Δ^{++} Spin Density Matrix Elements @GlueX

Vincent MATHIEU

University of Barcelona

Joint Physics Analysis Center
Exotic Hadron Topical Collaboration

FDSA

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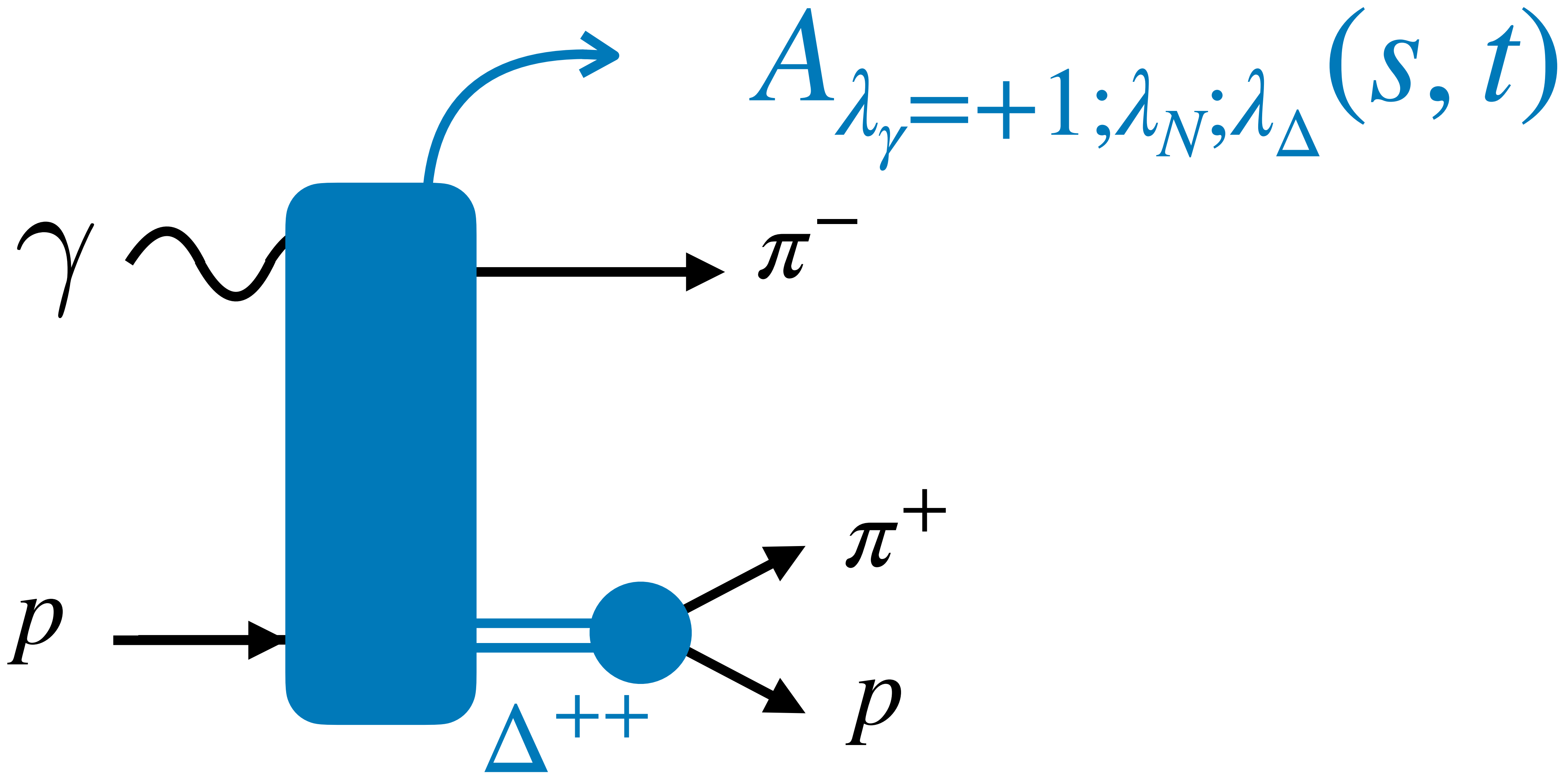
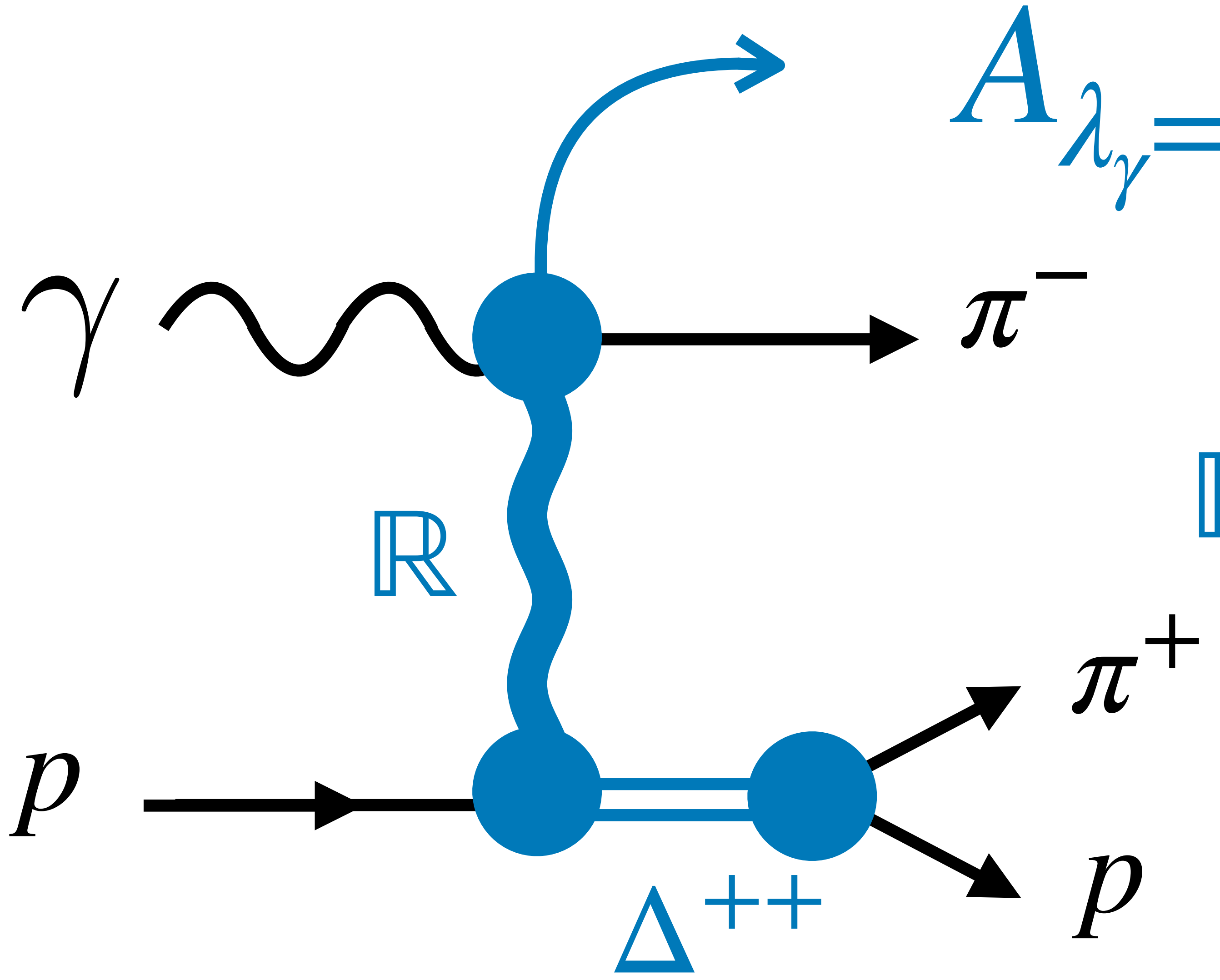
The Model



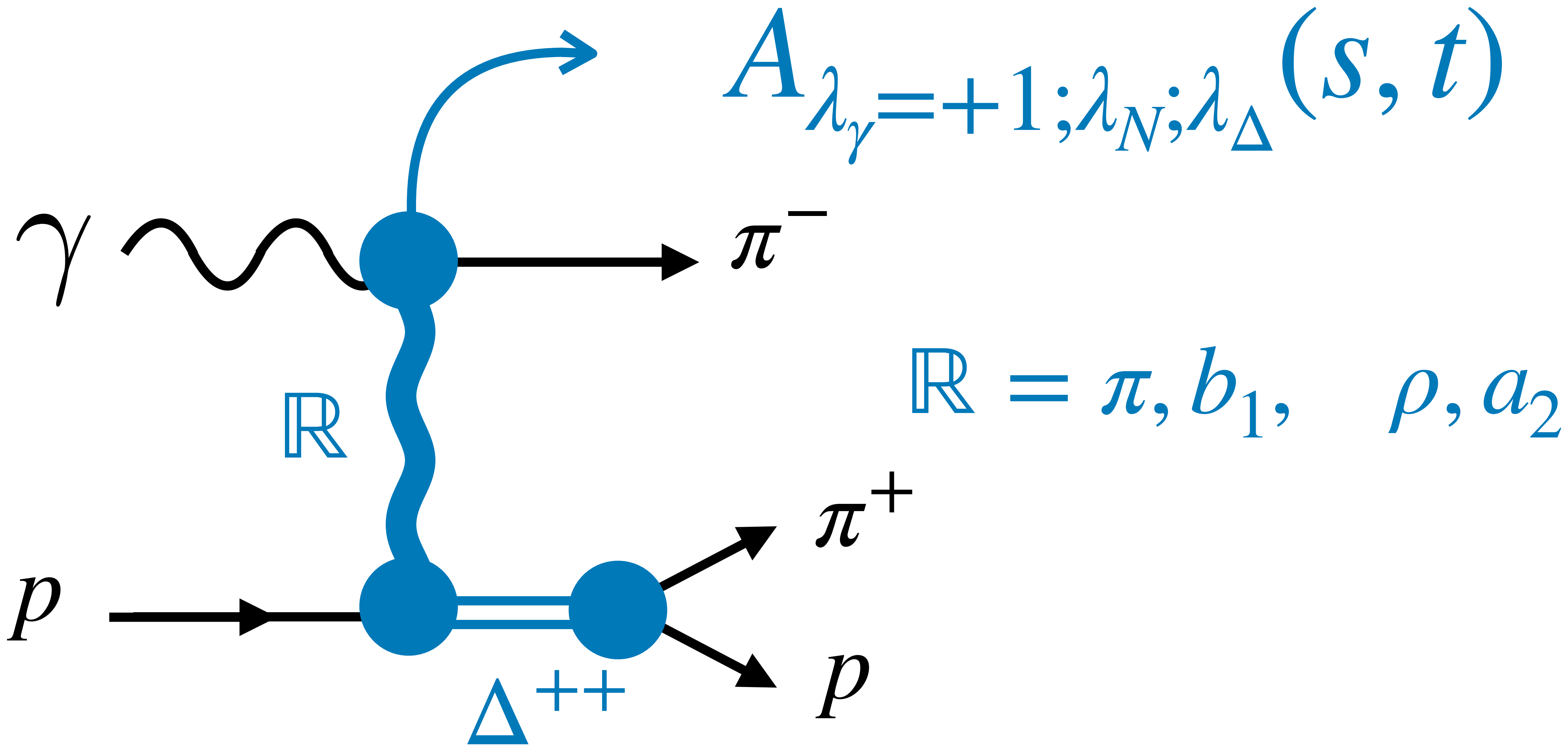
The Model



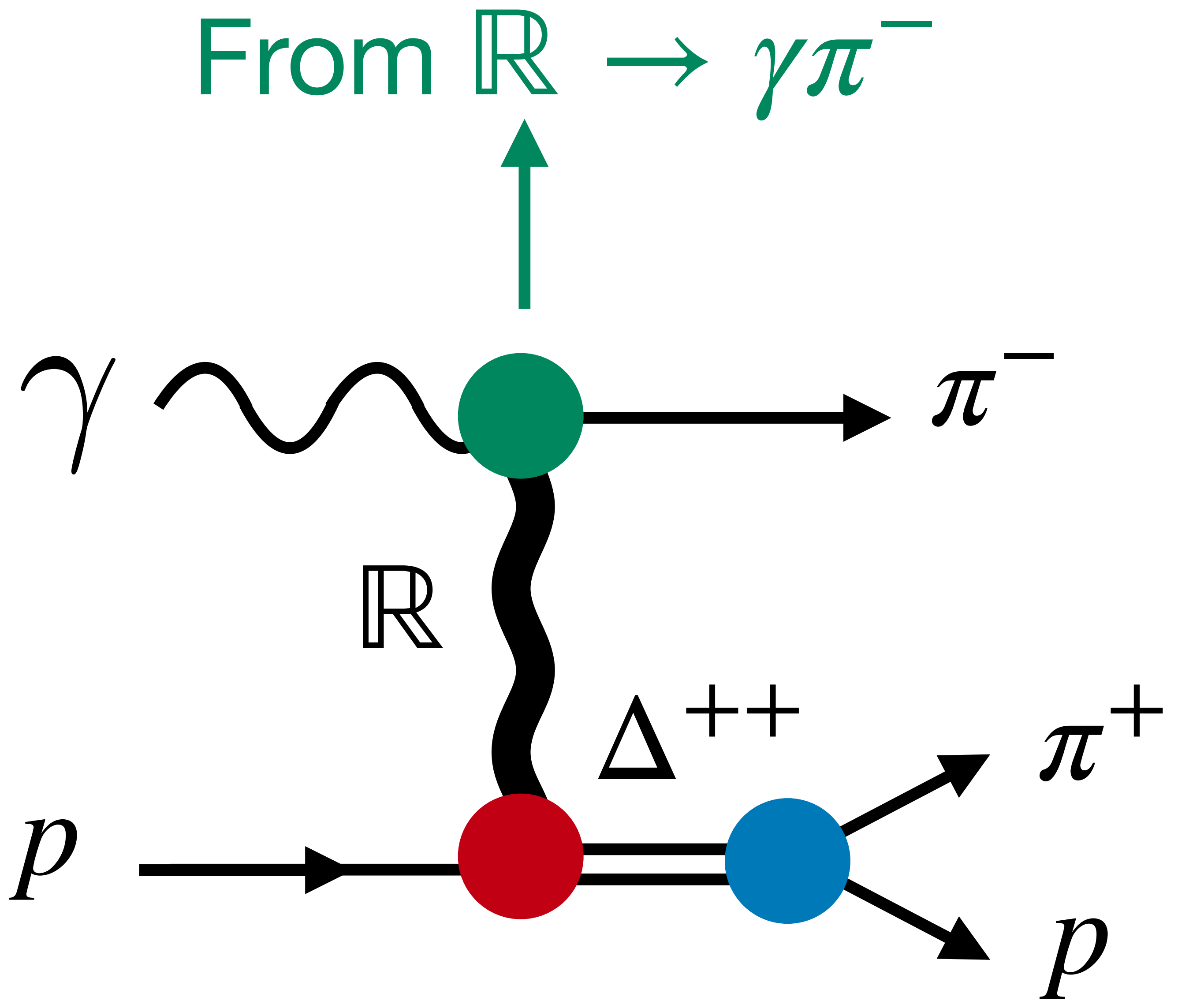
$A_{\lambda_\gamma=+1;\lambda_N;\lambda_\Delta}(s,t)$

$\mathbb{R} = \pi, b_1, \quad \rho, a_2$

The Model



$A_{\lambda_\gamma=+1;\lambda_N;\lambda_\Delta}(s,t)$
γ
π^-
$\mathbb{R}$
$\mathbb{R} = \pi, b_1, \quad \rho, a_2$
π^+
p
Δ^{++}
p



From $\mathbb{R} \to \gamma\pi^-$
γ
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$\mathbb{R}$
Δ^{++}
π^+
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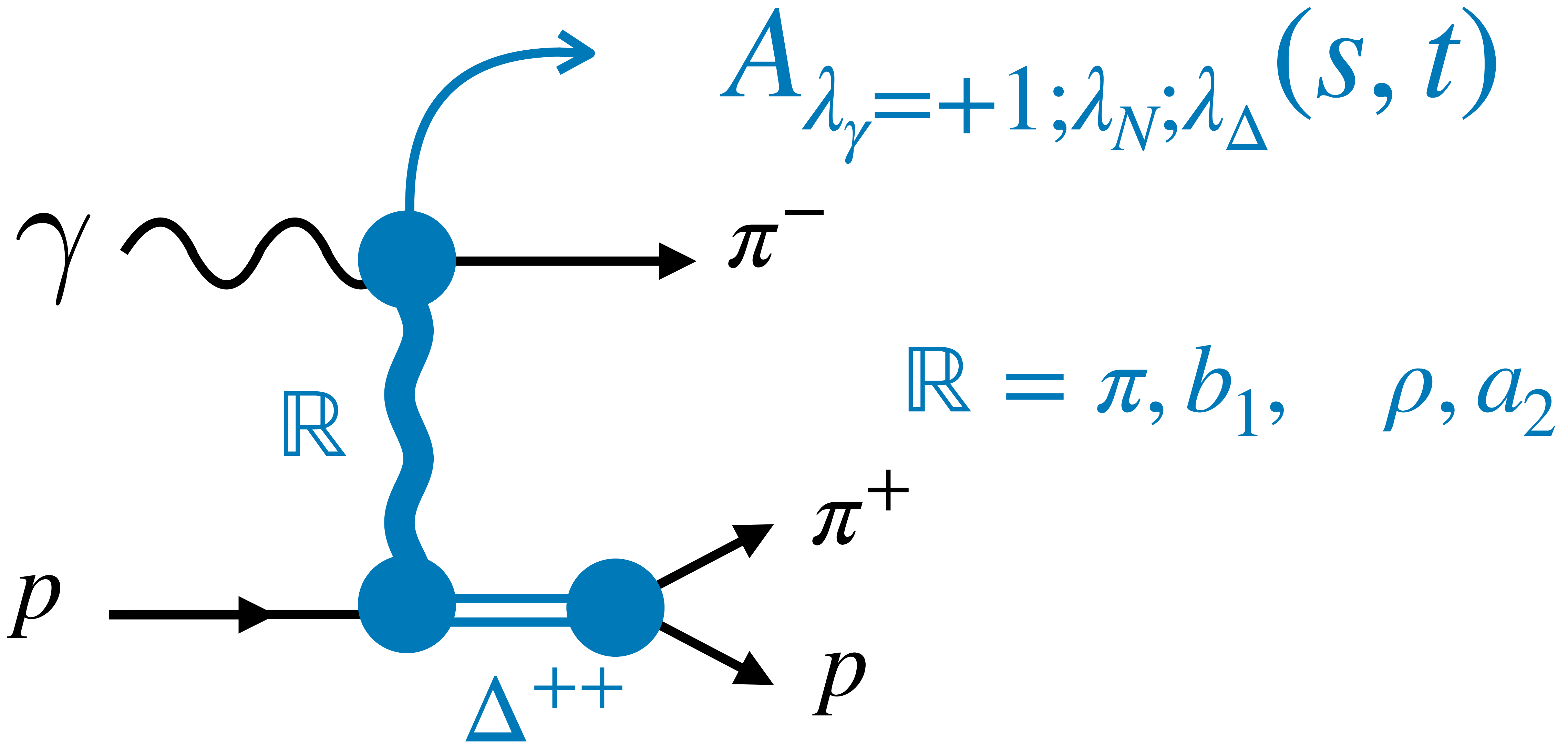
The Model



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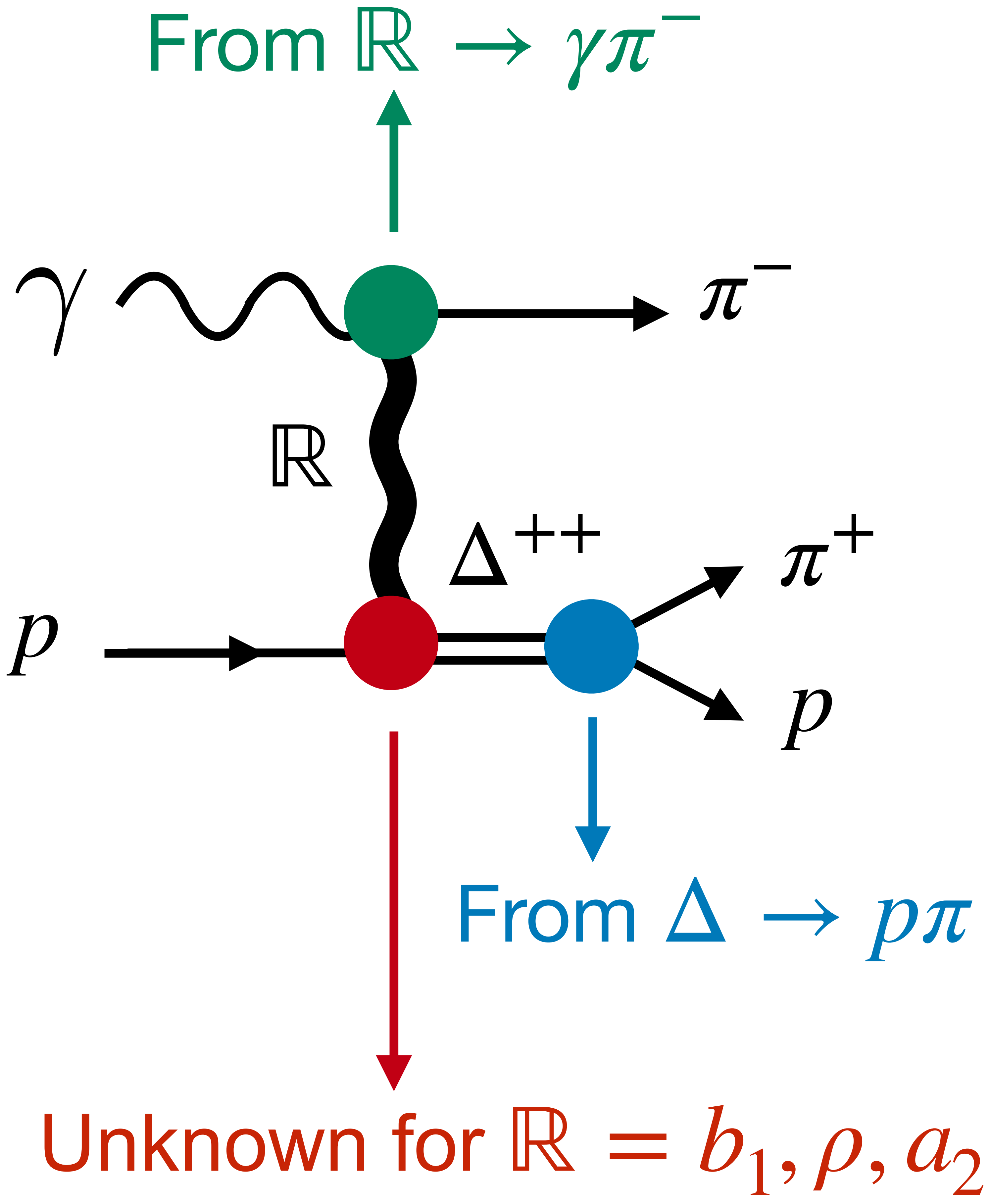

From $\mathbb{R} \to \gamma\pi^-$
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π^+
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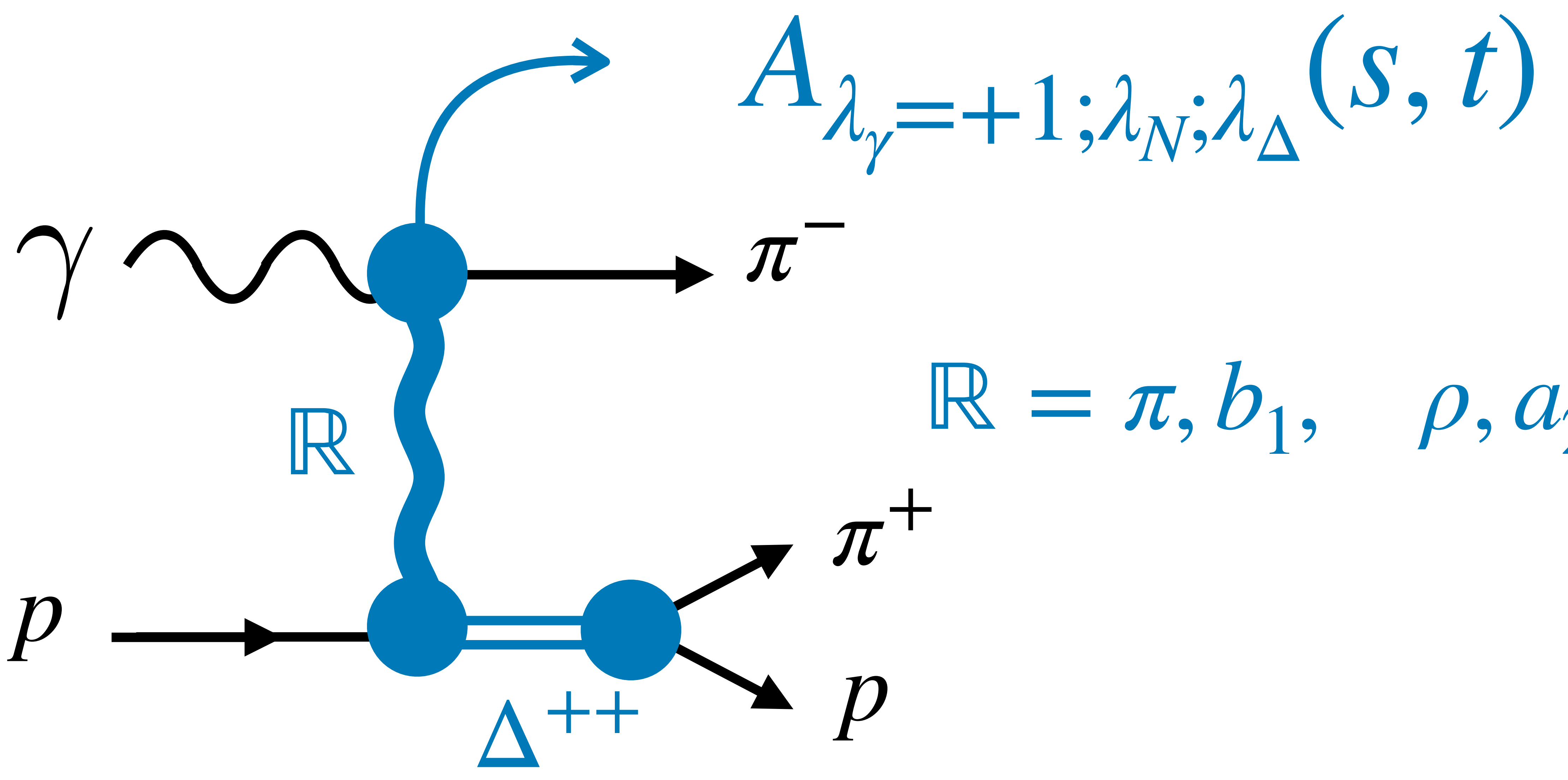
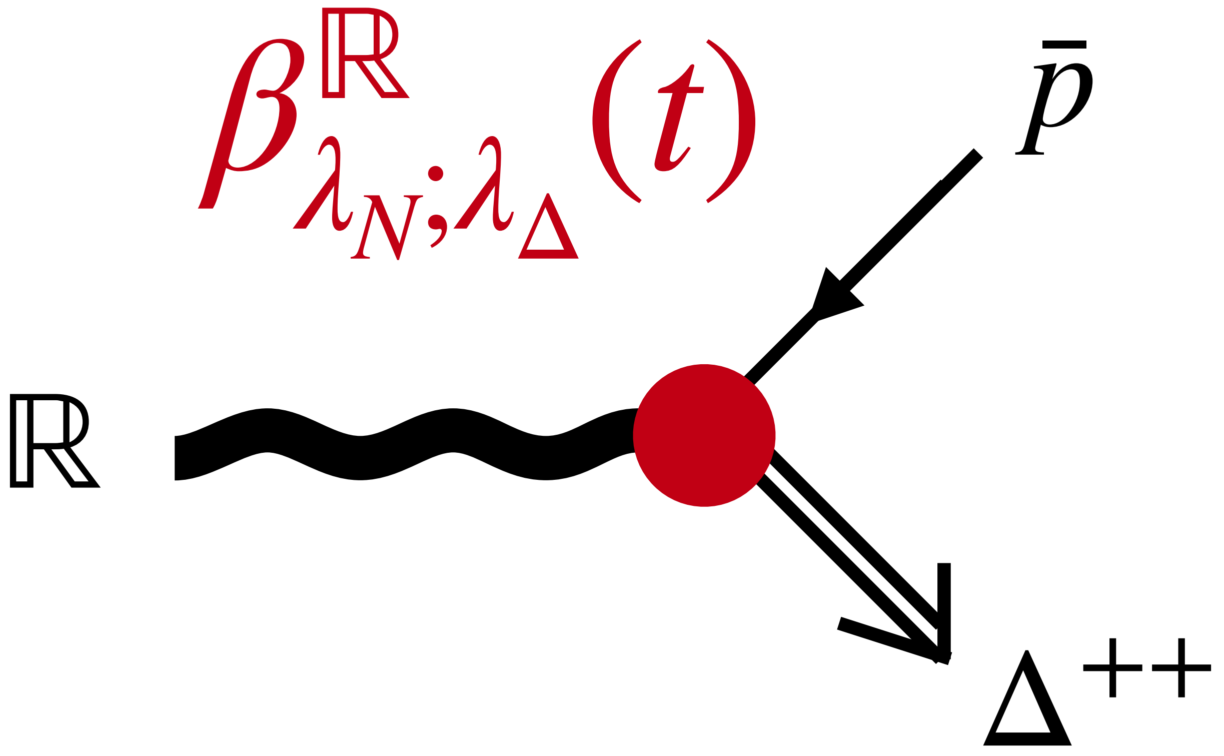
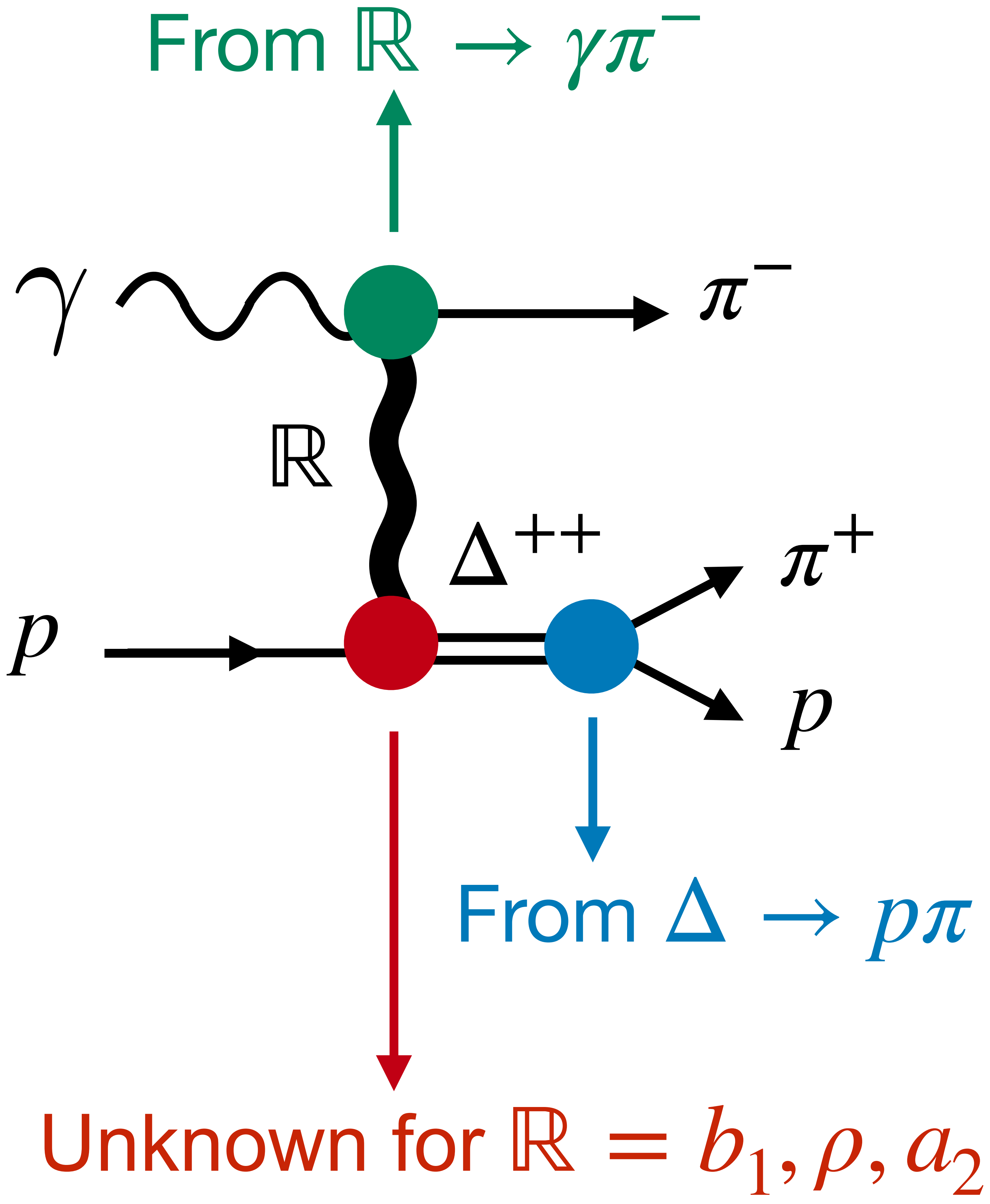
p
From $\Delta \to p\pi$
Unknown for $\mathbb{R} = b_1, \rho, a_2$

The Model



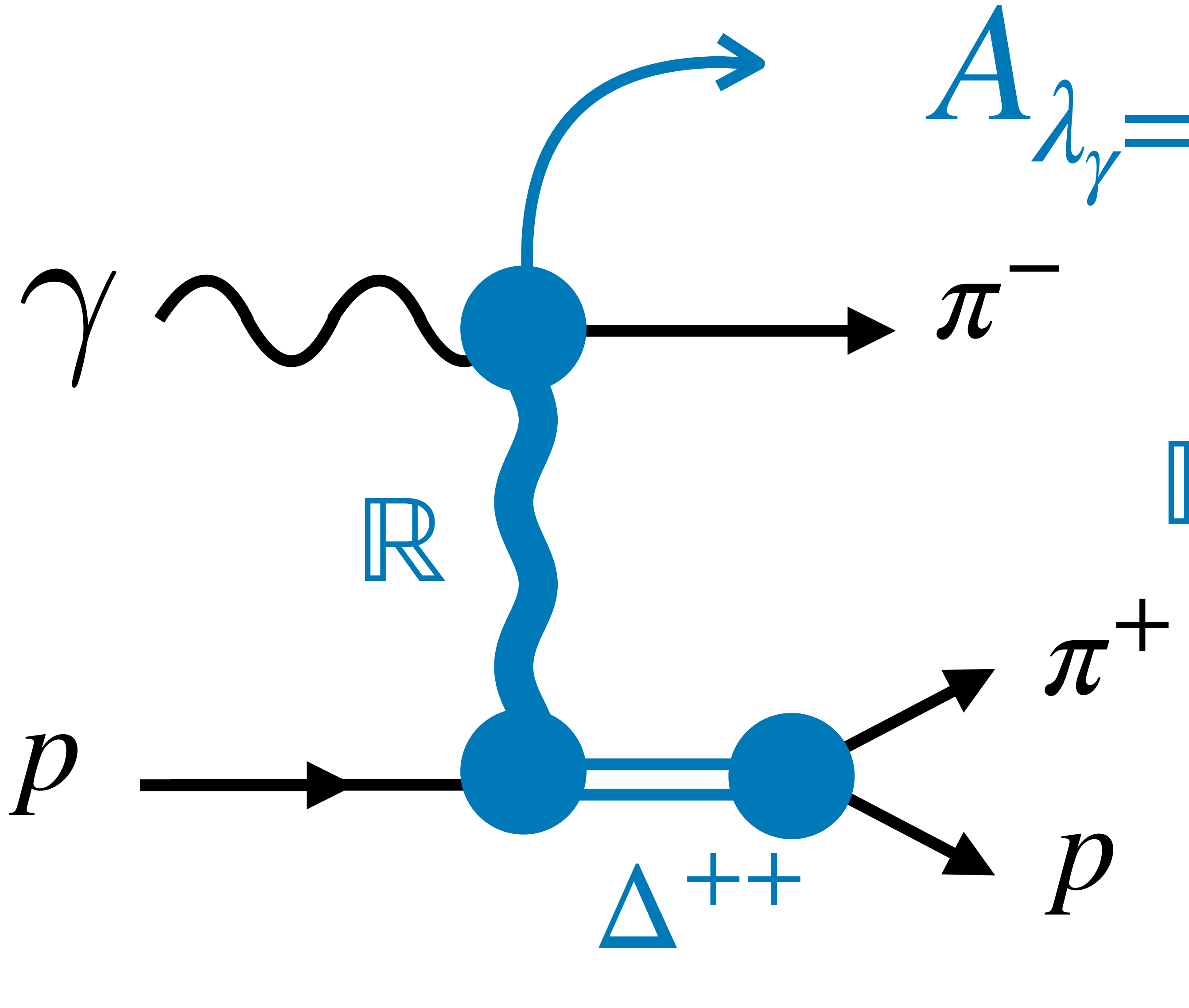
$\mathbb{R} = \pi, b_1, \quad \rho, a_2$





The Model

$A_{\lambda_\gamma=+1;\lambda_N;\lambda_\Delta}(s,t)$



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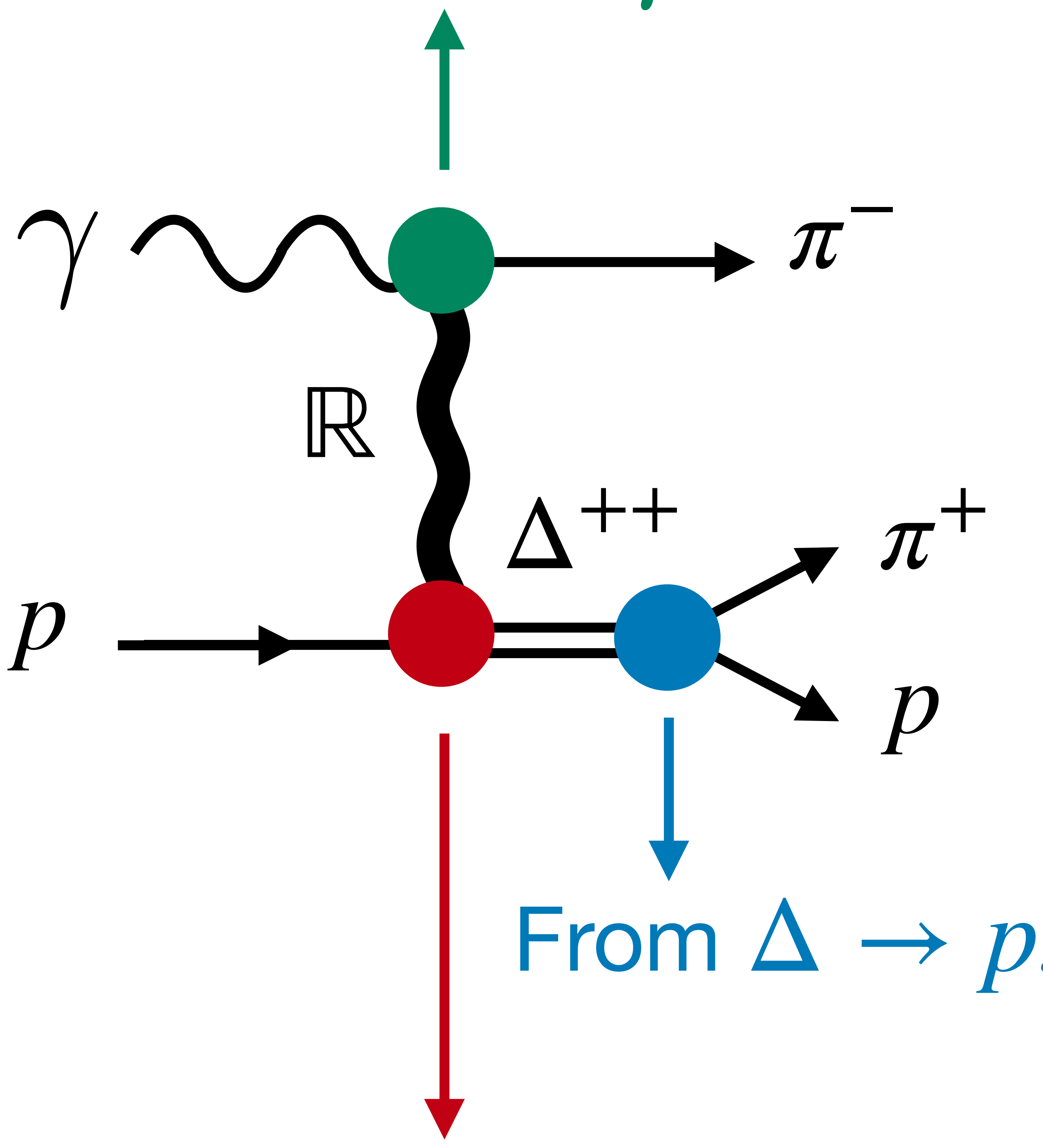
From $\mathbb{R} \to \gamma\pi^-$

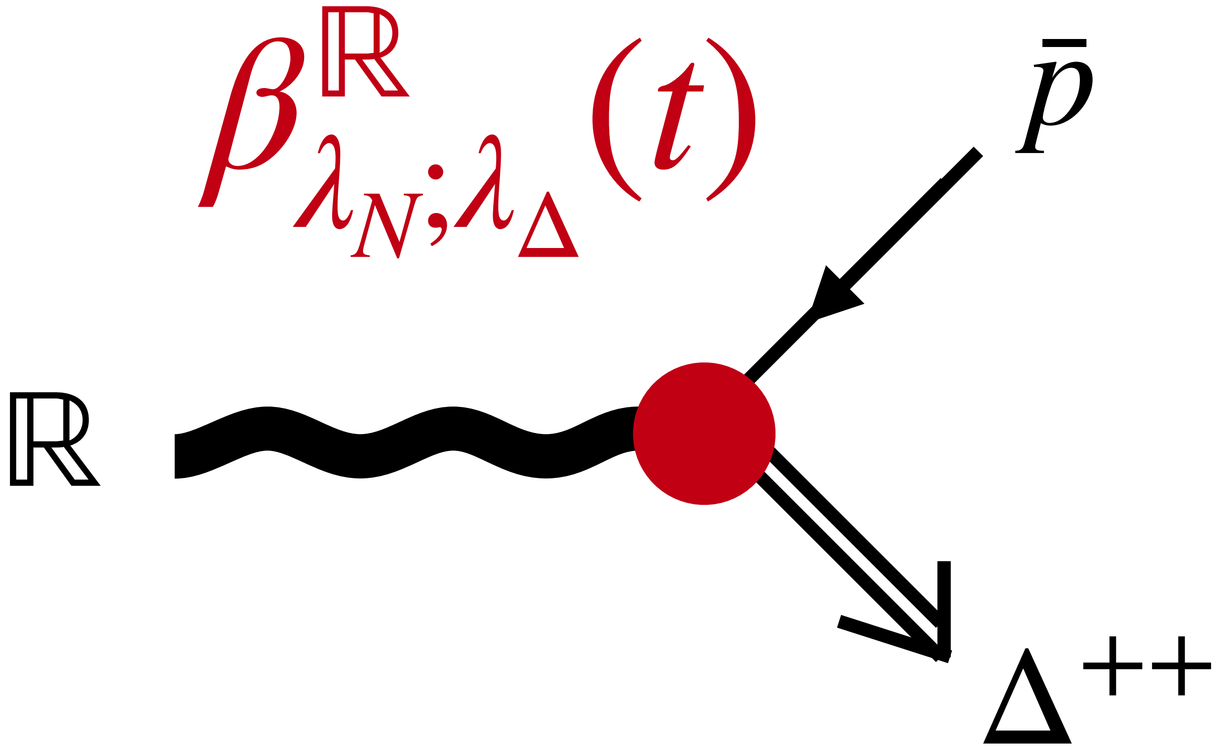


From $\Delta \to p\pi$

Unknown for $\mathbb{R} = b_1, \rho, a_2$

Constraints from spin-parity of $\mathbb{R}$

$\beta^{\mathbb{R}}_{\lambda_N;\lambda_\Delta}(t)$



$\beta^{\pi}_{\frac{1}{2};\frac{1}{2}}(t)$ Known for $\Delta \to p\pi$

The Model

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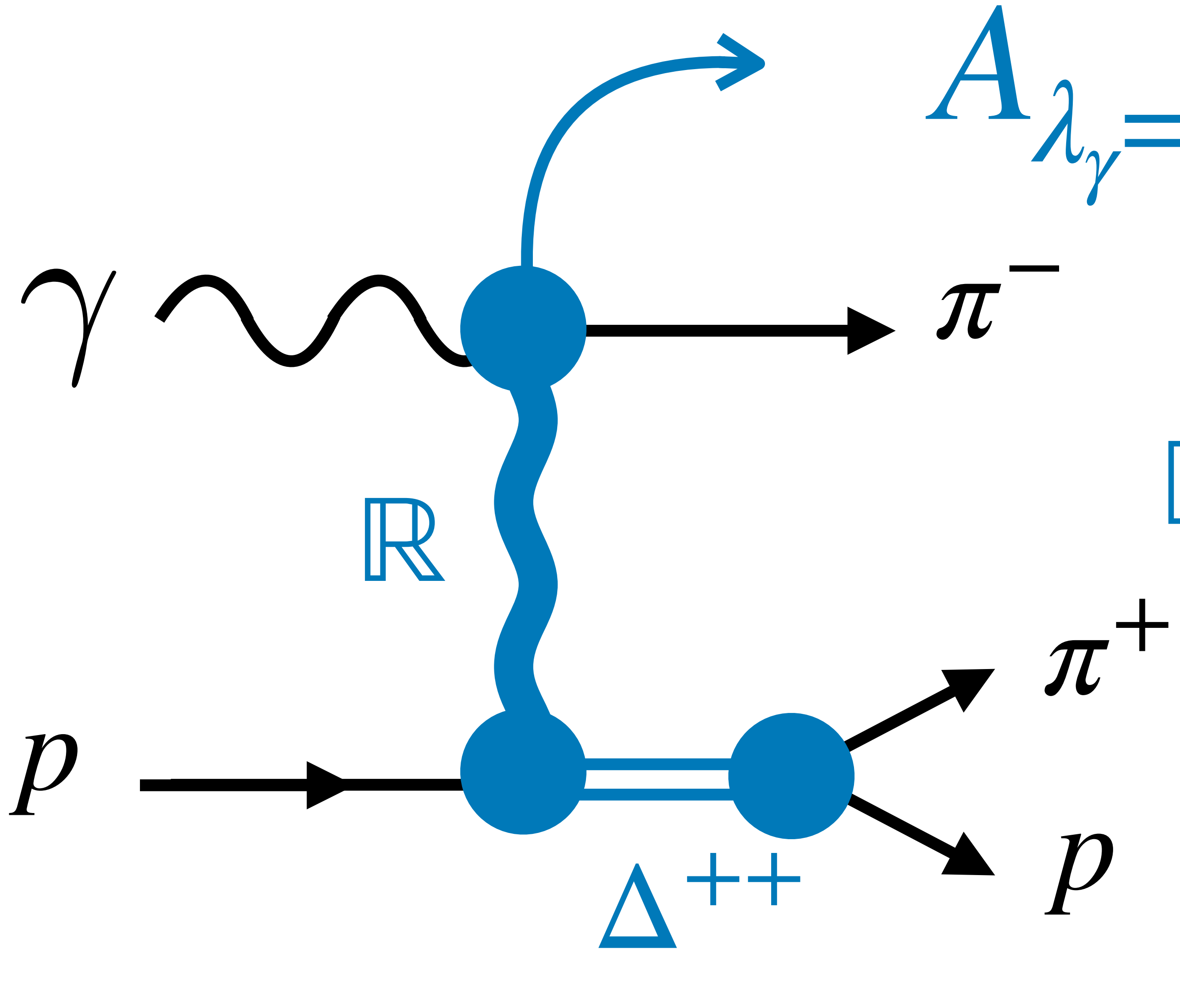

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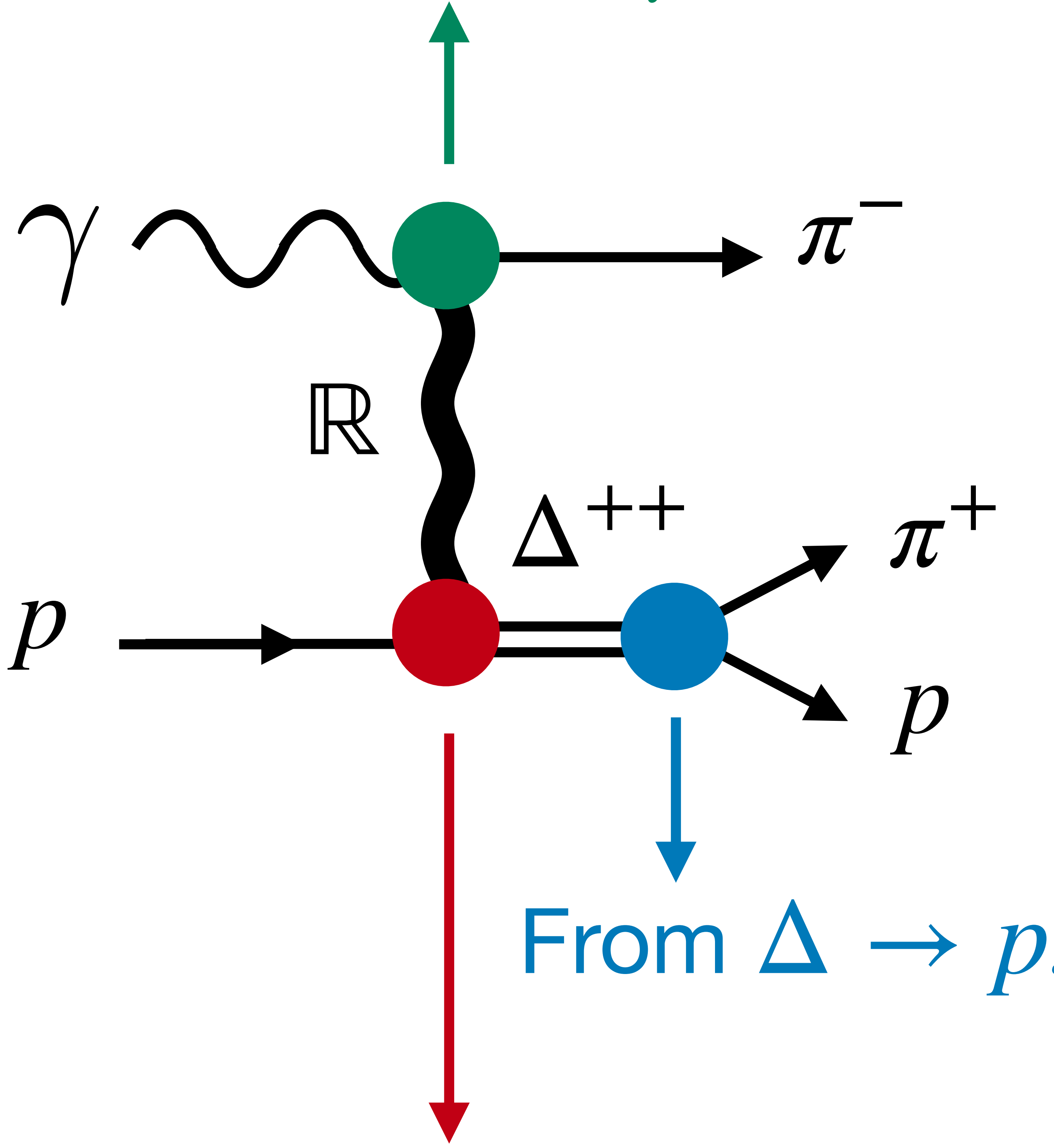

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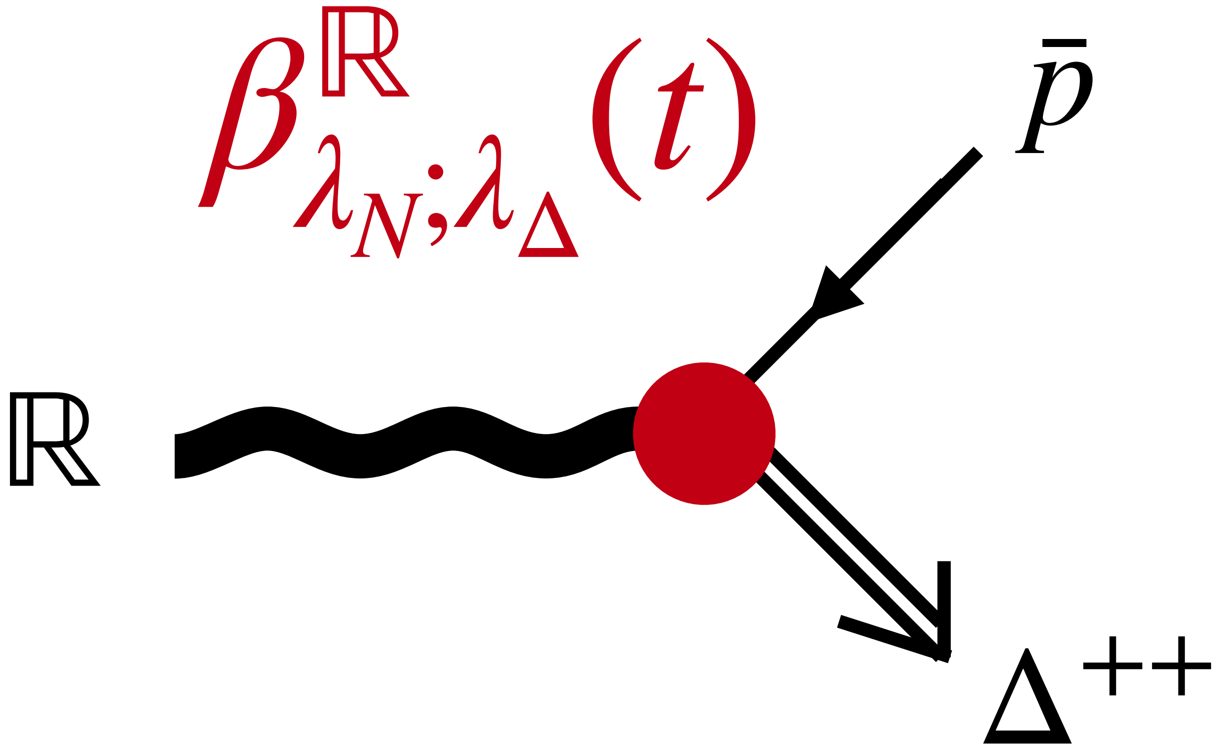

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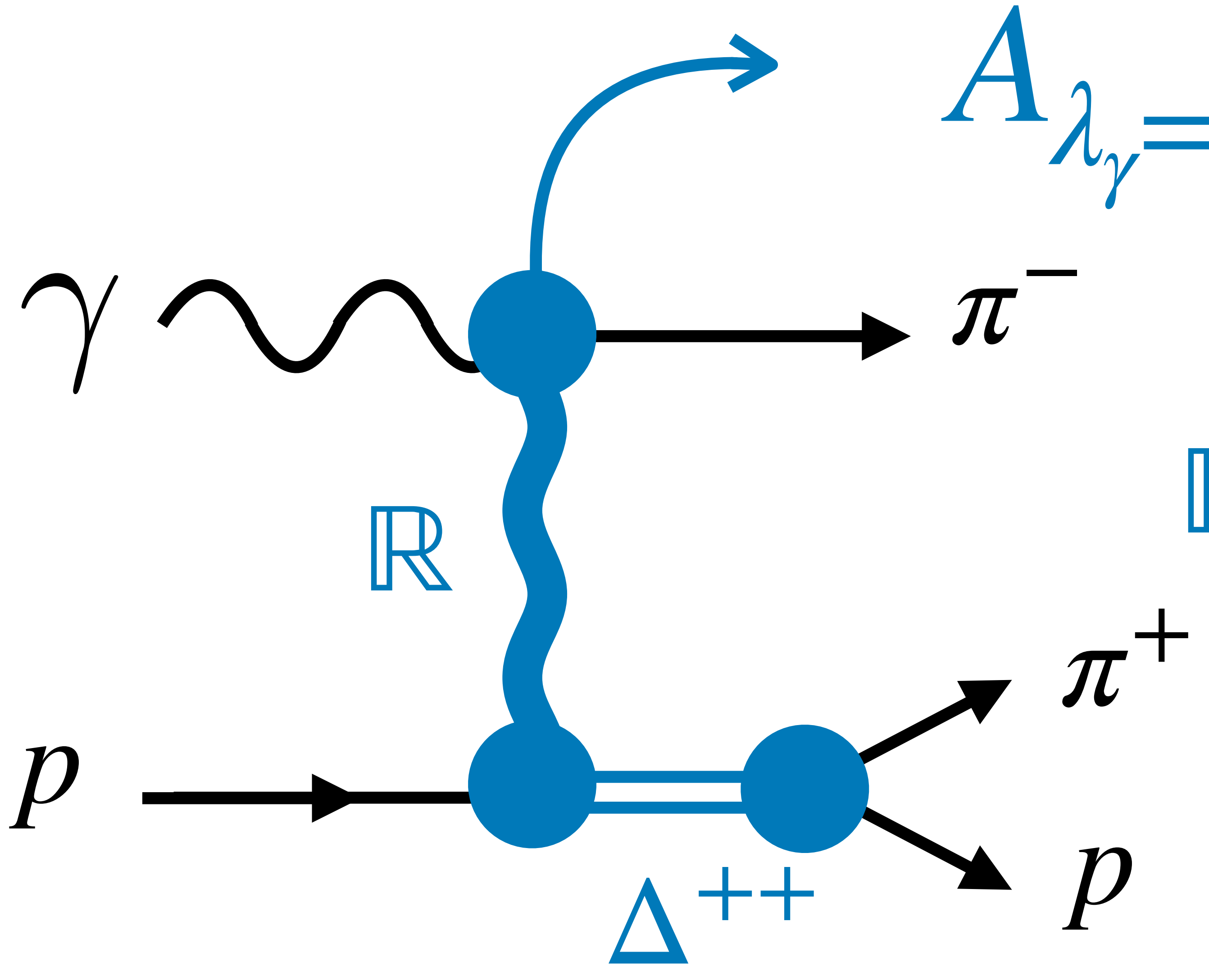
$\beta^{\pi}_{\frac{1}{2};\frac{1}{2}}(t)$ Known for $\Delta \to p\pi$

$\beta^{\rho}_{\frac{1}{2};\frac{1}{2}}(t)$ $\beta^{\rho}_{\frac{1}{2};-\frac{1}{2}}(t)$ $\beta^{\rho}_{\frac{1}{2};-\frac{3}{2}}(t)$

The Model



$$A_{\lambda_\gamma=+1;\lambda_N;\lambda_\Delta}(s,t)$$

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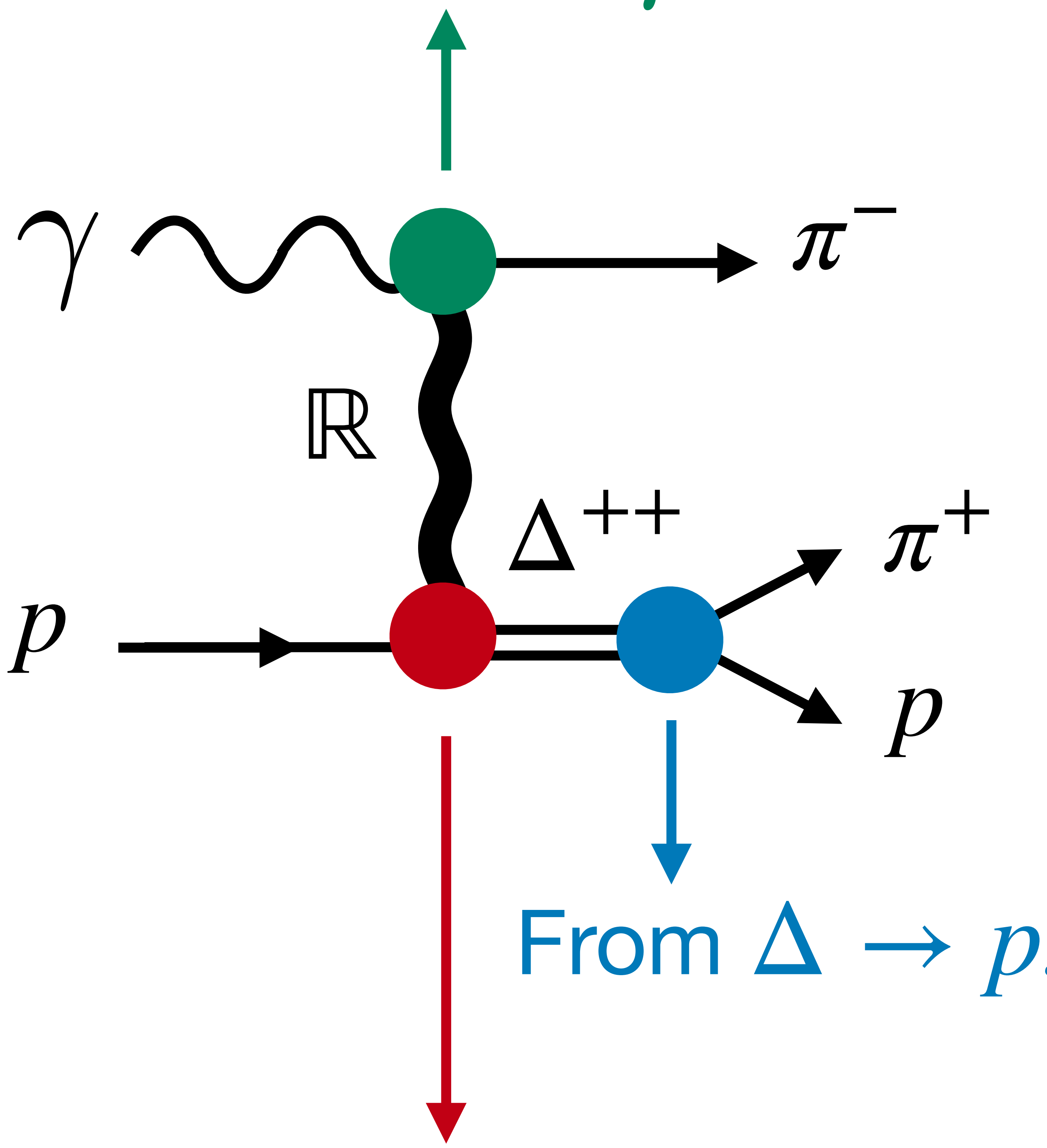
From $\mathbb{R} \to \gamma\pi^-$



From $\Delta \to p\pi$

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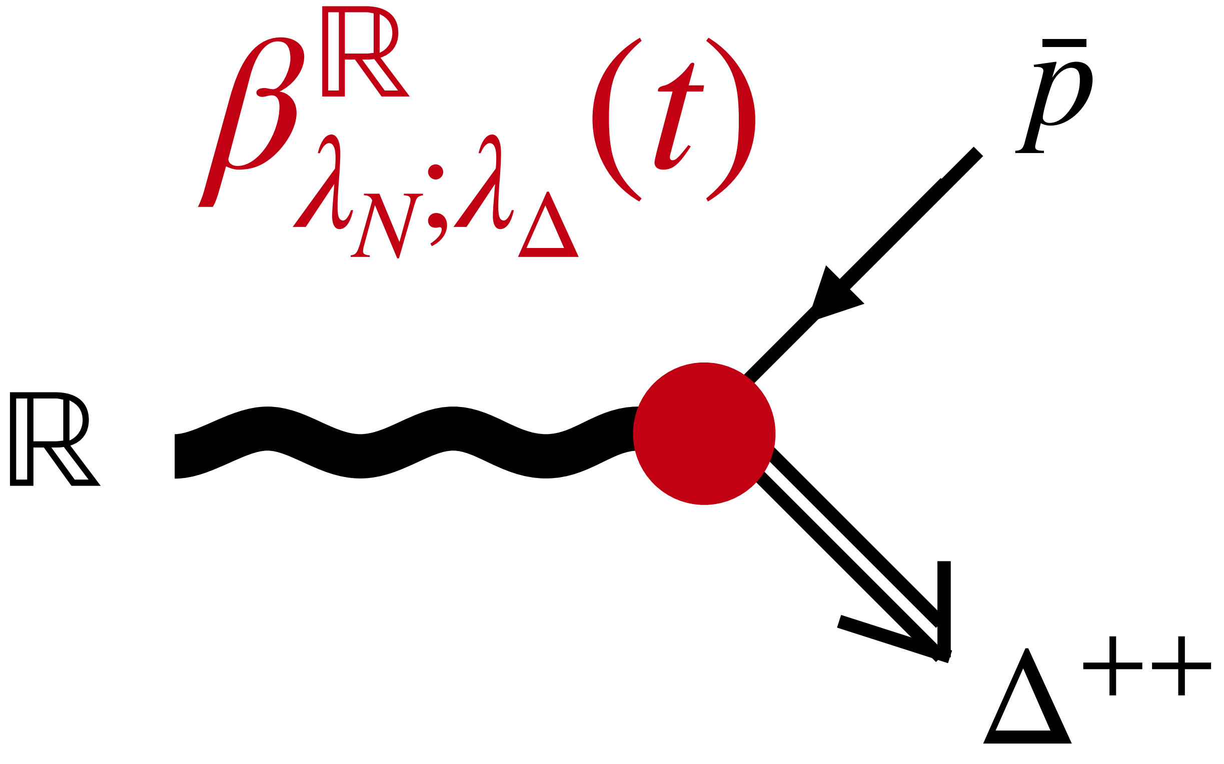
Constraints from spin-parity of $\mathbb{R}$

$$\beta^{\mathbb{R}}_{\lambda_N;\lambda_\Delta}(t)$$

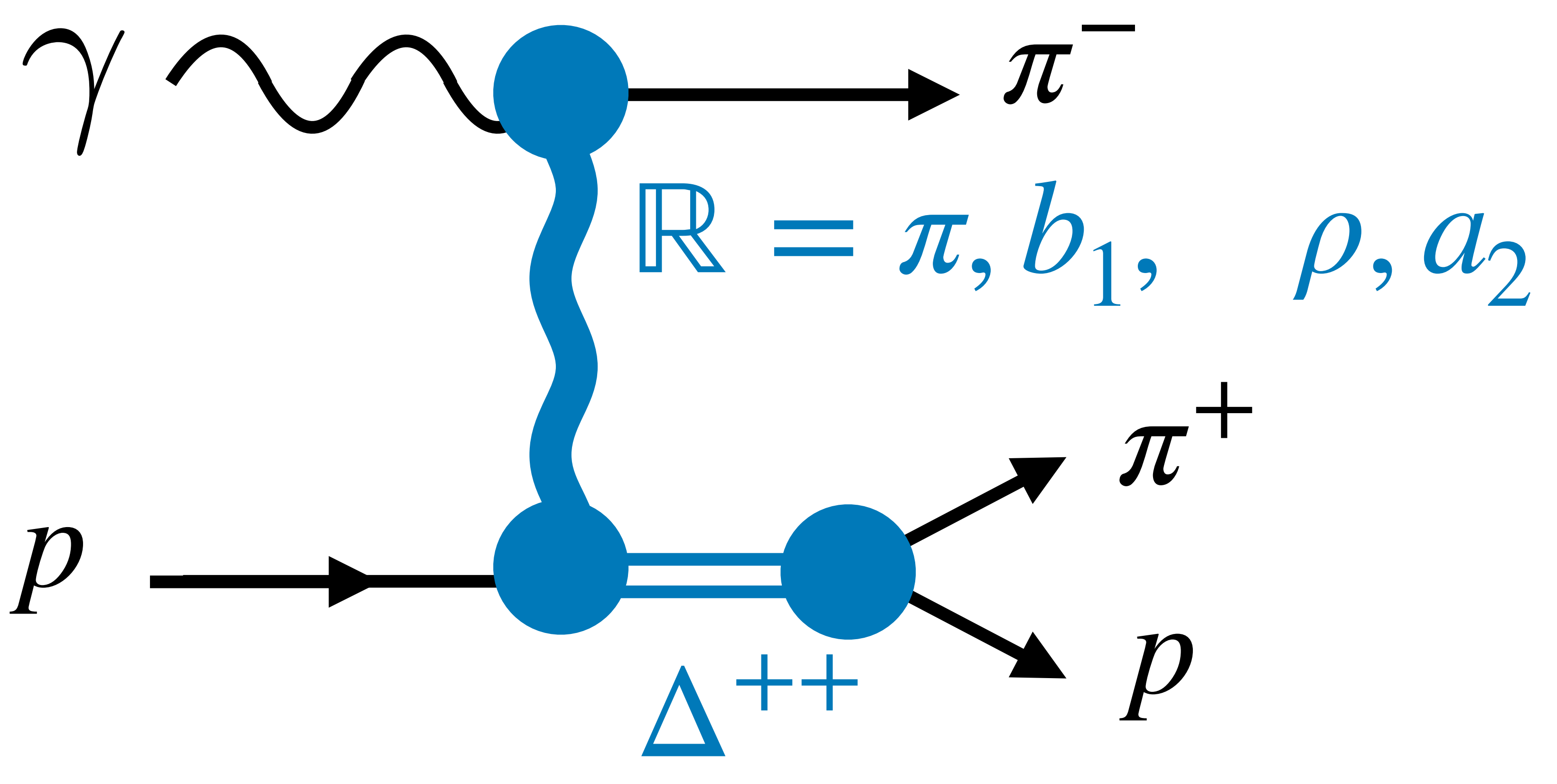


$\beta^{\pi}_{\frac{1}{2};\frac{1}{2}}(t)$ Known for $\Delta \to p\pi$

$\beta^{\rho}_{\frac{1}{2};\frac{1}{2}}(t)$ $\beta^{\rho}_{\frac{1}{2};-\frac{1}{2}}(t)$ $\beta^{\rho}_{\frac{1}{2};-\frac{3}{2}}(t)$

Argument for $\beta^{b_1}_{\lambda_N;\lambda_\Delta}(t) = \beta^{\pi}_{\lambda_N;\lambda_\Delta}(t)$ and $\beta^{a_2}_{\lambda_N;\lambda_\Delta}(t) = \beta^{\rho}_{\lambda_N;\lambda_\Delta}(t)$

Couplings



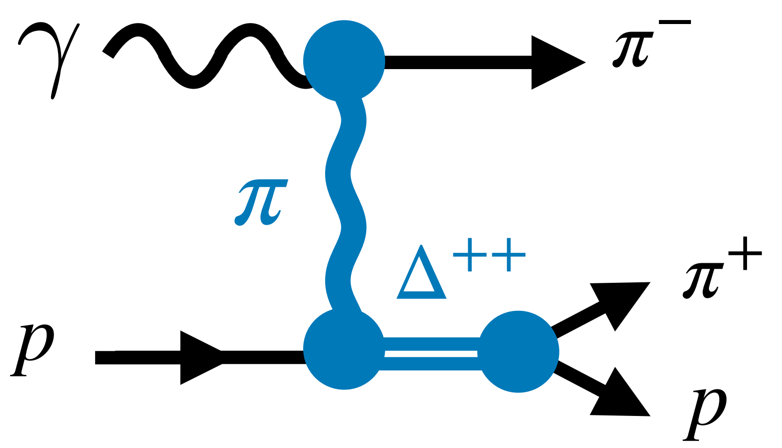
Photon couplings from radiative decays

Delta couplings from effective interactions

Delta couplings $g^{(1,2,3)}_{\rho N\Delta}$ fitted

$\hat{\beta}^{e,if}_{\mu_i\mu_f}$	Expression
$\hat{\beta}^{\pi,\gamma\pi}_{+1}(t)$	$\sqrt{2}e$
$\hat{\beta}^{\rho,\gamma\pi}_{+1}(t)$	$\frac{g_{\rho\pi\gamma}}{2m_\rho}$
$\hat{\beta}^{b_1,\gamma\pi}_{+1}(t)$	$\frac{g_{b_1\pi\gamma}}{2m_{b_1}}$
$\hat{\beta}^{a_2,\gamma\pi}_{+1}(t)$	$\frac{g_{a_2\pi\gamma}}{2m_{a_2}^2}$
$\hat{\beta}^{\pi,N\Delta}_{+\frac{1}{2}+\frac{3}{2}}(t)$	$\frac{g_{\pi N\Delta}(m_N+m_\Delta)}{\sqrt{2}m_\Delta}$
$\hat{\beta}^{\pi,N\Delta}_{-\frac{1}{2}+\frac{1}{2}}(t)$	$\frac{g_{\pi N\Delta}(-m_N^2+m_N m_\Delta+2m_\Delta^2+t)}{\sqrt{6}m_\Delta^2}$
$\hat{\beta}^{\pi,N\Delta}_{+\frac{1}{2}+\frac{1}{2}}(t)$	$\frac{-g_{\pi N\Delta}(-m_N^3-m_N^2 m_\Delta+m_\Delta^3+2m_\Delta t+m_N(m_\Delta^2+t))}{\sqrt{6}m_\Delta^2}$
$\hat{\beta}^{\pi,N\Delta}_{-\frac{1}{2}+\frac{3}{2}}(t)$	$\frac{-g_{\pi N\Delta}}{\sqrt{2}m_\Delta}$
$\hat{\beta}^{\rho,N\Delta}_{+\frac{1}{2}+\frac{3}{2}}(t)$	$\frac{-(2m_\Delta g^{(1)}_{\rho N\Delta}+g^{(2)}_{\rho N\Delta}(m_N-m_\Delta))}{2m_\Delta^2}$
$\hat{\beta}^{\rho,N\Delta}_{-\frac{1}{2}+\frac{1}{2}}(t)$	$\frac{-(2m_N m_\Delta g^{(1)}_{\rho N\Delta}+g^{(2)}_{\rho N\Delta}(-m_N m_\Delta+m_\Delta^2+2t)+2t g^{(3)}_{\rho N\Delta})}{2\sqrt{3}m_\Delta^3}$
$\hat{\beta}^{\rho,N\Delta}_{+\frac{1}{2}+\frac{1}{2}}(t)$	$\frac{-(2m_\Delta g^{(1)}_{\rho N\Delta}+g^{(2)}_{\rho N\Delta}(2m_N-3m_\Delta)+2g^{(3)}_{\rho N\Delta}(m_N-m_\Delta))}{2\sqrt{3}m_\Delta^3}(-t)$
$\hat{\beta}^{\rho,N\Delta}_{-\frac{1}{2}+\frac{3}{2}}(t)$	$\frac{g^{(2)}_{\rho N\Delta}}{2m_\Delta^2}$

Poor Man Absorption Model



$$A_\pi \propto \left(\sqrt{-t}\right)^{|\lambda_\gamma|+|\lambda_\Delta-\lambda_p|} \equiv \left(\sqrt{-t}\right)^{n+x} \quad \rightarrow \quad A_\pi \propto \left(\sqrt{-t}\right)^{n} \left(\sqrt{-m_\pi^2}\right)^{x}$$

λ_N	λ_Δ	n	$n+x$	λ_N	λ_Δ	n	$n+x$
$+1/2$	$+1/2$	1	1	$+1/2$	$+3/2$	2	2
$+1/2$	$-1/2$	0	2	$+1/2$	$-3/2$	1	3
$-1/2$	$+1/2$	2	2	$-1/2$	$+3/2$	3	3
$-1/2$	$-1/2$	1	1	$-1/2$	$-3/2$	0	2

Table 2: Pion photoproduction $\lambda_\gamma = 1$ and $\lambda_\pi = 0$.

$n = |\lambda_\gamma + \lambda_\Delta - \lambda_p|$ net helicity flip

$x = |\lambda_\gamma| + |\lambda_\Delta - \lambda_p| - n$

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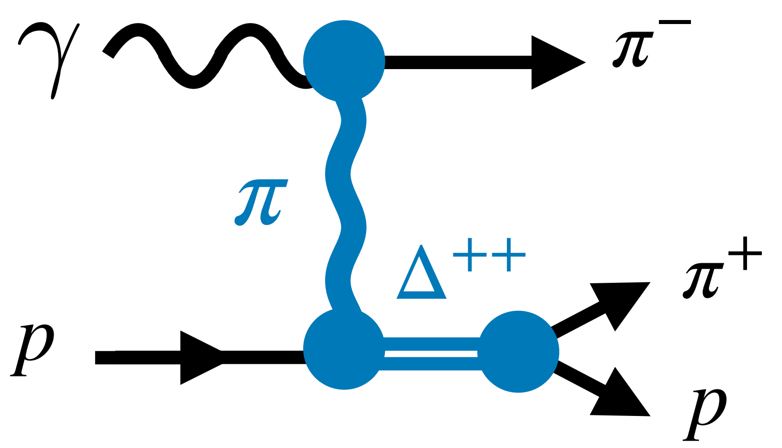

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PMA breaks parity relation for π exchange

$$A_{-1;+\frac{1}{2};-\frac{3}{2}} \neq -A_{+1;+\frac{1}{2};-\frac{3}{2}}$$

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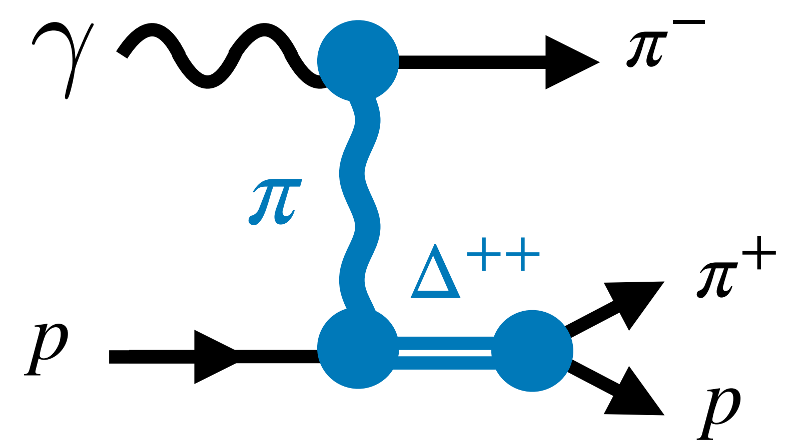

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$$A_{+1;+\frac{1}{2};-\frac{3}{2}} \propto (\sqrt{-t})(-m_\pi^2)$$

$$A_{-1;+\frac{1}{2};-\frac{3}{2}} = -A_{1;-\frac{1}{2};+\frac{3}{2}}$$

$$\propto -(\sqrt{-t})^3$$

Results

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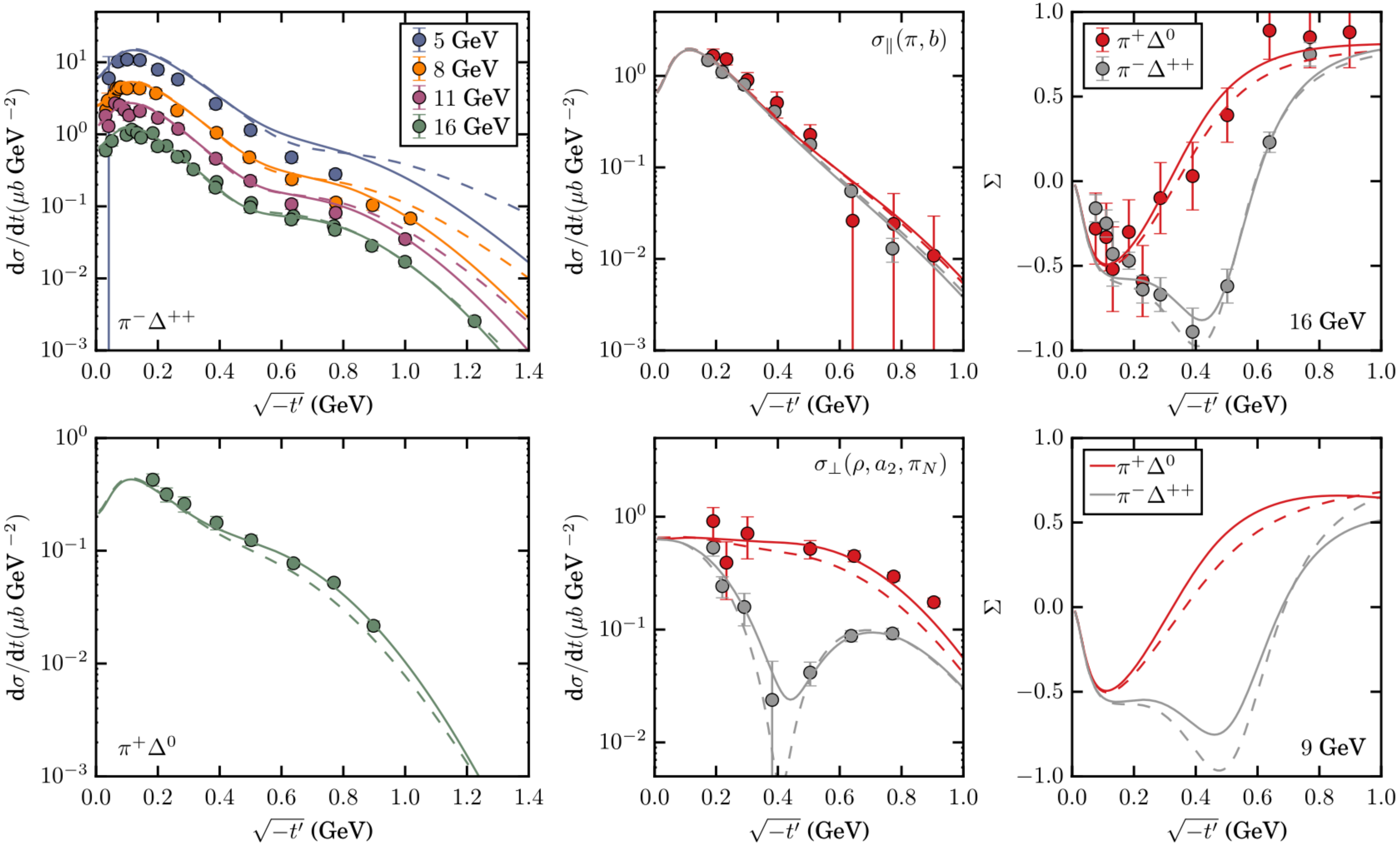

$$\Sigma = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel}$$

Σ and σ do not depend on the z-axis orientation

Results

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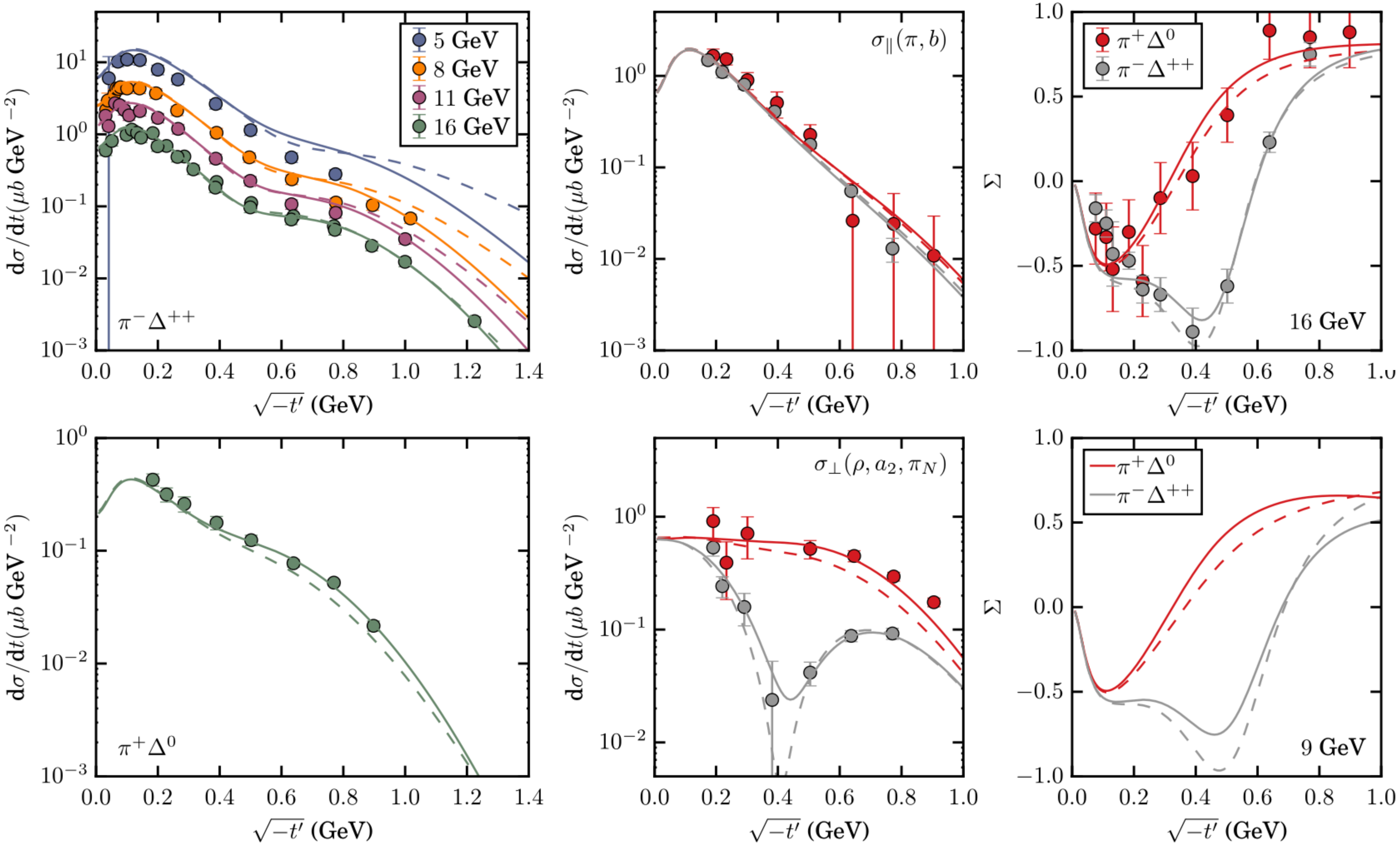

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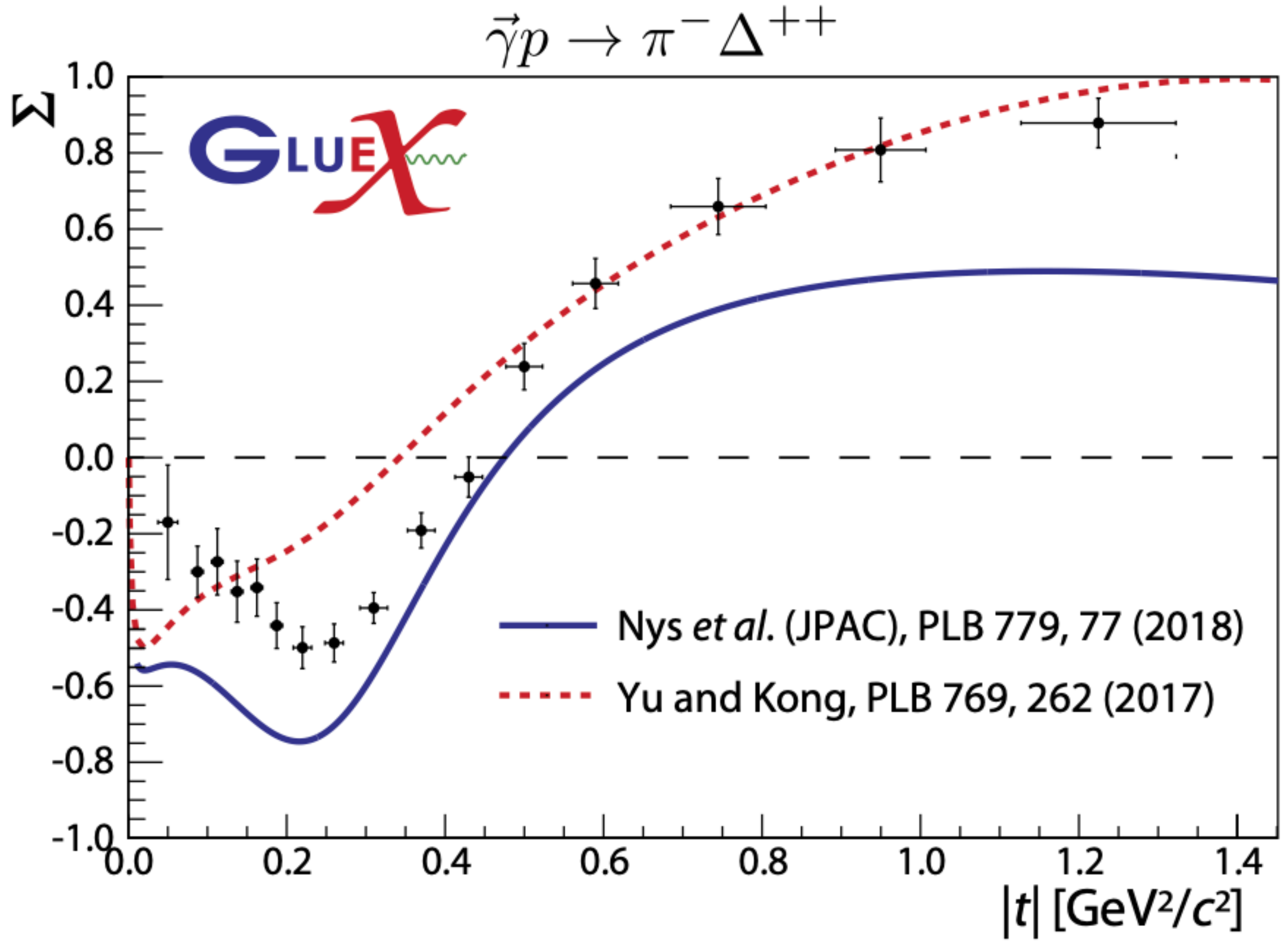

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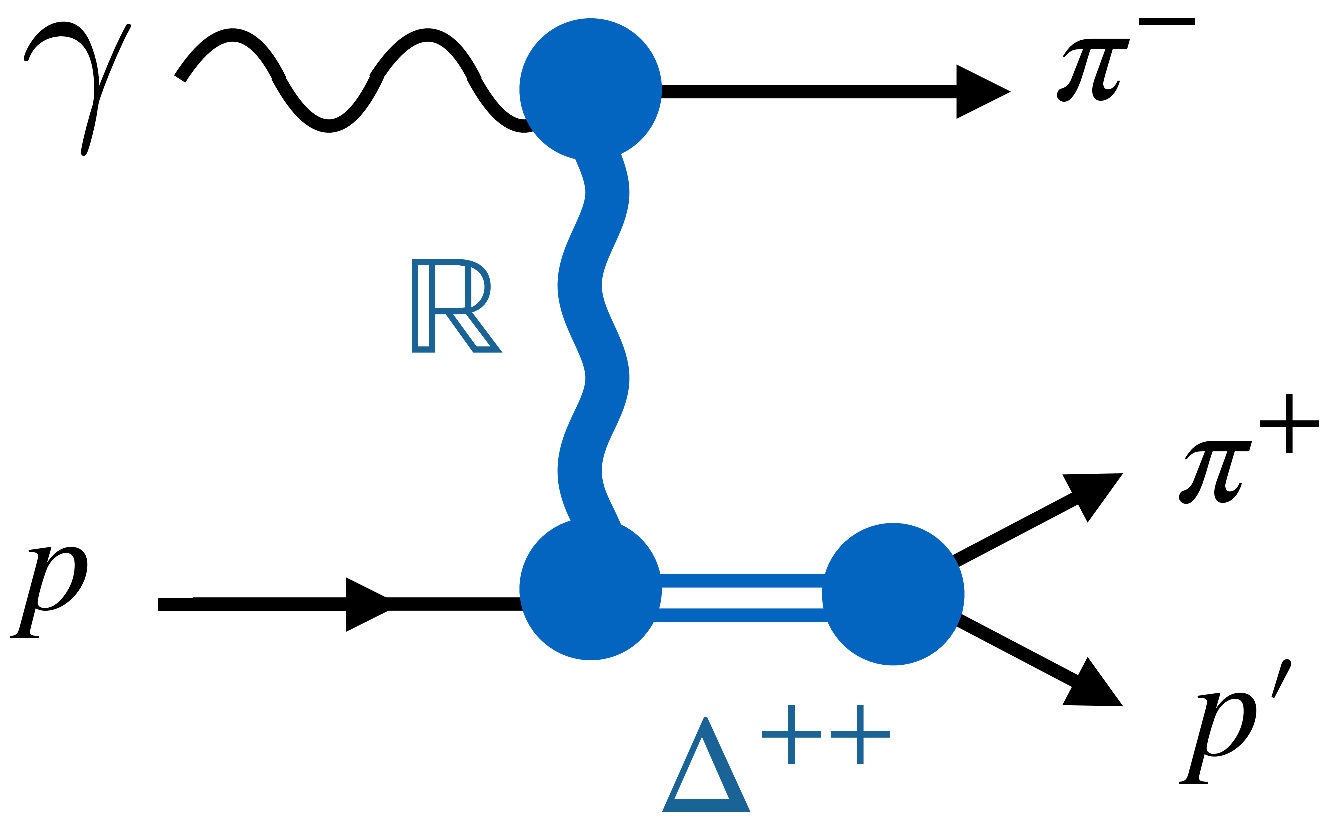
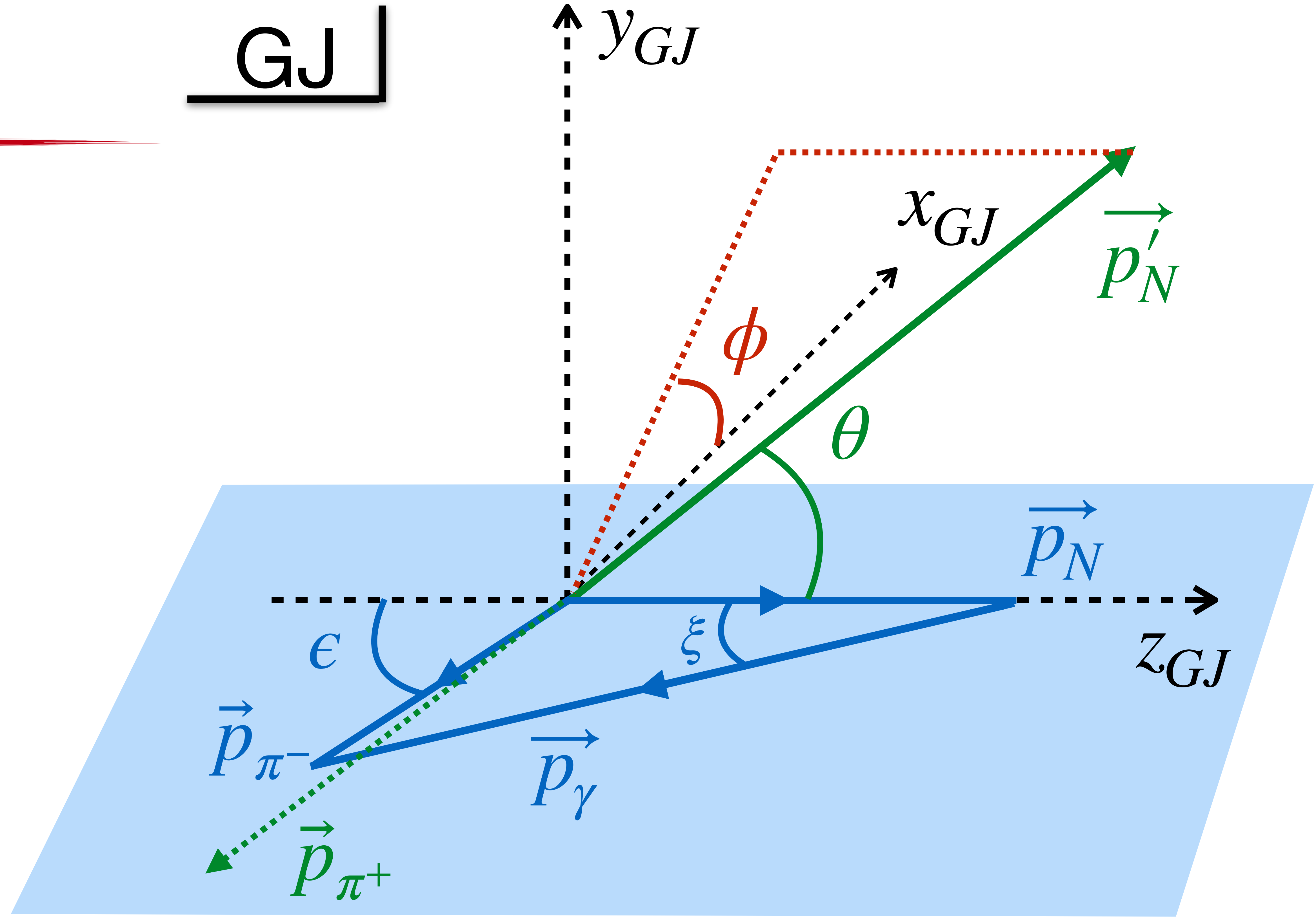
Gottfried-Jackson Frame





The z-axis is aligned with the target

$$W(\theta,\phi,\Phi) = \frac{1}{2\pi}\frac{d\sigma}{dt}\frac{3}{4\pi}\left\{\rho^0_{33}\sin^2\theta + \rho^0_{11}\left(\frac{1}{3}+\cos^2\theta\right) - \frac{2}{\sqrt{3}}\mathrm{Re}\rho^0_{31}\sin 2\theta\cos\phi - \frac{2}{\sqrt{3}}\mathrm{Re}\rho^0_{3-1}\sin^2\theta\cos 2\phi \right.$$
$$- P_\gamma\cos 2\Phi\left[\rho^1_{33}\sin^2\theta + \rho^1_{11}\left(\frac{1}{3}+\cos^2\theta\right) - \frac{2}{\sqrt{3}}\mathrm{Re}\rho^1_{31}\sin 2\theta\cos\phi - \frac{2}{\sqrt{3}}\mathrm{Re}\rho^1_{3-1}\sin^2\theta\cos 2\phi\right]$$
$$\left. - P_\gamma\sin 2\Phi\frac{2}{\sqrt{3}}\left[\mathrm{Im}\rho^2_{31}\sin 2\theta\sin\phi + \mathrm{Im}\rho^2_{3-1}\sin^2\theta\sin 2\phi\right]\right\}.$$

Amplitudes and SDME

Amplitudes N_σ and U_σ are positive and negative reflectivity with $\sigma = \lambda_p - \lambda_\Delta$

$$\rho^0_{\frac{1}{2}\frac{1}{2}} + \rho^1_{\frac{1}{2}\frac{1}{2}} = \frac{2}{N}\left(|N_0|^2 + |N_1|^2\right) \qquad \mathrm{Re}\left(\rho^0_{\frac{3}{2}\frac{1}{2}} + \rho^1_{\frac{3}{2}\frac{1}{2}}\right) = \frac{2}{N}\mathrm{Re}\left(N_{-1}N_0^* - N_1 N_2^*\right)$$

$$\rho^0_{\frac{1}{2}\frac{1}{2}} - \rho^1_{\frac{1}{2}\frac{1}{2}} = \frac{2}{N}\left(|U_0|^2 + |U_1|^2\right) \qquad \mathrm{Re}\left(\rho^0_{\frac{3}{2}\frac{1}{2}} - \rho^1_{\frac{3}{2}\frac{1}{2}}\right) = \frac{2}{N}\mathrm{Re}\left(U_{-1}U_0^* - U_1 U_2^*\right)$$

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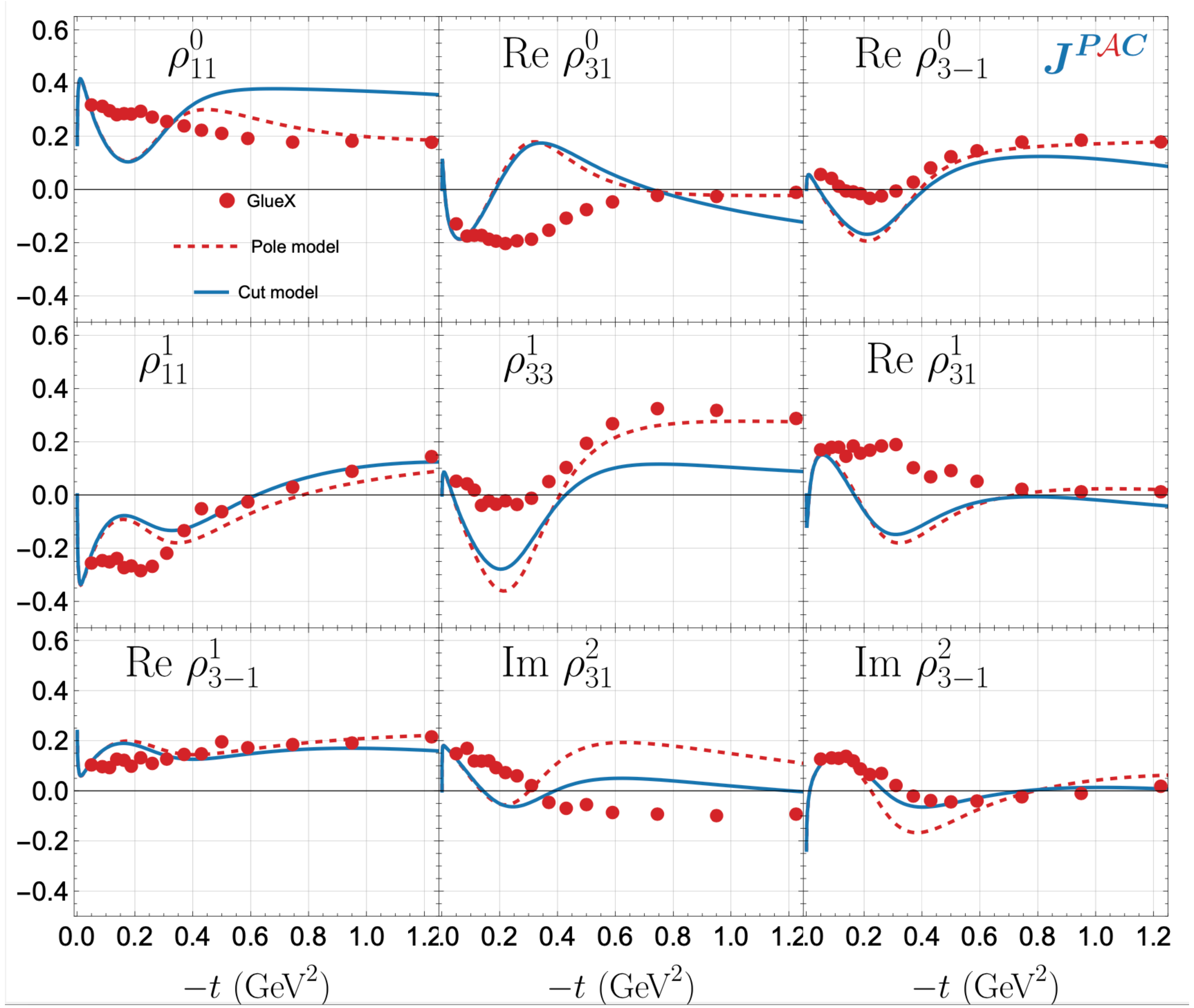
$$\mathrm{Im}\,\rho^2_{\frac{3}{2}\frac{1}{2}} = \frac{2}{N}\mathrm{Im}\left(N_1 U_2^* + N_0 U_{-1}^* - N_{-1}U_0^* - N_2 U_1^*\right)$$

$$\mathrm{Im}\,\rho^2_{\frac{3}{2}-\frac{1}{2}} = \frac{2}{N}\mathrm{Im}\left(N_1 U_{-1}^* - N_0 U_2^* - N_{-1}U_1^* + N_2 U_0^*\right)$$

$$N = 2\left(|N_{-1}|^2 + |N_0|^2 + |N_1|^2 + |N_2|^2 + |U_{-1}|^2 + |U_0|^2 + |U_1|^2 + |U_2|^2\right)$$

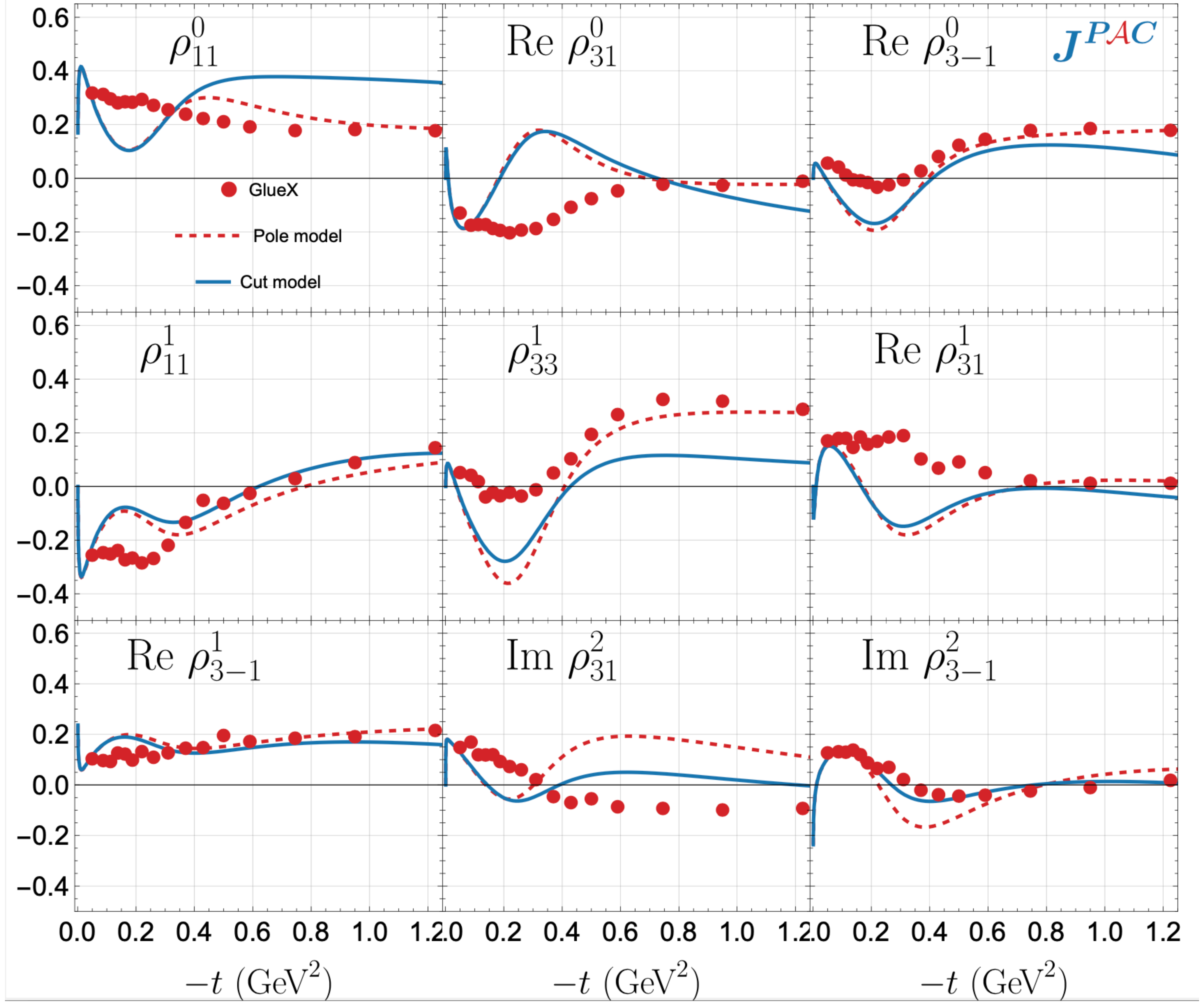
There are 8 complex amplitudes and 10 (= 9 sdme + 1 x-section) real observables

SDME @GlueX in the GJ frame



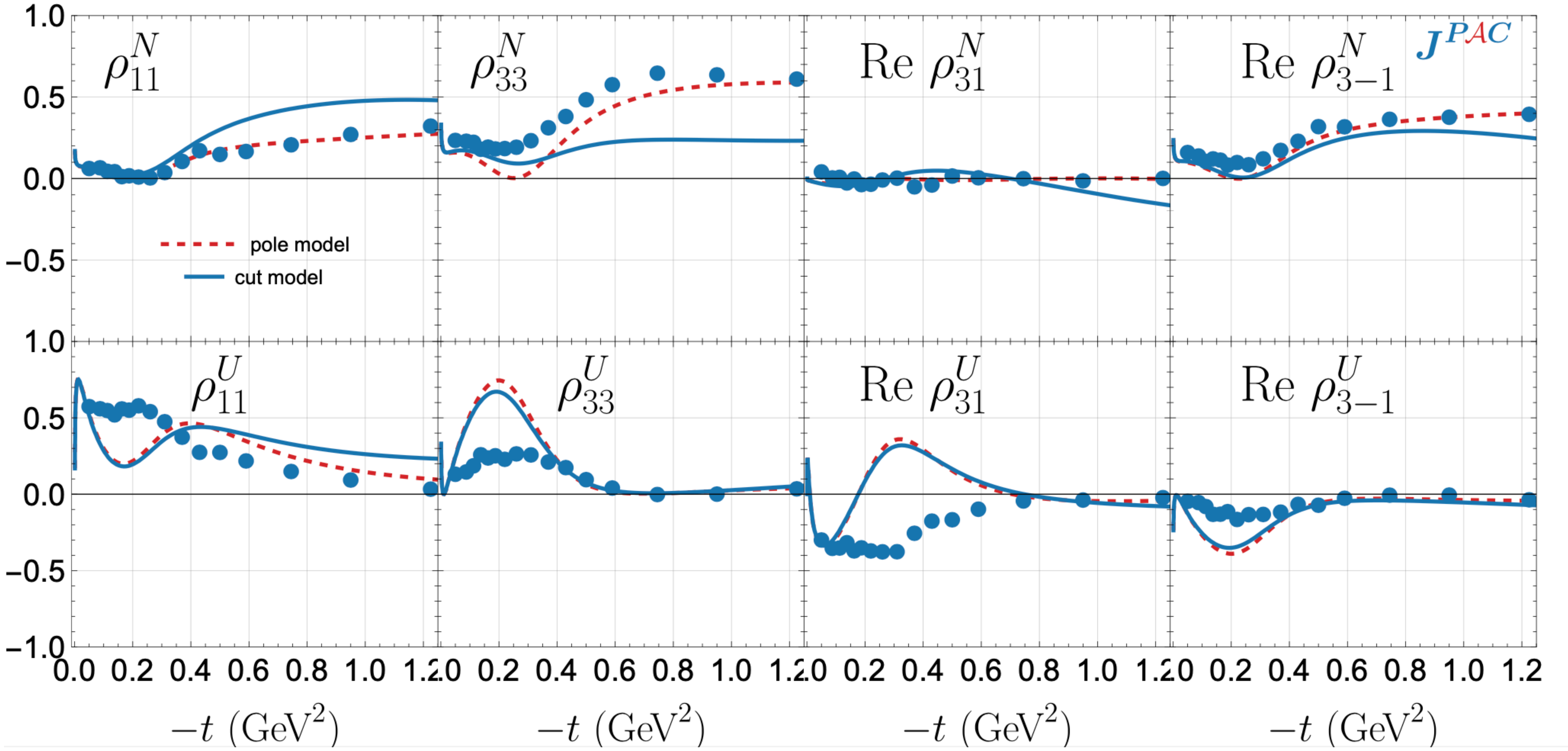
Dashed and solid lines are two versions of the model

SDME @GlueX in the GJ frame

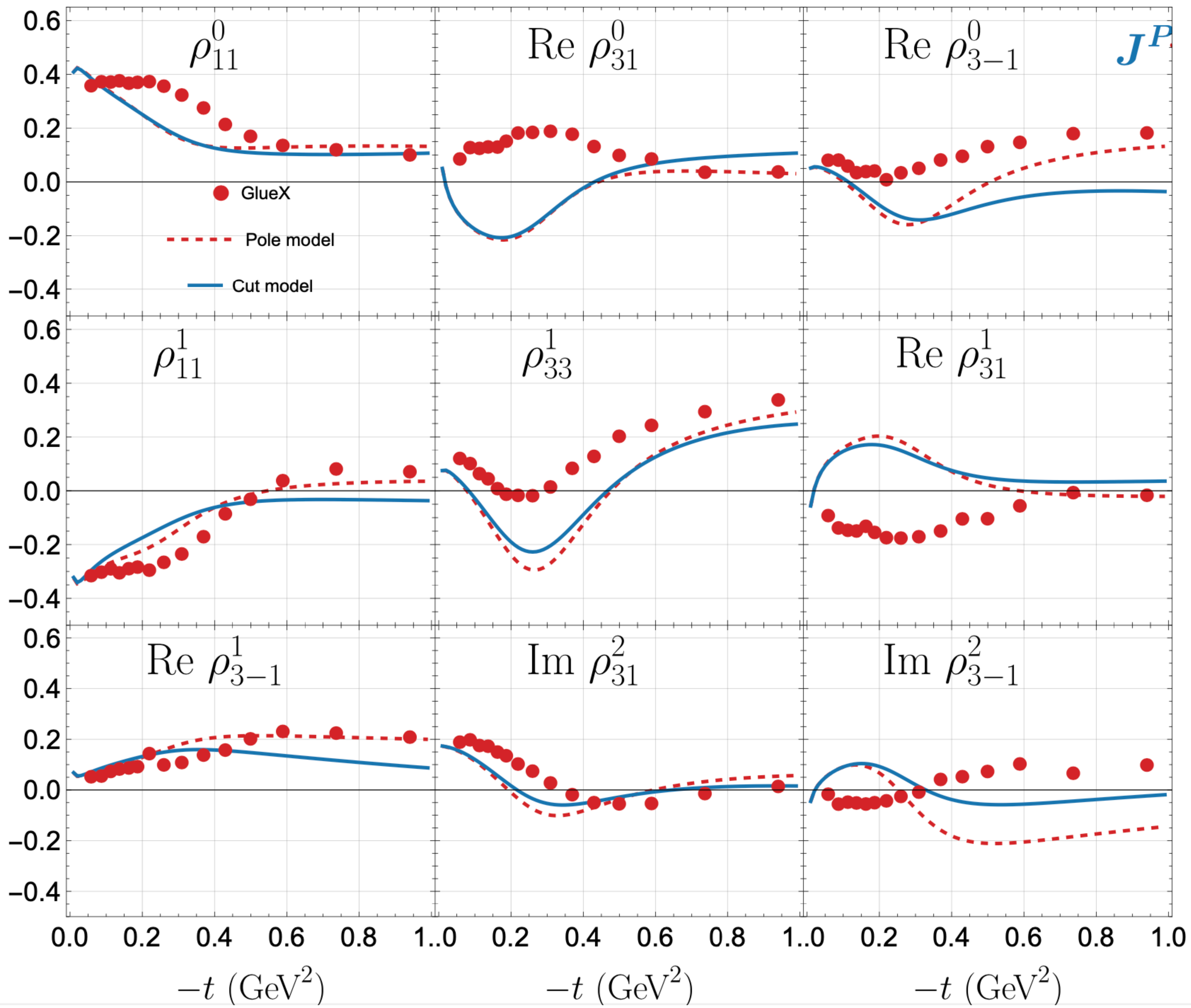


Dashed and solid lines are two versions of the model

Good agreement for the positive reflectivity components



SDME @GlueX in the Helicity frame



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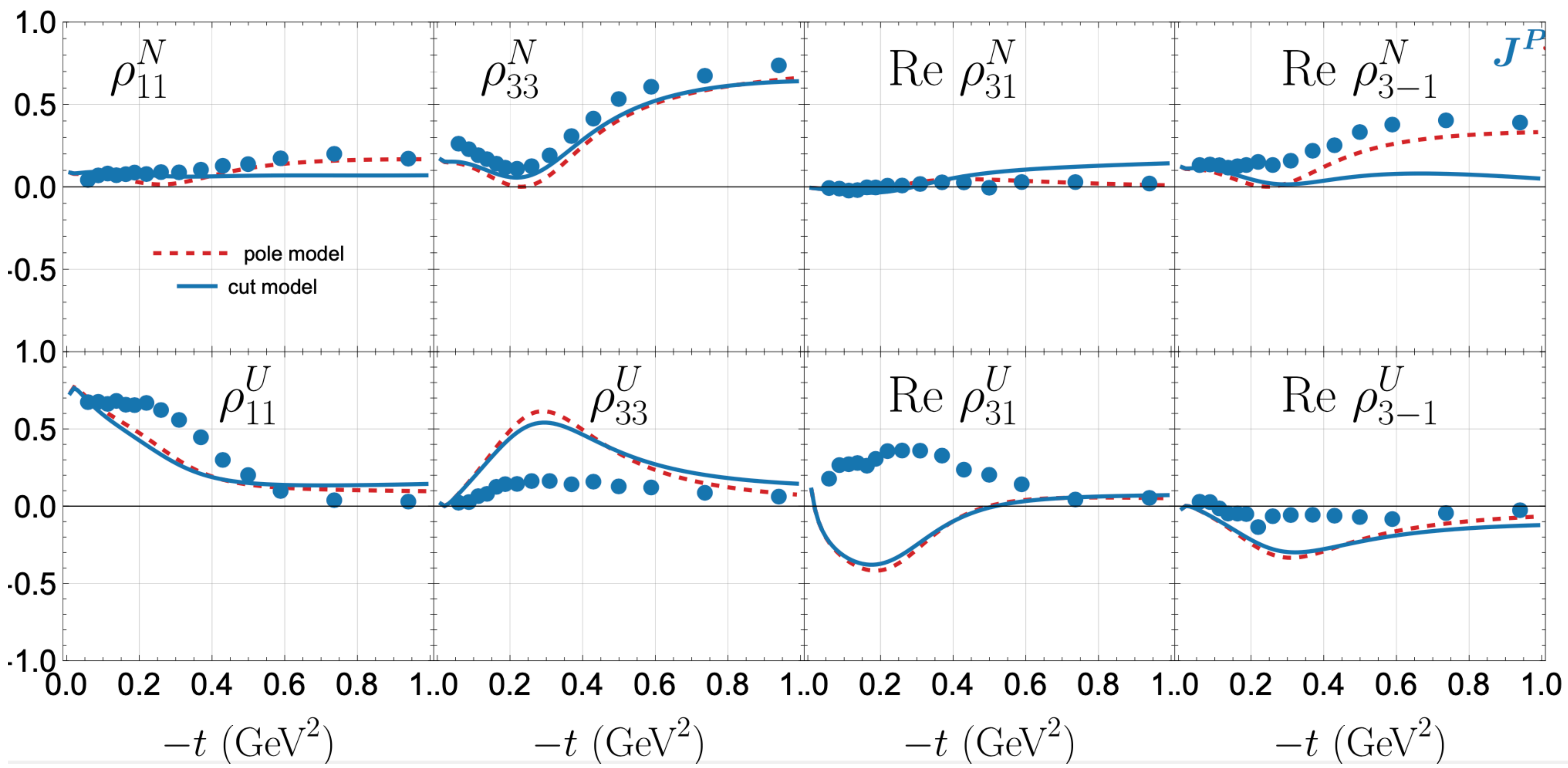
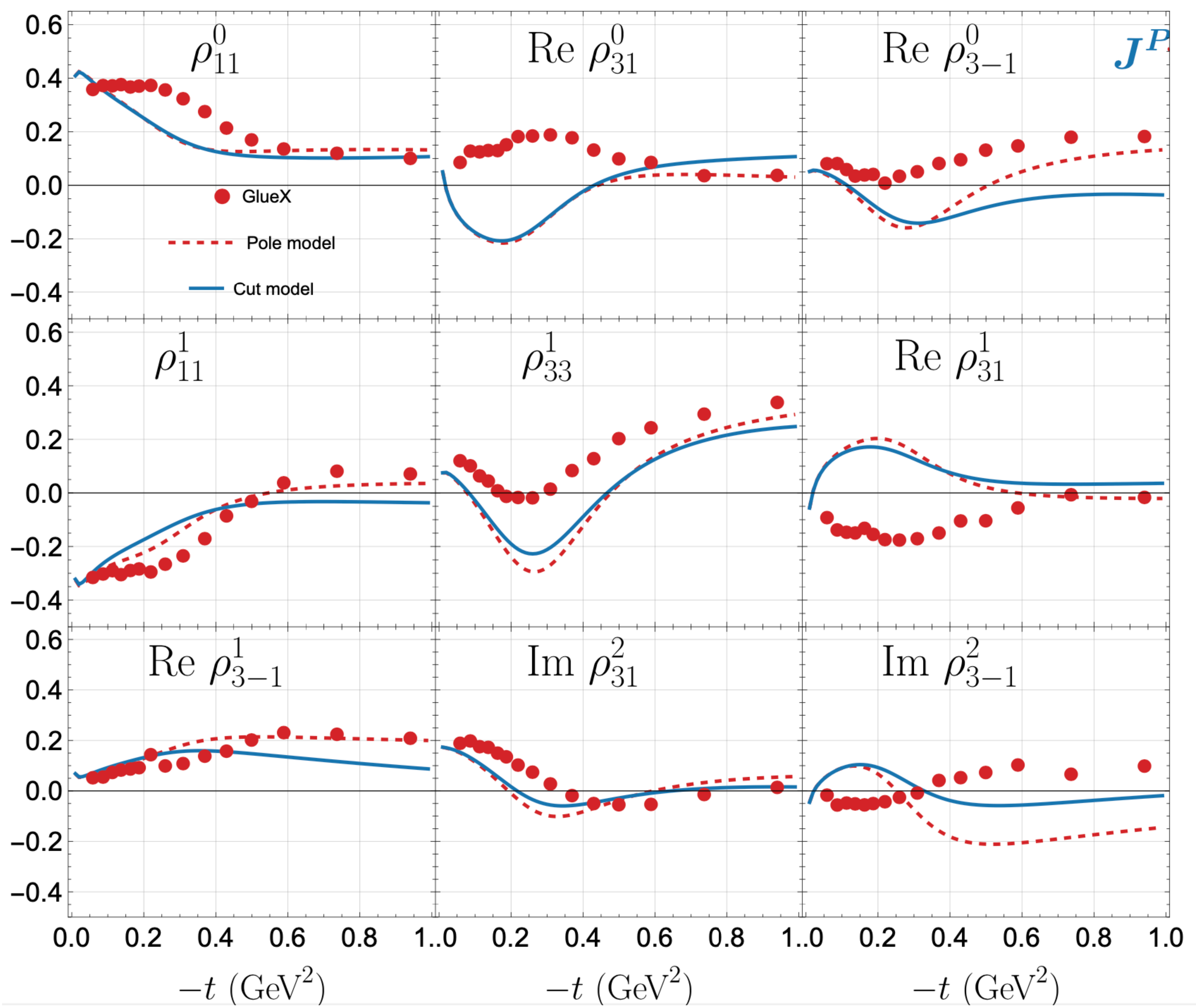
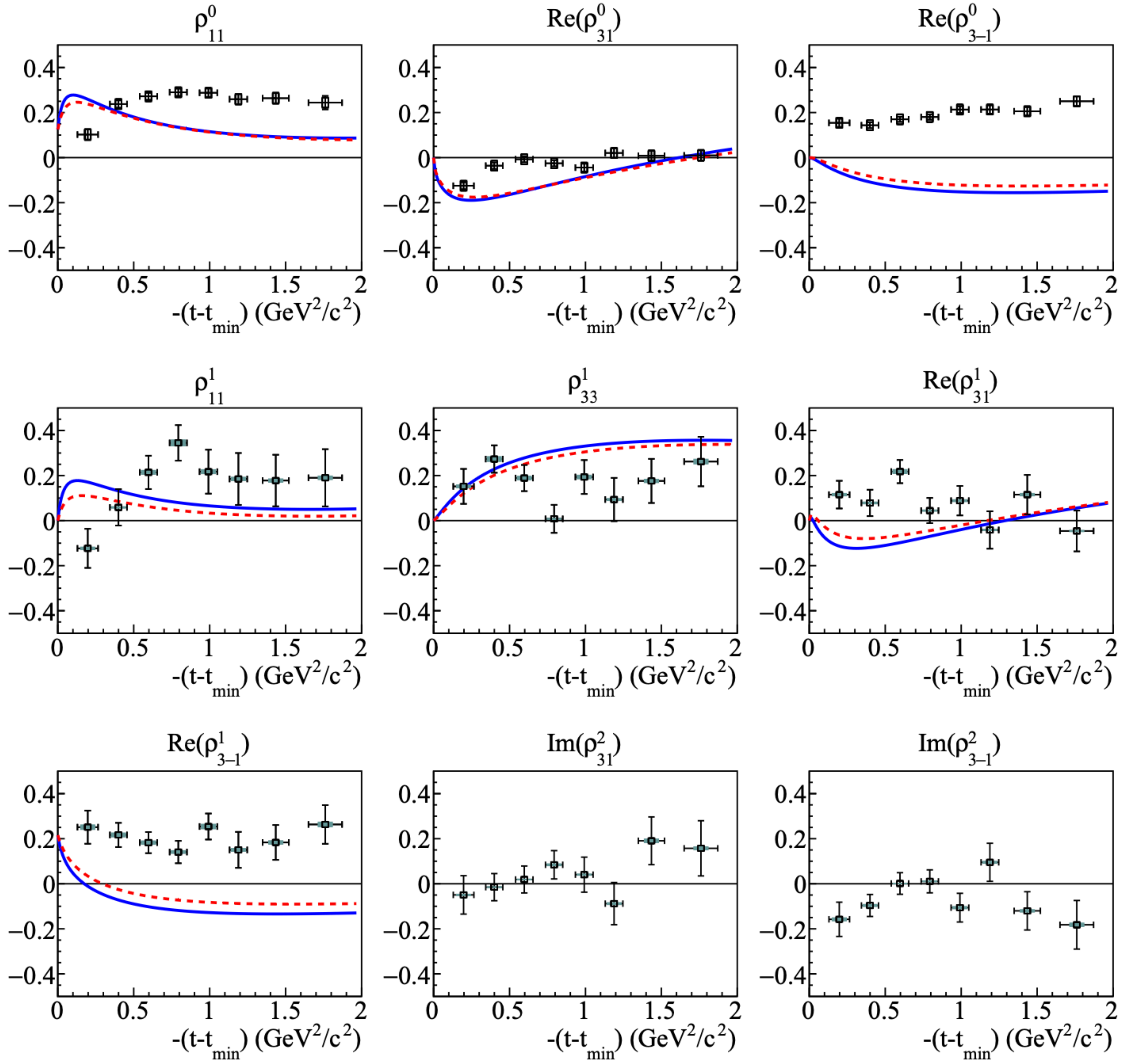


$\Lambda(1520)$SDME @GlueX in the GJ frame

$E_\gamma = 8.2 - 8.8$ GeV

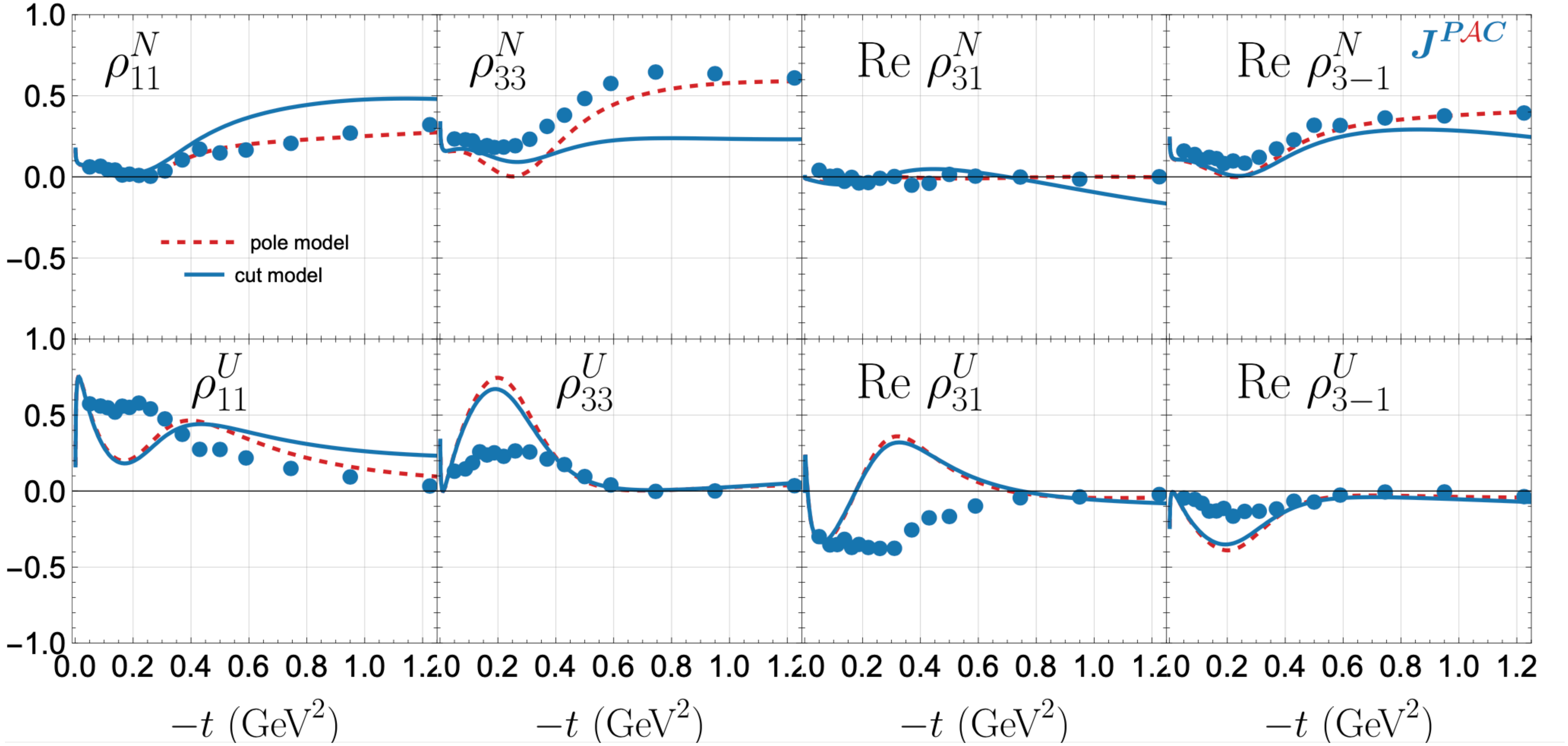




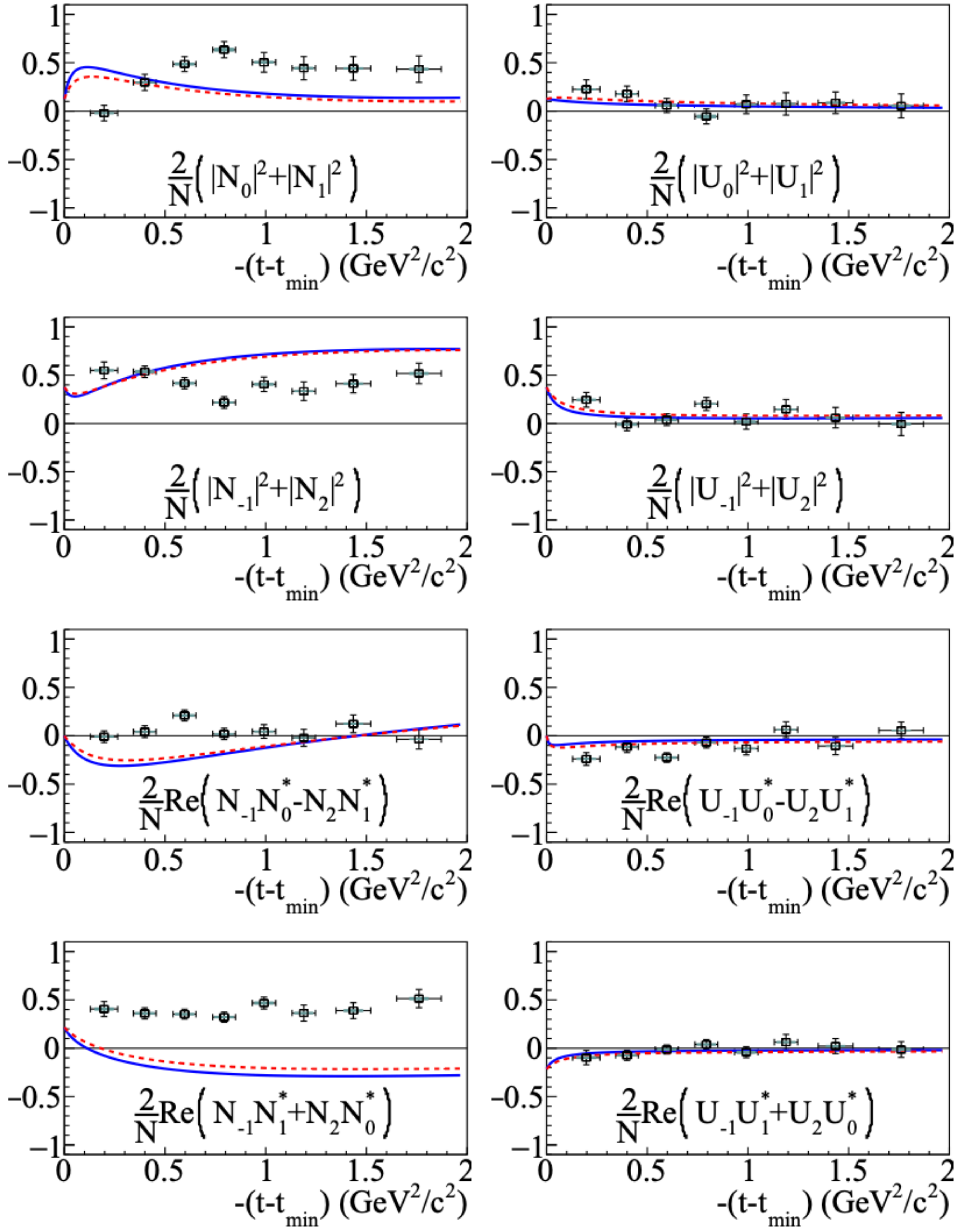
$\Lambda(1520)$SDME @GlueX in the GJ frame

Split between positive and negative reflectivity components

Δ^{++}

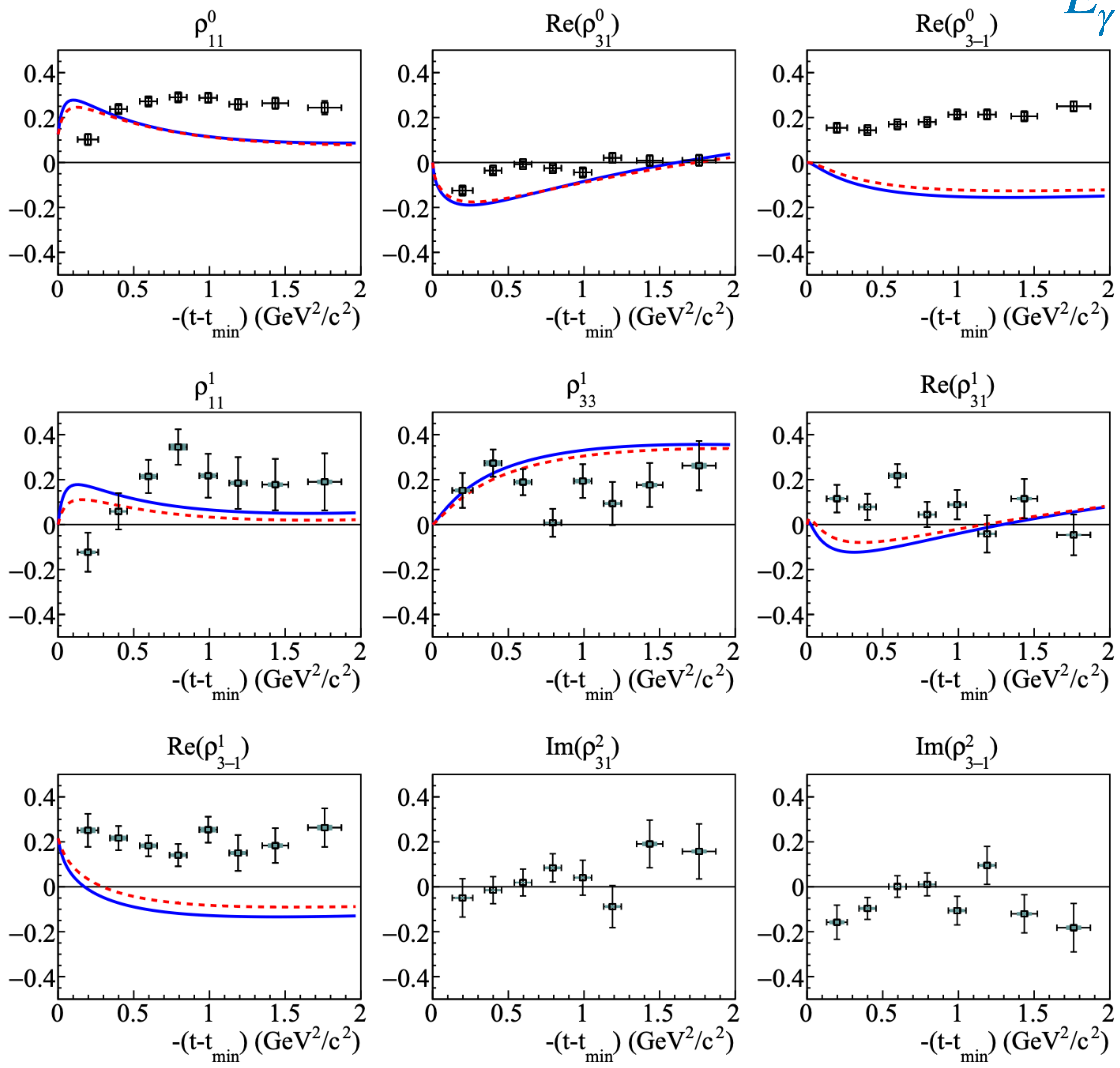


$\Lambda(1520)$



Λ(1520) SDME @GlueX

PHYSICAL REVIEW C **105**, 035201 (2022)



$E_\gamma = 8.2 - 8.8$ GeV