Probing ALPs with $K \to \pi a$

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based on 2308.16903 and work in progress with A. Galda, M. Neubert and D. Wyler

Looking for ALPs with flavor

Low-energy weak processes impose some of the most stringent "particle bounds" on ALP couplings. **Here focus on $K \to \pi a$:**

- has been studied for a long time, starting with Georgi, Kaplan & Randall in 1986
- previous calculations used implementation of weak currents in the chiral Lagrangian. "Real" branching ratio is ~ 37 times larger [Bauer, Neubert, Renner, Schnubel (2021)]
- strongest particle constraint on ALP couplings to gluons and light quarks for $m_a < m_K - m_\pi \sim 354$ MeV

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Only LO calculation in χPT. **What happens at NLO?**

In this talk:
- Inclusion of the ALP in the QCD and weak chiral Lag. at LO and NLO
- Some details on the calculation
- Discussion of the results

Effective ALP Lagrangian at low energies

Start from the ALP+SM effective Lagrangian around $\mu_\chi = 4\pi F_\pi \sim 1.6$ GeV.

Only light flavors: q=(u,d,s).

[Georgi, Kaplan, Randall (1986)]

soft breaking of shift symmetry $a \to a + c$

anomalous couplings to gauge fields

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a} + c_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$
$$+ \frac{\partial_\mu a}{f} \left(\bar{q}_L k_Q \gamma^\mu q_L + \bar{q}_R k_q \gamma^\mu q_R \right),$$

alp decay constant f >> v

derivative couplings to fermions

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A chiral transformation can be used to remove the ALP-gluon coupling:

[Srednicki (1985); Georgi, Kaplan, Randall (1986); Krauss, Wise (1986); Bardeen, Peccei, Yanagida (1987)]

$$q(x) \to e^{-i c_{GG} \frac{a}{f} \kappa_q \gamma_5} q(x) \qquad \langle \kappa_q \rangle = \kappa_u + \kappa_d + \kappa_s = 1$$

The κ_q must drop out of physical predictions!

QCD chiral Lagrangian at O(p²)

Below $\mu_\chi = 4\pi F_\pi \sim 1.6$ GeV, the light pseudo-scalar mesons π, η, K take the place of quarks and gluons. We describe them using **Chiral Perturbation Theory** (χ**PT**).

- Dofs $\quad \Sigma_0(x) = \exp\left[\frac{i\sqrt{2}}{F}\Phi(x)\right], \quad \Phi(x) = \lambda^a \pi^a(x) \quad \det \Sigma_0 = 1$

  meson decay constant $F \simeq F_\pi = 130\,\mathrm{MeV}$

- Power counting $\quad \lambda \sim \frac{p}{4\pi F} \quad \Sigma_0 \sim 1 \quad (\partial_\mu \Sigma_0)^n \sim \lambda^n \sim p^n$

- Symmetries $\quad$ gauge symmetry, $G_\chi = SU(3)_L \times SU(3)_R$ for $m_q \to 0$: $\Sigma_0 \to g_L \Sigma_0 g_R^\dagger$

The most general Lagrangian invariant under G_χ is

$$\mathcal{L}_{\mathrm{QCD}}^{(p^2)} = \frac{F^2}{8}\langle (D_\mu \Sigma_0)\,(D^\mu \Sigma_0^\dagger)\rangle$$

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$$\mathcal{L}_{\rm QCD}^{(p^2)} = \frac{F^2}{8}\langle (D_\mu \Sigma_0)(D^\mu \Sigma_0^\dagger)\rangle + \frac{F^2}{8}\langle \chi\Sigma^\dagger + \Sigma\chi^\dagger\rangle$$

$$\chi = 2B_0 m_q \qquad \chi \to g_L\,\chi\,g_R^\dagger$$

Quark masses break G_χ in a specific way: incorporated as **spurions**.

QCD chiral Lagrangian at O(p²), with arbitrary sources

Any interaction of quarks & gluons can be written in terms of **sources**

$$\mathcal{L}_{\rm eff} \ni \bar{q}(x)\big[l_\mu(x)\,\gamma^\mu P_L + r_\mu(x)\,\gamma^\mu P_R - s + i\gamma_5\,p\big]q(x) - \frac{\alpha_s}{8\pi}\,\theta(x)\,G^a_{\mu\nu}(x)\,\tilde{G}^{\mu\nu,a}(x)$$

These break G_χ in a specific way,
and can be treated as **spurions** by assigning them transformation properties under G_χ:

$$2B_0(s+ip) = \ \chi \to g_L \chi\, g_R^\dagger\,, \qquad l_\mu \to g_L\, l_\mu\, g_L^\dagger\,, \qquad r_\mu \to g_R\, r_\mu\, g_R^\dagger$$

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In presence of a θ source,

$$G_\chi = SU(3)_L \times SU(3)_R \quad \to \quad G'_\chi = U(3)_L \times U(3)_R$$

$$\det \Sigma_0 = 1 \quad \to \quad \det \Sigma(x) = e^{-i\,\theta(x)}$$

$$\Sigma_0(x) \;\to\; \Sigma(x) = e^{-\frac{i}{2}\theta(x)\,\kappa_q}\,\Sigma_0(x)\,e^{-\frac{i}{2}\theta(x)\,\kappa_q}$$

θ transforms non-linearly under G'_χ, instead $D_\mu\theta = \partial_\mu\theta - 2\,\langle a_\mu\rangle$ is a singlet of G'_χ

$$a_\mu = \frac{r_\mu - l_\mu}{2}$$

The ALP as a source

For the ALP, the sources are

$$s = m_q\,,\;\; p = 0\,,\quad l_\mu(x) = k_Q\,\frac{\partial_\mu a(x)}{f}\,,\qquad r_\mu(x) = k_q\,\frac{\partial_\mu a(x)}{f}\,,\;\; \theta(x) = -2c_{GG}\,\frac{a(x)}{f}\,.$$

So the QCD chiral Lagrangian at $\mathcal{O}(p^2)$ including the ALP is:

$$\mathcal{L}_{\rm QCD}^{(p^2)} = \frac{F^2}{8}\,\langle (D_\mu\Sigma)\,(D^\mu\Sigma)^\dagger + \chi\Sigma^\dagger + \Sigma\chi^\dagger\rangle + \frac{H_0}{2}\,(D_\mu\theta)(D^\mu\theta) + \frac{1}{2}m_{a,0}^2\,a^2$$

ALP kinetic energy and mass

$$\chi = 2B_0\,(s+ip) = 2B_0\,m_q$$

$$iD_\mu\Sigma = i\partial_\mu\Sigma + (QeA_\mu + l_\mu)\,\Sigma - \Sigma\,(QeA_\mu + r_\mu)$$

$$H_0 = \frac{f^2}{(2c_{GG} + \langle c^a\rangle)^2}$$

chiral ALP currents

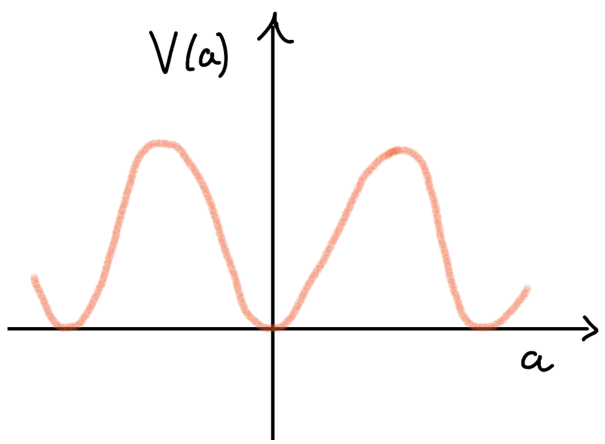
The ALP as a source

π_0, a and η_8 undergo both mass & kinetic mixing. As a consequence*,

- non-perturbative contribution to ALP mass (also for $m_{a,0} \neq 0$)

[Bardeen et al. (1978);
Shifman et al. (1980);
Di Veccia, Veneziano (1980)]

$$m_a^2 = m_{a,0}^2 \left\{1 - \frac{F^2}{4f^2}\left[\Delta + H_0\left(\langle c^a\rangle + 2c_{GG}\right)^2 + \frac{m_a^2}{2\left(\tilde{m}_{\pi^0}^2 - m_{a,0}^2\right)}\left(c_{uu}^a - c_{dd}^a\right)^2\right]\right\}$$
$$+ c_{GG}^2 \frac{F^2\,\tilde{m}_{\pi^0}^2}{2f^2} - \frac{F^2}{24f^2}\frac{\left(\left(c_{uu}^a + c_{dd}^a - 2c_{ss}^a\right)m_{a,0}^2 + 2c_{GG}\left(m_{a,0}^2 - \tilde{m}_{\pi^0}^2\right)\right)^2}{\tilde{m}_{\eta_8}^2 - m_{a,0}^2},$$



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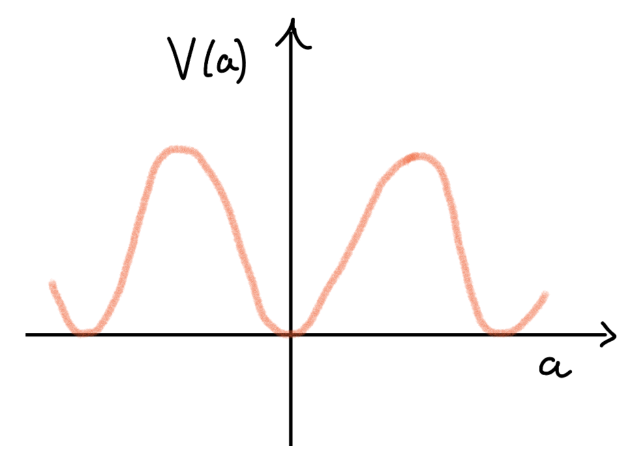


- Field redefinitions needed to work with physical fields:

$$\begin{pmatrix}\pi_0\\ \eta_8\\ a\end{pmatrix} = \boldsymbol{R}\begin{pmatrix}\pi_{0,\text{phys}}\\ \eta_{8,\text{phys}}\\ a_{\text{phys}}\end{pmatrix}, \qquad \boldsymbol{R} = \begin{pmatrix}1 & 0 & \frac{f_\pi}{f}\theta_{\pi_0 a}\\ 0 & 1 & \frac{f_\pi}{f}\theta_{\eta_8 a}\\ \frac{f_\pi}{f}\theta_{a\pi_0} & \frac{f_\pi}{f}\theta_{a\eta_8} & 1\end{pmatrix} + \mathcal{O}\left(\frac{f_\pi^2}{f^2}\right)$$

$$\theta_{\pi^0 a} = -\frac{M_{13}^2 - m_{a,0}^2\, Z_{13}}{\tilde{m}_{\pi^0}^2 - m_{a,0}^2} = \frac{F}{f}\,\frac{\left(\hat{c}^a_{uu} - \hat{c}^a_{dd}\right) m_{a,0}^2 - 2c_{GG}\left(\kappa_u - \kappa_d\right)\tilde{m}_{\pi^0}^2}{2\sqrt{2}\left(\tilde{m}_{\pi^0}^2 - m_{a,0}^2\right)}$$

κ_q dependence must drop in physical amplitudes!

*all expressions in the isospin-conserving limit $m_u = m_d$

QCD chiral Lagrangian at O(p⁴)

Gasser & Leutwyler constructed the O(p⁴) QCD chiral Lagrangian in 1985, under the assumption $\theta = \langle a_\mu \rangle = \langle v_\mu \rangle = 0$.

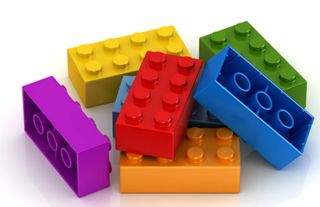
With an ALP, these assumptions no longer hold: need to **extend the basis** including operators containing θ , $\langle a_\mu \rangle$ in the invariant combination $D_\mu \theta$!

i	O_i	Γ_i
1	$\langle L^2 \rangle^2$	$\frac{3}{32}$
2	$\langle L^\mu L^\nu \rangle \langle L_\mu L_\nu \rangle$	$\frac{3}{16}$
3	$\langle L^4 \rangle$	0
4	$\langle L^2 \rangle \langle S \rangle$	$\frac{1}{8}$
5	$\langle L^2 S \rangle$	$\frac{3}{8}$
6	$\langle S \rangle^2$	$\frac{11}{144}$
7	$-\langle P \rangle^2$	0
8	$\frac{1}{2} \langle S^2 - P^2 \rangle$	$\frac{5}{48}$

$$\mathcal{L}^{(p^4)}_{\rm QCD} = \sum_{i=1}^{8} L_i\, O_i + \sum_{i=1}^{3} L_i^\theta\, O_i^\theta$$

[Herrera-Siklody et al. (1997)]

i	O_i^θ	Γ_i^θ
1	$-(\partial^\mu D_\mu \theta) \langle P \rangle$	0
2	$-(D_\mu \theta) \langle L^\mu S \rangle$	$-\frac{1}{4}$
3	$(D_\mu \theta) \langle L^\mu L^2 \rangle$	0



Building blocks

Object	Definition
S	$\chi \Sigma^\dagger + \Sigma \chi^\dagger$
P	$i(\chi \Sigma^\dagger - \Sigma \chi^\dagger)$
L_μ	$\Sigma\, i (D_\mu \Sigma)^\dagger$
$W_{\mu\nu}$	$2(D_\mu L_\nu + D_\nu L_\mu)$
$D_\mu \theta$	$\partial_\mu \theta - 2 \langle a_\mu \rangle$

$S \to m_q(\bar{q}_L q_R + \bar{q}_R q_L)$

$P \to i m_q(\bar{q}_L q_R - \bar{q}_R q_L)$

$L_\mu \to \bar{q}_L \gamma_\mu q_L$

QCD chiral Lagrangian at O(p⁴)

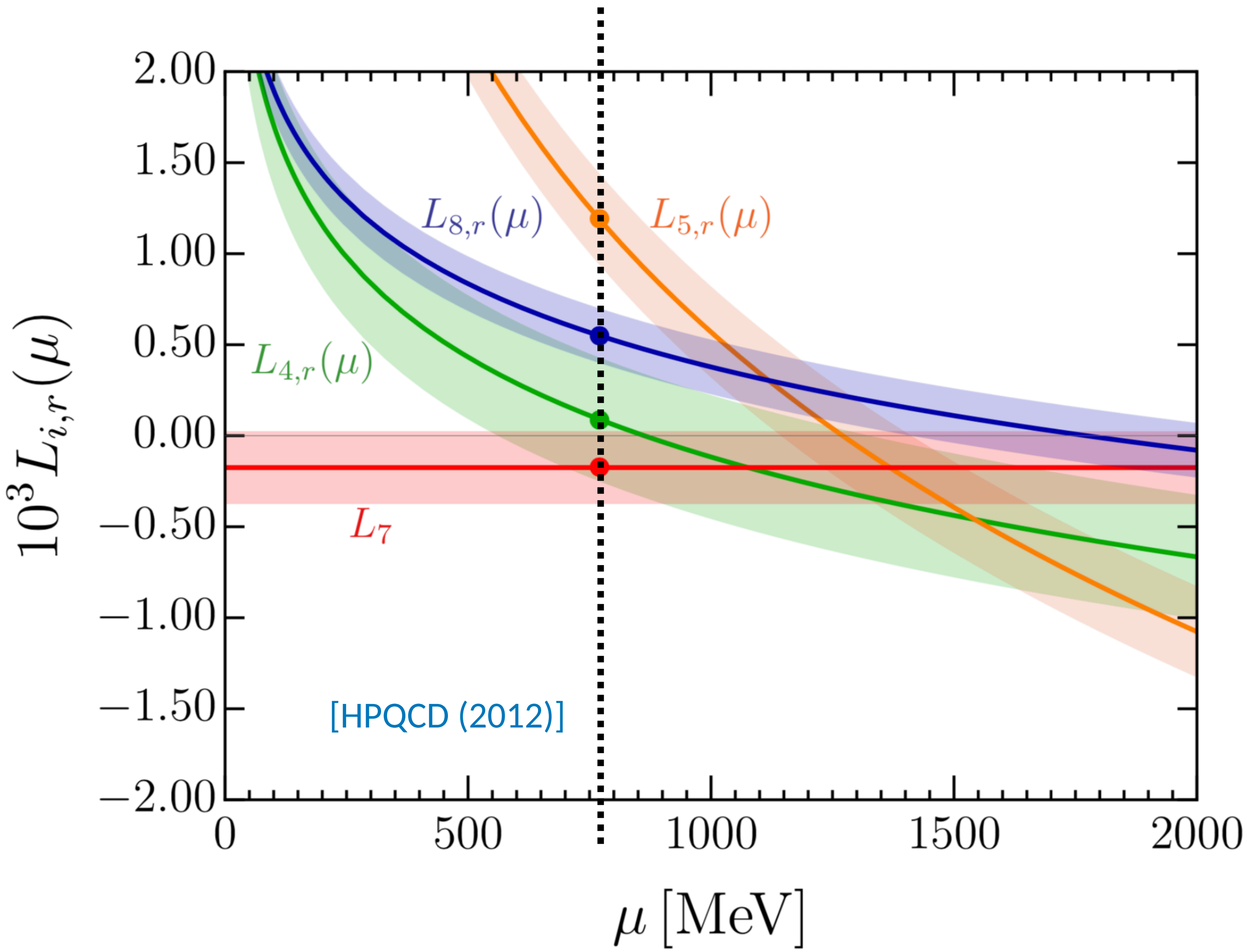
low-energy constants (**LECs**) anomalous dimensions (all known!)

$$L_i^{(\theta)} = L_{i,r}^{(\theta)}(\mu) + \lambda\,\Gamma_i^{(\theta)} \qquad \lambda = \frac{\mu^{d-4}}{32\pi^2}\left(\frac{2}{d-4} - \ln 4\pi + \gamma_E - 1\right)$$

No estimates for $L_{i,r}^{\theta}$ exist.

There are estimates for some $L_{i,r}$ at $\mu = m_\rho$ from fits to low-energy data & lattice, with $\sigma \sim$ 10 - 100%

[Bijnens et. al (2014); HPQCD (2012)]



Non-leptonic weak chiral Lagrangian at O(p²)

To build the **chiral Lagrangian** for **non-leptonic weak decays**, start from the $\Delta S = 1$ WET Lagrangian and classify its operators in reps of G_χ. [see e.g. Pich (1985), (1990)]

Neglecting em penguins, one has operators transforming as **(8_L, 1_R)** or **(27_L, 1_R)** of G_χ. [Cronin (1967), Bernard et al. (1985)]

Their chiral representation is:

$$\mathcal{L}_{\text{weak}}^{(p^2)} = \frac{F^4}{4}\left[G_8 \langle \lambda_+ L_\mu L^\mu \rangle + G_8' \langle \lambda_+ S \rangle + G_{27}^{1/2}\, \mathcal{O}_{27}^{1/2} + G_{27}^{3/2}\, \mathcal{O}_{27}^{3/2} + \text{h.c.}\right]$$

$$\mathcal{O}_{27}^{1/2} = L_{\mu\,32} L_{11}^\mu + L_{\mu\,31} L_{12}^\mu + 2 L_{\mu\,32} L_{22}^\mu - 3 L_{\mu\,32} L_{33}^\mu\,,$$
$$\mathcal{O}_{27}^{3/2} = L_{\mu\,32} L_{11}^\mu + L_{\mu\,31} L_{12}^\mu - L_{\mu\,32} L_{22}^\mu\,.$$

- proportional to Fermi constant: $G_i = -\dfrac{G_F}{\sqrt{2}} V_{ud}^* V_{us}\, g_i$
- $\lambda_+ = 1/2\,(\lambda_6 + i\,\lambda_7)$ selects $s \to d$ transitions
- L_μ, S represent the currents $\bar{q}_L \gamma_\mu q_L$ and $m_q(\bar{q}_L q_R + \bar{q}_R q_L)$
- "weak mass term" $\langle \lambda_+ S \rangle$ unobservable in the SM [Bernard, Draper, Soni, Politzer, Wise (1985); Crewther (1986); Leurer (1988)]

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$$+ G_8^\theta\,(D_\mu\theta)\,\langle\lambda_+ L^\mu\rangle$$

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And with the ALP?

- weak mass term is no longer unobservable
- **new octet** operator $\propto D_\mu\theta$!

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$$+ G_8^\theta \, (D_\mu \theta) \langle \lambda_+ L^\mu \rangle$$

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$$G_i = -\frac{G_F}{\sqrt{2}} V_{ud}^* V_{us}\, g_i$$

- from $K \to \pi\pi$:
  $$g_8 = 3.61 \pm 0.28$$
  $$g_{27}^{1/2} \approx 0.033 \pm 0.003$$
  $$g_{27}^{3/2} \approx 0.165 \pm 0.016$$
  octet strongly enhanced: $\Delta I = 1/2$ selection rule
- g_8^θ , g_8': don't enter processes without ALP, hence **unknown**

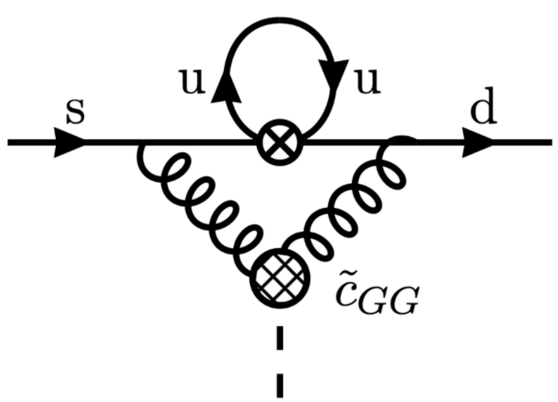
Interpreting the new octet operator

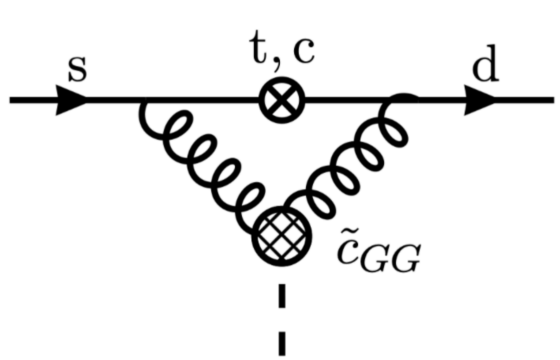
The new octet operator is the chiral representation of

$$\frac{F^2}{4}(D_\mu\theta)\langle\lambda_+ L^\mu\rangle \quad\longleftrightarrow\quad 2\,\tilde{c}_{GG}(\mu_\chi)\,\frac{(\partial_\mu a)}{f}\,\bar{d}_L\gamma^\mu s_L\,.$$

$$\tilde{c}_{GG} = c_{GG} + \frac{c^a_{uu} + c^a_{dd} + c^a_{ss}}{2}$$

It accounts for the gluon-induced interactions of the ALP with FCNCs:





Formally a two-loop effect, but can be enhanced at low energies:

$$c^{\rm eff}_{\gamma\gamma} \approx c_{\gamma\gamma} - (1.92 \pm 0.04)\, c_{GG}$$

[Villadoro et. al (2016)]

Non-leptonic weak chiral Lagrangian at O(p²)

Because of the large enhancement of the octet with respect to the 27plet, we **only** include **octet** operators at O(p⁴).

Started from the basis of **Ecker, Kambor & Wyler** (1993), obtained from the redundant basis of Kambor, Missimer, Wyler (1989) via eoms, Cayley-Hamilton, IBP.

- only 19 out of 37 ops. are needed because $F^L_{\mu\nu} = \partial_\mu l_\nu - \partial_\mu l_\mu - i[l_\mu, l_\nu] \sim \partial_\mu \partial_\nu a - \partial_\nu \partial_\mu a = 0$ (same for $F^R_{\mu\nu}$)
- …but the Ecker basis is derived for $\theta = \langle a_\mu \rangle = 0$.

i	W_i^8	Z_i	Z_i'
1	$\langle \lambda_6 L^2 L^2 \rangle$	2	0
2	$\langle \lambda_6 L_\mu L^2 L^\mu \rangle$	$-\frac{1}{2}$	0
3	$\langle \lambda_6 L_\mu L_\nu \rangle \langle L^\mu L^\nu \rangle$	0	0
4	$\langle \lambda_6 L_\mu \rangle \langle L^\mu L^2 \rangle$	1	0
5	$\langle \lambda_6 \{S, L^2\} \rangle$	$\frac{3}{2}$	$\frac{3}{4}$
6	$\langle \lambda_6 L_\mu \rangle \langle S L^\mu \rangle$	$-\frac{1}{4}$	0
7	$\langle \lambda_6 S \rangle \langle L^2 \rangle$	$-\frac{9}{8}$	$\frac{1}{2}$
8	$\langle \lambda_6 L^2 \rangle \langle S \rangle$	$-\frac{1}{2}$	0
9	$i \langle \lambda_6 [P, L^2] \rangle$	$\frac{3}{4}$	$-\frac{3}{4}$
10	$\langle \lambda_6 S^2 \rangle$	$\frac{2}{3}$	$\frac{5}{12}$
11	$\langle \lambda_6 S \rangle \langle S \rangle$	$-\frac{13}{18}$	$\frac{11}{18}$
12	$-\langle \lambda_6 P^2 \rangle$	$-\frac{5}{12}$	$\frac{5}{12}$
13	$-\langle \lambda_6 P \rangle \langle P \rangle$	0	0
19	$\langle \lambda_6 [l_\mu, [L^2, L^\mu]] \rangle$	$-\frac{5}{4}$	0
20	$\frac{i}{2} \langle \lambda_6 [l_\mu, \{L_\nu, W^{\mu\nu}\}] \rangle$	$\frac{3}{4}$	0
21	$-\langle \lambda_6 [l_\mu, [S, L^\mu]] \rangle$	$\frac{5}{6}$	0
23	$-i \langle \lambda_6 [l_\mu, \{P, L^\mu\}] \rangle$	$\frac{5}{12}$	0
24	$-i \langle \lambda_6 [l_\mu, L^\mu] \rangle \langle P \rangle$	0	0
28	$i\epsilon_{\mu\nu\rho\sigma} \langle \lambda_6 L^\mu \rangle \langle L^\nu L^\rho L^\sigma \rangle$	0	0

Non-leptonic weak chiral Lagrangian at O(p²)

We find that the **extension** to general case requires 9 additional operators!

i	$W_i^{\theta\,8}$	Z_i^θ	$Z_i'^\theta$	$Z_i^{\theta\theta}$
1	$(D_\mu\theta)\,\langle\lambda_6\{L^\mu,S\}\rangle$	(2.66)	$\frac{1}{2}$	(2.67)
2	$i(D_\mu\theta)\,\langle\lambda_6[L^\mu,P]\rangle$	(2.66)	$\frac{1}{2}$	(2.67)
3	$i(D_\mu\theta)\,\langle\lambda_6[L_\nu,W^{\mu\nu}]\rangle$	Z_3^θ	0	$Z_3^{\theta\theta}$
4	$(D_\mu\theta)\,\langle\lambda_6 L^\mu\rangle\,\langle S\rangle$	$-\frac{3}{4}$	0	$\frac{1}{2}$
5	$(\partial^\mu D_\mu\theta)\,\langle\lambda_6 P\rangle$	Z_5^θ	0	$Z_5^{\theta\theta}$
6	$(D_\mu\theta)\,\langle\lambda_6\{L^\mu,L^2\}\rangle$	–	0	–
7	$(D_\mu\theta)\,\langle\lambda_6 L^\mu\rangle\,\langle L^2\rangle$	–	0	–
8	$(D_\mu\theta)\,\langle\lambda_6 L_\nu\rangle\,\langle L^\mu L^\nu\rangle$	–	0	–
9	$i\epsilon_{\mu\nu\rho\sigma}(D^\mu\theta)\,\langle\lambda_6 L^\nu L^\rho L^\sigma\rangle$	–	0	–

[CC, Galda, Neubert, Wyler (2023)]

Non-leptonic weak chiral Lagrangian at O(p²)

[Ecker et. al (1993)] [CC, Galda, Neubert, Wyler (2023)]

$$\mathcal{L}_{\text{weak}}^{(p^4)} = \frac{G_8 F^2}{2} \left(\sum_{i \in \mathcal{S}} N_i \, W_i^8 + \sum_{i=1}^{9} N_i^\theta \, W_i^{\theta\, 8} \right)$$

$\mathcal{S} = \{1, \ldots, 13, 19, 20, 21, 23, 24, 28\}$

weak LECs

Anomalous dimensions

$$N_i = N_{i,r}(\mu) + \lambda \left(Z_i + \frac{G_8'}{G_8} Z_i' \right) ; \quad \text{Ecker (1993)} \qquad i \in S \, ,$$

...also practically unknown

$$N_i^\theta = N_{i,r}^\theta(\mu) + \lambda \left(Z_i^\theta + \frac{G_8'}{G_8} Z_i'^{\,\theta} + \frac{G_8^\theta}{G_8} Z_i^{\theta\theta} \right) ; \qquad i = 1, \ldots, 9 \, .$$

unknown

we determined* them requiring poles cancellation in $K \to \pi a$

On the weak mass term

Without external sources, the weak mass term $$\mathcal{L}_{\text{weak}}^{(p^2)}\Big|_{G_8'} = \frac{F^4}{4} G_8' \langle \lambda_+ S \rangle$$

can be removed with the redefinition $$\Sigma \to \Sigma' = g_L \Sigma \, g_R^\dagger$$

$$g_L = 1 + i\alpha_L \qquad \alpha_L = G_8' F^2 \left(\frac{m_s + m_d}{m_s - m_d} + \frac{m_s - m_d}{m_s + m_d} \right) \lambda_7$$

$$g_R = 1 + i\alpha_R \qquad \alpha_R = G_8' F^2 \left(\frac{m_s + m_d}{m_s - m_d} - \frac{m_s - m_d}{m_s + m_d} \right) \lambda_7$$

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With the ALP, the redefinition leaves a remnant:

$$\mathcal{L}^{(p^2)}\Big|_{G_8'} = \frac{F^2}{4} \langle [i\alpha_L, l_\mu] L^\mu + [i\alpha_R, r_\mu] R^\mu \rangle$$

$$\propto \frac{\partial_\mu a}{f} \left[\left([k_Q]_{ss} - [k_Q]_{dd} \right) \lambda_6 + \mathrm{Re}\, [k_Q]_{sd} \left(\sqrt{3}\, \lambda_8 - \lambda_3 \right) \right]$$

vanishes for flavor universal ALP!

We use the weak mass term in its transformed form to get rid of tadpoles at O(p^2).
Important: need to perform the redefinition also in the QCD O(p^4) Lagrangian!

$K \to \pi a$ at O(p²)

We work to first order in $1/f$ and G_F.

Depending on the origin of the $s \to d$ transition, distinguish:

"**direct**" contribution:
flavor-violating ALP coupling to s,d

"**indirect**" contribution:
G_F x flavor-conserving ALP couplings

$$\mathcal{A} = \mathcal{A}^{\mathrm{FV}} + \mathcal{A}^{\mathrm{FC}}$$

(neglect contributions with both)

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$$G \in \{G_8, G_8^\theta, G_8', G_{27}^{1/2}, G_{27}^{3/2}\}$$

$$c_{\rm ALP} \in \{\tilde{c}_{GG}, c^a_{uu}, (c^a_{dd} + c^a_{ss}), (c^a_{dd} - c^a_{ss}), (c^v_{dd} - c^v_{ss})\}$$

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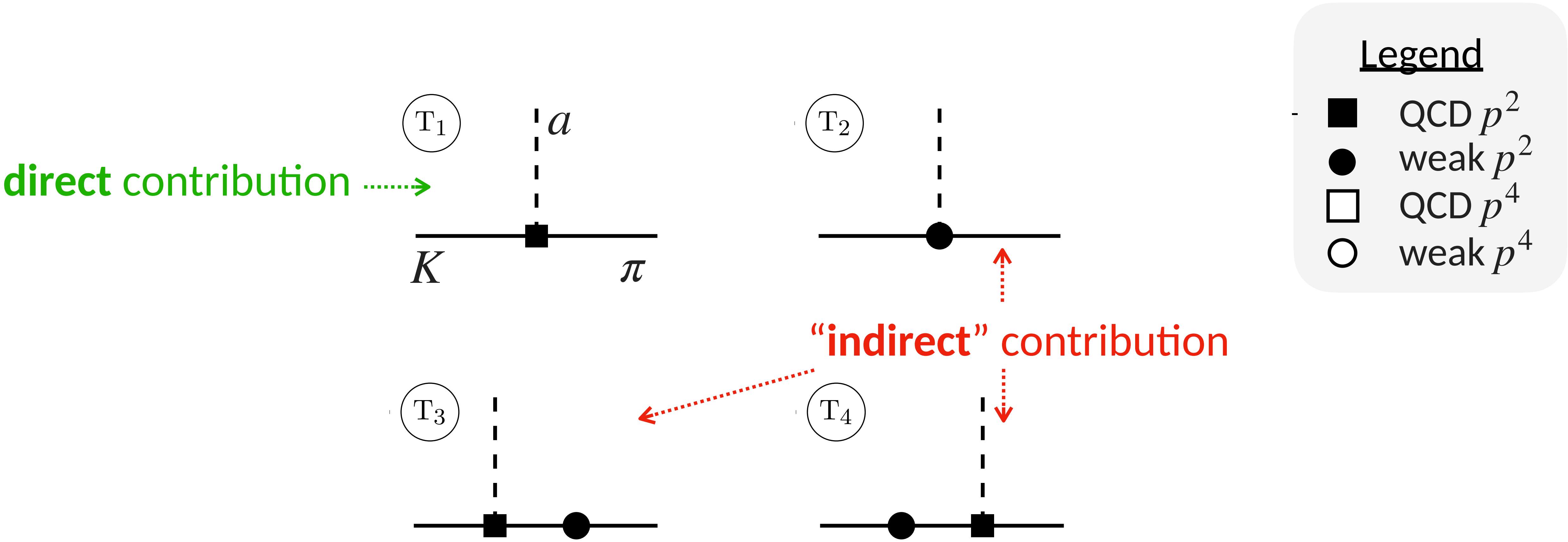
$\bar{d}\gamma^\mu d$ and $\bar{s}\gamma^\mu s$ are not individually conserved in presence of weak interactions

→ flavor-diagonal ALP vector couplings are observable in FCNCs!

$K \to \pi a$ at O(p²)



- the κ_q dependence cancels exactly
- Isospin-breaking corrections are below 1%, it's safe to neglect them at NLO

$K \to \pi a$ at O(p⁴)

We do not consider neither O(p^4) corrections to the 27-plets, nor isospin-breaking terms.

Legend

- ■ QCD p^2
- ● weak p^2
- □ QCD p^4
- ○ weak p^4

The NLO amplitude takes the standard form:

$$\mathcal{A}_{\rm NLO} = (\sqrt{Z_\pi Z_K} - 1)\,\mathcal{A}_{\rm LO} + \mathcal{A}^{(p^4)}_{\Delta m_i^2,\,\Delta F_\pi} + \mathcal{A}^{(p^4)}_{\text{1-loop}} + \mathcal{A}^{(p^4)}_{\rm tree}$$

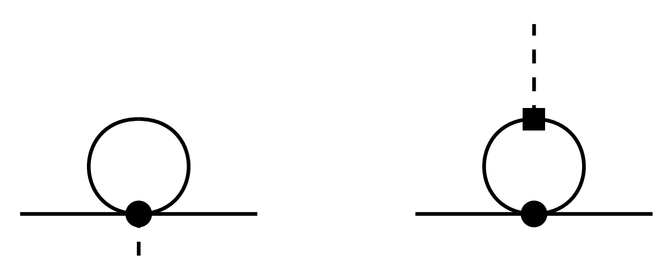
external leg corrections

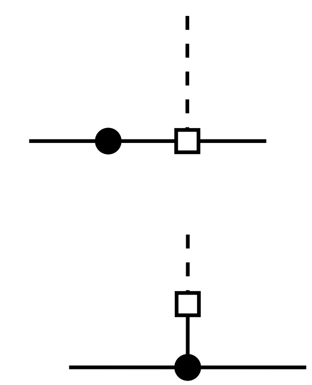
p⁴ shifts of masses & decay constant

1PI 1-loop diagrams with O(p²) insertions, at most 1 weak vertex

tree diagrams with one O(p⁴) insertion

$$Z_i = 1 - \left.\frac{d\Sigma_i(p^2)}{dp^2}\right|_{p^2=m_i^2}$$





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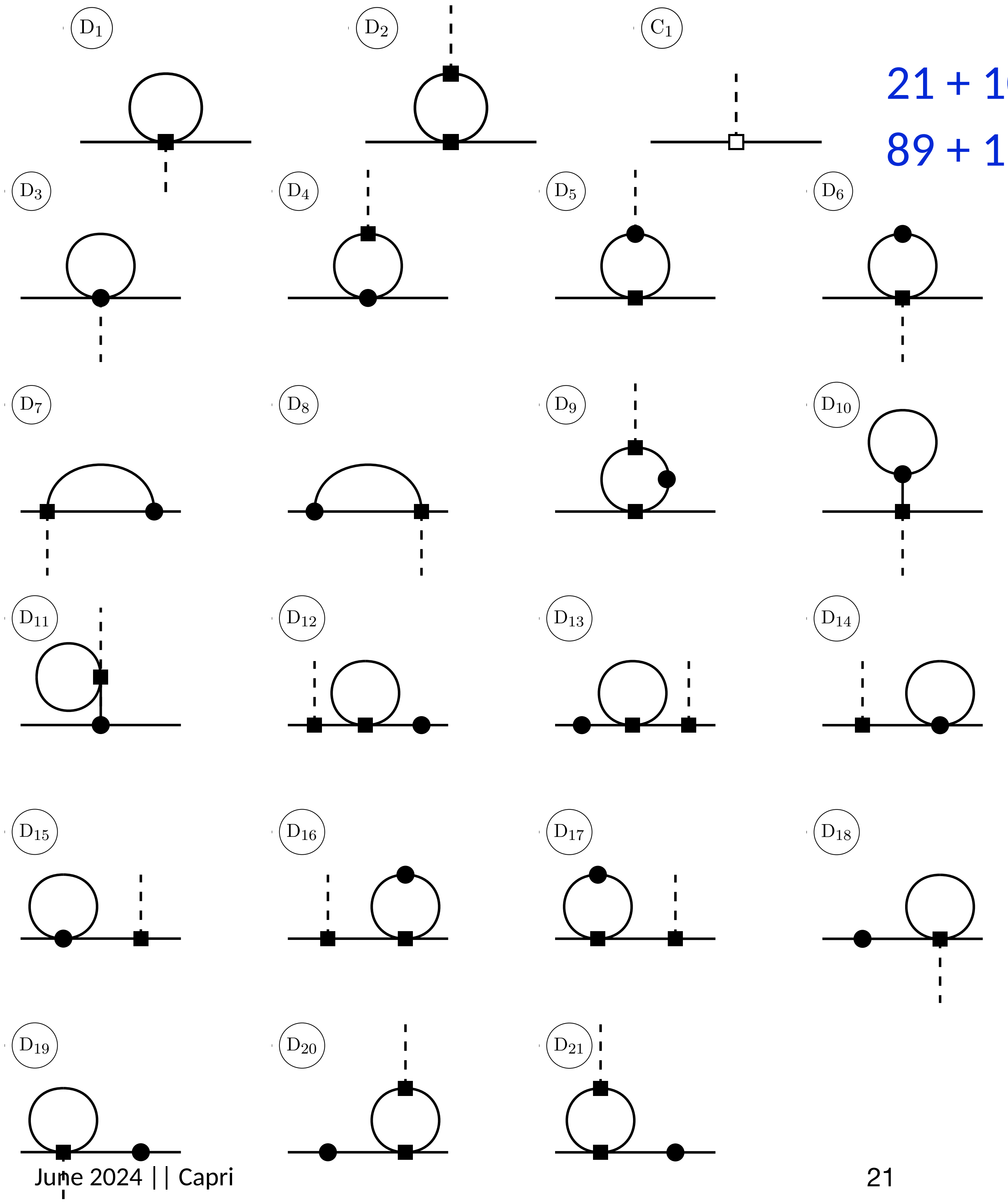

21 + 10 topologies

89 + 11 distinct diagrams

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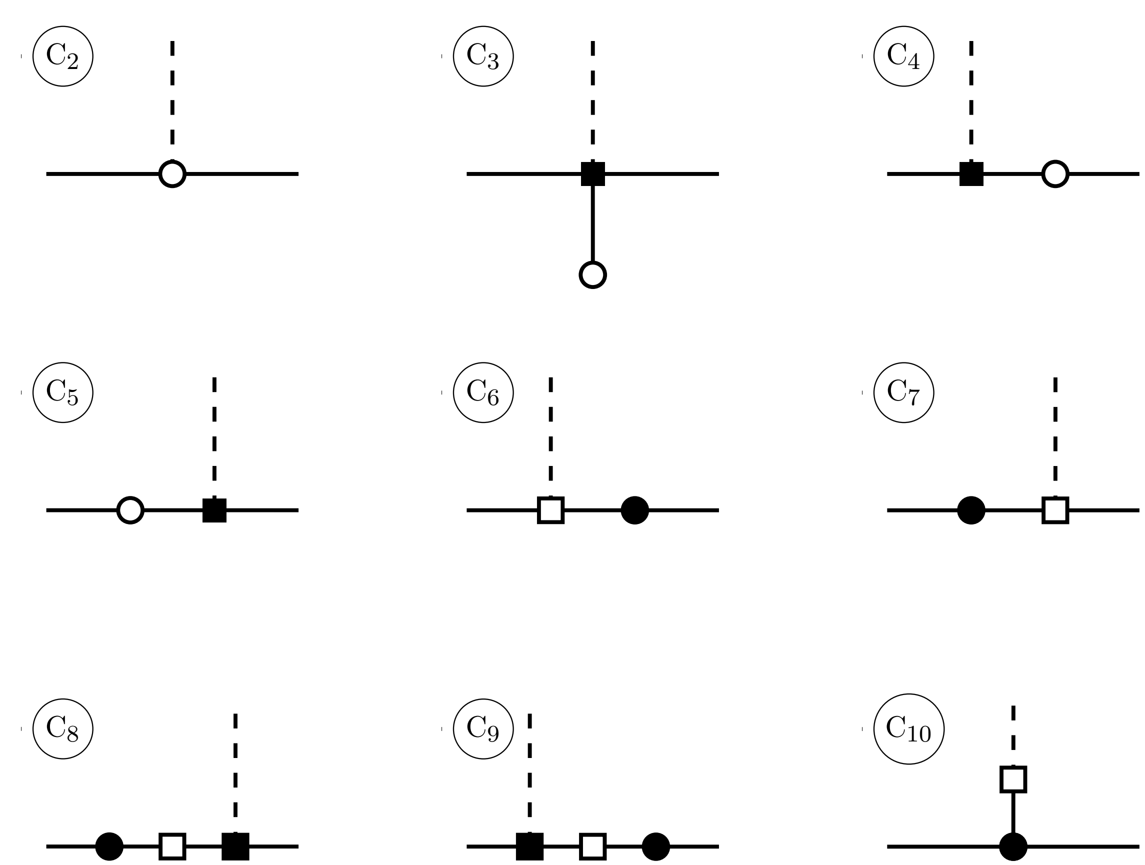

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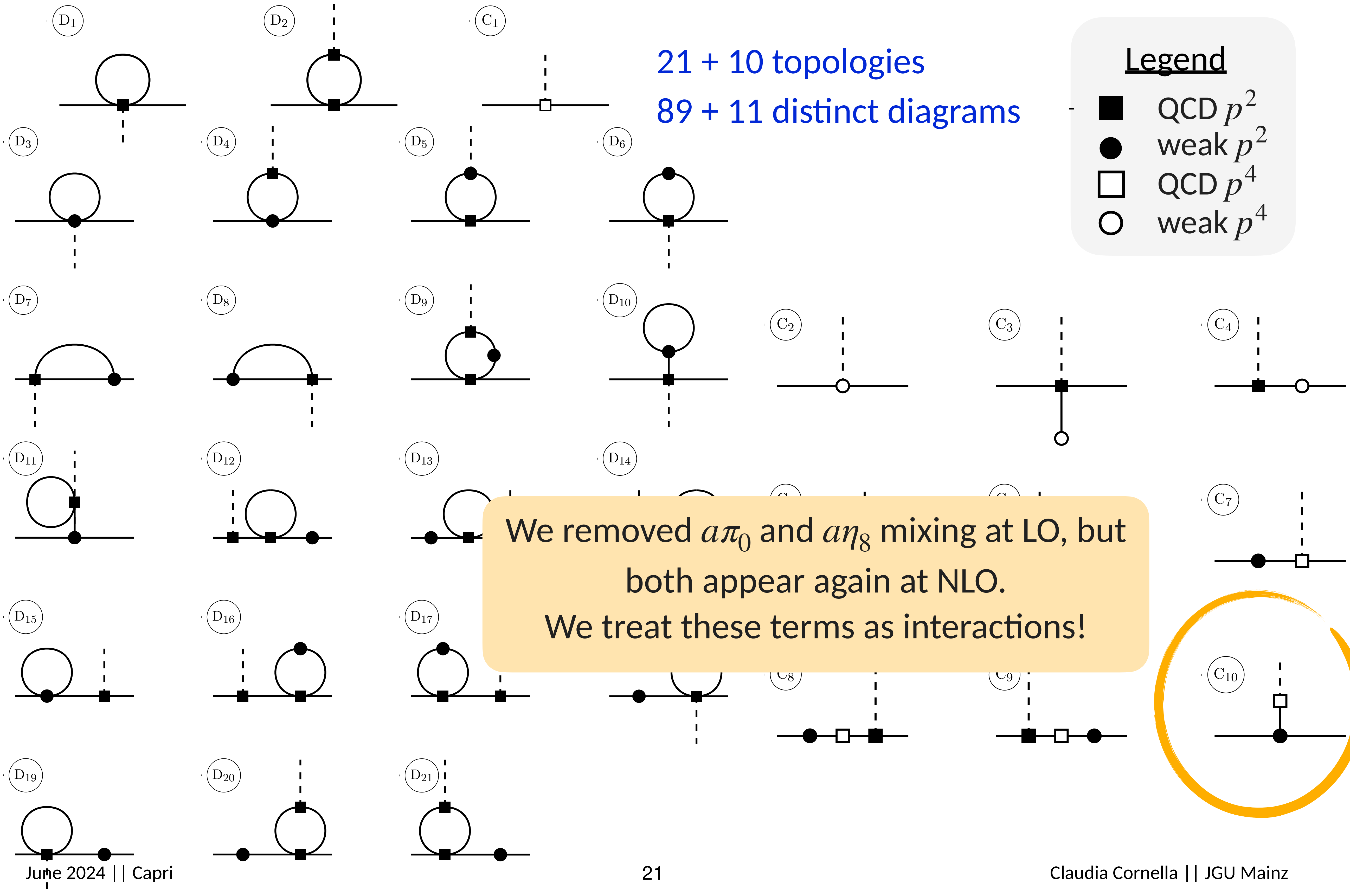

We removed $a\pi_0$ and $a\eta_8$ mixing at LO, but both appear again at NLO.
We treat these terms as interactions!

$K \to \pi a$ at O(p⁴)

The result satisfies two important **consistency checks**:

- again, the κ_q dependence cancels
- **UV divergences** cancel.

  As a byproduct, the explicit μ dependence of the remaining (finite) result is cancelled by the μ dependence of the various LECs.

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- poles with FV ALP couplings canceled by the QCD counterterms given in the literature
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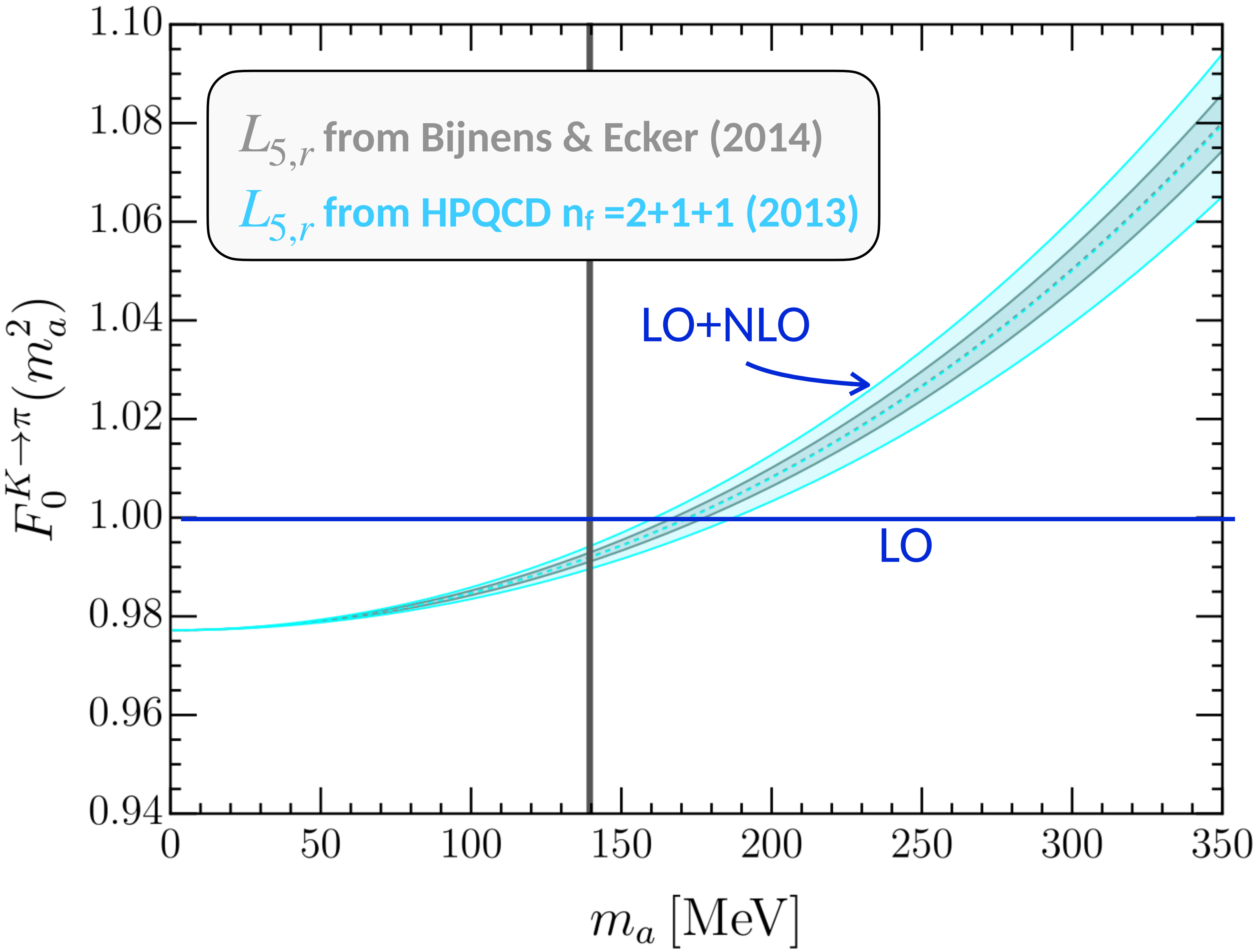
Surprisingly, the amplitude has **no absorptive part**, in contrast with $K_S \to \pi^+\pi^-$, where a strong rescattering phase is generated at NLO.

Size of O(p⁴) effects: direct contribution

$$\mathcal{A}^{\rm FV} = -(m_{K^-}^2 - m_{\pi^-}^2)\,\frac{[k_d + k_D]_{12}}{2f}\,F_0^{K\to\pi}(q^2 = m_a^2)$$



NLO correction:

small (**<10%**) over the entire kinematically allowed mass range

negligible uncertainty, since it depends on a single and quite well-measured LEC, $L_{5,r}$ that enters $\propto m_a$

$$F_0^{K\to\pi}(0) = (1_{\rm LO} - 0.023_{\rm NLO})$$

[Leutwyler, Roos (1984)]

Size of $O(p^4)$ effects: indirect contribution

Lots of unknown (mostly weak) LECs appear in the amplitude
$\rightarrow$ limitation on the predictive power of our results

The best we can do is to assume that at "some" scale μ_0 the unknown LECs are small, so that they can be neglected.

On theory grounds, one expects a reasonable choice for μ_0 to be the scale of chiral symmetry breaking, $\mu_\chi \approx 1.6$ GeV, where the LECs are free of large logs.

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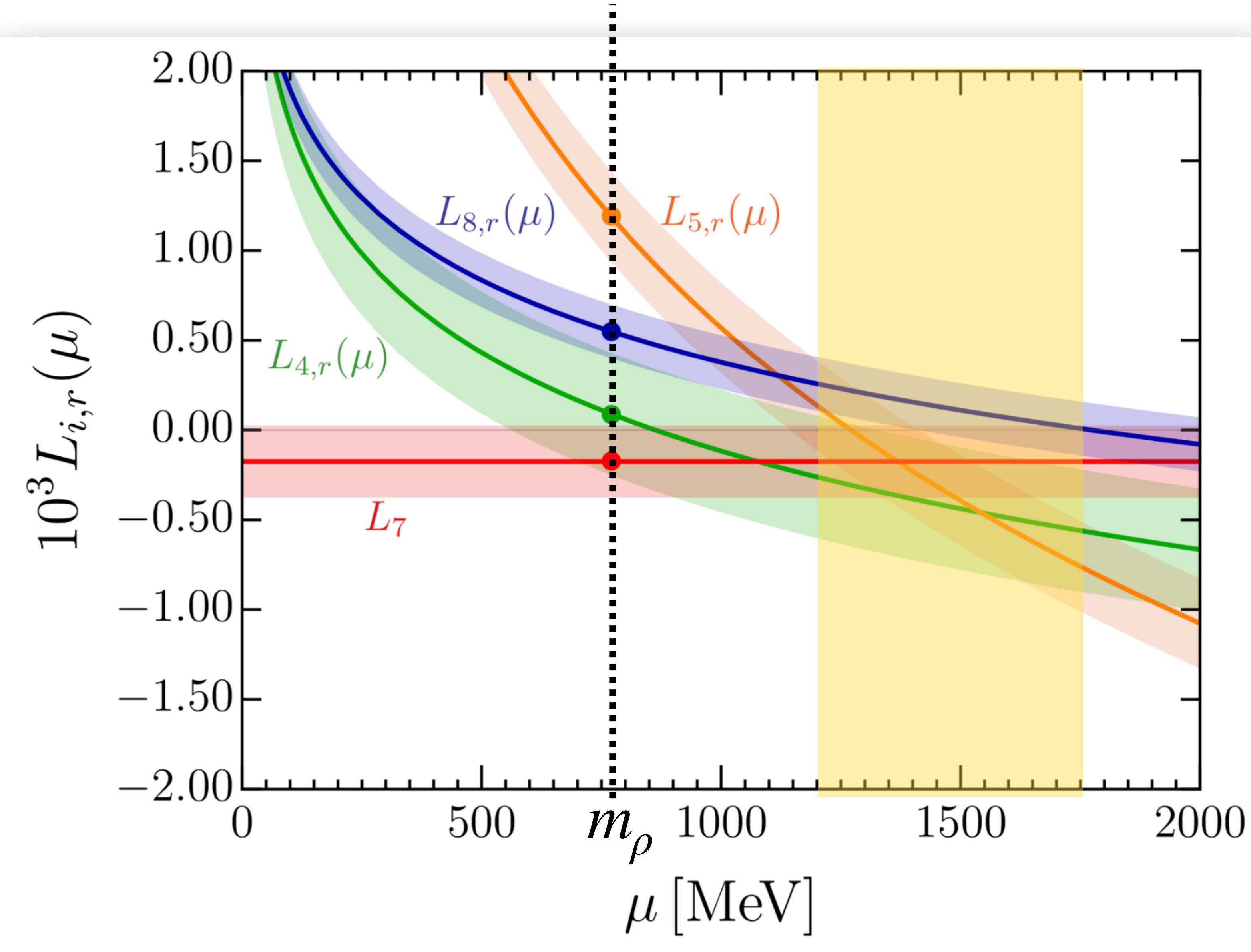

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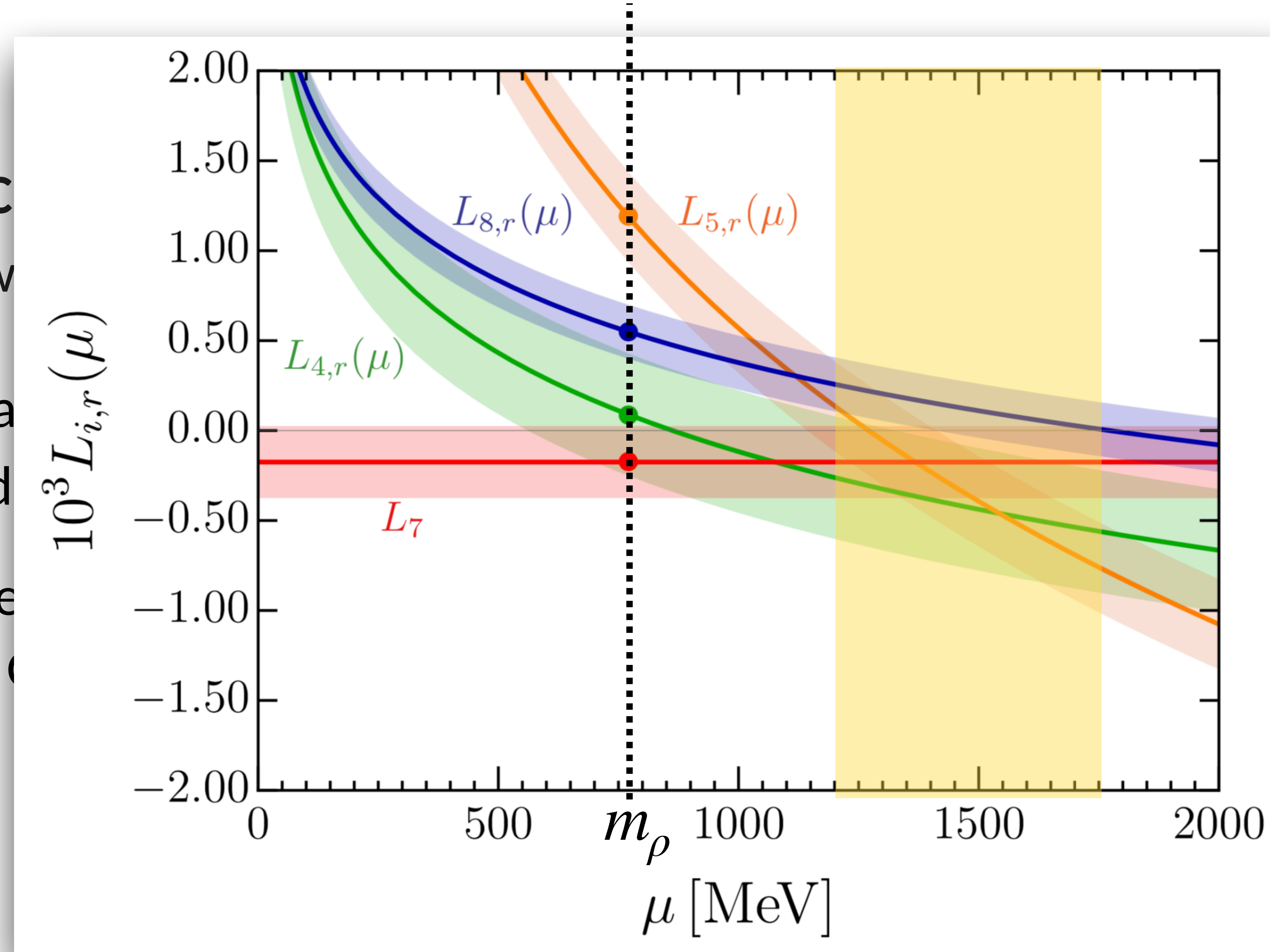
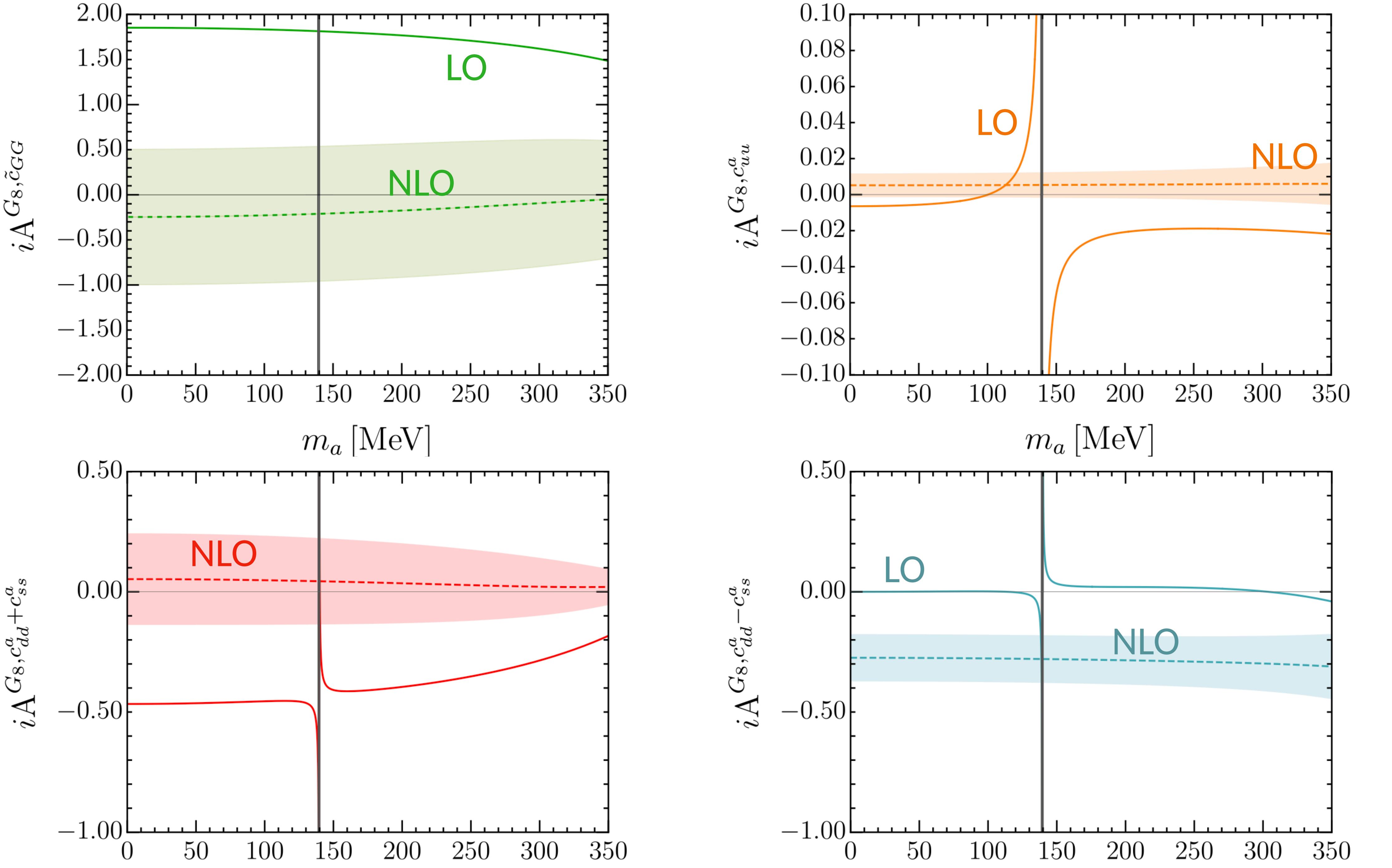


→ we assume the **unknown LECs** to have a **similar** behaviour:

- set them to 0 at $\mu_0 \approx 1.4$ GeV
- vary μ_0 by $\sqrt{2}$ in both directions (1 - 2 GeV) to estimate model uncertainty

Size of O(p⁴) effects: indirect contribution



Weak dependence from the alp mass, except for $m_a \approx m_\pi$

Bounds on ALP couplings

Experimental bound(s) set by NA62:

[NA62 2021]

$$\mathcal{B}(K^+ \to \pi^+ X) < (3-6) \times 10^{-11} \quad \text{@90\% CL} \quad m_X \in [0, 110]\ \text{MeV}$$

$$\mathcal{B}(K^+ \to \pi^+ X) < 1 \times 10^{-11} \quad \text{@90\% CL} \quad m_X \in [160, 260]\ \text{MeV}$$

Switching on one coupling at a time, they can be translated into a lower bound on the effective scale $\Lambda_i^{\text{eff}} = f/|c_i|$.

The probed NP scales range from few to tens of TeV.

Strong bounds on flavor-changing ALP couplings call for a flavor symmetry in the UV!

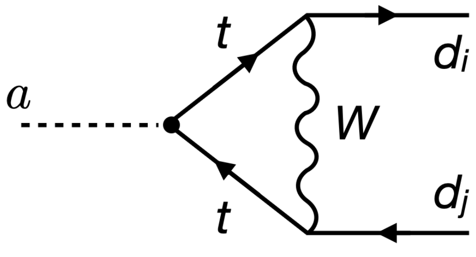
$c_i(\mu_\chi)$	$\Lambda_{c_i}^{\text{eff}}$ [TeV] $m_a = 0$	$m_a = 200\,\text{MeV}$
$[k_D + k_d]_{12}$	$2.9 \cdot 10^8$	$3.0 \cdot 10^8$
$\tilde{c}_{GG}{}^{(*)}$	43	39
c_{uu}^a	1.5	2.0
$c_{dd}^a + c_{ss}^a$	15	9
$c_{dd}^a - c_{ss}^a{}^{(**)}$	8	4
$c_{dd}^v - c_{ss}^v{}^{(**)}$	23	22

(*) assuming $g_{8\theta} = 0$

Bounds on ALP couplings for a flavor universal ALP

Consider a **flavor-universal ALP** at $\Lambda = 4\pi f$.

Reminder: flavor violating ALP couplings are anyway generated at low energies via RGE!



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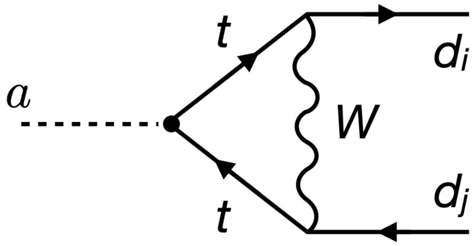

$c_i(\Lambda)$	$\Lambda^{\text{eff}}_{c_i}$ [TeV] $m_a = 0$	$\Lambda^{\text{eff}}_{c_i}$ [TeV] $m_a = 200\,\text{MeV}$
$\tilde{c}_{GG}(\Lambda)$	49	97
$\tilde{c}_{WW}(\Lambda)$	2.6	6
$\tilde{c}_{BB}(\Lambda)$	0.02	0.04
$\tilde{c}_{u}(\Lambda)$	$1.9 \cdot 10^3$	$4.2 \cdot 10^3$
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(assuming $G_8^\theta = 0$, $f = 1$ TeV)

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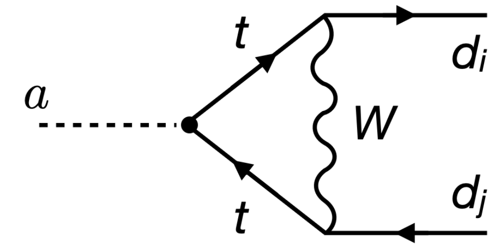

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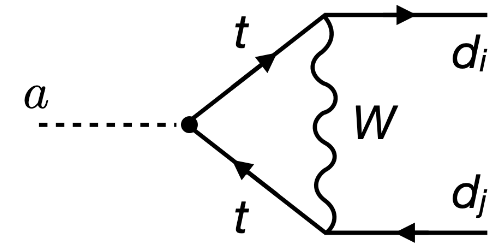

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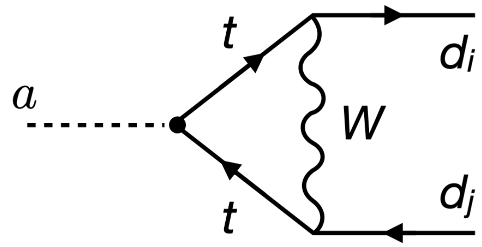

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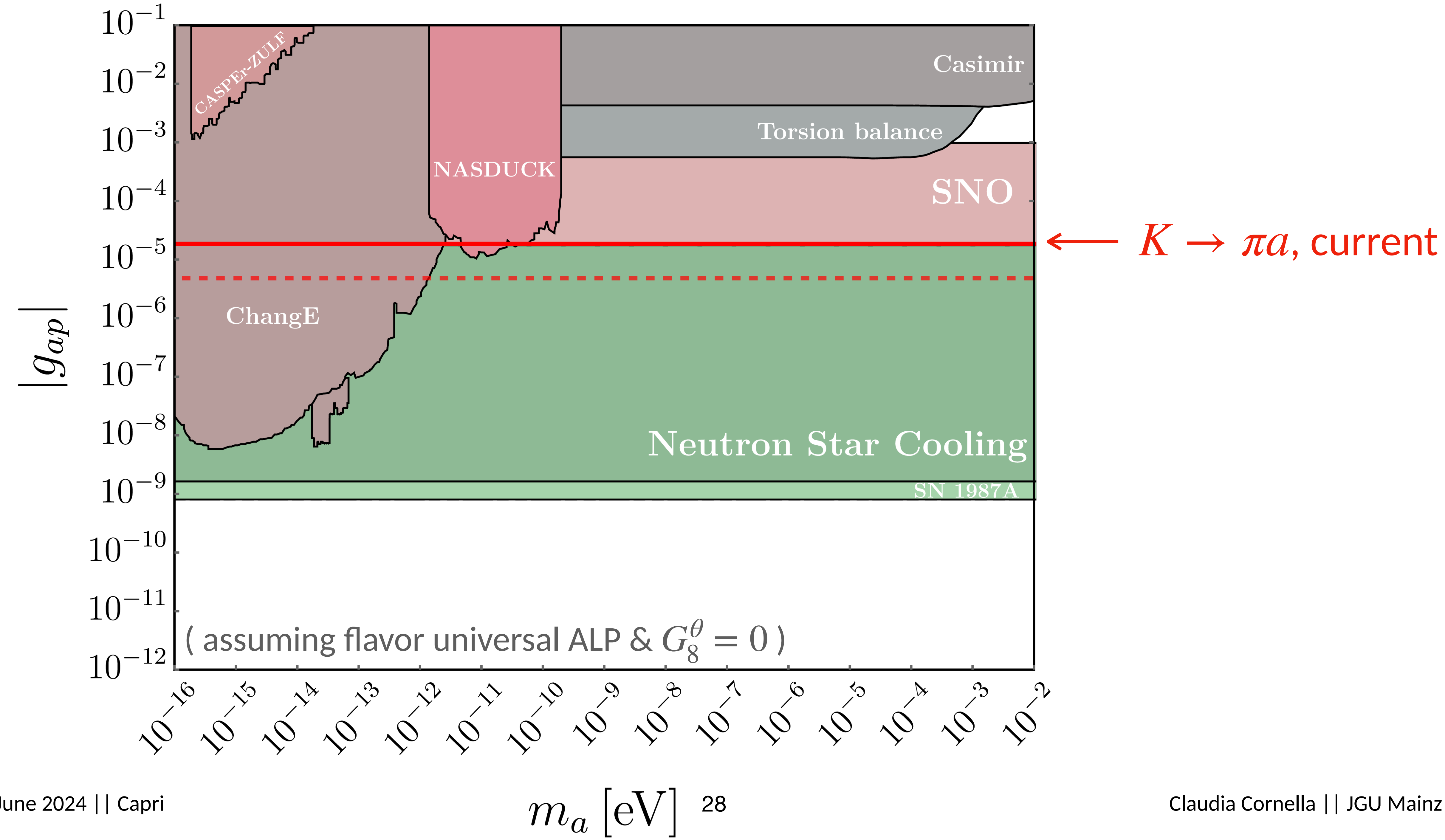
Still, $K \to \pi a$ remains the **strongest particle-physics probe** for $m_a \lesssim 300$ MeV !

Bounds on ALP couplings to nucleons

$K \to \pi a$ can be used to constrain the ALP couplings to nucleons.

Competes with several constraints from non-accelerator and astrophysical probes!



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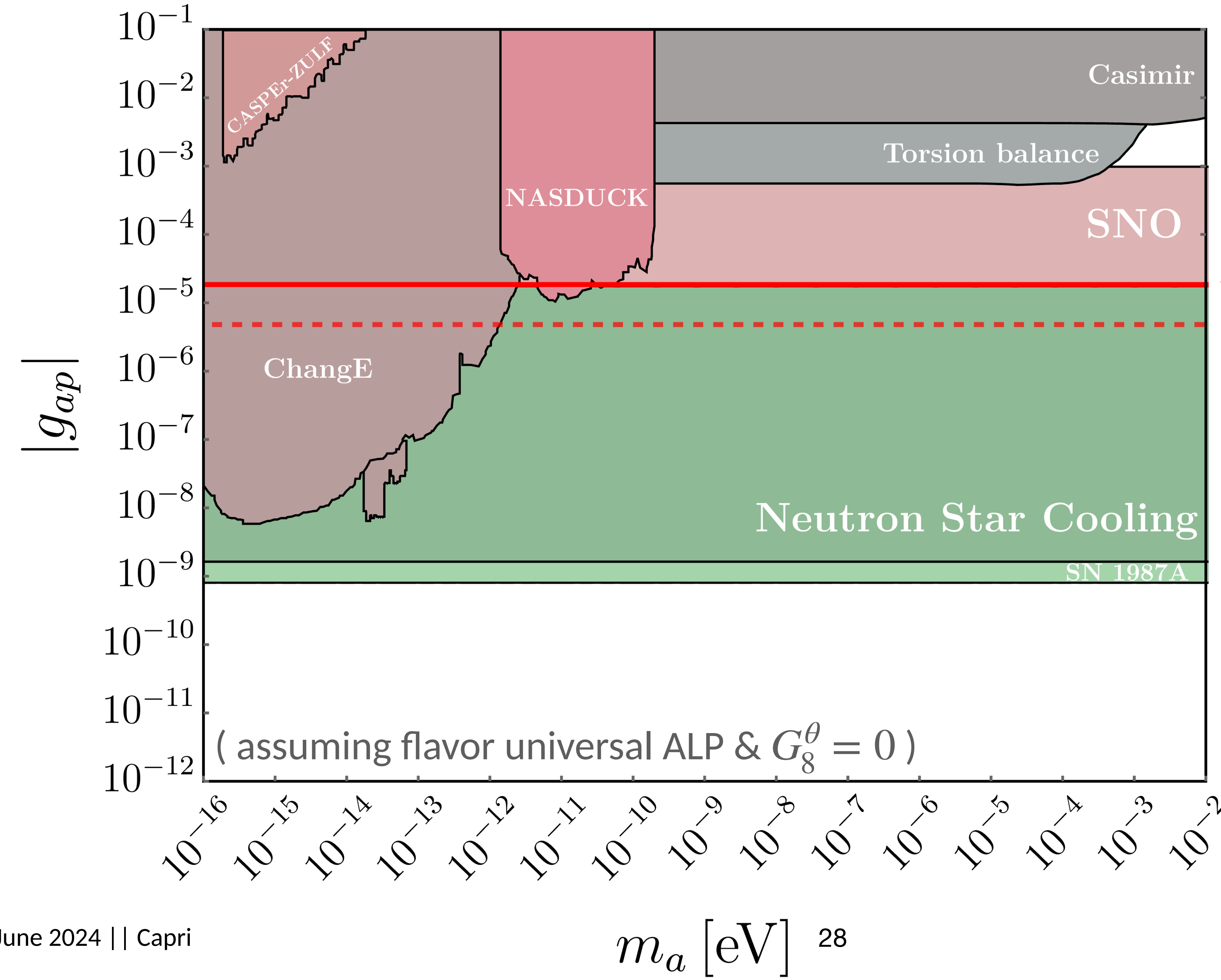


$\longleftarrow$ $K \to \pi a$, current

Conclusions

Weak meson decays are some of the most powerful probes of ALPs.

$K \to \pi a$ is a good example, and I discussed the associated challenges!

With this rigorous framework at hand, we aim to study other modes, e.g. $K_0 \to \pi_0 a$ and $\pi^- \to e^- \bar{\nu}\, a$.

Open Questions and Future Directions in Flavour Physics


The cat of Villa Orlandi ©Tiziano Pojer

Back-up

Size of O(p⁴) effects: indirect contribution

Contribution proportional to G_8 (for m_a = 0):

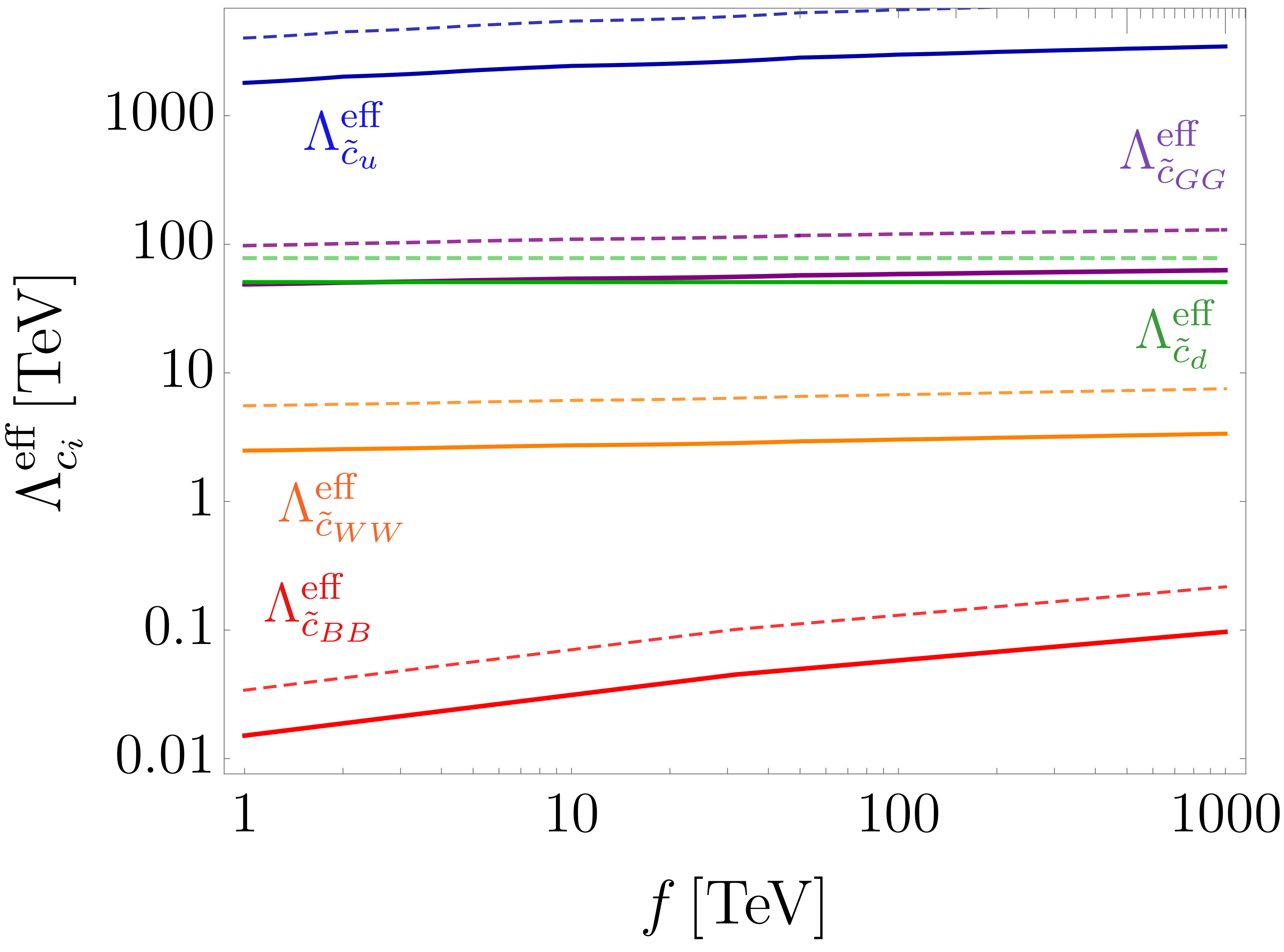
$$i\mathcal{A}_{\rm LO}^{G_8} = \frac{G_8 F_\pi^2\, m_K^2}{2f}\Big[\left(1.88 - 0.88\,\varepsilon^{(2)}\right)\tilde{c}_{GG} - \left(0.02 - 0.44\,\varepsilon^{(2)}\right)c^a_{uu} - \left(0.48 - 0.44\,\varepsilon^{(2)}\right)\left(c^a_{dd} + c^a_{ss}\right) + 0.54\left(c^v_{dd} - c^v_{ss}\right)\Big]\,,$$

$$i\mathcal{A}_{\rm NLO}^{G_8} = \frac{G_8 F_\pi^2\, m_K^2}{2f}\Big[(-0.25 \pm 0.43 \pm 0.61)\,\tilde{c}_{GG} + (5.21 \pm 1.03 \pm 6.52)\cdot 10^{-3}\, c^a_{uu} + (0.06 \pm 0.11 \pm 0.16)\left(c^a_{dd} + c^a_{ss}\right) - (0.27 \pm 0.10 \pm 0)\left(c^a_{dd} - c^a_{ss}\right) + (0.24 \pm 0.23 \pm 0.18)\left(c^v_{dd} - c^v_{ss}\right)\Big]\,,$$

- Moderate NLO corrections with sizeable uncertainties due to O(15) unknown weak LECs + uncertainty of some of the "measured" LECs, especially $L_{7,r}$
- Crossed-out terms vanish for a flavor-universal ALP in the UV

Bounds on ALP couplings: dependence on f

Bounds on the Λ_i^{eff} depend logarithmically on f:



$(G_8^\theta = 0)$

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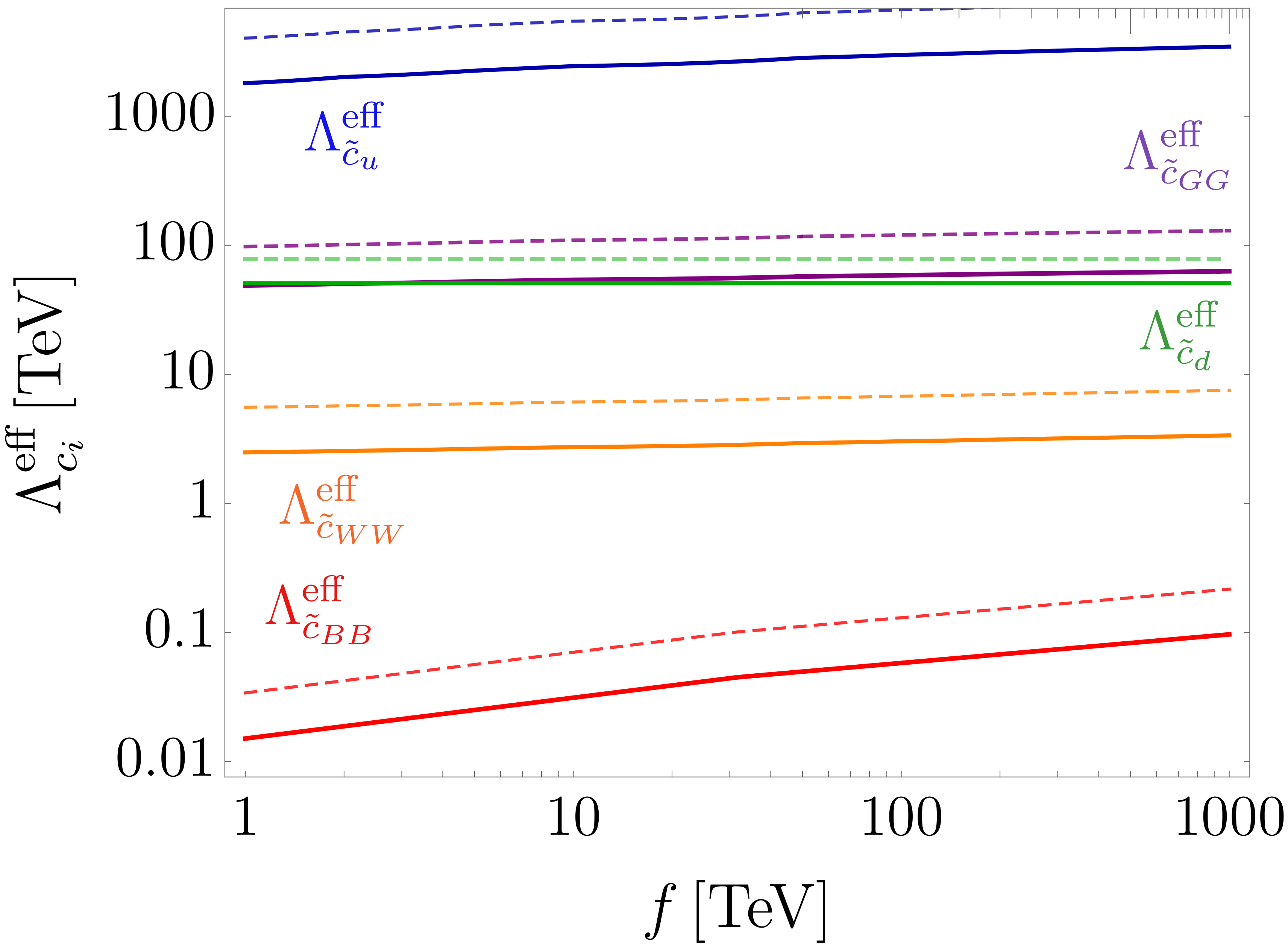

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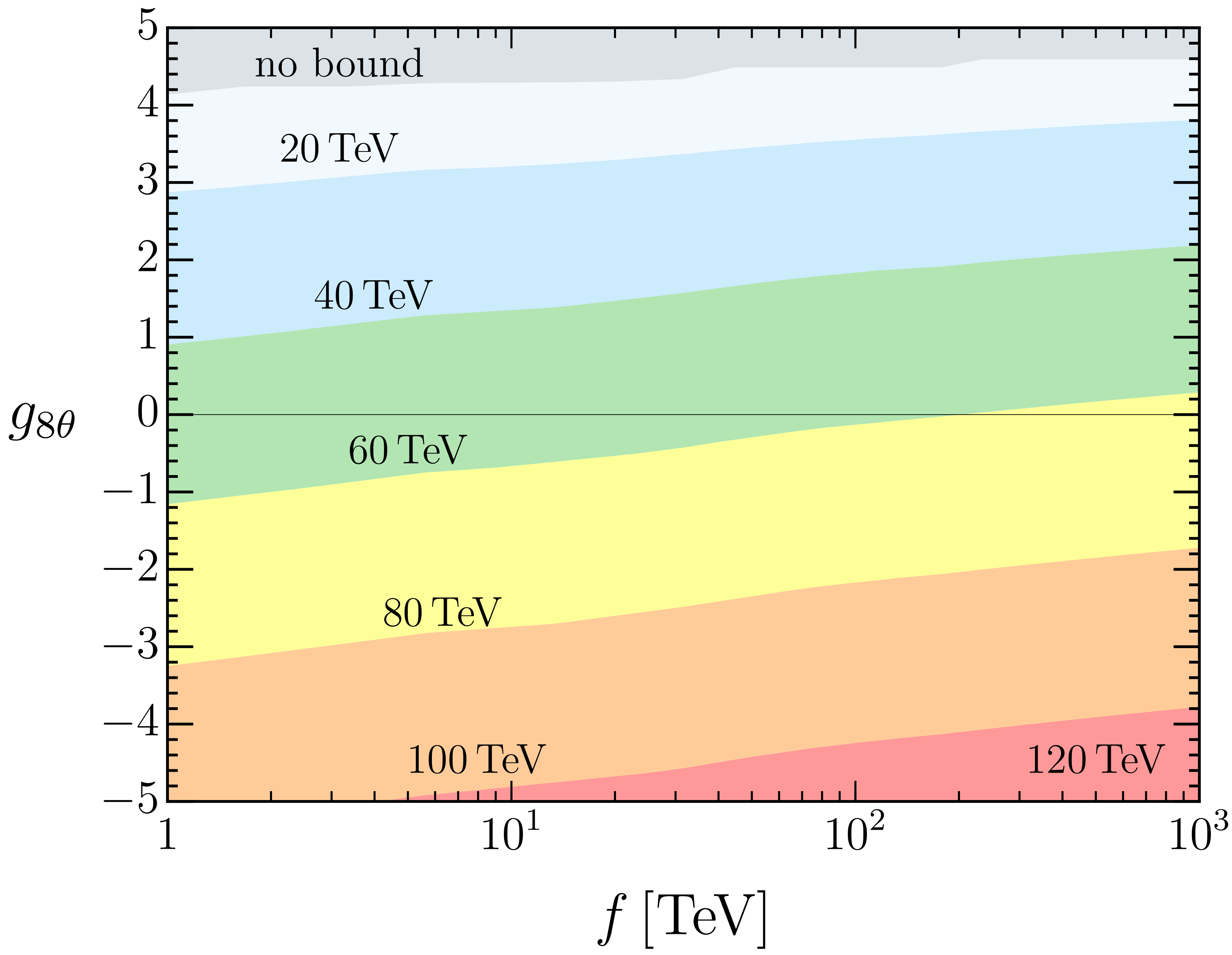
Bounds on ALP couplings: dependence on $g_{8\theta}$

Bounds on the $\Lambda^{\text{eff}}_{\tilde{c}_{GG}}$, $\Lambda^{\text{eff}}_{\tilde{c}_d}$ depend on the (unknown) low-energy coupling $g_{8\theta}$.

For $\tilde{c}_{GG}$:



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