Tracking Improvements from Cluster Counting

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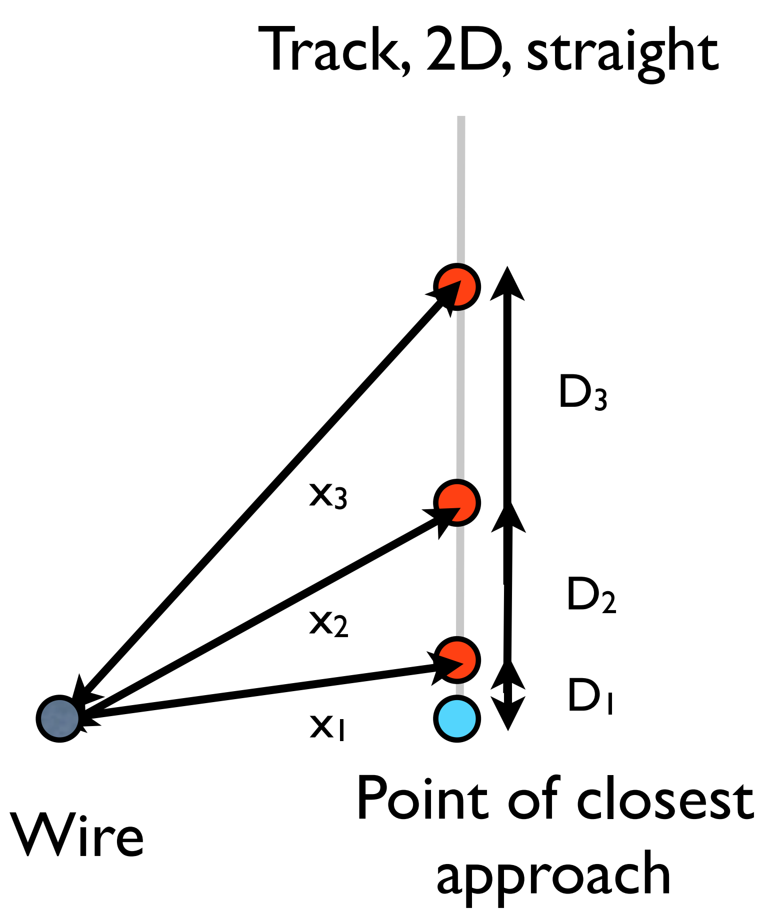
Simple Model



Infinite Cell

Probability density function (PDF) for distance of ionization event from point of closest approach.

$$f_{D_0}(D) = \delta(D)$$

$$f_{\Delta}(\Delta) = \begin{cases} \rho e^{-\rho\Delta} & \Delta \geq 0 \\ 0 & \Delta < 0 \end{cases}$$

$$D_n = D_{n-1} + \Delta$$

Convolutions

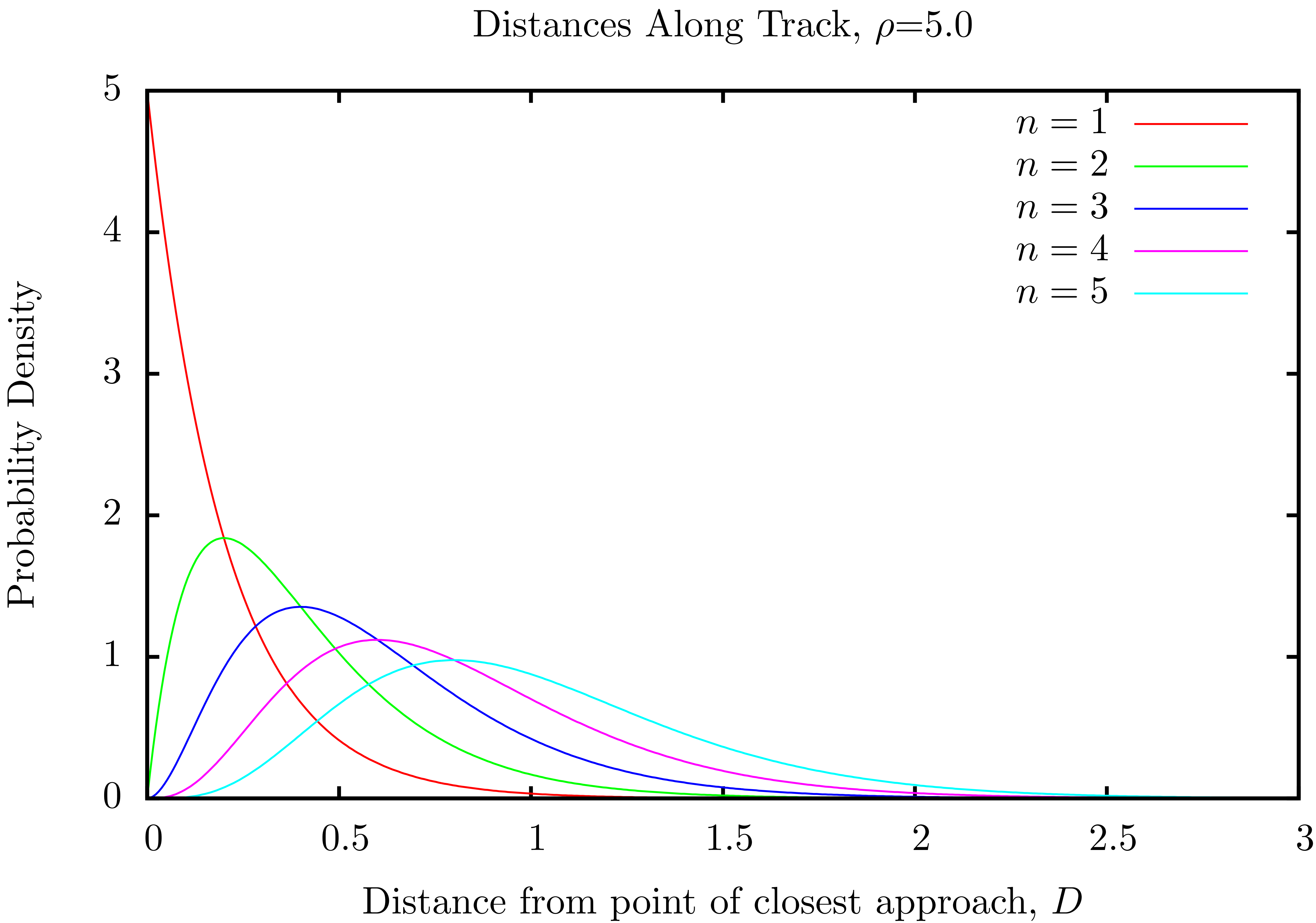
$$f_{D_n}(D) = (f_{D_{n-1}} \star f_\Delta)(D)$$

$$f_{D_1}(D) = \rho e^{-\rho D}$$

$$f_{D_2}(D) = \rho^2 D e^{-\rho D}$$

$$\vdots$$

$$f_{D_n}(D) = \rho \frac{(\rho D)^{n-1}}{(n-1)!} e^{-\rho D}$$


Distances Along Track, ρ=5.0
Probability Density
Distance from point of closest approach, D
$n = 1$
$n = 2$
$n = 3$
$n = 4$
$n = 5$
0
1
2
3
4
5
0
0.5
1
1.5
2
2.5
3

Change of Variables

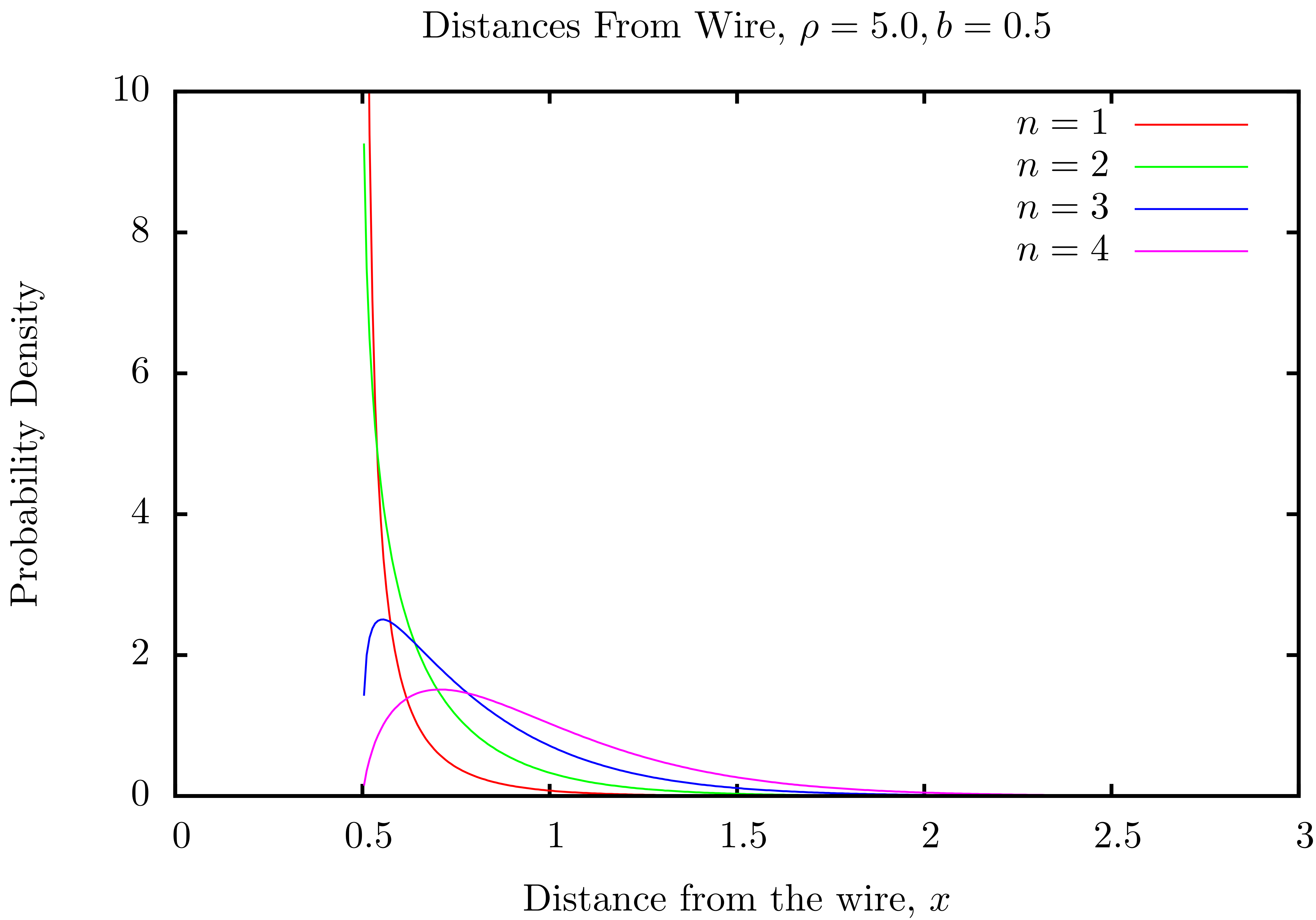
$$x_n = \sqrt{D_n^2 + b^2}$$

$$g_n(x) = \frac{\rho^n}{(n-1)!} x(x^2 - b^2)^{\frac{n-2}{2}} e^{-\rho\sqrt{x^2-b^2}}$$

$$x \to b^+$$

$$D \to 0^+$$

$$g_n(x) \to \begin{cases} \infty & n = 1 \\ \rho^2 b & n = 2 \\ 0 & n \geq 3 \end{cases}$$



Distances From Wire, $\rho = 5.0, b = 0.5$
Probability Density
Distance from the wire, x
$n = 1$
$n = 2$
$n = 3$
$n = 4$
0
2
4
6
8
10
0
0.5
1
1.5
2
2.5
3

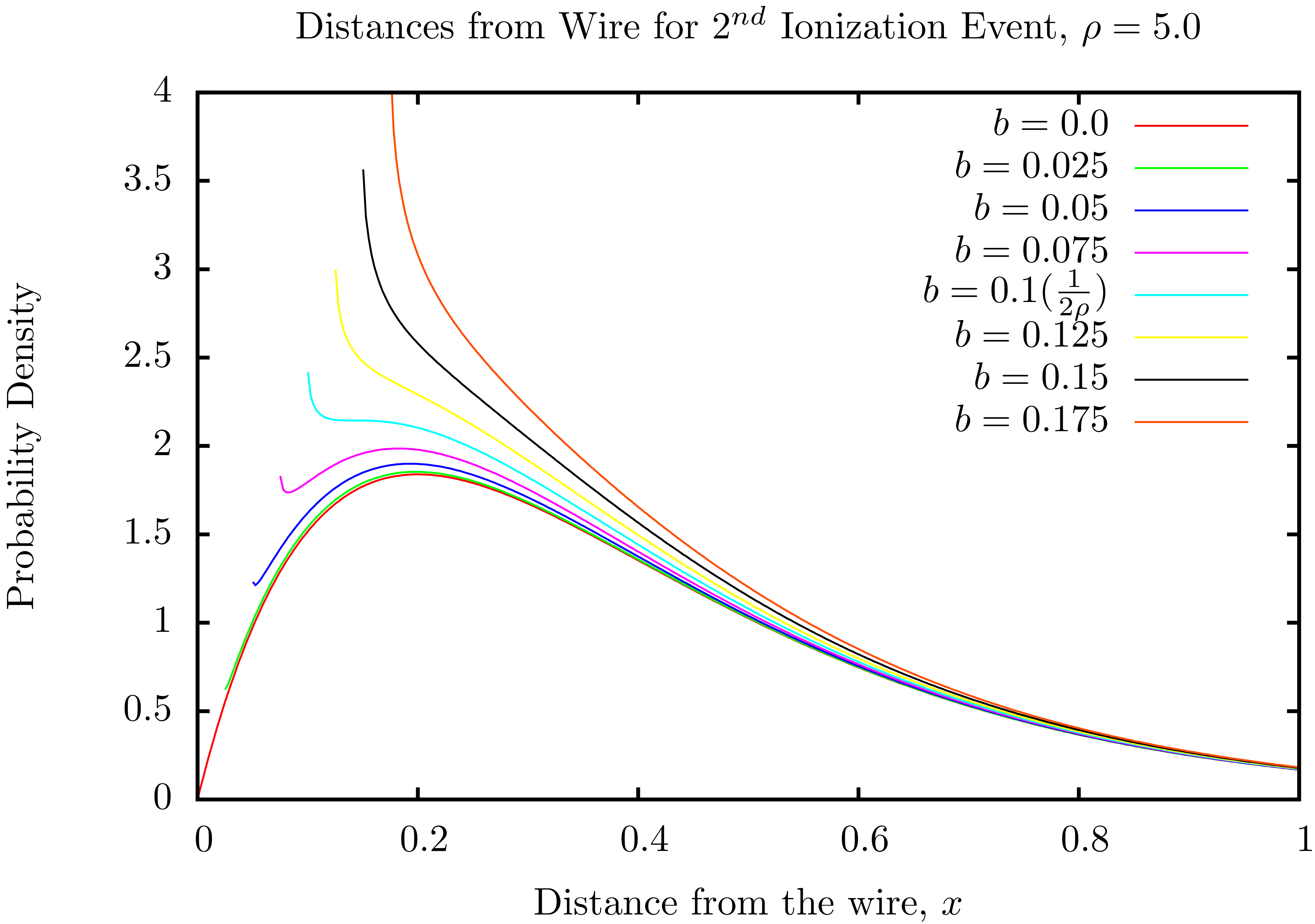
Peculiar Second Cluster

$$g_2(x) = \rho^2 x e^{-\rho\sqrt{x^2-b^2}}$$

$\rho b \geq \frac{1}{2}$ Maximum only at boundary.

$\rho b < \frac{1}{2}$ Critical points at $x_{\pm}^2 = \frac{1 \pm \sqrt{1-(2\rho b)^2}}{2\rho^2}$

The maximum is at x_+, and sometimes it is greater than the boundary maximum.



Distances from Wire for 2^{nd} Ionization Event, $\rho = 5.0$
Probability Density
Distance from the wire, x
$b = 0.0$
$b = 0.025$
$b = 0.05$
$b = 0.075$
$b = 0.1(\frac{1}{2\rho})$
$b = 0.125$
$b = 0.15$
$b = 0.175$
0
0.5
1
1.5
2
2.5
3
3.5
4
0
0.2
0.4
0.6
0.8
1

Actually...

If we already measured the first n clusters, the next one's distance from the wire actually follows:

$$g_n(x) = \frac{x_n \rho e^{-\rho(\sqrt{x_n^2 - b^2} - \sqrt{x_{n-1}^2 - b^2})}}{\sqrt{x_n^2 - b^2}}$$

Not $$g_n(x) = \frac{\rho^n}{(n-1)!} x(x^2 - b^2)^{\frac{n-2}{2}} e^{-\rho\sqrt{x^2 - b^2}}$$

with n=2.

Bayes' Theorem

Prior PDF

Likelihood

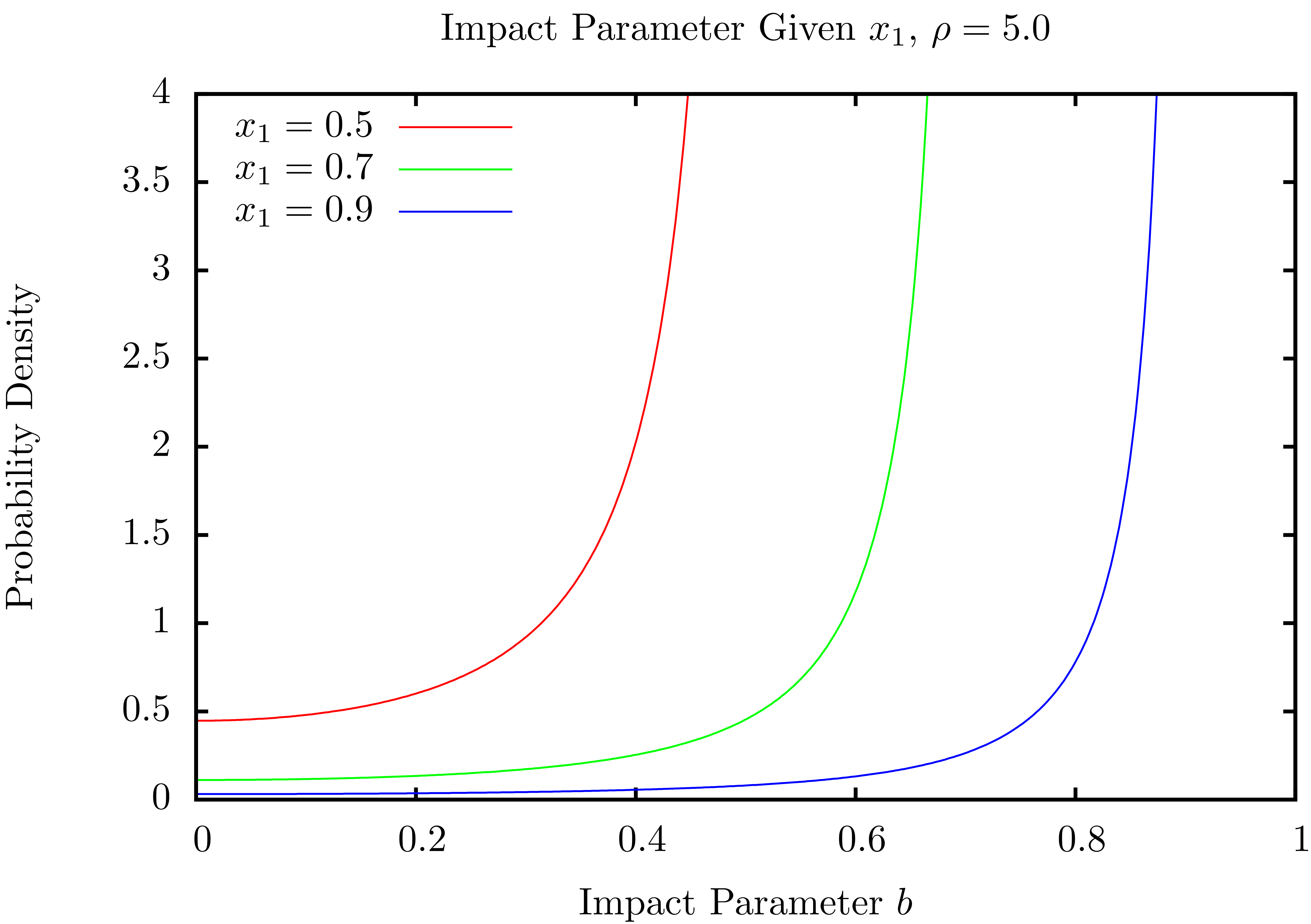
$$P(\alpha|\beta;\sigma) = \frac{P(\alpha;\sigma)P(\beta|\alpha;\sigma)}{P(\beta;\sigma)}$$

Normalization

$$P(b^2|x_1^2;\rho) = \frac{Cg_1(x_1)}{P(x_1^2;\rho)}$$

$$P(x_1^2;\rho) = \int_0^\infty Cg_1(x_1)d(b^2)$$

$$P(b^2|x_1^2;\rho) = \frac{\rho e^{-\rho\sqrt{x_1^2-b^2}}}{2\sqrt{x_1^2-b^2}(1-e^{-\rho x_1})}$$



Impact Parameter Given x_1, $\rho = 5.0$
4
3.5
3
2.5
2
1.5
1
0.5
0
Probability Density
$x_1 = 0.5$
$x_1 = 0.7$
$x_1 = 0.9$
0
0.2
0.4
0.6
0.8
1
Impact Parameter b

The Second Cluster

We just calculated that!

$$P(b^2|x_2^2;x_1^2,\rho) = \frac{P(b^2;x_1^2,\rho)P(x_2^2|b^2;x_1^2,\rho)}{P(x_2^2;x_1^2,\rho)}$$

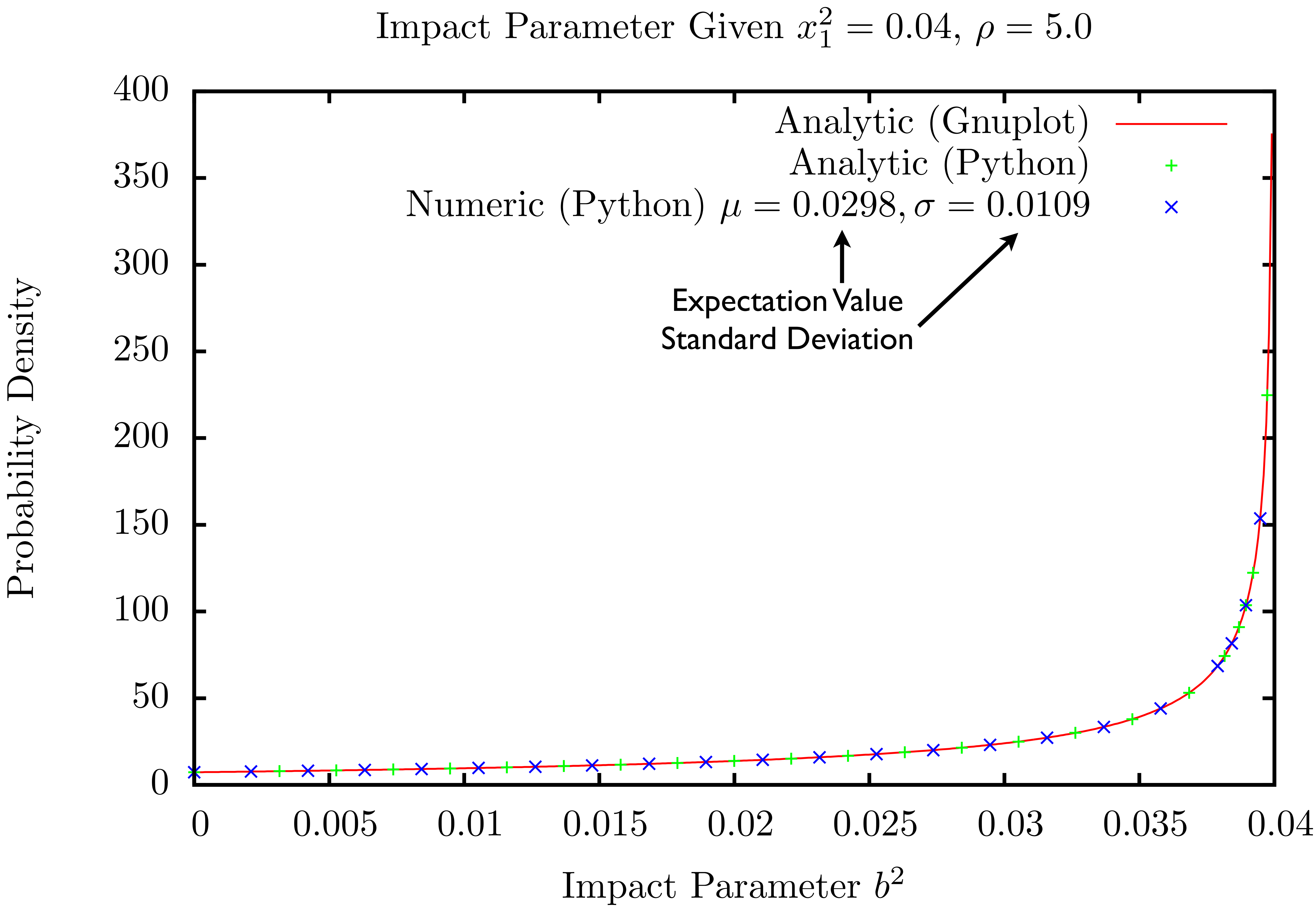
Remember, we already measured the first cluster!

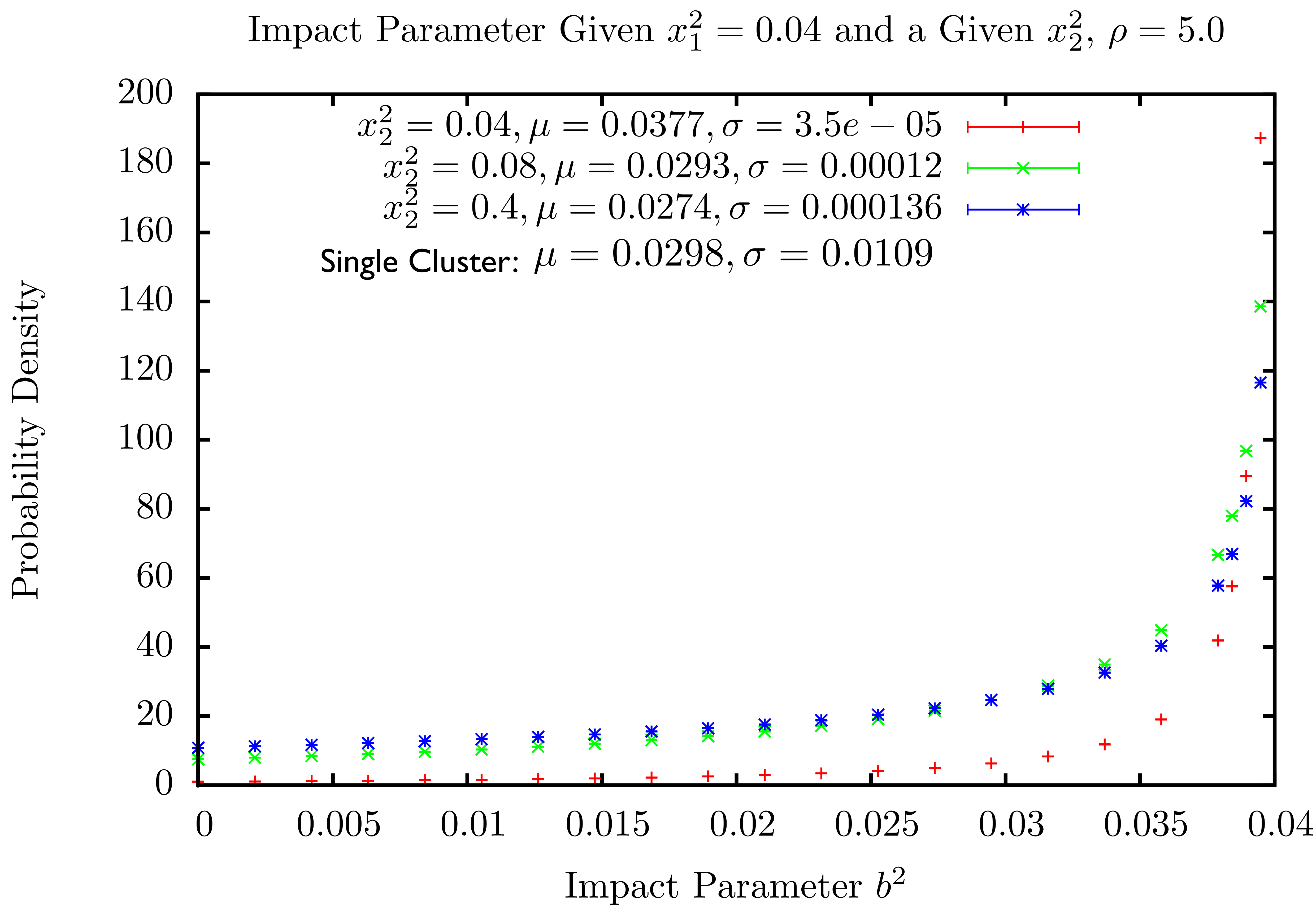
$$P(b^2|x_2^2;x_1^2,\rho) = \frac{\frac{e^{-\rho\sqrt{x_2^2-b^2}}}{\sqrt{x_1^2-b^2}\sqrt{x_2^2-b^2}}}{\int_0^{x_1^2} \frac{e^{-\rho\sqrt{x_2^2-b^2}}}{\sqrt{x_1^2-b^2}\sqrt{x_2^2-b^2}} d(b^2)}$$

Cannot be done analytically (by me)

Numerics

- Using Python, Scipy, Numpy
- Integration method is Gaussian Quadrature
- Normalizations check out
- Sanity checks passed


Impact Parameter Given $x_1^2 = 0.04$, $\rho = 5.0$
Probability Density
Impact Parameter b^2
Analytic (Gnuplot)
Analytic (Python)
Numeric (Python) $\mu = 0.0298, \sigma = 0.0109$
Expectation Value
Standard Deviation
0
50
100
150
200
250
300
350
400
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Impact Parameter Given $x_1^2 = 0.04$ and a Given x_2^2, $\rho = 5.0$
$x_2^2 = 0.04, \mu = 0.0377, \sigma = 3.5e-05$
$x_2^2 = 0.08, \mu = 0.0293, \sigma = 0.00012$
$x_2^2 = 0.4, \mu = 0.0274, \sigma = 0.000136$
Single Cluster: $\mu = 0.0298, \sigma = 0.0109$
Probability Density
Impact Parameter b^2
200
180
160
140
120
100
80
60
40
20
0
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

Lessons Learned

- The second cluster changes not the most likely impact parameter, but the expectation value and standard deviation
- Single-cell resolution is improved by considering multiple clusters

Next Steps

- Create a numerical recursive method for considering the first arbitrary n clusters
- Compare results to Garfield simulations to validate the model
- Use the lessons learned and real data (from proto2 at LNF) to develop a real procedure

Beyond Single-Cell & 2D

- If cluster counting is used for dE/dx, even if it is not used directly for tracking, the discriminator threshold can be set at 1 electron
- The number of clusters deposited is sensitive to the track length within the cell, possible to determine θ with single cells?
- Other approaches (throw variables into TMVA, neural net...)