Role of pion exchange in photoproduction: from current conservation to reggeization

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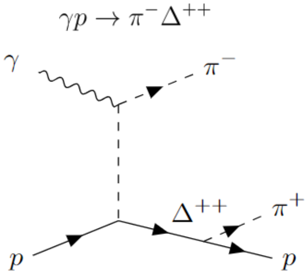
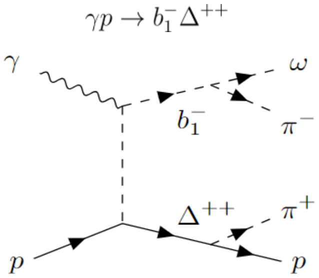
5th Workshop on Future Directions in Spectroscopy Analysis (FDSA2024)
Genova, January 22-24, 2024

Search for the hybrid mesons

- Identifying the spectrum of hybrid mesons in photoproduction is the primary purpose of the GlueX experiment.
- Understanding the production mechanism in light meson photoproduction reactions is essential for the successful analysis of the data.

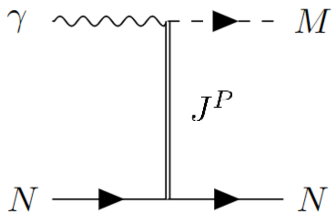


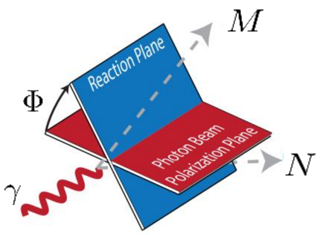
[Talks by F. Afzal and V. Mathieu]



[Talks by A. Schertz and V. Shastry]

Generalities of meson photoproduction at high photon energies



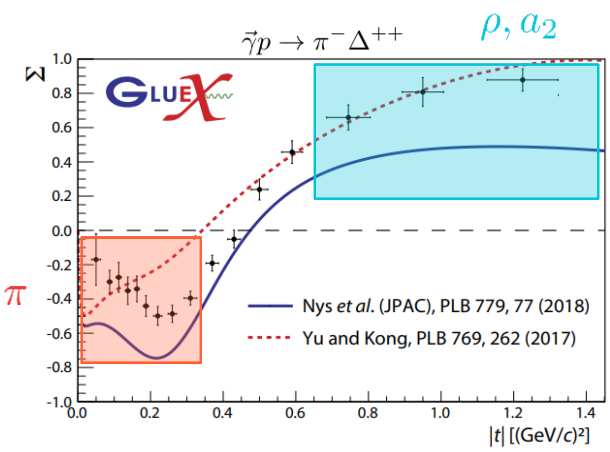


- At high energies, single meson photoproduction is dominated by the exchange of Regge trajectories in the t-channel.

- The beam polarization allows one to distinguish between exchange of

  → unnatural $(P(-1)^J = -1)$ parity

  → natural $(P(-1)^J = 1)$ parity



[GlueX Collaboration, *Phys.Rev.C* 103 (2021) 2, L022201]

- In peripheral high energy **pion photoproduction**, pion exchange dominates at small momentum transfer:

  → The t-channel pion exchange process is not gauge invariant by itself

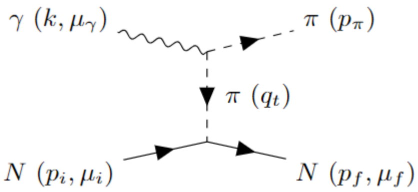
  → It is most susceptible to absorption corrections (longest range interaction)

  → Reggeization scheme

Pion Born diagram

- s-channel reaction: $\gamma(k,\mu_\gamma) + N(p_i,\mu_i) \to \pi(p_\pi) + N(p_f,\mu_f)$
- Helicity amplitude: $A_{\mu_\gamma\mu_i\mu_f} = \epsilon_{\mu_\gamma}(k)\cdot J_{\mu_i\mu_f}$

t-channel Born diagram



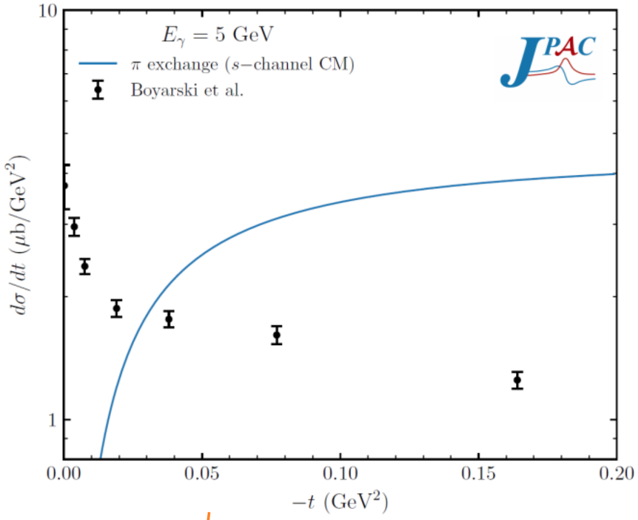
$$J^\mu_{\mu_i\mu_f,t} = -e_\pi g_{\pi NN}\frac{q_t^\mu - p_\pi^\mu}{t-\mu^2}\bar{u}_{\mu_f}(p_f)\gamma_5 u_{\mu_i}(p_i)$$

→ The current is not conserved

↓

the amplitude is not gauge invariant (frame dependent)

$g_{\pi NN} = 13.48$ → PS πNN coupling



[G.Montana et al. (in preparation)]

Pion exchange cannot reproduce experimental cross section at small momentum transfer

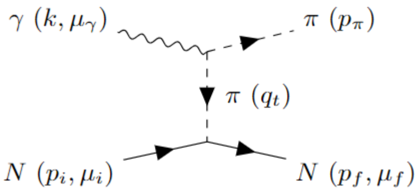
Adding the nucleon Born diagrams

- s-channel reaction: $\gamma(k,\mu_\gamma) + N(p_i,\mu_i) \to \pi(p_\pi) + N(p_f,\mu_f)$
- Helicity amplitude: $A_{\mu_\gamma\mu_i\mu_f} = \epsilon_{\mu_\gamma}(k)\cdot J_{\mu_i\mu_f}$

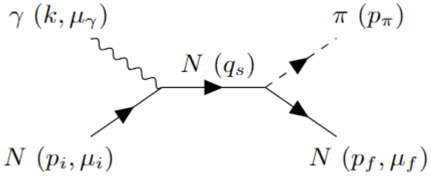
$$J^\mu_{\mu_i\mu_f} = J^\mu_{\mu_i\mu_f,t} + J^\mu_{\mu_i\mu_f,s} + J^\mu_{\mu_i\mu_f,u}$$

→ The total current is conserved

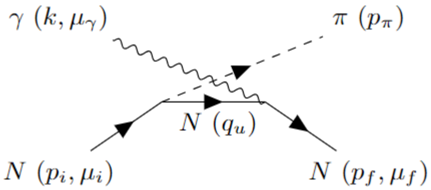
t-channel Born diagram



s-channel



u-channel



$$J^\mu_{\mu_i\mu_f,t} = -e_\pi g_{\pi NN}\frac{q_t^\mu - p_\pi^\mu}{t-\mu^2}\bar{u}_{\mu_f}(p_f)\gamma_5 u_{\mu_i}(p_i)$$

$$J^\mu_{\mu_i\mu_f,s} = e_N g_{\pi NN}\bar{u}_{\mu_f}(p_f)\gamma_5\frac{\not{q}_s + M}{s-M^2}\gamma^\mu u_{\mu_i}(p_i)$$

$$J^\mu_{\mu_i\mu_f,u} = e_N g_{\pi NN}\bar{u}_{\mu_f}(p_f)\gamma^\mu\frac{\not{q}_u + M}{u-M^2}\gamma_5 u_{\mu_i}(p_i)$$

- Separate electric and magnetic contributions: $A_{\mu_\gamma\mu_i\mu_f} = A^{\mathrm{e}}_{\mu_\gamma\mu_i\mu_f} + A^{\mathrm{m}}_{\mu_\gamma\mu_i\mu_f}$

$$A^{\mathrm{e}}_{\mu_\gamma\mu_i\mu_f} = 2g_{\pi NN}\left[e_\pi\frac{(\epsilon_{\mu_\gamma}\cdot p_\pi)}{t-\mu^2} + e_{N_i}\frac{(\epsilon_{\mu_\gamma}\cdot p_i)}{s-M^2} + e_{N_f}\frac{(\epsilon_{\mu_\gamma}\cdot p_f)}{u-M^2}\right]\bar{u}_{\mu_f}(p_f)\gamma_5 u_{\mu_i}(p_i)$$

$$A^{\mathrm{m}}_{\mu_\gamma\mu_i\mu_f} = g_{\pi NN}\left[\frac{e_{N_i}}{s-M^2} + \frac{e_{N_f}}{u-M^2}\right]\bar{u}_{\mu_f}(p_f)\gamma_5\not{k}\not{\epsilon}_{\mu_\gamma} u_{\mu_i}(p_i)$$

Electric term

$$A^{\rm e}_{\mu_\gamma\mu_i\mu_f} = 2g_{\pi NN}\left[e_\pi \frac{(\epsilon_{\mu_\gamma}\cdot p_\pi)}{t-\mu^2} + e_{N_i}\frac{(\epsilon_{\mu_\gamma}\cdot p_i)}{s-M^2} + e_{N_f}\frac{(\epsilon_{\mu_\gamma}\cdot p_f)}{u-M^2}\right]\bar{u}_{\mu_f}(p_f)\gamma_5 u_{\mu_i}(p_i)$$

- Using momentum conservation and electric charge conservation ($e_{N_i} = e_\pi - e_{N_f}$):

$$\begin{aligned} A^{\rm e}_{\mu_\gamma\mu_i\mu_f} = & \; g_{\pi NN}\Bigg[2e_\pi\left(\frac{(\epsilon_{\mu_\gamma}\cdot p_\pi)}{t-\mu^2} + \frac{(\epsilon_{\mu_\gamma}\cdot(p_i+p_f))}{s-u}\right) \\ & + e_{N_i}\left(\frac{(\epsilon_{\mu_\gamma}\cdot p_\pi)}{s-M^2} + \frac{(\epsilon_{\mu_\gamma}\cdot(p_i+p_f))}{s-u}\frac{t-\mu^2}{s-M^2}\right) \\ & - e_{N_f}\left(\frac{(\epsilon_{\mu_\gamma}\cdot p_\pi)}{u-M^2} + \frac{(\epsilon_{\mu_\gamma}\cdot(p_i+p_f))}{s-u}\frac{t-\mu^2}{u-M^2}\right)\Bigg]\bar{u}_{\mu_f}(p_f)\gamma_5 u_{\mu_i}(p_i) \end{aligned}$$

→ Minimal gauge invariant (m.g.i.)

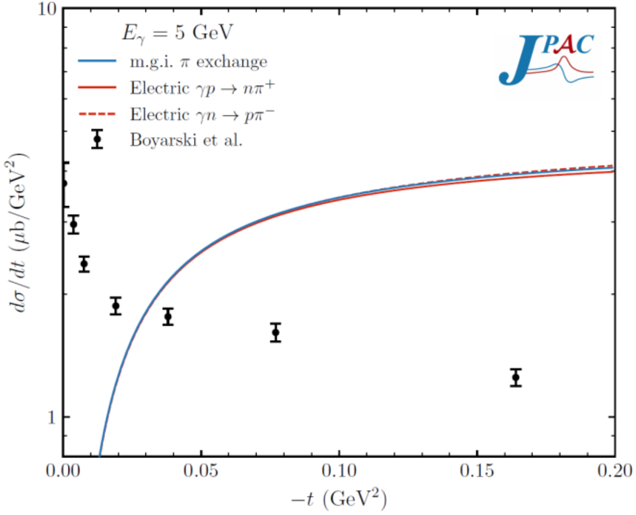
- Differential cross section

$$\left(\frac{d\sigma}{dt}\right)_{\pi\text{-m.g.i.}} = 4\left(\frac{s-M^2}{s-u}\right)^2\left(\frac{d\sigma}{dt}\right)_{\pi\text{-bare, CM}} \overset{t\to t_{\rm min}}{\approx} \left(\frac{d\sigma}{dt}\right)_{\pi\text{-bare, CM}}$$

$$\left(\frac{d\sigma}{dt}\right)_{\text{e, }\gamma p\to\pi^+ n} = \left(\frac{d\sigma}{dt}\right)_{\pi\text{-bare, CM}}$$

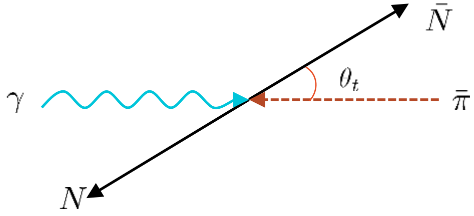
$$\left(\frac{d\sigma}{dt}\right)_{\text{e, }\gamma n\to\pi^- p} = 4\left(\frac{s-M^2}{M^2-u}\right)^2\left(\frac{d\sigma}{dt}\right)_{\pi\text{-bare, CM}} \overset{t\to t_{\rm min}}{\approx} \left(\frac{d\sigma}{dt}\right)_{\pi\text{-bare, CM}}$$

[G.Montana et al. (in preparation)]



Pion pole in the *t*-channel rest frame

- t-channel reaction: $\gamma(k,\lambda_\gamma)+\bar{\pi}(-p_\pi)\to\bar{N}(-p_i,\lambda_i)+N(p_f,\lambda_f)\,.$



$$A^{\mathrm{e}}_{\lambda_\gamma\lambda_i\lambda_f}=g_{\pi NN}\left[2e_\pi\left(\frac{1}{s-u}\right)+e_{N_i}\left(\frac{1}{s-M^2}-\frac{2}{s-u}\right)-e_{N_f}\left(\frac{1}{u-M^2}-\frac{2}{s-u}\right)\right](\epsilon_{\lambda_\gamma}\cdot(p_i+p_f))\,\bar{u}_{\lambda_f}(p_f)\gamma_5 v_{\lambda_i}(-p_i)$$

→ The nucleon Born terms contain a “pion pole” that arises from kinematical factors

$$\bar{\epsilon}_{\mu\nu}\equiv\sum_{\lambda_\gamma=\pm1}\epsilon_{\lambda_\gamma,\mu}(k)\epsilon^*_{\lambda_\gamma,\nu}(k)=-g_{\mu\nu}-\frac{k_\mu k_\nu}{(k\cdot n)^2}+\frac{k_\mu n_\nu+k_\nu n_\mu}{k\cdot n}\qquad\text{with}\qquad n=(1,\mathbf{0})$$

$$\bar{\epsilon}_{\mu\nu}P^\mu P^\nu=\frac{2(k\cdot P)(n\cdot P)(k\cdot n)-P^2(k\cdot n)^2-(k\cdot P)^2}{(k\cdot n)^2}\sim\frac{1}{t-\mu^2}\qquad\text{with}\quad P^\mu=(p_i+p_f)^\mu$$

→ Reggeization of the pion should involve the electric components of the nucleon Born terms

Magnetic term

$$A^{\mathrm{m}}_{\mu_\gamma \mu_i \mu_f} = g_{\pi NN} \left[\frac{e_{N_i}}{s - M^2} + \frac{e_{N_f}}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 \not{k} \not{\epsilon}_{\mu_\gamma} u_{\mu_i}(p_i)$$

- In the limit $t \to t_{\text{min}}$:

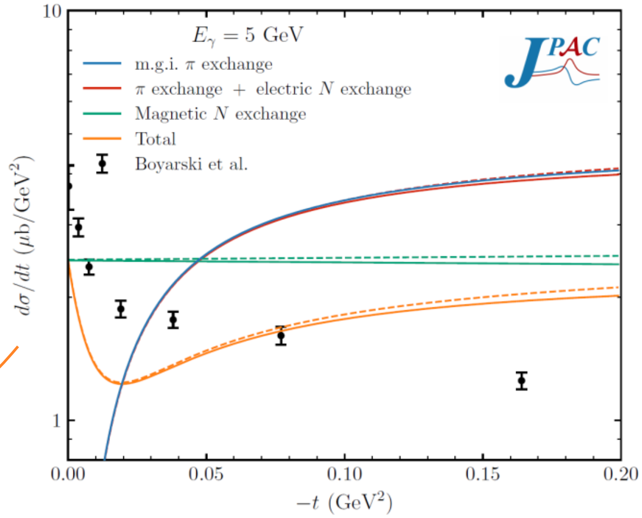
  → The pion exchange diagram vanishes.

  → The electric component of the nucleon exchanges also vanish.

  → The magnetic component of the nucleon exchanges has small dependence in t.

The size of the cross section agrees reasonably well with the data when the magnetic contribution of the nucleon-exchange diagrams is taken into account.

[G.Montana et al. (in preparation)]



Alternative explanations of the experimental data:

- Absorption corrections
- Pion conspiracy

Reggeization of pion exchange

- The exchanged pion is expected to reggeize:

  → Consider the exchange of all the members of the pion trajectory i.e. all the particles with different spin J but the same parity $P=-(-1)^J$, isospin, as the pion.

- In the Regge-pole approximation:

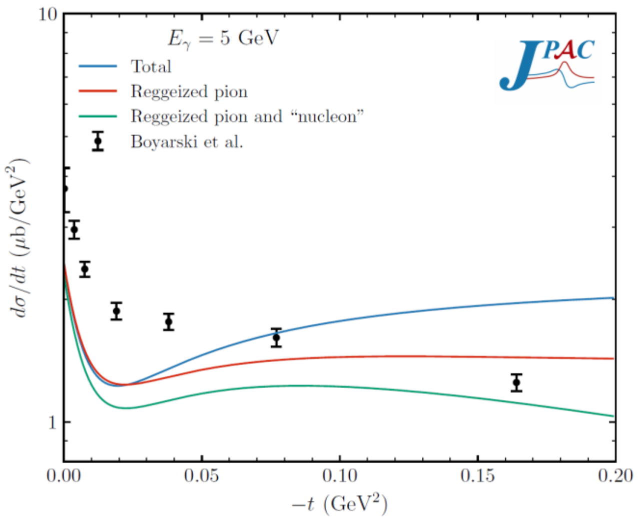
$$\frac{1}{t-\mu^2} \longrightarrow \mathcal{P}_\pi^{\text{Regge}} = \frac{\pi\alpha'_\pi}{2}\frac{1+e^{-i\pi\alpha_\pi(t)}}{\sin\pi\alpha_\pi(t)}\left(\frac{s}{s_0}\right)^{\alpha_\pi(t)}$$

Pion trajectory: $\alpha_\pi(t)=\alpha'_\pi(t-\mu^2)$ with $\alpha'_\pi=0.7$

- In the VGL model, the full Born amplitude (pion and nucleon exchanges, electric and magnetic) was reggeized.

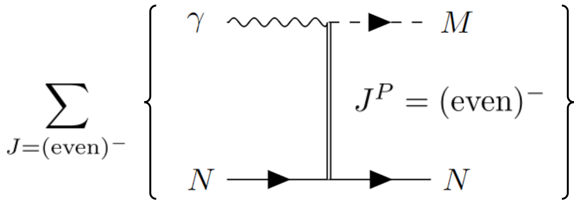
[M.Guidal, J.M.Laget and M. Vanderhaeghen, *Nucl.Phys.A* 627 (1997) 645-678]

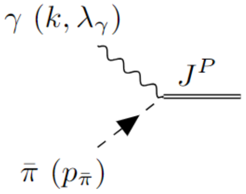
[G.Montana et al. (in preparation)]



New approach to Reggeization of pion exchange

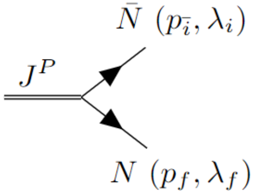
- Consider the explicit exchange of all the t-channel partial waves that have different spin J but the same parity, $P = -(-1)^J$, isospin, as the pion.
- Perform the summation over J
- Vertices coupling $\gamma\pi$ and $N\bar{N}$ to $J^P = (\text{even})^-$:





$$1^- \otimes 0^- = 1^+ \begin{cases} L = 1 & \text{for } J = 0 \\ L = \{J-1, J+1\} & \text{for } J \geq 2 \end{cases}$$

$$V^{\text{un}}_{\lambda_\gamma}(J) = 2e_{\bar{\pi}} \left[k^{\nu_1} \cdots k^{\nu_J} \epsilon_{\lambda_\gamma,\mu}(k) p^{\mu}_{\bar{\pi}} - k^{\nu_1} \cdots k^{\nu_{J-1}} \epsilon^{\nu_J}_{\lambda_\gamma}(k)(k \cdot p_{\bar{\pi}}) \right] \epsilon^*_{\nu_1,\cdots,\nu_J}(M)$$



$$\frac{1}{2}^+ \otimes \frac{1}{2}^- = 0^- \oplus 1^- \longrightarrow L = J$$

helicity non-flip (→ 0^-), helicity flip (→ 1^-)

$$V^{\text{un,NF}}_{\lambda_i\lambda_f}(J) = gP^{\nu_1} \cdots P^{\nu_J} \epsilon_{\nu_1,\cdots,\nu_J}(M) \bar{u}_{\lambda_f}(p_f) \gamma_5 v_{\lambda_i}(p_{\bar{i}})$$

$$V^{\text{un,F}}_{\lambda_i\lambda_f}(J) = gP^{\nu_1} \cdots P^{\nu_{J-1}} \epsilon_{\nu_1,\cdots,\nu_J}(M) \bar{u}_{\lambda_f}(p_f) \gamma^{\nu_J} \gamma_5 v_{\lambda_i}(p_{\bar{i}})$$

New approach to Reggeization of pion exchange

- Compute the full helicity amplitudes. We take the frame in which the exchanged particle is at rest (t-channel frame).

$$A^{\text{un,NF}}_{\lambda_\gamma \lambda_i \lambda_f}(J) = \left[(k\cdot P)(\epsilon_{\lambda_\gamma}\cdot p_{\bar\pi}) - (k\cdot p_{\bar\pi})(\epsilon_{\lambda_\gamma}\cdot P)\right] A_J(s,t)\, \bar u_{\lambda_f}(p_f)\gamma_5 v_{\lambda_i}(-p_i)$$
$$= -(k\cdot p_{\bar\pi})2|\mathbf{p}|d^1_{\lambda_\gamma 0}(\theta_t)A_J(s,t)\,\bar u_{\lambda_f}(p_f)\gamma_5 v_{\lambda_i}(-p_i)$$

$$A^{\text{un,F}}_{\lambda_\gamma \lambda_i \lambda_f}(J) = \bar u_{\lambda_f}(p_f)\left[(k\cdot\gamma)(\epsilon_{\lambda_\gamma}\cdot p_{\bar\pi}) - (k\cdot p_{\bar\pi})(\epsilon_{\lambda_\gamma}\cdot\gamma)\right]\gamma_5 v_{\lambda_i}(-p_i)A_J(s,t)$$
$$= (k\cdot p_{\bar\pi})\lambda'\delta_{\lambda'\pm1}4\sqrt{2}|\mathbf{p}|d^1_{\lambda_\gamma\lambda'}(\theta_t)A_J(s,t)$$

with the scalar function ($J \geq 2$) $\quad A_J(s,t) = -(-1)^{J+\lambda'}2e_{\bar\pi}g\mathcal{P}_J\left(2|\mathbf{k}||\mathbf{p}|\right)^{J-1}(c_J)^2\dfrac{J+1}{2J}P^{|\lambda_\gamma-\lambda'||\lambda_\gamma+\lambda'|}_{J-1}(z_t)$,

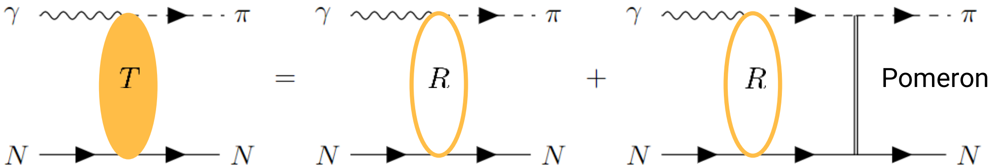
- Extend the definition to $J=0$ by comparing with m.g.i. pion exchange amplitude.

- Next, sum the Regge poles (work in progress):

$$\sum_{J=0,2,4\ldots} A_J(s,t)$$ using the Reggeon propagator $\mathcal{P}_J \to \mathcal{P}^{\text{Regge}}_J = \dfrac{\alpha'}{J-\alpha(t)}$

Absorption

- Multiple elastic rescattering of the final state particles.
- We can write a Bethe-Salpeter-like equation that combines the Reggeized exchange amplitude with an elastic scattering amplitude (Pomeron exchange):



$$A^T_{\lambda,\lambda'}(\mathbf{k},\mathbf{p}) = A^R_{\lambda,\lambda'}(\mathbf{k},\mathbf{p}) - i\sum_{\lambda''}\int \frac{dq^4}{(2\pi)^4}\frac{A^R_{\lambda,\lambda''}(\mathbf{k},\mathbf{q})A^P_{\lambda'',\lambda'}(\mathbf{k},\mathbf{p})}{[q^2-m^2][(P-q)^2-M^2]}$$
$$\equiv A^R_{\lambda,\lambda'}(\mathbf{k},\mathbf{p}) + \delta A^R_{\lambda,\lambda'}(\mathbf{k},\mathbf{p})$$

- For the partial waves:

$$a^{T,J}_{\lambda,\lambda'}(s) = a^{R,J}_{\lambda,\lambda'}(s) + \delta a^{R,J}_{\lambda,\lambda'}(s)$$

- Absorptive Regge cut models rely on the fit a free parameter, accounting for the "strength" of the intermediate inelastic channels

$$A^T_{\lambda,\lambda'}(s,t) = A^R_{\lambda,\lambda'}(s,t) + \lambda\,\delta A^R_{\lambda,\lambda'}(s,t) \qquad \text{with} \quad \lambda > 1$$

[F.Henyey, G.L.Kane, J.Pumplin, M.H.Ross, *Phys.Rev.* 182 (1969) 1579-1594]

- If λ is too large, the lowest partial waves undergo an unphysical sign change instead of being absorbed

→ over-absorption (no fundamental physics behind)

SUMMARY

- Understanding the features of pion exchange in hadronic reactions has been of fundamental interest for many decades, and not yet satisfactorily established.
- A precise comprehension of the production mechanisms is crucial for the light hybrid meson searches.
- At high energies, meson photoprodution reactions are dominated by the exchange of Regge trajectories, in particular, the pion trajectory plays a major role at low momentum transfer.
- In single pion photoproduction, pion exchange is not able to explain the cross-section data at small t:

  → The t-channel pion exchange process is not gauge invariant.

  → Electric current conservation requires to include the s- and u-channel nucleon exchanges.

  → The magnetic contribution of the nucleon exchanges gives a non-zero cross section at $t = 0$ of similar size than the data.

  → Proper Reggeization of the pion exchange has to take into account gauge invariance.

  → New approach to Reggeization is on its way:

    ↳ Explicit exchange of all the members of the pion trajectory.