Università degli Studi di Milano

Istituto Nazionale di Fisica Nucleare

Electroweak Physics, towards FCC-ee

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XXXVII Rencontres de Physique de la Vallée d'Aoste

La Thuile, March 7th 2024

Outline

The inclusive production of a fermion pair is a standard candle process both

at LHC (Drell-Yan) $\sigma(pp \to \mu^+\mu^- + X)$

and

at FCC-ee $\sigma(e^+e^- \to \mu^+\mu^- + X)$

the lowest order process, at partonic level, is in both cases $f\bar{f} \to \mu^+\mu^-$: they share very similar computational challenges

The evaluation of NNLO-EW corrections is needed not only at FCC-ee, but already at the LHC !

Motivation: statistical precision from small to large fermion-pair invariant masses

Statistical errors

FCC-ee $\sigma(e^+e^- \to \mu^+\mu^- + X)$

arXiv:2206.08326

sqrt(S) (GeV)	luminosity (ab^{-1})	σ (fb)	% error
91	150	$2.17595\ 10^6$	0.0002
240	5	1870.84 ± 0.612	0.03
365	1,5	787.74 ± 0.725	0.09

LHC and HL-LHC $\sigma(pp \to \mu^+\mu^- + X)$

arXiv:2106.11953

bin range (GeV)	% error 140 fb^{-1}	% error 3 ab^{-1}
91-92	0.03	$6\ 10^{-3}$
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

Theoretical systematics

EW input parameters

large QED corrections

increasingly large EW corrections

proton PDFs

increasingly large QCD, QCD-EW and EW corrections

Are we able to reach the 0.1% precision throughout the whole invariant mass range?

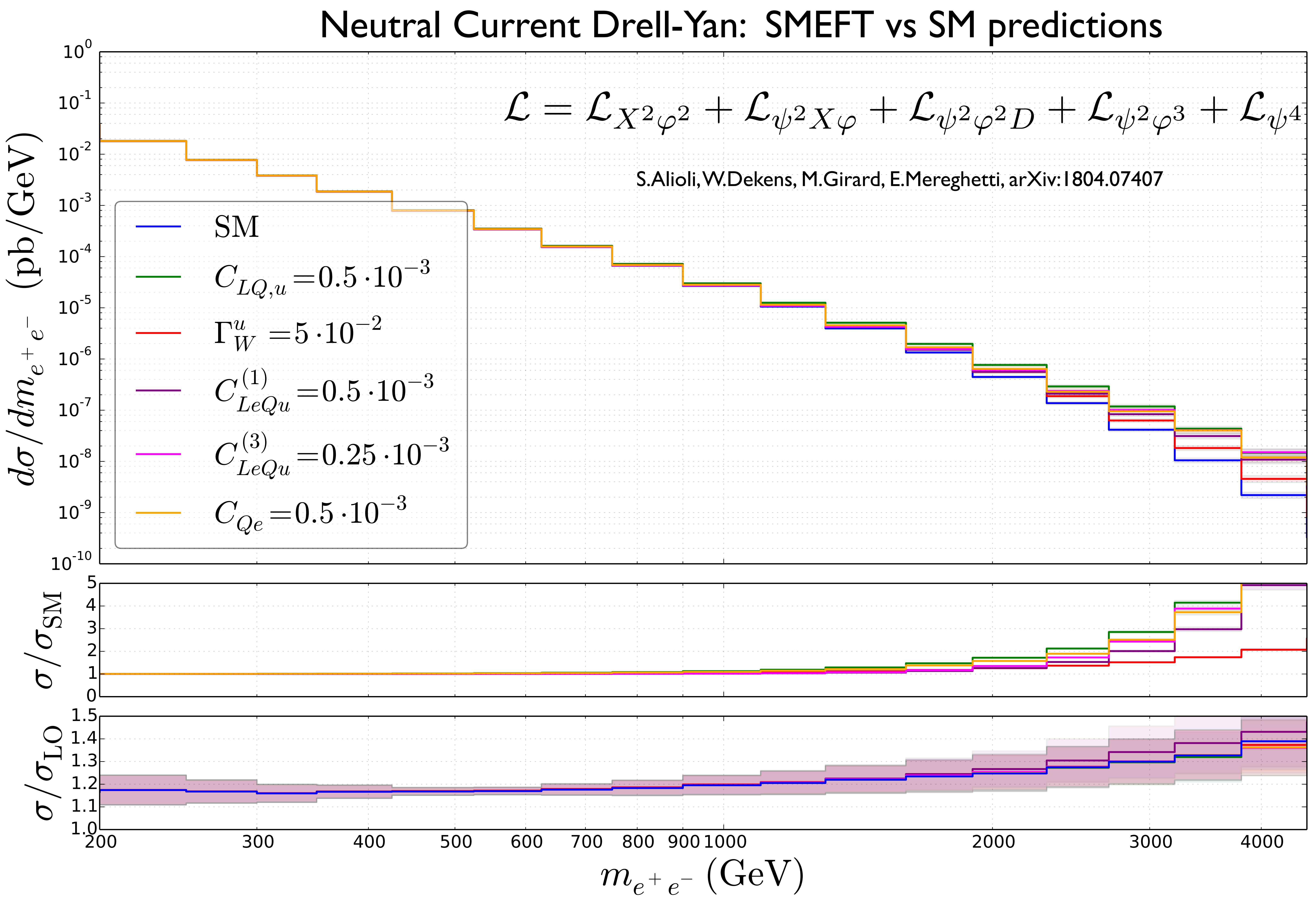
The Drell-Yan case poses the same challenges relevant for FCC-ee

Motivation: impact of higher dimension operators, as a function of the invariant mass

The parameterisation of BSM physics
in the SMEFT language
can be probed by studying the impact
of higher dimension operators
as a function of energy.

Deviations from the SM prediction
require the SM prediction to be
at the same precision level of the data
i.e. (sub) per mille level

HL-LHC and FCC-ee
have similar opportunities of testing
the presence of SMEFT contributions
in different mass windows



Motivation: interplay of precision measurements at Z resonance and low- and high-energy

The very high precision determination of EW parameters at the Z resonance is a cornerstone of the whole precision program but there is more…

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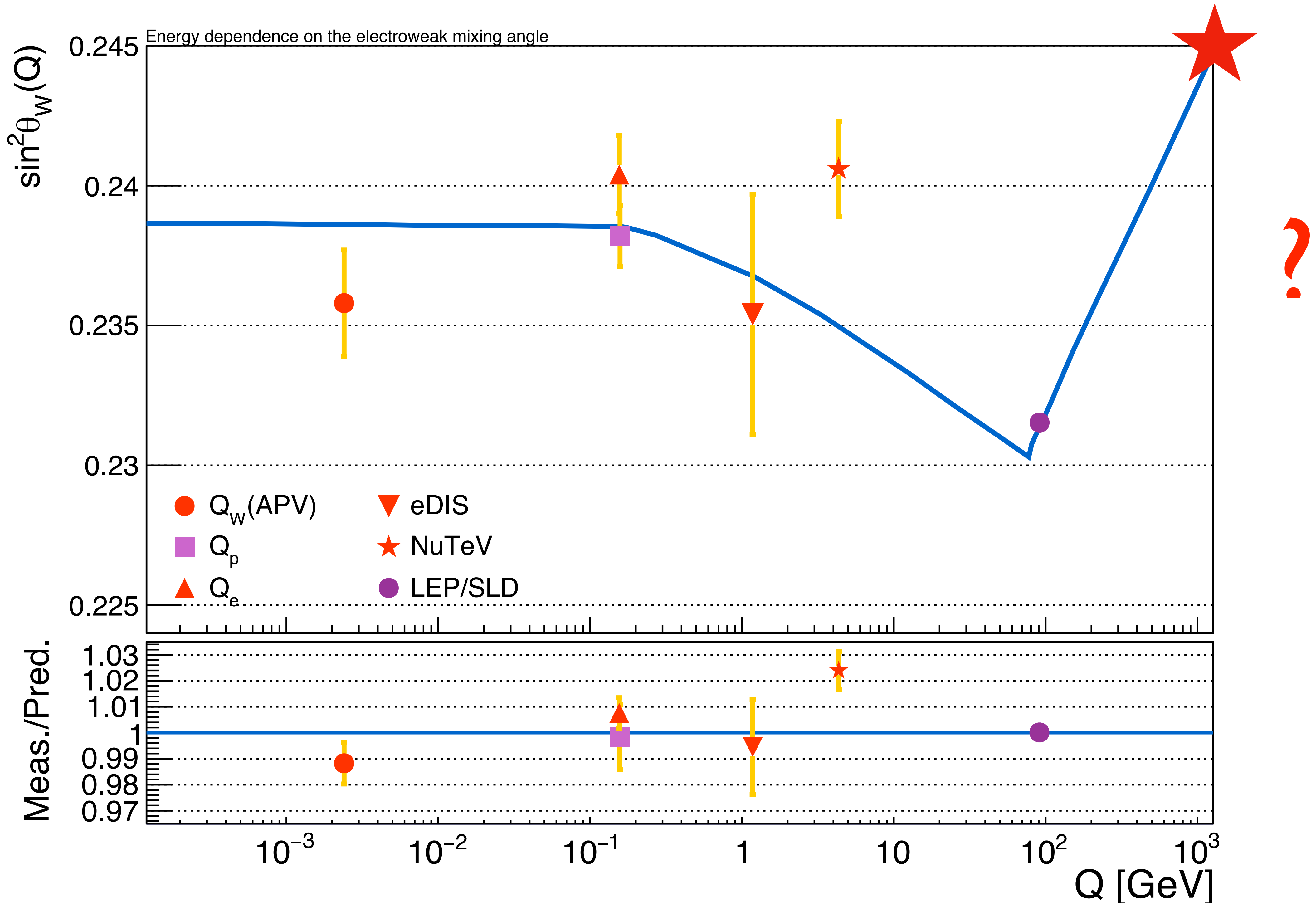
The SM predicts the running of its parameters, like e.g. $\sin^2\hat{\theta}(\mu_R^2)$, with non-trivial features and in turn complementary sensitivity to BSM physics

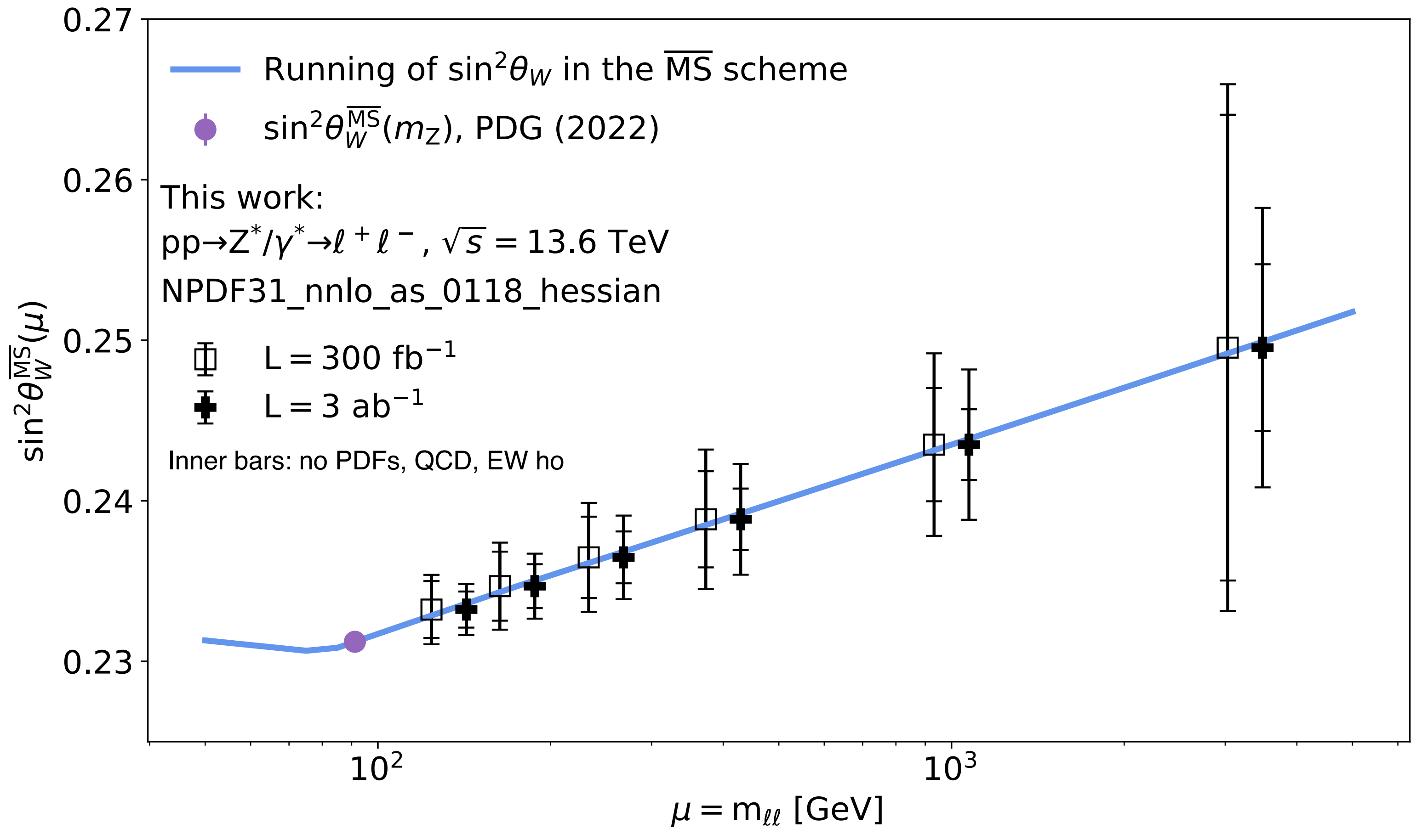
The running of an MSbar parameter is completely assigned once boundary and matching conditions are specified

low-energy (sub-GeV) determinations (P2 in Mainz, Møller at JLab)
high-energy (TeV) determinations (CMS, ATLAS)
offer a stringent test of the SM
complementary to the results at the Z resonance



Erler, J., Schott, M., Progr. in Part. and Nucl. Phys. 106 (2019) 68-119

Motivation: exploiting simultaneously Z-resonance and high-mass precision



The sensitivity to determine the running of $\sin^2\hat{\theta}(\mu_R^2)$ at the LHC
has been demonstrated in arXiv: 2302.10782

A dedicated POWHEG NCDY version has been implemented for this study,
with $\sin^2\hat{\theta}(\mu_R^2)$ among the input parameters, with NLO-EW renormalisation.
(when fitting the distributions to the data, we can only vary the input parameters of the calculation)

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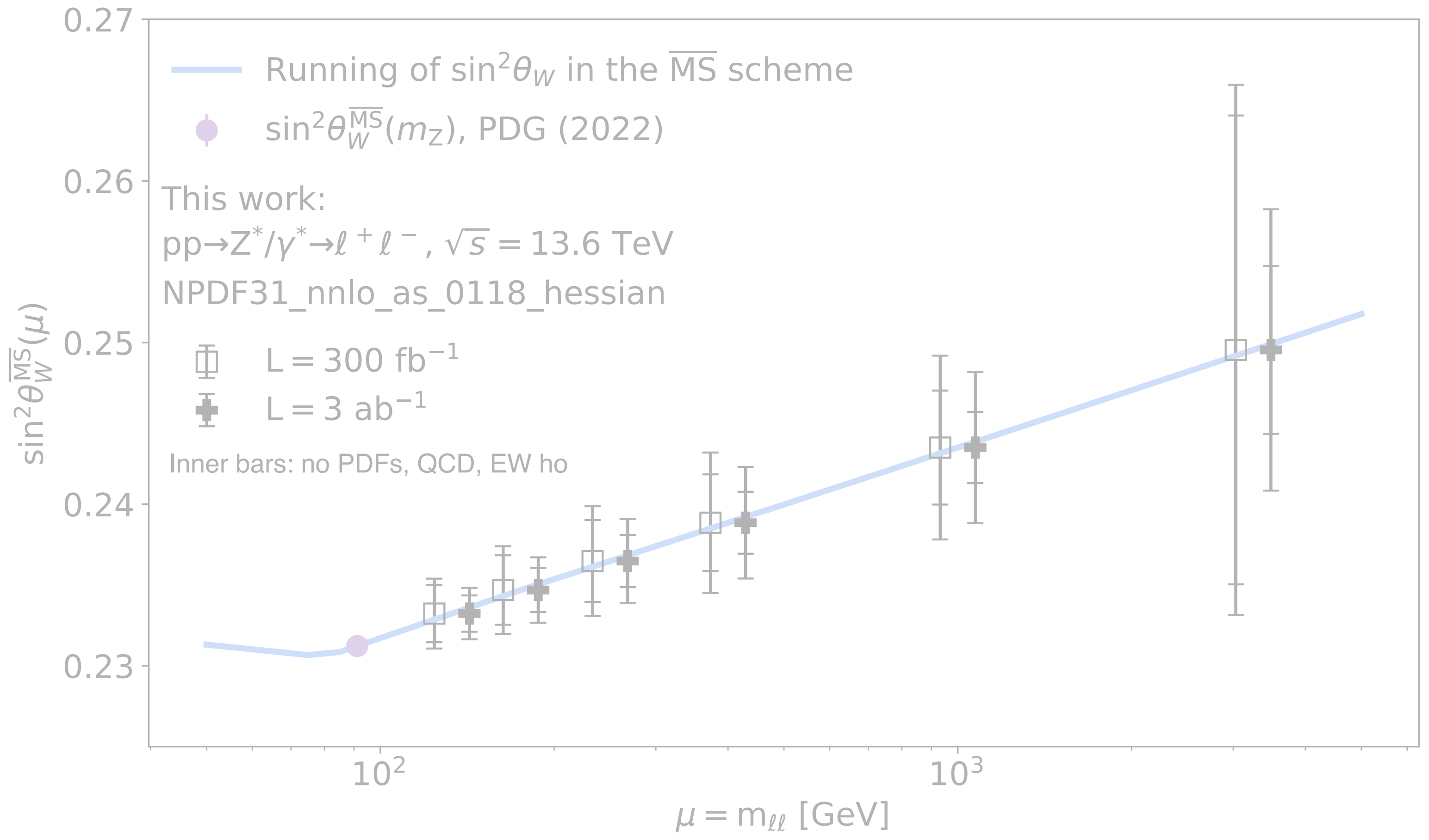

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The determinations of the
- $\sin^2\hat{\theta}(\mu_R^2)$ running
- Wilson coefficients of higher-dimension operators in SMEFT

share a problem:

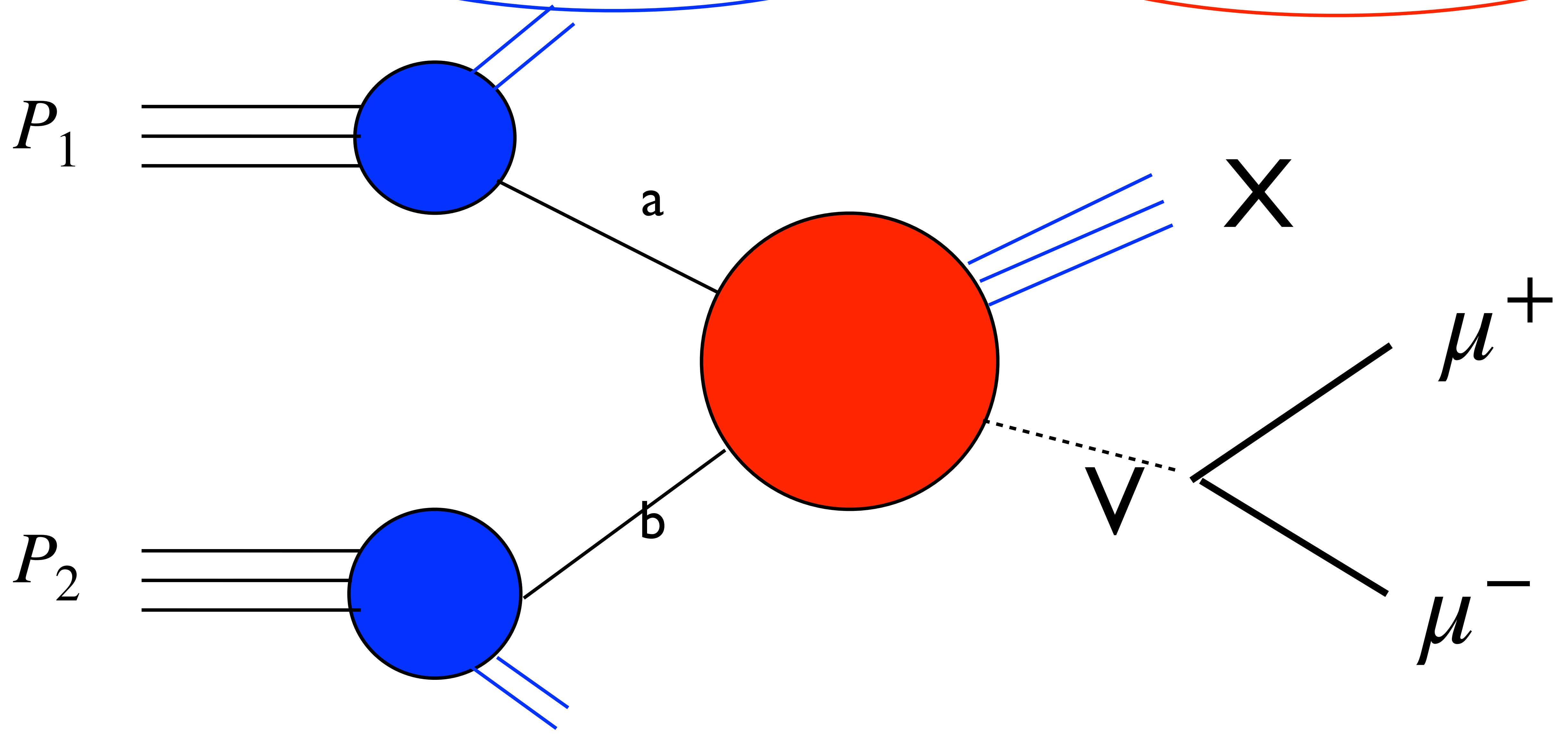
Missing SM higher-order effects, not related to the coupling definition, may be reabsorbed in these fitting parameters faking a BSM signal

examples: all the QCD corrections, the EW Sudakov logs, the corrections contributing to the electric charge running
→ we need the best SM description of the cross sections, before we move to the interpretation phase in terms of couplings

NNLO-EW corrections (with UV renormalisation) are needed both at the LHC and FCC-ee to tame this potential problem

Factorisation theorems and the cross section in the partonic formalism

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F)\, \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$



Particles $P_{1,2}$ can be protons (→ Drell-Yan @ LHC) or leptons (→ FCC-ee)

The partonic content of the scattering particles can be expressed in terms of PDFs (→ Maria's talk)

proton PDFs: ABM, CT18, MSHT,NNPDF,… lepton PDFs: Frixione et al. arXiv:1911.12040

The partonic scattering can be computed in perturbation theory,
exploiting the theoretical progress in QCD, in the understanding of its IR structure

Factorisation theorems guarantee the validity of the above picture up to power correction effects

Neutral current Drell-Yan in a fixed-order expansion

Drell-Yan (1970)

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} +$$

Altarelli, Ellis, Martinelli (1979)

$$\alpha_s\, \sigma^{(1,0)} + \alpha\, \sigma^{(0,1)} +$$

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

$$\alpha_s^2\, \sigma^{(2,0)} + \alpha\, \alpha_s\, \sigma^{(1,1)} + \alpha^2\, \sigma^{(0,2)} +$$

$$\alpha_s^3\, \sigma^{(3,0)} + \ldots$$

C.Duhr, B.Mistlberger, arXiv:2111.10379

still missing
Sudakov high-energy approximations

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, (2021)
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2022)
F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, (2022)

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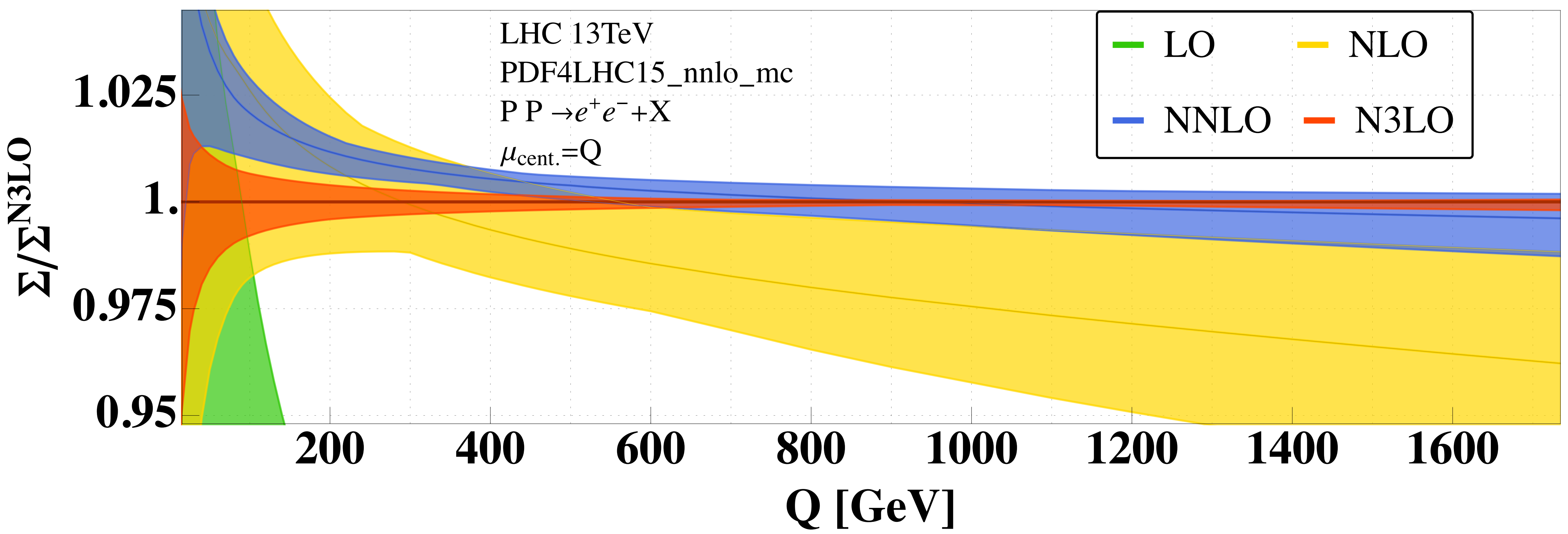
At the LHC, the need for a combined resummation of QCD and QED contributions, with QCD up to third logarithmic order in the relevant variables (e.g. $p_\perp^{\ell\ell}$, threshold variables,…) is crucial

At the FCC, QED resummation at least at third logarithmic order is needed.

Both deserve a separate talk. Here we focus on the description of the tails, above the Z resonance.

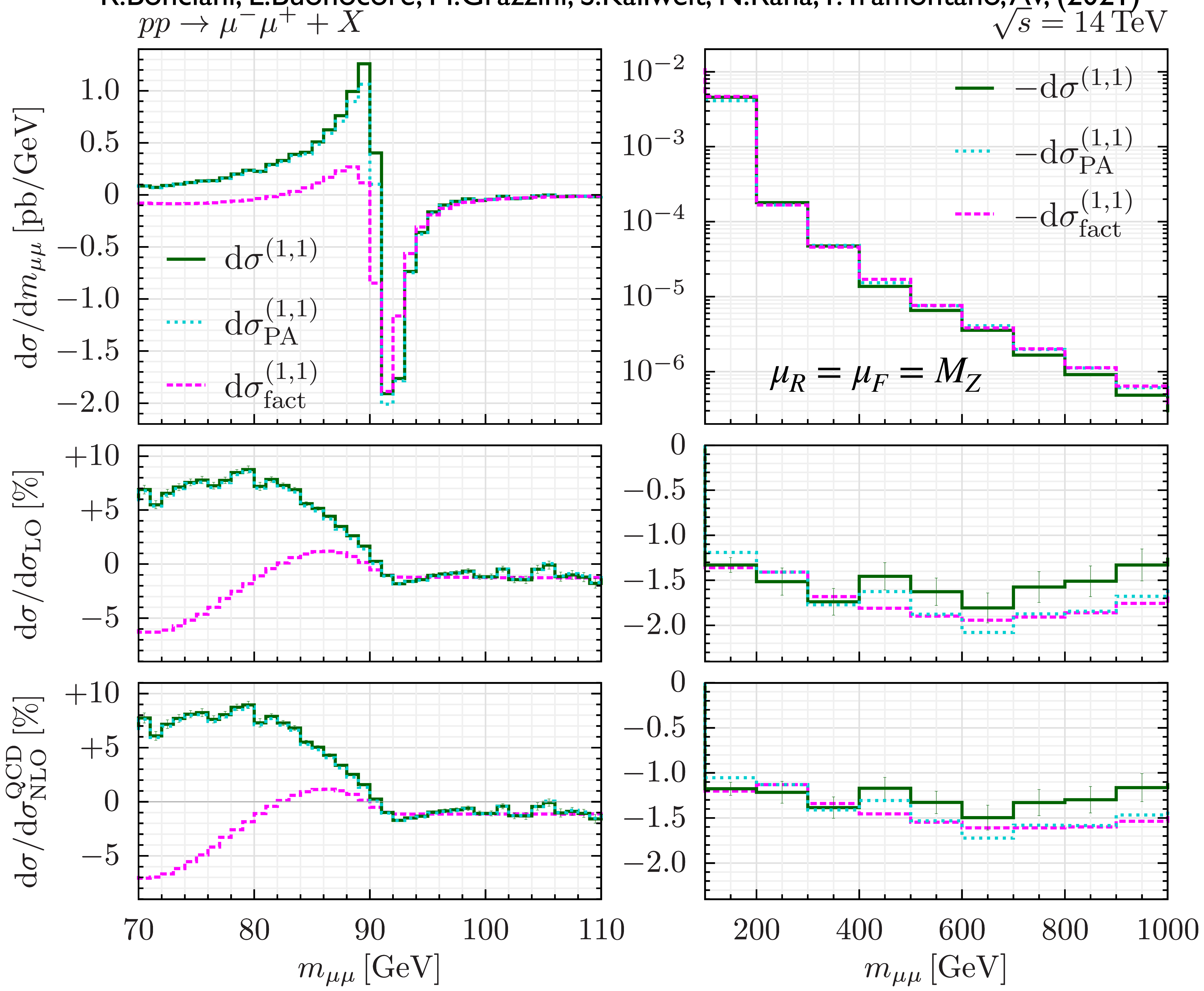
Impact of higher-order corrections in Drell-Yan production

C.Duhr, B.Mistlberger, arXiv:2111.10379



The N3LO corrections clearly stabilise the dependence on the choice of the QCD scales

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, (2021)



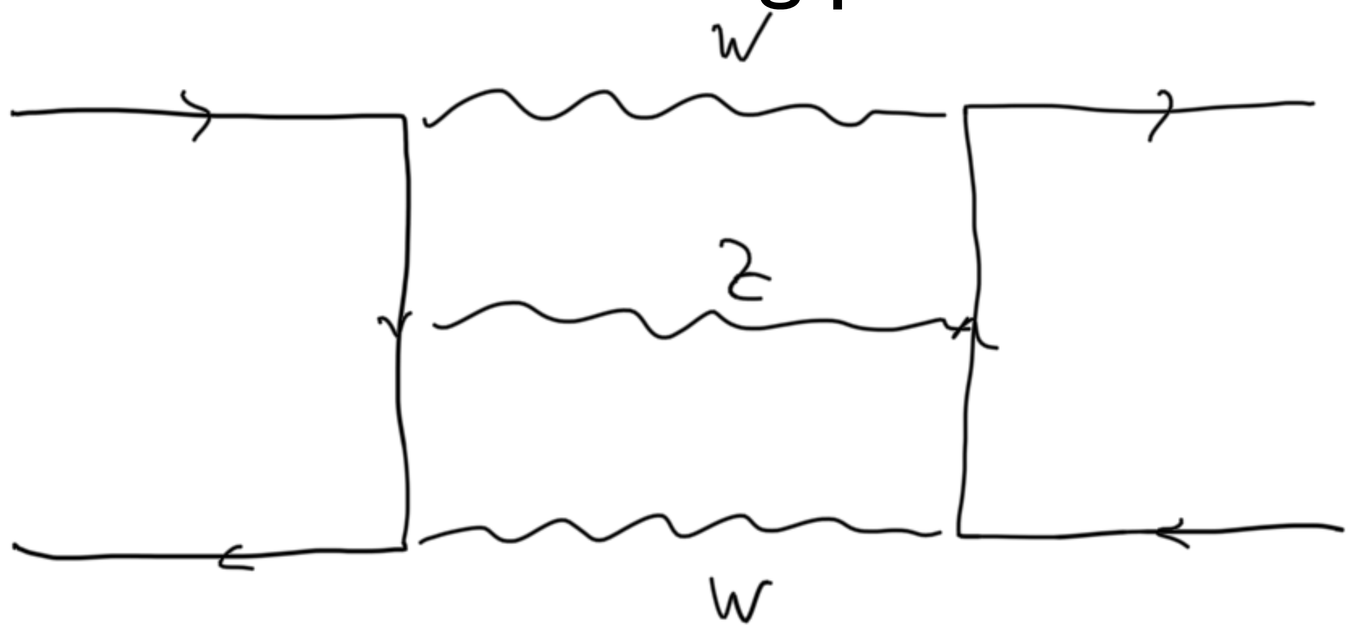
The mixed NNLO QCD-EW corrections feature a O(-1.5%) correction, up to 1 TeV of invariant mass missing in any additive combination available in simulation tools

At large invariant mass, QCD and EW show a factorisation pattern.

Next to the resonance, kinematic effects are important for a proper description

Need for a full NNLO-EW calculation to reduce the uncertainties to sub-percent level

The NNLO-EW corrections to scattering processes are still today one of the frontiers in QFT



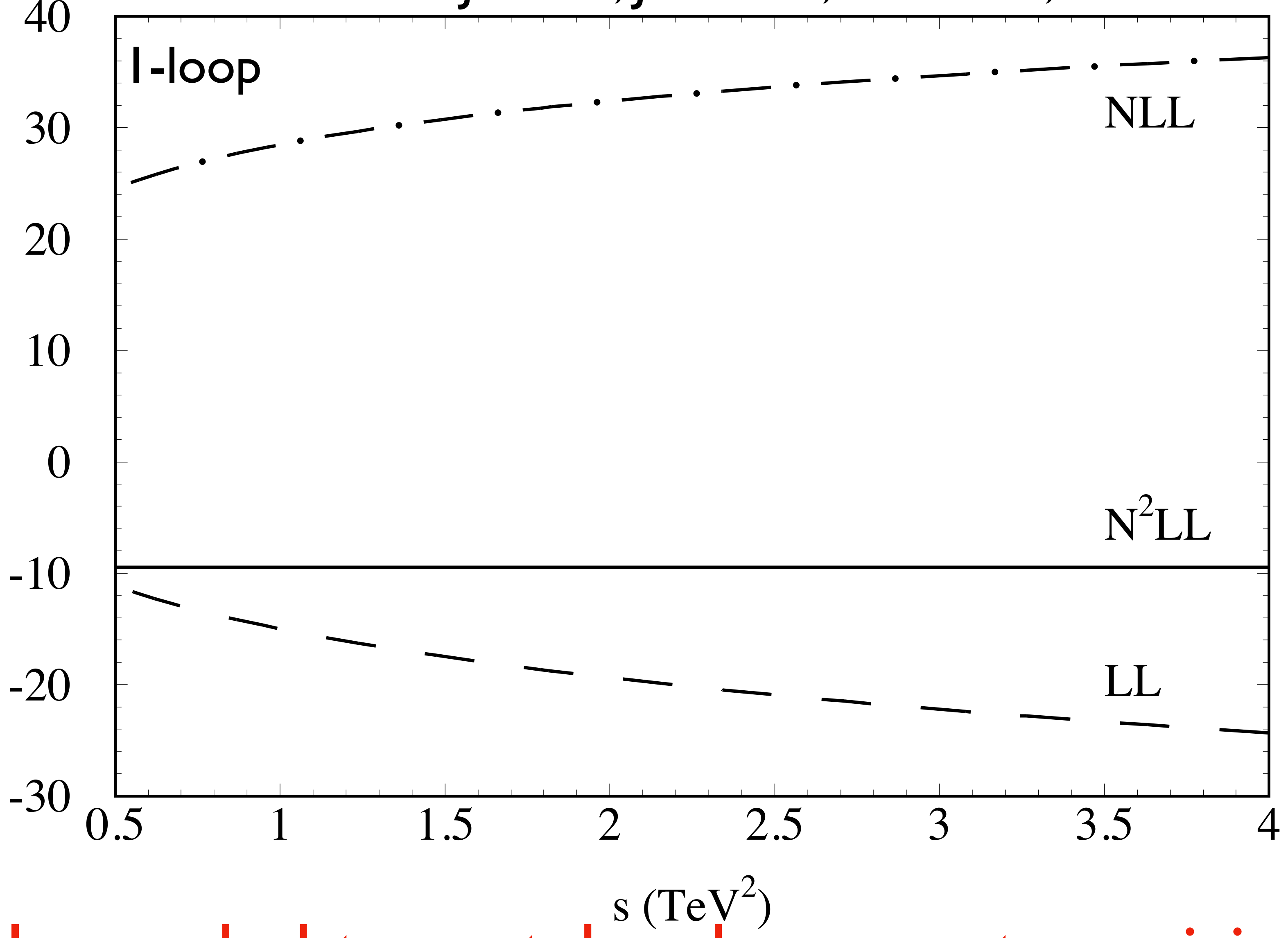
The NNLO-EW corrections could modify in a non-trivial way the large-mass/momentum tails of the distributions

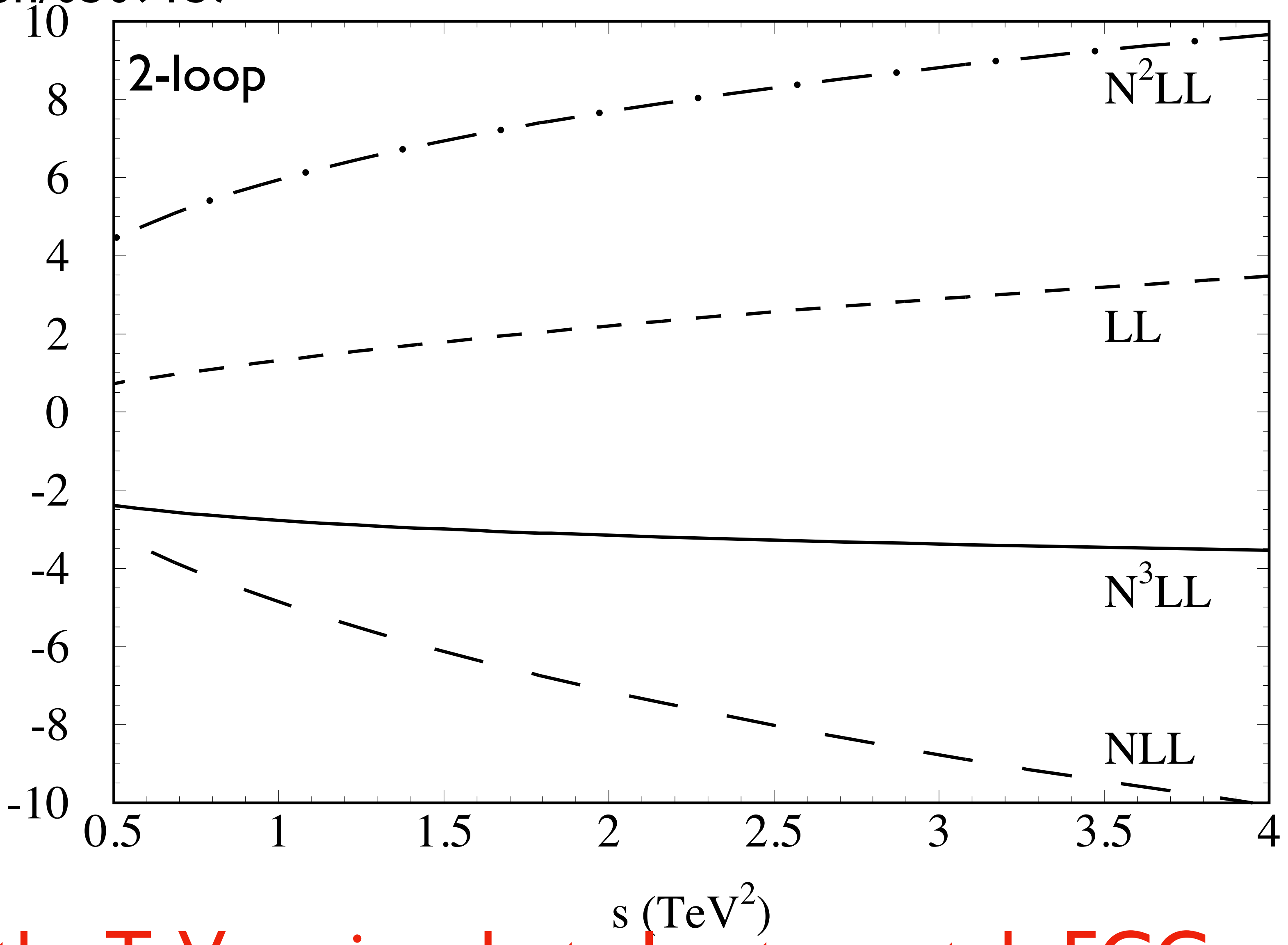
Large logarithmic corrections (EW Sudakov logs) appear in the virtual corrections

At two-loop level, we have up to the fourth power of $\log(s/m_V^2)$

The size of the constant term is not trivial

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157





corrections to $e^+e^- \to q\bar{q}$ due to EW Sudakov logs

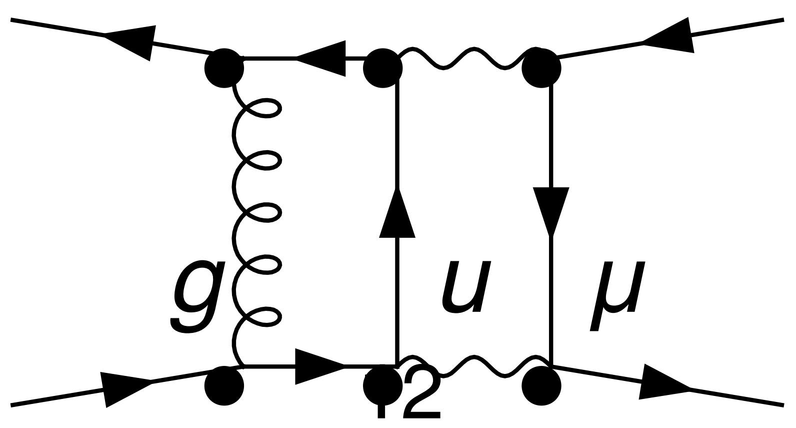
urgently needed to match sub-percent precision in the TeV region, but also to match FCC-ee precision at any energy

Towards the NNLO-EW corrections to $\sigma(f\bar{f} \to \mu^+\mu^- + X)$

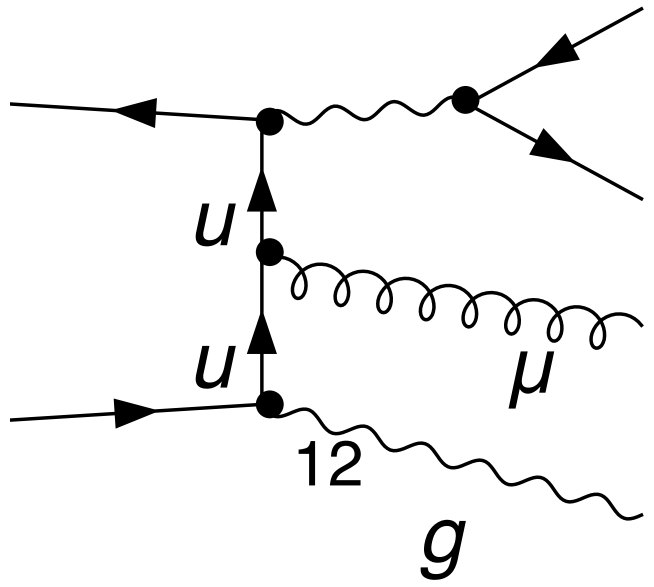
- The evaluation of NLO corrections (QCD and EW) can be accomplished with automatic tools

- At NNLO level different conceptual and technical problems arise:

 - evaluation of the 2-loop virtual amplitudes
   increasing complexity depending on the number of internal massive lines (# of energy scales)
   one of the main bottlenecks so far



 - phase-space integration of double-real and real-virtual contributions
   reaching 0.1% precision is already challenging (subtraction techniques)



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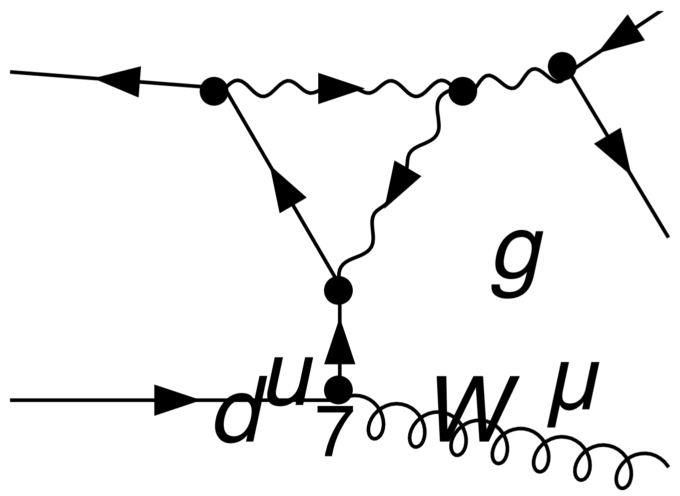

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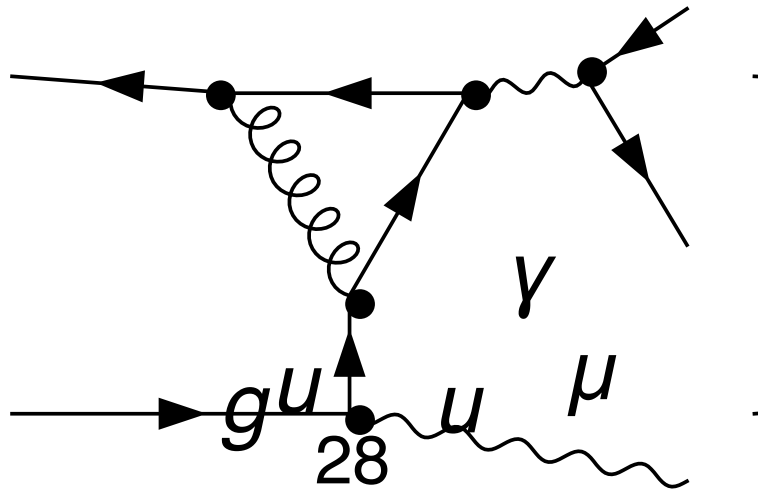
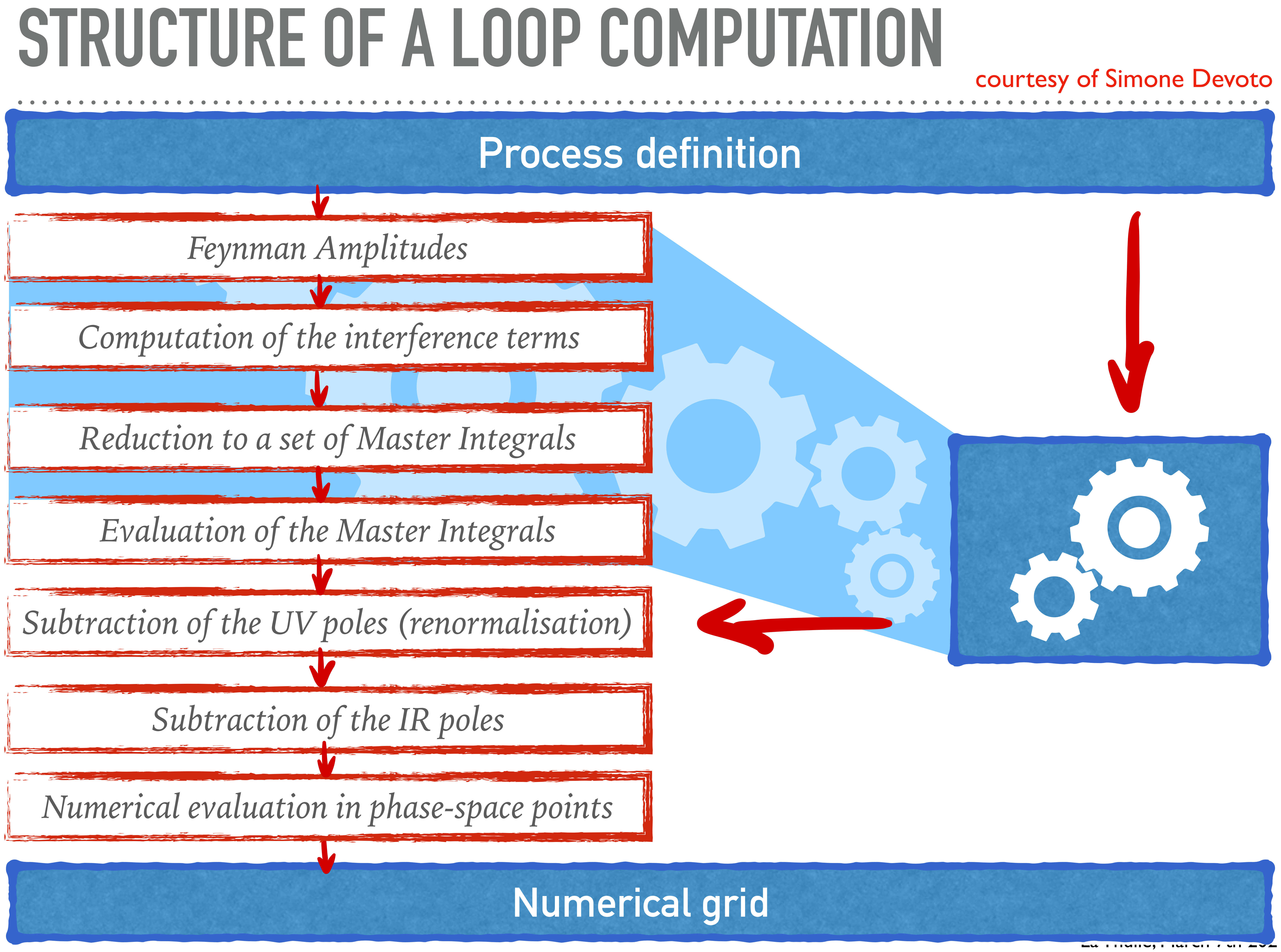


Towards the NNLO-EW corrections to $\sigma(f\bar{f} \to \mu^+\mu^- + X)$

The NNLO QCD-EW corrections to Drell-Yan are an excellent playground for many of these problems

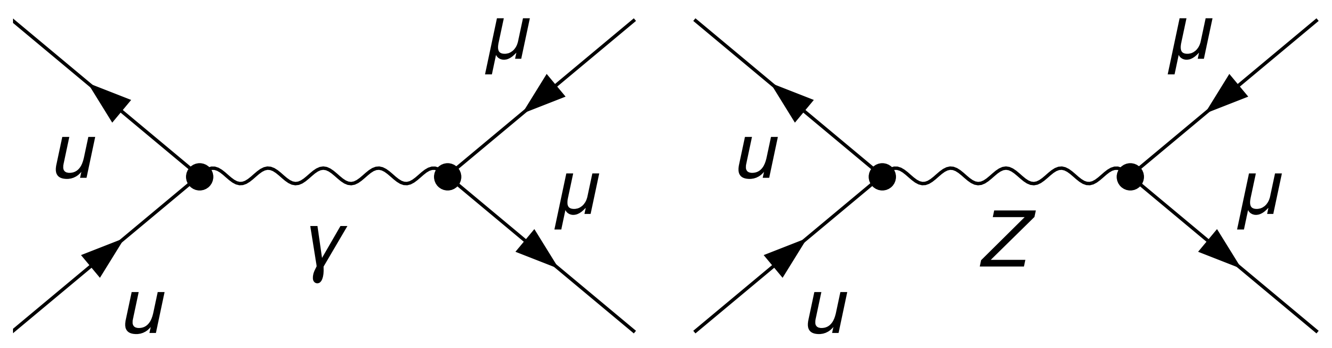
T.Armadillo, R.Bonciani, S.Devoto N.Rana,, AV, arXiv:2201.01754

→ in turn, directly relevant for $e^+e^- \to q\bar{q} + X$



The double virtual amplitude: generation of the amplitude

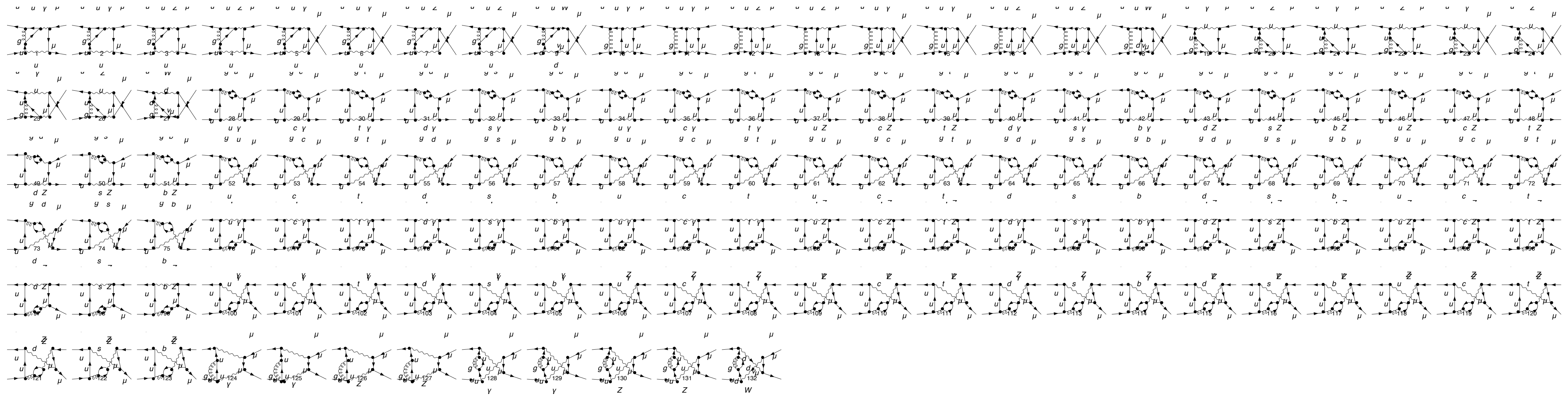
$$\mathcal{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$$

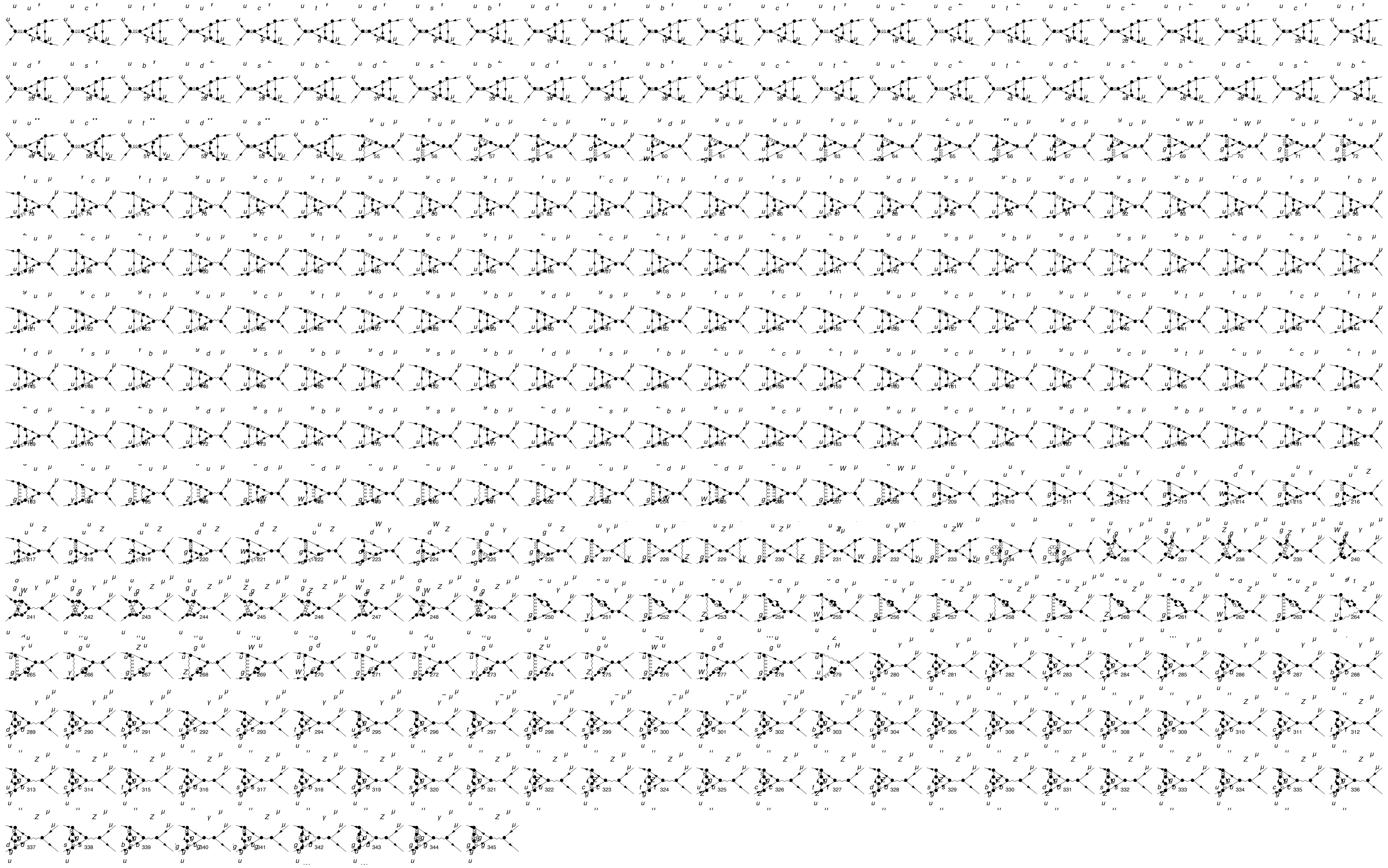


1 2

$$\mathcal{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$$

O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop
(before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)





The double virtual amplitude: reduction to Master Integrals

$$2\mathrm{Re}\left(\mathscr{M}^{(1,1)}(\mathscr{M}^{(0,0)})^{\dagger}\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathscr{T}_i(s,t,m;\varepsilon)$$

The coefficients c_i are rational functions of the invariants, masses and of ε

Their size can rapidly "explode" in the GB range

→ careful work to identify the patterns of recurring subexpressions, keeping the total size in the O(1-10 MB) range

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The Master Integrals $\mathcal{I}_i$ satisfy a system of first order linear differential equations

The solution can be obtained in several cases in closed analytical form in terms of special functions (GPLs, elliptic functions)

in general in semi-analytical form, via series expansions (with arbitrary precision)

using codes like DiffExp, SeaSyde, AMFlow

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The open question in view of 2-loop EW calculations with difficult 2-loop Master Integrals is the feasibility

of writing the differential equations in symbolic form → if yes, then the semi-analytical solution is available for any integral

The performance of such “solvers” can be optimised , in the most demanding cases with several internal masses

Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations.

The MIs are replaced by formal series with unknown coefficients → eqs for the unknown coefficients of the series.

DiffExp by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.

But we need complex-valued masses of W and Z bosons (unstable particles) → SeaSyde

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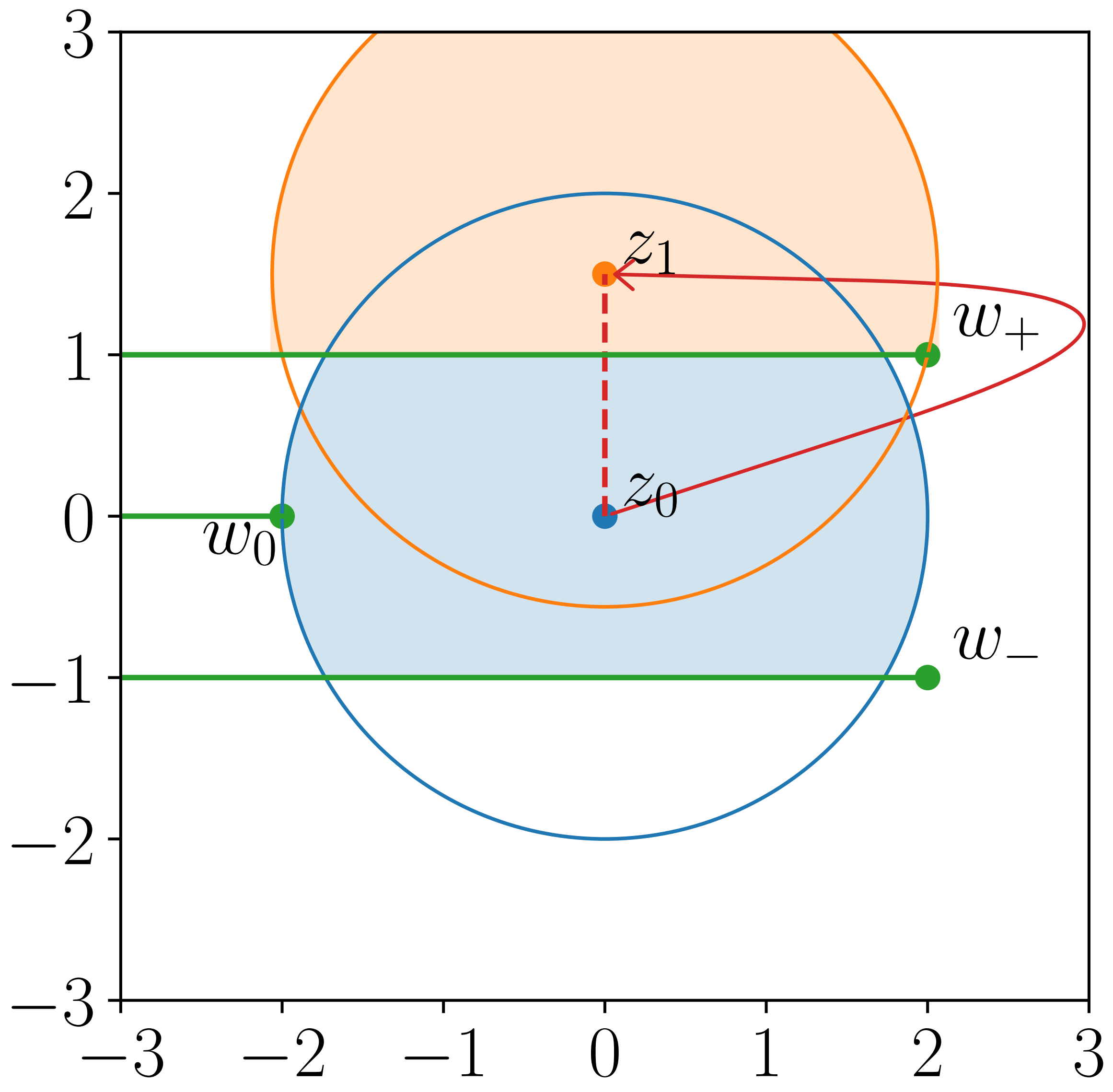

We implemented the series expansion approach, for arbitrary complex-valued masses, working in the complex plane of each kinematical variable, one variable at a time

Complete knowledge about the singular structure of the MI can be read directly from the differential equation matrix → interplay with S-matrix studies

The solution can be computed with an arbitrary number of significant digits, but not in closed form → semi-analytical

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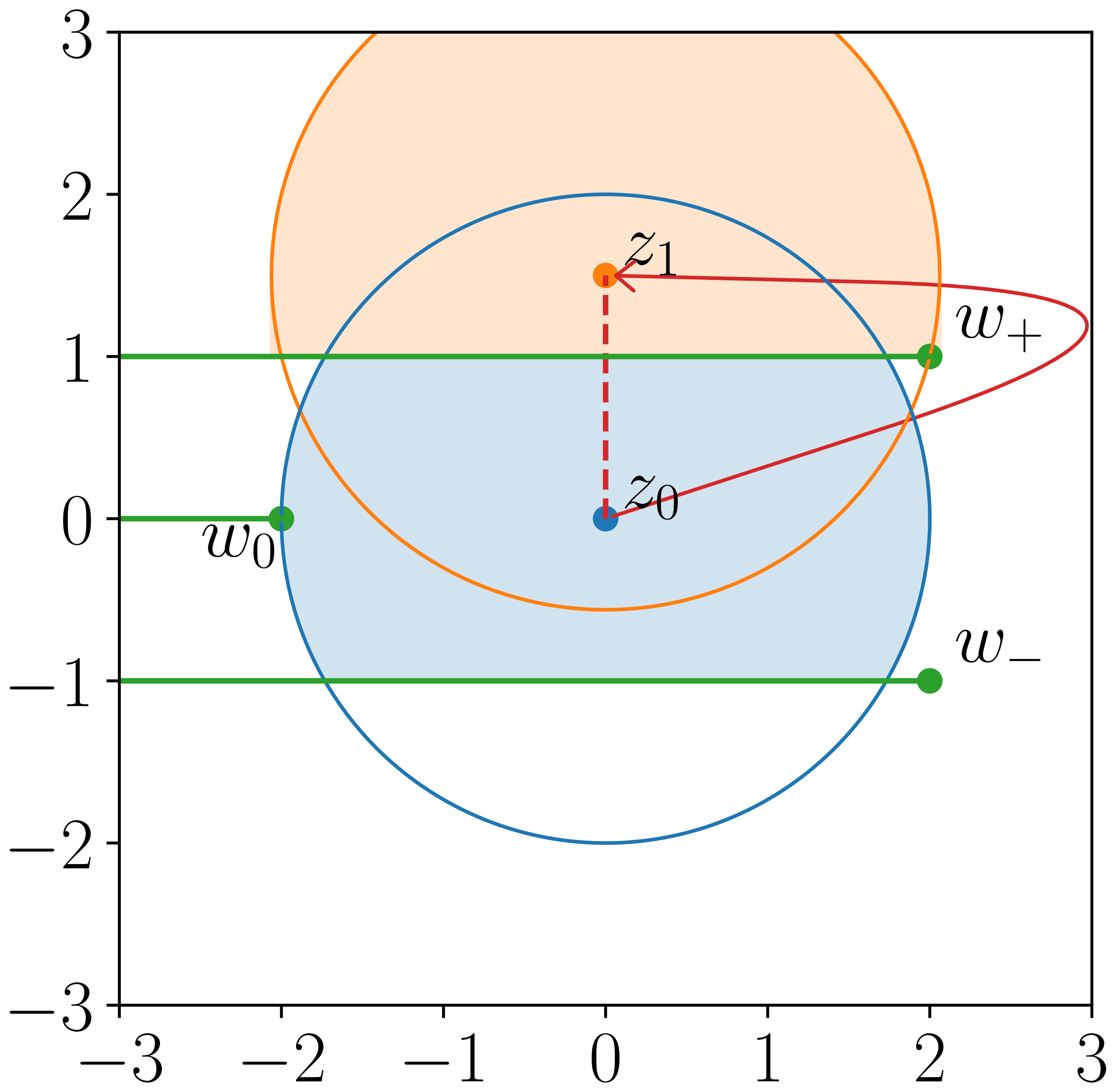

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Towards the NNLO-EW corrections to $\sigma(f\bar{f} \to \mu^+\mu^- + X)$

- Additional ingredients are needed at NNLO EW, in the 2-loop virtual sector

  - the complete implementation of the 2-loop EW renormalisation, in the complex mass scheme, using as input parameters precisely those that we plan to fit from the data (e.g. $\sin^2\theta^{\ell}_{eff}$ or $\sin^2\hat{\theta}(\mu_R^2)$)
    at the FCC-ee level of precision, the LEP/SLD pseudo-observables approach should be revised

  - a practical solution to handle the γ_5 problem (i.e. how far can we push the usage of naive-anti commuting γ_5)

  - an IR subtraction scheme (possibly inherited from QCD) fully consistent with gauge invariance

- Matching full NNLO (QCD, EW, QCD-EW) results with QCD+QED resummation
  is a must for any precision study at the LHC
  FCC studies can benefit of the LHC developments, but the precision level is extremely challenging

Conclusions

- The NNLO EW corrections to the Drell-Yan processes will be needed to match the final HL-LHC precision
  Steady progress is pushing the frontier of NNLO calculations from QCD-EW to full EW

- These results will be the core (starting point?) of the calculations needed at the FCC-ee
  to describe fermion-pair production in the whole energy range

- The availability of these corrections will establish the SM benchmark with precision comparable to the data
  → increase the significance of an observed deviation, as a function of energy → relevant to SMEFT studies

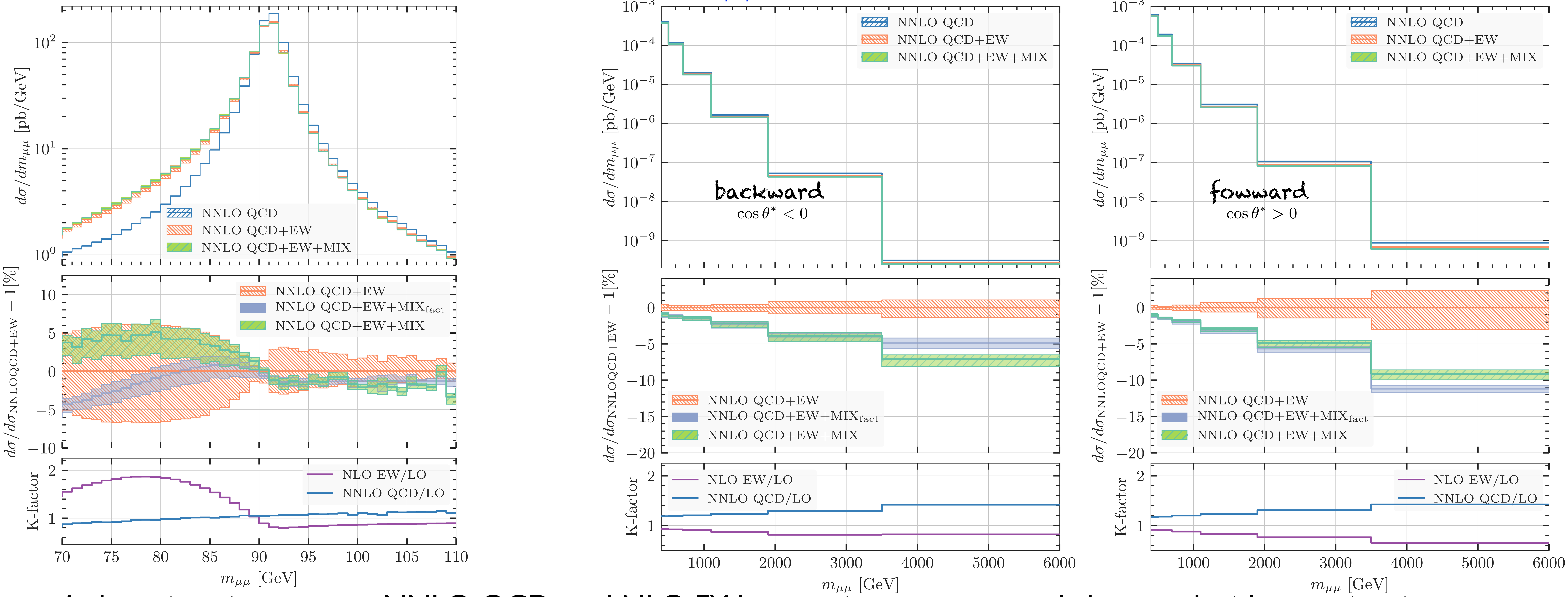
- As a starting example, the extraction of $\sin^2\hat{\theta}(\mu_R^2)$ at high-masses at the LHC shows
  the potential biases induced by neglecting SM higher-order effects
  → any BSM study must be done on top of the best SM results (NNLO-EW?) to avoid fake conclusions

Thank you

Back-up

Status of the perturbative prediction of the NC DY invariant mass distribution

R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953 and work in preparation



At large invariant masses, NNLO QCD and NLO EW corrections are separately large and with opposite signs
we also observe large NNLO QCD-EW corrections

If we do not simulate them explicitly, we reabsorb their effect in the value of the best fit $\sin^2\hat{\theta}_W(\mu_R^2 = M_{\ell\ell}^2)$

Which corrections do not contribute to the redefinition of the running coupling ?
- all the QCD corrections (same contribution to left- and right-handed couplings)
- more delicate breakdown of the EW contributions

Breakdown of EW radiative effects

Main subsets of EW corrections in the Drell-Yan process

- QED final state radiation
- universal corrections to the LO couplings
- EW Sudakov logarithms

Which ones do / do not contribute to the redefinition of the weak coupling at quantum level ?

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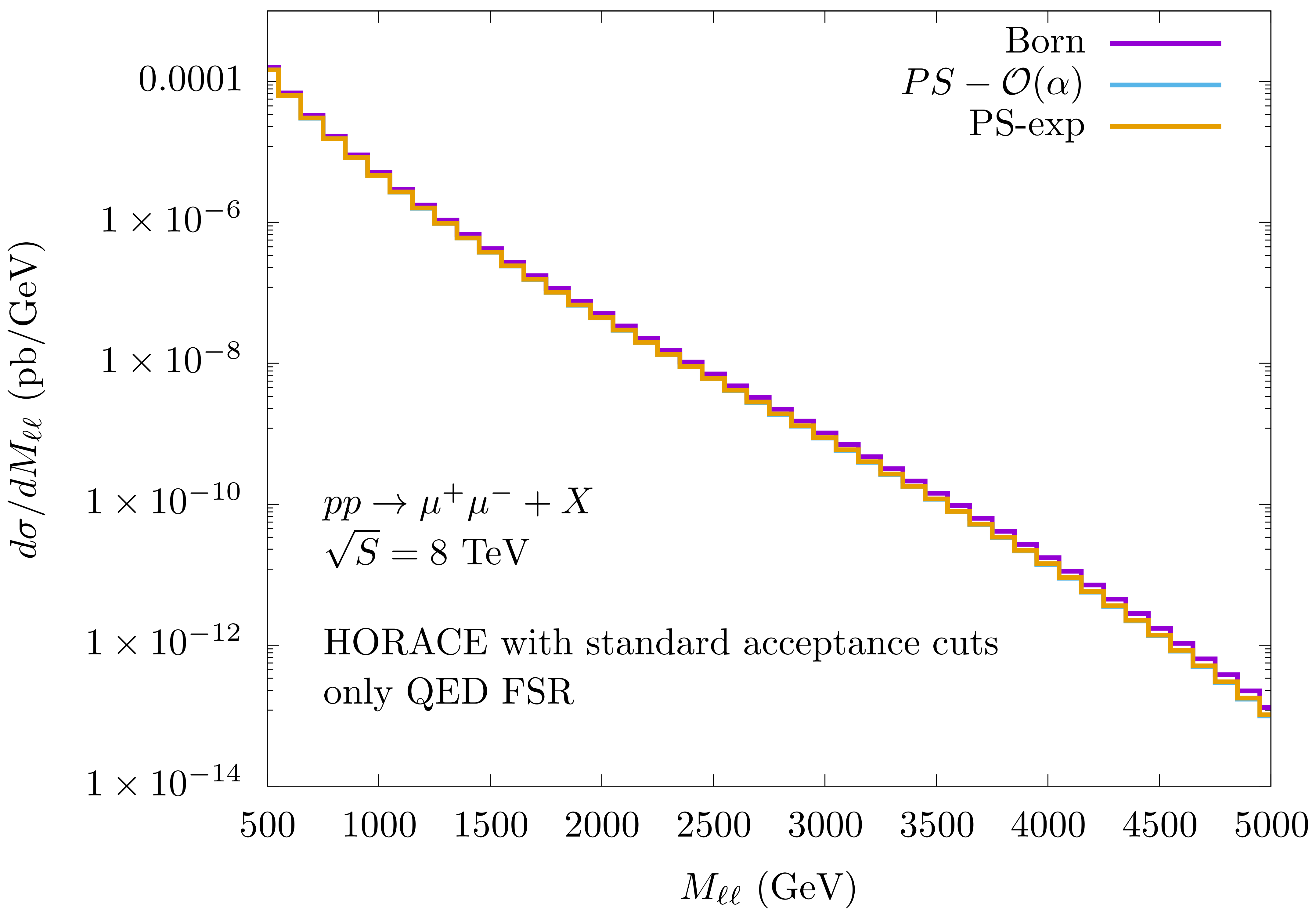

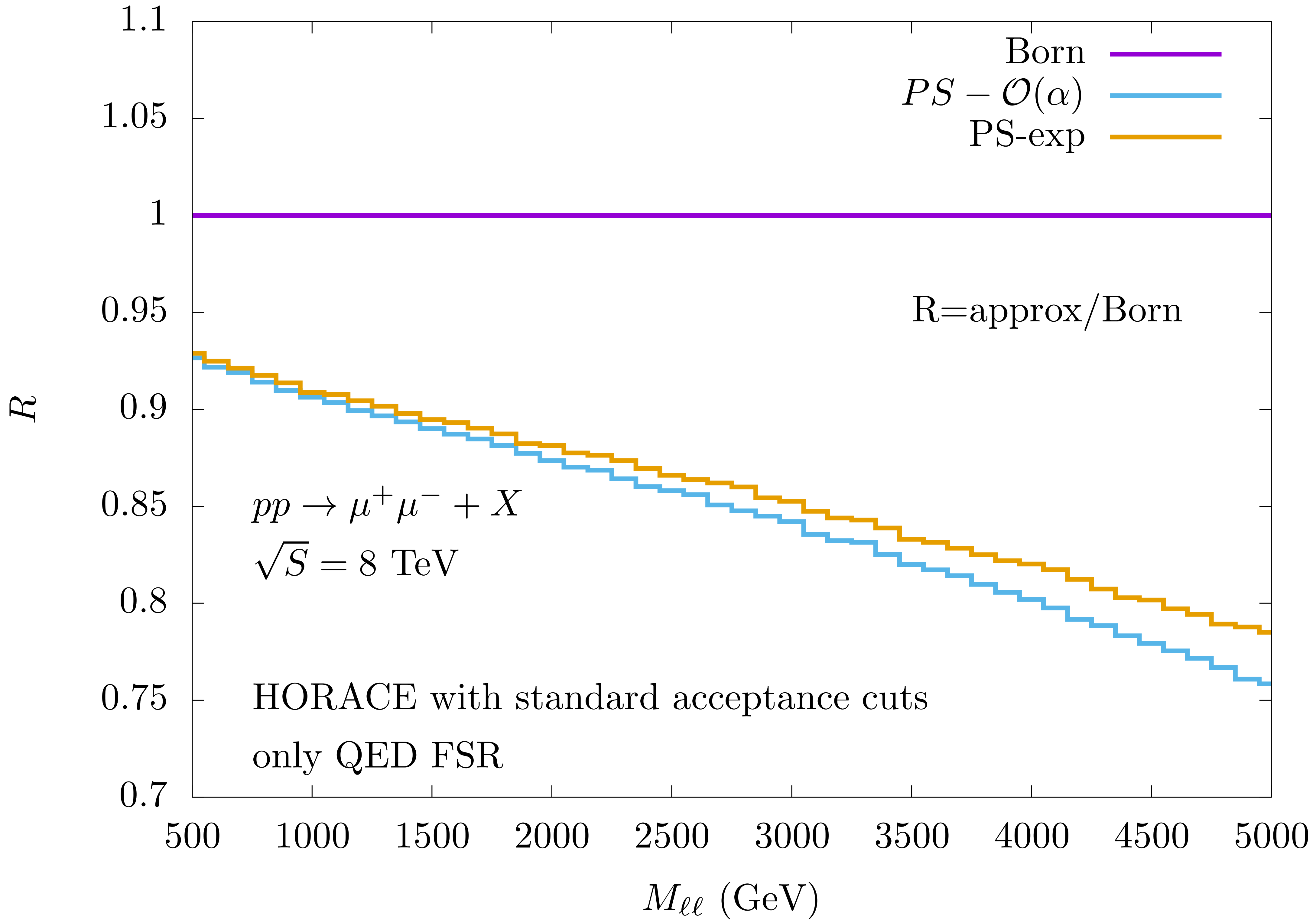

Do not contribute to the redefinition of the LO couplings (same contribution to left- and right-handed currents)

Not negligible kinematical effect moving events from higher to lower invariant mass bins

Same mechanism, with large effect, at the Z resonance, of $\mathcal{O}(80\%)$

Breakdown of EW radiative effects

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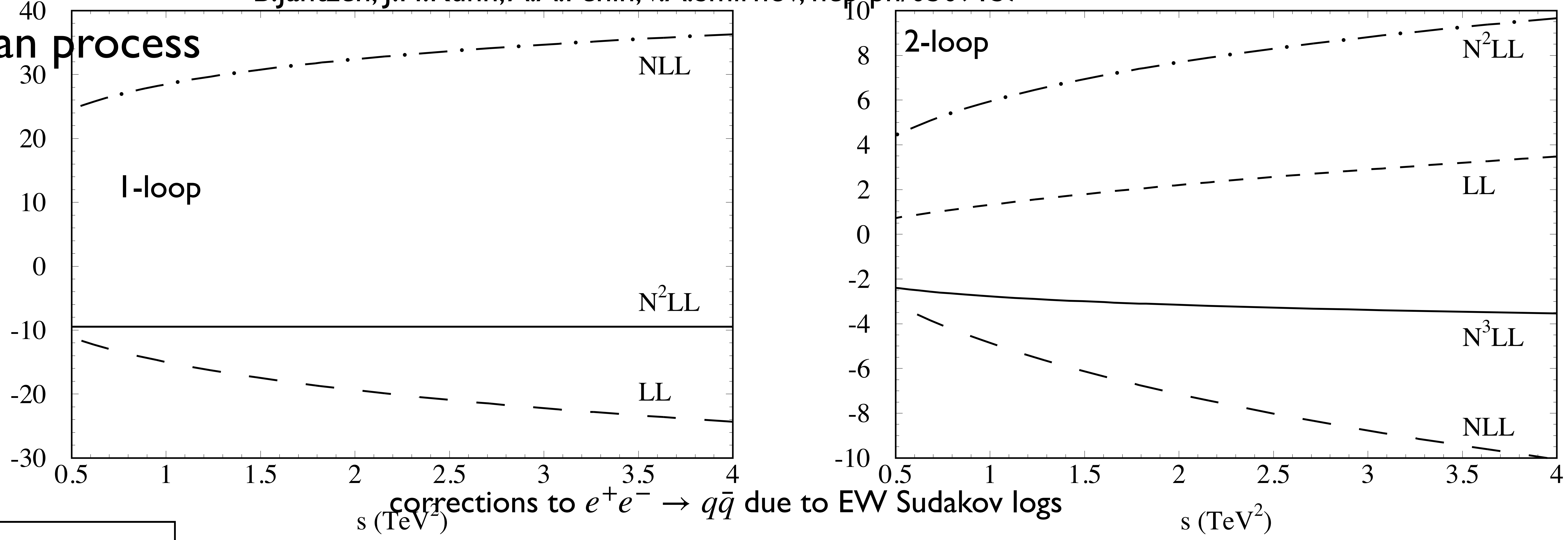
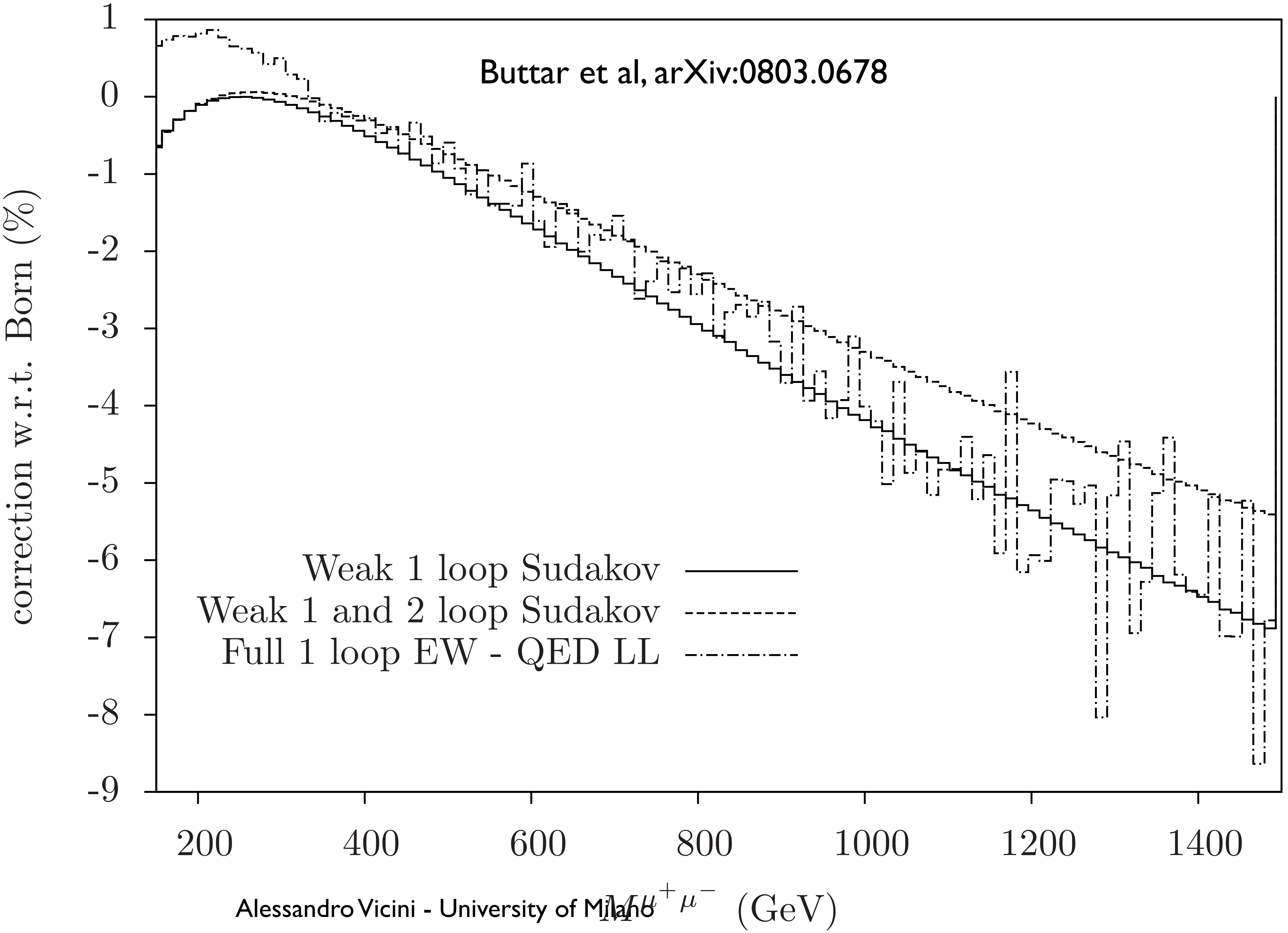
B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157



corrections to $e^+e^- \to q\bar{q}$ due to EW Sudakov logs



The EW Sudakov logs stem from vertex and box corrections

Their correction can be cast as
- one overall correction to the cross section
- one factor which distinguishes left- and right-handed currents
  → contributes to the definition of an effective mixing angle

Very large in the high-mass tail of the distribution (also at 2-loop level)

PDF-weighted combination of two alternating signs series of terms

Breakdown of EW radiative effects

Main subsets of EW corrections in the Drell-Yan process

- QED final state radiation
- universal corrections to the LO couplings
- EW Sudakov logarithms

Relevant in the accurate description of the Z resonance

The values of the couplings at $\mu_R = m_Z$ are initial conditions of the running of $\hat{\alpha}(\mu)$ and $\sin^2\hat{\theta}(\mu)$ → relevant for our test EW precision tests at the LHC from the simultaneous comparison of 100 and 1000 GeV regions

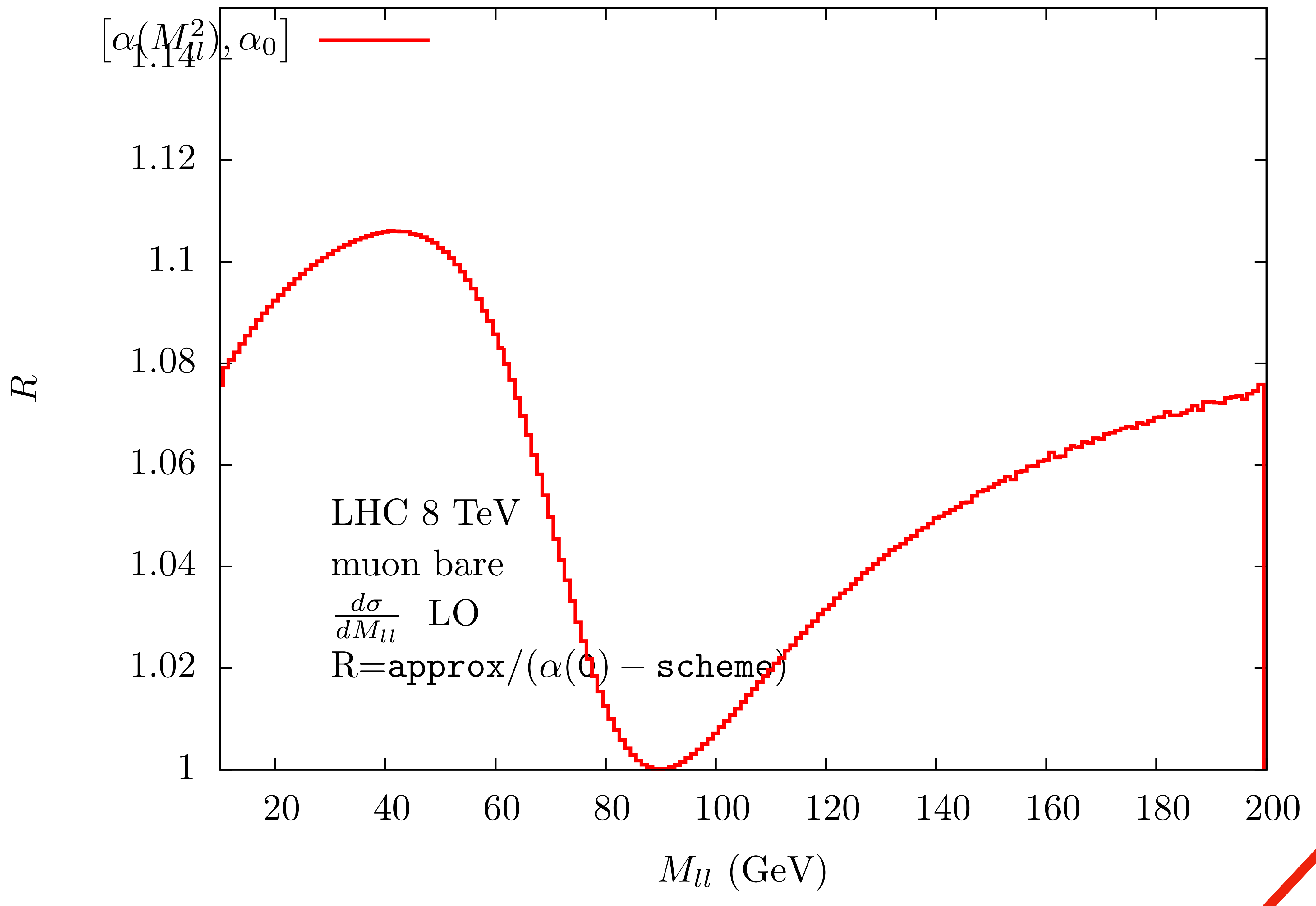
The impact of different universal corrections to the LO couplings can be illustrated via an Improved Born Approximation

The interplay of photon- and Z-exchange diagrams is modulated by the precise values of their respective couplings

In the following slides, the reference is given by LO results in the $\left(\alpha(0), m_W, m_Z\right)$ input scheme

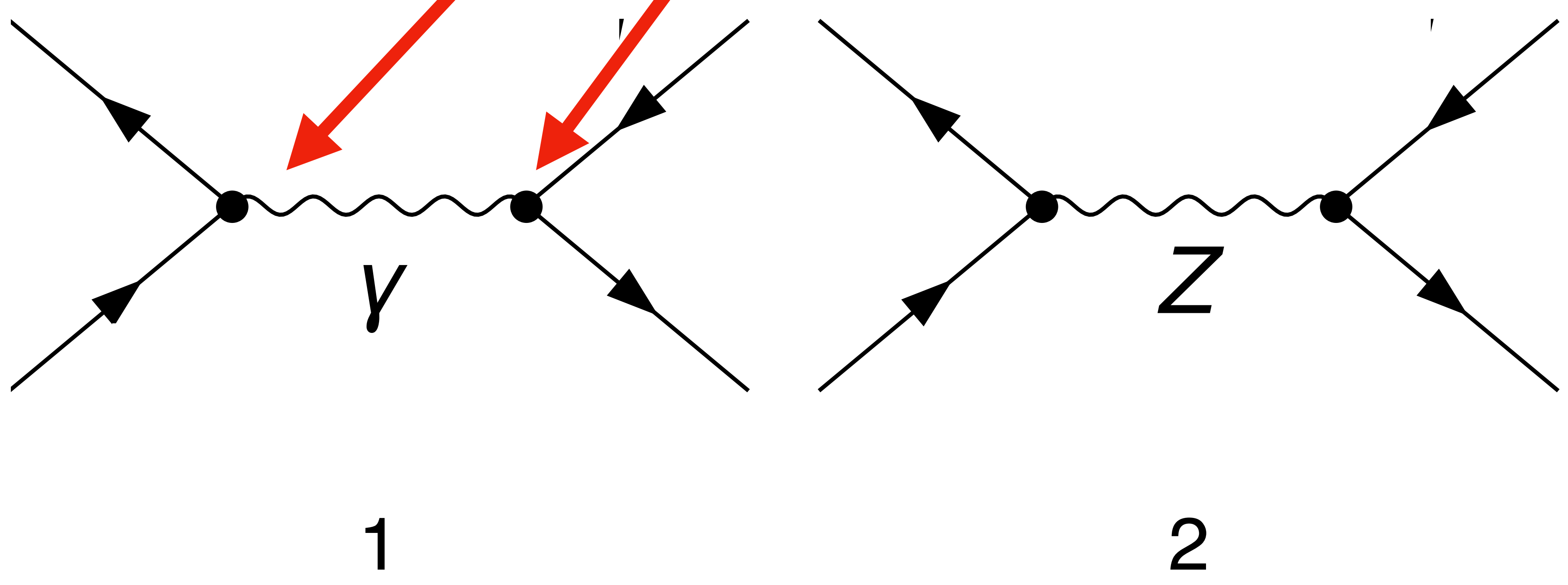
Each input replacement effectively introduces higher-order corrections, which should otherwise be computed in pert. theory

Beyond LO approximation in neutral current Drell Yan

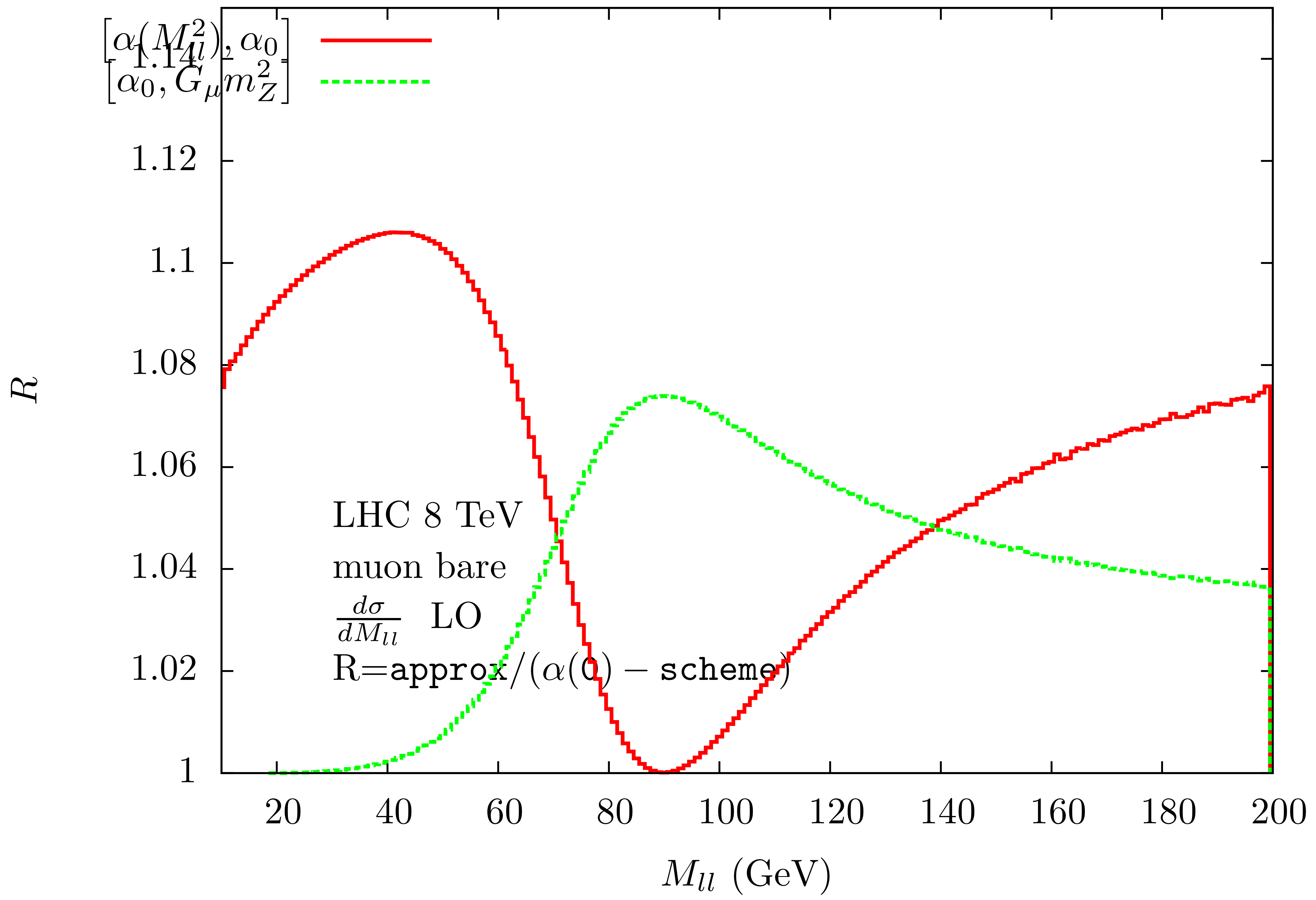


Running of α only in the photon diagram
enhances the photon exchange contribution
which grows with the invariant mass

$$\alpha(0) \to \alpha(M_{\ell\ell}^2)$$

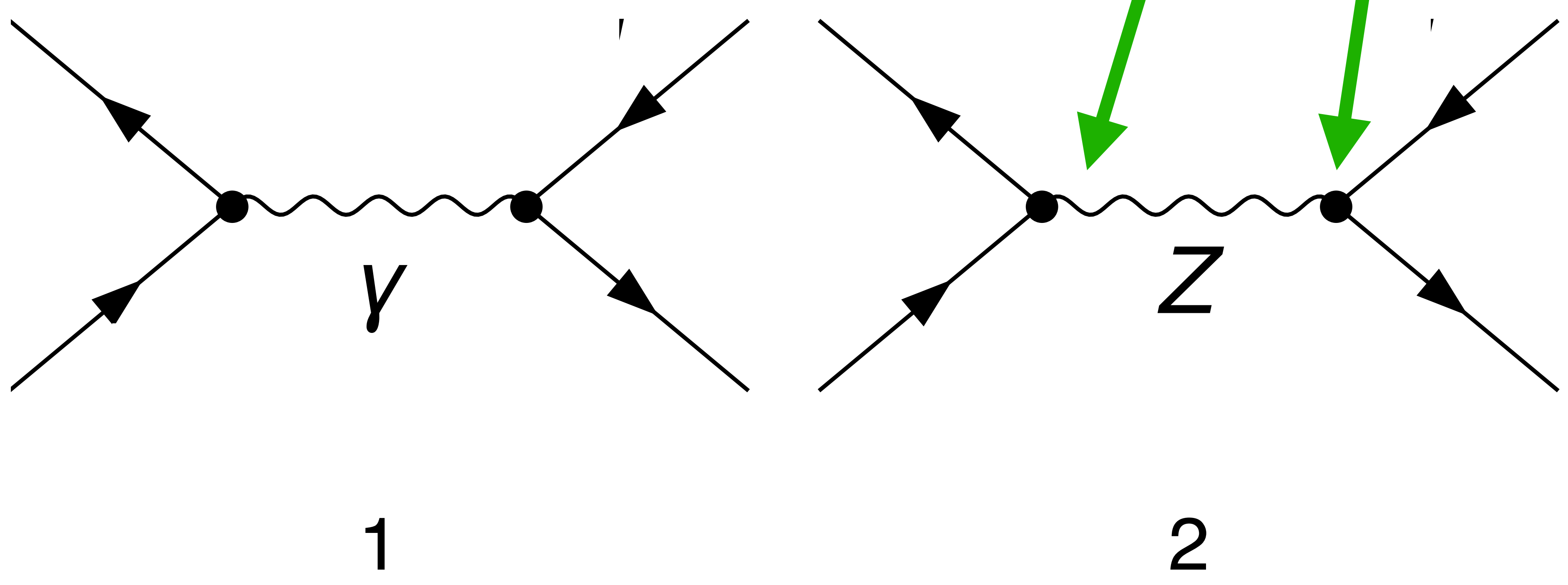


Beyond LO approximation in neutral current Drell Yan

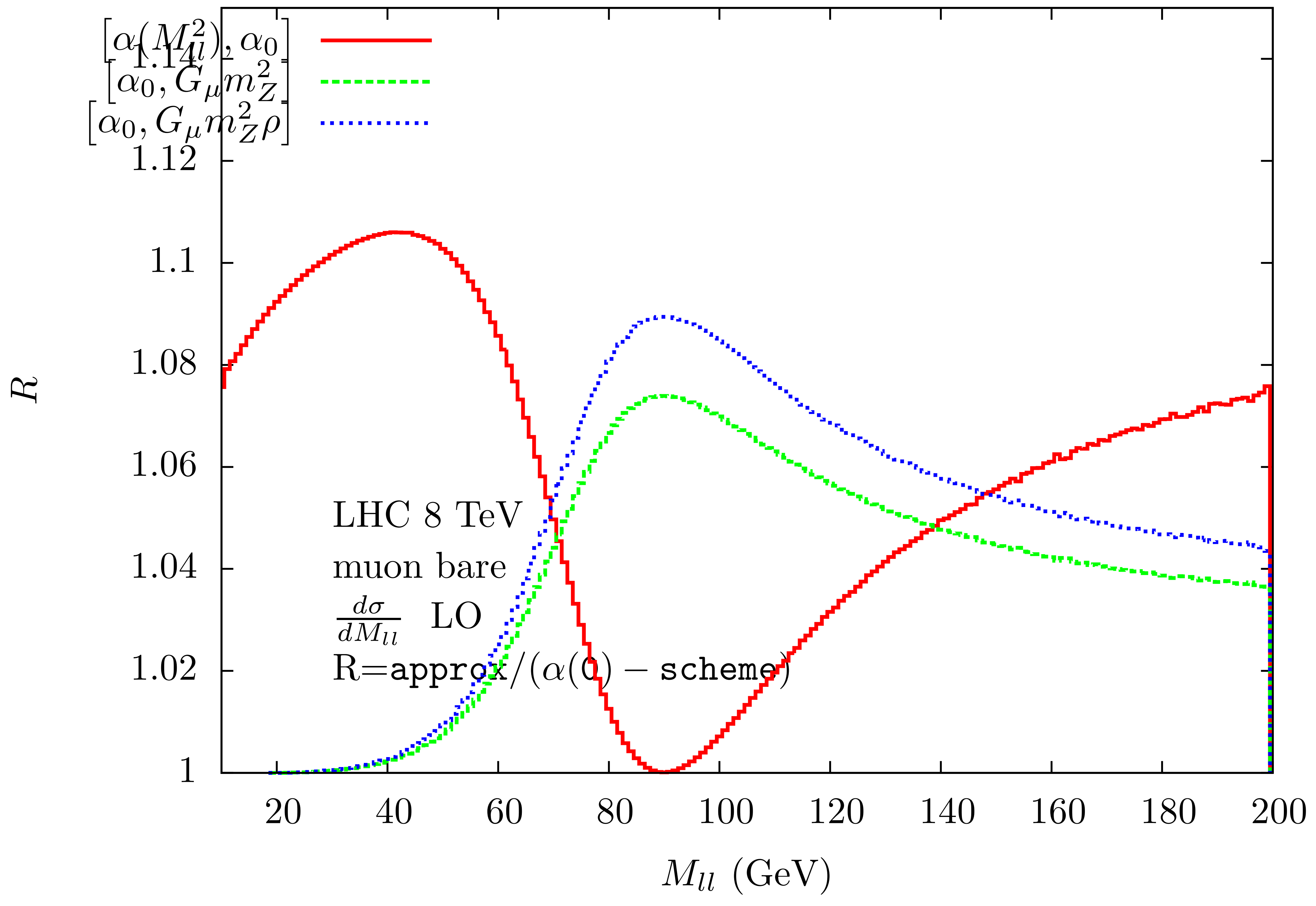


Use of G_μ only in the Z diagram enhances the peak of the Z resonance

$\alpha(0) \to G_\mu$

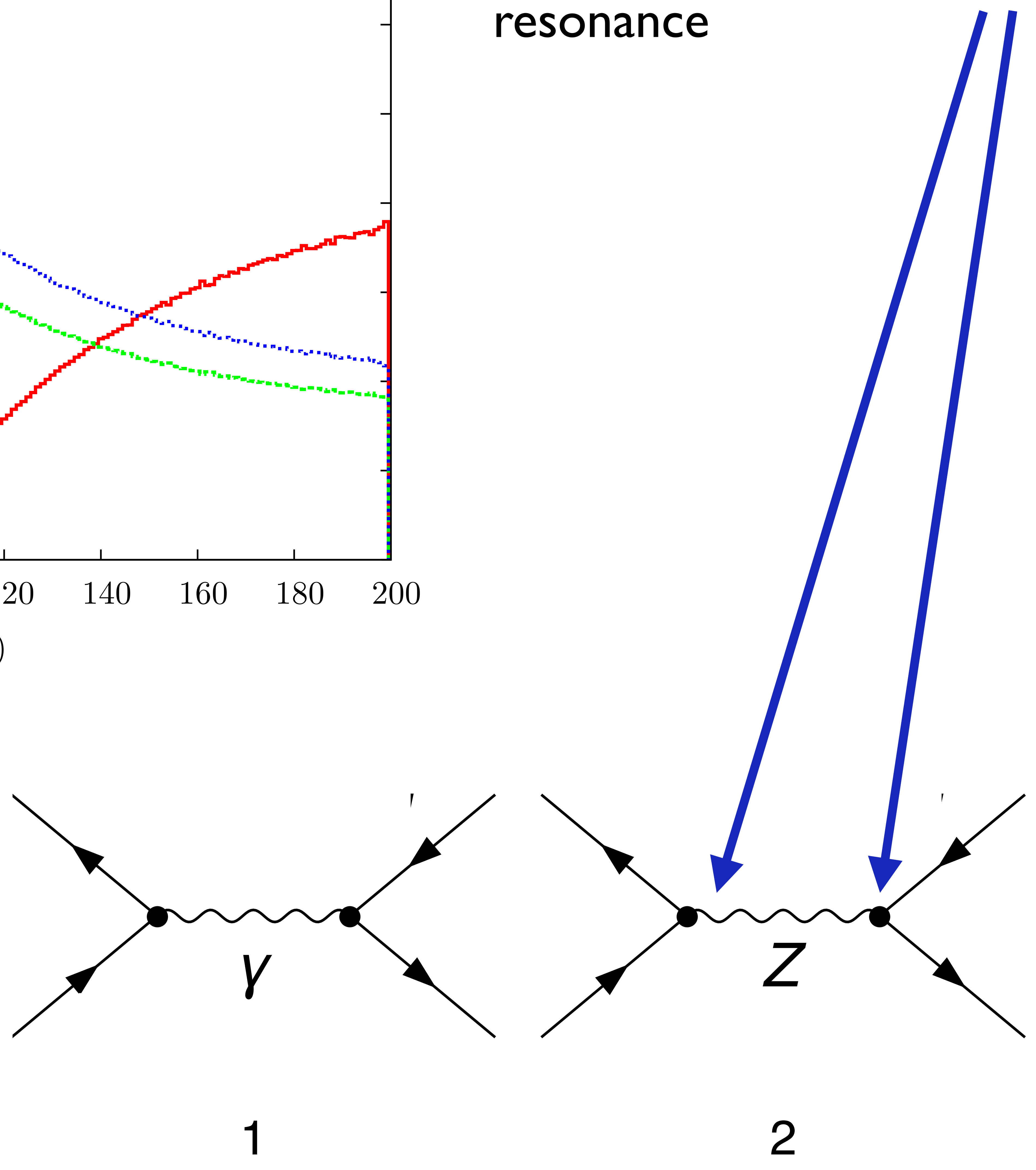


Beyond LO approximation in neutral current Drell Yan

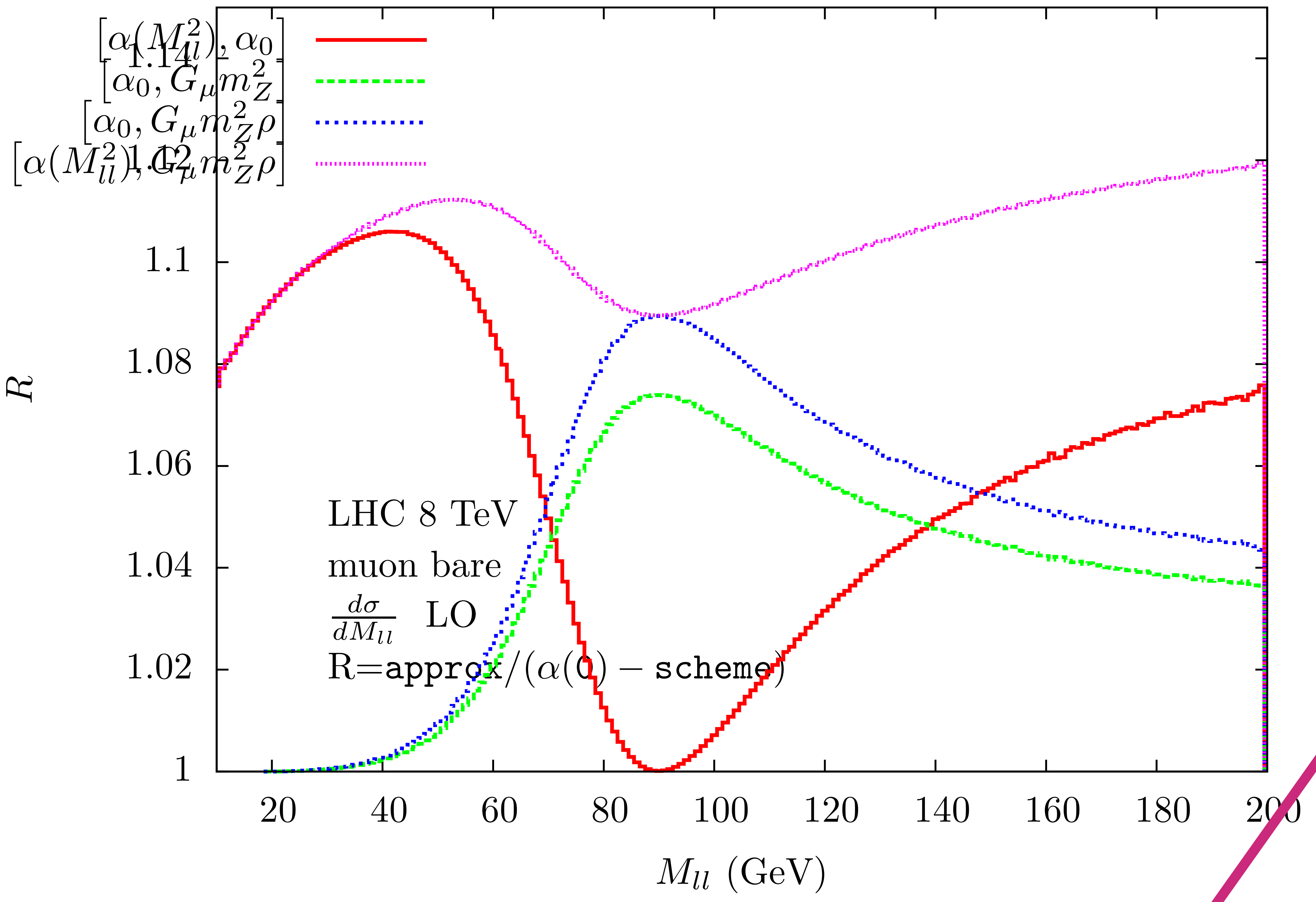


Use of G_μ and ρ only in the Z diagram enhances the peak of the Z resonance

$\alpha(0) \to G_\mu \ \rho$



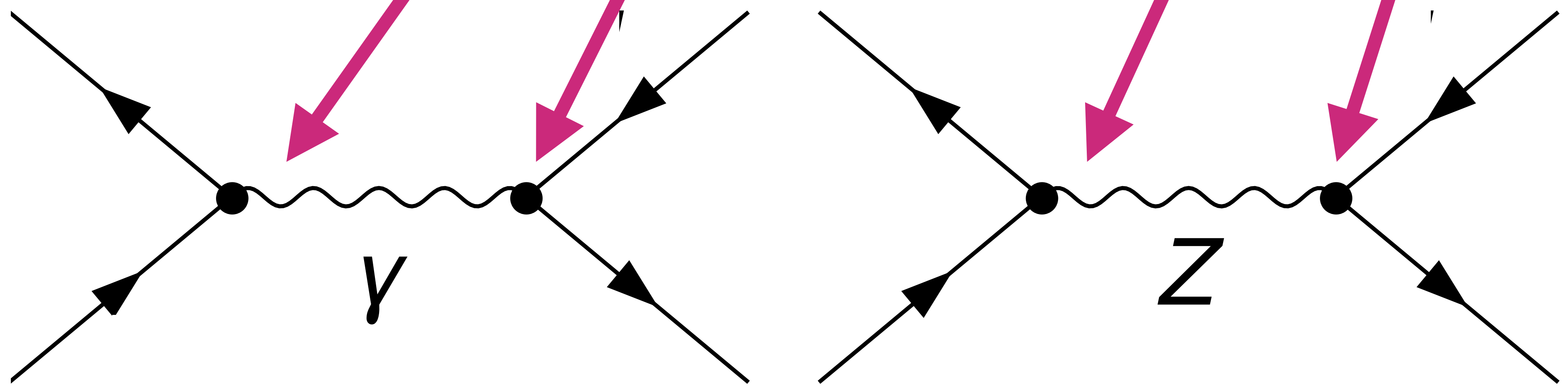
Beyond LO approximation in neutral current Drell Yan



- running of α only in the photon diagram
- use of G_μ and ρ only in the Z diagram:

$\alpha(0) \to G_\mu\ \rho$

$\alpha(0) \to \alpha(M^2_{\ell\ell})$



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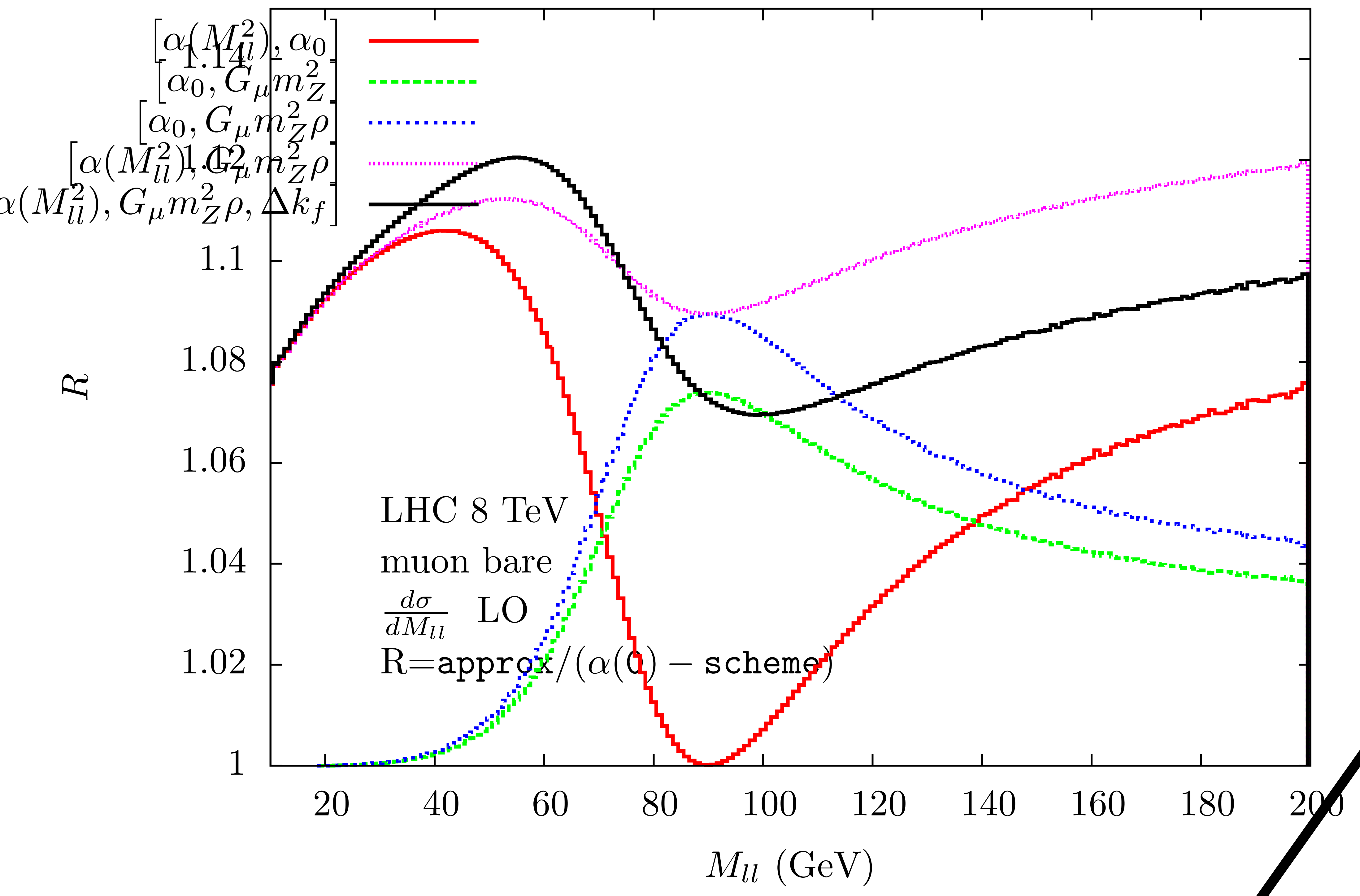
Beyond LO approximation in neutral current Drell Yan

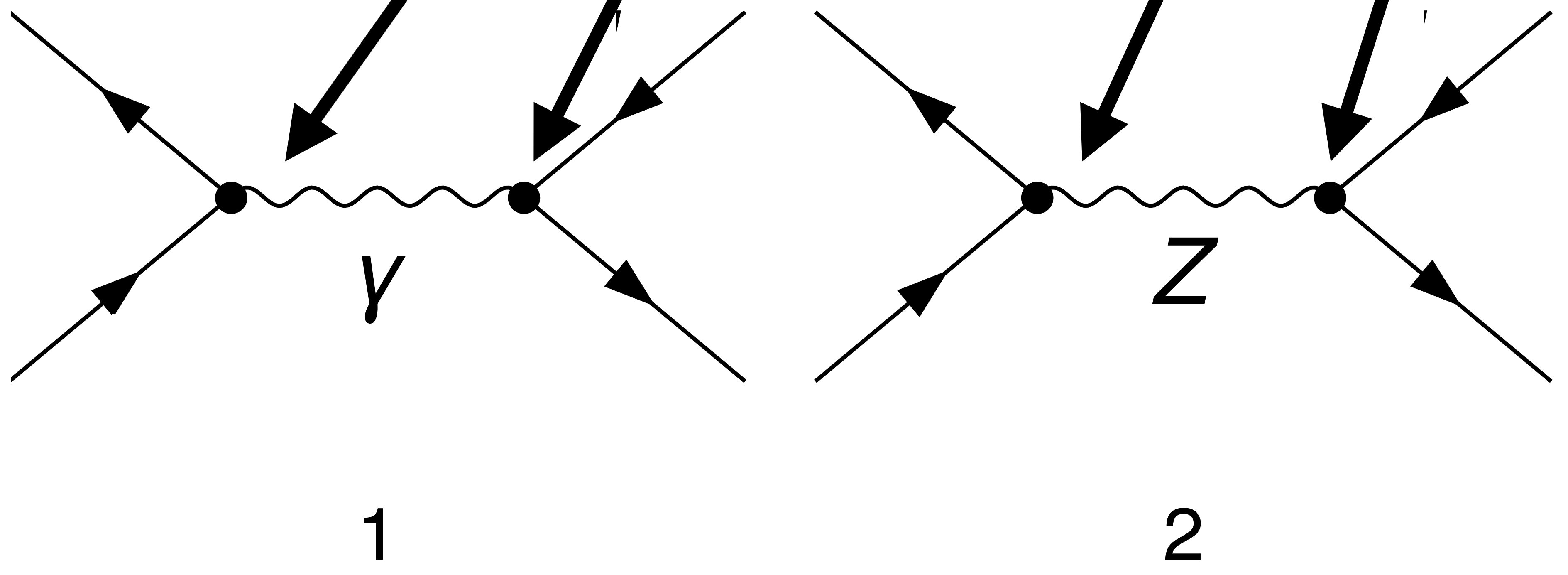


- running of α only in the photon diagram
- use of G_μ and ρ only in the Z diagram:
- rescaling of $\sin^2\theta_W$ in the Z vector coupling by $1 + \Delta\kappa_f$

$\alpha(0) \rightarrow \alpha(M_{\ell\ell}^2)$

rescaling $\alpha(0) \rightarrow G_\mu\ \rho$

$\sin^2\theta_W \rightarrow \left(1 + \Delta\kappa_f\right)\ \sin^2\theta_W$



Several effects enter in the coupling redefinition

NLO-EW contains at first order all these effects but not the higher-order corrections

$\Delta\kappa_f$ is the only correction which modifies the precise $\sin^2\theta_W$ value