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Positivity Constraints on Lorentz-breaking EFTs

with O. Janssen and L. Senatore 2207.14224 (JHEP) +
also with M. Delladio, A. Longo 2312.08441 +
work in progress also with B. Salehian
+ ....

Genova, 17.1.24

Positivity: LI case

Coefficients of EFT operators must satisfy inequalities
(if there is a "standard" UV completion)

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi 06

For example: $$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 + \frac{c}{\Lambda^4}(\partial\pi)^4 \qquad c \geq 0$$

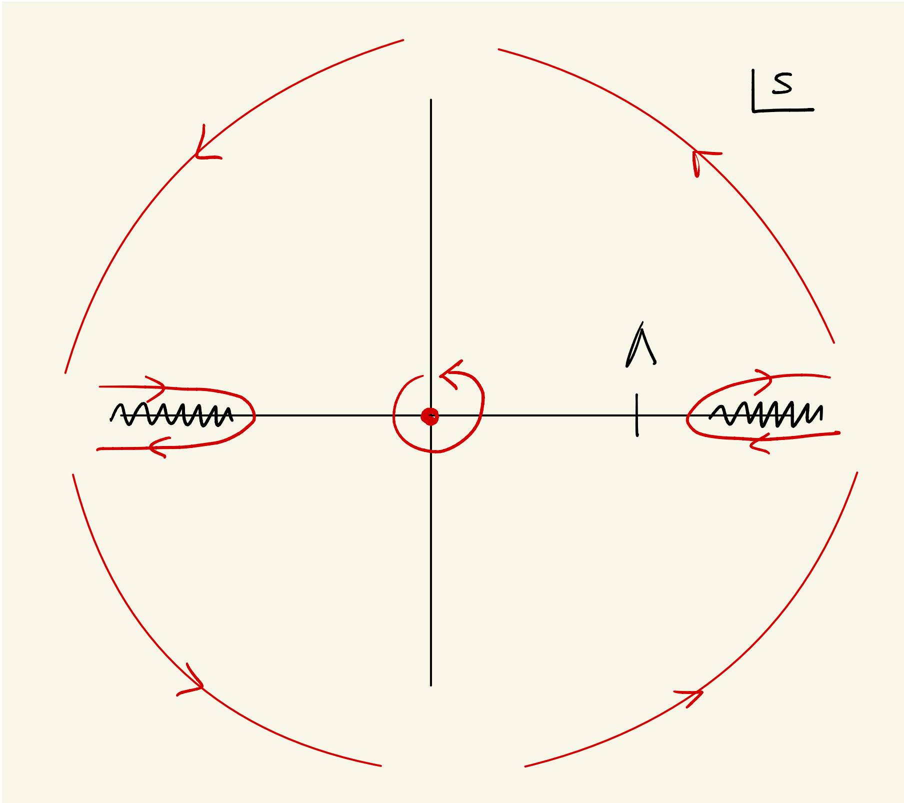
$$\mathcal{A}(s) \equiv \mathcal{M}(s, t \to 0) \qquad \mathcal{A}(s) = c\frac{s^2}{\Lambda^4} + \ldots$$

Crossing: $\mathcal{A}(s) = \mathcal{A}^*(-s^*)$

$$\oint \frac{ds}{2\pi i}\frac{\mathcal{A}(s)}{s^3} = \frac{c}{\Lambda^4}$$

$$\frac{c}{\Lambda^4} = \frac{2}{\pi}\int ds\frac{s\sigma(s)}{s^3} \geq 0$$

Froissart bound: $|\mathcal{A}(s)| < s\log^2 s$



Origin of analyticity

Consequence of microcausality: commutators vanish outside lightcone

See e.g. Itzykson Zuber's book

LSZ: $$S_{fi} = -\int d^4x\, d^4y\; e^{i(q_2 \cdot y - q_1 \cdot x)}(\Box_y + m_a^2)(\Box_x + m_a^2)\langle p_2 | T\varphi^\dagger(y)\varphi(x) | p_1 \rangle$$

Up to disconnected pieces: $T\varphi^\dagger(y)\varphi(x) \to \theta(y^0 - x^0)[\varphi^\dagger(y), \varphi(x)]$

$$S_{fi} = (2\pi)^4 \delta^4(p_2 + q_2 - p_1 - q_1) i\mathscr{T}$$

$$\mathscr{T} = i \int d^4z\; e^{iq \cdot z} \langle p_2 | \, \theta(z^0) \left[j^\dagger\left(\frac{z}{2}\right), j\left(-\frac{z}{2}\right)\right] | p_1 \rangle \qquad (\Box + m_a^2)\varphi(x) = j(x)$$

$$q = \tfrac{1}{2}(q_1 + q_2)$$

Commutator vanishes outside FLC → $\mathcal{T}(q^\mu)$ analytic for Im q^μ in FLC

Similar bounds for non-LI theories?

Motivation: in many interesting situations Lorentz is spontaneously broken

1. Cosmology. In particular Inflation and Dark Energy/Modifications of Gravity

   We are particularly interested in "peculiar" theories (Galileon, Ghost Condensate...): are they consistent?

2. Condensed Matter. Can we deduce general inequalities for a system?

3. QFT at finite T or finite Q

In general the theory is <u>defined</u> with non-linearly realised Lorentz

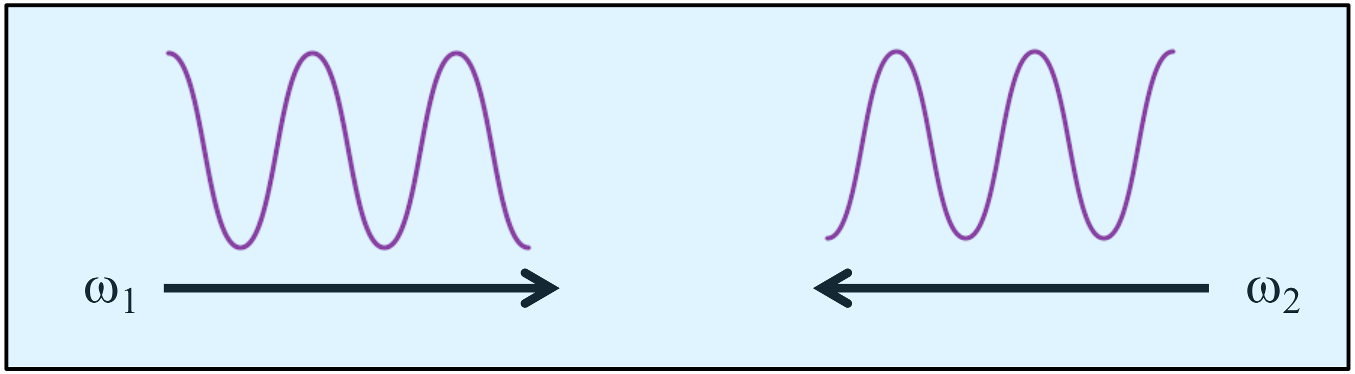
Cannot be "extrapolated" from a LI invariant theory: think about a fluid

Simply do the same?

Baumann, Green, Lee, Porto 15
Grall, Melville 21

$$\mathcal{L} = \frac{1}{2}\dot{\pi}^2 - \frac{1}{2}c_s^2(\partial_i\pi)^2 + \frac{\alpha_1}{\Lambda^2}\dot{\pi}^3 + \frac{\alpha_2}{\Lambda^2}\dot{\pi}(\partial_i\pi)^2 + \dots$$

Look at π scattering



In a LI theory this is well-defined at arbitrary high energy
(calculable in EFT only at low energy)

If LI is broken, π is not a good asymptotic state at high energy:
scatter phonons at 10 TeV?

What is the object whose analyticity we want to study?
What is the analogue of the Froissart bound?

S - Matrix

with Delladio, Janssen, Longo, Senatore 23
Also Hui, Kourkoulou, Nicolis, Podo, Zhou 23

What if the low energy states *do* exist at high energy?

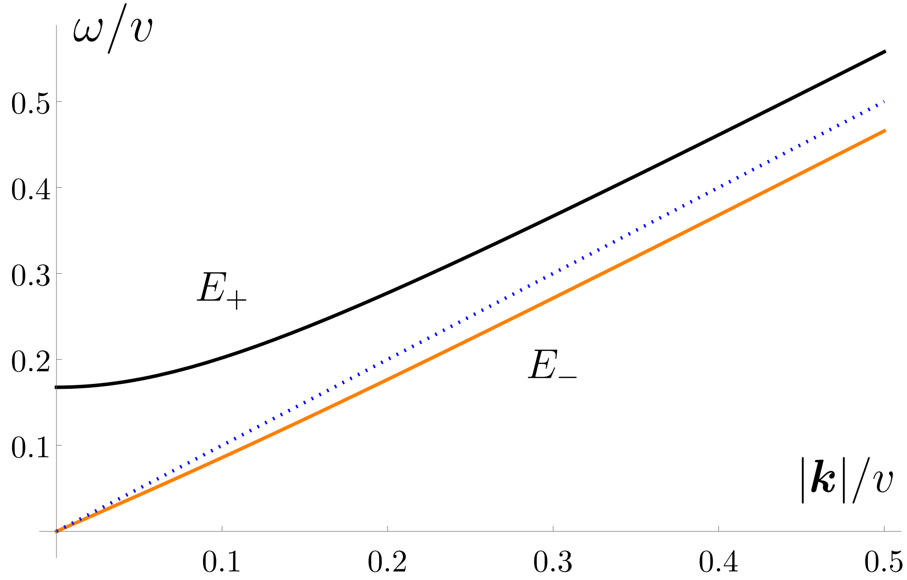
$$\mathcal{L} = \partial\Phi^\dagger \cdot \partial\Phi + m^2\,\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \qquad \Phi = \frac{\rho}{\sqrt{2}}e^{i\theta/v} \qquad \theta = \mu^2 t/2 + \pi$$

$$\rho = v + h$$

$$\mathcal{L} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(\partial\pi)^2 + \frac{1}{2v^2}\left(\mu^2\dot{\pi} + (\partial\pi)^2\right)(h^2 + 2vh) - \frac{\lambda}{4}(h^2+2vh)^2$$

Integrating out h one gets low energy EFT for Goldstone π

$$\frac{1}{2}\begin{pmatrix} \tilde{\pi}_{-k} & \tilde{h}_{-k} \end{pmatrix}\begin{pmatrix} k^2 & i\mu^2\omega/v \\ -i\mu^2\omega/v & k^2 - M^2 \end{pmatrix}\begin{pmatrix} \tilde{\pi}_k \\ \tilde{h}_k \end{pmatrix}$$



$$E_\pm(\boldsymbol{k})^2 \equiv \boldsymbol{k}^2 + \frac{1}{2}\left(M^2 + \frac{\mu^4}{v^2}\right) \pm \sqrt{\frac{\mu^4}{v^2}\boldsymbol{k}^2 + \frac{1}{4}\left(M^2 + \frac{\mu^4}{v^2}\right)^2}$$

LSZ reduction

$$\phi^a(t,\boldsymbol{x}) \equiv \sum_{l=\pm} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 2E_l(\boldsymbol{k})} \left(Z_l^a(\boldsymbol{k}) a_l(\boldsymbol{k}) e^{-i(E_l(\boldsymbol{k})t - \boldsymbol{k}\cdot\boldsymbol{x})} + \mathrm{h.c.}\right) , \quad a \in \{\pi, h\}$$

Imposing EOM and CCR one gets e.g. $\quad Z_-^\pi(\boldsymbol{k}) = \sqrt{\dfrac{M^2 + \boldsymbol{k}^2 - E_-(\boldsymbol{k})^2}{E_+(\boldsymbol{k})^2 - E_-(\boldsymbol{k})^2}}$

LSZ formula, using polology

$$\prod_i^n \int \mathrm{d}^4 y_i \, e^{ip_i\cdot y_i} \prod_j^m \int \mathrm{d}^4 x_j \, e^{-ik_j\cdot x_j} \langle 0 | T(\pi(y_1)\dots\pi(y_n)\pi(x_1)\dots\pi(x_m))|0\rangle \sim$$

$$\prod_i^n \frac{iZ_-^\pi(\boldsymbol{p}_i)}{p_i^{0\,2} - E_-^2(\boldsymbol{p}_i) + i\varepsilon} \prod_j^m \frac{i\bar{Z}_-^\pi(\boldsymbol{k}_j)}{k_j^{0\,2} - E_-^2(\boldsymbol{k}_j) + i\varepsilon} \langle \boldsymbol{p}_1 \dots \boldsymbol{p}_n | S | \boldsymbol{k}_1 \dots \boldsymbol{k}_m \rangle$$

$$Z_\pm^h(\boldsymbol{k}) \equiv \langle \Omega | h(0) | \boldsymbol{k}, \pm \rangle$$

(Another procedure is to write creation/annihilation operators in terms of fields: different LSZ expression, but same conclusions)

Lack of analyticity

The usual arguments of S-matrix analyticity breaks down

$$S = -\int \mathrm{d}^4x \mathrm{d}^4y \, e^{i(q_2\cdot y - q_1\cdot x)} \frac{-\partial_{y^0}^2 - E_-^2(-i\partial_{y_i})}{Z_-^\pi(-i\partial_{y_i})} \frac{-\partial_{x^0}^2 - E_-^2(-i\partial_{x_i})}{\bar{Z}_-^\pi(-i\partial_{x_i})} \langle \boldsymbol{p}_2 | T(\pi(y)\pi(x)) | \boldsymbol{p}_1 \rangle$$

$$S = (2\pi)^4 \delta^{(4)}(p_2 + q_2 - p_1 - q_1) i\mathcal{T}$$

$$\mathcal{T} = i \int \mathrm{d}^4z \, e^{iqz} \frac{-\partial_{z^0/2}^2 - E_-^2(-i\partial_{z_i/2})}{Z_-^\pi(-i\partial_{z_i/2})} \frac{-\partial_{z^0/2}^2 - E_-^2(-i\partial_{z_i/2})}{\bar{Z}_-^\pi(-i\partial_{z_i/2})} \langle \boldsymbol{p}_2 | \theta(z^0)[\pi(\frac{z}{2})\pi(-\frac{z}{2}))] | \boldsymbol{p}_1 \rangle$$

Vanishes outside FLC in z

$\mathcal{T}(q^\mu)$ analytic for Im q^μ in FLC

Without Lorentz invariance Z(k) and E(k) introduce non-analyticities

Conserved currents

with Janssen, Senatore 22

UV: CFT

IR: ~~LI~~

1. Assume to flow to a CFT in UV (general?)

2. UV behaviour of $\langle J^\mu J^\nu \rangle$ and $\langle T^{\mu\nu} T^{\alpha\beta} \rangle$ are known:

$$\langle J^\mu(p) J^\nu(-p) \rangle = c_J \frac{p^2 g^{\mu\nu} - p^\mu p^\nu}{p^{4-d}}$$

3. At low energy EFT (is not conformal and) breaks LI spontaneously

4. Analyticity and unitarity of $\langle J^\mu J^\nu \rangle$ and $\langle T^{\mu\nu} T^{\alpha\beta} \rangle$ + UV limit above

→ positivity properties in EFT

Superfluid

Superfluid: $$\int d^4x\sqrt{-g}\,P(X) \qquad X \equiv -\partial_\mu\phi\partial^\mu\phi$$

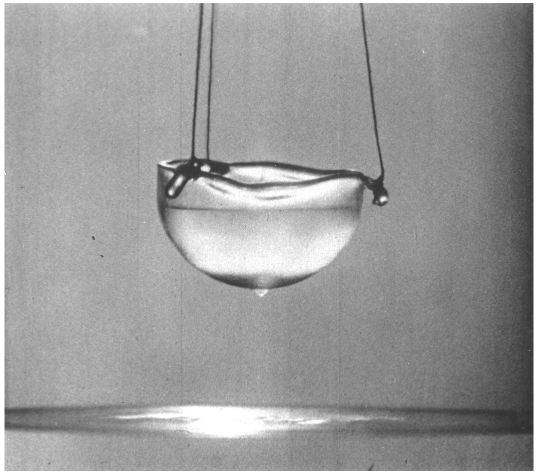
Expanded as $\phi = c\,t + \pi(t,x)$ it describes a perfect fluid w/o vortices

$$T_{\mu\nu} = 2P'(X)\partial_\mu\phi\partial_\nu\phi + P(X)g_{\mu\nu} \quad \rho = 2P'X - P \quad p = P \quad u_\mu = \frac{\partial_\mu\phi}{\sqrt{-X}}$$

E.g. $P(X) = X^2$ gives $w = 1/3$: radiation fluid

Poincarè x U(1) → Spacetime translations x Rotations (x Shift)

Real superfluids



Given approximate shift symmetry, good starting point for inflation and dark energy (K-inflation, K-essence)

Conformal superfluid

Superfluid: $$\int d^4x\sqrt{-g}\,P(X) \qquad X \equiv -\partial_\mu\phi\partial^\mu\phi$$

Expanded as $\phi = c\,t + \pi(t,x)$ it describes a perfect fluid w/o vortices

$$T_{\mu\nu} = 2P'(X)\partial_\mu\phi\partial_\nu\phi + P(X)g_{\mu\nu} \quad \rho = 2P'X - P \quad p = P \quad u_\mu = \frac{\partial_\mu\phi}{\sqrt{-X}}$$

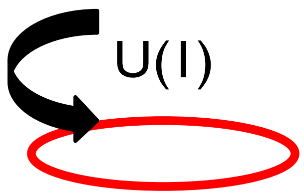
E.g. $P(X) = X^2$ gives $w = 1/3$: radiation fluid

Poincarè x U(1) → Spacetime translations x Rotations (x Shift)

Conformal superfluid: add non-linearly realised conformal symmetry

SO(d,2) x U(1) → Spacetime translations x Rotations (x Shift)

CFT at finite chemical potential μ. U(1) spontaneously broken.
At E << μ system is described by an EFT



CFTs at large charge

Action by coset construction or by using the metric $\hat{g}_{\mu\nu} \equiv g_{\mu\nu}|g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi|$

$$\chi(t,\vec{x}) = \mu t + \pi(t,\vec{x}) \qquad S^{(1)} = \frac{c_1}{6}\int \mathrm{d}^3x\sqrt{-\hat{g}} = \frac{c_1}{6}\int \mathrm{d}^3x\sqrt{-g}|\partial\chi|^3$$

Expansion in $\dfrac{\partial}{\mu}$

$$S^{(2)} = \int \mathrm{d}^3x\sqrt{-\hat{g}}\left(-c_2\hat{R} + c_3\hat{R}^{\mu\nu}\hat{\partial}_\mu\chi\hat{\partial}_\nu\chi\right)$$

Large Q limit of: $\langle\mathcal{O}_{-\vec{Q},a}(x_{out})\mathcal{O}_m(x_m)\ldots\mathcal{O}_1(x_1)\mathcal{O}_{\vec{Q},a}(x_{in})\rangle$

Hellerman, Orlando, Reffert, Watanabe 15; Monin, Pirtskhalava, Rattazzi, Seibold 16

By operator-state correspondence, one is in a state at large Q on S^{d-1}x R

For large Q, one has an EFT with a single Goldstone and CFT results can be obtained as an expansion in 1/Q: $1/\mu R \sim 1/Q^{1/2}$

$$\Delta_Q = \frac{2}{3}\frac{Q^{3/2}}{\sqrt{2\pi c_1}} + 8\pi c_2\sqrt{\frac{Q}{2\pi c_1}} - \underline{0.0937256} + \mathcal{O}\left(Q^{-1/2}\right)$$

1-loop correction

Constraints from Current

Focus on 3d: $$\mathcal{L} = \frac{c_1}{6}|\nabla\chi|^3 - 2c_2\frac{(\partial|\nabla\chi|)^2}{|\nabla\chi|} + c_3\left(2\frac{(\nabla^\mu\chi\partial_\mu|\nabla\chi|)^2}{|\nabla\chi|^3} + \partial_\mu\left(\frac{\nabla^\mu\chi\nabla^\nu\chi}{|\nabla\chi|^2}\right)\partial_\nu|\nabla\chi|\right)$$
$$- \frac{b}{4}\frac{F_{\mu\nu}F^{\mu\nu}}{|\nabla\chi|} + \frac{d}{2}\frac{F^\mu{}_i F^{\nu i}}{|\nabla\chi|^3}\nabla_\mu\chi\nabla_\nu\chi\,,$$ ← Contact terms for A_μ

Can we get constraints on $c_{1,2,3}$, b and d using $\langle J^\mu J^\nu\rangle$?

Most general using conservation: $$i\langle J^\mu(-k)J^\nu(k)\rangle = \mathsf{A}\left(k^\mu k^\nu - \eta^{\mu\nu}k^2\right) + \mathsf{B}\left(k^i k^j - \delta^{ij}\boldsymbol{k}^2\right)$$

$$\mathsf{A} = -\frac{\mu c_1}{2\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)} + \frac{c_2}{\mu}\frac{\left(\omega^2 - \boldsymbol{k}^2\right)\boldsymbol{k}^2}{\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)^2} - \frac{c_3}{\mu}\frac{\omega^2\boldsymbol{k}^2}{\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)^2} + \frac{b}{\mu} + \frac{d}{\mu}\,,$$
$$\mathsf{B} = \frac{\mu c_1}{4\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)} + \frac{c_2}{\mu}\frac{\left(\omega^2 - \boldsymbol{k}^2\right)^2}{\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)^2} - \frac{c_3}{\mu}\frac{\omega^2(\omega^2 - \boldsymbol{k}^2)}{\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)^2} - \frac{d}{\mu}\,.$$

Current analyticity

Retarded Green function: $G_R^{\mu\nu}(x-y) = i\theta(x^0-y^0)\,\langle 0|[J^\mu(x), J^\nu(y)]|0\rangle$

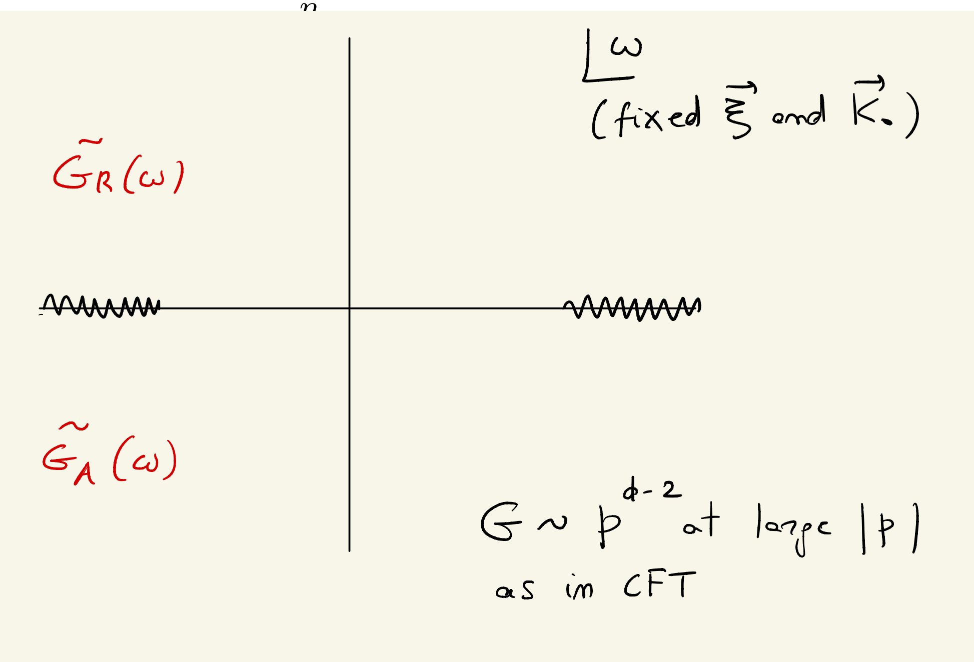
$\tilde{G}_R^{\mu\nu}(\omega,\boldsymbol{p})$ analytic for $p_{\text{Im}}{}^{\mu}$ in FLC $\qquad \boldsymbol{p} = \boldsymbol{k}_0 + \omega\boldsymbol{\xi} \qquad |\boldsymbol{\xi}| \equiv \xi < 1$

$$\tilde{G}^{\mu\nu}(\omega) = \begin{cases} \tilde{G}_R^{\mu\nu}(\omega,\boldsymbol{p}) & \text{if } \omega^{\text{Im}} \geq 0 \\ \tilde{G}_A^{\mu\nu}(\omega,\boldsymbol{p}) & \text{if } \omega^{\text{Im}} < 0 \end{cases}$$

$$\lim_{\varepsilon\to 0}\left(\tilde{G}^{\mu\nu}(\omega+i\varepsilon) - \tilde{G}^{\mu\nu}(\omega-i\varepsilon)\right) = i\int_{\mathbb{R}^d} \mathrm{d}^d x\, e^{-ip\cdot x}\,\langle 0|[J^\mu(x), J^\nu(0)]|0\rangle$$

$$= i(2\pi)^d \sum_n \left\{\delta^{(d)}(p-P_n)\,\langle 0|J^\mu(0)|P_n\rangle\,\langle P_n|J^\nu(0)|0\rangle - \delta^{(d)}(p+P_n)\,\underline{\langle 0|J^\nu(0)|P_n\rangle\,\langle P_n|J^\mu(0)|0\rangle}\right\}$$

Positive!



Assuming mass gap, or working at tree level

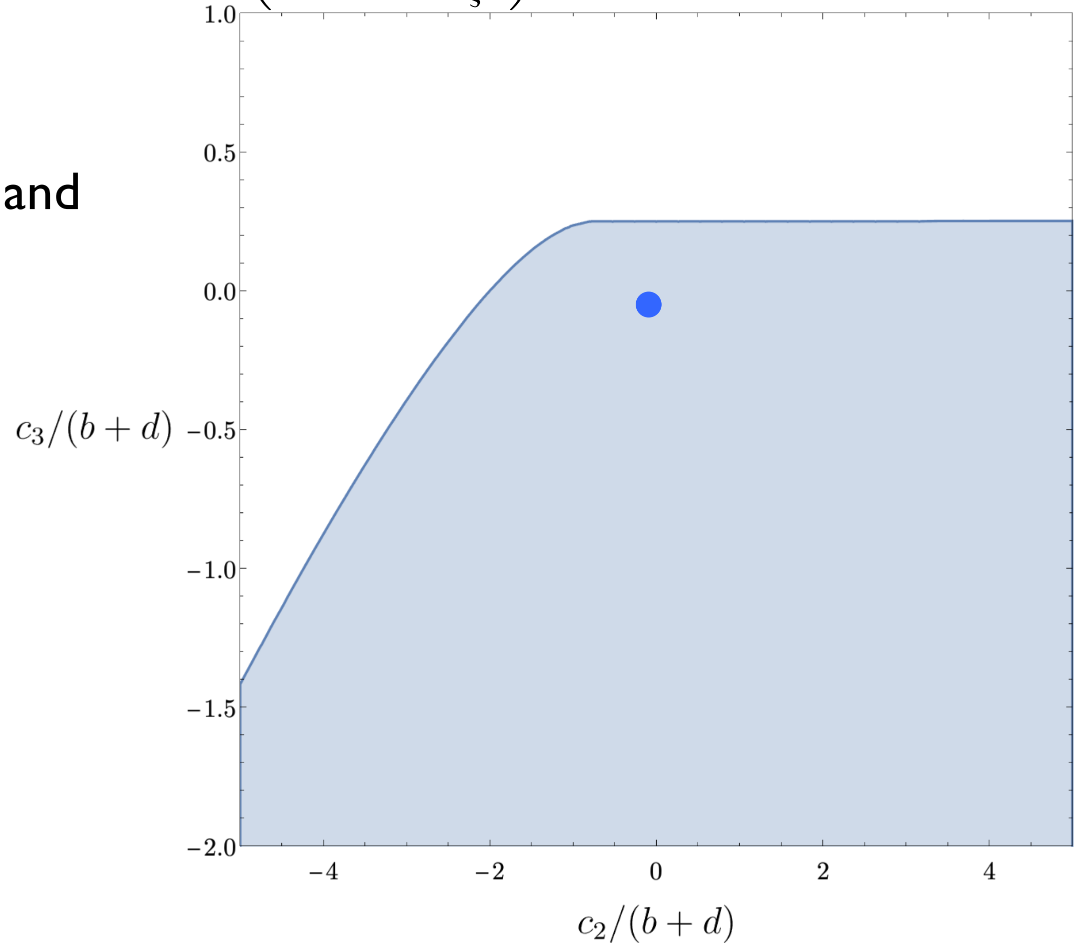
Current analyticity 2

$$\frac{1}{2\pi i}\oint \frac{G^{\mu\nu}(\omega;\vec{p}=\vec{\xi}\omega)V^\mu V^\nu}{\omega^3} = \frac{1}{2\pi i}\int_{\mathrm{CUT}} \frac{G^{\mu\nu}(\omega;\vec{p}=\vec{\xi}\omega)V^\mu V^\nu}{\omega^3} \geq 0$$ for any ξ and V^μ (β, γ)

$$c_2\frac{\xi^2(1-\xi^2)}{(1-\xi^2/2)^2}\beta^2 - c_3\frac{\xi^2}{(1-\xi^2/2)^2}\beta^2 + b(\beta^2+\gamma^2) + d\left(\beta^2 + \frac{\gamma^2}{1-\xi^2}\right) \geq 0$$

$$\Longrightarrow \qquad d \geq 0, \qquad b + d \geq 0$$

and



Unable to constrain the sign of c_2
(most relevant for CFT use)

Nothing more from k_0

Same for <$T^{\mu\nu}$ $T^{\alpha\beta}$>

Given the behavior at infinity I have to divide by ω^5: NNLO

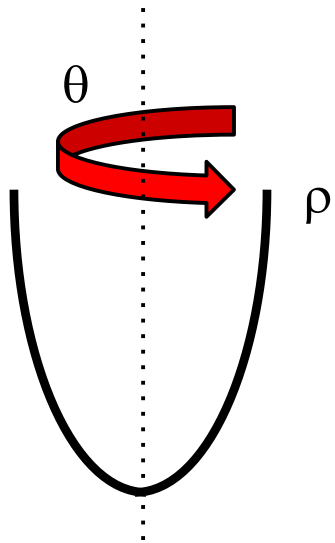
$$S^{(3)} = \int \mathrm{d}^3x \sqrt{-\hat{g}} \left(c_4 \hat{R}^2 + c_5 \hat{R}_{\mu\nu} \hat{R}^{\mu\nu} + c_6 \hat{R}^0_\mu \hat{R}^{\mu 0} \right)$$

Only operators that start quadratic in perturbations at NNLO (up to field redefiniton)

$$\frac{1}{2\pi i} \oint \frac{\langle T^{\mu\nu} T^{\alpha\beta} \rangle (\omega; \vec{p} = \vec{\xi}\omega) A_{\mu\nu} A_{\alpha\beta}}{\omega^5} \geq 0$$

$$\begin{aligned} 4c_4 + 2c_5 + c_6 &\geq 4(c_2 + c_3)^2 / c_1 \,, \\ c_5 &\geq 0 \,, \\ c_6 &\geq 0 \,. \end{aligned}$$

UV checks



$$\mathcal{L}_{\mathsf{UV}} = \sqrt{-g}\left(-|\partial\phi|^2 - \lambda|\phi|^6 - \frac{1}{8}R|\phi|^2\right)$$
$$= \sqrt{-g}\left(-(\partial\rho)^2 + \rho^2|\partial\theta|^2 - \lambda\rho^6 - \frac{1}{8}R\rho^2\right)$$

Conformal at tree level. Integrate out ρ:

$$\frac{\mathcal{L}_{\mathsf{EFT}}}{\sqrt{-g}} = \frac{2}{3\sqrt{3\lambda}}|\partial\theta|^3 - \frac{1}{8\sqrt{3\lambda}}\left(|\partial\theta|R + 2\frac{(\partial|\partial\theta|)^2}{|\partial\theta|}\right) + \frac{1}{4|\partial\theta|^2}\left(\Box\rho_0 - \frac{R}{8}\rho_0\right)^2$$

$$c_1 = \frac{4}{\sqrt{3\lambda}}\,, \quad c_2 = \frac{1}{8\sqrt{3\lambda}}\,, \quad c_4 = \frac{1}{256\sqrt{3\lambda}}\,, \quad c_3 = c_5 = c_6 = b = d = 0$$

More general:

$$\mathcal{L}_{\mathsf{UV}} = \sqrt{-g}\left(-|\partial\phi|^2 \underline{- (\partial\varphi)^2} - \lambda_1|\phi|^6 \underline{- \lambda_2\varphi^6 - \beta_1|\phi|^2\varphi^4 - \beta_2|\phi|^4\varphi^2} - \frac{R}{8}\left(|\phi|^2 \underline{+ \varphi^2}\right)\right)$$

Bounds are satisfied (of course!)

Positivity bounds for EM response of media

Re-explore the old problem of light in a material

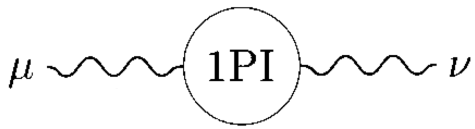
in progress with Janssen, Salehian, Senatore

Fields are small (compared with the atomic ones) → linear optic

We want EOM for <E> and <B> after you integrate out the medium

IN-IN Effective action

$$\int \mathrm{d}^4x \; J^\mu(\psi) A_\mu$$



$$\Gamma_M[A_1, A_2] = \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y \left[A_{1\mu}(x) \;\; A_{2\mu}(x)\right] S^{\mu\nu}(x,y) \begin{bmatrix} A_{1\nu}(y) \\ A_{2\nu}(y) \end{bmatrix}$$

$$S^{\mu\nu}(x,y) = i \begin{bmatrix} \langle T J^\mu(x) J^\nu(y)\rangle & -\langle J^\nu(y) J^\mu(x)\rangle \\ -\langle J^\mu(x) J^\nu(y)\rangle & \left\langle \tilde{T} J^\mu(x) J^\nu(y)\right\rangle \end{bmatrix}_{\mathrm{1PI}}$$

Macroscopic Maxwell equations

$$\frac{1}{g^2}\partial_\nu F^{\nu\mu} + \int \mathrm{d}^4y\, \Pi^{\mu\nu}(x,y) A_\nu(y) = -J^\mu_{\mathrm{ext}}(x)$$

$$\Pi^{\mu\nu}(x,y) = i\theta(x^0 - y^0)\, \langle [J^\mu(x), J^\nu(y)]\rangle_{\mathrm{1PI}}$$

Susceptibilities

$$p_\mu \Pi^{\mu\nu} = 0 \qquad\qquad \Pi^{\mu\nu} = \pi_L(\omega,k) p^2 \mathcal{P}_L^{\mu\nu} + \pi_T(\omega,k) k^2 \mathcal{P}_T^{\mu\nu}$$

$$\varepsilon - 1 = -g^2 \pi_L \,, \qquad 1 - \frac{1}{\mu} = g^2 \left(\pi_T + \frac{\omega^2}{k^2} \pi_L \right)$$

Usual electric and magnetic susceptibility, now function of ω, k

Analyticity of Π gives analyticity of π_L and π_T

!! Cheating Warning !!

$$\pi_L(\omega, \vec{k} + \omega\vec{\xi}) = \frac{1}{i\pi} \mathrm{PV} \int_{-\infty}^{+\infty} \frac{\mathrm{d}z}{z - \omega} \, \pi_L(z, \vec{k} + z\vec{\xi})$$

Generalisation of Kramers-Kronig relation

Leontovich 61

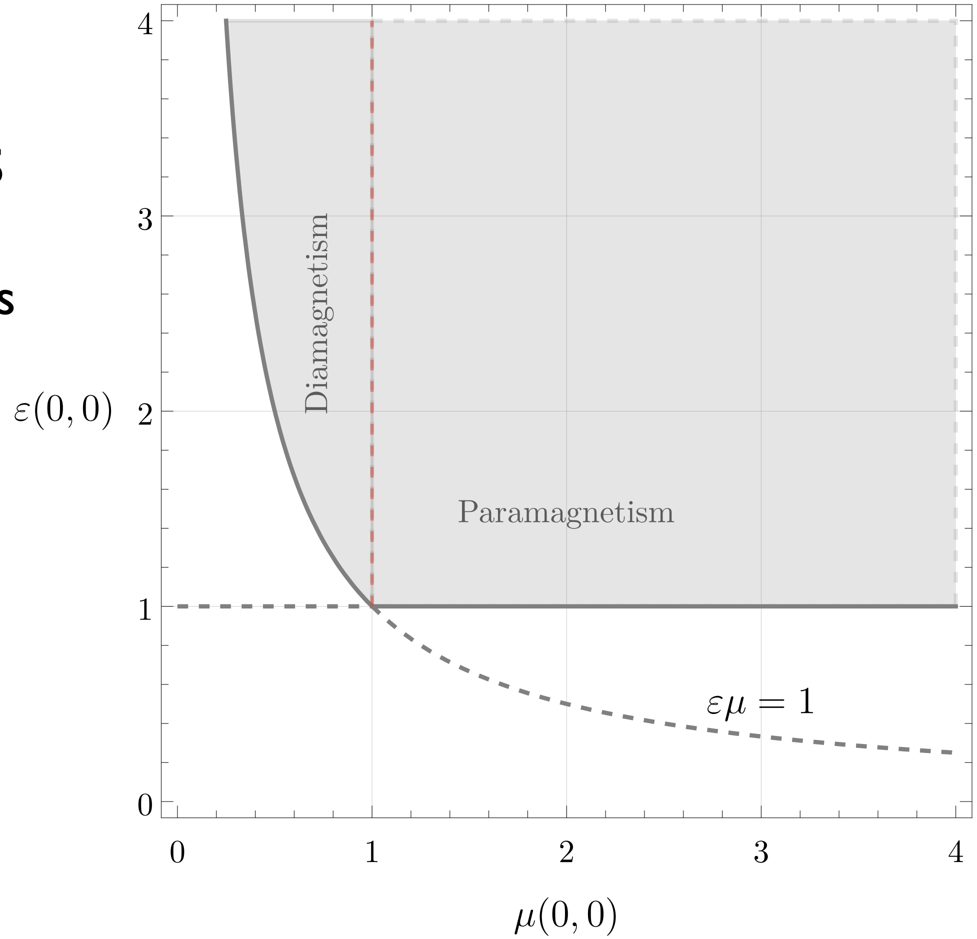
The sign of the imaginary part is fixed by imposing material can only absorb light (laser is an exception for example)

Positivity bounds for EM response of media

$$\varepsilon(0,0) - 1 = \frac{g^2}{\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{d}z}{z}\mathrm{Im}\ \pi_L(z, z\vec{\xi}) > 0$$

$$(\varepsilon(0,0) - 1) + \xi^2\left(1 - \frac{1}{\mu(0,0)}\right) = \frac{g^2}{\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{d}z}{z}\xi^2\mathrm{Im}\ \pi_T(z, z\vec{\xi}) > 0$$

- Not only bounds but given positive RHS
- If the medium is "slow": stronger bounds
- Bounds on derivatives?



see also Dolgov, Kirzhnits, Losyakov 82

Conclusions and Future

- Robust constraints on non-LI EFTs are possible but:

    a. No constraint for 1-derivative per field ~ P(X). Only more irrelevant than CFT
    b. Only operators that start quadratic (but for any background)

- General bounds on "CM" systems deriving from $\langle J^\mu J^\nu \rangle$ and $\langle T^{\mu\nu} T^{\alpha\beta} \rangle$
  Superconductivity, fluids, fluctuations...

- *We do not know anything without Lorentz.*
  Khallen-Lehman representation? Not every spectral density is ok

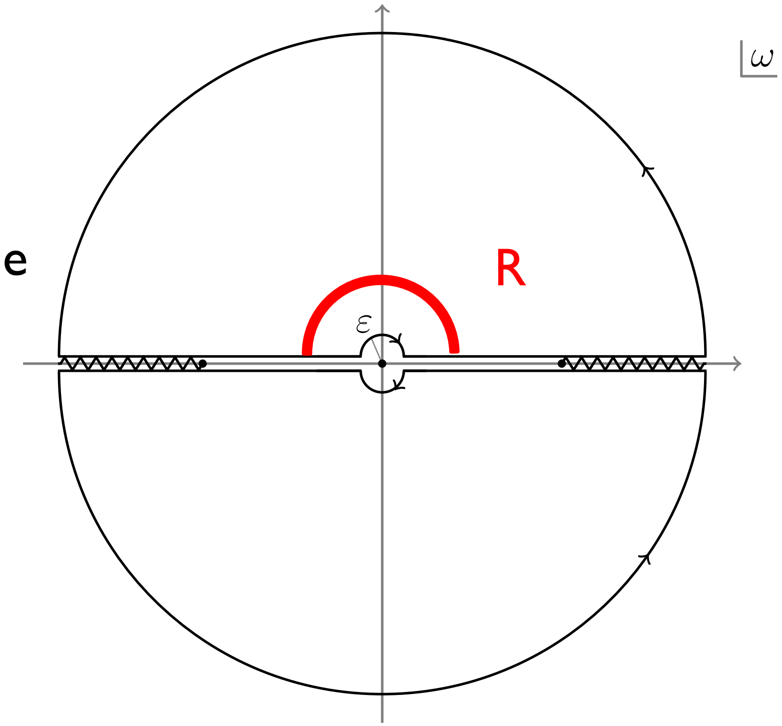
- Back to S-matrix ?
  Weakly gauge U(1) and look at π A —> π A

Backup slides

Loops?

Loops open a cut all along the real axis.
Can be treated using a countour in upper half plane

Bellazzini, Elias-Miró, Rattazzi, Riembau, Riva 20



In the particular case of 3d conformal superfluid

$$\mathcal{L} = \frac{\mu c_1}{2}\left[\dot\pi^2 - \frac{1}{2}(\partial_i\pi)^2\right] + c_1\dot\pi^3 + \frac{c_1}{\mu}\dot\pi^4 + \ldots \frac{c_{2;3}}{\mu}\left[\partial^2\pi\partial^2\pi + \partial^2\pi\partial^2\pi\dot\pi + \ldots\right] +$$
$$+ \frac{c_{4;5;6}}{\mu^3}\left[\partial^3\pi\partial^3\pi + \ldots\right]$$
$$= \frac{1}{2}\left[\dot{\pi_c}^2 - \frac{1}{2}(\partial_i\pi_c)^2\right] + \frac{1}{c_1^{1/2}\mu^{3/2}}\dot\pi_c^3 + \frac{1}{c_1\mu^3}\dot\pi_c^4 + \frac{c_{2;3}}{c_1\mu^2}\partial^2\pi_c\partial^2\pi_c + \frac{c_{2;3}}{c_1^{3/2}\mu^3}\partial^2\pi_c\partial^2\pi_c\dot\pi_c$$
$$+ \frac{c_{4;5;6}}{c_1\mu^4}\partial^3\pi_c\partial^3\pi_c + \ldots$$

Loops do not generate μ^{-2} or μ^{-4} (they start at μ^{-6})

Positivity bounds for EM response of media

$$\boldsymbol{P}(t) = \int_{-\infty}^{+\infty} \mathrm{d}t' \, \chi_e(t-t') \boldsymbol{E}(t') \qquad \boldsymbol{P}(\omega) = \tilde{\chi}_e(\omega) \boldsymbol{E}(\omega)$$

Electric susceptibility
(Linear response in general)

$\chi_e(t)$ is retarded $\longrightarrow$ $\tilde{\chi}_e(\omega)$ is analytic in upper half plane

$\longrightarrow$ Kramers-Kronig: $\tilde{\chi}_e(\omega) = \frac{1}{i\pi} \, \mathrm{PV} \int_{\mathbb{R}} \frac{\mathrm{d}\zeta}{\zeta - \omega} \, \tilde{\chi}_e(\zeta)$

In general the response of the medium is k-dependent: $\tilde{\chi}_e(\omega, \mathbf{k})$

Impose response vanishes outside the lightcone (and convergence at infinity):

$$\tilde{\chi}_e(\omega, \mathbf{k}) = \frac{1}{i\pi} \mathrm{PV} \int \frac{\mathrm{d}\zeta}{\zeta - \omega} \tilde{\chi}_e(\zeta, \mathbf{k} + (\zeta - \omega)\boldsymbol{\xi})$$

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Two responses: longitudinal and transverse

$$\mathrm{Re} \, \tilde{\chi}_e^{L;T}(\omega, \mathbf{k}) = \frac{1}{\pi} \mathrm{PV} \int \frac{\mathrm{d}\zeta}{\zeta - \omega} \underbrace{\mathrm{Im} \, \tilde{\chi}_e^{L;T}(\zeta, \mathbf{k} + (\zeta - \omega)\boldsymbol{\xi})}_{\text{Positive}}$$

(medium in thermodynamic eq. can only absorb)

Positivity in a medium

Magnetic susceptibility

$$1-\tilde{\mu}(\omega,\mathbf{k})^{-1}=\frac{\omega^2}{k^2}\left[\tilde{\chi}_e^T(\omega,\mathbf{k})-\tilde{\chi}_e^L(\omega,\mathbf{k})\right]$$

< 0 diamagnetic
> 0 paramagnetic

$$\mathrm{Re}[\tilde{\chi}_e^L+\xi^2(1-\tilde{\mu}^{-1})](\omega,\xi\omega)=\frac{1}{\pi}\mathrm{PV}\int\frac{\mathrm{d}\zeta}{\zeta-\omega}\mathrm{Im}\,[\tilde{\chi}_e^L+\xi^2(1-\tilde{\mu}^{-1})](\zeta,\xi\zeta)$$

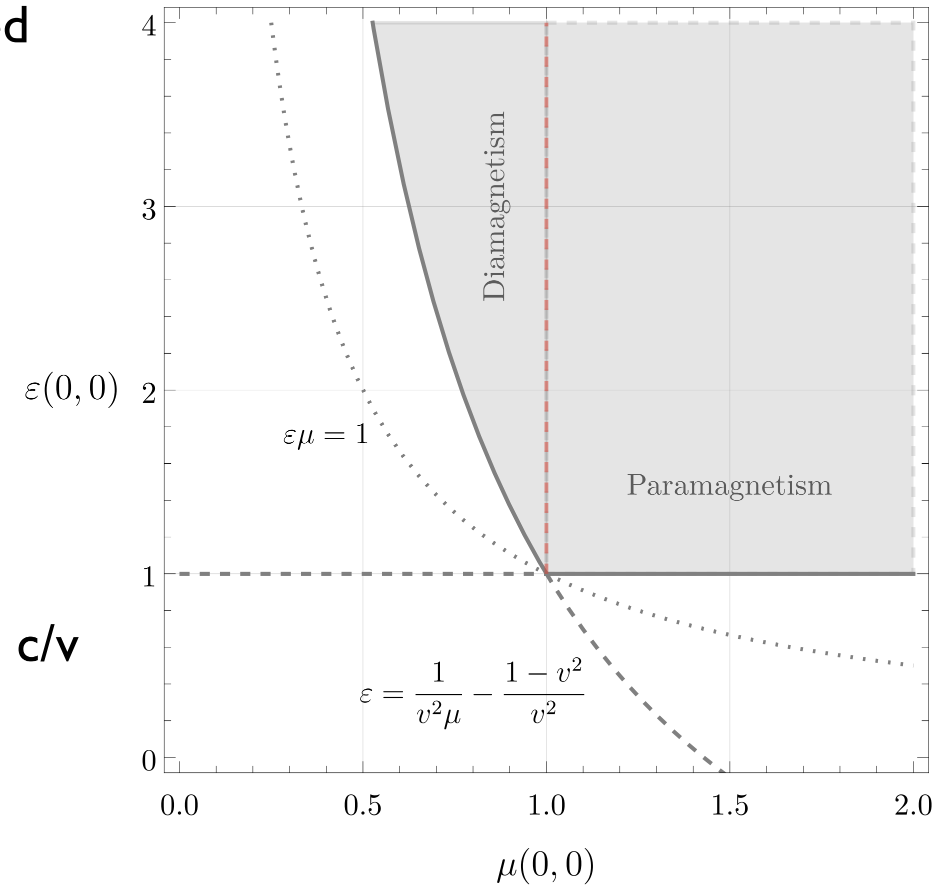
Non-trivial constraints, not fully explored

(but see Dolgov, Kirzhnits, Losyakov 82)

$$\omega\to 0\qquad \xi\to 1$$

$$\mathrm{Re}[\tilde{\chi}_e^L+(1-\tilde{\mu}^{-1})](0,0)=$$

$$\frac{1}{\pi}\int\frac{d\zeta}{\zeta}\mathrm{Im}\left[\tilde{\chi}_e^L+(1-\tilde{\mu}^{-1})\right](\zeta,\zeta)>0$$



Usually cone is much narrower than c: $\xi\to c/v$
E.g. Lieb-Robinson velocity

in progress with Janssen, Salehian, Senatore