Sensitivity studies for mixing and CPV in charm at ψ(3770) vs ϒ(4S)

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1st SuperB Collaboration Meeting, QMUL (UK), 2011

Outline

- General considerations
- Time dependences
- FastSim studies
- Sensitivity studies and results
- Summary and next steps

General considerations

- At Υ(4S)
  - Flavor tagged D^0 through $D^{*+} \rightarrow D^0\pi^+$ decay. Flavor mistag ≈ 0.2%
  - We denote the D* flavor tag with label *l*X
  - D^0 can be reconstructed in flavor *l*X, CP, Kπ and multibody (e.g. Ksππ) final states. Relatively high purity due to $m(D^0)$ and $\Delta m = m(D^{*+}) - m(D^0)$
  - Proper time resolution is about $\tau(D^0)/4 \approx 0.1$ ps
- At ψ(3770)
  - Coherent $D^0\bar{D}^0$ production
  - Both D mesons can be reconstructed in *l*X, CP, Kπ and Ksππ final states, with very low background
  - Flavor mistag ≈ 0.2% with eX, but ≈ 2% with μX (large μ misid @ low p)
  - Time-dependent measurements require larger CM boost compared to the Υ(4S) case to achieve time resolution, but reconstruction efficiency decreases with large CM boost. Need to determine the optimal boost value.

Double tags @ Ψ(3770)

Modes with D* tag @ Υ(4S)

	CP–	Kπ	*l*X	Ksππ
CP+	X	X	XX	X
CP–		X	XX	X
Kπ		X	XX	X
*l*X			XX	XX
Ksππ				X

Time dependences

- We have derived the time-dependence for all combination of double tags

	CP−	Kπ	*l*X	Ksππ
CP+	X	X	XX	X
CP−		X	XX	X
Kπ		X	XX	X
*l*X			XX	XX
Ksππ				X

- Complete expressions
- Simplified expressions with CPT invariance, CP conserved in decay, and second order in x, y

Example: flavor tag

At ψ(3770):

Identical time-dependence wrt Υ(4S) when using flavor tag !

$$\frac{d\Gamma[V_{\rm phys}(t_1,t_2)\to f_1 f_2]/dt}{e^{-\Gamma|\Delta t|}\mathcal{N}_{f_1 f_2}} =$$

$$(|a_+|^2+|a_-|^2)\cosh(y\Gamma\Delta t)+(|a_+|^2-|a_-|^2)\cos(x\Gamma\Delta t)$$
$$-2\mathcal{R}e((a_+^* a_-)\sinh(y\Gamma\Delta t)+2\mathcal{I}m(a_+^* a_-)\sin(x\Gamma\Delta t)$$

$$a_+ \equiv \bar{A}_{f_1}A_{f_2}-A_{f_1}\bar{A}_{f_2},$$
$$a_- \equiv -\sqrt{1-\mathrm{z}^2}\left(\frac{q}{p}\bar{A}_{f_1}\bar{A}_{f_2}-\frac{p}{q}A_{f_1}A_{f_2}\right)+\mathrm{z}\left(\bar{A}_{f_1}A_{f_2}+A_{f_1}\bar{A}_{f_2}\right)$$

z = CPT violation parameter
q, p = indirect CP violation parameters

At Υ(4S) using D^{*+} tagged events:

$$\frac{d\Gamma[M^0_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} =$$

$$(|A_f|^2+|(q/p)\bar{A}_f|^2)\cosh(y\Gamma t)+(|A_f|^2-|(q/p)\bar{A}_f|^2)\cos(x\Gamma t)$$
$$+2\mathcal{R}e((q/p)A_f^*\bar{A}_f)\sinh(y\Gamma t)-2\mathcal{I}m((q/p)A_f^*\bar{A}_f)\sin(x\Gamma t)$$

Example: double Kπ and *l*X tags

Double $K^{\mp}\pi^{\pm}$ decays

$$R_{odd}(K^-\pi^+, K^-\pi^+; \Delta t) = |A_{K^-\pi^+}|^4 \left|\frac{p}{q}\right|^2 \left[1 + \left|\frac{q}{p}\right|^4 R_D^2 - 2R_D \left|\frac{q}{p}\right|^2 \cos[2(\delta_{K\pi} - \phi)]\right] \frac{x^2+y^2}{2}(\Gamma\Delta t)^2$$

$$R_{odd}(K^+\pi^-, K^+\pi^-; \Delta t) = |A_{K^+\pi^-}|^4 \left|\frac{q}{p}\right|^2 \left[1 + \left|\frac{p}{q}\right|^4 R_D^2 - 2R_D \left|\frac{p}{q}\right|^2 \cos[2(\delta_{K\pi} + \phi)]\right] \frac{x^2+y^2}{2}(\Gamma\Delta t)^2$$

Double semileptonic decays

$$R_{odd}(l^+X^-, l^+X^-; \Delta t) = |A_{l^+X^-}|^4 \left|\frac{p}{q}\right|^2 \frac{x^2+y^2}{2}(\Gamma\Delta t)^2$$

$$R_{odd}(l^-X^+, l^-X^+; \Delta t) = |A_{l^-X^+}|^4 \left|\frac{q}{p}\right|^2 \frac{x^2+y^2}{2}(\Gamma\Delta t)^2$$

Example: Kπ vs CP tag

$K^{\mp}\pi^{\pm}$ **decays with** CP **tag**

$$
\begin{aligned}
&R_{odd}(S_\eta, K^-\pi^+;\Delta t) = \left|A_{S_\eta} A_{K^-\pi^+}\right|^2 \left\{ 2\left(1 + 2\eta\sqrt{R_D}\cos\delta_{K\pi} + R_D\right) \right. \\
&+\left[\left(\eta\left|\frac{p}{q}\right|\cos\phi + \sqrt{R_D}\cos(\delta_{K\pi}-\phi)\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) + R_D\left|\frac{q}{p}\right|\cos\phi\right) y \right. \\
&+\left. \left(-\eta\left|\frac{p}{q}\right|\sin\phi + \sqrt{R_D}\sin(\delta_{K\pi}-\phi)\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) + R_D\left|\frac{q}{p}\right|\sin\phi\right) x \right](\Gamma\Delta t) \\
&+\frac{1}{2}\left[\left(\left(1 + \left|\frac{p}{q}\right|^2\right) + 2\eta\sqrt{R_D}\left(\cos\delta_{K\pi} + \cos(\delta_{K\pi}-2\phi)\right) + R_D\left(1 + \left|\frac{q}{p}\right|^2\right)\right) y^2 \right. \\
&- \left. \left. \left(\left(1 - \left|\frac{p}{q}\right|^2\right) + 2\eta\sqrt{R_D}\left(\cos\delta_{K\pi} - \cos(\delta_{K\pi}-2\phi)\right) + R_D\left(1 - \left|\frac{q}{p}\right|^2\right)\right) x^2 \right] (\Gamma\Delta t)^2 \right\}
\end{aligned}
$$

Example: Ksππ vs CP tag

$$R_{odd}(S_\eta, K^0_S h^+ h^-; \Delta t) = \left|A_{S_\eta}\right|^2 \Big\{ 2\left(|A_f|^2 + |\bar{A}_f|^2 - 2\eta \mathcal{R}e(A^*_f \bar{A}_f)\right)$$

$$+2\Bigg[\Bigg(\left|\frac{p}{q}\right|\left(\cos\phi \mathcal{R}e(A^*_f \bar{A}_f) - \sin\phi \mathcal{I}m(A^*_f \bar{A}_f) - \eta\cos\phi |A_f|^2\right)\right) +$$

$$+\left|\frac{q}{p}\right|\left(\cos\phi \mathcal{R}e(A^*_f \bar{A}_f) - \sin\phi \mathcal{I}m(A^*_f \bar{A}_f) - \eta\cos\phi |\bar{A}_f|^2\right)\Bigg) y$$

$$-\Bigg(\left|\frac{p}{q}\right|\left(-\cos\phi \mathcal{I}m(A^*_f \bar{A}_f) - \sin\phi \mathcal{R}e(A^*_f \bar{A}_f + \eta\sin\phi |A_f|^2)\right) +$$

$$+\left|\frac{q}{p}\right|\left(\cos\phi \mathcal{I}m(A^*_f \bar{A}_f) + \sin\phi \mathcal{R}e(A^*_f \bar{A}_f) - \eta\sin\phi |\bar{A}_f|^2\right)\Bigg) x\Bigg](\Gamma\Delta t)$$

$$+\frac{1}{2}\Bigg[\left(|A_f|^2\left(1 + \left|\frac{p}{q}\right|^2\right) + |\bar{A}_f|^2\left(1 + \left|\frac{q}{p}\right|^2\right) - 4\eta\cos\phi\left(\cos\phi \mathcal{R}e(A^*_f \bar{A}_f) - \sin\phi \mathcal{I}m(A^*_f \bar{A}_f)\right)\right) y^2$$

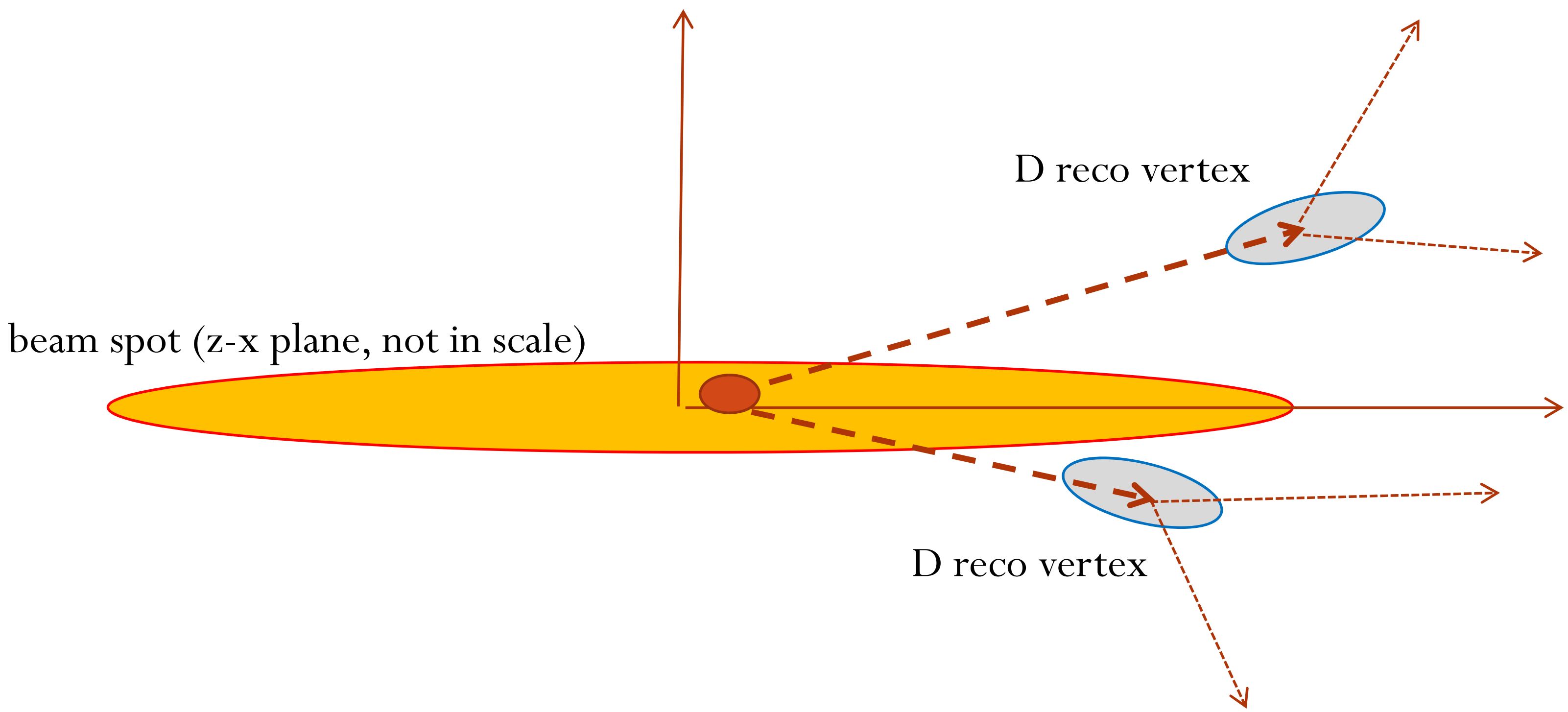
$$-\left(|A_f|^2\left(1 - \left|\frac{p}{q}\right|^2\right) + |\bar{A}_f|^2\left(1 - \left|\frac{q}{p}\right|^2\right) - 4\eta\sin\phi\left(\sin\phi \mathcal{R}e(A^*_f \bar{A}_f) + \cos\phi \mathcal{I}m(A^*_f \bar{A}_f)\right)\right) x^2\Bigg](\Gamma\Delta t)^2\Big\}$$

Example: double Ksππ tag

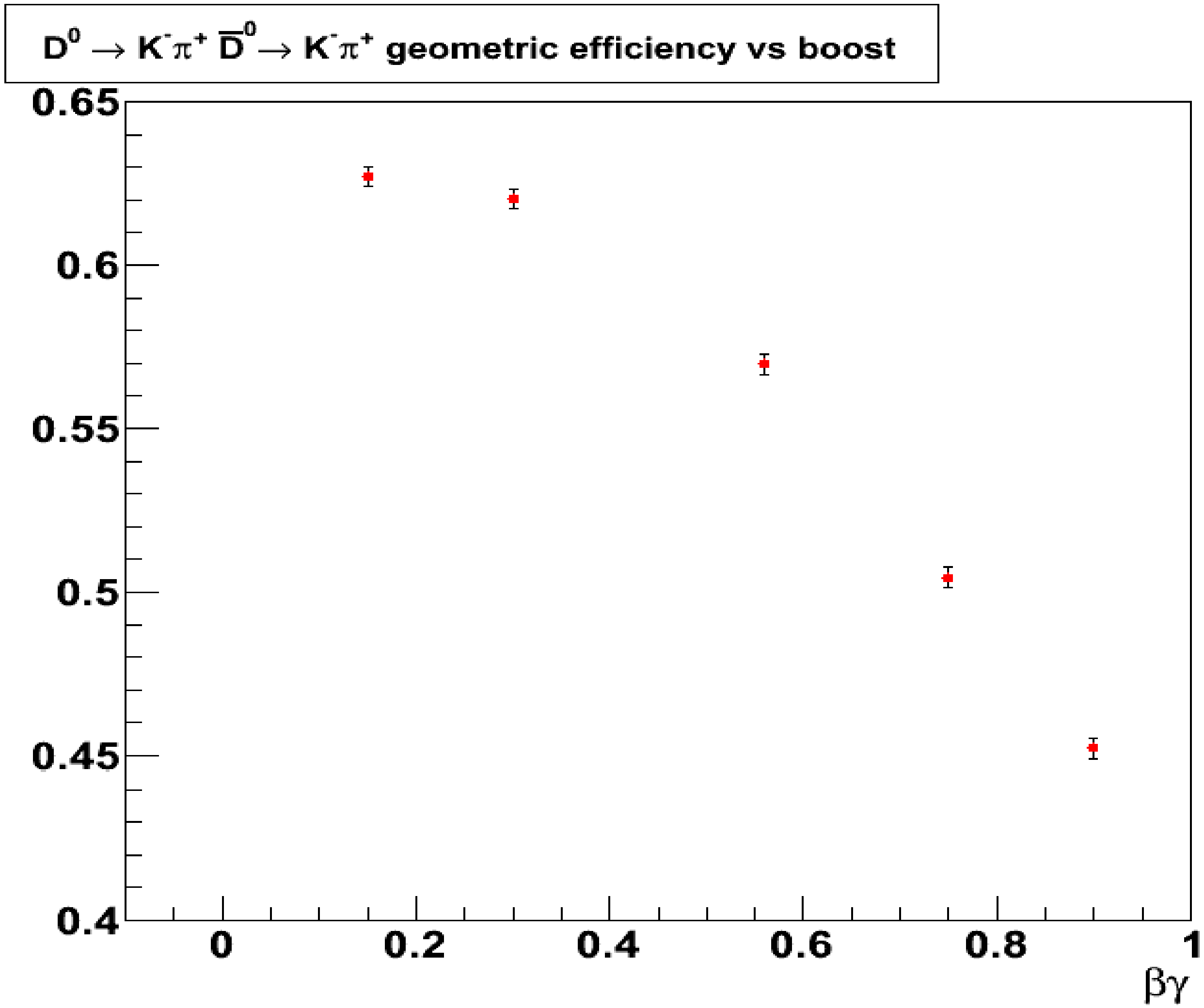
$$
\begin{aligned}
&R_{odd}(K_S^0 h^+h^-, K_S^0 h^+h^-; \Delta t) = \\
&2\left[|\bar{A}_1 A_2|^2 + |A_1 \bar{A}_2|^2 - 2\mathcal{R}e(\bar{A}_1^* A_2^* A_1 \bar{A}_2)\right] \\
&-2\Bigg\{\Bigg[|A_2|^2\left(\left|\frac{p}{q}\right|(\cos\phi\mathcal{R}e(A_1\bar{A}_1^*) + \sin\phi\mathcal{I}m(A_1\bar{A}_1^*) - \mathcal{R}e(A_1\bar{A}_1^*))\right) \\
&\qquad -|A_1|^2\left(\left|\frac{p}{q}\right|(\cos\phi\mathcal{R}e(A_2\bar{A}_2^*) + \sin\phi\mathcal{I}m(A_2\bar{A}_2^*))\right) \\
&\qquad +|\bar{A}_2|^2\left(\left|\frac{q}{p}\right|(\cos\phi\mathcal{R}e(\bar{A}_1 A_1^*) - \sin\phi\mathcal{I}m(\bar{A}_1 A_1^*))\right)\Bigg] y \\
&\quad -\Bigg[|A_2|^2\left(\left|\frac{p}{q}\right|(\cos\phi\mathcal{I}m(A_1\bar{A}_1^*) - \sin\phi\mathcal{R}e(A_1\bar{A}_1^*) - \mathcal{I}m(A_1\bar{A}_1^*))\right) \\
&\qquad -|A_1|^2\left(\left|\frac{p}{q}\right|(\cos\phi\mathcal{I}m(A_2\bar{A}_2^*) - \sin\phi\mathcal{R}e(A_2\bar{A}_2^*))\right) \\
&\qquad +|\bar{A}_2|^2\left(\left|\frac{q}{p}\right|(\cos\phi\mathcal{I}m(\bar{A}_1 A_1^*) + \sin\phi\mathcal{R}e(\bar{A}_1 A_1^*))\right)\Bigg] x\Bigg\}(\Gamma\Delta t) \\
&+\frac{1}{2}\Bigg\{\left[|\bar{A}_1 A_2|^2 + |A_1\bar{A}_2|^2 - 2\mathcal{R}e(\bar{A}_1^* A_2^* A_1\bar{A}_2)\right](y^2 - x^2) \\
&\quad +\left[\left|\frac{p}{q}\right|^2 |A_1 A_2|^2 + \left|\frac{q}{p}\right|^2 |\bar{A}_1\bar{A}_2|^2 - 2\left(\cos(2\phi)\mathcal{R}e(A_1^* A_2^* \bar{A}_1\bar{A}_2) - \sin(2\phi)\mathcal{I}m(A_1^* A_2^* \bar{A}_1\bar{A}_2)\right)\right](x^2 + y^2) \\
&\qquad \Bigg\}(\Gamma\Delta t)^2
\end{aligned}
$$

FastSim studies: Δt reconstruction

- The flight lengths of the two Ds are reconstructed through a combined beam spot constrained vertex fit
- Proper times are computed from the flight lengths and the D momenta

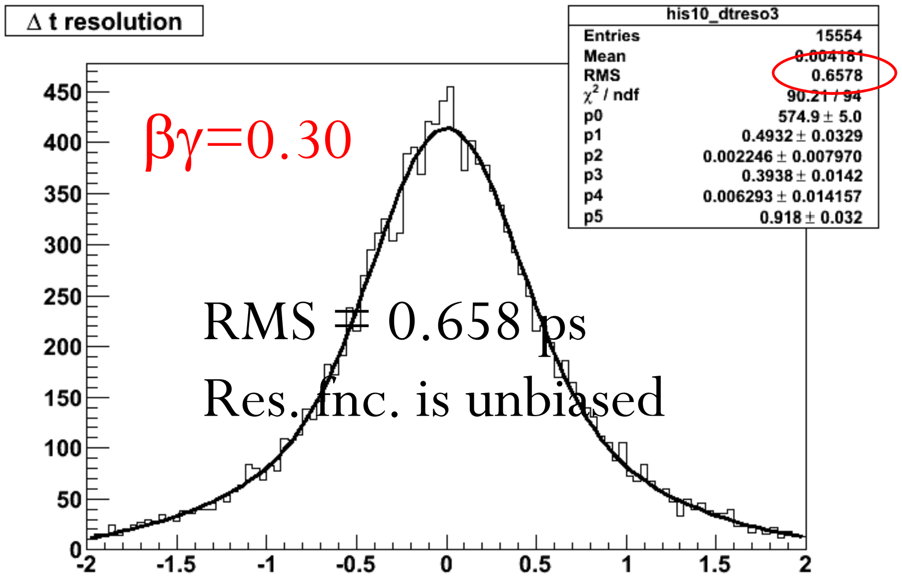


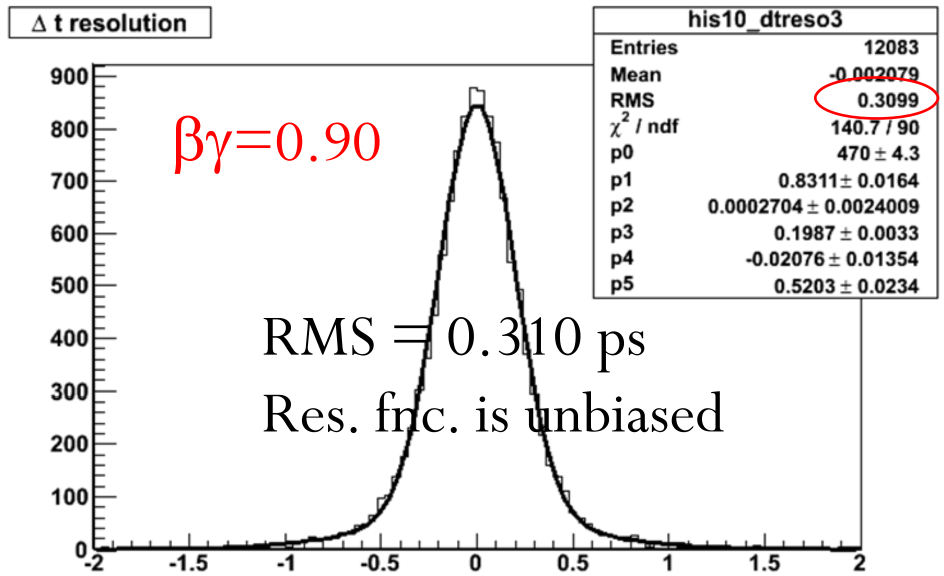
FastSim studies: ε_{geo} vs CM boost

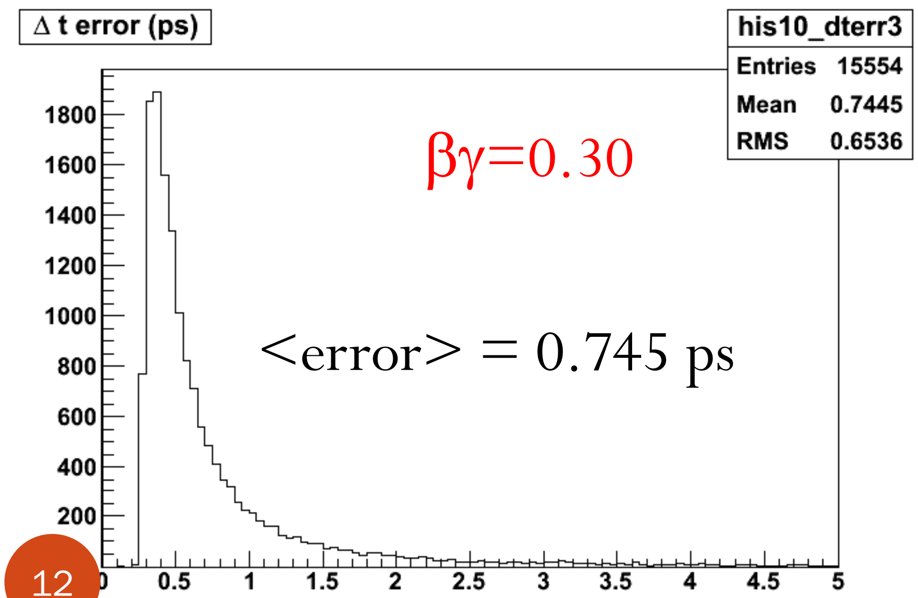


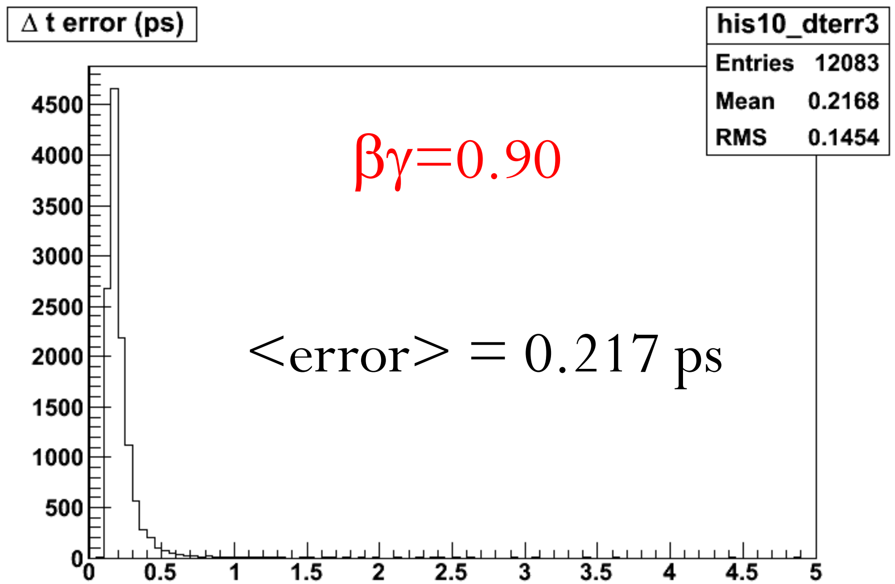
From Elba meeting

FastSim studies: Δt resolution









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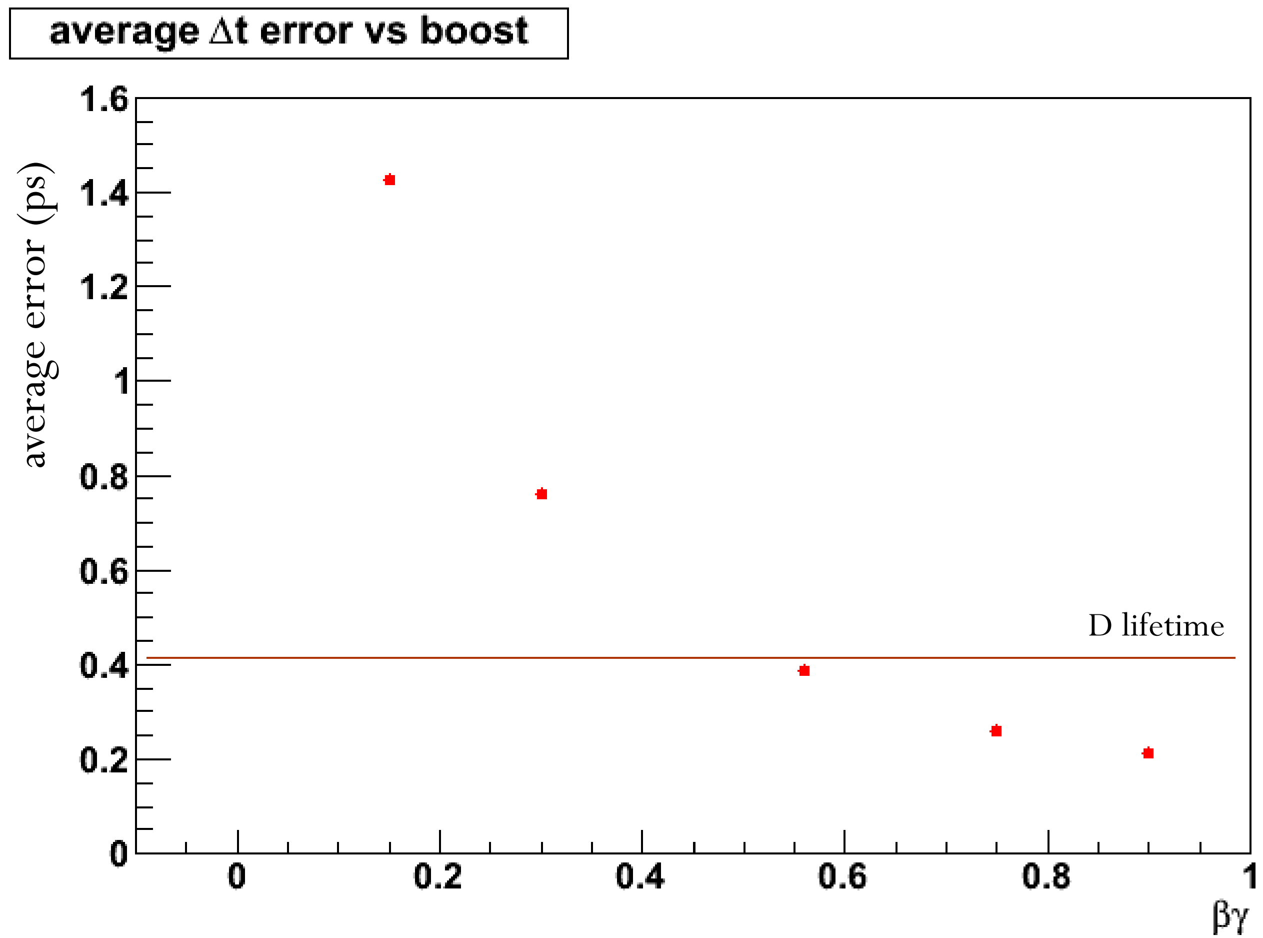
FastSim studies: Δt resolution vs CM boost



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Sensitivity studies: overview

- For ψ(3770) modes
  - Extrapolate CLEOc yields (includes cross-sections and selection efficiencies)
  - Correct by SuperB geometrical efficiency vs CM boost
  - Evaluate tripe Gaussian (TG) resolution function from FastSim vs CM boost
- For ϒ(4S) modes, extrapolate BaBar yields
  - TG proper time resolution of ~0.15 ps (0.1 ps core)
- Toy MC generator and fitter developed
  - <u>For now focus on 2-body decays</u>
- Strategy:
  - Generate *O*(100) experiments for each double tag
  - Perform combined UML fit of given ensemble of 2-body double tags, fitting simultaneously for the mixing and CPV parameters: x, y, arg(q/p), |q/p|
  - Generated values are current HFAG averages
  - Assumed CP conservation in decay
  - D→Kπ strong phase kept fixed

Double tags @ Ψ(3770)
Modes with D* tag @ ϒ(4S)
used in this study

	CP–	Kπ	*l*X
CP+	X	X	XX
CP–		X	XX
Kπ		X	XX
*l*X			XX

Sensitivity studies: expected # of events

Selected decays	$\Upsilon(4S)$ 75 ab^{-1}	LB $\psi(3770)$: $\Psi(3770)$ 0.5 ab^{-1}, $\beta\gamma = 0.238$	IB $\psi(3770)$: $\Psi(3770)$ 0.5 ab^{-1}, $\beta\gamma = 0.56$	HB $\psi(3770)$: $\Psi(3770)$ 0.5 ab^{-1}, $\beta\gamma = 0.91$
$l^{\pm}X^{\mp}$, $CP+$	19600000	569395	525890	418331
$l^{\pm}X^{\mp}$, $CP-$	30900000	685053	612430	491599
$l^{\pm}X^{\mp}$, $K^{\pm}\pi^{\mp}$	222900000	4181494	3862011	3072118
	(790000)	(13798)	(12744)	(10137)
$l^{\pm}X^{\mp}$, $K_S^0\pi^+\pi^-$	86600000	828850	689557	498370
$l^{\pm}X^{\mp}$, $l^{\mp}X^{\pm}$	85300000	1067615	986045	784370
	(50)	(51)	(47)	(38)
$K^{\mp}\pi^{\pm}$, $K^{\pm}\pi^{\mp}$	N/A	1067615	986045	784370
	(N/A)	(51)	(47)	(38)
$CP+$, $K^{\mp}\pi^{\pm}$	N/A	309608	285953	227467
$CP-$, $K^{\mp}\pi^{\pm}$	N/A	291814	260879	209408
$CP+$, $CP-$	N/A	92526	82717	66397
$CP+$, $K_S^0\pi^+\pi^-$	N/A	113691	91553	66770
$CP-$, $K_S^0\pi^+\pi^-$	N/A	115525	93030	67847
$K_S^0\pi^+\pi^-$, $K_S^0\pi^+\pi^-$	N/A	290342	217578	142875

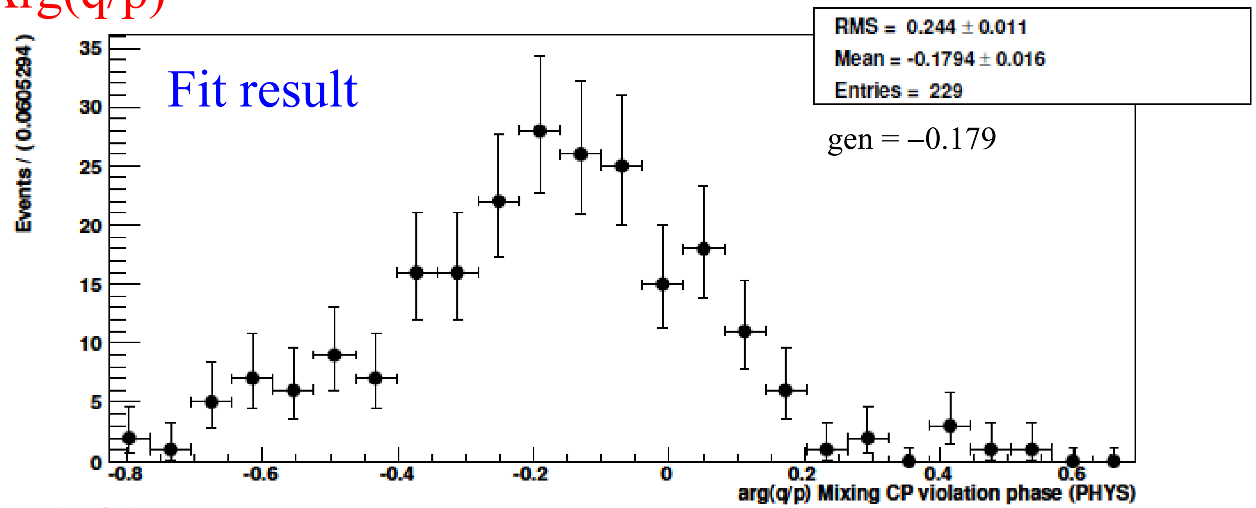
Favored # of events

Suppressed # of events

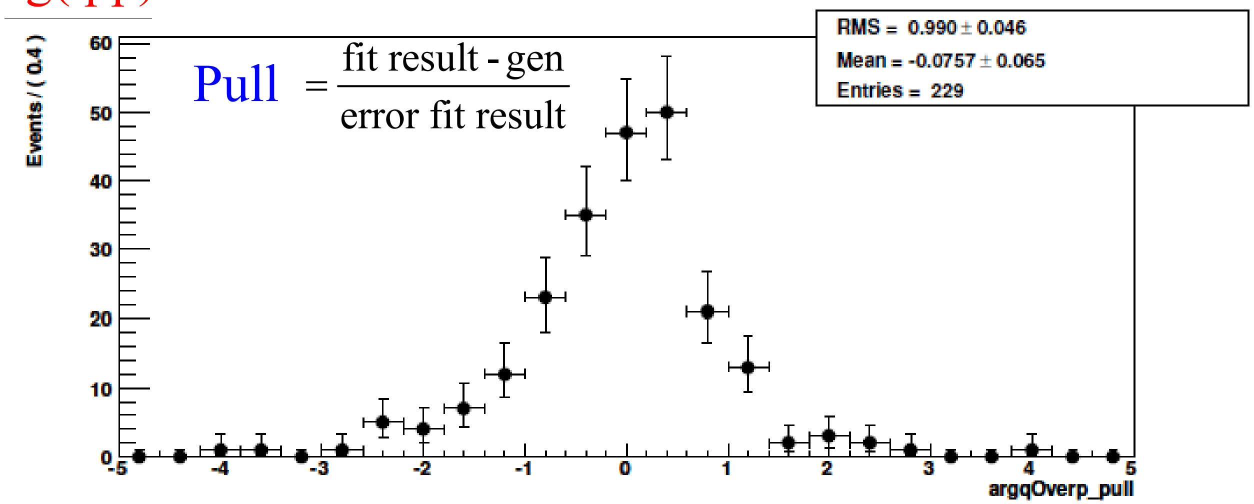
$\Upsilon(4S)$ $\psi(3770)$

Sensitivity studies: Toy MC @ ψ(3770)

Arg(q/p)



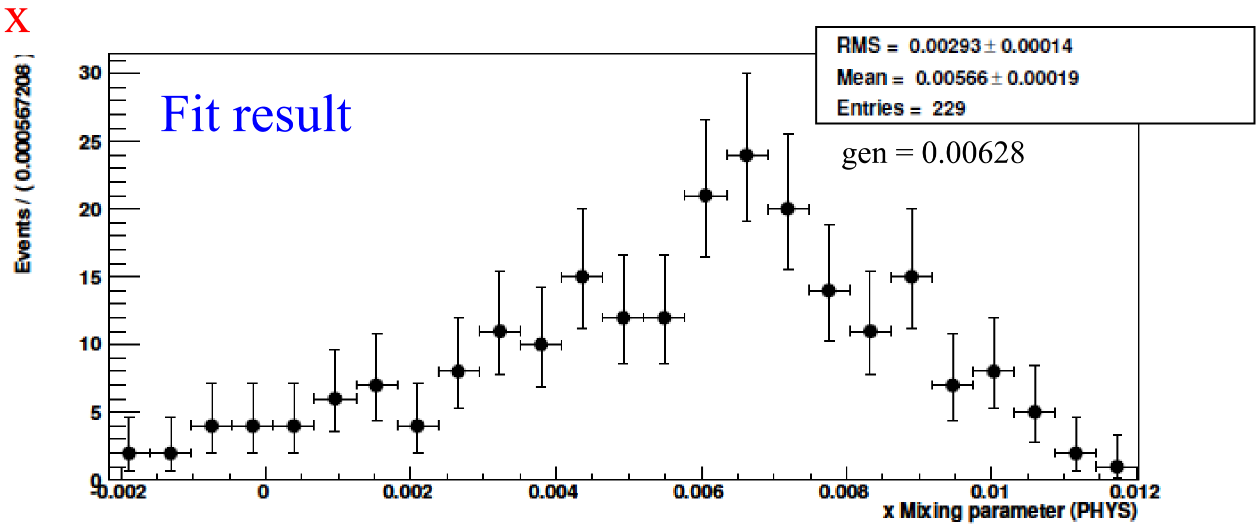
Arg(q/p)

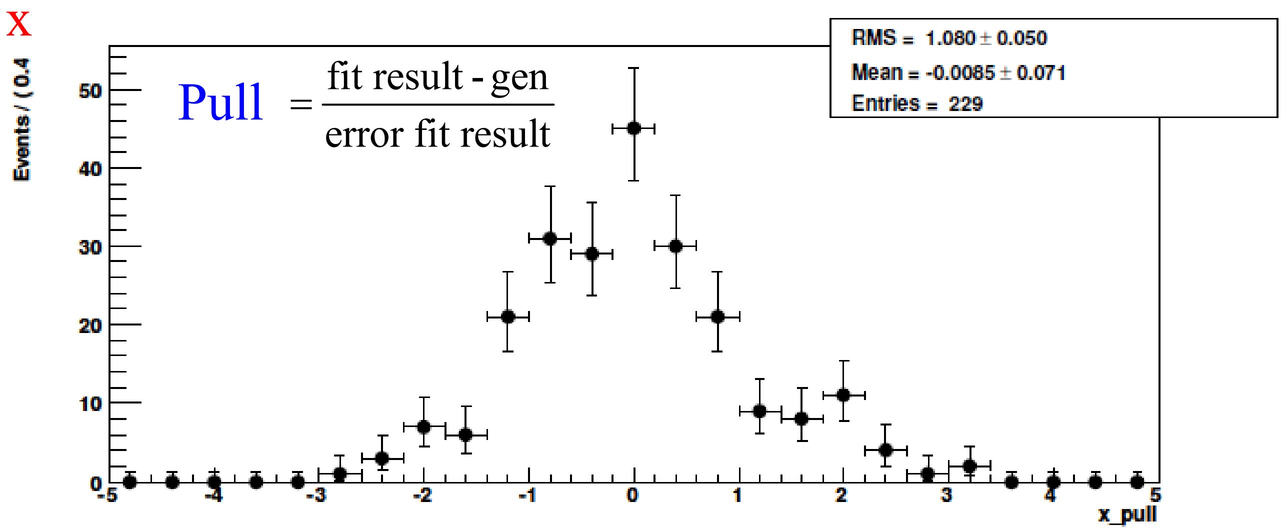


- LB Ψ(3770) , perfect resolution, no mistag

- 300 experiments generated
- ×10 less events (CPU memory limitation)
- 24% converge with error matrix not definite positive
- Understood due to the smallness of x,y while also fitting for Arg(q/p) and |q/p|. With mixing 10 times larger all fits return correct error matrix

Sensitivity studies: Toy MC @ ψ(3770)





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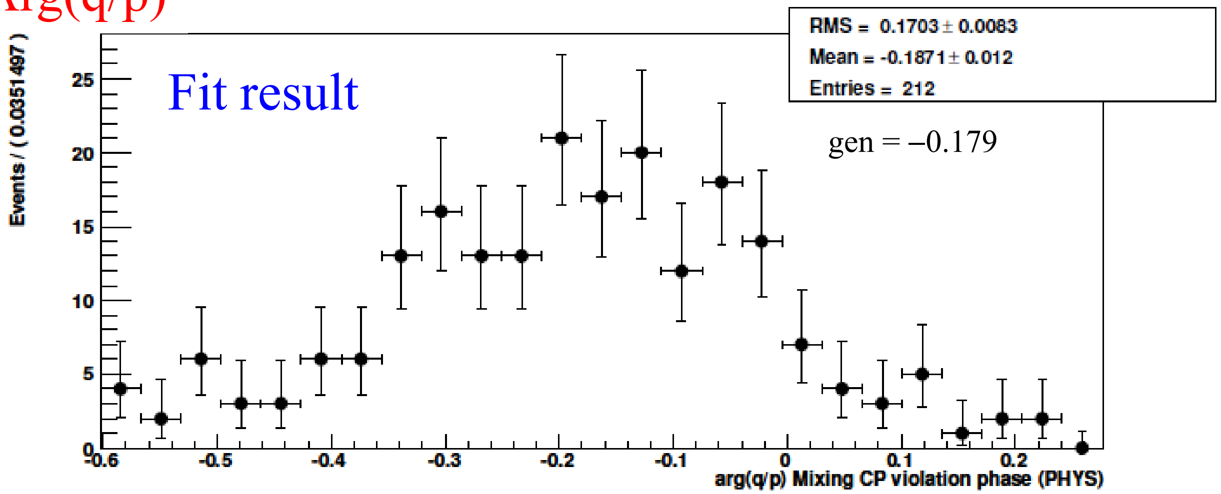
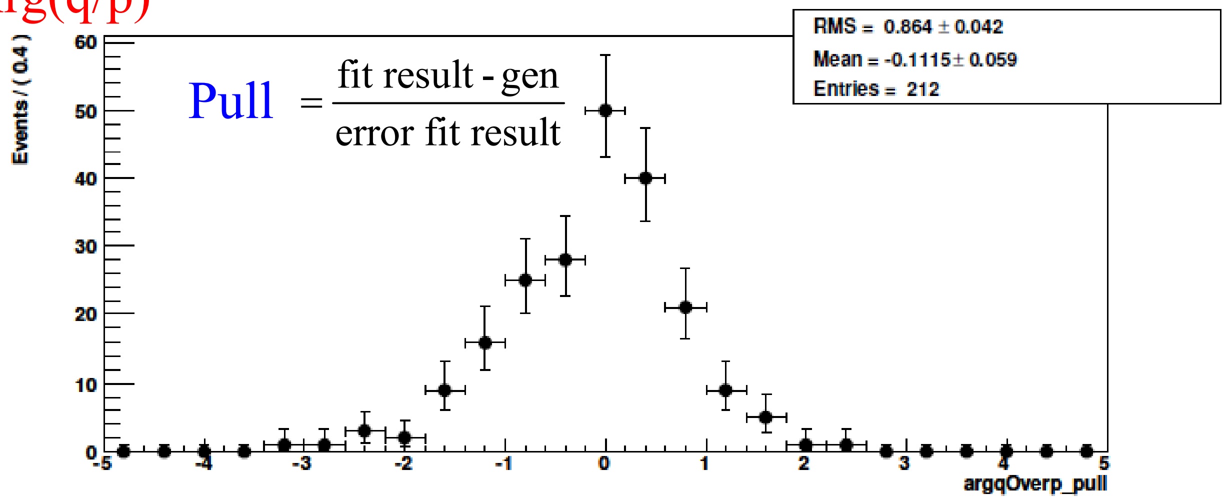
Sensitivity studies: Toy MC @ ϒ(4S)

Arg(q/p)



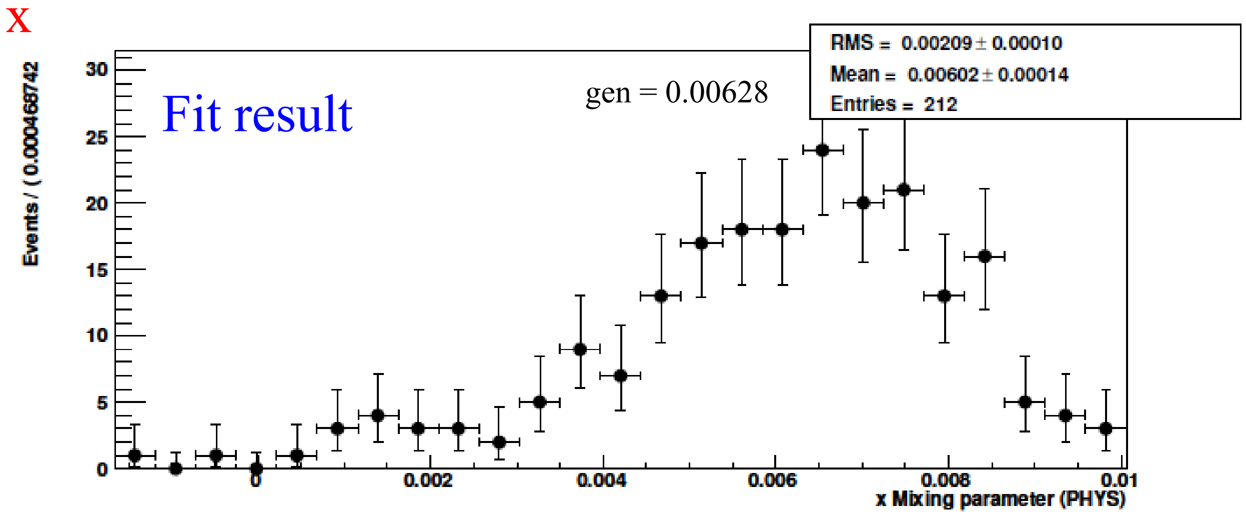
Arg(q/p)

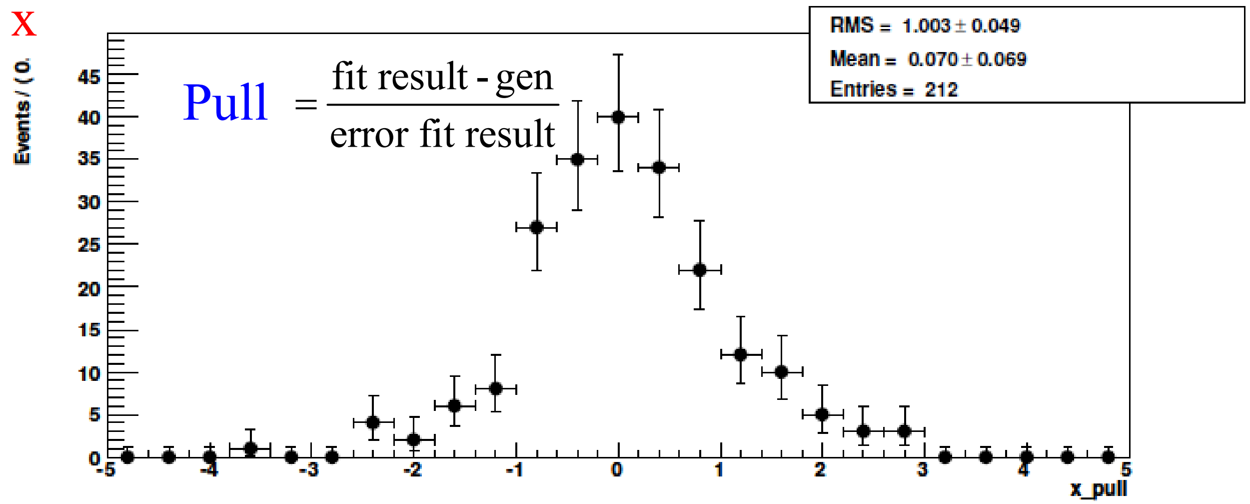


- ϒ(4S) , perfect resolution, no mistag

- 300 experiments generated
- ×200 less events (CPU memory limitation)
- 29% converge with error matrix not definite positive
- Understood due to the smallness of x,y while also fitting for Arg(q/p) and |q/p|. With mixing 10 times larger all fits return correct error matrix

Sensitivity studies: Toy MC @ ϒ(4S)





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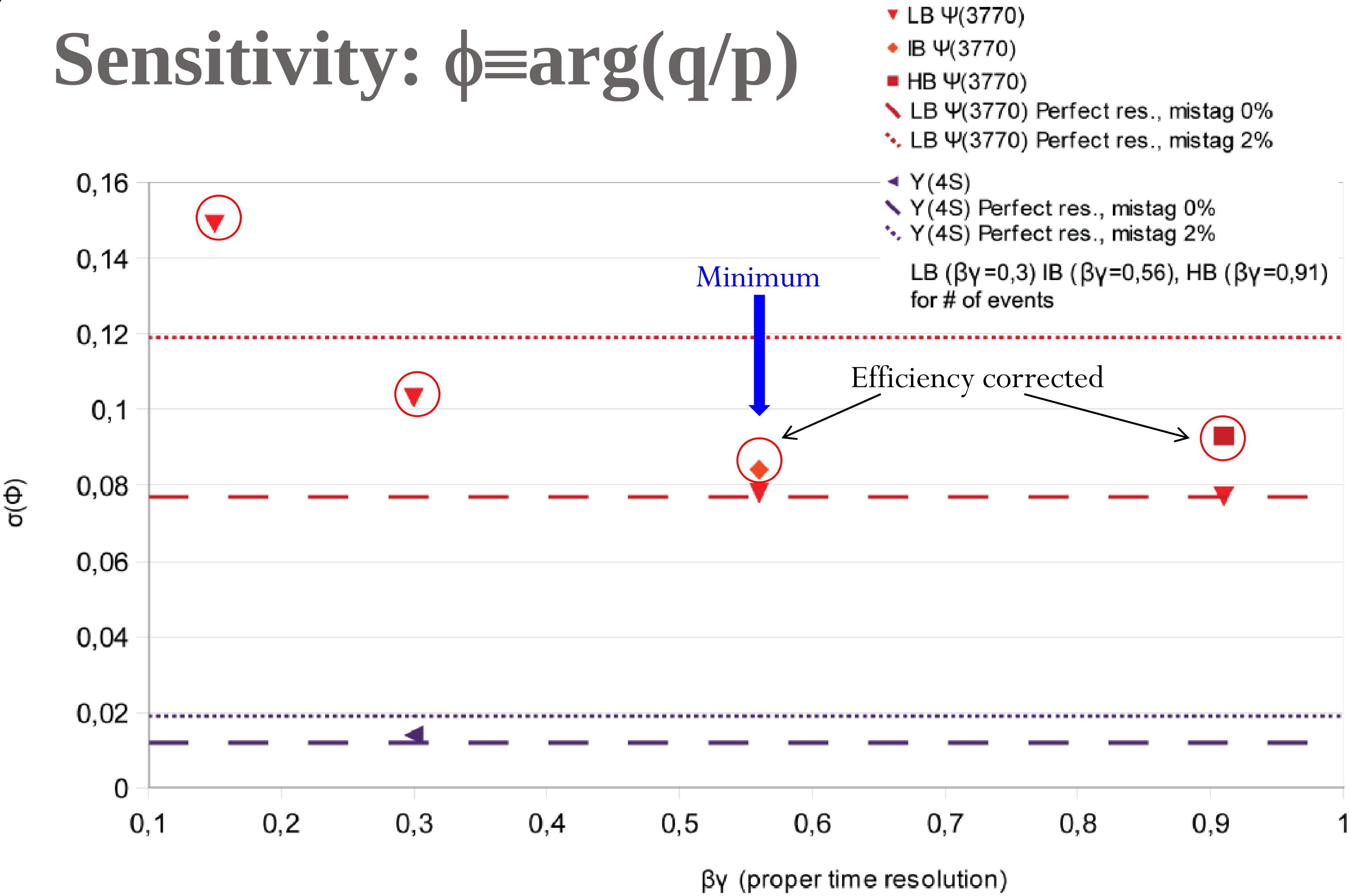
Sensitivity studies: summary of results

Data	Time resolution	Mistag	$\sigma(x)$	$\sigma(y)$	$\sigma(\phi)$	$\sigma(\lvert q/p\rvert)$
LB $\Psi(3770)$	Perfect	0	0.00076	0.00044	0.077	0.031
LB $\Psi(3770)$ large mixing	Perfect	0	0.00059	0.00043	0.007	0.003
LB $\Psi(3770)$ no CPV	Perfect	0	0.00081	0.00027	0	0
LB $\Psi(3770)$	HB TG (0.25/0.20 ps)	0	0.00098	0.00046	0.077	0.034
LB $\Psi(3770)$	IB TG (0.40/0.28 ps)	0	0.00100	0.00051	0.078	0.035
LB $\Psi(3770)$	LB TG (0.66/0.39 ps)	0	0.00104	0.00054	0.103	0.037
LB $\Psi(3770)$	VLB TG (1.27/0.76 ps)	0	0.00107	0.00067	0.149	0.061
HB $\Psi(3770)$	HB TG (0.25/0.20 ps)	0	0.00118	0.00056	0.093	0.041
IB $\Psi(3770)$	IB TG (0.40/0.28 ps)	0	0.00108	0.00054	0.084	0.037
LB $\Psi(3770)$	Perfect	2%	0.00210	0.00062	0.119	0.097
$\Upsilon(4S)$	Perfect	0	0.00021	0.00007	0.012	0.005
$\Upsilon(4S)$ large mixing	Perfect	0	0.00010	0.00008	0.001	0.001
$\Upsilon(4S)$ no CPV	Perfect	0	0.00012	0.00004	0	0
$\Upsilon(4S)$	TG (0.17/0.10 ps)	0	0.00017	0.00008	0.014	0.005
$\Upsilon(4S)$	Perfect	2%	0.00030	0.00011	0.019	0.014

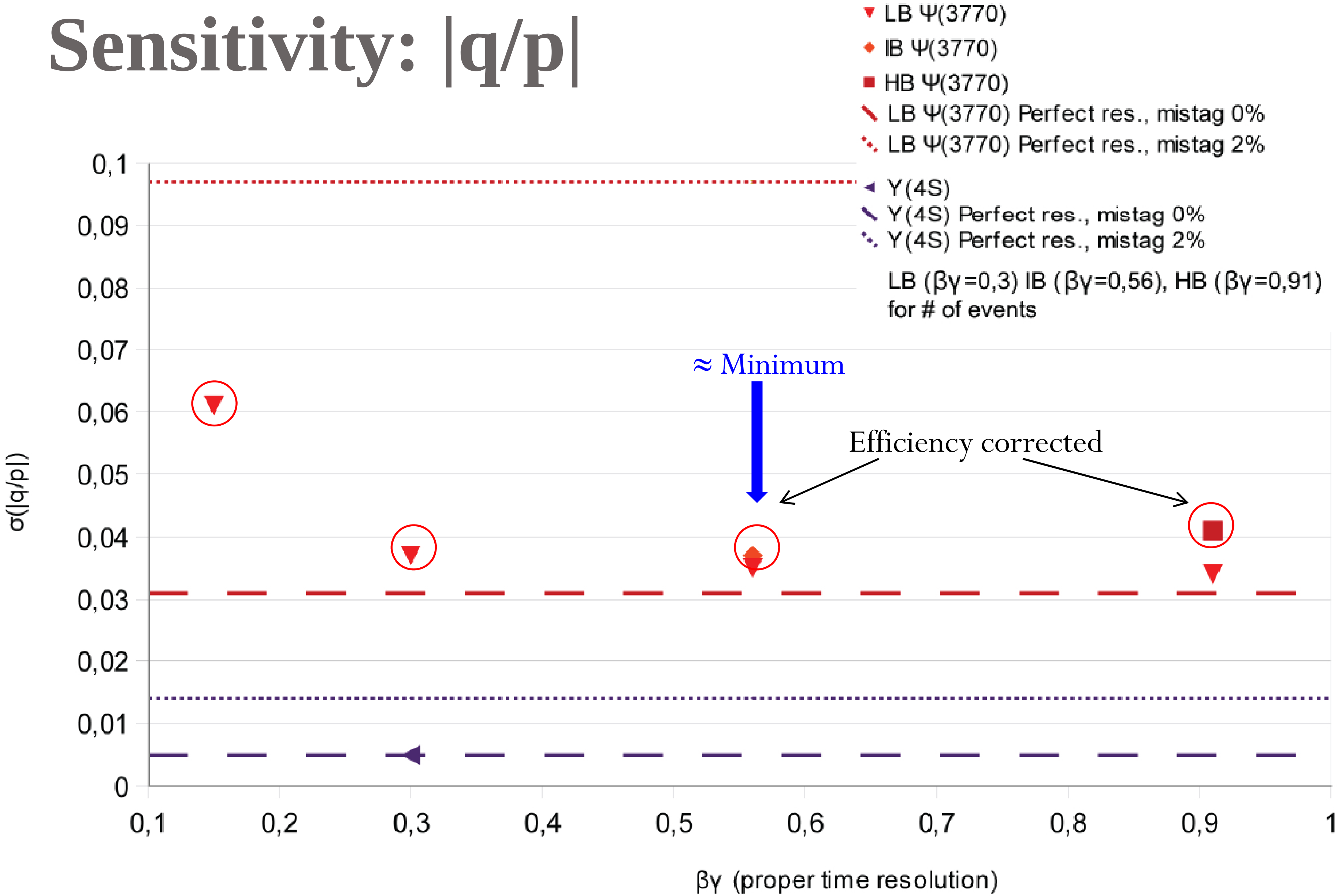
Parameter	Sensitivity @ Υ(4S) with time resolution, no mistag. 75 ab^{-1}	Best sensitivity @ ψ(3770) with time resolution ($\beta\gamma$=0.56), no mistag. 0.5 ab^{-1}
x	0.017%	0.11%
y	0.008%	0.05%
Arg(q/p)	0.8 deg	4.8 deg
|q/p|	0.5%	3.7%

Relative effect of flavor mistag similar at Ψ(3770) and Υ(4S)

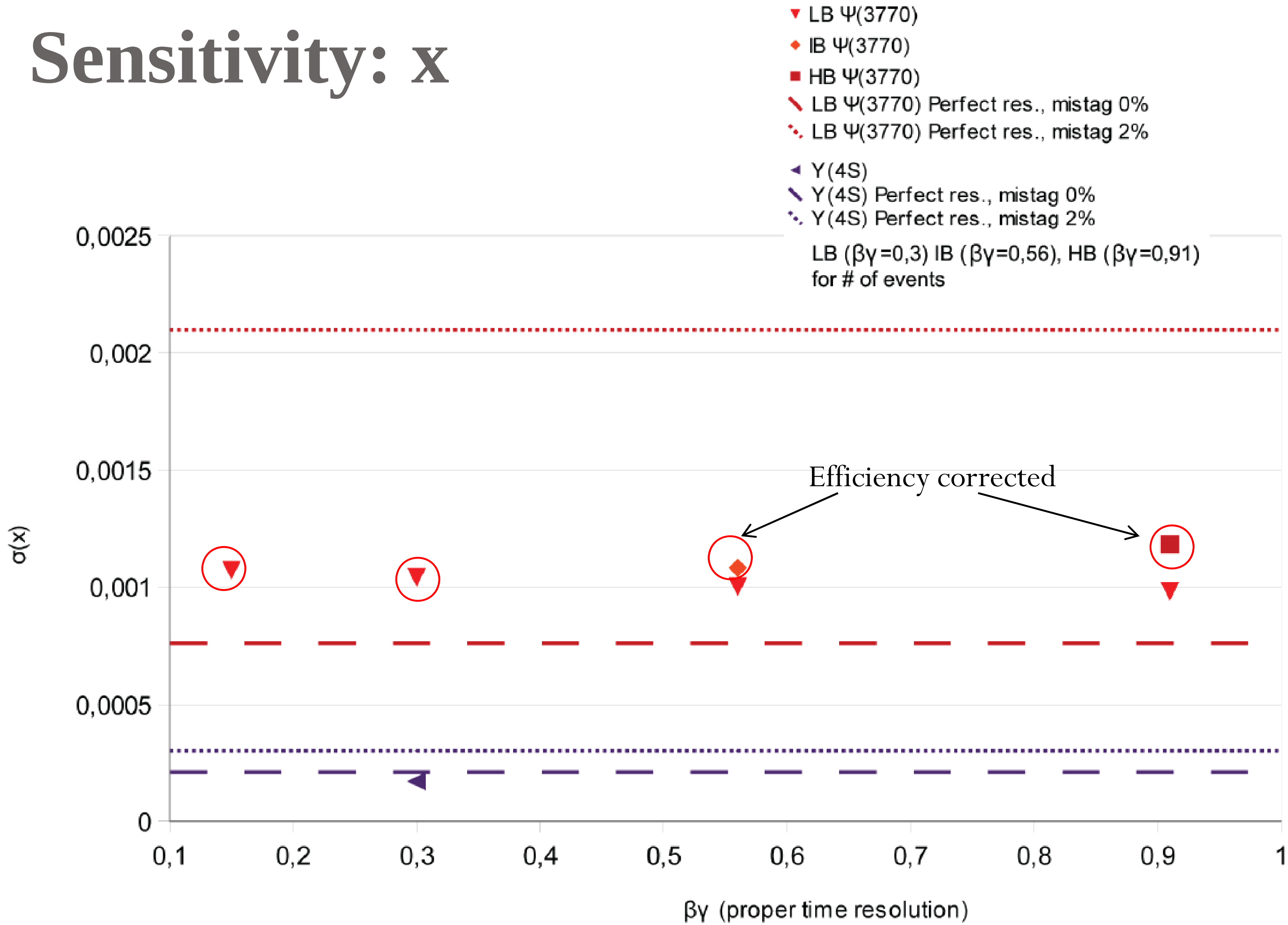
Sensitivity: $\phi \equiv \arg(q/p)$



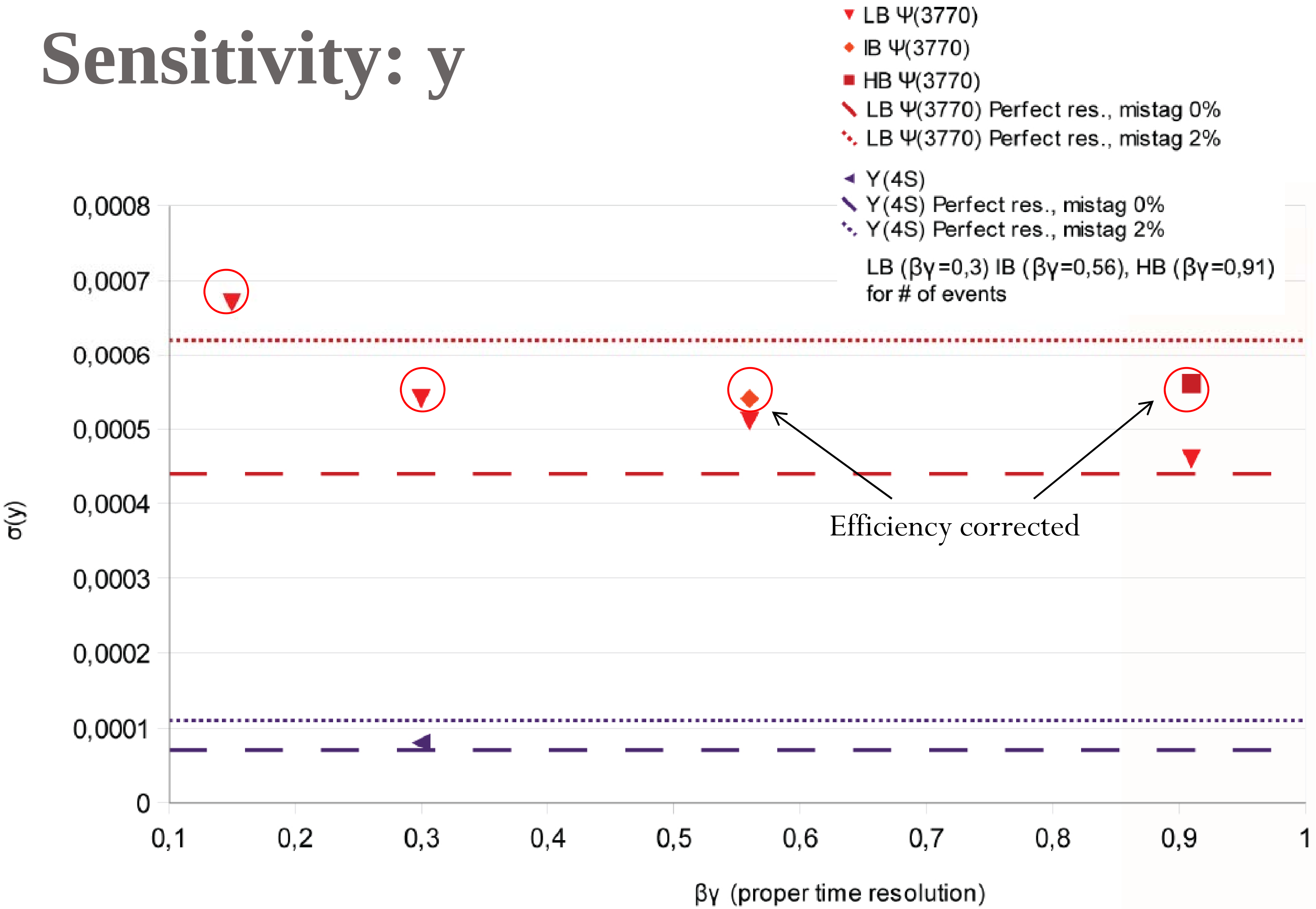
Sensitivity: |q/p|



Sensitivity: x



Sensitivity: y



Summary

- Flavor tag at D^0-$\bar{D}^0$ threshold provides identical time-dependence than at $\Upsilon(4S)$ using D* tagging, and less events, although in a different environment
- D^0-$\bar{D}^0$ threshold is unique to provide CP, $K\pi$ and $K_s\pi\pi$ tags
- Variation of Δt resolution and geometrical acceptance vs CM boost evaluated
- Estimated the impact on physics with 2-body decays
  - Combined fit to all 2-body double-tags allows determination of x, y, Arg(q/p), |q/p|
  - Best sensitivity at $\Psi(3770)$ for intermediate boost, $\beta\gamma \approx 0.56$

Parameter	Sensitivity @ $\Upsilon(4S)$ with time resolution, no mistag. 75 ab^{-1}	Best sensitivity @ $\psi(3770)$ with time resolution ($\beta\gamma$=0.56), no mistag. 0.5 ab^{-1}
x	0.017%	0.11%
y	0.008%	0.05%
Arg(q/p)	0.8 deg	4.8 deg
|q/p|	0.5%	3.7%

Relative effect of flavor mistag similar at $\Psi(3770)$ and $\Upsilon(4S)$

  - Sensitivity at $\Psi(3770)$ with 2-body decays 5 times worse than at $\Upsilon(4S)$
  - Mistag has to be understood very well. At $\Psi(3770)$ critical good separation between pions and muons at low momentum

Next steps

- Sensitivity studies on mixing and CPV parameters for 3-body decays with a time-dependent Dalitz plot analysis:
  - Dalitz plot model independent approach is to be pursued at SuperB. For this, it is crucial to have access to Ψ (3770) data.
- Consider two different scenarios:
  - Time-dependent measurements at Ψ(3770);
  - Time-dependent measurements at Υ(4S) with model independent coefficients (c_i, s_i) obtained with time-integrated Psi(3770) data.
- Setting up simulation technology for 3-body Toy MC studies.

Time-dependent Dalitz plot analyses

- Self-conjugate modes allow to extract mixing and CP violation parameters without D^0-$\overline{D}^0$ relative phase ambiguity when assuming CP is conserved in the decay.

$$A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3))$$
$$= A(\overline{D}^0 \to K_S(p_1)\pi^+(p_2)\pi^-(p_3))$$

  - In SM we expect CPV in the D^0 decay due to CPV in K_S mixing at the level of $3\text{x}10^{-3}$.
    - Is the above assumption still valid for the precision that we aim at SuperB?
  - Dalitz model uncertainty can be reduced using Psi(3770) data. Is it possible to perform a TDDP analysis in a model independent way for extracting mixing and CPV parameters? Can we relax the assumption of CP conservation in decays?
  - *Yes, it is possible a Dalitz plot model independent approach. In this case no assumptions for CP conservation in the decay are necessary.*

Model independent approach at Υ(4S)

A. Bondar, A. Poluektov, "The Use of quantum-correlated D^0 decays for phi3 measurement," Eur. Phys. J. **C55**, 51-56 (2008). [arXiv:0801.0840 [hep-ex]].

A. Bondar, A. Poluektov, V. Vorobiev, "Charm mixing in the model-independent analysis of correlated $D^0\bar{D}^0$ decays," Phys. Rev. **D82**, 034033 (2010). [arXiv:1004.2350 [hep-ph]].

- c_i, s_i from Ψ(3770) time-integrated data.
- No assumption of CP conservation in decay.

$$A_f = |A_f|e^{i\delta_f} \qquad \bar{A}_f = |\bar{A}_f|e^{i\bar{\delta}_f}$$

$$\int_i |A_f|^2 d\mathcal{P} = T_i \qquad \int_i |\bar{A}_f|^2 d\mathcal{P} = \bar{T}_i$$

$$\frac{\int_i \mathcal{R}e(A_f^*\bar{A}_f)d\mathcal{P}}{\sqrt{T_i\bar{T}_i}} = \frac{\int_i |A_f||\bar{A}_f|\cos(\bar{\delta}_f - \delta_f)d\mathcal{P}}{\sqrt{T_i\bar{T}_i}} = c_i$$

$$\frac{\int_i \mathcal{I}m(A_f^*\bar{A}_f)d\mathcal{P}}{\sqrt{T_i\bar{T}_i}} = \frac{\int_i |A_f||\bar{A}_f|\sin(\bar{\delta}_f - \delta_f)d\mathcal{P}}{\sqrt{T_i\bar{T}_i}} = s_i$$

$$\begin{aligned}\frac{d\Gamma_i[D^0_{\rm phys} \to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \Bigg[\left(T_i + |\frac{q}{p}|^2\bar{T}_i\right)\cosh(\Gamma y t) + \left(T_i - |\frac{q}{p}|^2\bar{T}_i\right)\cos(\Gamma x t)\\ &+ 2\left(c_i\sqrt{T_i\bar{T}_i}\left|\frac{q}{p}\right|\cos\phi - s_i\sqrt{T_i\bar{T}_i}\left|\frac{q}{p}\right|\sin\phi\right)\sinh(\Gamma y t)\\ &- 2\left(c_i\sqrt{T_i\bar{T}_i}\left|\frac{q}{p}\right|\sin\phi + s_i\sqrt{T_i\bar{T}_i}\left|\frac{q}{p}\right|\cos\phi\right)\sin(\Gamma x t)\Bigg]\end{aligned}$$

Model independent approach at ψ(3770)

- c_i, s_i from Ψ(3770) data.
- No assumption of CP conservation in decay.

$$\int_i\int_j |a_+|^2 = \bar{T}_{1i}T_{2j} + T_{1i}\bar{T}_{2j} - 2\sqrt{T_{1i}\bar{T}_{1i}T_{2j}\bar{T}_{2j}}(c_{1i}c_{2j} + s_{1i}s_{2j})$$

$$\int_i\int_j |a_-|^2 = \left|\frac{p}{q}\right|^2 T_{1i}T_{2j} + \left|\frac{q}{p}\right|^2 \bar{T}_{1i}\bar{T}_{2j} - 2\sqrt{T_{1i}\bar{T}_{1i}T_{2j}\bar{T}_{2j}}\left(\cos 2\phi\,(c_{1i}c_{2j} - s_{1i}s_{2j}) - \sin 2\phi\,(c_{1i}s_{2j} + s_{1i}c_{2j})\right)$$

$$\int_i\int_j \mathcal{R}e(a_+^* a_-) = \left(\left|\frac{p}{q}\right| T_{2j} + \left|\frac{q}{p}\right| \bar{T}_{2j}\right)\sqrt{T_{1i}\bar{T}_{1i}}(\cos\phi\; c_{1i} + \sin\phi\; s_{1i}) - \left(\left|\frac{p}{q}\right| T_{1i} + \left|\frac{q}{p}\right| \bar{T}_{1i}\right)\sqrt{T_{2j}\bar{T}_{2j}}(\cos\phi\; c_{2j} + \sin\phi\; s_{2j})$$

$$\int_i\int_j \mathcal{I}m(a_+^* a_-) = \left(\left|\frac{p}{q}\right| T_{2j} + \left|\frac{q}{p}\right| \bar{T}_{2j}\right)\sqrt{T_{1i}\bar{T}_{1i}}(\cos\phi\; s_{1i} - \sin\phi\; c_{1i}) - \left(\left|\frac{p}{q}\right| T_{1i} + \left|\frac{q}{p}\right| \bar{T}_{1i}\right)\sqrt{T_{2j}\bar{T}_{2j}}(\cos\phi\; s_{2j} - \sin\phi\; c_{2j});$$

$$\frac{d\Gamma[V_{\rm phys}(t_1,t_2)\to f_1 f_2]/dt}{e^{-\Gamma|\Delta t|}\mathcal{N}_{f_1 f_2}} = \left(|a_+|^2 + |a_-|^2\right)\cosh(y\Gamma\Delta t) + \left(|a_+|^2 - |a_-|^2\right)\cos(x\Gamma\Delta t) - 2\mathcal{R}e((a_+^* a_-)\sinh(y\Gamma\Delta t) + 2\mathcal{I}m(a_+^* a_-)\sin(x\Gamma\Delta t)$$