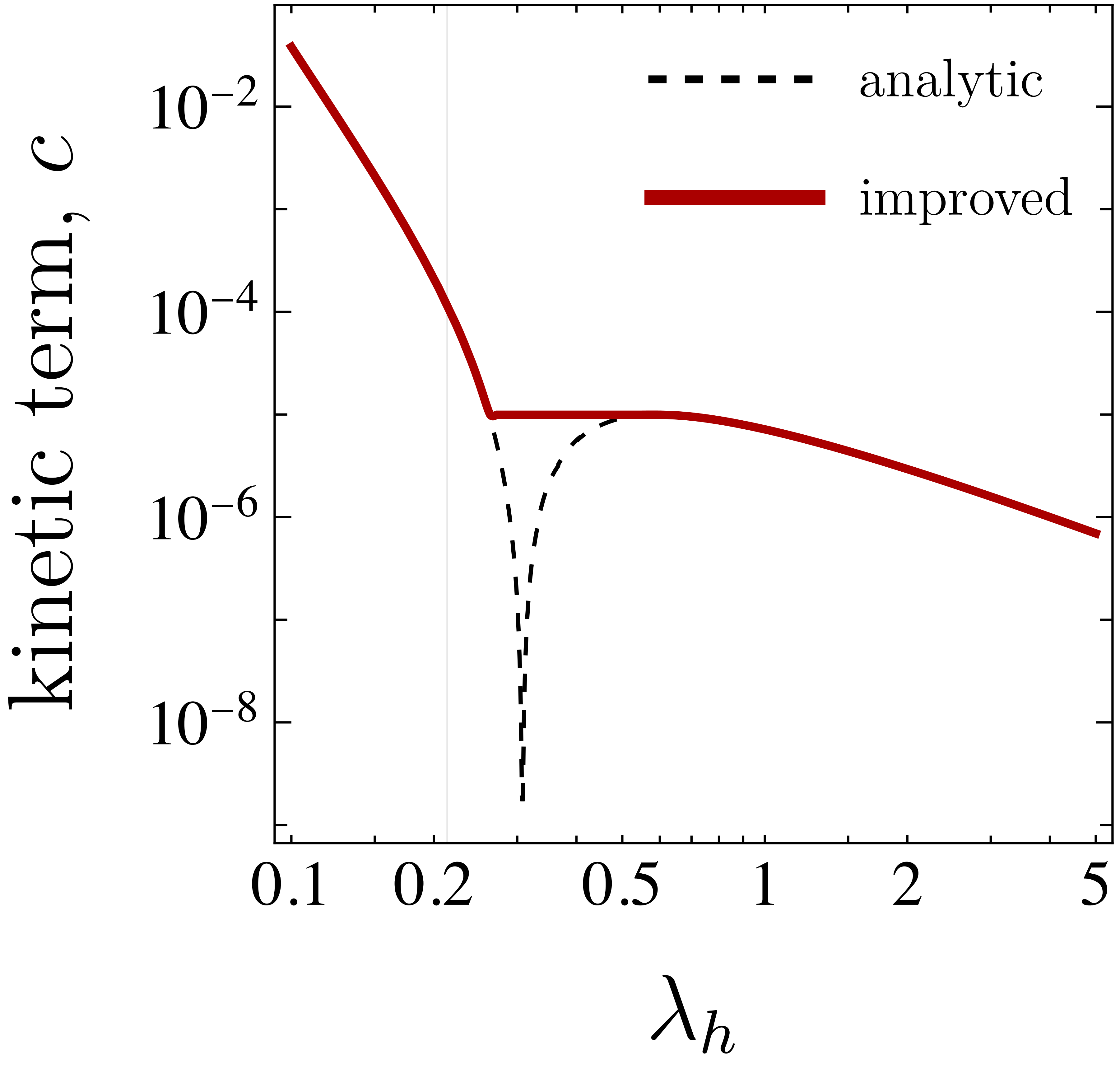
evaluate the action on a bubble configuration

$$\mathcal{S} \supset \int d^5x\ \ldots(\vec{\nabla}\lambda_h)^2 + \ldots(\vec{\nabla}\lambda_h)^4 + \ldots$$

Enrico Morgante
Trieste U. & INFN

Kinetic term

PRELIMINARY



$$\mathcal{S}_{\rm eff} = \frac{4\pi}{T}\int d\rho\rho^2\left[c\frac{N_c^2}{16\pi^2}(\partial_r\lambda_h(r))^2 + V_{\rm eff}(\lambda_h(r))\right]$$

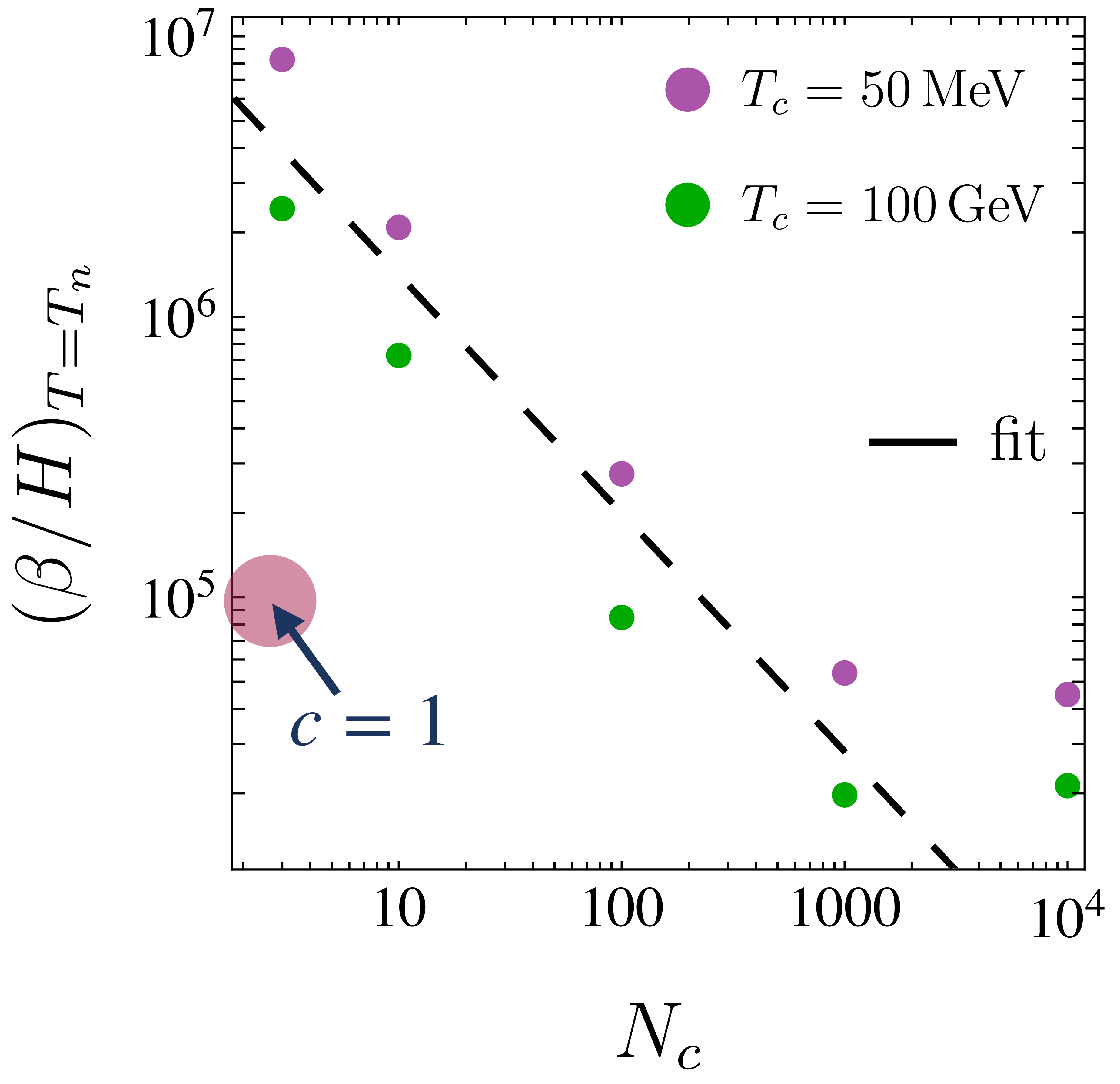
Small kinetic term

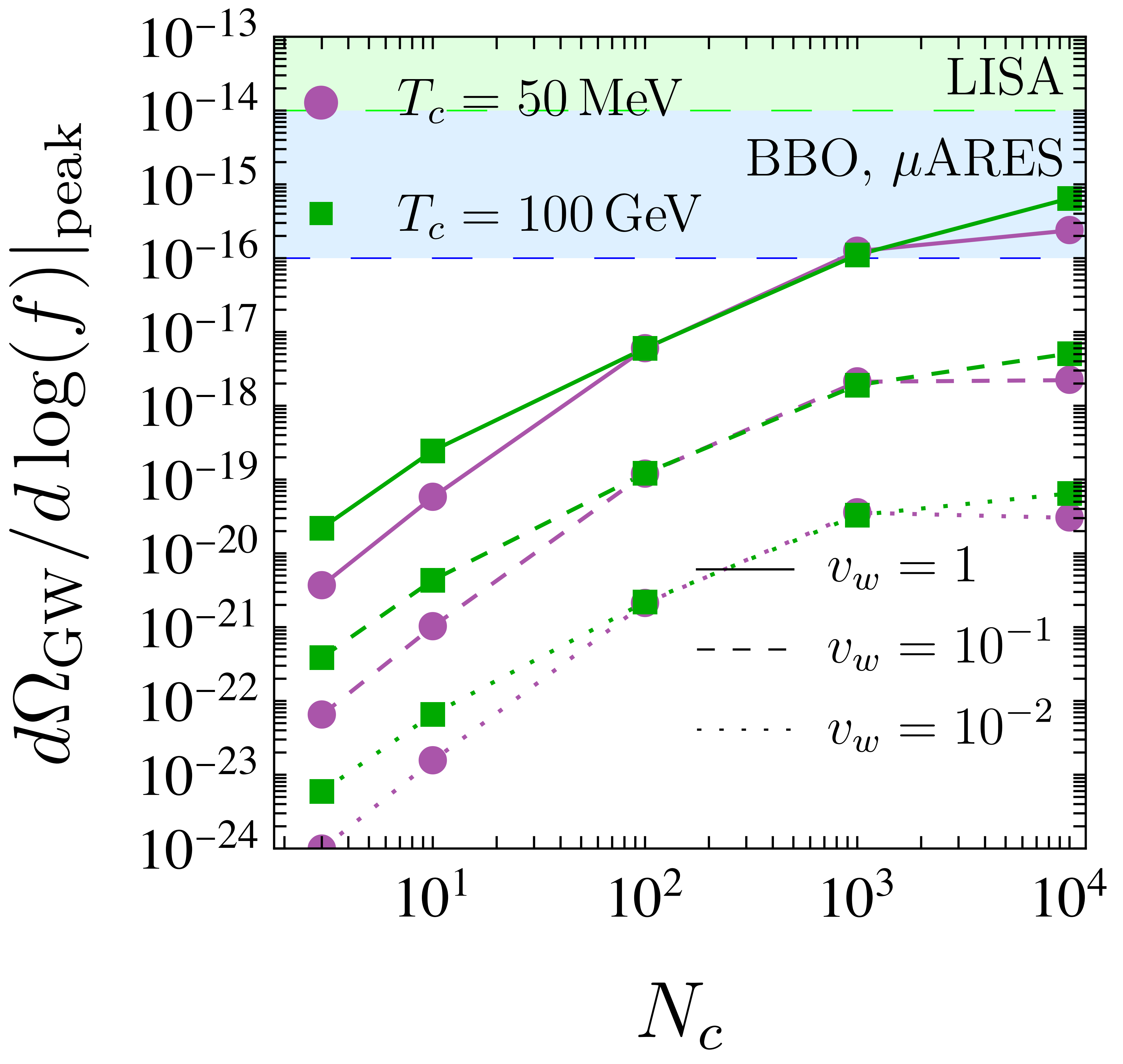
- expect fast transition

Enrico Morgante
Trieste U. & INFN

Effect of the kinetic term

PRELIMINARY





Enrico Morgante
Trieste U. & INFN

Summary

- GW signal from confinement in a dark $SU(3)$ gauge group
  - theoretically robust model based on holography
  - fitted to resemble real-world QCD
- Uncertainties
  - wall velocity **`but see 2312.09964`**
  - (kinetic term of the effective action) **`in progress`**

OUTLOOK

- wall velocity, fit to lattice for large N_c, (add flavour/axion)
- kinetic term $\leftrightarrow$ physical properties of the theory

Backup

Enrico Morgante
Trieste U. & INFN

UNIVERSITÀ DEGLI STUDI DI TRIESTE

UV: asymptotic freedom

β function of the 't Hooft coupling $\lambda_t = g^2 N_c$

$$\beta(\lambda_t) \equiv \frac{d\lambda_t}{d\log E} = -\beta_0 \lambda_t^2 - \beta_1 \lambda_t^3 + \cdots \to 0 \quad \text{in the UV}$$

$$\beta_0 = \frac{22}{3(4\pi)^2}$$

$$\beta_1 = \frac{51}{121}\beta_0^2$$

conformal sym broken by logarithmic running

Holographic equivalent: **geometry → AdS space (logarithmically)**

Enrico Morgante
Trieste U. & INFN

Geometry in the UV

The solution asymptotes to AdS for $r \to 0$

$$b(r) = \frac{\ell}{r}\left[1 + \frac{4}{9}\frac{1}{\log r\Lambda} - \frac{4}{9}\frac{b_1}{b_0^2}\frac{\log(-\log r\Lambda)}{(\log r\Lambda)^2} + \dots\right],$$

$$b_0\lambda(r) = -\frac{1}{\log r\Lambda} + \frac{b_1}{b_0^2}\frac{\log(-\log r\Lambda)}{(\log r\Lambda)^2} + \dots$$

$$V(\lambda) = \frac{12}{\ell^2}(1 + v_0\lambda + v_1\lambda^2 + \dots)$$

$$b_0 = \frac{9}{8}v_0 \qquad b_1 = \frac{9}{4}v_1 - \frac{207}{256}v_0^2$$

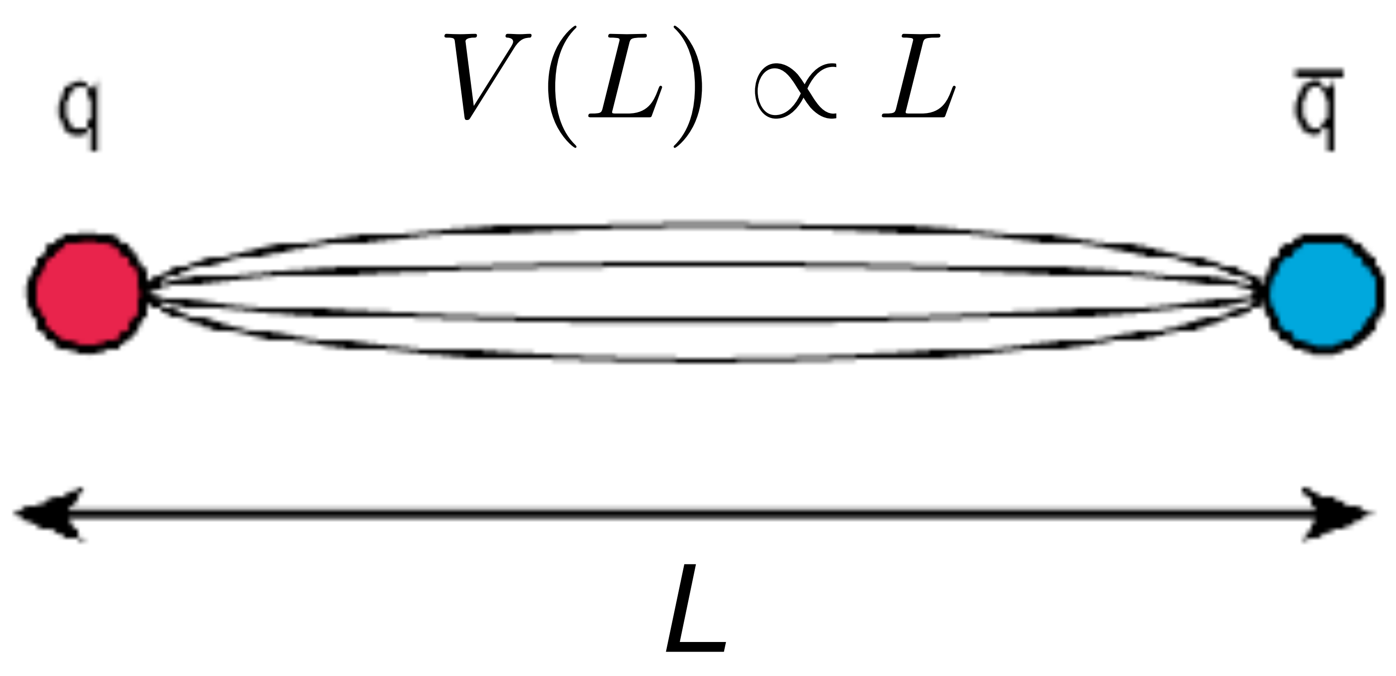
Identify b ↔ E, $\lambda \leftrightarrow \lambda_t$ —> β function:

$$\beta(\lambda) \equiv \frac{d\lambda}{d\log E} = -b_0\lambda^2 - b_1\lambda^3 + \dots = \beta(\lambda_t) \Rightarrow \begin{cases} b_0 = \beta_0 \\ b_1 = \beta_1 \end{cases}$$

Enrico Morgante
Trieste U. & INFN

IR: Confinement

1. The static potential of a probe $q\,\bar{q}$ pair is linear in the distance



(flux tube w/ constant energy/length)

2. Mass gap: no asymptotic massless state (no free gluon)

$$m^2_{\text{glue}} > 0$$

3. Linear glueball spectrum

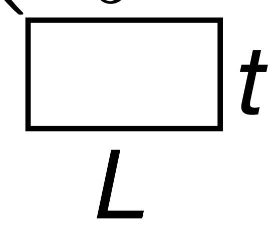
$$m_n^2 \sim n$$

Enrico Morgante
Trieste U. & INFN

Confinement
in holography

in QFT:
Wilson loop follows area law:

$q\,\bar{q}$ potential: $\longrightarrow W = \mathrm{tr}\left[\mathcal{P}\exp\left(i\oint A_\mu dx^\mu\right)\right] \propto L \times t \longrightarrow$



Holography:
Nambu-Goto string action
on a rectangle of sides $L \times t$

glueball masses: $\longrightarrow$ Spectrum of fluctuations $\Phi(r)$

equations of motion

$$b(r)\,,\Phi(r)\ \leftrightarrow\ V(\lambda)$$

$$V(\lambda) \sim \lambda^{4/3}(\log\lambda)^{1/2}\,, \qquad \lambda \to \infty$$

Enrico Morgante
Trieste U. & INFN

Potential

UV: $\lambda \to 0$

$$V(\lambda) = \frac{12}{\ell^2}(1 + v_0\lambda + v_1\lambda^2 + \dots)$$

IR: $\lambda \to \infty$

$$V(\lambda) \sim \lambda^{4/3}(\log\lambda)^{1/2}$$

Ansatz:

$$V(\lambda) = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}[\log(1 + V_2\lambda^{4/3} + V_3\lambda^2)]^{1/2}\right\}$$

$V_0\,,\;V_1 V_2^{1/2}$

fixed by UV

$V_1\,,\;V_3$

fit to lattice data
(thermodyn. or glueballs)

Enrico Morgante
Trieste U. & INFN

Confined / deconfined

Witten hep-th/9803131

Thermal gas <—> confined

$$ds^2 = b^2(r)(dr^2 - dt^2 + d\vec{x}^2)$$

$$\Phi = \Phi_0(r)\,, \quad r \in (0, \infty)$$

Black Hole <—> deconfined

$$ds^2 = b^2(r)\left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2\right]$$

$$\Phi = \Phi(r)\,, \quad r \in (0, r_h)$$

Polyakov loop

(finite T version of Wilson loop)

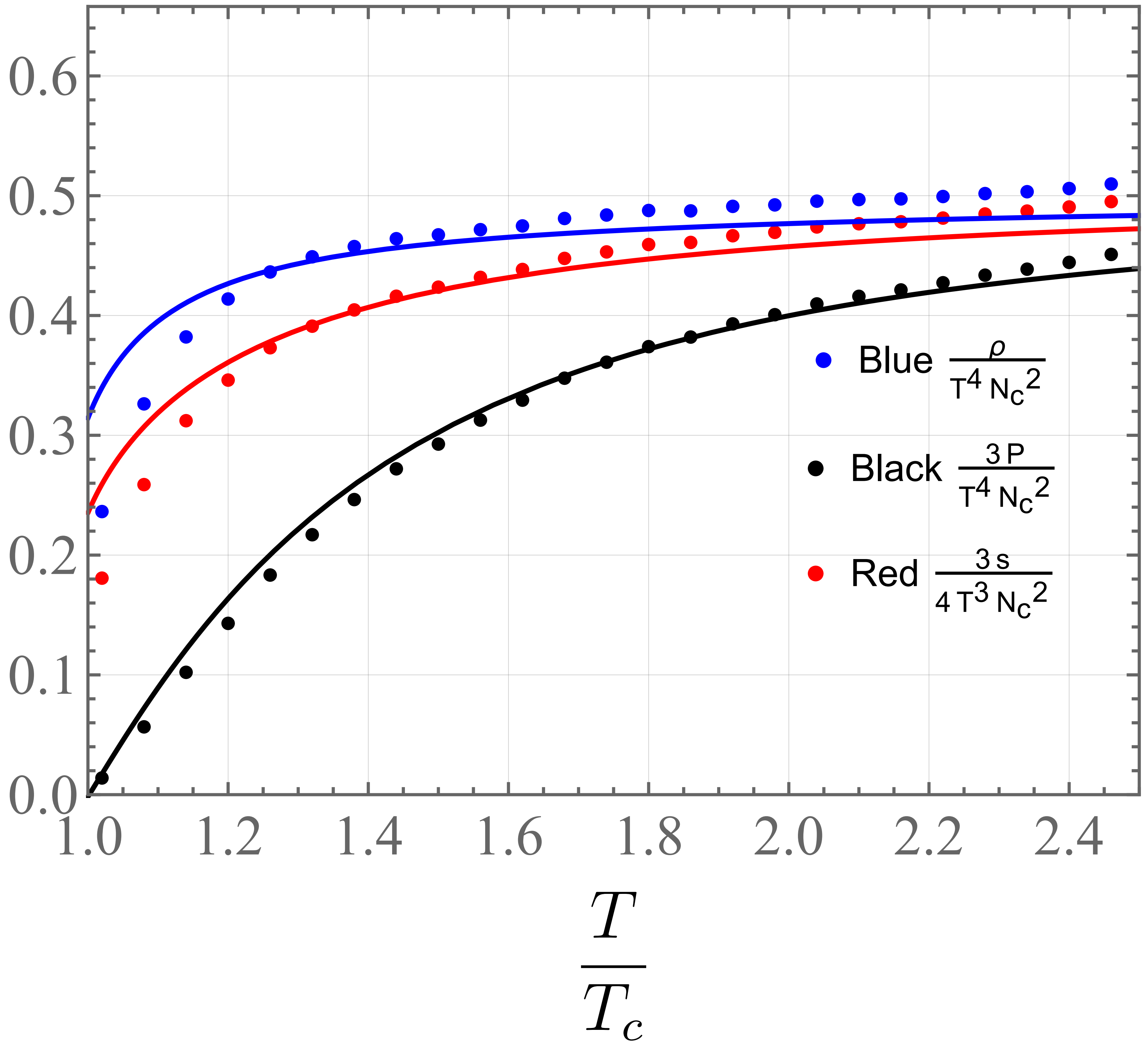
$$\mathbf{PL} = \frac{1}{N_c}\mathrm{tr}\left[\mathcal{T}\exp\left(i\int_0^\beta A_0 d\tau\right)\right]$$

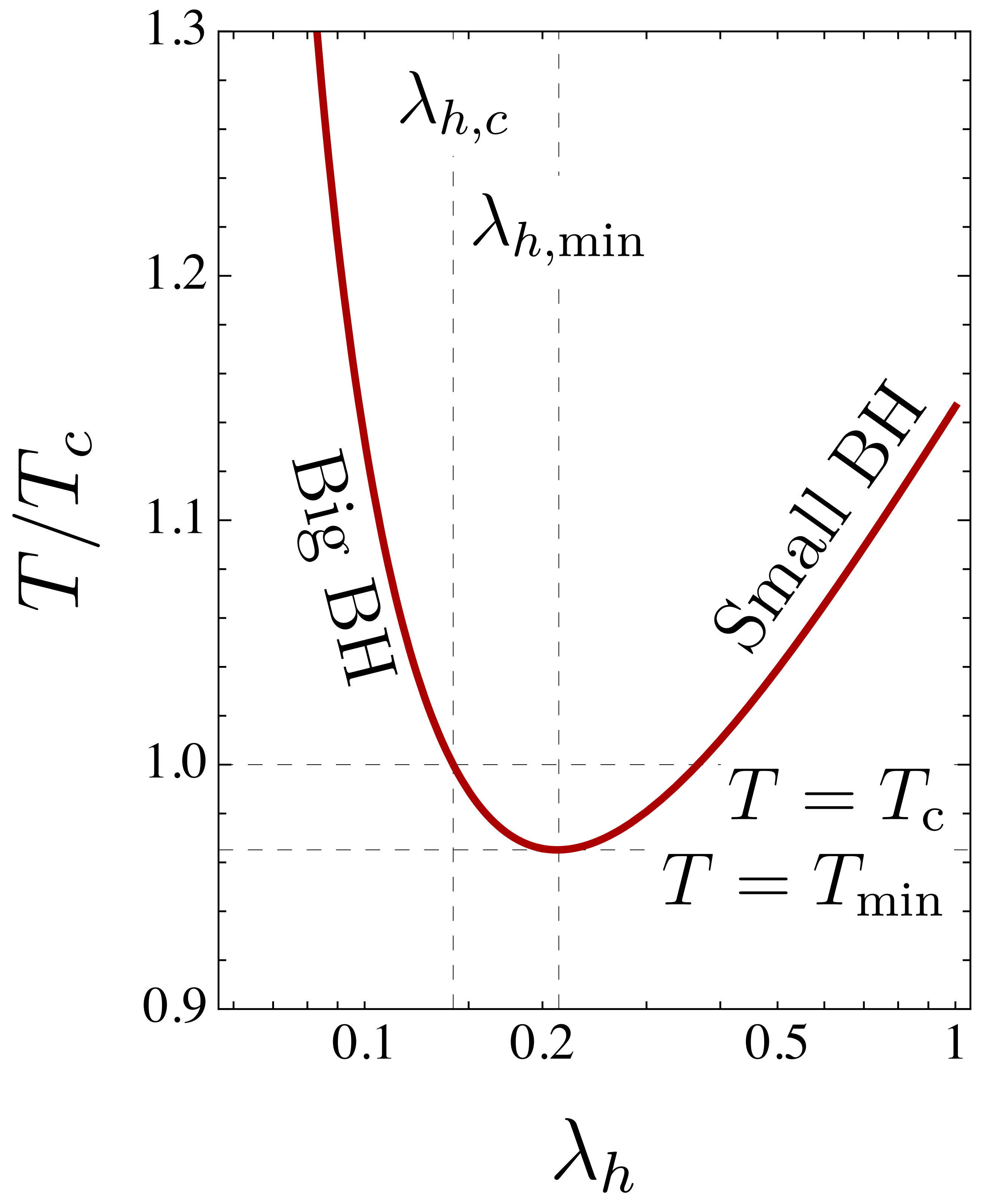
$$\begin{cases} = 0 & \text{confined} \\ \neq 0 & \text{deconfined} \end{cases}$$

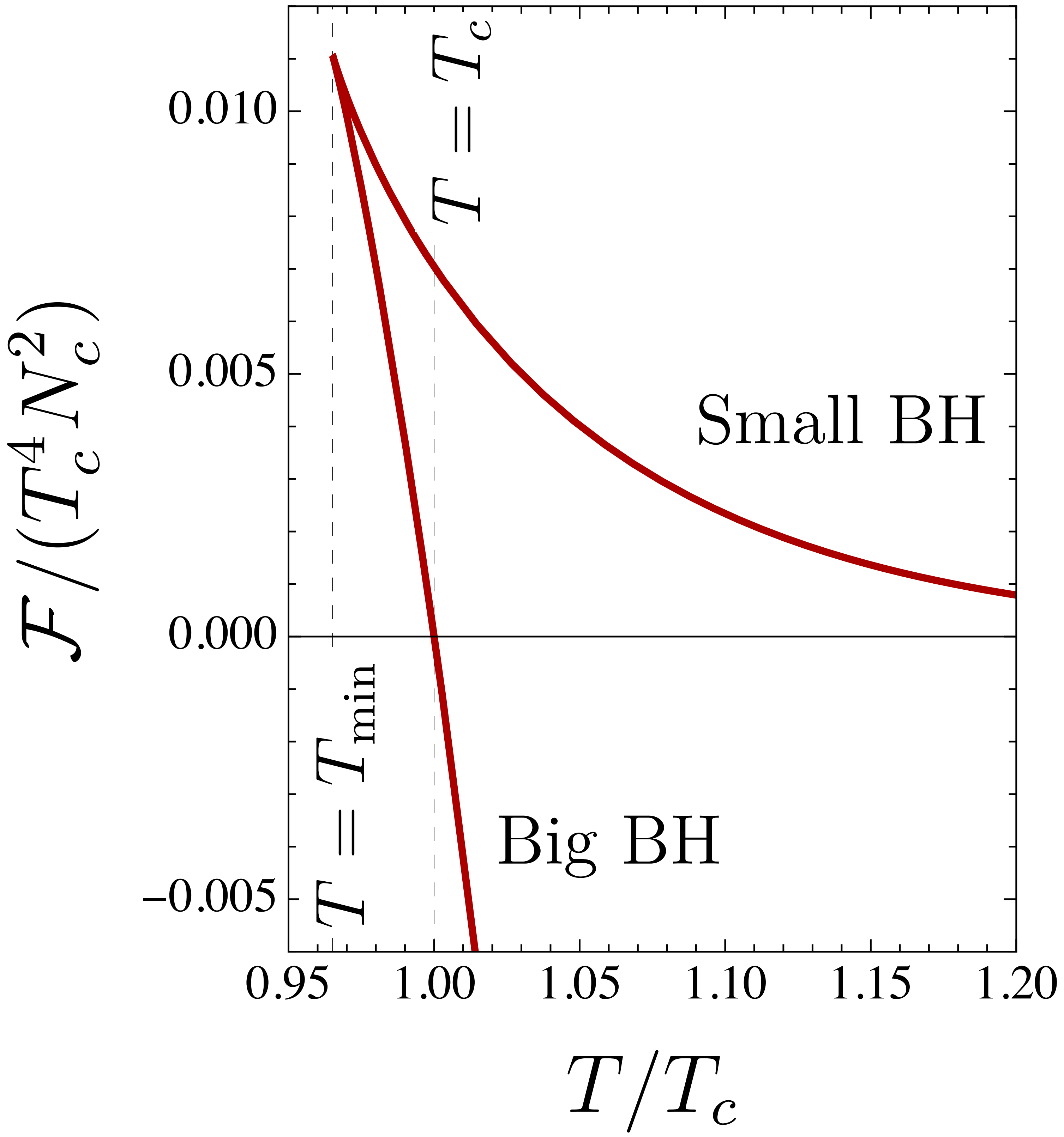
$|\mathbf{PL}| = \exp(-\beta\, F)$ $\longleftrightarrow$ energy cost of removing one colour charge

Enrico Morgante
Trieste U. & INFN

Thermodynamics







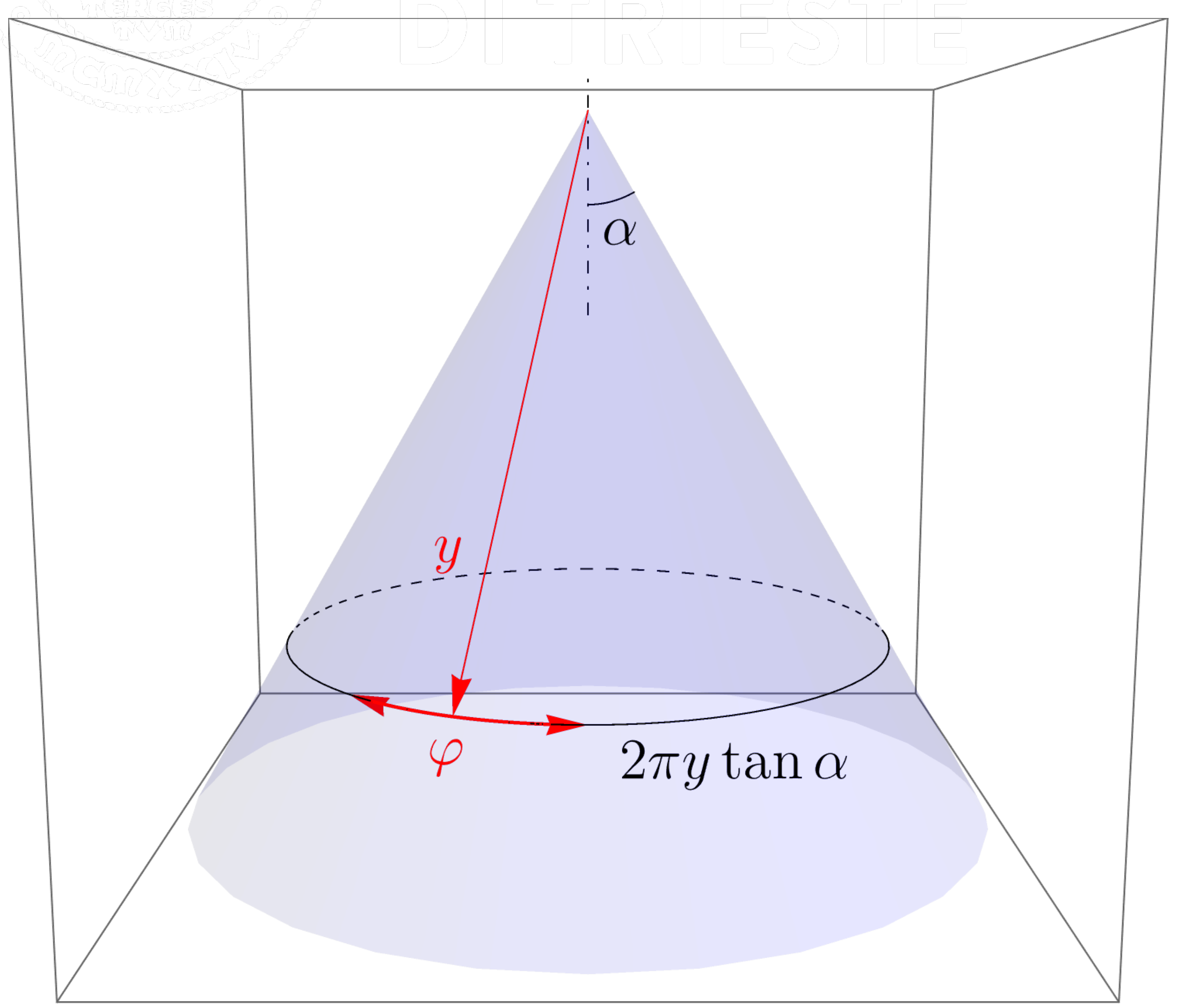
Enrico Morgante
Trieste U. & INFN

Conical singularity

Expand BH metric at the horizon

$$ds^2 = b(r)^2\left(\frac{dr^2}{f(r)} + f(r)dt^2 + d\vec{x}^2\right) \qquad f(r) \approx \dot{f}_h(r - r_h)\,, \qquad b(r) \approx b_h$$

define
$$\begin{cases} y = \dfrac{2b_h}{\dot{f}_h^{1/2}}\sqrt{r - r_h}\,, & y > 0 \\ \varphi = 2\pi T t\,, & 0 < \varphi < 2\pi \end{cases}$$



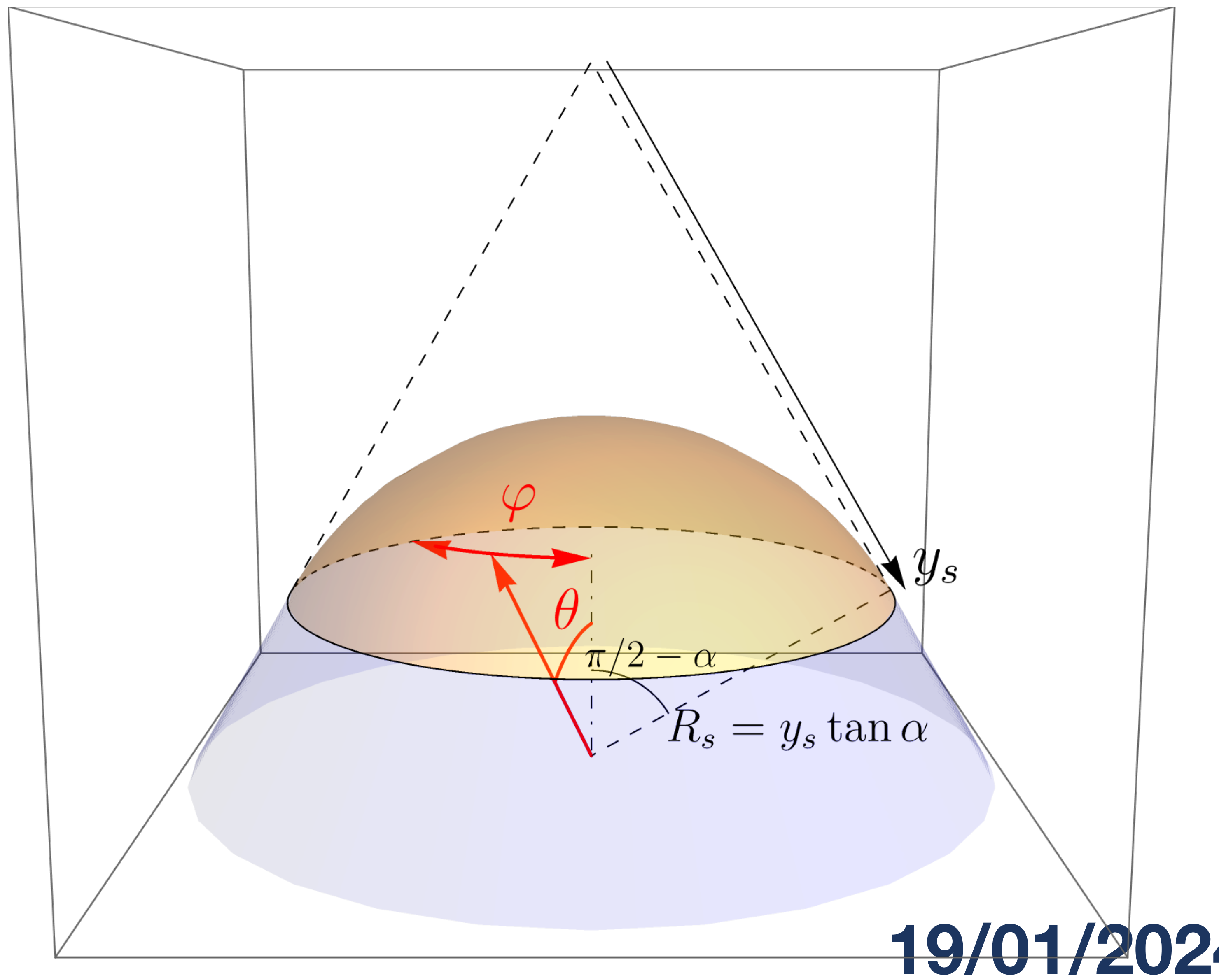
The metric describes a cone —> regular for $\sin\alpha = 1$

$$ds^2 = dy^2 + y^2\left(\frac{\dot{f}_h}{4\pi T}\right)^2 d\varphi^2 \qquad \sin\alpha = \frac{\dot{f}_h}{4\pi T} \equiv \frac{T_h}{T}$$

Regularize with a small spherical cap

$$\mathcal{S}^{\rm BH}_{\rm cap} = -M_p^3 N_c^2 \int d^5x\sqrt{g}[\mathcal{R} - \frac{4}{3}(\partial\phi)^2 + V(\phi)]$$

finite
vanish as $R_s \to 0$



Enrico Morgante
Trieste U. & INFN

GW - sound waves

1910.13125 LISA Cosmology Working Group

Normalized to photon density

$$F_{\mathrm{gw},0} = \Omega_{\gamma,0} \left(\frac{g_{s0}}{g_{s*}}\right)^{\frac{4}{3}} \frac{g_*}{g_0} \sim 10^{-5} \left(\frac{100}{g_*}\right)^{\frac{1}{3}}$$

Duration of linear fluid evolution (source duration)

$$\tau_{\mathrm{sh}} \sim R_* / \sqrt{\langle U_f^2 \rangle} \quad \propto (\beta/H)^{-1}$$

typical fluid length / av. plasma velocity

$$\frac{d\Omega_{\mathrm{gw},0}}{d\log f} = 0.687 F_{\mathrm{gw},0} K^2 \left(\frac{H_* R_*}{c_s}\right) \left(\frac{H_* R_*}{K^{1/2}}\right) \tilde{\Omega}_{\mathrm{gw}}\, C\left(\frac{f}{f_{p,0}}\right)$$

Kinetic energy fraction

$$K = \frac{\alpha\kappa}{1+\alpha} \qquad \kappa = \frac{3}{\epsilon v_w^3} \int w(\xi) v^2 \gamma^2 \xi^2 d\xi$$

$$\xi = r/t$$

$$\kappa^2 \sim 10^{-5} - 10^{-1}$$

Long-lasting source:
correlation time of
acoustic production

$$\propto (\beta/H)^{-1}$$

$$\tilde{\Omega}_{\mathrm{gw}} \sim 10^{-2}$$

$$C \sim \begin{cases} f^3 & \text{for } f < f_p \\ f^{-4} & \text{for } f > f_p \end{cases}$$

$$f_{p,0} \simeq 26 \left(\frac{1}{H_* R_*}\right) \left(\frac{z_p}{10}\right) \left(\frac{T_*}{100\,\mathrm{GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6} \mu\mathrm{Hz}$$