Advanced Calculus for Precision Physics

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Informations [ACPP]

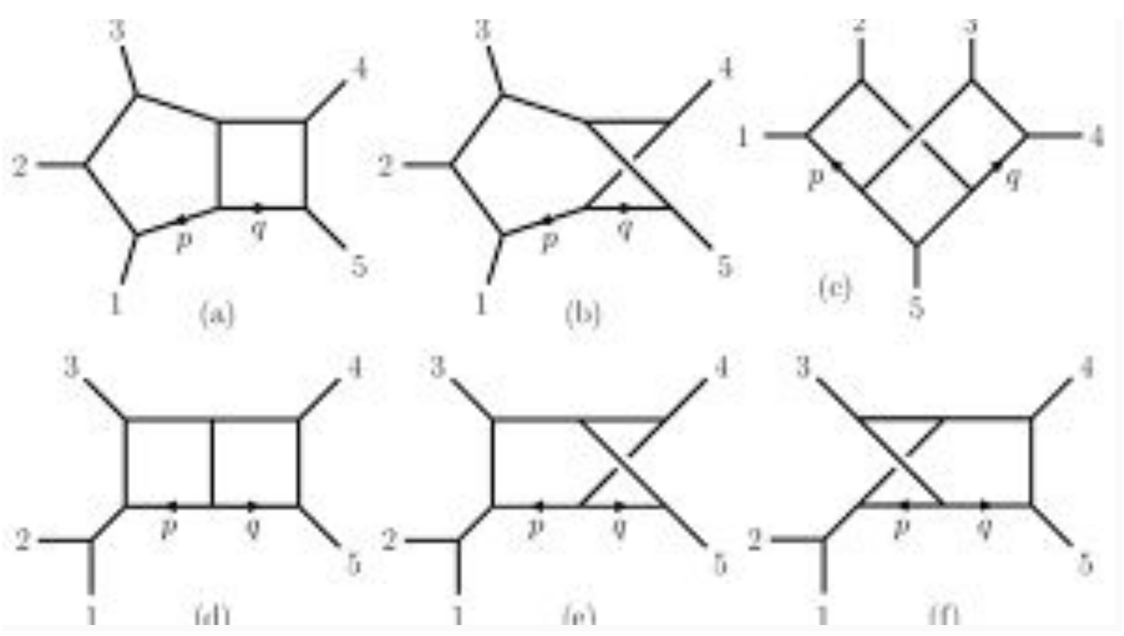
Flagship Use Cases [UC2.1.3]

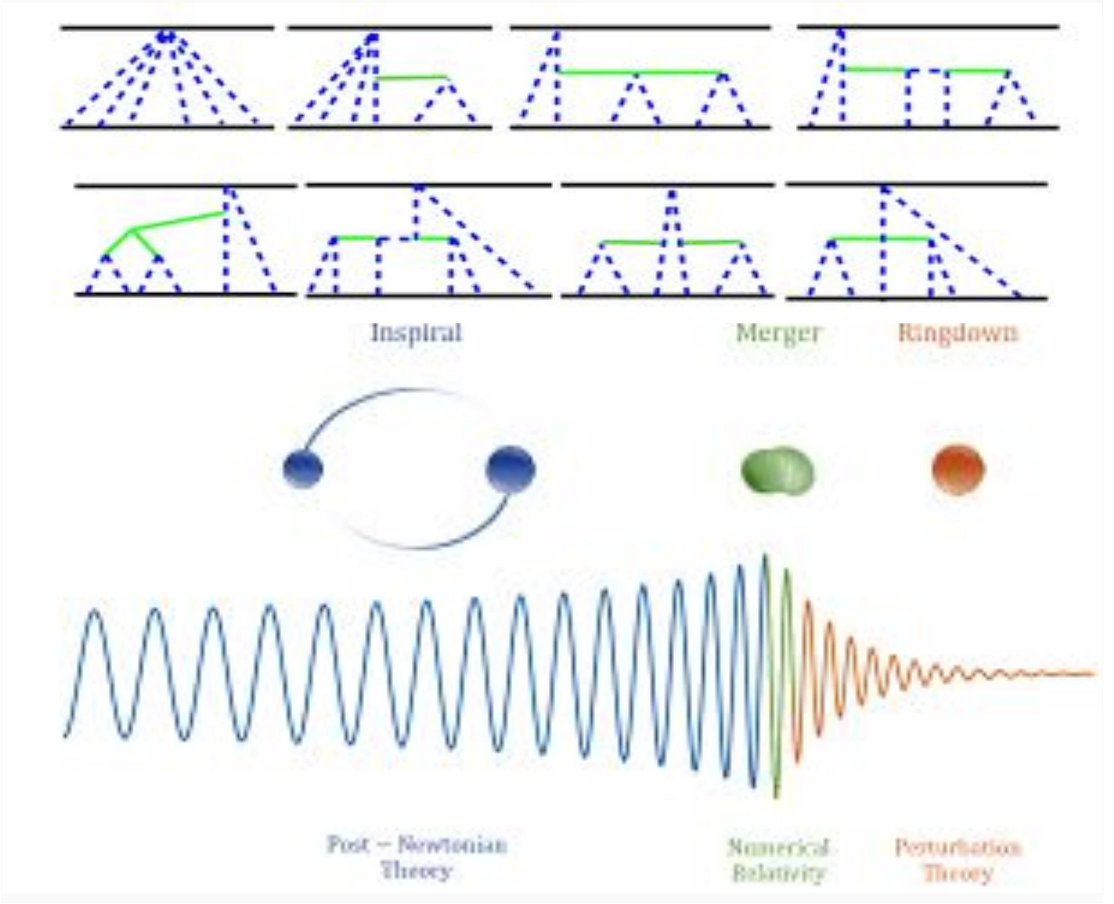
RTD-A at the University of Padova

ACPP

UNIBO, UNIPD

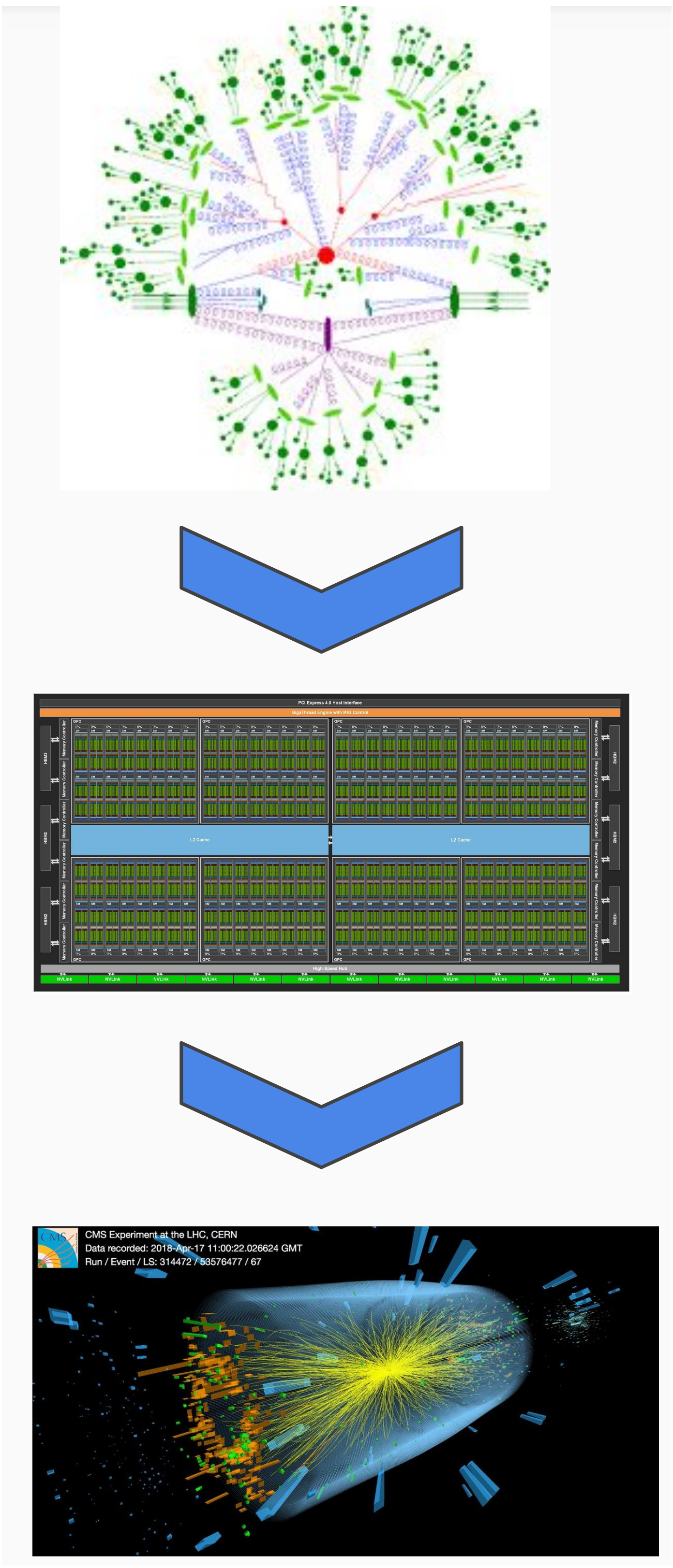
Amplitudes





UNIMIB

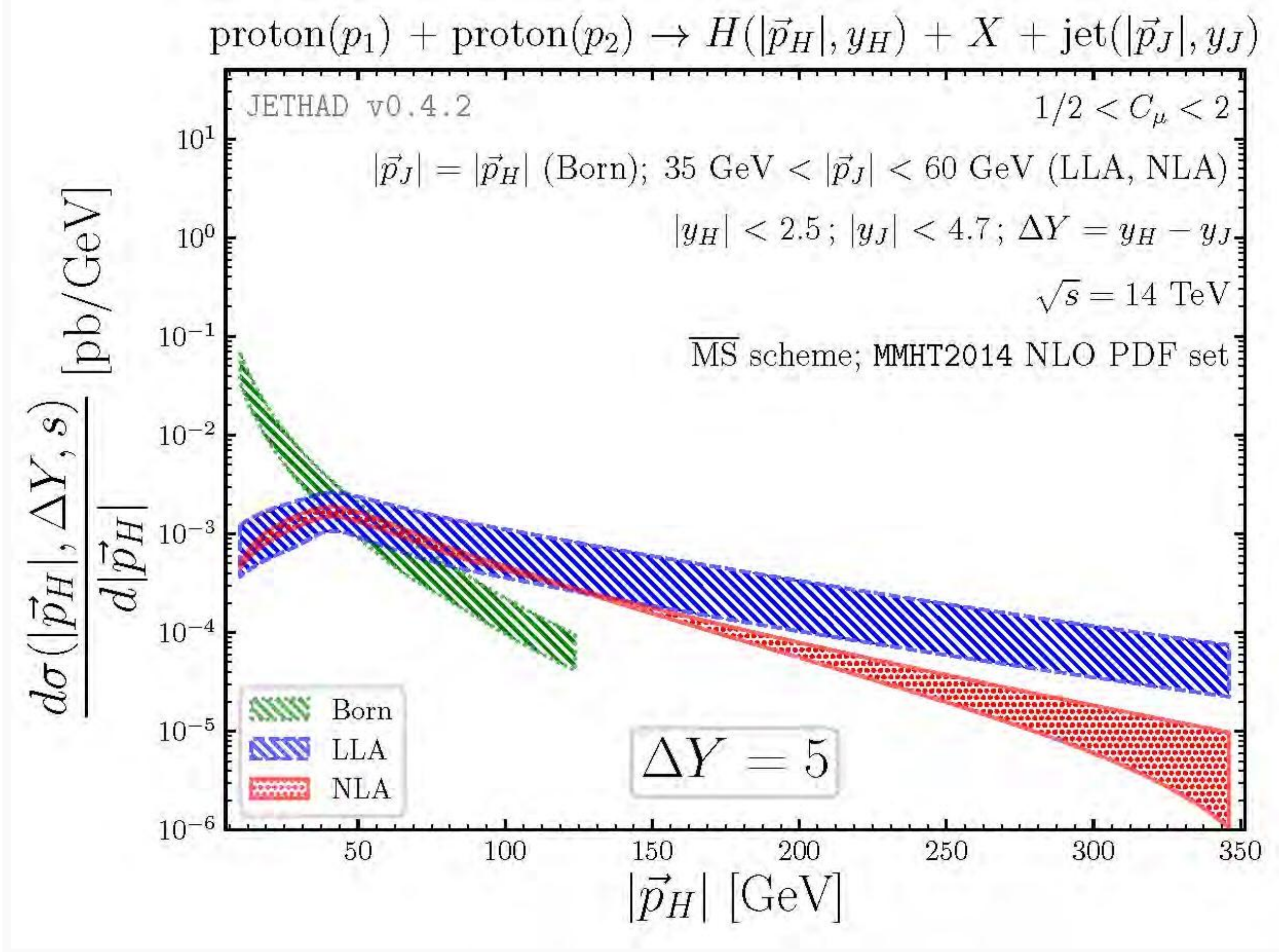
Event Generators



UNICAL

Monte Carlo Sample

Parton Distributions

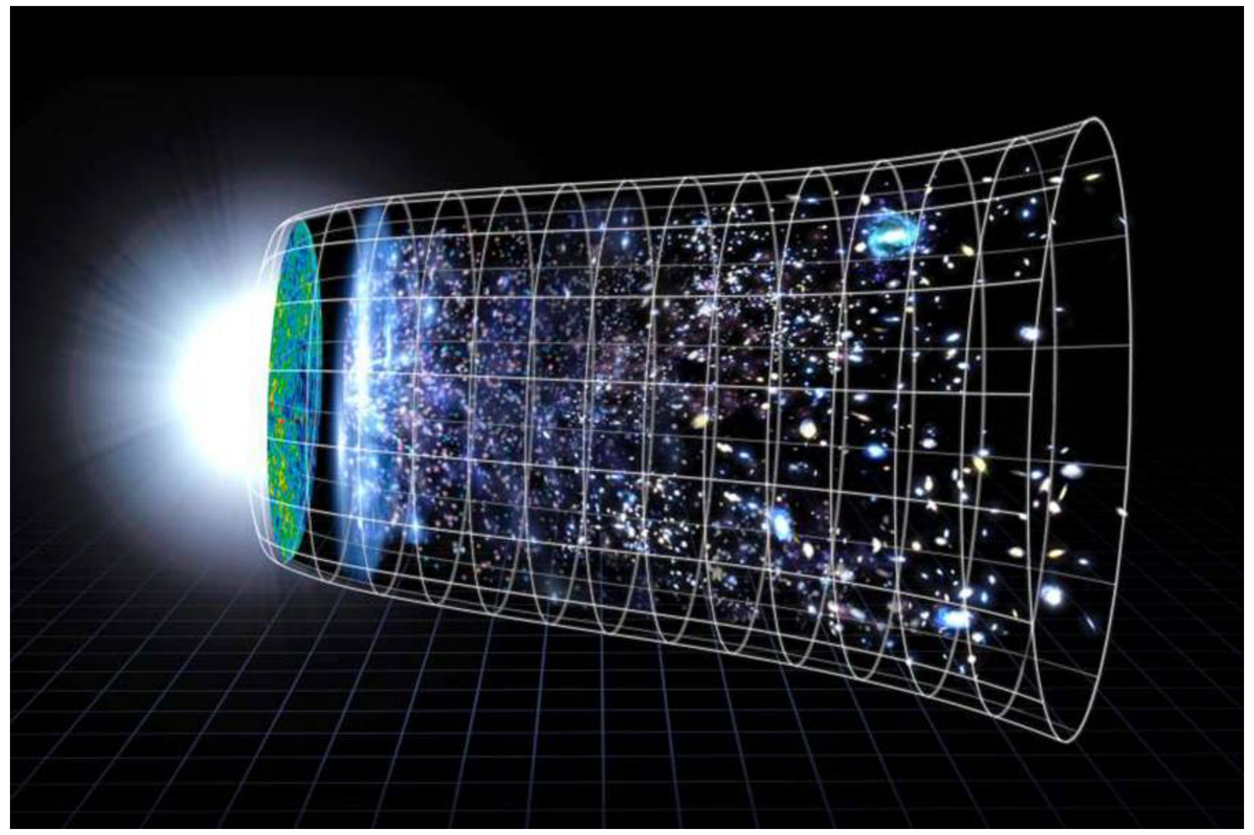


Motivation

Amplitudes

Collider Phenomenology

Gravitational Waves



Cosmology

Computation of the Loop Amplitude

TEAM LoopIn : Pierpaolo Mastrolia, Manoj K. Mandal, Tiziano Peraro, Jonathan Ronca, William J. Torres Bobadilla

Generation of Feynman Diagram

↓

Spin sums, Dirac Algebra, Trace

↓

Tensor Reduction

↓

Integral-By-Parts Reduction

↓

Evaluation of Master Integrals

Mathematica Based Package LoopIn

Tensor Reduction →

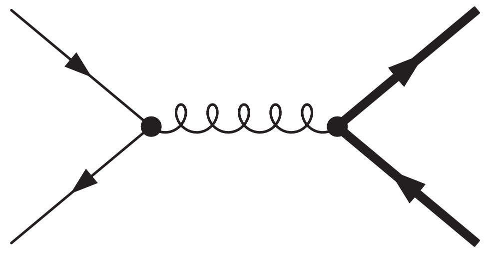
$$\mathcal{M}_{\mathrm{b}}^{(n)} = (S_\epsilon)^n \int \prod_{i=1}^{n} \frac{d^d k_i}{(2\pi)^d} \sum_G \frac{1}{\prod_{\sigma \in G} D_\sigma}$$

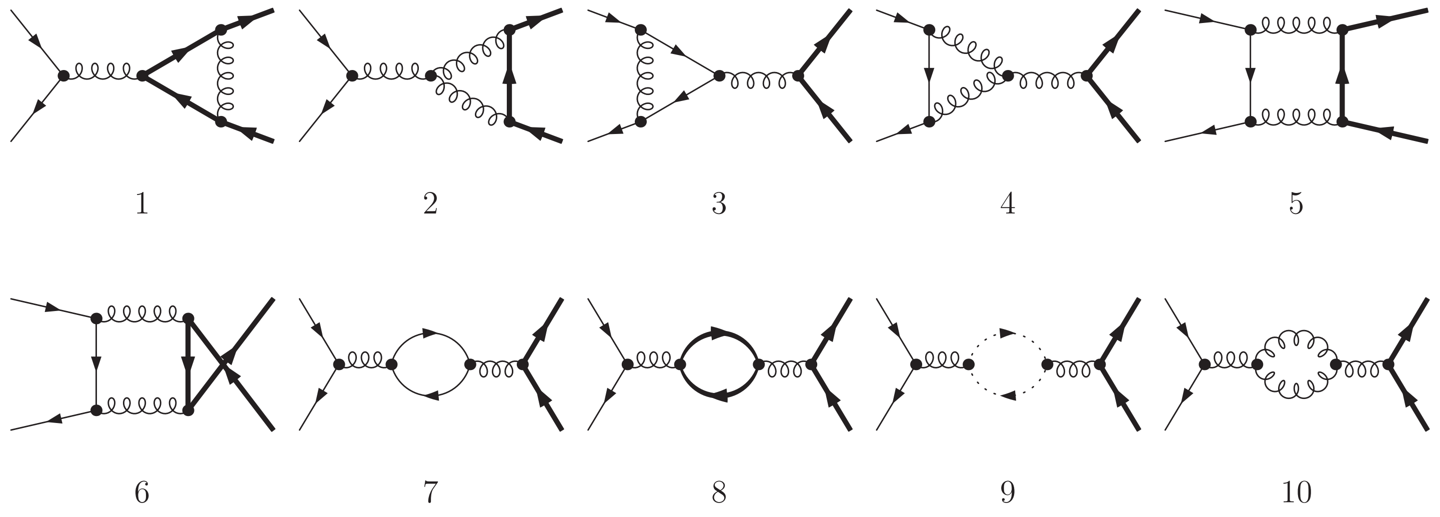
Integral-By-Parts Reduction →

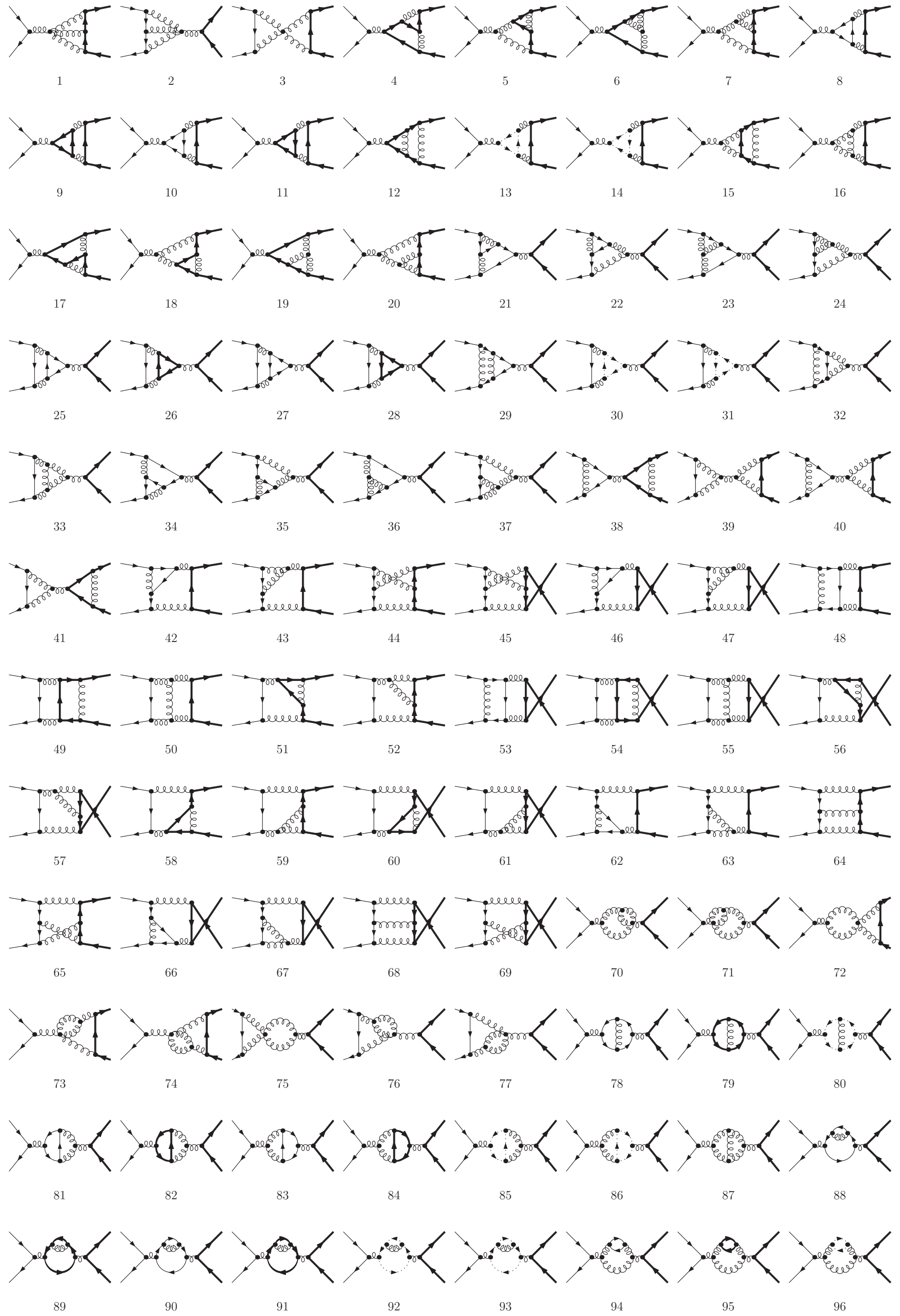
$$\mathcal{M}_{\mathrm{b}}^{(n)} = \mathbb{C}^{(n)} \cdot \mathbf{I}^{(n)}$$

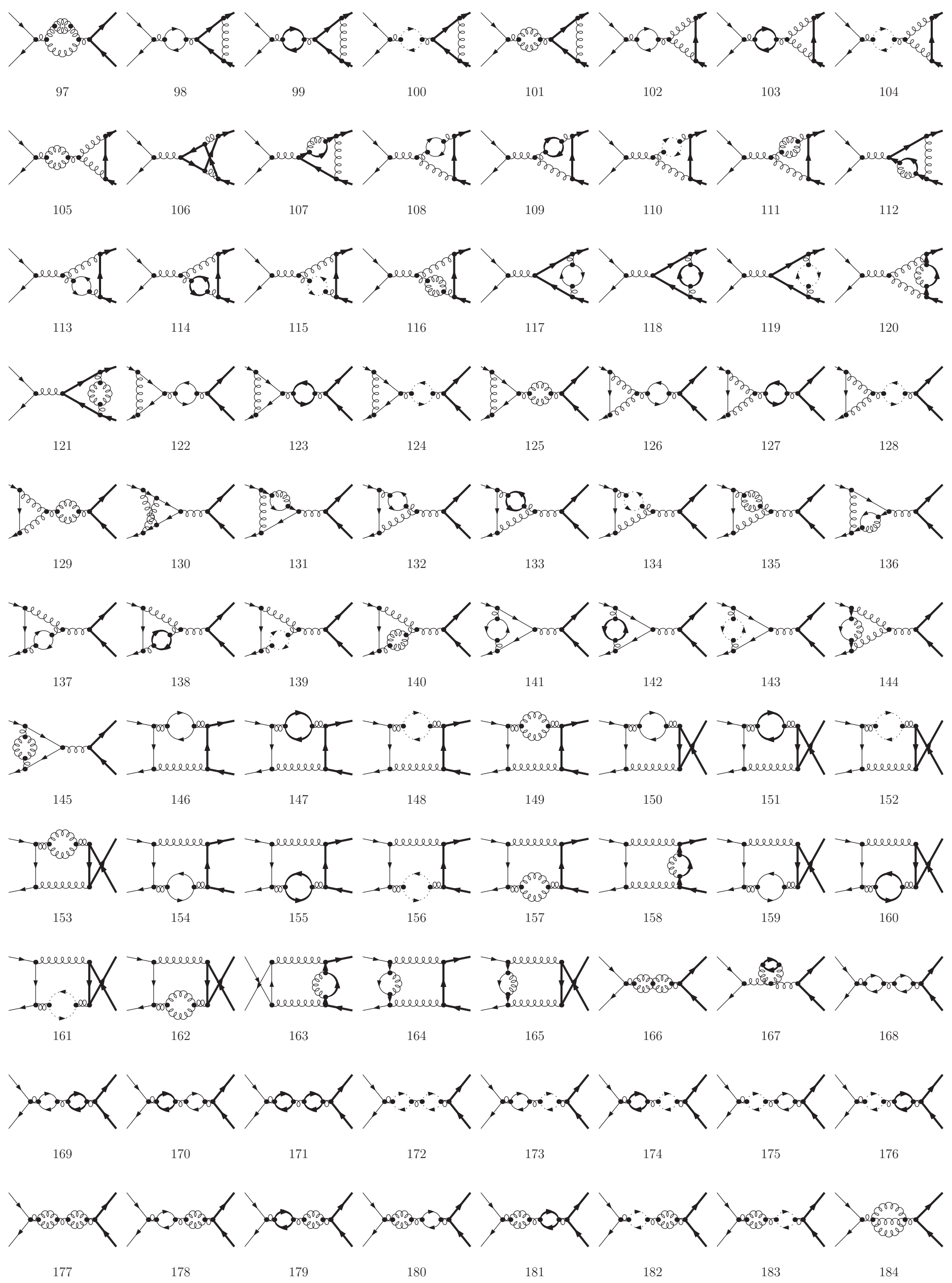
Master Integrals

Generation of Integrands









Softwares: Qgraf, FeynArts, Feyncalc

$$\mathcal{M} = \sum_i a_i I_i \qquad\qquad i = \mathcal{O}(10^5)$$

Integration-By-Parts Identity

Chetyrkin, Tkachov

$$\int \prod_{\alpha=1}^{l} d^d k_\alpha \frac{\partial}{\partial k_{j,\mu}} \left(\frac{v^\mu}{D_1^{a_1} \cdots D_N^{a_N}} \right) = 0$$

Loop momenta

Loop and external momenta

$$\int_{\alpha=1}^{l} \prod d^d k_\alpha \frac{\partial}{\partial k_{j,\mu}} \left(\frac{v^\mu}{D_1^{a_1} \cdots D_N^{a_N}} \right) = \int_{\alpha=1}^{l} \prod d^d k_\alpha \left[\frac{\partial v^\mu}{\partial k_{j,\mu}} \left(\frac{1}{D_1^{a_1} \cdots D_N^{a_N}} \right) - \sum_{j=1}^{N} \frac{a_j}{D_j} \frac{\partial D_j}{\partial k_{j,\mu}} \left(\frac{v^\mu}{D_1^{a_1} \cdots D_N^{a_N}} \right) \right]$$

$$C_1 \, I(a_1, \cdots a_N - 1) + \cdots + C_r \, I(a_1 + 1, \cdots a_N) = 0$$

- Gives relations between different scalar integrals with different exponents
- l(l+E) number of equations
- Solve the system symbolically : Recursion relations — LiteRed
- Solve for specific integer value of the exponents : Laporta Algorithm — Fire, Reduze, Kira,..

Integrand Reduction

KIRA + FIREfly

README.rst

Kira - A Feynman Integral Reduction Program

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  - 1.2 Compiling Kira with the Meson build system
  - 1.3 Compiling Kira with the Autotools build system

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FireFly

build unknown | tag v2.0.3

FireFly is a reconstruction library for rational functions written in C++.

LiteRed + FiniteFlow

README.md

LiteRed2 MATHEMATICA package

LiteRed2 is conceived as an essential update of the LiteRed package by the same author (Roman Lee). Its purpose is the IBP reduction of the multiloop diagrams.

Please, check Discussions for announcements and feedback.

README.md

FiniteFlow

A proof-of-concept implementation of the `FiniteFlow` framework.

FiniteFlow is a framework for defining and executing numerical algorithms over finite fields and reconstructing multivariate rational functions from numerical evaluations. Within this framework, complex algorithms are easily defined by combining basic building blocks into computational graphs, known as dataflow graphs. This allows to easily implement complex methods over finite fields from high-level languages and computer algebra systems, without being concerned with the low-level details of the numerical implementation. Multivariate analytic results are then reconstructed from these numerical evaluations. The algorithm sidesteps the appearance of large intermediate expressions and can be massively parallelized.

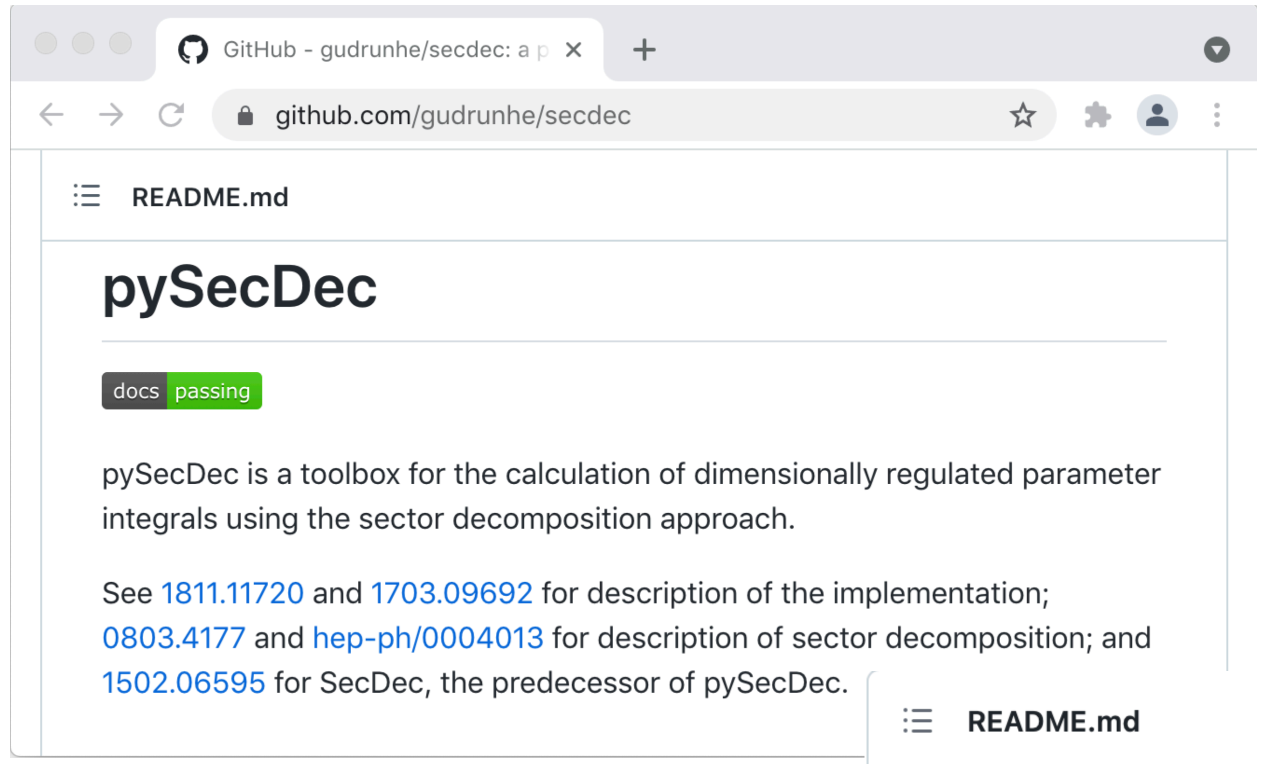
$$\mathcal{M} = \sum_i c_i J_i \qquad i = \mathcal{O}(10^2)$$

Evaluation of Integrals

Numerical Evaluation of the Master Integrals

PySecDec [Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke]

FeynTrop [Borinsky, Munch, Tellander]


GitHub - gudrunhe/secdec: a p
github.com/gudrunhe/secdec

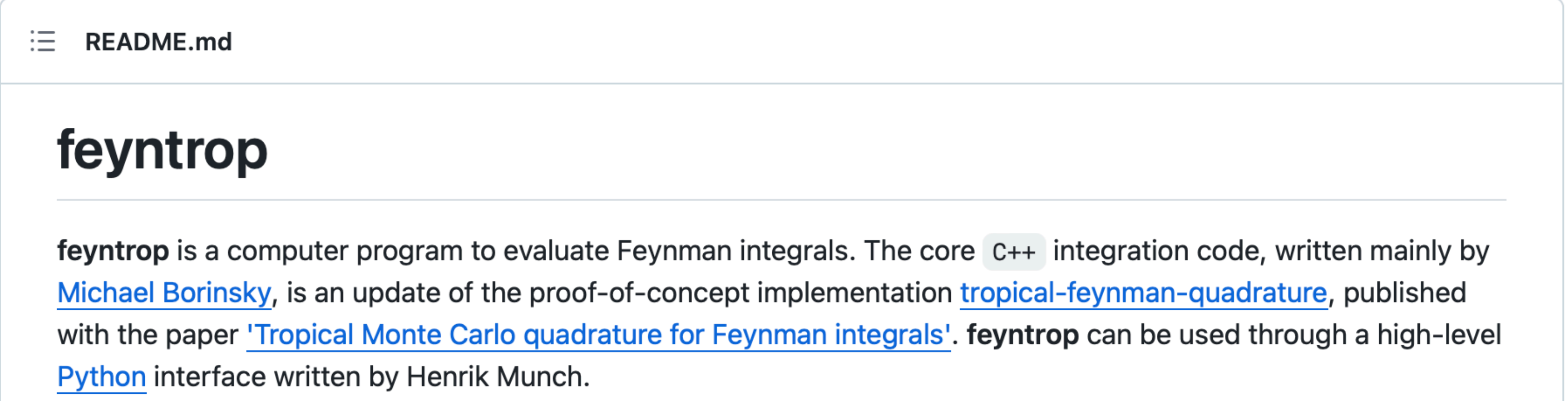
README.md

pySecDec

docs passing

pySecDec is a toolbox for the calculation of dimensionally regulated parameter integrals using the sector decomposition approach.

See 1811.11720 and 1703.09692 for description of the implementation; 0803.4177 and hep-ph/0004013 for description of sector decomposition; and 1502.06595 for SecDec, the predecessor of pySecDec.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feyntrop

feyntrop is a computer program to evaluate Feynman integrals. The core C++ integration code, written mainly by Michael Borinsky, is an update of the proof-of-concept implementation tropical-feynman-quadrature, published with the paper 'Tropical Monte Carlo quadrature for Feynman integrals'. **feyntrop** can be used through a high-level Python interface written by Henrik Munch.

DiffExp [Hidding]

SeaSyde [Armadillo, Bonciani, Devoto, Rana, Vicini]

Numerical Solution [MKM, Zhao]

AMFlow [Liu, Ma]

$$I(D;\{\nu_\alpha\};\eta) \equiv \int \prod_{i=1}^{L} \frac{\mathrm{d}^D \ell_i}{\mathrm{i}\pi^{D/2}} \prod_{\alpha=1}^{N} \frac{1}{(\mathcal{D}_\alpha + \mathrm{i}\eta)^{\nu_\alpha}}$$

$$\frac{\partial I(x;\epsilon)}{\partial x_i} = J_i(x;\epsilon) I(x;\epsilon) \qquad \frac{\partial}{\partial \eta} \vec{I}(\eta) = A(\eta)\vec{I}(\eta)$$

Opportunities at ICSC

- Our overarching objective is to build the LoopIn code, which can be potentially used for application in collider physics, Gravitational waves and cosmology

- Interfaces between different public softwares and internal routines has been done to obtain the integrand in terms of Master Integrals

- Interfaces are being built for the evaluation of Master Integrals

- Submitted the computing resources request

- ☆ Leveraging the substantial resources provided by ICSC presents an opportunity to assess the new algorithms and the efficacy

- ☆ Rigorous testing will be conducted to evaluate the stability of the code, performance across a high number of cores and for extended job durations.

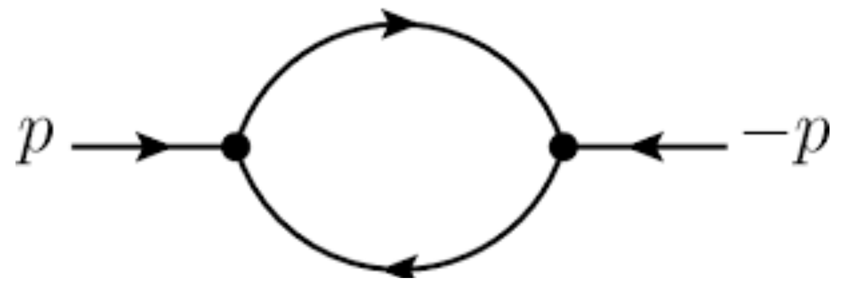
- ☆ The available resources align with the infrastructure required to produce results

Thank You

BackUp

Integration-By-Parts Identity (Example)

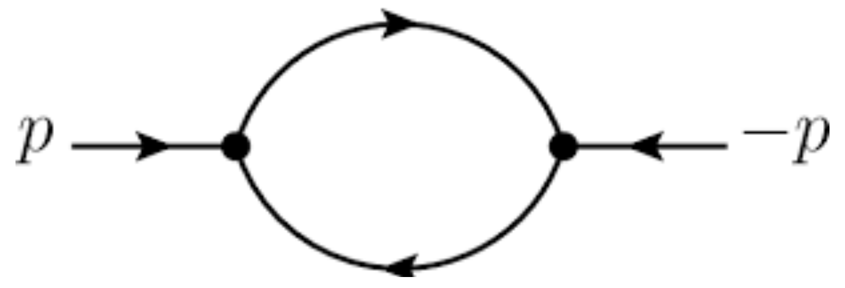
One Loop Massless Bubble



$$I(a_1, a_2) = \int \frac{d^d k_1}{(k_1^2)^{a_1}((k_1+p)^2)^{a_2}}$$

Integration-By-Parts Identity (Example)

One Loop Massless Bubble



$$I(a_1, a_2) = \int \frac{d^d k_1}{(k_1^2)^{a_1} ((k_1+p)^2)^{a_2}}$$

IBP Identity

$$I(a_1, a_2) = \frac{a_1 + a_2 - d - 1}{p^2(a_2 - 1)} I(a_1, a_2 - 1) + \frac{1}{p^2} I(a_1 - 1, a_2)$$