Recent Advances in pre Big Bang String Cosmology: Bouncing solutions in non-perturbative String Theory

Speaker: **Eliseo Pavone**

Co-authors: **P. Conzinu, G. Fanizza, M. Gasperini, L. Tedesco, G. Veneziano**

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Why String Theory in Cosmology?

- Consistent theory of Quantum Gravity
- Avoid the Classical Big Bang Singularity
- Go beyond EFT approach to Inflation (Transplanckian problem)
- No need for ad-hoc fields to drive Inflation
- Possible observational signature of String Theory

What does String Theory suggest?

- The existence of a low energy massless multiplet $\{\phi, g_{\mu\nu}, B_{\mu\nu}\}$
- Pre Big-Bang scenario instead of Slow-Roll (Bouncing Cosmology)
- A systematic way to implement higher order curvature corrections (α' expansion)
- A systematic way to implement higher order string coupling expansion in a perturbative way $g_s^2 = exp(\phi)$ (Genus Expansion)
- Additional simmetry in space-time with d abelian isometries, O(d,d) invariance in the field space (Continuous generalisation of T-duality)

Main Topics

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- Dilaton stabilisation, FLRW and de-Sitter attractor from non-perturbative dilaton potential

- Isotropisation mechanism via α' corrections and Dilaton potential

All order α' O(d,d) invariant HZ String Action in a anisotropic cosmology (Bianchi-I)

$$S = \int L dt = -\frac{1}{2}\int dt N e^{-\bar{\phi}} \left(\boxed{N^{-2}\dot{\bar{\phi}}^{2} + F(N^{-1}\dot{\beta}_i)} + \boxed{2(\alpha')^{(d-1)/2} V(\phi)} \right)$$

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$$ds^2 = N^2(t)dt^2 - \sum_i \delta_{ij} e^{2\beta_i} dx^i dx^j$$

$F = -N^{-2}\sum_i \dot{\beta}_i^2 + \mathcal{O}(\alpha') = -\sum_i H_i^2 + \mathcal{O}(\alpha')$ Even function of the Hubble functions

$\bar{\phi} = \phi - \sum_i \beta_i$ Shifted Dilaton

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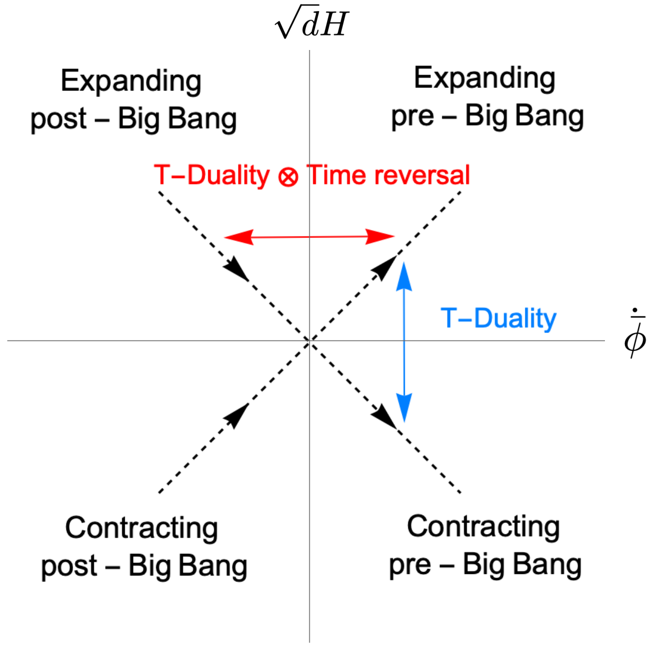

The Routhian formalism and the all order α' Hamiltonian.

Legendre transform only a subset of the variables of the Lagrangian β_i

$$\mathcal{R}(N, \bar{\phi}, \pi_i) = \sum_i \pi_i \dot{\beta}_i - L = N e^{\bar{\phi}} \left[\frac{1}{2} N^{-2} \dot{\bar{\phi}}^2 + h(z_i) + V(\bar{\phi} + \sum_i \beta_i) \right]$$

$$h(z_i) \equiv \frac{1}{2} \left(F - \sum_i \dot{\beta}_i \frac{\partial F}{\partial H_i} \right) = \frac{1}{2} \sum z_i^2 + \mathcal{O}(\alpha') + \dots \qquad \frac{\partial h}{\partial z_i} = H_i$$

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EoM

$$\boxed{\dot{\bar{\phi}}^2 = 2h(z_i) + 2V, \qquad \dot{z}_i = z_i \dot{\bar{\phi}} - \frac{\partial V}{\partial \phi}, \qquad \ddot{\bar{\phi}} = \sum_i z_i \frac{\partial h}{\partial z_i} + \frac{\partial V}{\partial \phi}}$$

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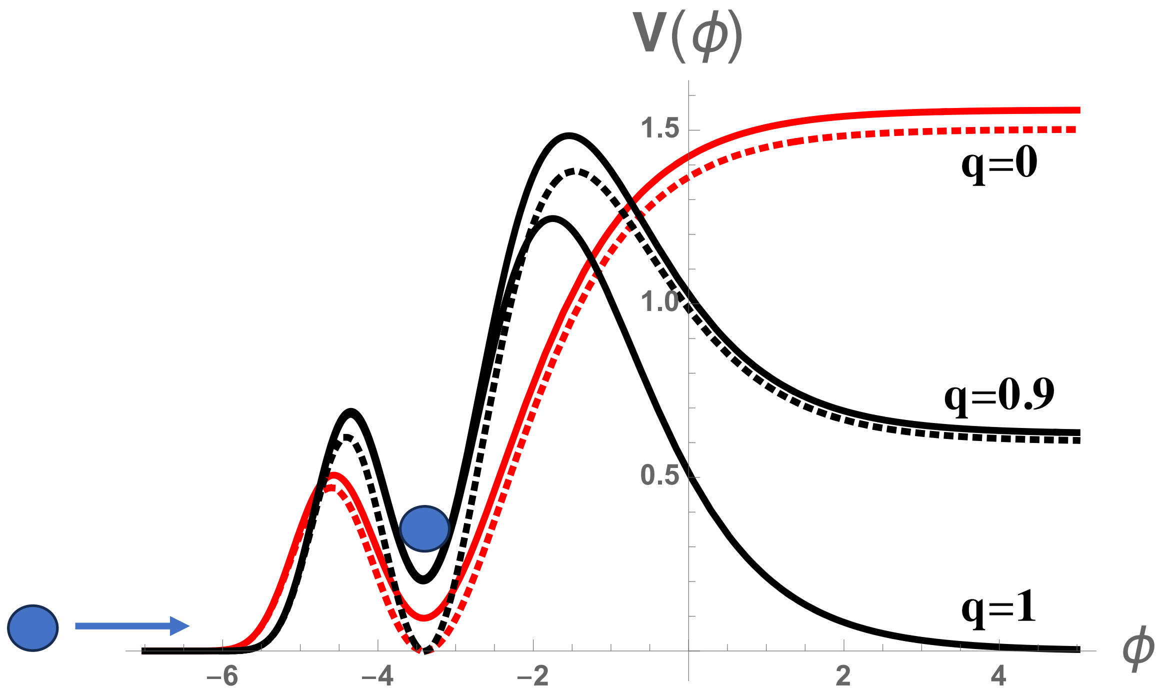
Advantages:

- Specify only two functions to have a non-perturbative description of the system.
- Simple 'hamiltonians' capture all α' corrections and generate bouncing solutions they have another zero except the trivial one. In general they come from non-holomorphic F. In the isotropic case we used:

$$h(z) = \frac{d}{2}(z^2 - \alpha'\frac{z^4}{2})$$

The phenomenological non-perturbative dilaton potential

Dilaton Potential with Istantonic behaviour not captured by string coupling expansion (Non-perturbative Potential)



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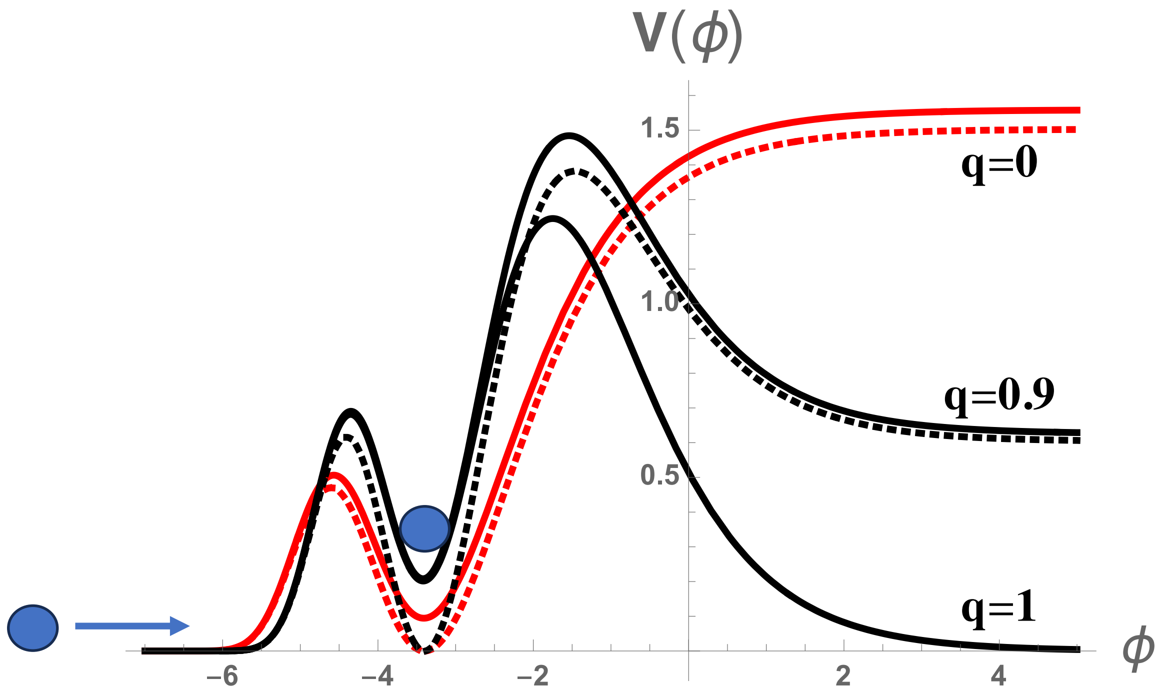

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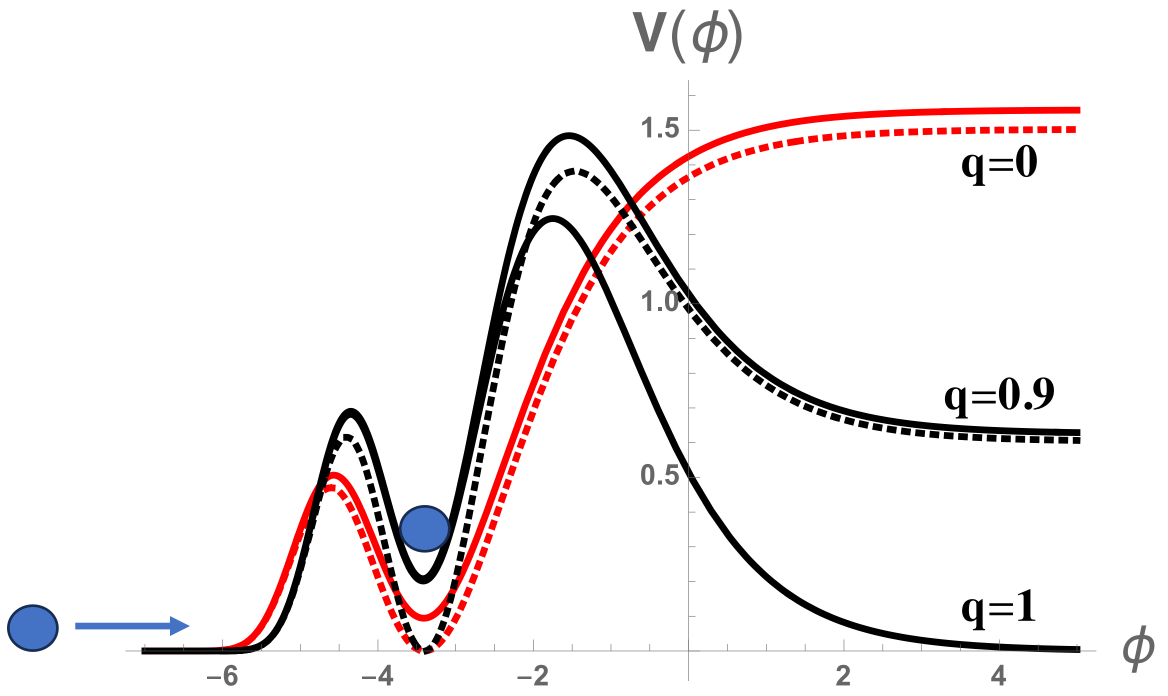
$$V(\phi) = A\, e^{-B(\phi)/\beta} \left[\left(c^2 - B(\phi) \right)^2 + \delta B(\phi) \right] \left[1 - q\, B^{-1}(\phi) \right]$$

$$B(\phi) = \frac{1 + \alpha\, g_s^2}{\alpha\, g_s^2} = \frac{1 + \alpha\, e^{\phi}}{\alpha\, e^{\phi}}$$



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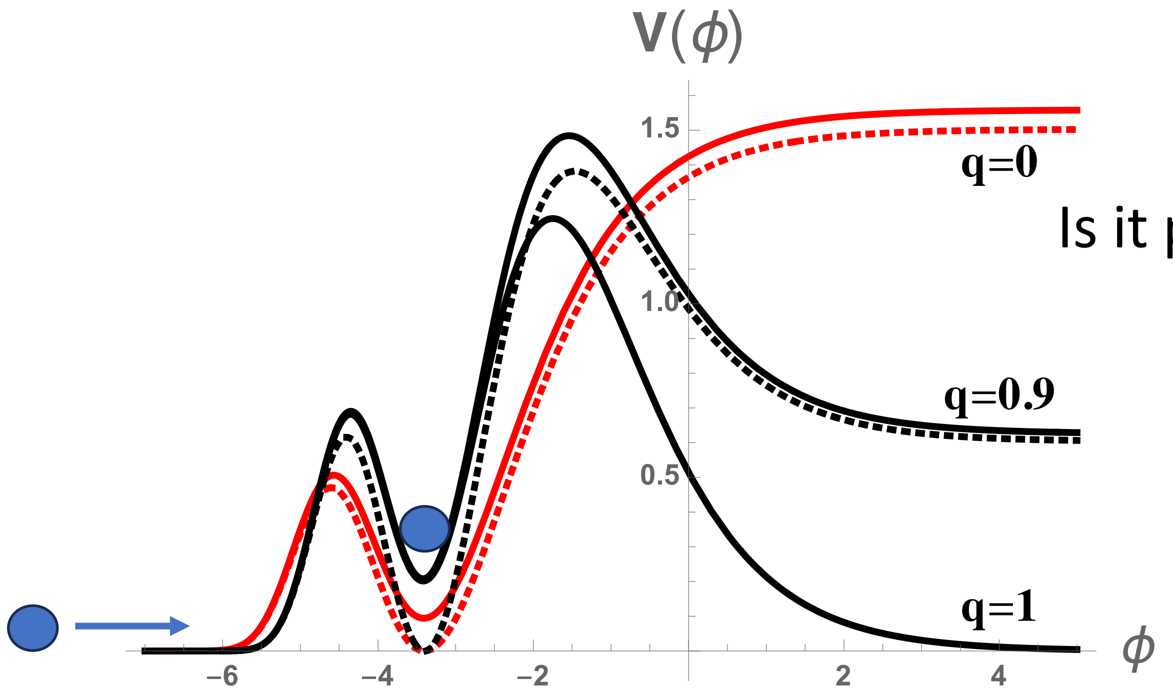

Is it possible to reach a final state with a frozen dilaton?

- Stabilisation of the dilaton implies stabilisation of the Newton's constant
- What is the final geometry?
- Is the final configuration an attractor?

Dilaton Stabilisation and FLRW attractor

Asymptotic Matter FLRW geometry δ=0

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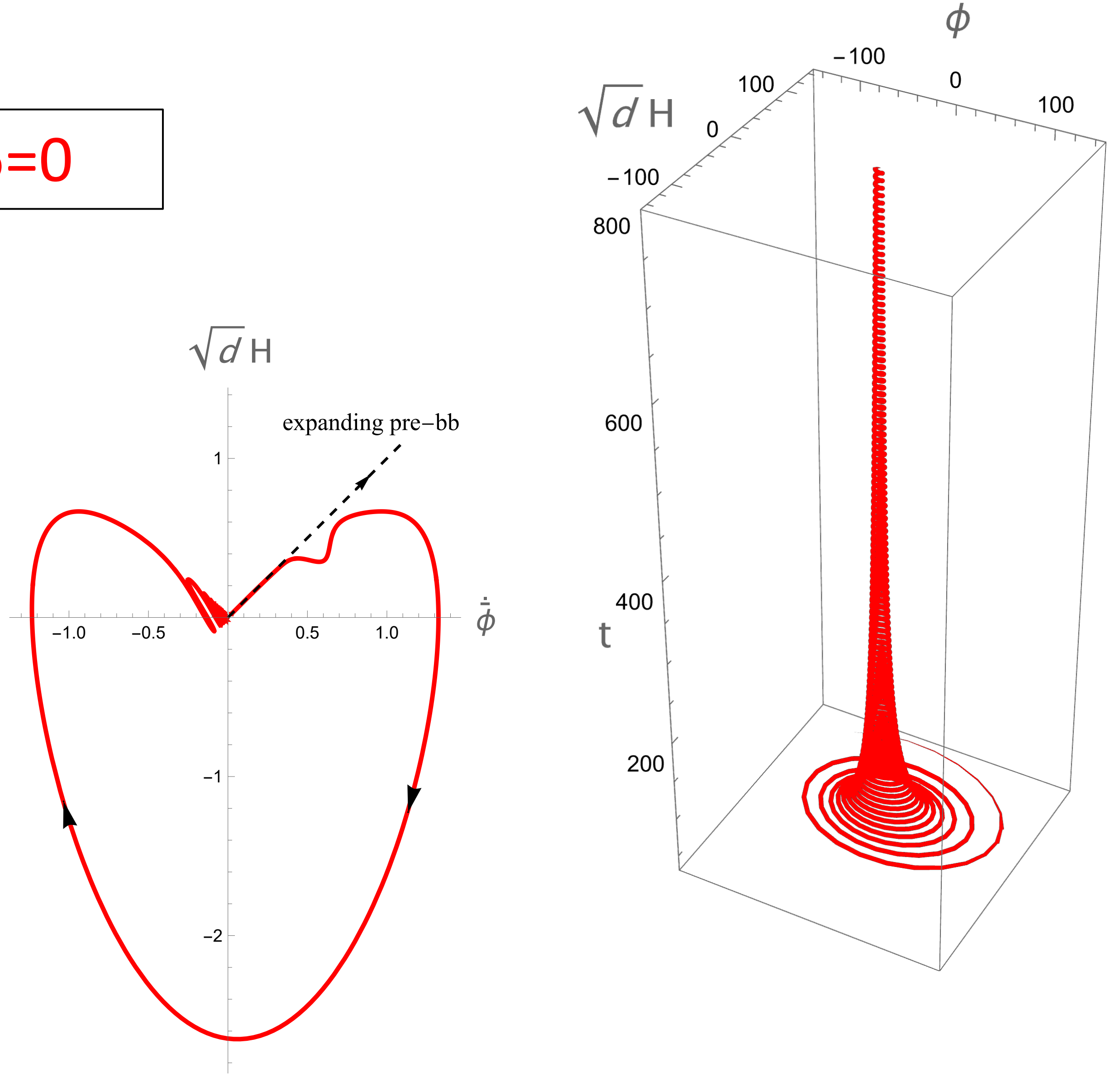

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Asymptotic Matter FLRW geometry δ=0

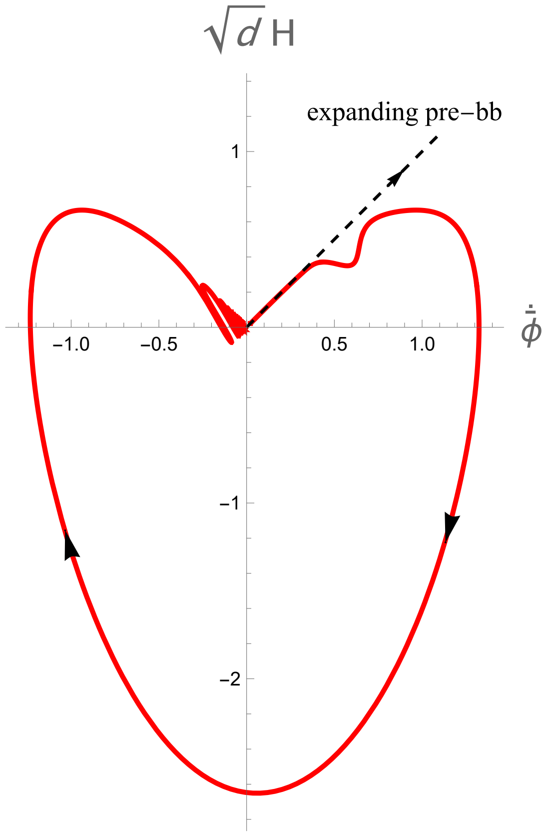
$$V_0 = 0$$

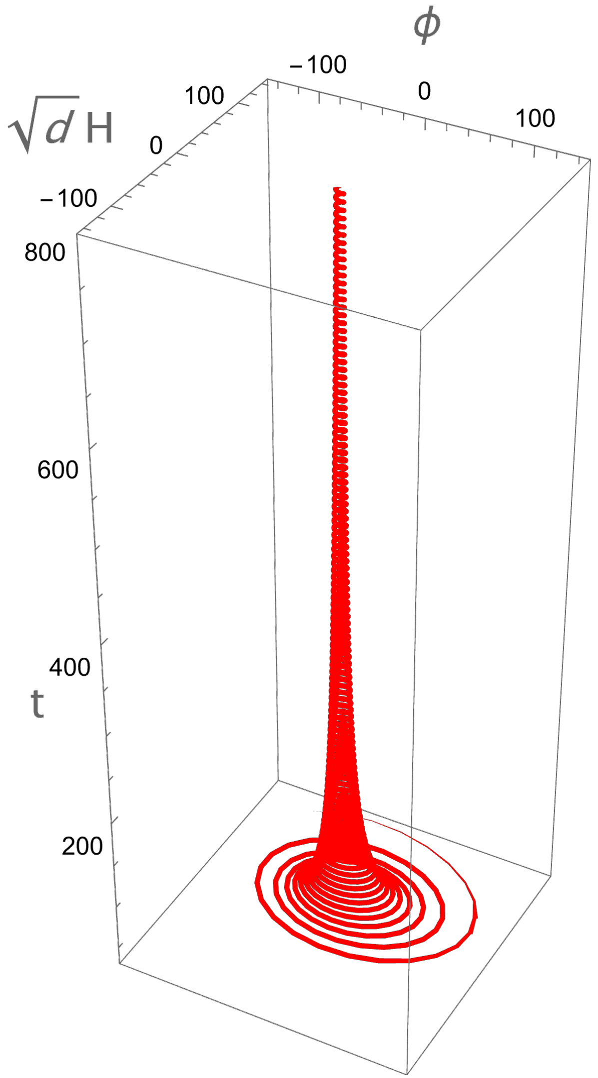
$$\phi(t) = \quad \phi_0 + \frac{2(d-1)}{t\omega\sqrt{d}}\sin(\omega t+\theta)$$

$$H(t) = \quad \frac{2}{td}\left[1+\sqrt{d}\cos(\omega t+\theta)\right]$$

$$\omega = m\sqrt{d-1}$$

$$m^2 = V''(\phi_m)$$





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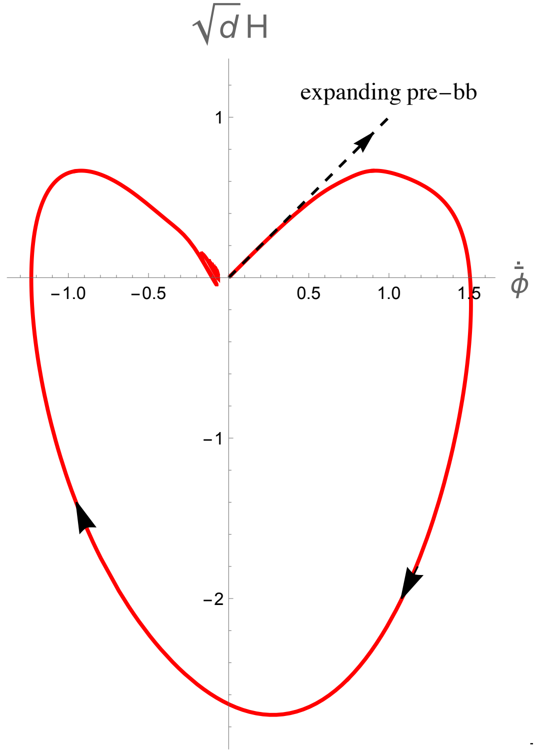

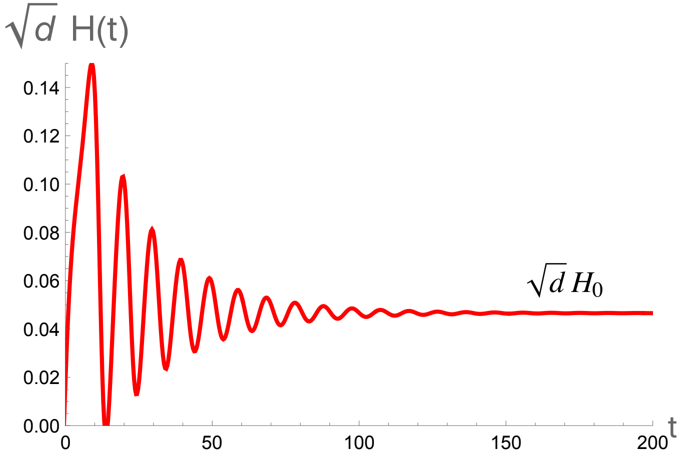

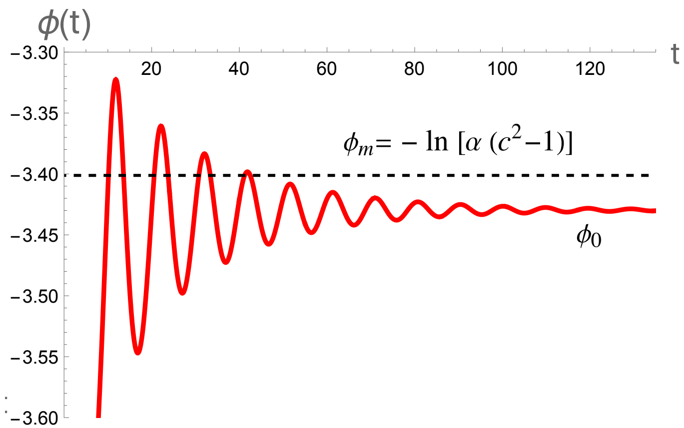

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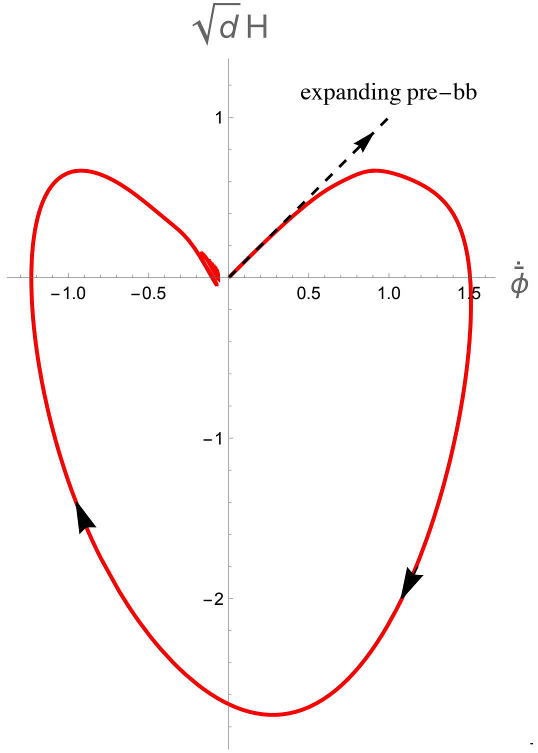

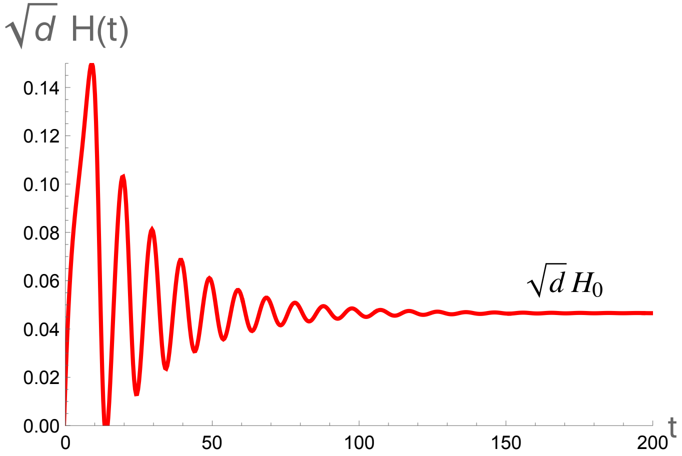

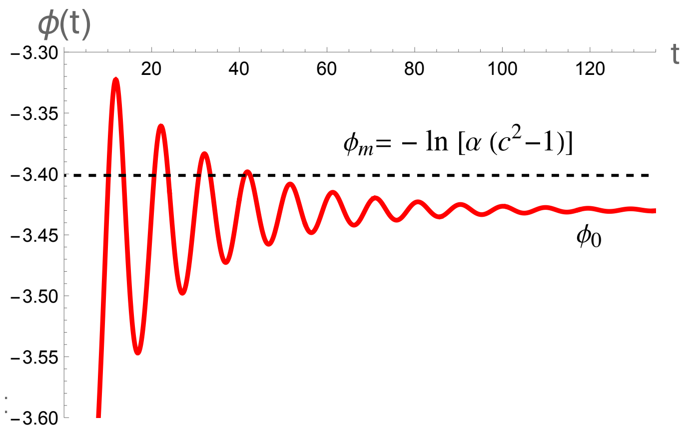

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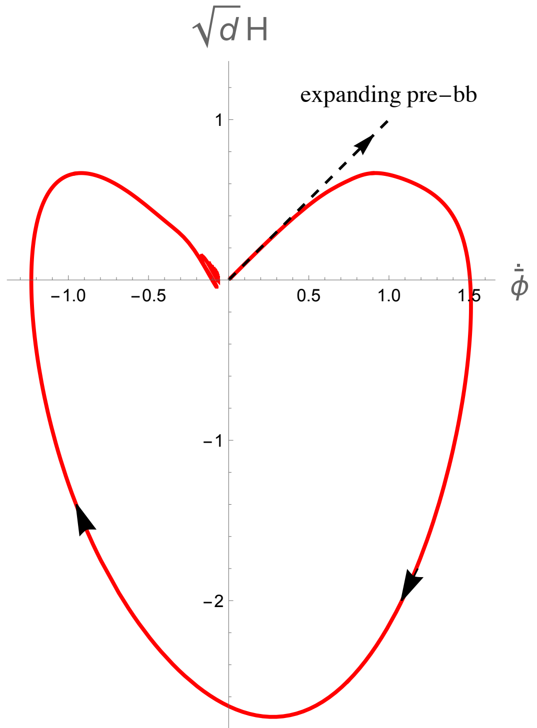
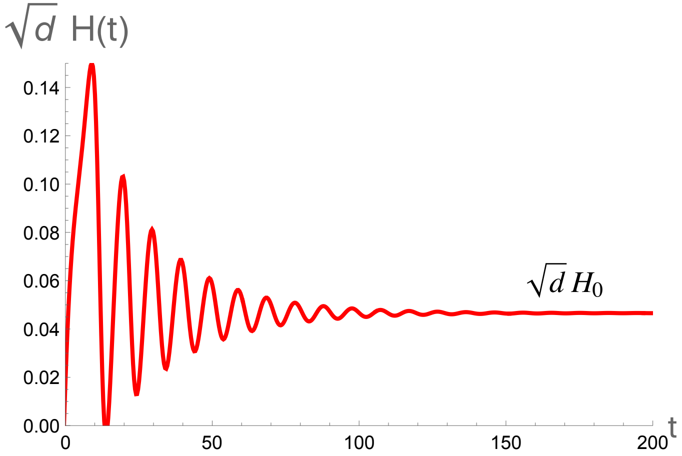
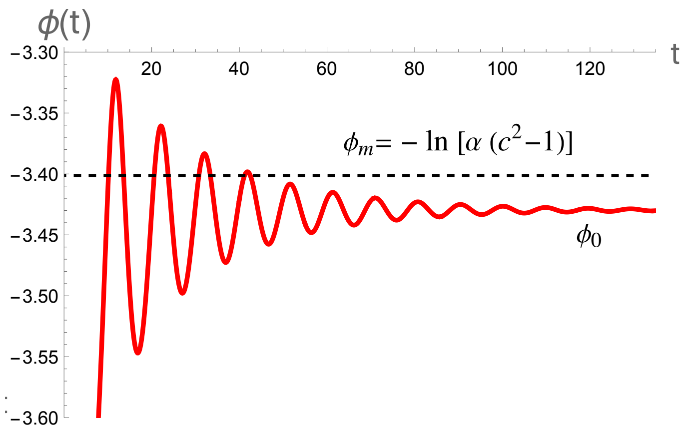
E-frame dilaton potential



Minimum of the dilaton potential in the E-frame (The dilaton is a minimally coupled scalar in the E-frame)





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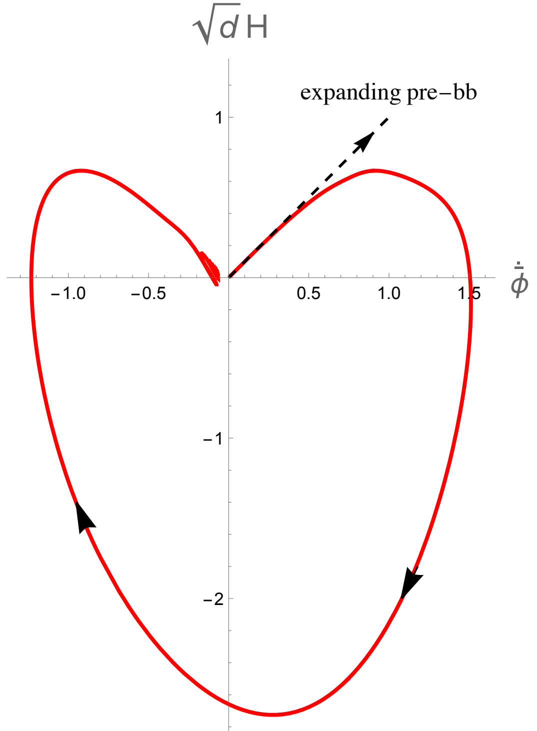

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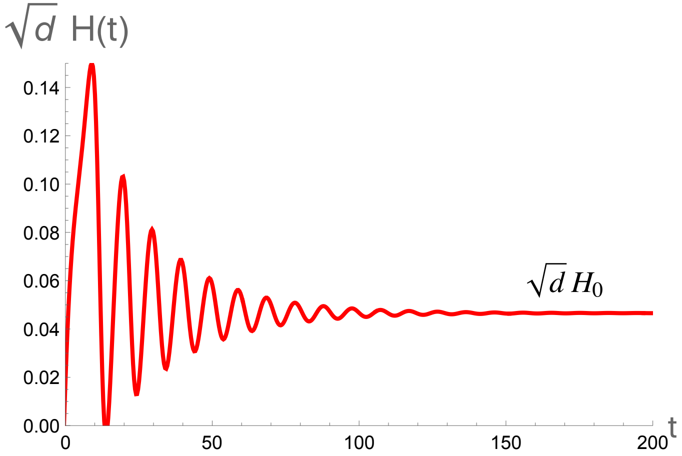
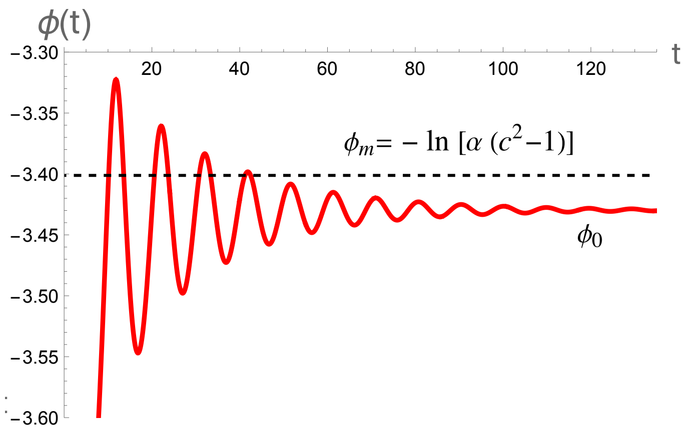




Is this solution a stable attractor?

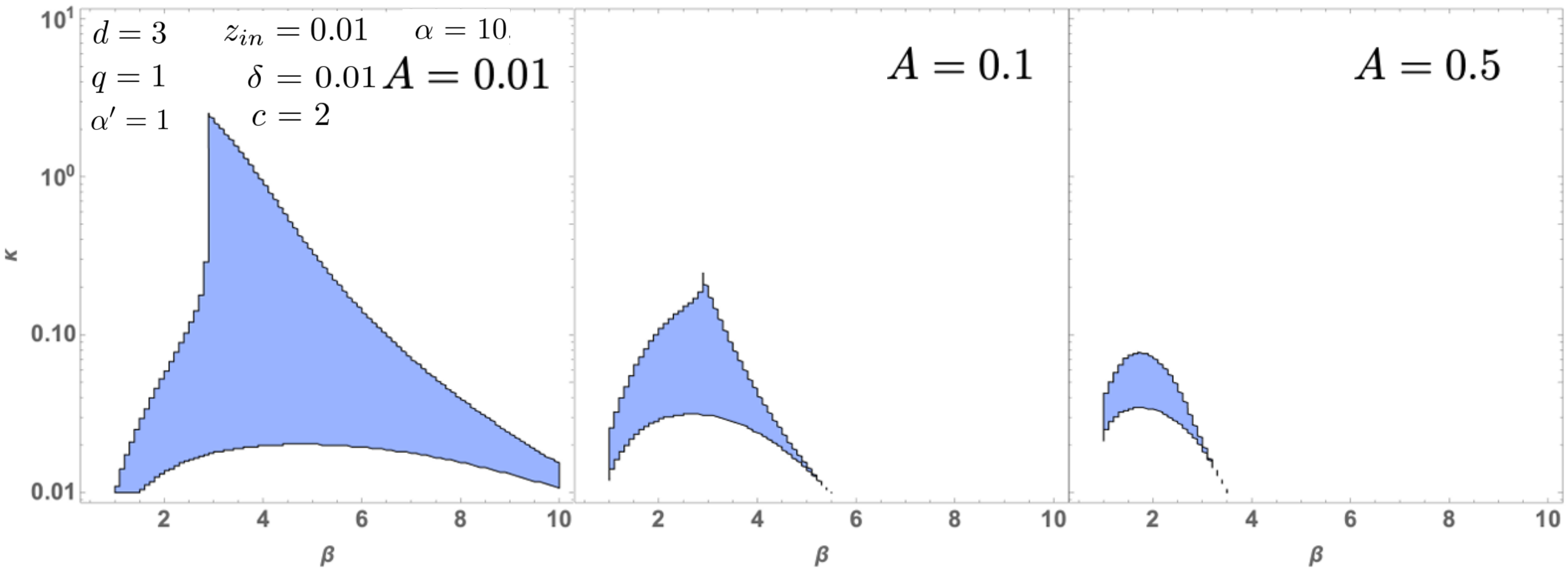
Numerical analysis of the attraction basin: Isotropic case

β controls the height of the first peak.

- Conserved quantity from the EoM used to fix the initial condition of the dilaton.

$$\boxed{e^{-\overline{\phi}}z = \kappa^{-1}}$$

Variation of the height of the first peak (β) and the amplitude A



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q controls the asymptotic behaviour of the dilaton potential.

- **q**=1: Asymptotically vanishing potential
- 0 ≤ **q** < 1: Asymptotically constant potential

: de Sitter, stabilised dilaton

: Growing dilaton, Runaway

: Time-reversal post Bounce, decreasing dilaton

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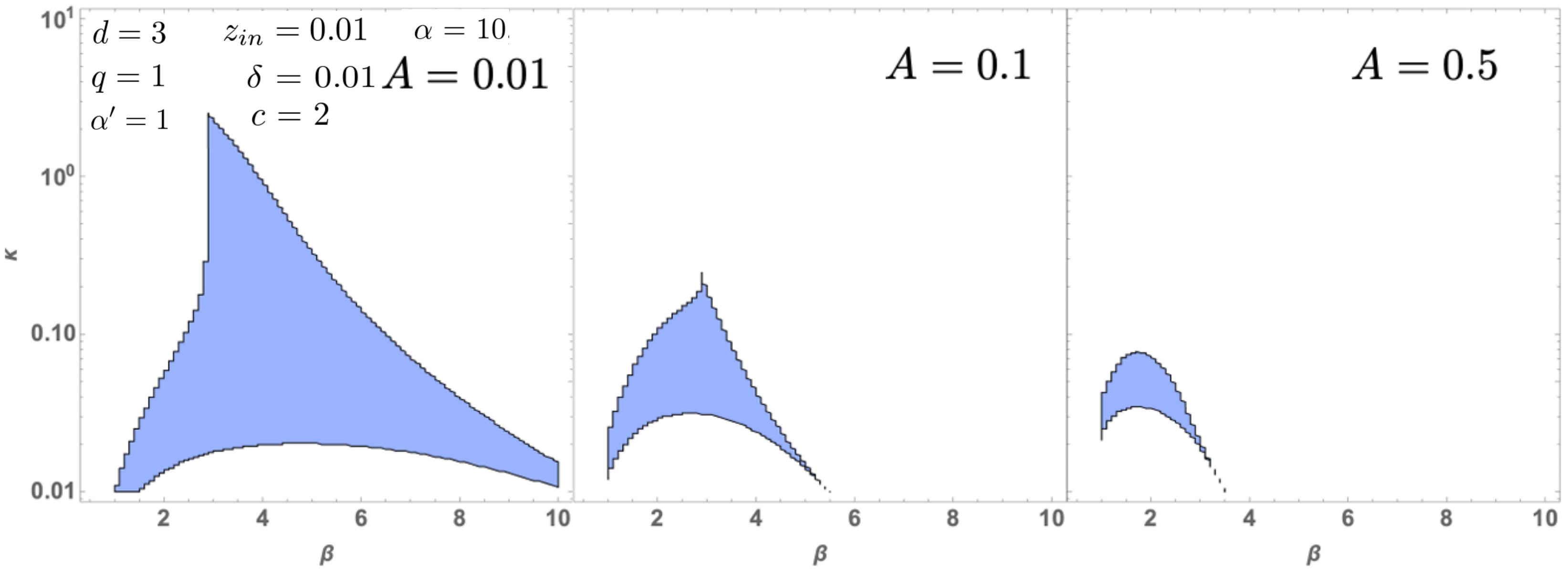
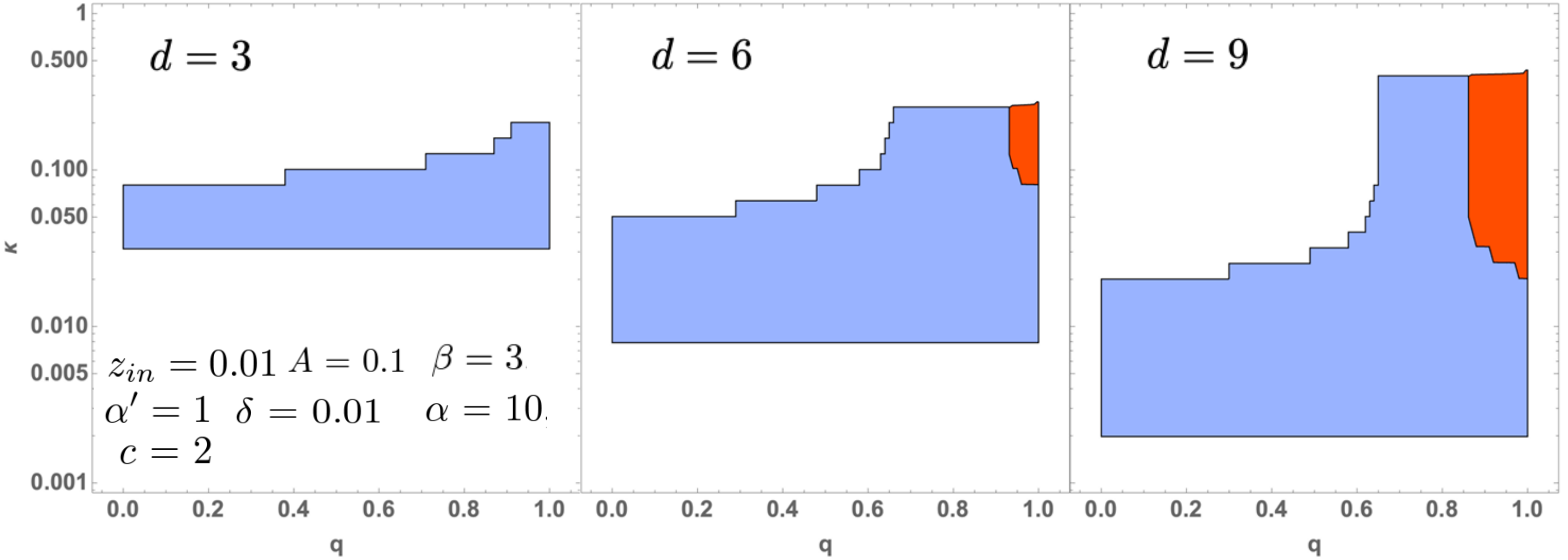


Variation of the asymptotic behaviour (q) and the number of spatial dimensions d



Anisotropic case: Isotropisation mechanism

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Case study: Bianchi I geometry with two isotropic *d* and *n* dimensional subspaces

$$\boxed{h(z_i) = \frac{1}{2}\sum_i z_i^2 - \frac{\alpha'}{4}\sum_i z_i^4}$$

Anisotropic ''Hamiltonian'' consistent at the first order in α' with the perturbative result obtained for F in heterotic string theory

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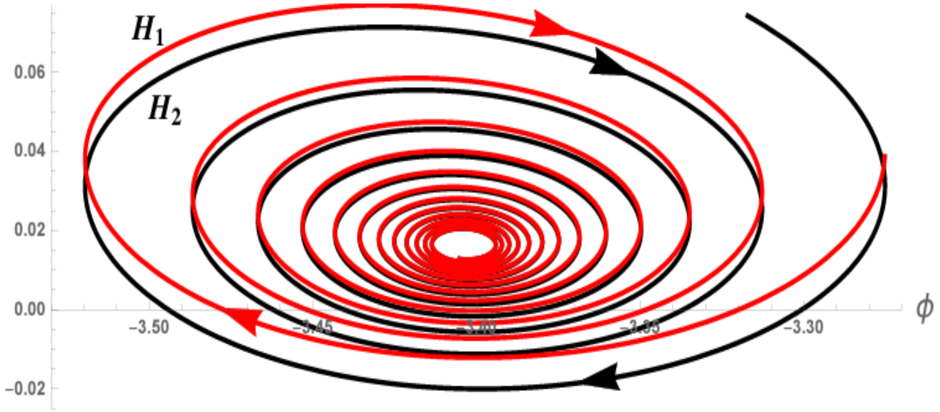
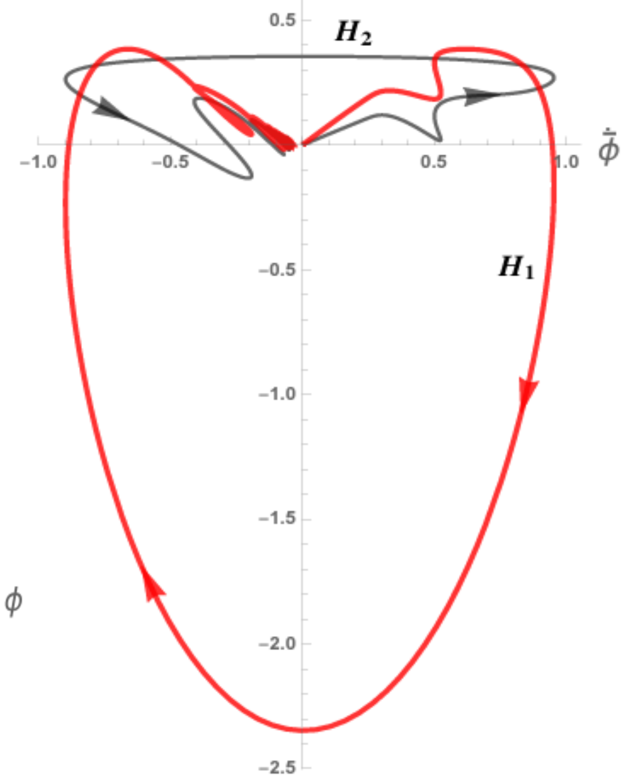


$d = 1 \qquad n = 2$



Anisotropic case: numerical analysis of the attraction basin

- Constant quantity

$$\kappa = e^{\phi} \left[\frac{d\, H_1 + n\, H_2}{\sqrt{d+n}} \right]^{-(1+d\,\gamma_1+n\,\gamma_2)}$$

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Past-asymptotic low curvature solutions.

- Scale factors:

$$a_i \sim (-t)^{-\gamma_i}$$

- Hubble functions:

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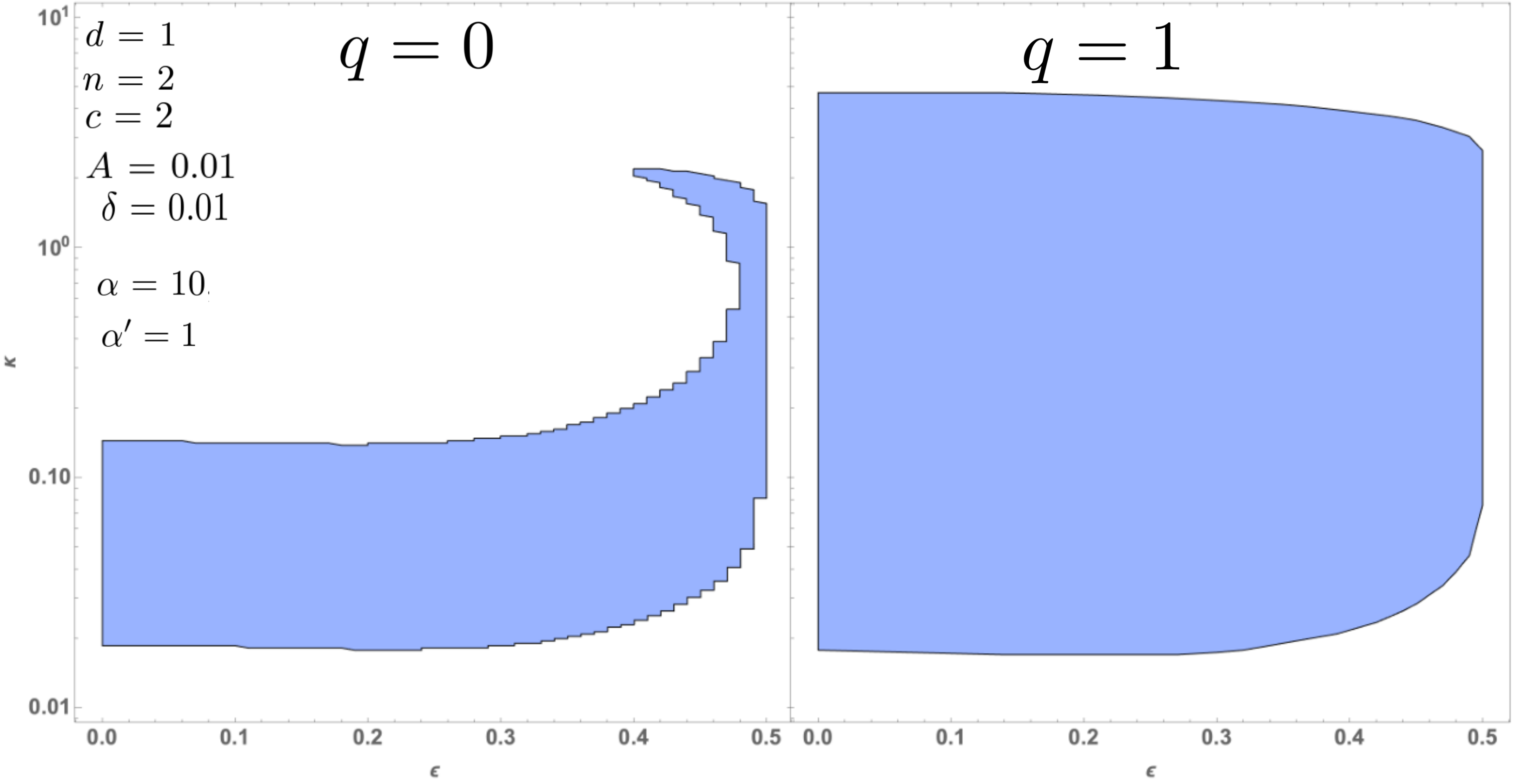
de Sitter attraction basin in the $\{\epsilon, \kappa\}$ space



Future research and developments

- Stabilizing mechanism for the internal compact dimensions
- Add the Kalb-Ramond $B_{\mu\nu}$ field
- Compute perturbations (scalar and tensor) to bound the model with observational data
- Can the Lagrangians, Hamiltonians, or Routhians that implement a regular bounce correspond to the dimensional reduction of some general covariant action.

Thanks for your attention!

E-mail: eliseo.pavone@ba.infn.it

This presentation was based on:

P. Conzinu, G.Fanizza, M.Gasperini, E. Pavone, L. Tedesco, G. Veneziano, *From the string vacuum to FLRW or de Sitter via α' corrections* ,JCAP **12**, 019 (2023).