S. Di Noi

Renormalisation group running effects in $pp \to t\bar{t}h$ in the Standard Model Effective Field Theory

(based on 2312.11327 with R. Gröber)

Les Rencontres de Physique de la Vallée d'Aoste

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Introduction

- The **Standard Model** (**SM**) is one of the biggest scientific successes of our time, but leaves some phenomena unexplained (baryon asymmetry, dark matter. . .)
- Many **New Physics** (**NP**) theories have been proposed, but it is not clear which is the correct direction.
- **Effective Field Theories** (**EFTs**) offer a powerful and pragmatic approach to the search for NP with minimal UV assumptions.
- This talk focuses on Standard Model Effective Field Theory (**SMEFT**).

The SMEFT

- Parametrise heavy NP effects with a tower of gauge/Lorentz invariant operators:

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{\mathfrak{D}_i > 4} \frac{C_i}{\Lambda^{\mathfrak{D}_i - 4}} \mathcal{O}_i,$$

$$O_i \sim \partial^{n^i_d}\,\phi^{n^i_\phi}\,A^{n^i_A}\,\psi^{n^i_\psi}, \qquad \mathfrak{D}_i = n^i_d + n^i_\phi + n^i_A + \frac{3}{2} n^i_\psi.$$

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- ϕ, A, ψ: SM fields.
- Gauge group: $\text{SU}(3)_\text{C} \otimes \text{SU}(2)_\text{W} \otimes \text{U}(1)_\text{Y}$.

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- The dominant effects in collider physics arise at $\mathfrak{D} = 6$ (**Warsaw basis**, [Grzadkowski,Iskrzynski,Misiak,Rosiek,'10]).

SMEFT: how should we use it?

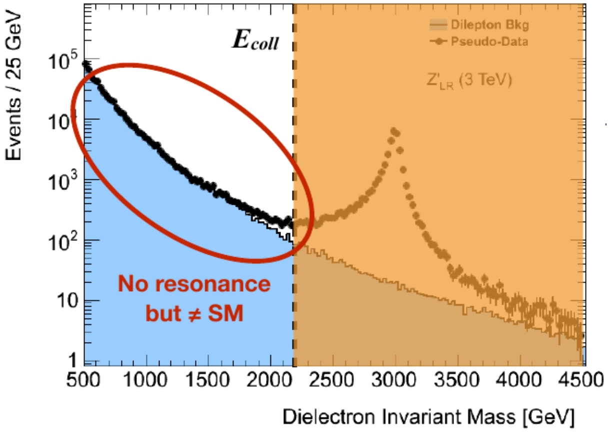

Figure: Courtesy of P. Azzi.

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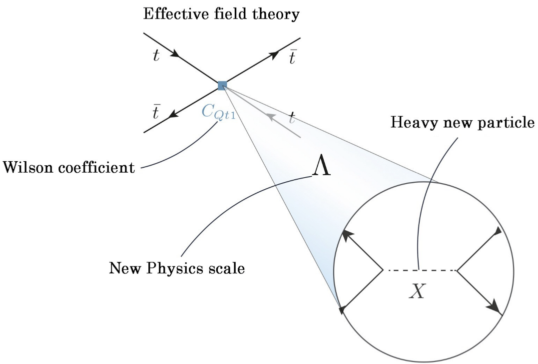

Figure: courtesy of L. Alasfar

- Information on UV mediator from IR dynamics: see [Altmannshofer, Gori, Lehmann, Zuo,'23]

Running effects

- Renormalization procedure induces energy-dependent parameters.
- Crucial ingredient to connect different energy scales (e.g.: matching scale Λ, experiment $\sim v \ll \Lambda$).
- The scale dependence of the coefficients is encoded in the **Renormalization Group Equations (RGEs)** (1-loop):

$$\mu \frac{dC_i(\mu)}{d\mu} = \frac{1}{16\pi^2} \Gamma_{ij}(\mu) C_j(\mu).$$

- $\Gamma_{ij}(\mu)$: **Anomalous Dimension Matrix (ADM)**.

Structure of ADM in the SMEFT

- Known at 1-loop [(Alonso),Jenkins,Manohar,Trott,'13].
- $\Gamma_{ij}(\mu)$ depends on μ through the couplings:

$$\Gamma_{ij}(\mu) = g_1^2(\mu)\Gamma_{ij}^{(g_1^2)} + g_2^2(\mu)\Gamma_{ij}^{(g_2^2)} + \dots$$

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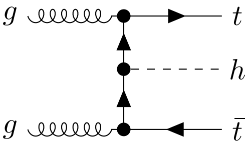
- $\Gamma^{(g_i)}$ do not commute: **analytical solution is impossible**.

RGESolver

- A C++ library that performs RG evolution of SMEFT coefficients ([S.D.N.,Silvestrini,'22]).
- General flavour structure (assuming L, B conservation).
- Tested against DsixTools [Fuentes-Martín, et al.'20].
- High time efficiency: (numerical running: $\mathcal{O}\left(0.1\,s\right)$ vs $\mathcal{O}\left(10\,s\right)$ (DsixTools)).
- Flavour back-rotation implemented.

- Authors:
  - Stefano Di Noi,
  - Luca Silvestrini.

$pp \to \bar{t}th$@LHC (SM)



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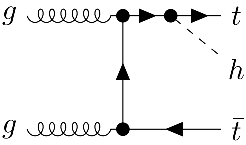

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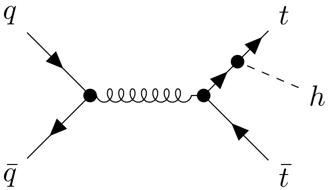

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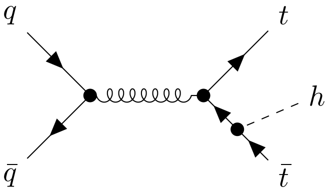
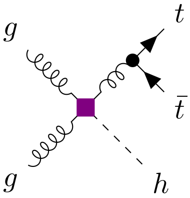


(d)

$pp \to \bar{t}th$@LHC (SMEFT)

- SMEFT introduce new vertices and rescale SM couplings.



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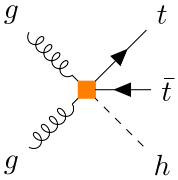

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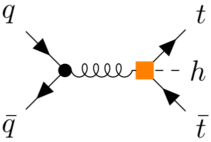

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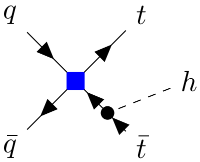

(d)

Dynamical vs Fixed scale I

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- How to choose μ_{R}?

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- We set some Wilson coefficients at the scale $\Lambda = 2\,\mathrm{TeV}$ and test their impact on differential distributions.
- We compare two different choices:
  - Fixed scale: $\mu_\mathrm{R} = m_t$ (same for all the events).
  - Dynamical scale: $\mu_\mathrm{R} = (p_{T,h} + p_{T,t} + p_{T,\bar{t}})/2$ (changes event by event).

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- Factorisation scale $\mu_{\mathrm{F}} = (p_{T,h} + p_{T,t} + p_{T,\bar{t}})/2$ in both cases.
- In the fixed scale case, we use $g_s = g_s(\mu_{\mathrm{F}})$ (SM one-loop running).

Dynamical vs Fixed scale II

- Important effect for large coefficients ($\sim 100\,\mathrm{TeV}^{-2}$)

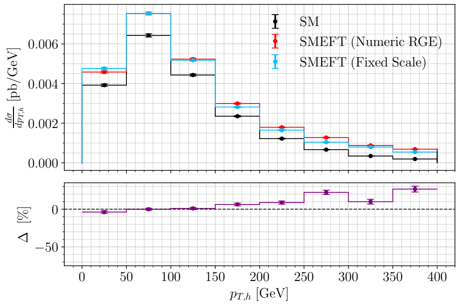

Figure: Conservative scenario: $C_{4t} \sim 1\,\mathrm{TeV}^{-2}$.

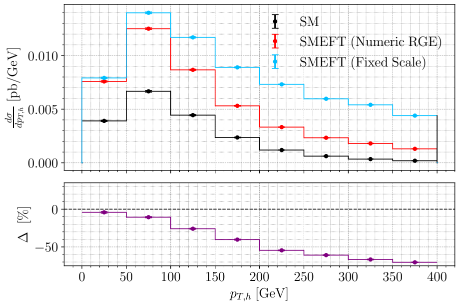

Figure: Extreme scenario: $C_{4t} \sim 100\,\mathrm{TeV}^{-2}$.

g_s vs y_t I

- The β-functions of $C_{t\phi}$ contains a term $\propto y_t^3 \left(C_{Qt}^{(1)} + (4/3)C_{Qt}^{(8)}\right)$.
- $g_{ht\bar{t}} = \frac{m_t}{v}\left(1 - \frac{v^2}{\sqrt{2}}C_{t\phi}\right)$ is the effective Higgs-top coupling.

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- We set $C_{Qt}^{(1,8)} \neq 0$ (inside the bounds in [Ethier et al.,'21]) individually in such a way they contribute (almost) in the same way to the Yukawa-induced running.

g_s vs y_t II

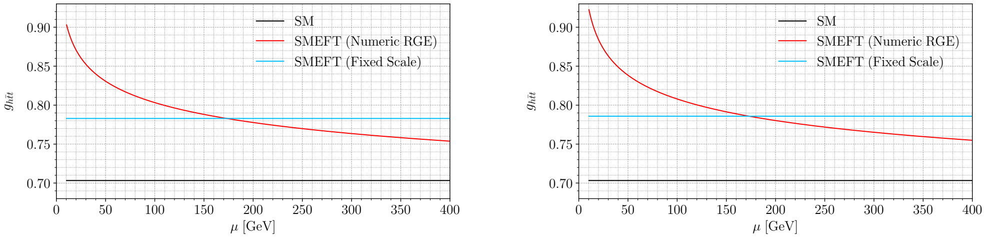

Figure: $C_{Qt}^{(1)}(\Lambda) = \frac{4}{3} \times 20\,/\mathrm{TeV}^2$.

Figure: $C_{Qt}^{(8)}(\Lambda) = 20\,/\mathrm{TeV}^2$.

g_s vs y_t III

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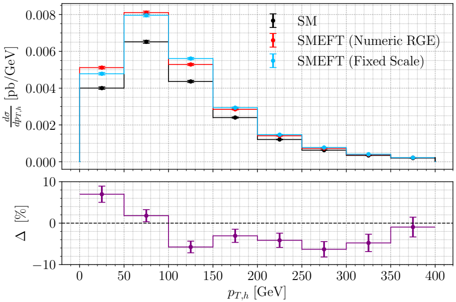

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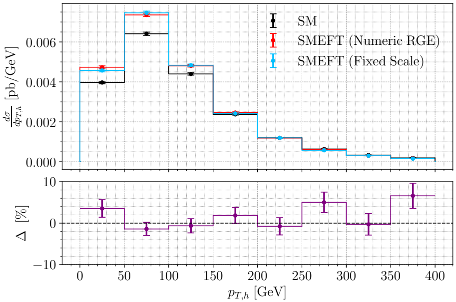

Figure: $C_{Qt}^{(8)}(\Lambda) = 20\,/\mathrm{TeV}^2$.

Conclusions

- Running effects are a crucial ingredient for precision physics in the next future.
- Appreciable differences can arise when employing a dynamical renormalisation scale.
- Yukawa contributions can be as important as strong ones in some cases.
- In presence of large Wilson coefficients, the leading-log solution of the RGEs shows sizeable differences w.r.t. the numeric integration.

Thank you for your attention!

Backup

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Solving the RGEs

① **Approximate solution** (first leading log):

$$C_i(\mu_F) = C_i(\mu_I) + \Gamma_{ij}(\mu_I) C_j(\mu_I) \frac{\log(\mu_F/\mu_I)}{16\pi^2}.$$

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Solving the RGEs

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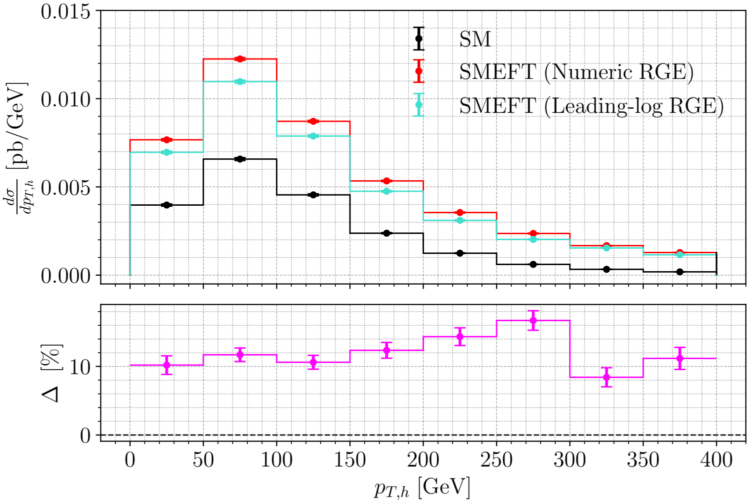
2. **Numeric solution**:

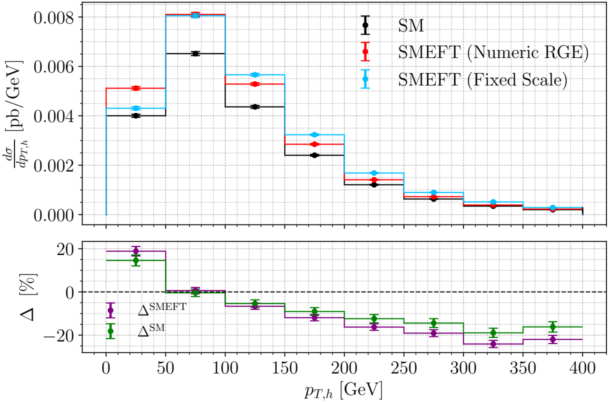
- More precise.
- Slow! Problem for extensive phenomenological analyses.

Numeric vs. 1LL



Impact of SM running of g_s

- $g_s = g_s(m_t)$ in the fixed case scenario (instead of $g_s = g_s(\mu_F) = g_s((p_{T,h} + p_{T,t} + p_{T,\bar{t}})/2)$ in the SM@1 loop).
- $C_{Qt}^{(1)} = 20\,\text{TeV}^{-2}$.



$p_{T,h}$ vs μ_F

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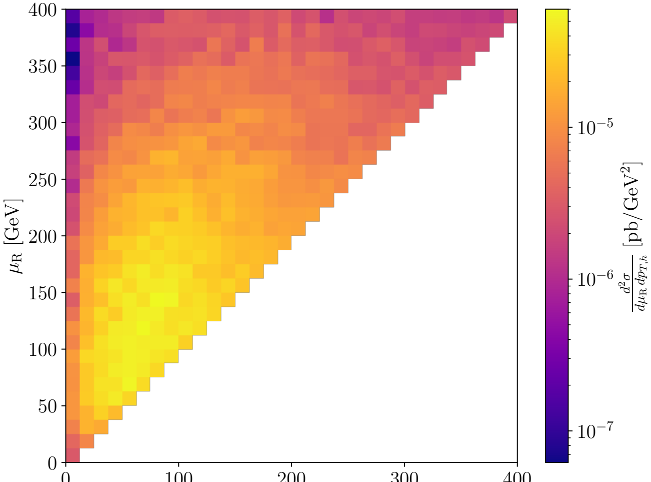

Sum over external polarizations in QCD

- Ex: 2 external gauge bosons, $\mathcal{M} = \epsilon^{\mu_1}(p_1)\epsilon^{\mu_2}(p_2)\mathcal{M}_{\mu_1\mu_2}$.
- **Ward identity**: $p_1^{\mu_1}\mathcal{M}_{\mu_1\mu_2} = p_2^{\mu_2}\mathcal{M}_{\mu_1\mu_2} = 0$ ("each photon is independent").
- In QED (abelian), we can use:

$$\sum_{\text{Pol}} \epsilon_\mu(p)\epsilon_\nu(p)^* = -g_{\mu\nu} + \cancel{\frac{n_\mu p_\nu}{(n\cdot p)}} + \cancel{\frac{p_\mu n_\nu}{(n\cdot p)}} - \cancel{\frac{p_\mu p_\nu}{(n\cdot p)^2}}.$$

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- ~~**Ward identity**~~$\to$ **Slavnov-Taylor identity**: $p_1^{\mu_1}\epsilon^{\mu_2}(p_2)\mathcal{M}_{\mu_1\mu_2} = \epsilon^{\mu_1}(p_1)p_2^{\mu_2}\mathcal{M}_{\mu_1\mu_2} = 0$.
- Weaker than QED: all the particles must be on-shell.
- Terms $\propto p_1, p_2$ cannot be dropped.
- If we do, we must compensate subtracting (incoherently) MEs with external ghosts ([Malmquist,Kachelrieß,'21]).