# Flavour deconstruction: from the electroweak scale to the GUT scale

Les Rencontres de Physique de la Vallée d'Aoste,
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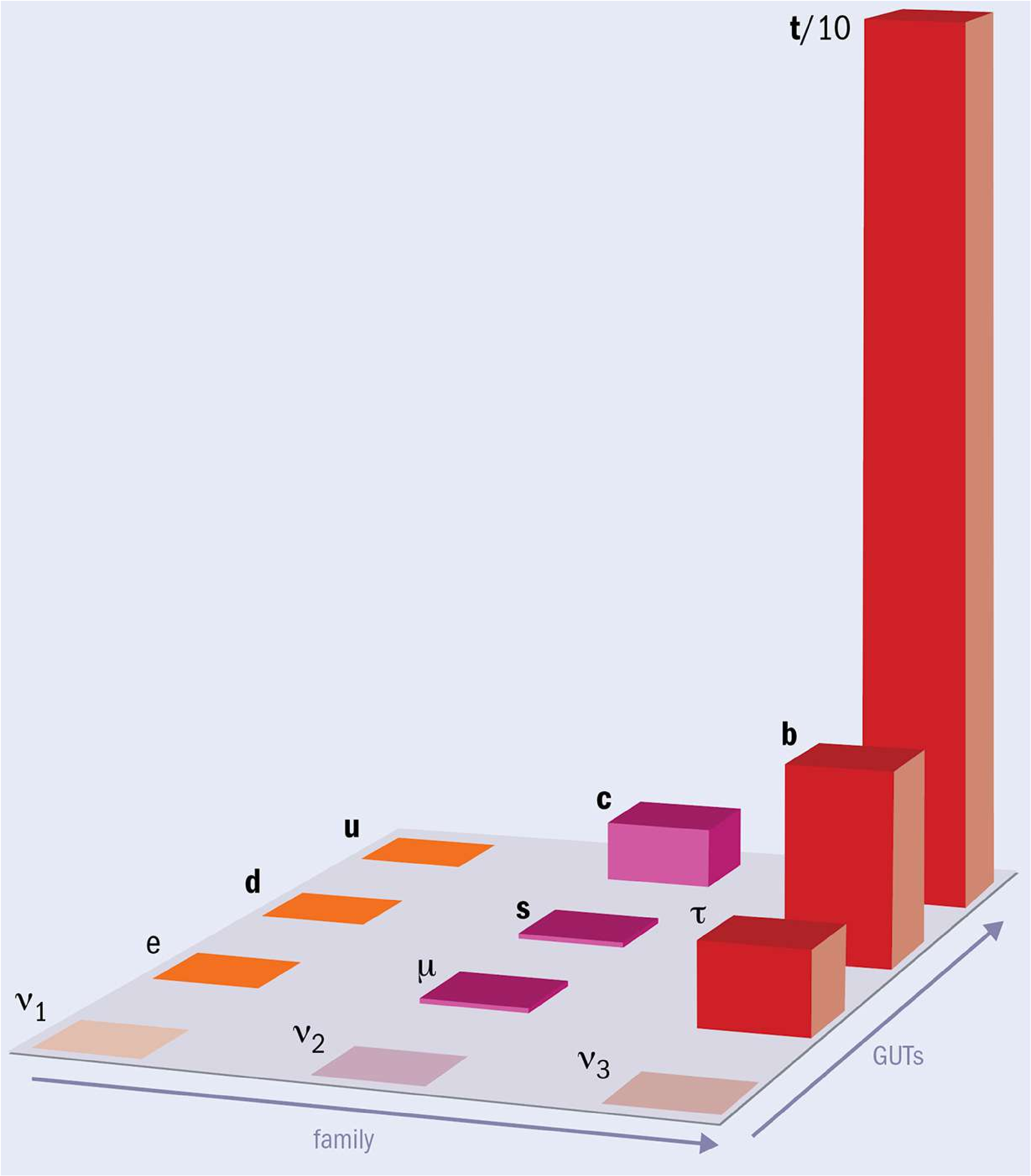
La Thuile, Italy

Mario Fernández Navarro

Based on:
MFN, Stephen F. King: [2305.07690] hep-ph, JHEP 08 (2023) 020
MFN, Stephen F. King and Avelino Vicente: [2311.05683] hep-ph

# The flavour puzzle




$$m_t \sim \frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_c \sim \lambda^{3.3}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_u \sim \lambda^{7.5}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$
$$m_b \sim \lambda^{2.5}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_s \sim \lambda^{5.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_d \sim \lambda^{7.0}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$
$$m_\tau \sim \lambda^{3.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_\mu \sim \lambda^{4.9}\frac{v_{\rm SM}}{\sqrt{2}}\,, \; m_e \sim \lambda^{8.4}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$

$$\tan\theta^\nu_{23} \sim 1\,, \; \tan\theta^\nu_{12} \sim \frac{1}{\sqrt{2}}\,, \; \theta^\nu_{13} \sim \frac{\lambda}{\sqrt{2}}\,, \quad V_{us} \sim \lambda\,, \; V_{cb} \sim \lambda^2\,, \; V_{ub} \sim \lambda^3$$

where $v_{\rm SM} \simeq 246\,\mathrm{GeV}$ and $\lambda \simeq \sin\theta_C \simeq 0.224$

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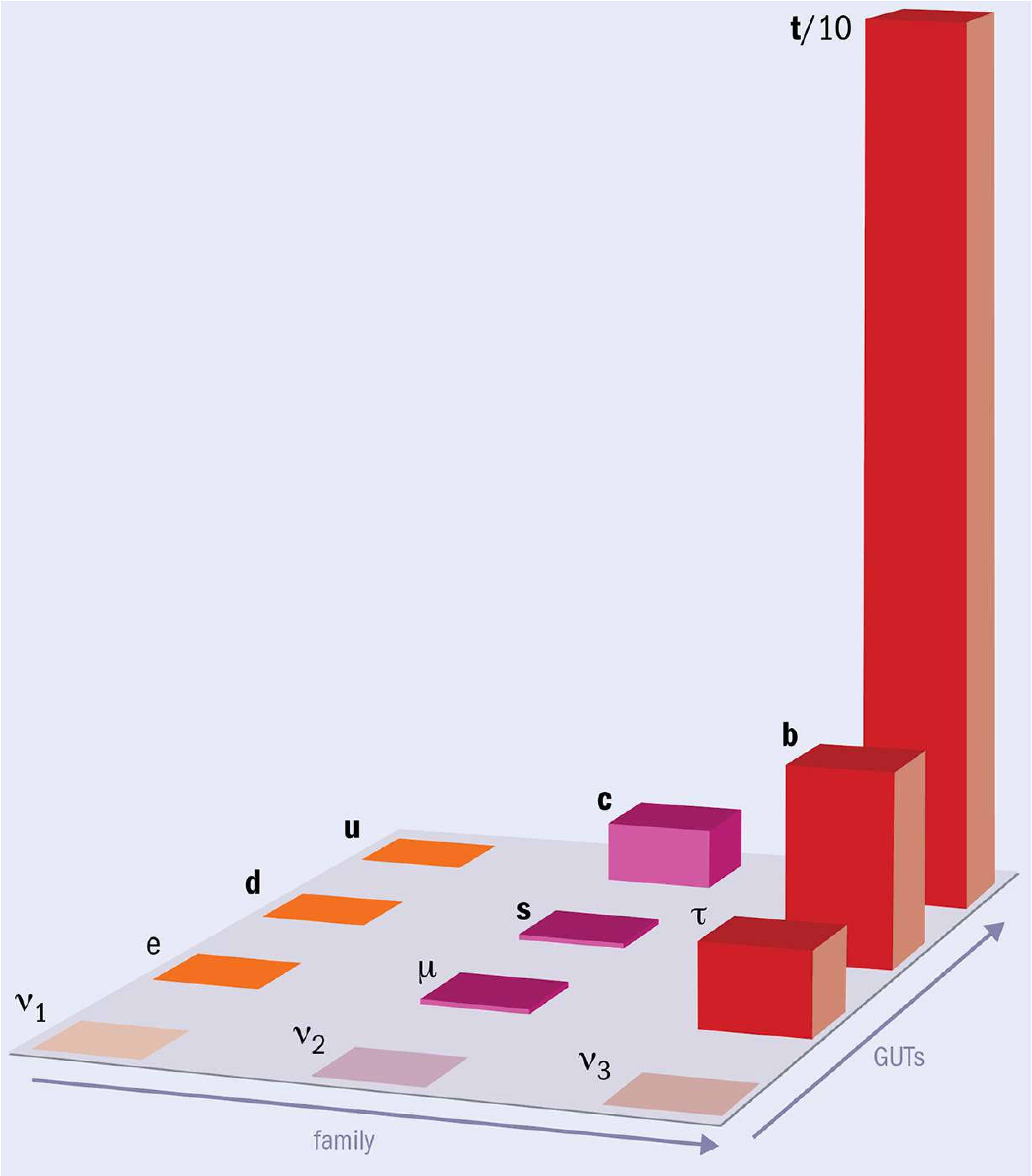


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where $v_{\rm SM} \simeq 246\,\text{GeV}$ and $\lambda \simeq \sin\theta_C \simeq 0.224$

- Why three (replicated) families?
- Why the three families interact so differently with the Higgs?
- What is the origin of very small neutrino masses, and why the PMNS mixing is so different from the CKM mixing?

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- SM is embedded in a gauge symmetry that contains a separate factor for each family:

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- $SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$ [This talk]
- $SU(3)_c \times SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \times U(1)_Y$ [Li and Ma, PRL 81'; Muller and Nandi, hep-ph/9602390; Chiang *et al*, 0911.1480; Allwicher *et al*, 2011.01946; Davighi *et al* 2312.13346; Capdevila *et al*, 2401.00848 .....]
- $SU(3)_{c,1} \times SU(3)_{c,2} \times SU(3)_{c,3} \times SU(2)_L \times U(1)_Y$ [Carone and Murayama, hep-ph/9504393]
- $SU(4)_{c,3} \times SU(3)_{c,12} \times SU(2)_L \times U(1)_{Y_1+Y_2+R_3}$ [Bordone *et al*, 1712.01368; Greljo and Stefanek, 1802.04274; Cornella *et al*, 1903.11517; Fuentes-Martín *et al*, 2006.16250 .....]
- $SU(3)_c \times SU(2)_L \times U(1)_{R_1} \times U(1)_{R_2} \times U(1)_{(B-L)_{12}} \times U(1)_{Y_3}$ [Barbieri and Isidori, 2312.14004]

# Tri-hypercharge: basics

| Field | $SU(3)_c$ | $SU(2)_L$ | $U(1)_{Y_1}$ | $U(1)_{Y_2}$ | $U(1)_{Y_3}$ |
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| $d_1^c$ | $\bar{\mathbf{3}}$ | $\mathbf{1}$ | $1/3$ | $0$ | $0$ |
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$\psi^c \equiv$ CP-conjugated (2-component) right-handed fields

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- Type II 2HDM can take care of $m_{b,\tau}/m_t$ mass hierarchies via $\tan\beta = v_u/v_d \approx \lambda^{-2} \simeq 20$.
- Light charged fermion masses and mixing arise from the tri-hypercharge SSB down to SM hypercharge:

$$Y_{\mathrm{SM}} \equiv Y = Y_1 + Y_2 + Y_3$$

# Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

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$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Phi(0, -\frac{1}{6}, \frac{1}{6}) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

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- Promote spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$ to the physical scalar ("hyperon") , $\phi_{q23} \sim (0, -\frac{1}{6}, \frac{1}{6})$

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- We also promote the spurion in the (2,2) entry to hyperon $\Phi \sim (0,-\frac{1}{2},\frac{1}{2}) \sim \phi_{\ell 23}$:

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Flavour structure dynamically generated via tri-hypercharge SSB

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- So far we have two scalars (hyperons) $\phi_{\ell 23} \sim (0,-\frac{1}{2},\frac{1}{2})$ and $\phi_{q23} \sim (0,-\frac{1}{6},\frac{1}{6})$

$$Y_d = \begin{pmatrix} \approx 0 & \approx 0 & \approx 0 \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_{\ell 23}} & \frac{\phi_{q23}}{\Lambda_{q23}} \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_{\ell 23}}\frac{\phi_{q23}}{\Lambda_{q23}} & 1 \end{pmatrix}$$

# Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2}, 0, \frac{1}{2}) & \Phi(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{2}) & \Phi(-\frac{1}{6}, 0, \frac{1}{6}) \\ \Phi(-\frac{1}{3}, -\frac{1}{6}, \frac{1}{2}) & \Phi(0, -\frac{1}{2}, \frac{1}{2}) & \Phi(0, -\frac{1}{6}, \frac{1}{6}) \\ \Phi(-\frac{1}{3}, 0, \frac{1}{3}) & \Phi(0, -\frac{1}{3}, \frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d + \text{h.c.}$$

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- Need to generate remaining quark mixing:

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$$Y_d = \begin{pmatrix} \approx 0 & \frac{\phi_{q12}\phi_{\ell 23}}{\Lambda_{q12}\Lambda_{\ell 23}} & \frac{\phi_{q12}\phi_{q23}}{\Lambda_{q12}\Lambda_{q23}} \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_{\ell 23}} & \frac{\phi_{q23}}{\Lambda_{q23}} \\ \approx 0 & \frac{\phi_{\ell 23}\phi_{q23}}{\Lambda_{\ell 23}\Lambda_{q23}} & 1 \end{pmatrix}$$

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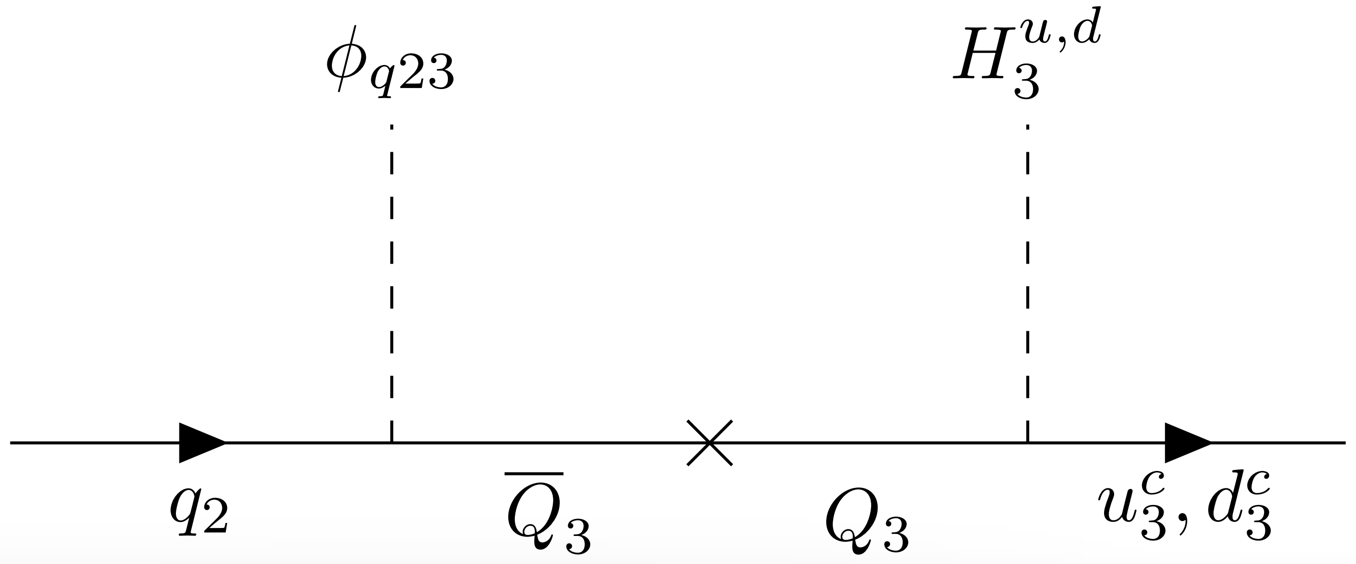
$$Y_d = \begin{pmatrix} \approx 0 & \frac{\phi_{q12}\phi_{\ell 23}}{\Lambda_{q12}\Lambda_{\ell 23}} & \frac{\phi_{q12}\phi_{q23}}{\Lambda_{q12}\Lambda_{q23}} \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_{\ell 23}} & \frac{\phi_{q23}}{\Lambda_{q23}} \\ \approx 0 & \frac{\phi_{\ell 23}\phi_{q23}}{\Lambda_{\ell 23}\Lambda_{q23}} & 1 \end{pmatrix}$$

- Promote (1,1) spurion to hyperon $\phi_{\ell 13} \sim \left(-\frac{1}{2},0,\frac{1}{2}\right)$

Heavy messengers needed for $\Lambda$!

# Tri-hypercharge: UV-complete model

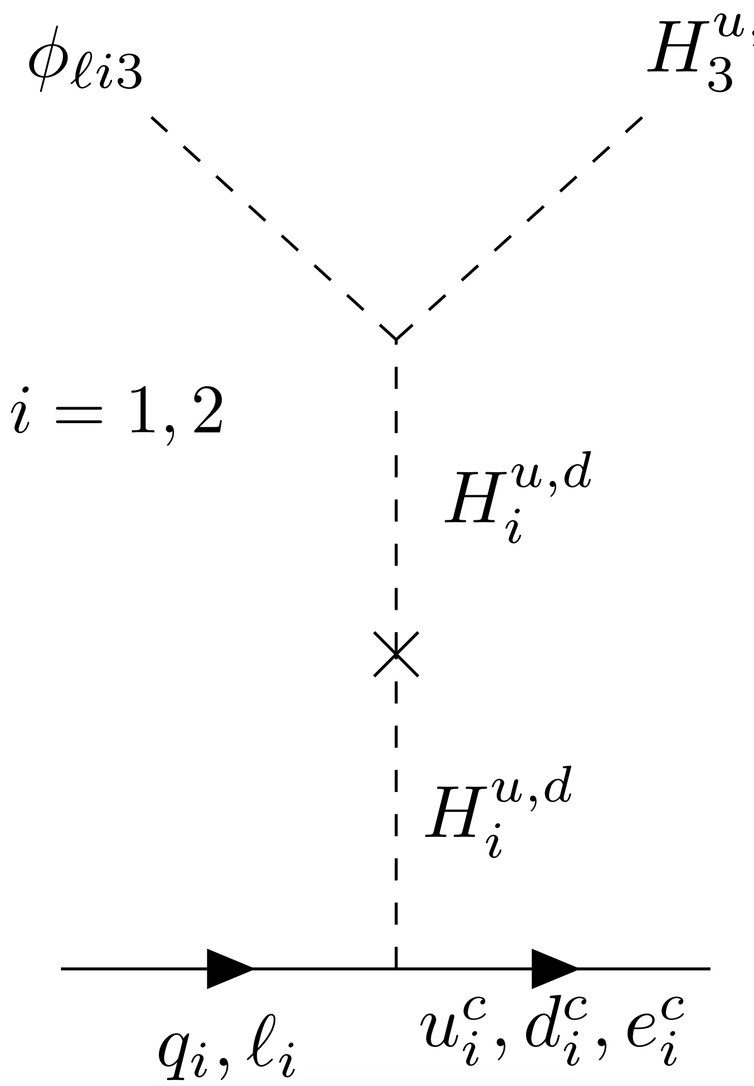
| | $U(1)_{Y_1}$ | $U(1)_{Y_2}$ | $U(1)_{Y_3}$ | $SU(3)_c \times SU(2)_L$ |
|---|---|---|---|---|
| $H_3^{u,d}$ | 0 | 0 | $\pm 1/2$ | $(\mathbf{1},\mathbf{2})$ |
| $H_2^{u,d}$ | 0 | $\pm 1/2$ | 0 | $(\mathbf{1},\mathbf{2})$ |
| $H_1^{u,d}$ | $\pm 1/2$ | 0 | 0 | $(\mathbf{1},\mathbf{2})$ |
| $\phi_{q12}$ | -1/6 | 1/6 | 0 | $(\mathbf{1},\mathbf{1})$ |
| $\phi_{q23}$ | 0 | -1/6 | 1/6 | $(\mathbf{1},\mathbf{1})$ |
| $\phi_{\ell 13}$ | -1/2 | 0 | 1/2 | $(\mathbf{1},\mathbf{1})$ |
| $\phi_{\ell 23}$ | 0 | -1/2 | 1/2 | $(\mathbf{1},\mathbf{1})$ |
| $Q_1$ | 1/6 | 0 | 0 | $(\mathbf{3},\mathbf{2})$ |
| $Q_2$ | 0 | 1/6 | 0 | $(\mathbf{3},\mathbf{2})$ |
| $Q_3$ | 0 | 0 | 1/6 | $(\mathbf{3},\mathbf{2})$ |




Heavy messengers chosen to have approx. same matter under each hypercharge

$$\mathcal{L} = Y_u^{ij} q_i H_3^u u_j^c + Y_d^{ij} q_i H_3^d d_j^c + Y_e^{ij} \ell_i H_3^d e_j^c + \text{h.c.}$$

$$Y_d = \begin{pmatrix} c_{11}^d \dfrac{\phi_{\ell 13}}{M_{H_1^d}} & c_{12}^d \dfrac{\phi_{q12}}{M_{Q_2}} \dfrac{\phi_{\ell 23}}{M_{H_2^d}} & c_{13}^d \dfrac{\phi_{q12}}{M_{Q_2}} \dfrac{\phi_{q23}}{M_{Q_3}} \\ c_{21}^d \dfrac{\phi_{\ell 13}}{M_{H_1^d}} \dfrac{\phi_{q12}}{M_{Q_1}} & c_{22}^d \dfrac{\phi_{\ell 23}}{M_{H_2^d}} & c_{23}^d \dfrac{\phi_{q23}}{M_{Q_3}} \\ c_{31}^d \dfrac{\phi_{\ell 13}}{M_{H_1^d}} \dfrac{\phi_{q12}}{M_{Q_2}} \dfrac{\phi_{q23}}{M_{Q_3}} & c_{32}^d \dfrac{\phi_{\ell 23}}{M_{H_2^d}} \dfrac{\phi_{q23}}{M_{Q_2}} & y_b \end{pmatrix}$$




$$Y_u = Y_d(d \to u)$$

$$Y_e = \begin{pmatrix} c_{11}^e \dfrac{\phi_{\ell 13}}{M_{H_1^d}} & 0 & 0 \\ 0 & c_{22}^e \dfrac{\phi_{\ell 23}}{M_{H_2^d}} & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

# Tri-hypercharge: UV-complete model

Spectrum up to $\mathcal{O}(1)$ variations:

$E$

$\mathcal{O}(10^5\,\mathrm{TeV})$ — $M_{H_1^{u,d}},\ M_{Q_{1,2}}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$

$\mathcal{O}(1000\,\mathrm{TeV})$ — $v_{12} \sim \langle \phi_{q12} \rangle,\ M_{Q_3},\ M_{H_2^{u,d}}$

$\mathcal{O}(10\,\mathrm{TeV})$ — $v_{23} \sim \langle \phi_{q23} \rangle,\ \langle \phi_{\ell 23} \rangle,\ \langle \phi_{\ell 13} \rangle$

$174\,\mathrm{GeV}$ — $v_{\mathrm{SM}} \sim \langle H_3^{u,d} \rangle$

$$\frac{\langle \phi_{\ell 23} \rangle}{M_{H_2^{u,d}}} \sim \lambda^3 \qquad \frac{\langle \phi_{q23} \rangle}{M_{Q_3}} \sim \lambda^2$$

$$\frac{\langle \phi_{\ell 13} \rangle}{M_{H_1^{u,d}}} \sim \lambda^6 \qquad \frac{\langle \phi_{q12} \rangle}{M_{Q_{1,2}}} \sim \lambda$$

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$\downarrow$

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$SU(3)_c \times SU(2)_L \times U(1)_{Y_{12} \equiv Y_1 + Y_2} \times U(1)_{Y_3}$

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$SU(3)_c \times SU(2)_L \times U(1)_{Y \equiv Y_1 + Y_2 + Y_3}$

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$\downarrow$

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Different CKM alignments are possible (via $\mathcal{O}(1)$ coefficients)

$$\lambda \simeq \sin\theta_C \simeq 0.224$$

$$\begin{aligned}\mathcal{L} = {} & \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \lambda^7 & \lambda^4 & \lambda^3 \\ \lambda^8 & \lambda^3 & \lambda^2 \\ \lambda^{10} & \lambda^5 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} v_{\rm SM} \\ & + \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^7 & \lambda^3 & \lambda^2 \\ \lambda^9 & \lambda^5 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 v_{\rm SM} \\ & + \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \end{pmatrix} \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 v_{\rm SM} \\ & + \text{h.c.}\end{aligned}$$

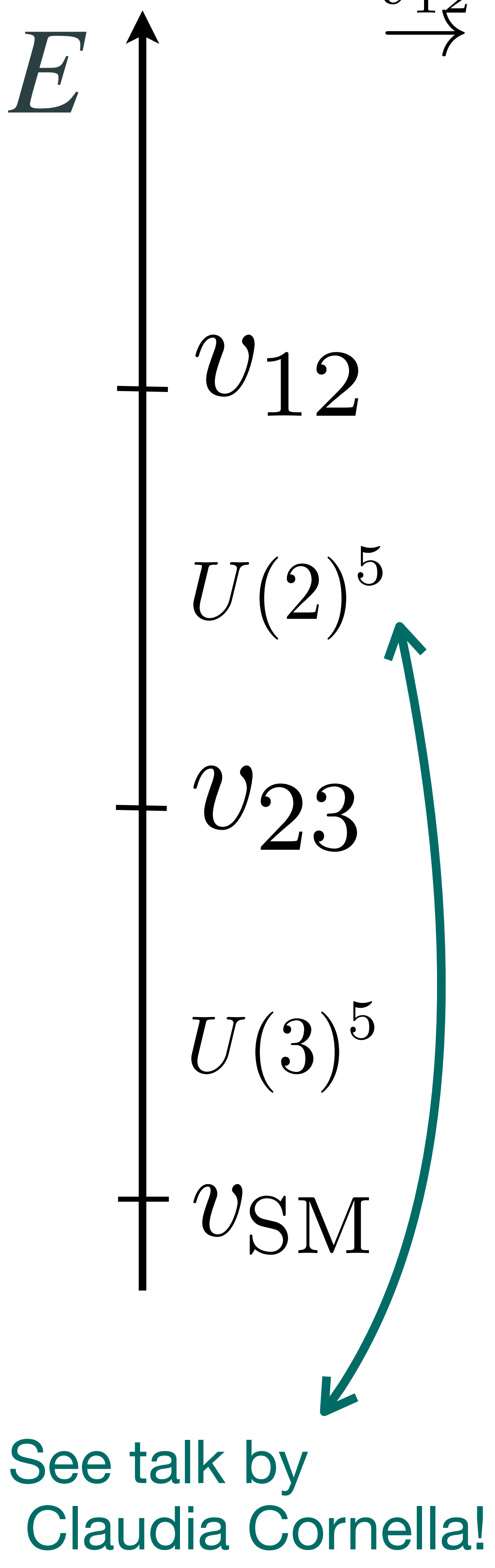
up to $\mathcal{O}(1)$ coefficients!

Charged leptons approx. diagonal

# Phenomenology

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

$$\xrightarrow{v_{12}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + {\color{red} Z'_{12}}$$



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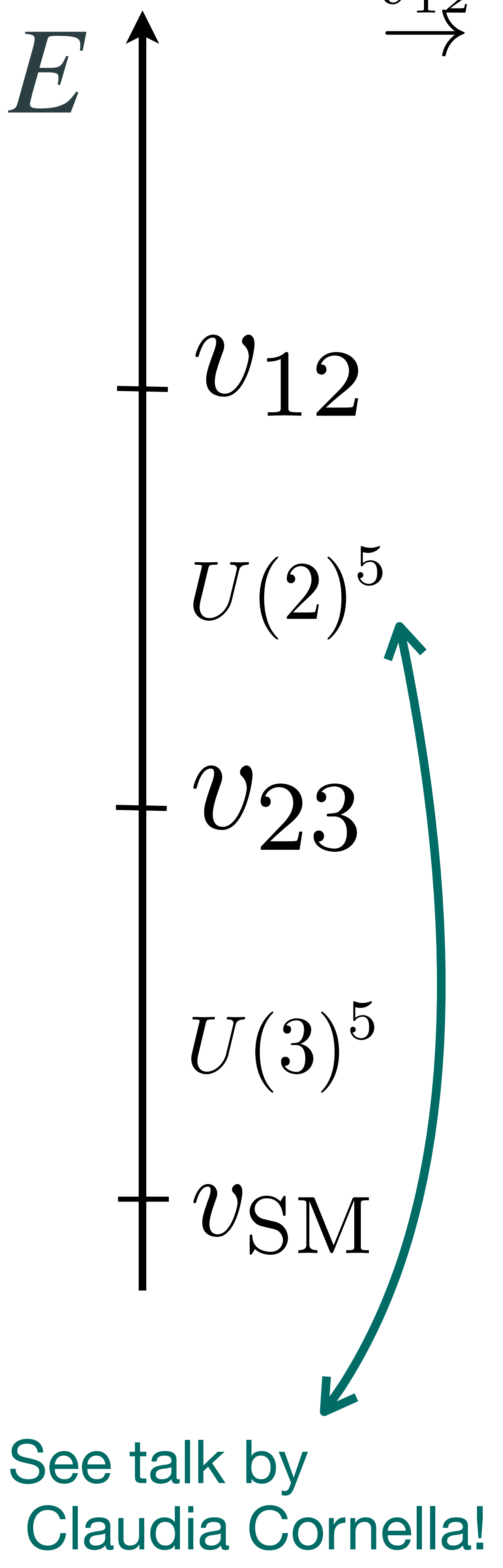


- $Z'_{12}$ potentially mediating $K-\overline{K}$ $(D-\overline{D})$ mixing and $\mu \to e\gamma$ $(3e)$, typically:

$$\boxed{v_{12} \gtrsim \mathcal{O}(100\ \mathrm{TeV})}$$

  - Higher for large $(\theta_R^d)_{12}$ and $(\theta_R^e)_{12}$.

# Phenomenology

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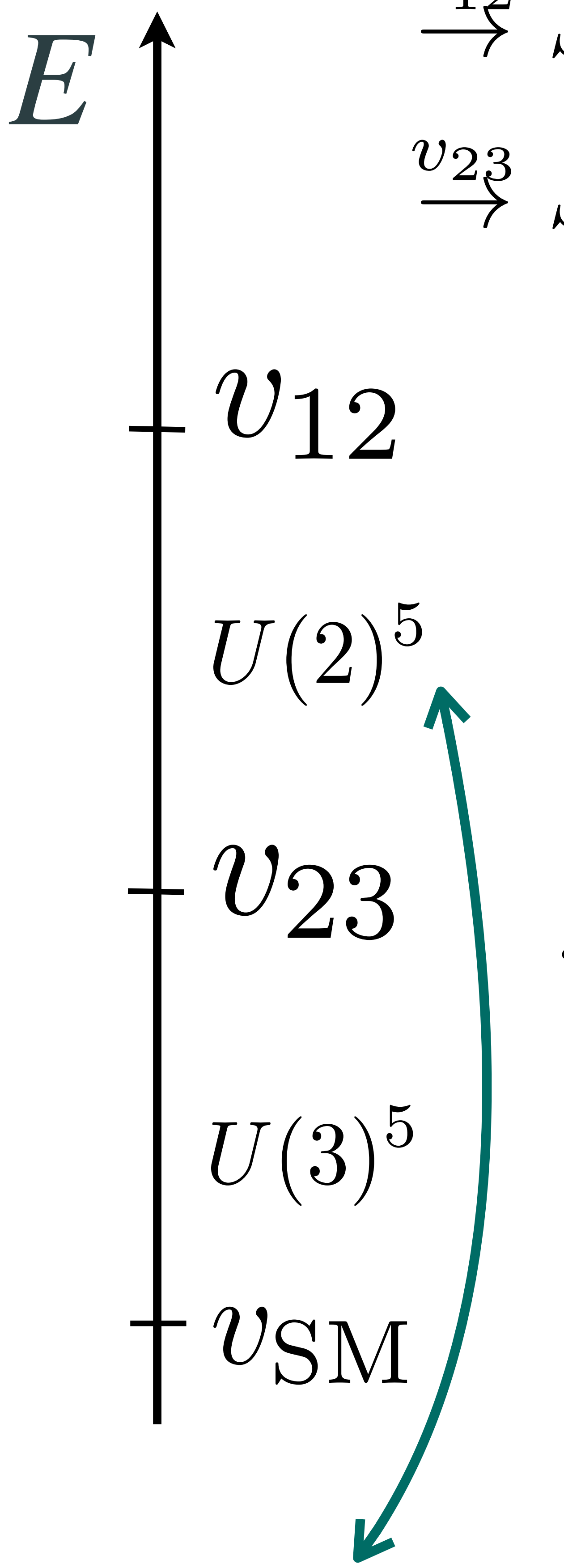
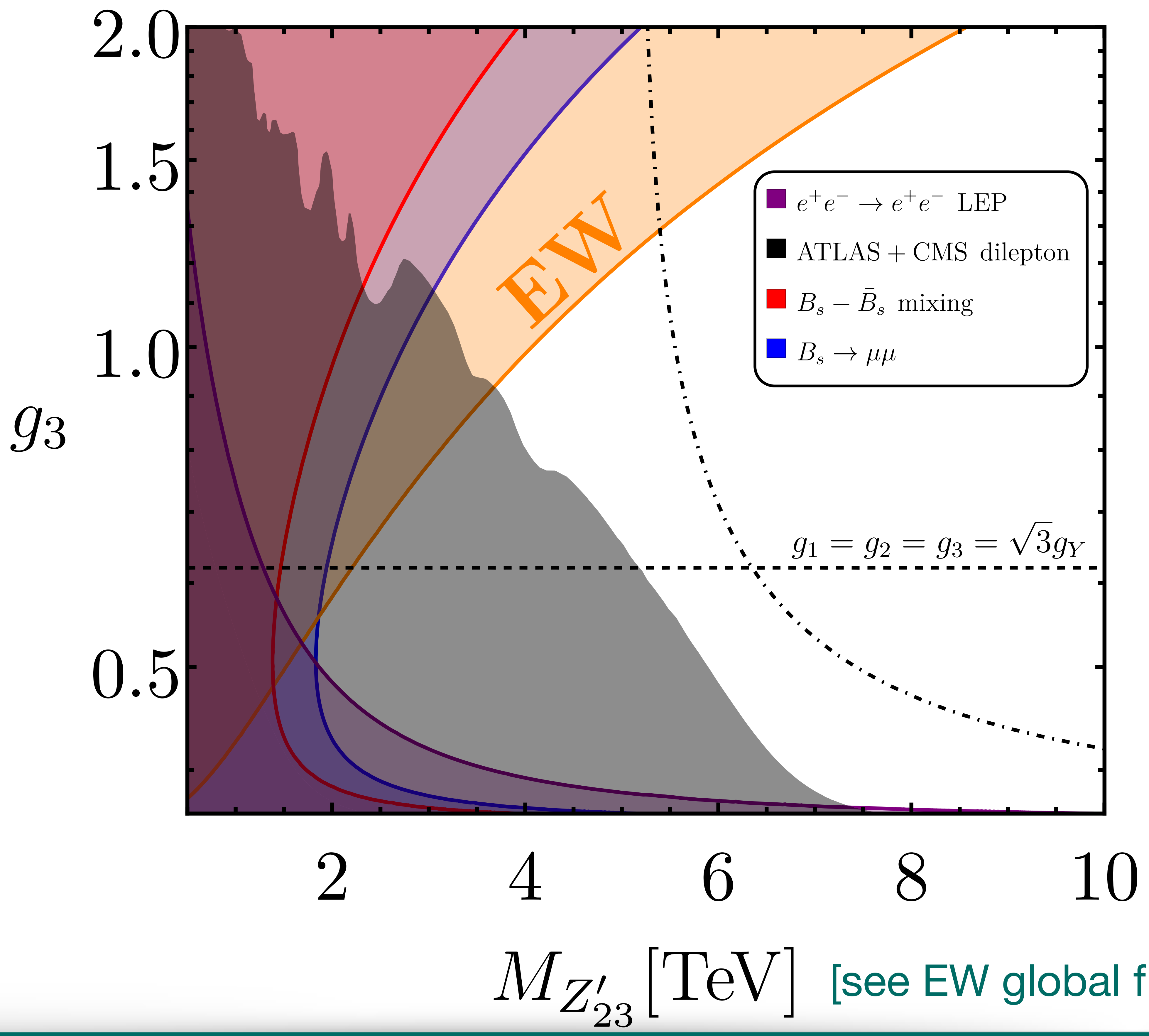
$$\overset{v_{12}}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$
$$\overset{v_{23}}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$




See talk by
Claudia Cornella!




[see EW global fit and FCC-ee projections in Davighi and Stefanek, 2305.113344]

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$$g_Y = \frac{g_{12} g_3}{\sqrt{g_{12}^2 + g_3^2}} \simeq 0.36\,(M_Z) \qquad g_{12} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}$$

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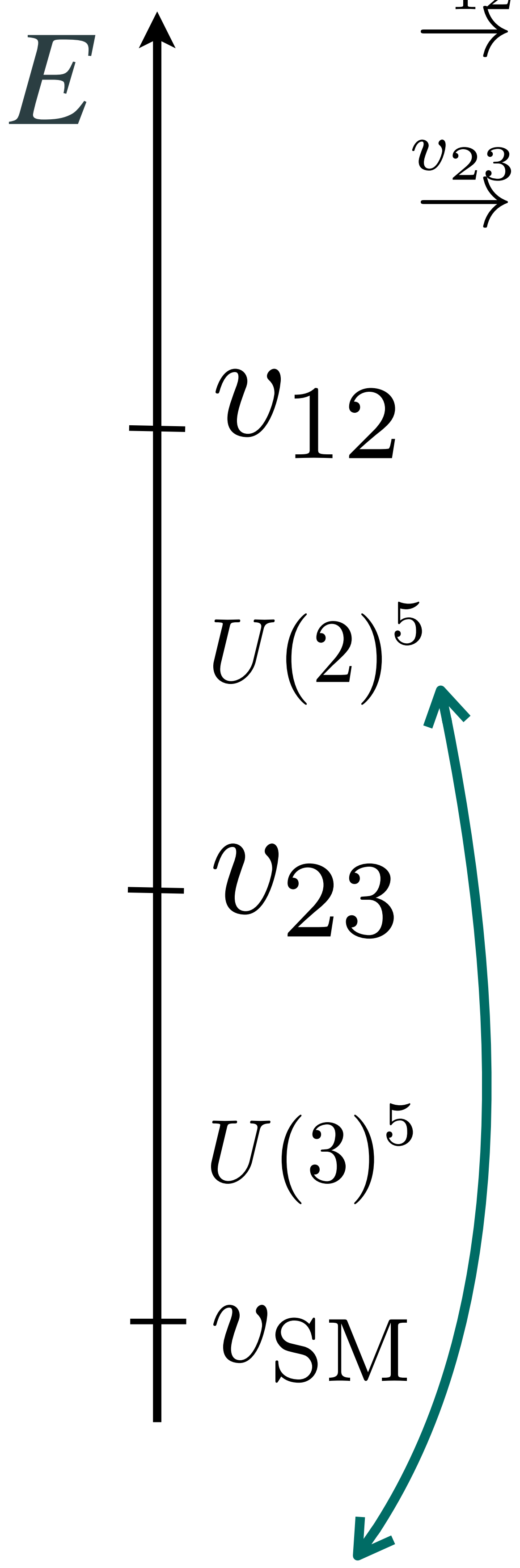


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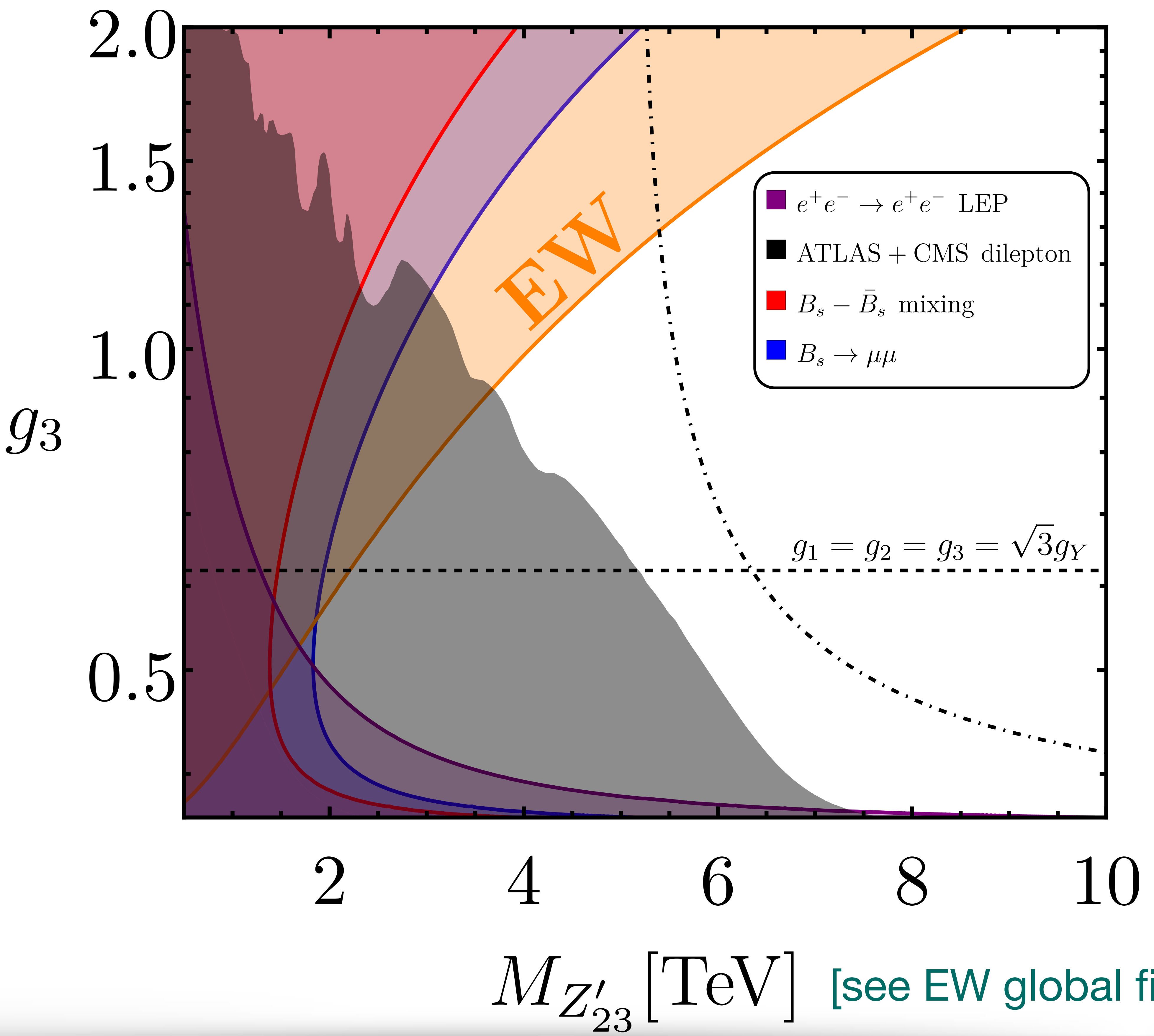


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- if $g_1 \simeq g_2 \simeq g_3 \simeq \sqrt{3} g_Y$

$$\Rightarrow \boxed{v_{23} \gtrsim 5\ \text{TeV}}$$

# Extension to neutrinos

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- Typically, flavour deconstruction imposes selection rules on the Weinberg operator.
- However, this difficulty can be alleviated in UV-complete tri-hypercharge:

$$\mathcal{L} = c_{3i}\ell_3 H_3^u \nu_i^c + c_{2i}\frac{\phi_{\ell 23}}{M_{N_{23}}}\ell_2 H_3^u \nu_i^c + c_{1i}\frac{\phi_{\ell 13}}{M_{N_{13}}}\ell_1 H_3^u \nu_i^c$$

$$+ M_{ij}\nu_i^c \nu_j^c + \text{h.c.} \qquad i = 1, 2$$

| | $U(1)_{Y_1}$ | $U(1)_{Y_2}$ | $U(1)_{Y_3}$ | $SU(3)_c \times SU(2)_L$ |
|---|---|---|---|---|
| $\nu_1^c$ | 0 | 0 | 0 | $(\mathbf{1}, \mathbf{1})$ |
| $\nu_2^c$ | 0 | 0 | 0 | $(\mathbf{1}, \mathbf{1})$ |
| $N_{23}$ | 0 | 1/2 | -1/2 | $(\mathbf{1}, \mathbf{1})$ |
| $N_{13}$ | 1/2 | 0 | -1/2 | $(\mathbf{1}, \mathbf{1})$ |

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$\approx 1$ $\approx 1$

$$+ M_{ij}\nu_i^c \nu_j^c + \text{h.c.} \qquad i = 1, 2$$

| | $U(1)_{Y_1}$ | $U(1)_{Y_2}$ | $U(1)_{Y_3}$ | $SU(3)_c \times SU(2)_L$ |
|---|---|---|---|---|
| $\nu_1^c$ | 0 | 0 | 0 | $(\mathbf{1},\mathbf{1})$ |
| $\nu_2^c$ | 0 | 0 | 0 | $(\mathbf{1},\mathbf{1})$ |
| $N_{23}$ | 0 | 1/2 | -1/2 | $(\mathbf{1},\mathbf{1})$ |
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# Extension to neutrinos

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✓ $M \approx 10^{15}\,\text{GeV}$

✓ No need of small couplings nor $v_{12},\ v_{23}$ being very heavy

✓ No need of adding extra scalars

✓ $M_{N_{23,13}} \approx v_{23} \approx \mathcal{O}(10\,\text{TeV})$

# “Deconstructed” GUT?

- Gauge sector of flavour deconstructed models may contain up to 9 gauge couplings:

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  - E.g. $\{\phi_{\ell 12}^{(\frac{1}{2},-\frac{1}{2},0)}, \phi_{\ell 13}^{(\frac{1}{2},0,-\frac{1}{2})}, \phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}\}, \{H_1^{(\frac{1}{2},0,0)}, H_2^{(0,\frac{1}{2},0)}, H_3^{(0,0,\frac{1}{2})}\}, \; \{Q_1^{(\frac{1}{6},0,0)}, Q_2^{(0,\frac{1}{6},0)}, Q_3^{(0,0,\frac{1}{6})}\}$

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✓ Deconstructed GUTs may be the origin of low energy flavour deconstructed models.

# Deconstructed GUT example

| **Field** | $\boldsymbol{SU(5)_1}$ | $\boldsymbol{SU(5)_2}$ | $\boldsymbol{SU(5)_3}$ |
|---|---|---|---|
| $F_1$ | $\bar{\mathbf{5}}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $F_2$ | $\mathbf{1}$ | $\bar{\mathbf{5}}$ | $\mathbf{1}$ |
| $F_3$ | $\mathbf{1}$ | $\mathbf{1}$ | $\bar{\mathbf{5}}$ |
| $T_1$ | $\mathbf{10}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $T_2$ | $\mathbf{1}$ | $\mathbf{10}$ | $\mathbf{1}$ |
| $T_3$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{10}$ |
| $\Omega$ | $\mathbf{24}$ | $\mathbf{24}$ | $\mathbf{24}$ |
| $H_1$ | $\mathbf{5}$ | $\mathbf{1}$ | $\mathbf{1}$ |
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+ hyperons and VL fermions of tri-hypercharge model

$F_i \to d_i^c \oplus \ell_i \quad T_i \to q_i \oplus u_i^c \oplus e_i^c$

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- For tri-hypercharge, need of deconstructing $SU(3)_c$ and $SU(2)_L$ as well for embedding in $SU(5)^3$. This suggests a possible SM$^3$ intermediate scale, i.e. two options:

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$$
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# Gauge coupling unification

- Discontinuities due to gauge coupling matching conditions:

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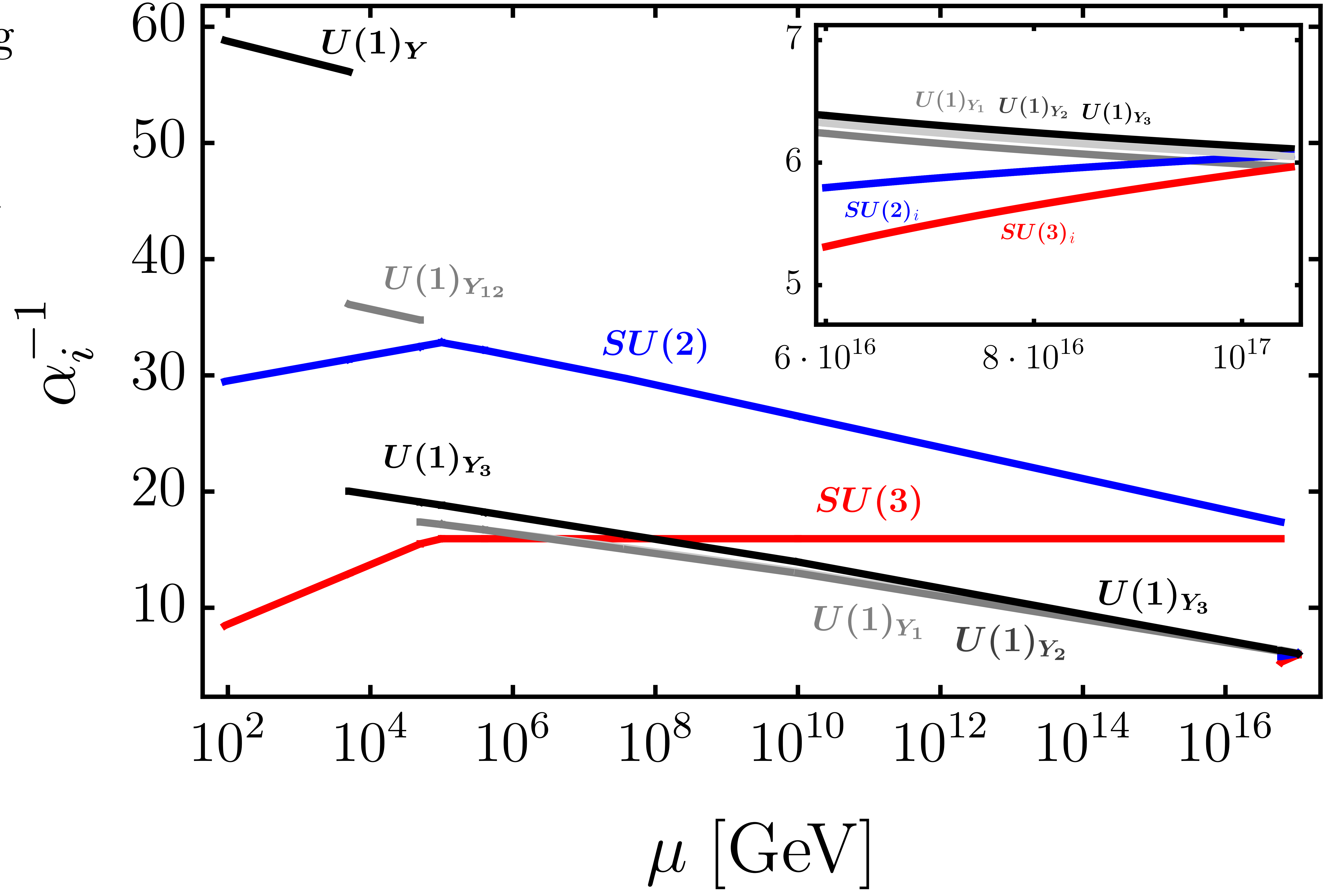

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- Colour octet $\Theta_i \sim (\mathbf{8},\mathbf{1},0)_i$ from cyclic $\mathbf{24}$ at $v_{12}$ scale to bend SU(3) (non-SUSY).
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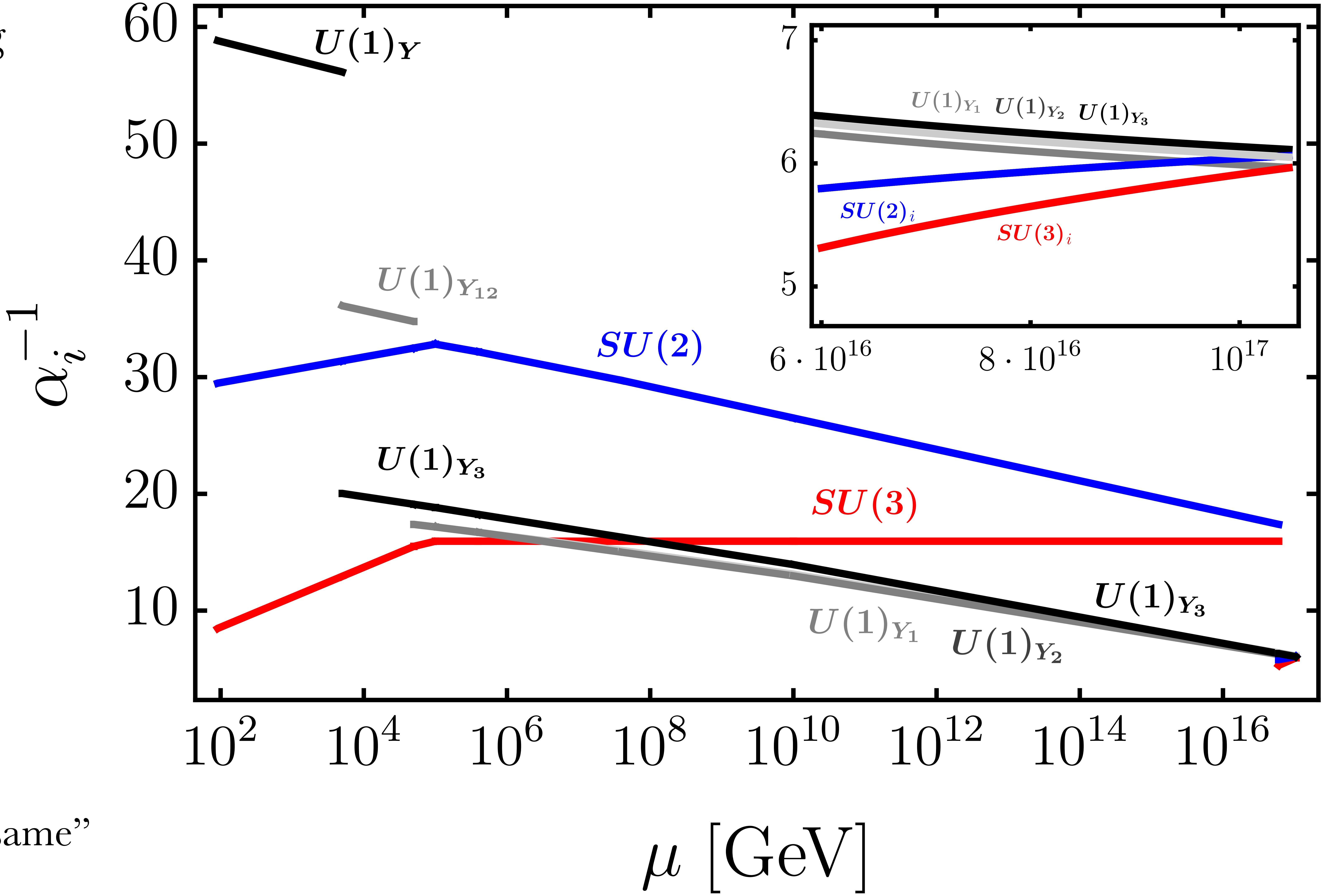

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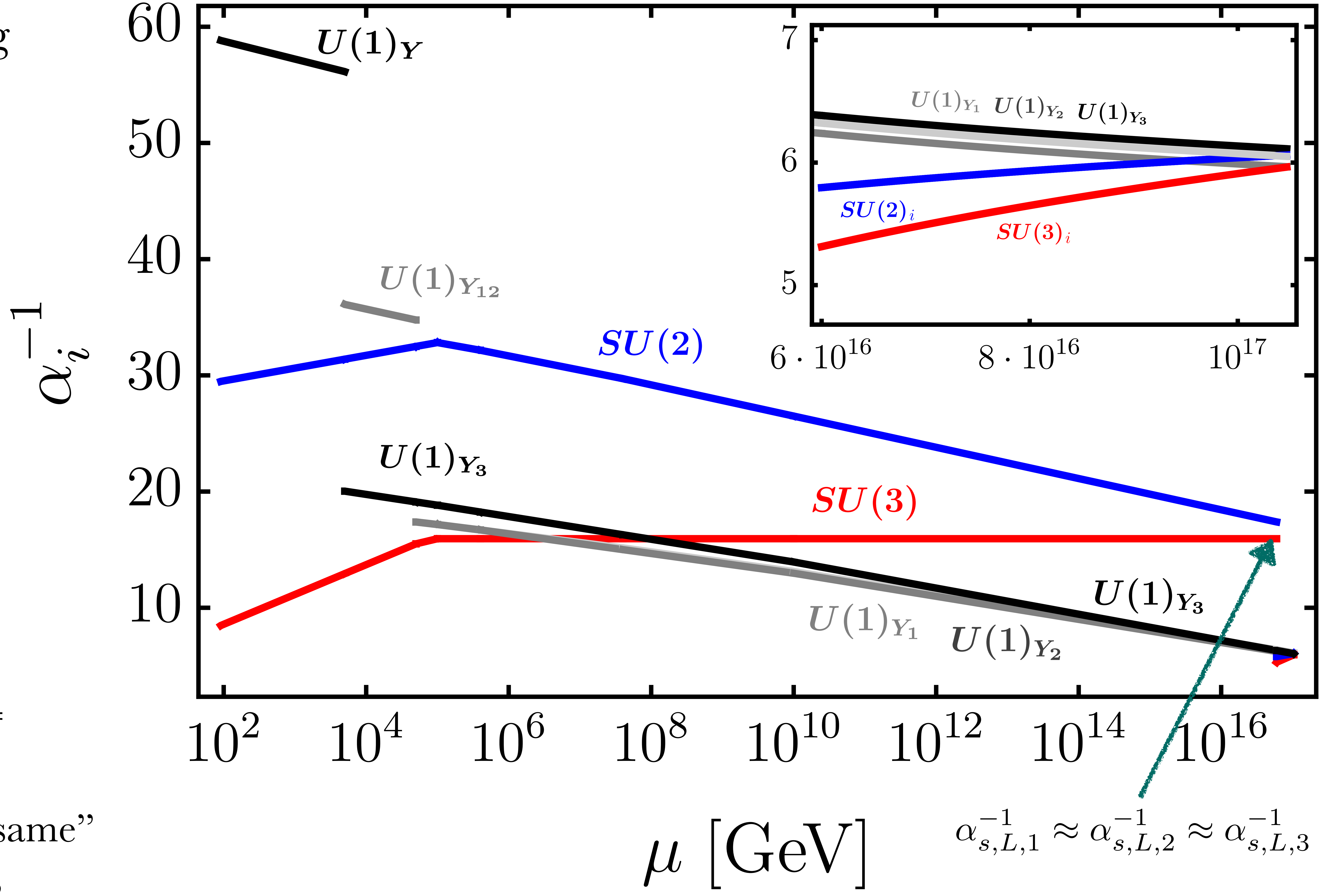


$v_{\mathrm{SM}^3} = 6\times 10^{16}$ GeV $\longrightarrow$ $M_{\mathrm{GUT}} = 10^{17}$ GeV

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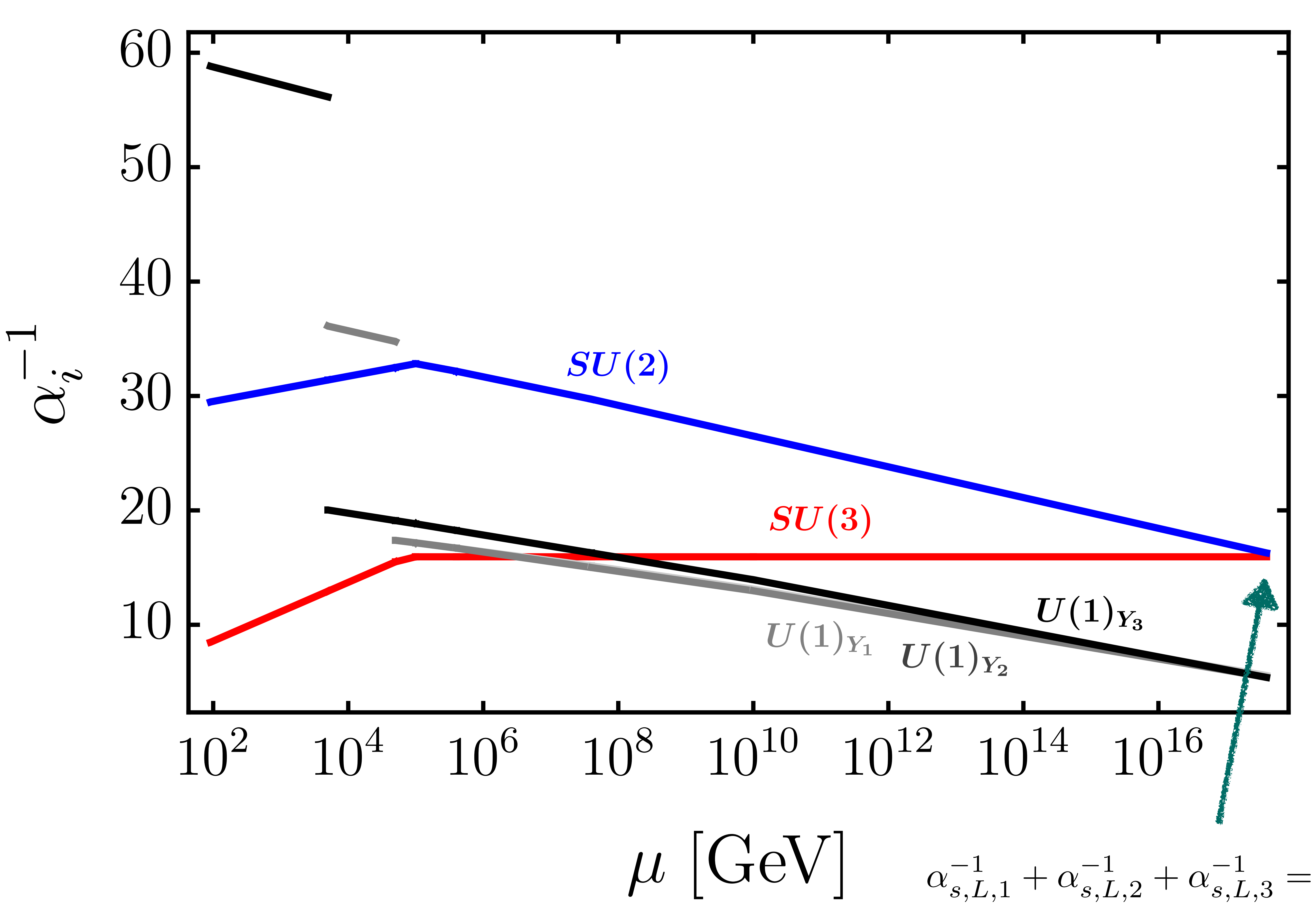
No intermediate SM$^3$ scale: $SU(5)^3 \xrightarrow{M_{\rm GUT}} SU(3)_c \times SU(2)_L \times U(1)_Y^3$

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Robust prediction for GUT scale: $M_{\rm GUT} \simeq 5 \times 10^{17}$ GeV

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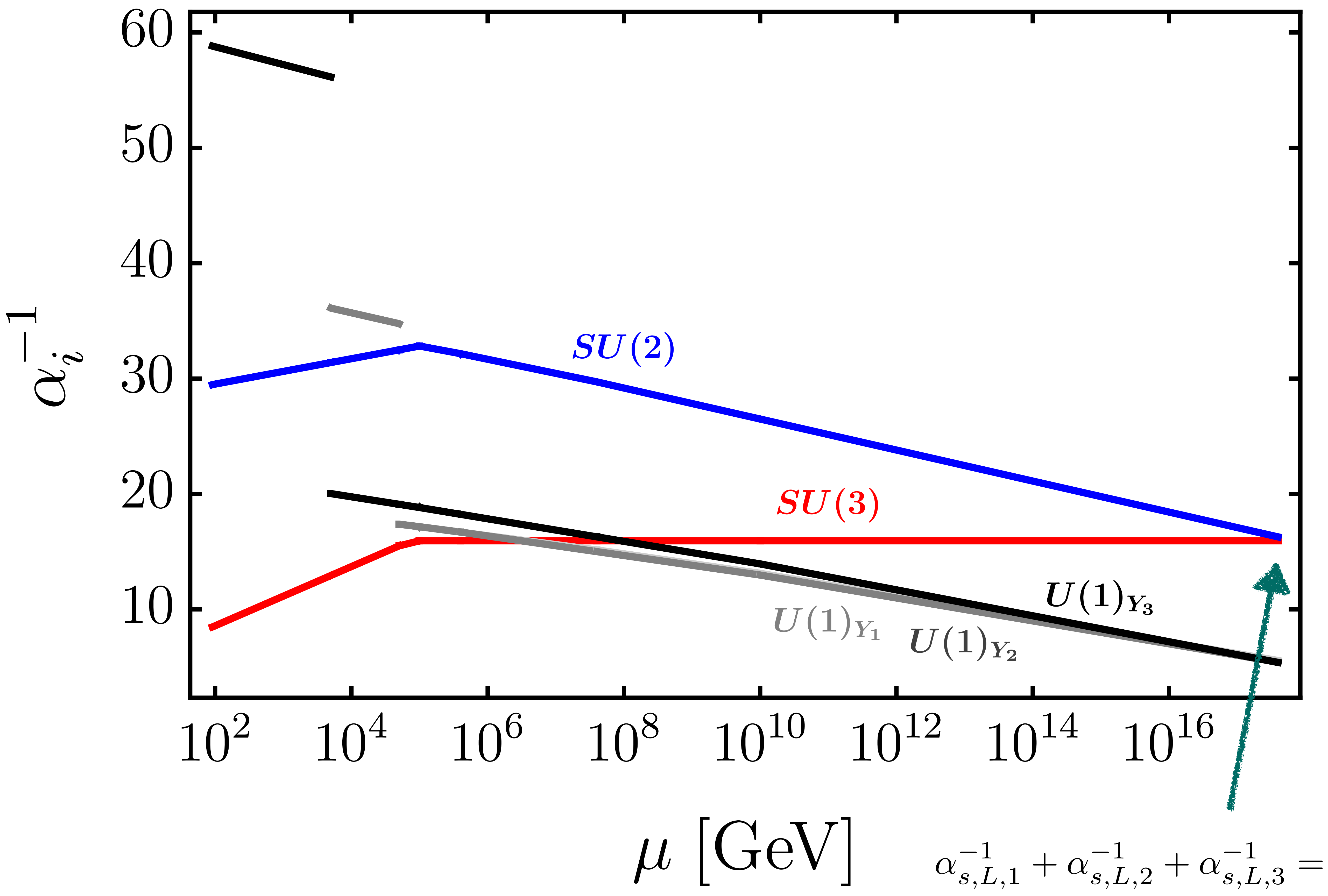
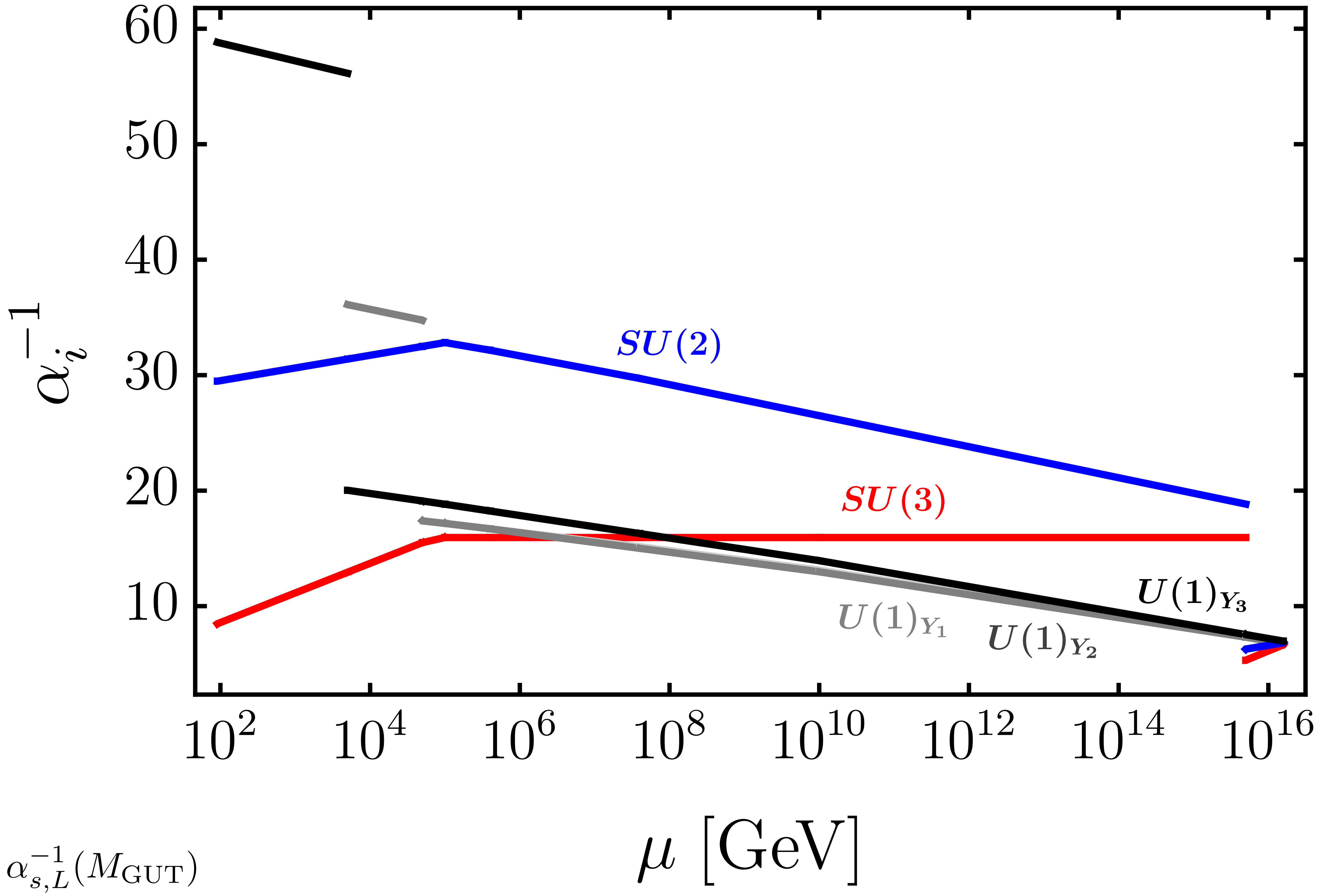
How low can can we deconstruct $SU(3)_c$ and $SU(2)_L$ ?

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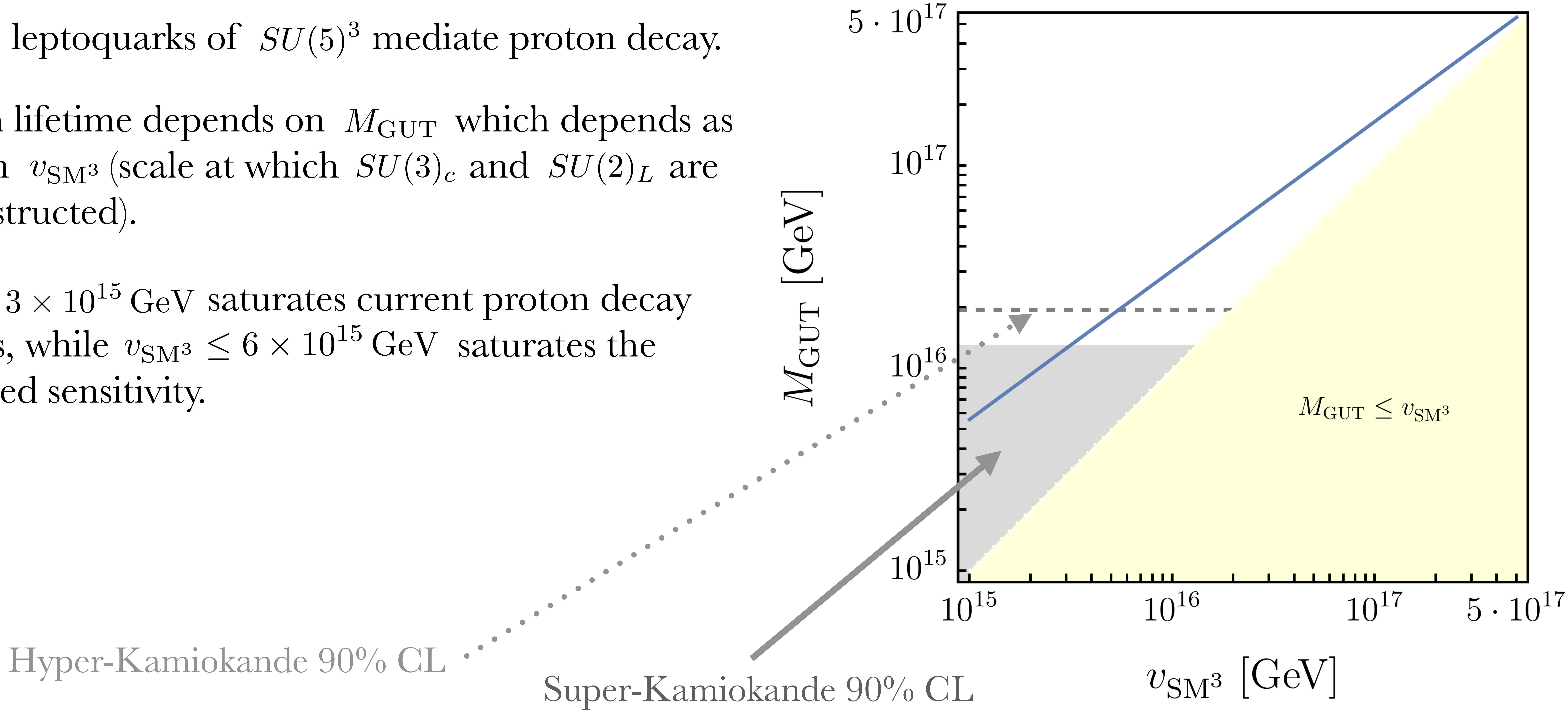
$v_{\mathrm{SM}^3} = 5 \times 10^{15}$ GeV $\longrightarrow$ $M_{\mathrm{GUT}} \simeq 1.8 \times 10^{16}$ GeV

proton decay!

# Proton decay

- Gauge leptoquarks of $SU(5)^3$ mediate proton decay.
- Proton lifetime depends on $M_{\rm GUT}$ which depends as well on $v_{\rm SM^3}$ (scale at which $SU(3)_c$ and $SU(2)_L$ are deconstructed).
- $v_{\rm SM^3} \leq 3\times 10^{15}\,{\rm GeV}$ saturates current proton decay bounds, while $v_{\rm SM^3} \leq 6\times 10^{15}\,{\rm GeV}$ saturates the projected sensitivity.



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# Backup: Yukawa couplings UV-complete model

$$\mathcal{L} = Y_u^{ij} q_i H_3^u u_j^c + Y_d^{ij} q_i H_3^d d_j^c + Y_e^{ij} \ell_i H_3^d e_j^c + \text{h.c.}$$

$$Y_u = \begin{pmatrix} c_{11}^u \frac{\tilde{\phi}_{\ell 13}}{M_{H_u^1}} & c_{12}^u \frac{\tilde{\phi}_{\ell 23}}{M_{H_u^2}} \frac{\phi_{q12}}{M_{Q_2}} & c_{13}^u \frac{\phi_{q12}}{M_{Q_2}} \frac{\phi_{q23}}{M_{Q_3}} \\ c_{21}^u \frac{\phi_{\ell 13}}{M_{H_u^1}} \frac{\phi_{q12}}{M_{Q_2}} & c_{22}^u \frac{\tilde{\phi}_{\ell 23}}{M_{H_u^2}} & c_{23}^u \frac{\phi_{q23}}{M_{Q_3}} \\ c_{31}^u \frac{\phi_{\ell 13}}{M_{H_u^1}} \frac{\tilde{\phi}_{q23}}{M_{Q_2}} \frac{\tilde{\phi}_{q12}}{M_{Q_1}} & c_{32}^u \frac{\phi_{\ell 23}}{M_{H_u^2}} \frac{\tilde{\phi}_{q23}}{M_{Q_3}} & c_{33}^u \end{pmatrix}$$

$$Y_e = \begin{pmatrix} c_{11}^e \frac{\phi_{\ell 13}}{M_{H_1^d}} & 0 & 0 \\ 0 & c_{22}^e \frac{\phi_{\ell 23}}{M_{H_2^d}} & 0 \\ 0 & 0 & c_{33}^e \end{pmatrix}$$

$$Y_d = \begin{pmatrix} c_{11}^d \frac{\phi_{\ell 13}}{M_{H_1^d}} & c_{12}^d \frac{\phi_{\ell 23}}{M_{H_2^d}} \frac{\phi_{q12}}{M_{Q_2}} & c_{13}^d \frac{\phi_{q12}}{M_{Q_2}} \frac{\phi_{q23}}{M_{Q_3}} \\ c_{21}^d \frac{\tilde{\phi}_{\ell 13}}{M_{H_1^d}} \frac{\phi_{q12}}{M_{Q_1}} & c_{22}^d \frac{\phi_{\ell 23}}{M_{H_2^d}} & c_{23}^d \frac{\phi_{q23}}{M_{Q_3}} \\ c_{31}^d \frac{\tilde{\phi}_{\ell 13}}{M_{H_1^d}} \frac{\tilde{\phi}_{q23}}{M_{Q_2}} \frac{\tilde{\phi}_{q12}}{M_{Q_1}} & c_{32}^d \frac{\tilde{\phi}_{\ell 23}}{M_{H_2^d}} \frac{\tilde{\phi}_{q23}}{M_{Q_2}} & c_{33}^d \end{pmatrix}$$

# Backup: Neutrinos

- Avoid complete singlet neutrinos but introduce neutrino $N_{23}\left(0,\frac{1}{4},-\frac{1}{4}\right)$ and $\phi_{\text{atm}}\left(0,-\frac{1}{4},\frac{1}{4}\right)$:

$$\mathscr{L} \supset \frac{1}{\Lambda}(\phi_{\text{atm}}\ell_3 H_u N_{23} + \widetilde{\phi}_{\text{atm}}\ell_2 H_u N_{23}) + \phi_{\ell 23} N_{23} N_{23}$$

# Backup: Gauge coupling unification

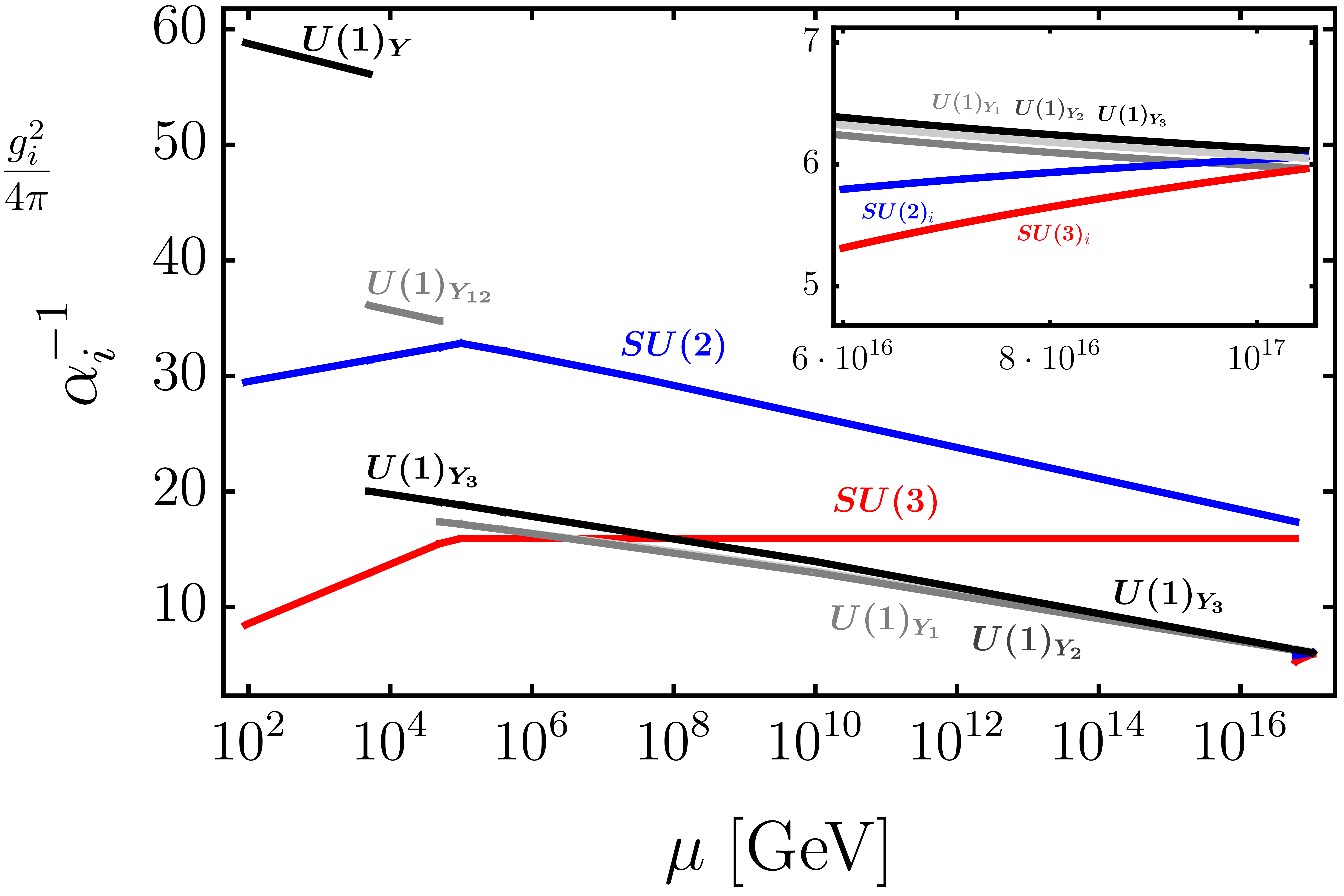
- Discontinuities due to gauge coupling matching conditions:

$$\alpha_{Y_{12}}^{-1} + \alpha_{Y_3}^{-1} = \alpha_Y^{-1}(v_{23}) \qquad \alpha_i = \frac{g_i^2}{4\pi}$$

$$\alpha_{Y_1}^{-1} + \alpha_{Y_2}^{-1} = \alpha_{Y_{12}}^{-1}(v_{12})$$

$$\alpha_{Y_1}^{-1} \approx \alpha_{Y_2}^{-1} \approx \alpha_{Y_3}^{-1} \approx \sqrt{3}\alpha_Y^{-1}(v_{12})$$

$$\alpha_{s,L,1}^{-1} + \alpha_{s,L,2}^{-1} + \alpha_{s,L,3}^{-1} = \alpha_{s,L}^{-1}(v_{\mathrm{SM}^3})$$



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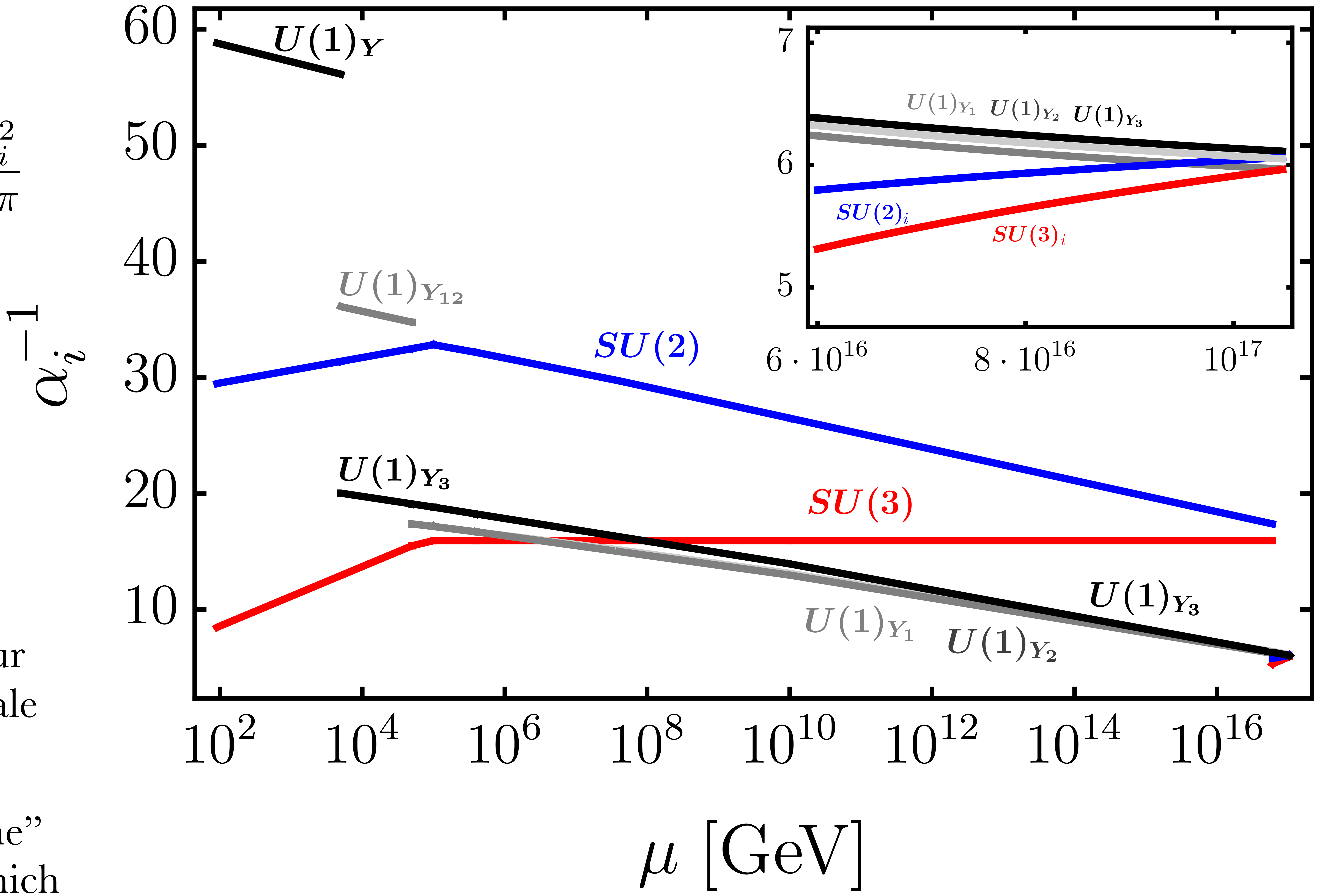
- VL quarks $Q_i$ help bend SU(2), plus colour octet $\Theta_i \sim (\mathbf{8},\mathbf{1},0)_i$ from cyclic $\mathbf{24}$ at $v_{12}$ scale to bend SU(3) (non-SUSY).
- Gauge couplings approximately "run the same" thanks to approximate $\mathbb{Z}_3$ at low energies, which becomes exact at high energies.



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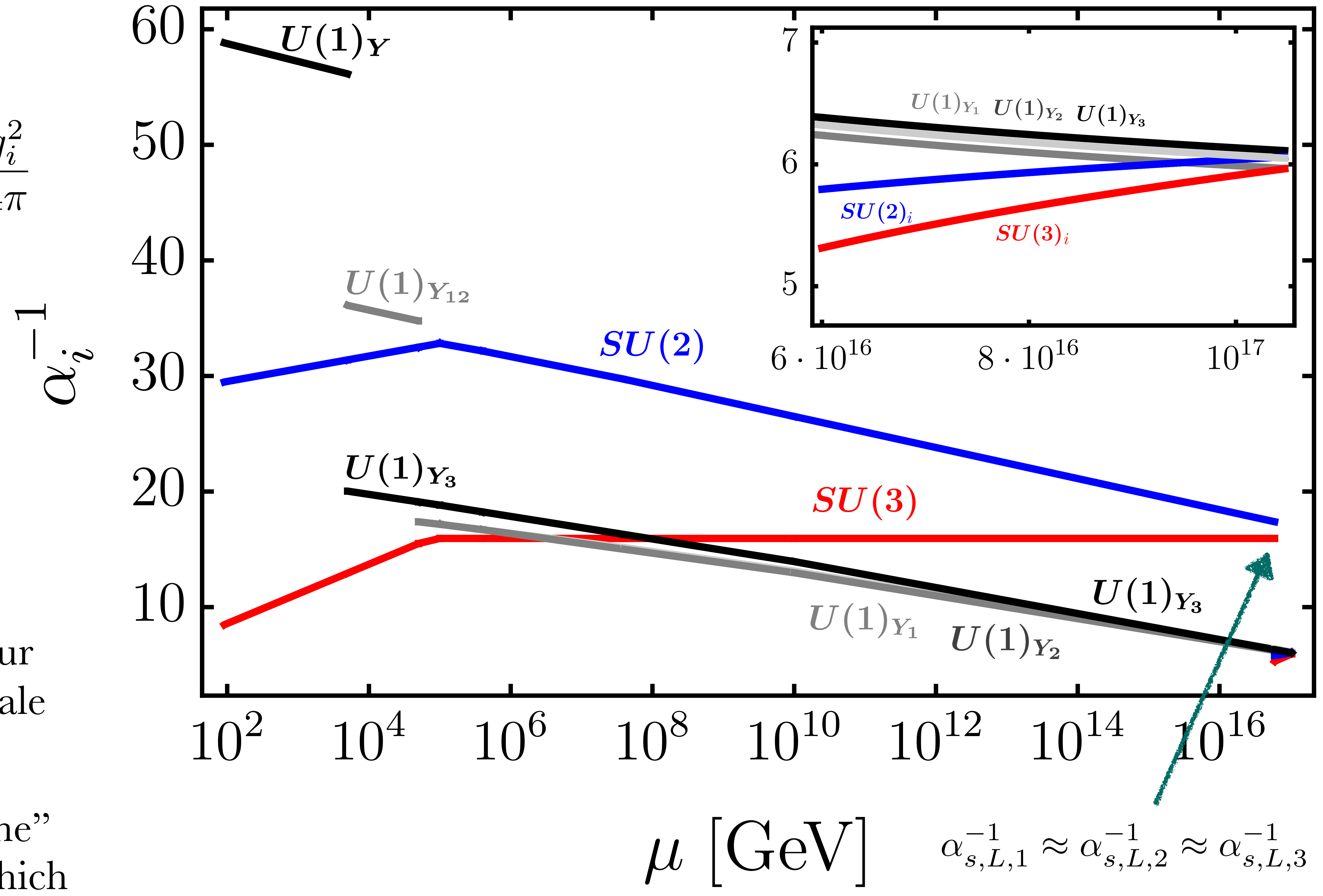


$v_{\mathrm{SM}^3} = 6 \times 10^{16}$ GeV $\longrightarrow$ $M_{\mathrm{GUT}} = 10^{17}$ GeV

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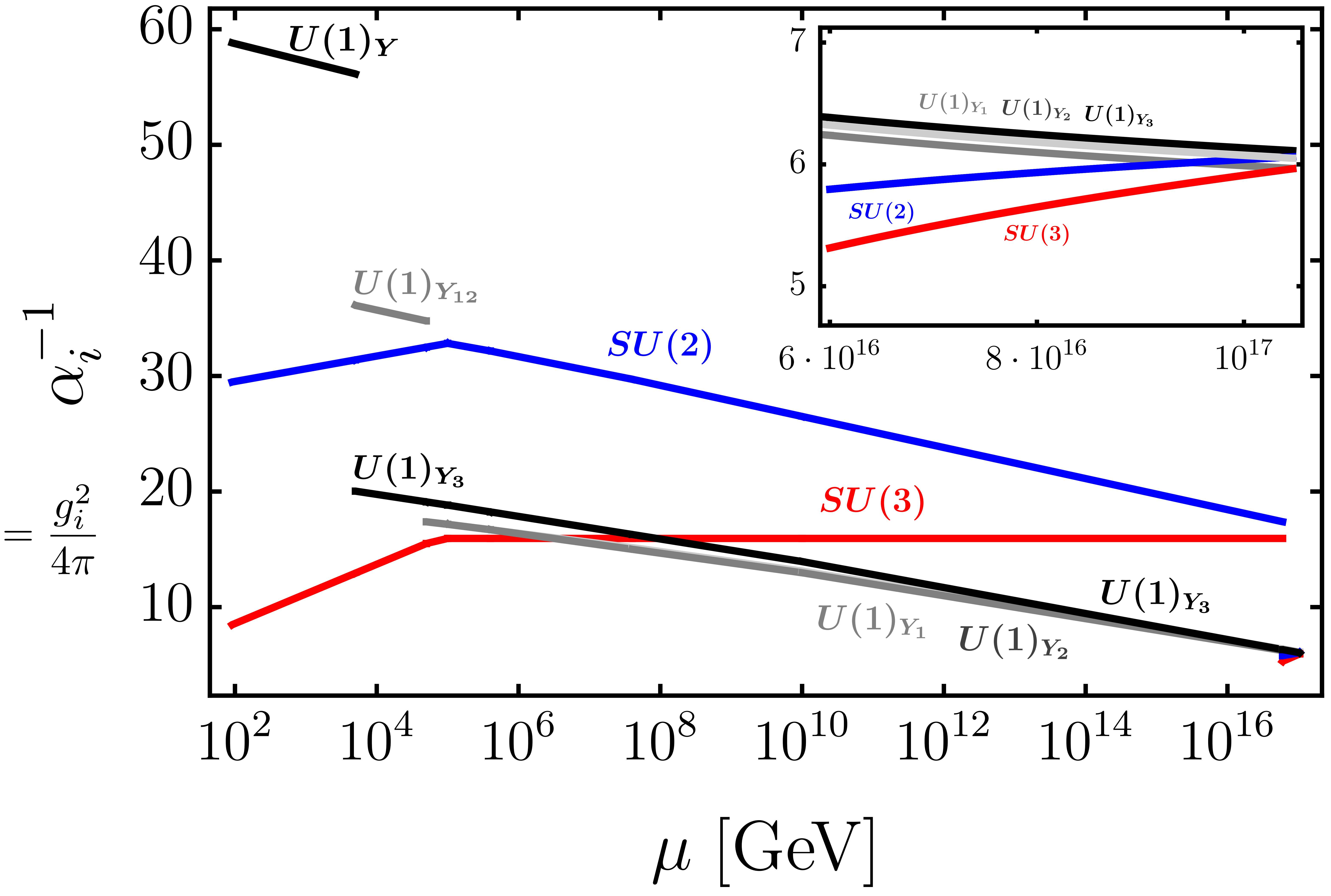


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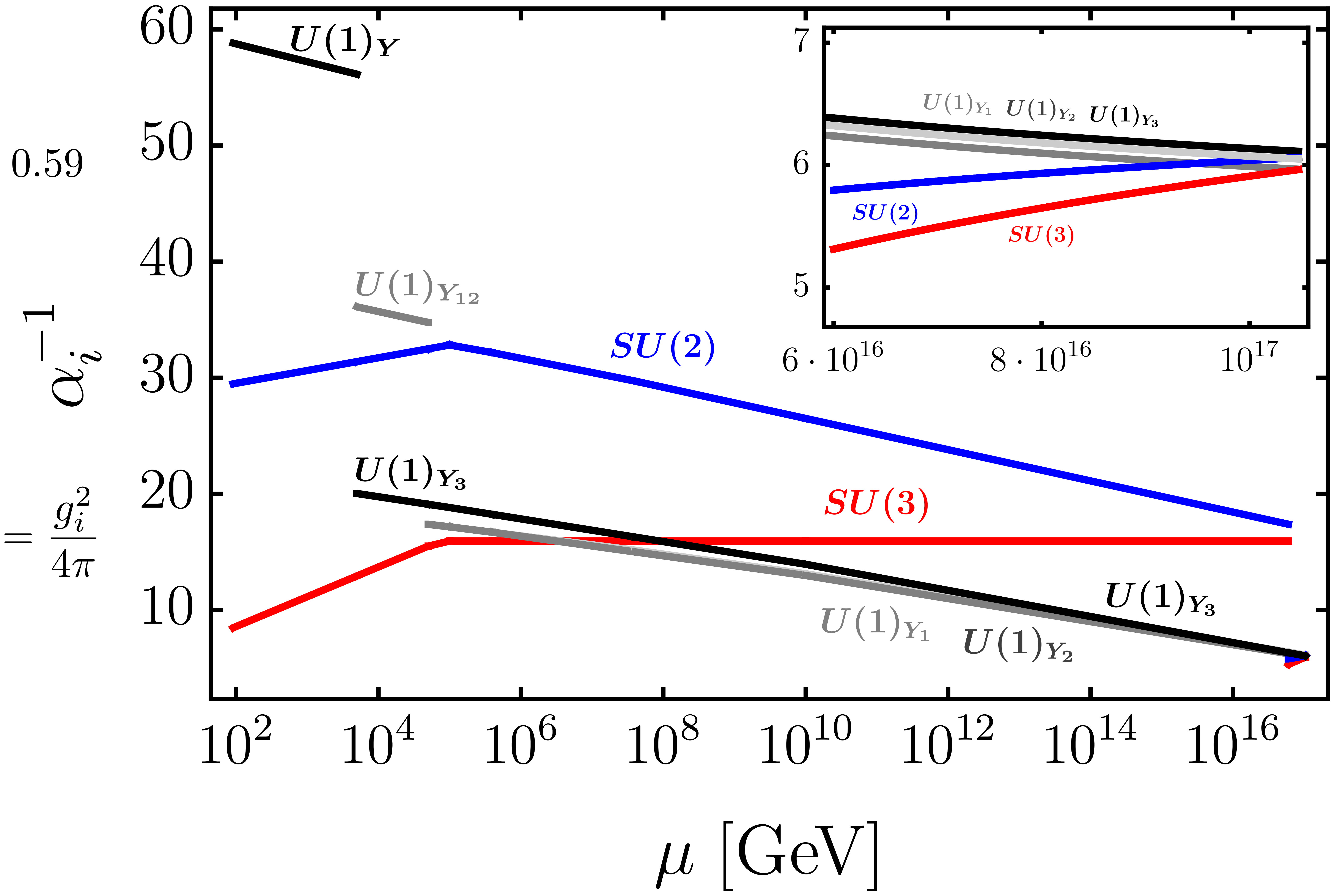
# Backup: Gauge coupling unification

- Discontinuities due to gauge coupling matching conditions:

$g_{Y_1}(v_{12}) \approx g_{Y_2}(v_{12}) \longrightarrow g_{Y_{12}}(v_{23}) \approx 0.59$

$g_{Y_3}(v_{23}) \approx \sqrt{3} g_Y(v_{23}) \approx 0.79$

$\alpha_i = \frac{g_i^2}{4\pi}$

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$$\frac{g_{s,1} g_{s,2} g_{s,3}}{\sqrt{g_{s,1}^2 + g_{s,2}^2 + g_{s,3}^2}} = g_s(v_{\mathrm{SM}^3})$$

same for $g_{L,i}$ couplings via $s \to L$

$$g_{s,1} \simeq g_{s,2} \simeq g_{s,3} \simeq \sqrt{3} g_s(v_{\mathrm{SM}^3})$$

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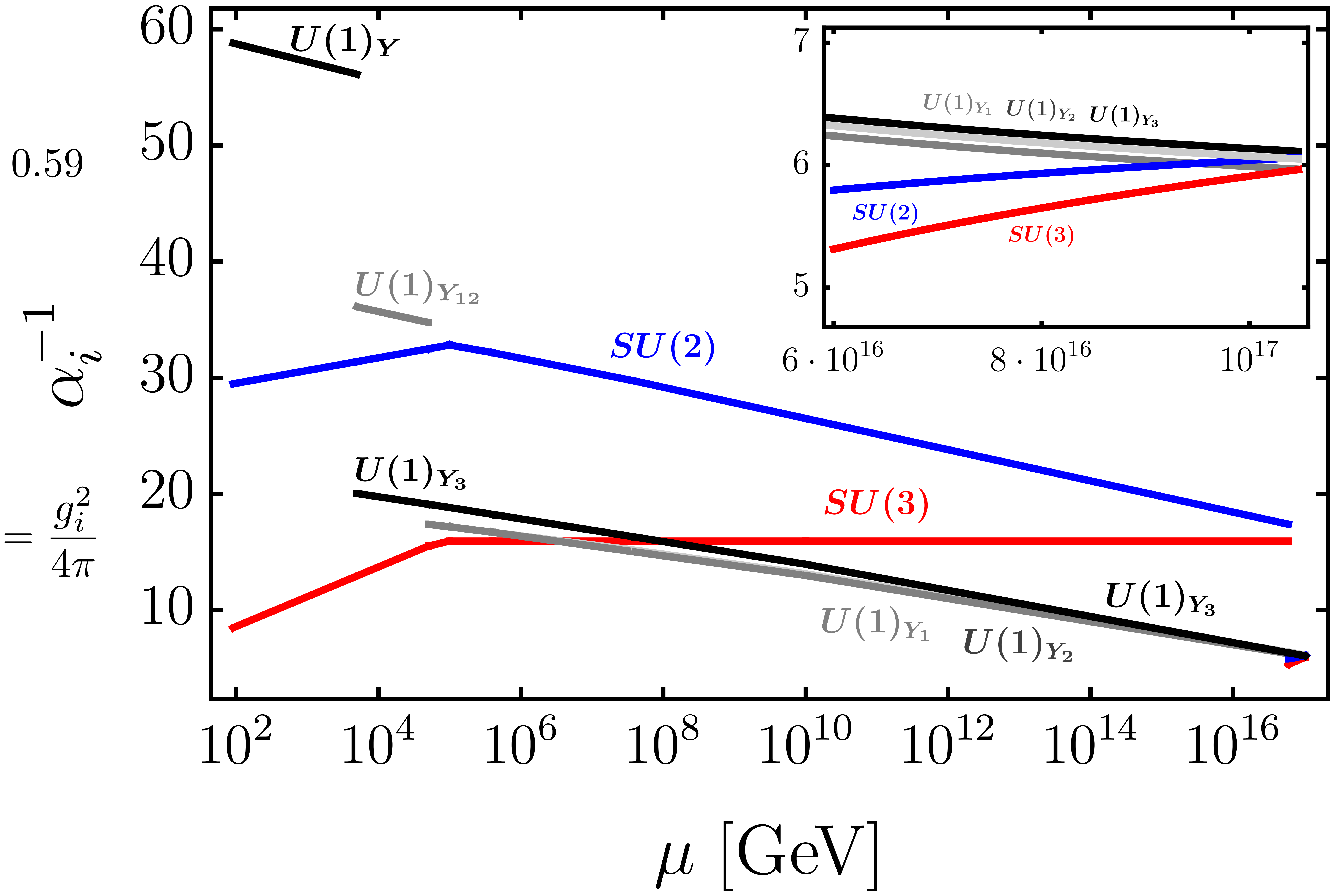


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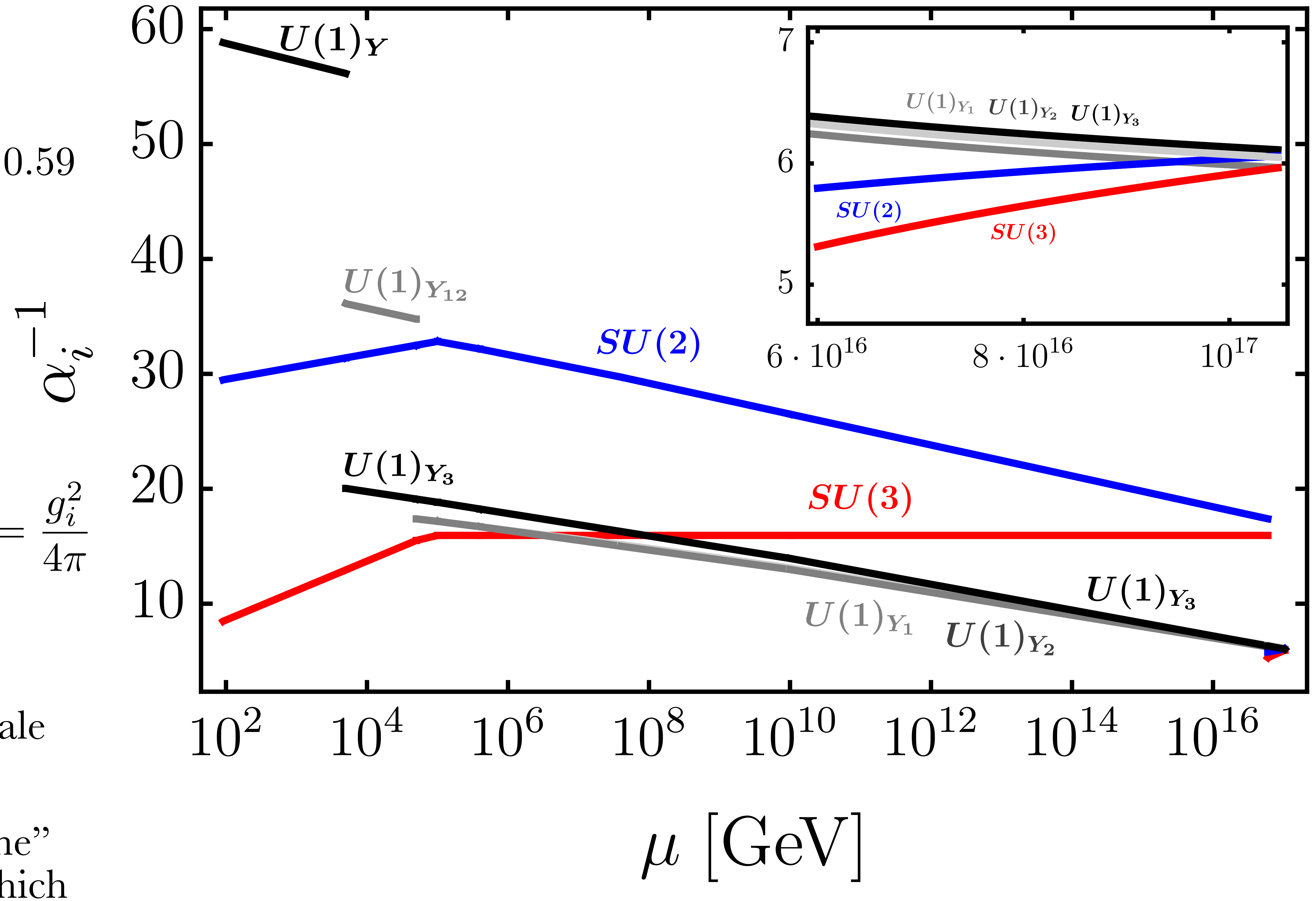
- Unification achieved with tri-hypercharge content (only 3 VL quarks) plus cyclic colour octet $\Theta_i \sim (\mathbf{8}, \mathbf{1}, 0)_i$ from cyclic $\mathbf{24}$ at $v_{12}$ scale (non-SUSY).
- Gauge couplings approximately "run the same" thanks to approximate $\mathbb{Z}_3$ at low energies, which becomes exact at high energies.

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# Backup: Literature review

**GUT product groups**

"tribal group" to motivate multiple scales vs SO(10)

- **1979** Abdus Salam; EPS conference 1979, footnote 41 ⟶ $SU(5)_1 \times SU(5)_2 \times SU(5)_3 \xrightarrow{M_i} \mathrm{SM}_i \times SU(5)_j$
- **1981** Subhash Rajpoot; PRD 24 (1981) 1890. ⟶ numerics + study different breakings + discrete symmetries for 1 gauge coupling
- **1982** Howard Georgi; Nucl. Phys. B 202 (1982) 397 ⟶ $SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times SU(5)_{\mathrm{TC}}$ + cyclic permutation (also $SO(10)^5$ )
- **1984** de Rújula, Georgi, Glashow; Fifth worksop on GUT ⟶ $SU(3)_c \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3$
- **1995** Barbieri, Dvali and Strumia, hep-ph/9407239 ⟶ SUSY $SU(5)^3$ $SO(10)^3$ + $(\mathbf{5}_i, \overline{\mathbf{5}}_j)$ scalars + discrete symmetries ⟶ d=5 proton decay!
- **1998-2007** C.L. Chou, [hep-ph/9804325]; Asaka and Takanashi, [hep-ph/0409147]; Babu, Barr and Gogoladze [0709.3491]
- **2023** MFN, Stephen F. King, Avelino Vicente [2311.05683]; ⟶ $SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3$ Broken via **24** to "deconstructed" theory of flavour (e.g. tri-hypercharge)

# Backup: SU(5) cube model table

| Field | $SU(5)_1$ | $SU(5)_2$ | $SU(5)_3$ |
|---|---|---|---|
| $F_1$ | $\overline{\mathbf{5}}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $F_2$ | $\mathbf{1}$ | $\overline{\mathbf{5}}$ | $\mathbf{1}$ |
| $F_3$ | $\mathbf{1}$ | $\mathbf{1}$ | $\overline{\mathbf{5}}$ |
| $T_1$ | $\mathbf{10}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $T_2$ | $\mathbf{1}$ | $\mathbf{10}$ | $\mathbf{1}$ |
| $T_3$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{10}$ |
| $\chi_1$ | $\mathbf{10}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\chi_2$ | $\mathbf{1}$ | $\mathbf{10}$ | $\mathbf{1}$ |
| $\chi_3$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{10}$ |
| $N_{12}$ | $\mathbf{5}$ | $\overline{\mathbf{5}}$ | $\mathbf{1}$ |
| $N_{13}$ | $\mathbf{5}$ | $\mathbf{1}$ | $\overline{\mathbf{5}}$ |
| $N_{23}$ | $\mathbf{1}$ | $\mathbf{5}$ | $\overline{\mathbf{5}}$ |
| $\Omega$ | $\mathbf{24}$ | $\mathbf{24}$ | $\mathbf{24}$ |
| $H_1^{u,d}$ | $\mathbf{5}$, $\overline{\mathbf{5}}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $H_2^{u,d}$ | $\mathbf{1}$ | $\mathbf{5}$, $\overline{\mathbf{5}}$ | $\mathbf{1}$ |
| $H_3^{u,d}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{5}$, $\overline{\mathbf{5}}$ |
| $\Phi_{12}^F$ | $\mathbf{5}$ | $\overline{\mathbf{5}}$ | $\mathbf{1}$ |
| $\Phi_{13}^F$ | $\overline{\mathbf{5}}$ | $\mathbf{1}$ | $\mathbf{5}$ |
| $\Phi_{23}^F$ | $\mathbf{1}$ | $\mathbf{5}$ | $\overline{\mathbf{5}}$ |
| $\Phi_{12}^T$ | $\overline{\mathbf{10}}$ | $\mathbf{10}$ | $\mathbf{1}$ |
| $\Phi_{13}^T$ | $\mathbf{10}$ | $\mathbf{1}$ | $\overline{\mathbf{10}}$ |
| $\Phi_{23}^T$ | $\mathbf{1}$ | $\overline{\mathbf{10}}$ | $\mathbf{10}$ |

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| $F_2$ | $\mathbf{1}$ | $\overline{\mathbf{5}}$ | $\mathbf{1}$ |
| $F_3$ | $\mathbf{1}$ | $\mathbf{1}$ | $\overline{\mathbf{5}}$ |
| $T_1$ | $\mathbf{10}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $T_2$ | $\mathbf{1}$ | $\mathbf{10}$ | $\mathbf{1}$ |
| $T_3$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{10}$ |
| $\chi_1$ | $\mathbf{10}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\chi_2$ | $\mathbf{1}$ | $\mathbf{10}$ | $\mathbf{1}$ |
| $\chi_3$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{10}$ |
| $N_{12}$ | $\mathbf{5}$ | $\overline{\mathbf{5}}$ | $\mathbf{1}$ |
| $N_{13}$ | $\mathbf{5}$ | $\mathbf{1}$ | $\overline{\mathbf{5}}$ |
| $N_{23}$ | $\mathbf{1}$ | $\mathbf{5}$ | $\overline{\mathbf{5}}$ |
| $\Omega$ | $\mathbf{24}$ | $\mathbf{24}$ | $\mathbf{24}$ |
| $H_1^{u,d}$ | $\mathbf{5}$, $\overline{\mathbf{5}}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $H_2^{u,d}$ | $\mathbf{1}$ | $\mathbf{5}$, $\overline{\mathbf{5}}$ | $\mathbf{1}$ |
| $H_3^{u,d}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{5}$, $\overline{\mathbf{5}}$ |
| $\Phi_{12}^F$ | $\mathbf{5}$ | $\overline{\mathbf{5}}$ | $\mathbf{1}$ |
| $\Phi_{13}^F$ | $\overline{\mathbf{5}}$ | $\mathbf{1}$ | $\mathbf{5}$ |
| $\Phi_{23}^F$ | $\mathbf{1}$ | $\mathbf{5}$ | $\overline{\mathbf{5}}$ |
| $\Phi_{12}^T$ | $\overline{\mathbf{10}}$ | $\mathbf{10}$ | $\mathbf{1}$ |
| $\Phi_{13}^T$ | $\mathbf{10}$ | $\mathbf{1}$ | $\overline{\mathbf{10}}$ |
| $\Phi_{23}^T$ | $\mathbf{1}$ | $\overline{\mathbf{10}}$ | $\mathbf{10}$ |

+ cyclic **45** to split down/charged lepton masses as in conventional $SU(5)$